EVALUATION OF ION DENSITY AND PLASMA POTENTIAL FROM LANGMUIR PROBE DATA

by Roman Krawec

Lewis Research Center
Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1971
The collisionless probe theory of Laframboise is used to calculate values of probe current for a plasma composed of electrons and ions with equal temperatures and Maxwellian velocity distribution functions. Specifically, the value of probe current is given as a function of ion density for a probe biased at a voltage $5kT/q$ below probe floating potential, where $k$ is Boltzmann's constant, $T$ is temperature, and $q$ is the electron charge.
EVALUATION OF ION DENSITY AND PLASMA POTENTIAL FROM LANGMUIR PROBE DATA

by Roman Krawec

Lewis Research Center

SUMMARY

The collisionless probe theory of Laframboise is used to calculate values of probe current for a plasma composed of electrons and ions with equal temperatures and Maxwellian velocity distribution functions. Specifically, the value of probe current is given as a function of ion density for a probe biased at a voltage $5kT/q$ below probe floating potential, where $k$ is Boltzmann's constant, $T$ is temperature, and $q$ is the electron charge.

Calculations are performed for four values of probe diameter and cover the density and temperature regions from $10^9$ to $10^{14}$ cm$^{-3}$ and 1 to 30 eV, respectively. Ion masses from 1 to 4 are considered.

INTRODUCTION

Although the problem of ion collection by a cylindrical probe in a collisionless plasma has been solved exactly by Laframboise (ref. 1), the manner in which the results are presented is inconvenient for use by the experimenter. Attempts at simplifying the data analysis have recently been made by Corbin and Oss (ref. 2) and by Battle and Bell (ref. 3). Corbin and Oss compute complete probe current-voltage characteristics for a low-density cesium plasma at three electron temperatures and for two different probes. These calculations were performed for an extremely restricted range of parameters and thus cannot be used by most experimenters. A further problem lies in the fact that any attempt to plot complete current-voltage characteristics for a wide range of parameters would lead to an undesirably large number of figures.

The work of Battle and Bell, on the other hand, is presented in the form of dimensionless parameters for helium and argon. These are given as figures 2 and 3 of reference 3 and cover a wide region of ratios of probe radius to Debye length. However, their
work does not apply to a hydrogen plasma.

The results of Laframboise are presented in this report in a form which should be useful to the experimenter. The data are presented in the form of probe current per unit length as functions of ion density for cylindrical probes of various diameters biased 5kT/q below floating potential, where k is Boltzmann’s constant, T is temperature, and q is the electron charge. The choice of 5kT/q below floating potential is arbitrary; it was chosen to be sufficiently far from plasma potential so that the probe would operate in a region where the electron current is negligible. The electron temperature is assumed to be available from other probe measurements.

The ranges of plasma parameters covered are (1) electron temperature, 1 to 30 eV; (2) ion number density, $10^9$ to $10^{14}$ cm$^{-3}$; (3) ion mass, 1 to 4; and (4) probe radius, 0.0127 to 0.0318 cm (radius from 0.005 to 0.0125 in. in steps of 0.0025 in.).

Although the calculations were performed for a plasma with equal electron and ion temperatures ($T_e = T_i$), the use of the results for $T_i \leq T_e$ is justified by the weak dependence of ion current on ion temperature.

RELATIONS USED IN THE CALCULATIONS

The total current to the probe is given (ref. 2) as

$$I_p = I_e + I_i = -I_{se} \exp\left(-\frac{qV}{kT}\right) + I_i$$

$$= 2\pi R L q n \sqrt{\frac{kT}{2\pi M}} \left[ \psi(\varphi, \beta) - \sqrt{\frac{M}{m}} \exp(-\varphi) \right] \quad \text{(SI units)} \quad (1a)$$

where

- $I_p$ probe current
- $I_e$ electron current
- $I_i$ ion current
- $I_{se}$ saturation electron current
- $q$ electron charge
- $V$ probe potential measured with respect to plasma potential
- $k$ Boltzmann’s constant
- $T$ temperature

2
The probe current per unit length is

\[ \frac{I_p}{L} = 3.91 \times 10^{-13} \frac{nRT^{1/2}}{Z^{1/2}} \left[ \psi(\varphi, \beta) - 42.8 Z^{1/2} \exp(-\varphi) \right] \quad \text{(A/cm)} \]  

(1b)

where \( Z \) is the atomic number of the ion, and temperature is in eV, ion number density is in \( \text{cm}^{-3} \), and probe radius and length are in cm. Values of \( \psi(\varphi, \beta) \) are presented in table I, which was taken from reference 2.

RESULTS AND DISCUSSION

The value of \( \varphi \) at which \( I_p \) equalled zero (eq. (1b)) was found for given values of \( n, T, \) and \( Z \) by using an iteration procedure. (This is the probe floating potential, denoted by \( \varphi_o \).) The value of probe current at \( \varphi = \varphi_o + 5 \) was then calculated. Values of \( \psi(\varphi, \beta) \) for values of \( \beta \) other than those given in table I were extrapolated by fitting a parabola through three points in the immediate vicinity of the desired value of \( \beta \).

The Debye length is given as

\[ \lambda_d = 785 \sqrt{\frac{T}{n}} \quad \text{cm} \]

where the temperature is expressed in eV and the number density in \( \text{cm}^{-3} \).

3
The calculations are limited to values of \( n \) and \( T \) such that \( 1 \leq \beta \leq 50 \) in order to stay within regions of validity of the theory. The limitations imposed on the density for a given temperature by these restrictions are presented in figure 1. Figure 1(a) presents the minimum densities for which the calculations are valid as a function of probe radius and particle temperature, while figure 1(b) presents the maximum densities.

Figures 2 to 5 present the probe currents at \( \varphi = \varphi_0 + 5 \) for various probe radii and ion masses as functions of temperature and density.

With the solution of equation (1b) used for \( \varphi_0 \), the plasma potential can be obtained in terms of the floating potential. Figure 6 gives the value of the dimensionless probe floating potential for different plasma conditions.

As an example of how the curves are to be used, assume an atomic hydrogen plasma and the following: probe radius, 0.0254 cm; probe length, 0.1 cm; electron temperature, 8 eV; and probe floating potential, -20 V.

The atomic hydrogen plasma \((Z = 1)\) and the probe radius tell us that figure 2(c) is the proper one to use. The first thing to do is to measure the probe current when the probe is biased at -60 V (five times the electron temperature or 40 V below floating potential).

If this probe current is 9 mA, the probe current per cm would then be 90 mA. Referring to figure 2(c), 90 mA at a temperature of 8 eV corresponds to a number density of \( 1.35\times10^{12} \) cm\(^{-3} \).

Furthermore, figure 6(c) tells us that under these conditions the dimensionless floating potential is near 3.1, which means that the plasma potential is 3.1\( \times \)8 above the probe floating potential. The plasma potential in this case is thus 4.8 V.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 17, 1971,
129-02.
REFERENCES


TABLE I. - DIMENSIONLESS ION CURRENT FOR A CYLINDRICAL PROBE WHERE ION AND ELECTRON TEMPERATURES ARE EQUAL AND BOTH SPECIES ARE MAXWELLIAN

[Data from ref. 2.]

<table>
<thead>
<tr>
<th>Dimensionless probe potential, $\varphi$</th>
<th>Dimensionless probe radius, $\beta$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0804</td>
<td>1.0803</td>
<td>1.0803</td>
<td>1.0803</td>
<td></td>
</tr>
<tr>
<td>.3</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2101</td>
<td>1.2100</td>
<td>1.208</td>
<td>1.205</td>
<td></td>
</tr>
<tr>
<td>.6</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.3721</td>
<td>1.371</td>
<td>1.362</td>
<td>1.348</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.5560</td>
<td>1.5560</td>
<td>1.5560</td>
<td>1.5560</td>
<td>1.554</td>
<td>1.549</td>
<td>1.549</td>
<td>1.523</td>
<td>1.488</td>
<td>1.439</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1.7551</td>
<td>1.7551</td>
<td>1.7551</td>
<td>1.754</td>
<td>1.747</td>
<td>1.735</td>
<td>1.677</td>
<td>1.605</td>
<td>1.523</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>1.9320</td>
<td>1.9320</td>
<td>1.9320</td>
<td>1.928</td>
<td>1.913</td>
<td>1.893</td>
<td>1.798</td>
<td>1.689</td>
<td>1.576</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>2.2417</td>
<td>2.2417</td>
<td>2.2417</td>
<td>2.237</td>
<td>2.226</td>
<td>2.192</td>
<td>2.151</td>
<td>1.98</td>
<td>1.801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>2.7555</td>
<td>2.7555</td>
<td>2.731</td>
<td>2.731</td>
<td>2.701</td>
<td>2.626</td>
<td>2.544</td>
<td>2.22</td>
<td>1.940</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>3.2346</td>
<td>3.2346</td>
<td>3.266</td>
<td>3.227</td>
<td>3.174</td>
<td>3.050</td>
<td>2.920</td>
<td>2.442</td>
<td>2.060</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. - Region of validity - minimum and maximum densities to be measured.
(a) Probe radius, 0.0127 cm (0.005 in.).

Figure 2. - Probe ion currents for mass 1 ions at probe potential $\phi = \phi_0 + 5$. 
Figure 2. - Continued.

Probe current per unit length, A/cm

Ion number density, cm$^{-3}$

Electron temperature, eV

(b) Probe radius, 0.01905 cm (0.0075 in.)
Figure 2. - Continued.

Electron temperature, eV

Probe current per unit length, A/cm

Ion number density, cm$^{-3}$

(c) Probe radius, 0.0254 cm (0.010 in.).

Figure 2. - Continued.
Figure 2. - Concluded.
Figure 3. - Probe current for mass 2 ions at probe potential $V = V_0 + 5$. 

(a) Probe radius, 0.0127 cm (0.005 in.).
Figure 3. - Continued.

(b) Probe radius, 0.01905 cm (0.0075 in.)
Figure 3 - Continued.
Figure 3. - Concluded.

(d) Probe radius, 0.0318 cm (0.0125 in.).

Ion number density, cm⁻³

Probe current per unit length, A/cm

Electron temperature, eV

10, 8, 6, 4, 2, 1

10⁻⁹, 10⁻⁸, 10⁻⁷, 10⁻⁶, 10⁻⁵

10¹⁰, 10¹¹, 10¹², 10¹³, 10¹⁴
Figure 4. - Probe current for mass 3 ions at probe potential $\varphi = \varphi_0 + 5$.

(a) Probe radius, 0.0127 cm (0.005 in.).
Figure 4, - Continued.

(b) Probe radius, 0.01905 cm (0.0075 in.).
Figure 4. - Continued.

(c) Probe radius, 0.0254 cm (0.010 in.).
Ion number density, $n_i$

Electron temperature, eV

(d) Probe radius, 0.0318 cm (0.0125 in.).

Figure 4 - Concluded.
Figure 5. - Probe current for mass 4 ions at probe potential $\varphi = \varphi_0 + 5$. 
(a) Probe radius, 0.0127 cm.
Figure 5. - Continued.

(b) Probe radius, 0.01905 cm (0.0075 in.).
Figure 5 - Continued.
Figure 5. - Concluded.

(d) Probe radius, 0.0318 cm (0.0125 in.).
Figure 6. - Probe floating potential.
Figure 6. - Concluded.
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546