Dynamic Upper Atmospheric Force Model on Stabilized Vehicles for a High-Precision Trajectory Computer Program

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This article summarizes the results of research carried out at JPL for the design and implementation of dynamic upper atmosphere and lift and drag models into the advanced Double-Precision Trajectory Program (DPTRAJ). The upper atmosphere model draws heavily on the behavior of the Earth's upper atmosphere which exhibits cyclic as well as irregular variations in density profile, temperature, pressure, and composition in unison with solar activities as deduced from the more recent land-based and satellite observations.

The lift and drag model is designed specifically for inertially stabilized vehicles of the Mariner class, with possible extension to gravity gradient stabilized vehicles of the GEOS class. The model considers operation in the free molecular flow regimes with large Knudsen numbers. The vehicle is considered a composite structure with basic components having well-defined shapes, each with its own surface characteristics in terms of temperature, reflectivity, and accommodation of free stream molecules. The model takes into account both the calculation of precise aerodynamic force coefficients in terms of expansion of modified Bessel functions in speed ratios and angle of attack, and approximate force coefficients when the speed ratios approach infinity. Other considerations include specular and diffused reflectivity, shielding, and shadow effects.

Introduction

Improvements in the tracking data and orbit determination accuracies at JPL in conjunction with low-altitude orbiter missions having upper atmospheric penetration have resulted in the need for accurate modeling of the aerodynamic forces which influence the trajectories of these vehicles. Accurate lift and drag force models require an accurate, dynamic atmosphere model, as Earth satellite drag analysis have made it obvious that no static model can approximate the atmospheric parameters in the general case. Although modeling complexity is unconstrained in principle when solving equations of motion with special perturbation techniques (i.e., numerical integration), practical limiting factors on the complexity are determined by computer hardware performance (i.e., precision of the numerical integration) and the accuracy of other trajectory force models. It is also important to design efficient general models which may be used for a wide spectrum of missions in this decade.

The atmospheric model selected draws heavily on the behavior of the Earth's upper atmosphere due to the availability of data obtained in the past decade. Although the amount of factual data on many of the important parameters necessary for the determination of the general variation of atmospheres of other planets is scarce, the mechanisms and inferences implied in the generality of a model based on the behavior of the Earth's upper atmosphere remain valid to the extent of assuming that atmospheric behavior is grossly comparable when projecting dynamic effects to other planets.

The lift and drag force model selected is one that is applicable in a free molecular flow regime, where the atmospheric molecular mean-free-path is larger than the characteristic dimension of the vehicle (i.e., large Knudsen number). For practical considerations, the spacecraft is considered a composite structure of basic well-defined shapes with analytic properties. While ignoring multiple reflections, shielding factors are introduced to approximate the nonlinear interaction between components. The analysis is simplified by ignoring torques and inertia considerations, thus limiting the model to inertially stabilized types of vehicles (e.g., Mariner class) with possible extension to gravity gradient stabilized types (e.g., GEOS class).

Upper Atmosphere

Survey

Instruments and drag analysis methods of investigations revealed the existence of both periodic and irregular variations in the basic parameters of the Earth's upper atmosphere. These variations proved to be in unison with solar activities.

Although the microscopic details of the process of radiation absorption and reradiation are well understood, such processes are generally complicated, especially when evaluating variable solar radiation effects on multicomponent atmospheres.

Several major considerations due to efforts during the past decade can now be stated.

(1) Higher regions of the atmosphere are best represented by a vertical isothermy in a state of diffusion equilibrium. The ratio of the principal constituents is essentially fixed at the beginning of the diffusion level. Heat conduction is the principal transport mode.

(2) The atmosphere is heated by all radiations of the solar spectrum. Principal contributors in terms of the total kinetic energy of a thermospheric column belong to the region 1 to 1700 Å (ultraviolet and soft X-ray). Thermal emission from Coronal condensation clustering above Sun spots is an emitter of radiation (EUV). A single wave length is usually used as an index of solar decimetric flux ($F_{10.7}$). The choice of this index is influenced by its high correlation relative to solar activity and atmospheric temperatures.

(3) Although satisfactory explanations for meridional transport and auroral zone heating due to magnetic storms are not available (Reference 1), satellite drag analysis shows a linear correlation between geomagnetic indices and the upper-atmospheric temperatures with an observed time lag.

(4) The observed variations can be categorized as follows:

(a) Altitude and latitude position.

(b) Diurnal (planetary rotation).

(c) Monthly (27-day solar rotation).

(d) Semi-annual and seasonal (planetary orbit).

(e) Solar decimetric radiation (11-yr Sun-spot cycle).

(f) Magnetic storms (irregular) and auroral activity (high latitudes).

Major approaches used in constructing upper-atmosphere models include: an empirical approach, derived from satellite drag analysis, generally in agreement with physical laws, and a theoretical approach based on conservation laws. The empirical one-dimensional model of Jacchia (Reference 2) is a quasi-static, multiple-component model. The model is based on the assumption of hydrostatic equilibrium, an empirical temperature profile, and a fixed set of boundary conditions for temperature and number densities. Empirical temperature profiles reflecting atmospheric temperature variations due to solar activities are used to compute number densities. Knowledge of thermal diffusion ratios and diffusive equilibrium is assumed. Contrary to observations, a nearly isopycnic layer with a constant density at some level results in models based on the above approach as a consequence of the use of a fixed set of boundary conditions (Reference 3). Another approach taken by Harris and Priester (Reference 4) is in computing the temperature from an energy conservation equation by utilizing all known (and some virtual) heat sources to generate the temperature profile, which is then used simultaneously with number densities assumed in hydrostatic equilibrium. Other more recent models include those by Thomas (Reference 5) and Friedman (Reference 6). The model of Thomas is analytic and one dimensional, where the undimensional conservation equations are transformed to an isobaric frame, thus enabling solution by simple analytic Green functions. Friedman's model utilizes simplified expressions for radiation heating in solution of the conservation equations. This three-dimensional model allows boundary conditions to vary until agreement with observed data is achieved. This last approach is more physically realistic in that it accounts for horizontal heat conduction and mass diffusion.

General Computer Model

Due to operational flexibility, the empirical model was chosen for the high-precision trajectory program DPTRAJ (Reference 7). The user supplies density and average molecular weight versus height for the full range of exospheric temperatures. The table can be made up of patched set of one-dimensional atmospheres corresponding to given top exospheric temperatures. The range of temperatures extends from the absolute minimum night time to the maximum observed and modeled temperatures. Empirical equations relating variations of exospheric temperature to dynamic causes are evaluated yielding the required temperature at the evaluation epoch. The required parameters at the given height are thus obtained. The empirical form of the temperature equation (Reference 8) is

$$T = (T_{\min} + \Delta T_D) f(\theta), \text{ in kelvins}$$

where

$T_{\min}$ = minimum known temperature

$f(\theta) = 1 + L \cos^M \theta$ = diurnal bulge effect, where the angle θ is measured from the bulge maximum to the considered location, and L and M are constants

ΔT_D = contribution of dynamic effects

$$= (a - T_{\min}) + b\Delta F_{10.7} + \left[c\Delta F'_{10.7} + d(\Delta F'_{10.7})^2\right]$$

$$+ \left[(e + f \sin 2\pi D_1)(\bar{F}_{10.7} \sin 4\pi D_2)\right]$$

$$+ \left[g a_p - h(1 - \exp k a_p)\right]$$

and $a, b, c, d, e, f, g, h,$ and k are constants

$\Delta F_{10.7}$ = decimetric flux less $\bar{F}_{10.7}$, 10^{-22} W-m^{-2} (cps)$^{-1}$ per bandwidth

$\bar{F}_{10.7}$ = average over three solar rotations

$\Delta F'_{10.7}$ = average over one solar rotation less the average over the 11-yr Sun-spot cycle

D_1, (D_2) = days past observed min, (max) seasonal variations

a_p = the 3-h geomagnetic index, properly scaled for atmospheric response time lag, 2×10^{-5} gauss

The dynamic equation terms empirically account for the effects of the solar rotation, solar cycle, semiannual, and magnetic storms, respectively.

Aerodynamic Forces

Survey

At higher altitude, when the ratio of the mean-free-path of free stream molecules to a basic characteristic body dimension (Knudsen number) is large, the rarefied gases do not behave like a continuum. Computation of aerodynamic forces and moments are based on a concept from the kinetic theory of gases known as the free molecular flow regime. The theory takes into account uniform mass motion of gases superimposed upon the thermal motion (Maxwellian distribution) of gas molecules. Ignoring collisions between impinging and re-emitted molecules, the total force on a body is made up of components from surface bombardment by impinging molecules and components due to specular or diffuse re-emission of those molecules. In the specular reflection the normal velocity component is reversed, while the tangential component with shear effects on the surface remains unchanged. In the diffuse reflection, all previous directional history is erased. Molecules leaving the surface have speeds with Maxwellian distribution dependent on re-emitted stream temperature, and direction controlled by the Knudsen cosine law.

The recent methods developed by Heineman (Reference 9), Stalder (Reference 10), and Blick (Reference 11) involve integration of components of the momentum force, imparted to a differential plane area by impinging and reflected molecules, in defined directions. The direction cosines chosen are related to the component of force being computed, which in turn yield the desired aerodynamic coefficients for the type of shape being considered.

The basic equation for the total incident momentum (front and rear sides) per elemental area per unit time is

$$dG_i = \left\{ \frac{\beta^3 mN}{\pi 3/2} \int_{-\infty}^{\infty} \int \left[\int_0^{\infty} c_1 (\mathbf{c} \cdot \hat{\boldsymbol{\ell}}) \exp(-\beta^2 \mathbf{V} \cdot \mathbf{V}) \right.\right.$$

$$\left.\left. - \int_{-\infty}^{0} c_1 (\mathbf{c} \cdot \hat{\boldsymbol{\ell}}) \exp(-\beta^2 \mathbf{V} \cdot \mathbf{V}) \right] dc_1 dc_2 dc_3 \right\} dA$$

where

$\hat{\ell}$ = desired direction of momentum force vector

mN = ρ = number density

β = $1/c$ = reciprocal of most probable molecular speed

$\mathbf{V}$ = thermal velocity

$\mathbf{c}$ = $\mathbf{V} + \mathbf{U}$ = total velocity = sum of thermal and mass velocities

U = Sc = $(\gamma/2)^{1/2} M$ = stream mass speed, and S is the speed ratio, γ is ratio of the specific heats, and M is the Mach number

For example, application of the above equation yields the impinging front side pressure on an elemental area of flat plate along **N** (see Figure 1).

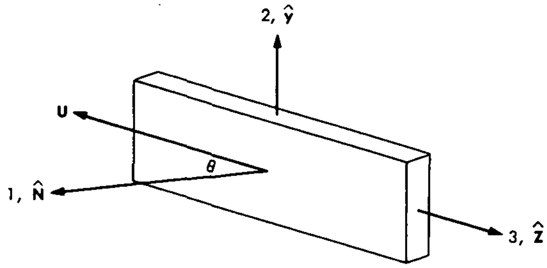
Let

$$\mathbf{U} = U(\sin\theta\, \hat{\mathbf{N}} - \cos\theta\, \hat{\mathbf{y}})$$

$$\hat{\boldsymbol{\ell}} = \hat{\mathbf{N}}$$

Therefore,

$$P_i = \frac{\rho U^2}{2} \left\{ \left(\sin^2\theta + \frac{1}{2S^2}\right) [1 + \operatorname{erf}(S \sin\theta)] + \frac{\sin\theta}{\sqrt{\pi}\, S} \exp[-S^2 \sin^2\theta] \right\}$$



ANGLE OF ATTACK $\alpha \doteq 90 - \theta$

Figure 1. Coordinate system definition

Simplifying assumptions leading to approximate expressions were used by Blick (Reference 11) to obtain aerodynamic coefficients for a larger set of geometric shapes and bodies of revolution. Large speed ratios are assumed (zero kinetic temperature) allowing elimination of error function and exponential terms. As a result, the flow approaches that of an elastic or inelastic Newtonian flow for specular or diffuse reflection, respectively.

General Computer Model

The vehicle is considered a composite structure of components with known aerodynamic properties. The components include: flat plate, sphere, cylinder, and segment of a sphere. For each component, the front and back side surface temperatures, the effective areas, the accommodation coefficients, and the proportions of each type of reflection are assumed known. The orientation of each component relative to slip stream is calculated and the angle of attack is determined. Tables supplying shadow effects in terms of reduction of effective area of each component, based on its orientation relative to the slip stream, are supplied externally.

The total lift and drag forces are then calculated as a vector sum of forces on each component. The fact that this model is designed for inertially stabilized spacecraft simplifies the calculations. Forces are translated and torque considerations are not needed except when the model is utilized in conjunction with GEOS class vehicles. In the case of gravity gradient stabilized GEOS, the restoring gravity torque is needed for the determination of the offset angle created by drag. The total aerodynamic force is then expressed in the proper inertial frame and added to the other perturbative forces in the equations of motion of the spacecraft for numerical integration by DPTRAJ (Reference 7). More complete details of the model and analysis are planned.

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