A GENERAL STAGNATION-POINT CONVECTIVE-HEATING EQUATION FOR ARBITRARY GAS MIXTURES

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**Abstract**

The stagnation-point convective heat transfer to an axisymmetric blunt body for arbitrary gases in chemical equilibrium was investigated. The gases considered were base gases of nitrogen, oxygen, hydrogen, helium, neon, argon, carbon dioxide, ammonia, and methane and 22 gas mixtures composed of the base gases. In the study, enthalpies ranged from 2.3 to 116.2 MJ/kg, pressures ranged from 0.001 to 100 atmospheres, and the wall temperatures were 300 and 1111 K.

A general equation for the stagnation-point convective heat transfer in base gases and gas mixtures was derived and is a function of the mass fraction, the molecular weight, and a transport parameter of the base gases. The relation compares well with present boundary-layer computer results and with other analytical and experimental results. In addition, the analysis verified that the convective heat transfer in gas mixtures can be determined from a summation relation involving the heat-transfer coefficients of the base gases. The basic technique developed for the prediction of stagnation-point convective heating to an axisymmetric blunt body could be applied to other heat-transfer problems.
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SUMMARY

The stagnation-point convective heat transfer to an axisymmetric blunt body for arbitrary gases in chemical equilibrium was investigated. The gases considered were base gases of nitrogen, oxygen, hydrogen, helium, neon, argon, carbon dioxide, ammonia, and methane and 22 gas mixtures composed of the base gases. In the study, enthalpies ranged from 2.3 to 116.2 MJ/kg, pressures ranged from 0.001 to 100 atmospheres, and the wall temperatures were 300 and 1111 K.

A general equation for the stagnation-point convective heat transfer in base gases and gas mixtures was derived and is a function of the mass fraction, the molecular weight, and a transport parameter of the base gases. The relation compares well with present boundary-layer computer results and with other analytical and experimental results. In addition, the analysis verified that the convective heat transfer in gas mixtures can be determined from a summation relation involving the heat-transfer coefficients of the base gases. The basic technique developed for the prediction of stagnation-point convective heating to an axisymmetric blunt body could be applied to other heat-transfer problems.

INTRODUCTION

Convective heating during planetary entry depends on the atmospheric gas composition surrounding the planet. For the planets presently considered for entry, the atmospheres are primarily binary mixtures of nitrogen and oxygen (air) for Earth; carbon dioxide and nitrogen for Mars and Venus; carbon dioxide and argon for Mercury; and helium and hydrogen for Jupiter. (See refs. 1 to 7.) Trace amounts of neon, methane, and ammonia have also been considered for several of the planets. Except for Earth, the actual composition of the atmospheres are not accurately known. Consequently, numerous calculations will be required to determine the convective heating during planetary entry unless a general relation can be determined. Generally, existing analytical (refs. 8 to 16) and experimental (refs. 17 to 27) studies have emphasized only particular gases or gas mixtures and a solution for arbitrary gases was not obtained. In references 14, 16,
and 28, general relations for convective heating are presented but the analyses were
developed for a limited number of gases and gas mixtures.

The purpose of the present study is to develop a general relation for calculating the
convective heating to the stagnation point of a blunt axisymmetric body for gases or gas
mixtures in chemical equilibrium, especially those gases encountered during high-velocity
planetary entry. In the analytical study, the approach is to derive a simple approximate
relation from the basic theoretical definition of heat transfer in gases. Also, the convective
heating for numerous gases is calculated by an existing computer code (computer
study) to obtain results for use in developing the approximate general relation and as a
basis of comparison with the approximate relation. For the computer study, stagnation
enthalpies ranged from 2.3 to 116.2 MJ/kg, stagnation pressures ranged from 0.001 to
100 atmospheres (1 atmosphere equals 101.325 kN/m²), and wall temperatures were 300
and 1111 K. The gases considered were base gases of nitrogen, oxygen, hydrogen, helium,
neon, argon, carbon dioxide, ammonia, and methane and 22 mixtures composed of the base
gases. Radiative heating was neglected in the present study.

SYMBOLS

- c: mass fraction
- \( c_p \): specific heat at constant pressure, J/kg-K
- \( \overline{D} \): reference diffusion coefficient, see equation (6), m²/s
- \( D_{eff} \): effective diffusion coefficient defined by equation (16)
- \( D_{ij} \): binary diffusion coefficient, m²/s
- E: energy factor, see equation (C2)
- F: diffusion factor, see equation (6)
- h: enthalpy, MJ/kg
- \( h^0 \): standard enthalpy of formation, MJ/kg
- j: species mass flux, kg/m²-s
- K: heat-transfer coefficient defined by equation (33), kg/s-m³/2-atm¹/2
- 2
k  Boltzmann constant, $1.380622 \times 10^{-23}$ J/K

M  molecular weight

$N_{Le}$  Lewis number defined by equation (15)

$N_{Pr}$  Prandtl number defined by equation (13)

$N_{Sc}$  Schmidt number defined by equation (14)

p  pressure, atm (1 atmosphere equals 101.325 kN/m$^2$)

$q$  convective heating rate, MW/m$^2$

R  equivalent body radius, m

$\mathcal{R}$  universal gas constant, $8.31484 \times 10^3$ J/kmole-K

r  body coordinate, m

s  distance along body surface from stagnation point, m

T  temperature, K

$T^*$  nondimensional temperature defined by equation (B2)

u  velocity component parallel to body surface, m/s

y  normal distance through boundary layer from wall surface, m

$z_i = \frac{M c_i}{F_i \beta_2}$

$\alpha$  constant, see equations (36) and (38), $2.2621 \times 10^{-6}$

$\beta$  velocity gradient defined by equation (23), s$^{-1}$

$\beta_1 = M \sum \frac{c_i F_i}{M_i}$
\( \beta_2 = M \sum \frac{c_i}{F_i} \)

\( \gamma \) transport parameter defined by equation (37), Å\(^{-2}\) K\(^{-0.15}\)

\( \epsilon \) maximum energy of attraction, J

\( \eta \) transformed coordinate defined by equation (20)

\( \lambda \) thermal conductivity, J/m-s-K

\( \mu \) viscosity, N-s/m\(^2\)

\( \xi \) transformed coordinate defined by equation (21)

\( \rho \) density, kg/m\(^3\)

\( \sigma \) molecular collision diameter, Å (1 angstrom equals 10\(^{-10}\) meter)

\( \Omega^{(2,2)}_* \) reduced collision integral for viscosity

Subscripts:

- \( e \) external edge of boundary layer
- \( i \) \( i \)th species or component
- \( j \) \( j \)th species
- \( max \) maximum
- \( o \) initial composition
- \( s \) stagnation condition at boundary-layer edge
- \( w \) wall value
- \( \infty \) free-stream value
DESCRIPTION OF COMPUTER PROGRAM

As part of the present study, the convective heating of the base gases and gas mixtures was calculated by an existing computer code which is a numerical solution of the multicomponent boundary-layer equations for gases in chemical equilibrium. Results from these computations were used in developing the approximate general relation and as a basis of comparison with the approximate relation. Descriptions of the computer code and method of solution of the boundary-layer equations are presented in references 29 to 32. The routines for the thermodynamic and transport properties are so constructed that any arbitrary gas mixture can be considered. Hereinafter, these solutions are referred to as computer results.

The thermodynamic properties are calculated by the method described in reference 33, and, for the present study, the necessary inputs were obtained from references 34 to 37. The inputs are for a common basis, and a comparison of this method with other methods is given in reference 33.

The mass diffusion is calculated by using the bifurcation approximation as presented in reference 38. The accuracy of the method depends on the bifurcation diffusion factors, and, for the present study, the factors are calculated by the method given in reference 39. For a few species, the basic data required were not available, so the diffusion factors were taken from the correlation of reference 38. Comparisons of the bifurcation diffusion approximation with exact multicomponent diffusion are given in references 30, 38, and 39.

The viscosity and thermal conductivity values are calculated by approximate relations of the Sutherland-Wassiljewa type as presented in reference 38. The method requires the previously mentioned bifurcation diffusion factors and also the self-diffusion factors calculated by the exact method presented in reference 34. Comparisons of the transport properties calculated by this method and other methods are given in reference 34.

The computer program provided solutions for arbitrary gas mixtures and thus gave a consistent base for developing and evaluating the approximate relation. In the present study, the computer program was used to obtain numerical solutions for the stagnation-point convective heating for nine base gases and 22 gas mixtures. They are as follows –

Base gases:

\[
\begin{align*}
N_2 & \quad He \\
O_2 & \quad Ne \\
H_2 & \quad Ar \\
\end{align*}
\]

\[
\begin{align*}
CO_2 & \\
NH_3 & \\
CH_4 & \\
\end{align*}
\]
Gas mixtures (composition by mass fraction):

- 0.2320 O₂ - 0.7680 N₂ (air)
- 0.6714 CO₂ - 0.3286 Ar
- 0.1345 CO₂ - 0.8655 N₂
- 0.5000 CO₂ - 0.5000 N₂
- 0.8500 CO₂ - 0.1500 N₂
- 0.7500 O₂ - 0.2500 N₂
- 0.5000 O₂ - 0.5000 N₂
- 0.1500 H₂ - 0.8500 He
- 0.3500 H₂ - 0.6500 He
- 0.6500 H₂ - 0.3500 He
- 0.5000 H₂ - 0.5000 Ar
- 0.1500 N₂ - 0.8500 H₂
- 0.5000 N₂ - 0.5000 H₂
- 0.2000 CO₂ - 0.8000 H₂
- 0.4000 CO₂ - 0.6000 H₂
- 0.6000 CO₂ - 0.4000 H₂
- 0.8000 CO₂ - 0.2000 H₂
- 0.3626 CO₂ - 0.3077 N₂ - 0.3297 Ar
- 0.1339 CO₂ - 0.8525 N₂ - 0.0136 Ar
- 0.3000 Ne - 0.3000 Ar - 0.4000 He
- 0.3500 H₂ - 0.3500 CO₂ - 0.3000 N₂
- 0.352 H₂ - 0.423 He - 0.176 Ne - 0.024 CH₄ - 0.025 NH₃
The 81 chemical species considered in the computer study are as follows:

<table>
<thead>
<tr>
<th>Species</th>
<th>Chemical Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>C&lt;sub&gt;3&lt;/sub&gt;</td>
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<tr>
<td>C</td>
<td>C&lt;sub&gt;3&lt;/sub&gt;O&lt;sub&gt;2&lt;/sub&gt;</td>
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<tr>
<td>CH</td>
<td>C&lt;sub&gt;4&lt;/sub&gt;</td>
</tr>
<tr>
<td>CHNO</td>
<td>C&lt;sub&gt;4&lt;/sub&gt;N&lt;sub&gt;2&lt;/sub&gt;</td>
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<tr>
<td>CH&lt;sub&gt;2&lt;/sub&gt;</td>
<td>C&lt;sub&gt;5&lt;/sub&gt;</td>
</tr>
<tr>
<td>CH&lt;sub&gt;2&lt;/sub&gt;O</td>
<td>He</td>
</tr>
<tr>
<td>CH&lt;sub&gt;3&lt;/sub&gt;</td>
<td>H</td>
</tr>
<tr>
<td>CH&lt;sub&gt;4&lt;/sub&gt;</td>
<td>HCN</td>
</tr>
<tr>
<td>C&lt;sub&gt;2&lt;/sub&gt;H</td>
<td>HCO</td>
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<td>HNO</td>
</tr>
<tr>
<td>C&lt;sub&gt;2&lt;/sub&gt;H&lt;sub&gt;4&lt;/sub&gt;</td>
<td>HNO&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>C&lt;sub&gt;2&lt;/sub&gt;H&lt;sub&gt;4&lt;/sub&gt;O</td>
<td>HNO&lt;sub&gt;3&lt;/sub&gt;</td>
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<tr>
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<td>HO&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>C&lt;sub&gt;2&lt;/sub&gt;N&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>O</td>
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</table>

For each base gas or gas mixture, the heating was calculated for a combination of conditions of –

- Stagnation enthalpy: 2.3, 4.6, 9.3, 13.9, 18.6, 23.2, 27.9, 32.5, 37.2, 46.5, 58.1, 69.7, 81.4, 93.0, 104.6, and 116.2 MJ/kg

- Stagnation pressure: 0.001, 0.01, 0.1, 1.0, 10.0, and 100.0 atm

- Wall temperature: 300 and 1111 K
ANALYSIS

A simple general expression for the convective heating to the stagnation point of a blunt axisymmetric body for high-enthalpy gases in chemical equilibrium was derived from the basic theoretical definition of heat transfer in gases. To meet the criterion of simplicity, it was necessary to approximate several of the more complex expressions that occurred in the theoretical development with more tractable expressions.

From reference 40, the convective heating to a wall from a gas is

\[ \dot{q}_w = \left( \lambda \frac{\partial T}{\partial y} - \sum j_i h_i \right)_w \]  

(1)

The first term on the right-hand side of the equation is the heating due to conduction and the second term is the heating due to diffusion. The mass diffusion due to thermal gradients is small in comparison with mass diffusion due to concentration gradients (ref. 40) and has been neglected. The specific heat and enthalpy of a mixture are defined as

\[ c_p = \sum c_i c_{p,i} \]  

(2)

\[ h = \sum c_i h_i \]  

(3)

where

\[ h_i = \int c_{p,i} \ dT + h_i^0 \]  

(4)

With these definitions, equation (1) becomes

\[ \dot{q}_w = \left( \frac{\lambda}{c_p} \frac{\partial h}{\partial y} - \frac{\lambda}{c_p} \sum h_i \frac{\partial c_i}{\partial y} - \sum j_i h_i \right)_w \]  

(5)

The species mass flux \( j_i \) is based on a bifurcation approximation to binary diffusion coefficients. This approximation has been used in references 29, 30, 38, and 39 and was shown to be in good agreement with exact multicomponent diffusion. The present form of the bifurcation approximation is taken from reference 38. The approximation is expressed as

\[ D_{ij} \approx \frac{D}{F_i F_j} \]  

(6)
where $\overline{D}$ is a reference diffusion coefficient and $F_i$ and $F_j$ are diffusion factors for the $i$th and $j$th species, respectively.

As given in reference 38, the species mass flux for the bifurcation approximation can be expressed as

$$j_1 = -\frac{\rho \overline{D} \beta_2 \theta z_1}{M \beta_1} \frac{\partial}{\partial y}$$  \hspace{1cm} (7)

For unequal diffusion, the quantity $z_1$ lies between a mass and a mole fraction and is defined by

$$z_1 = \frac{Mc_i}{F_i \beta_2}$$  \hspace{1cm} (8)

and $\beta_1$ and $\beta_2$ are system quantities defined by

$$\beta_1 = M \sum c_i \frac{F_i}{M_i}$$  \hspace{1cm} (9)

$$\beta_2 = M \sum c_i \frac{F_1}{F_i}$$  \hspace{1cm} (10)

From equation (8),

$$\frac{M}{F_i} \frac{\partial c_i}{\partial y} = \beta_2 \frac{\partial z_1}{\partial y} + z_1 \frac{\partial \beta_2}{\partial y} - c_i \frac{\partial M}{\partial y} \left(\text{Neglected}\right)$$  \hspace{1cm} (11)

The last two terms on the right-hand side of equation (11) are neglected in the present analysis. The neglected terms involve variations in system quantities and the variations of these quantities should be much less than the variations of individual species. Thus, in the present analysis, it is assumed that

$$\frac{\partial c_i}{\partial y} = \frac{F_i \beta_2}{M} \frac{\partial z_1}{\partial y}$$  \hspace{1cm} (12)

With the following nondimensional parameters —

$$\text{Prandtl number: } N_{Pr} = \frac{c_p \mu}{\lambda}$$  \hspace{1cm} (13)
where
\[ D_{\text{eff}} = \frac{\overline{D}\beta_2}{M\beta_1} \] (16)

and equations (5), (7), and (12), the convective heating can be expressed as

\[ \dot{q}_w = \frac{\mu_w}{N_{\text{Pr},w}} \left( \frac{\partial h}{\partial y} - \frac{\beta_2}{M} \sum F_i h_i \frac{\partial z_i}{\partial y} + N_{\text{Le}} \sum h_i \frac{\partial z_i}{\partial y} \right) \] (17)

In the present study, an analogy between energy and mass transfer is used in the form

\[ \frac{\partial z_i}{\partial y} = \frac{1}{N_{\text{Le}}^{1/3}} \frac{z_{i,e} - z_{i,w}}{h_s - h_w} \frac{\partial h}{\partial y} \] (18)

This analogy is based on the analyses presented in references 30, 38, 40, 41, and 42. By using equation (18), equation (17) becomes

\[ \dot{q}_w = \frac{\mu_w}{N_{\text{Pr},w}} \left( \frac{\partial h}{\partial y} \right)_w \left[ 1 - \frac{\beta_{2,w}}{M_w(N_{\text{Le},w})^{1/3}} \frac{\sum F_i h_i,w(z_{i,e} - z_{i,w})}{h_s - h_w} + \left( N_{\text{Le},w} \right)^{2/3} \frac{\sum h_i,w(z_{i,e} - z_{i,w})}{h_s - h_w} \right] \] (19)

The present equation is transformed into the usual boundary-layer coordinate system for a blunt axisymmetric body by the Lees-Dorodnitsyn transformation (ref. 43):

\[ \eta = \frac{u_r}{\sqrt{2\xi}} \int_0^y \rho \, dy \] (20)
\[ \xi = \int_0^S \rho e \mu \nu u e r^2 ds \] (21)

For a stagnation-point solution

\[ \xi = \frac{\rho e \mu \nu u e 2r^2}{4\beta} \] (22)

where the velocity gradient \( \beta \) is

\[ \beta = \frac{du_e}{ds} \] (23)

Thus, the transformation from the \( y \) coordinate to the \( \eta \) coordinate for the stagnation-point solution of a blunt axisymmetric body is

\[ \eta = \sqrt{\frac{2\beta}{\rho e \mu}} \int_0^y \rho dy \] (24)

and applying this transformation, equation (19) becomes for the stagnation point

\[ \dot{q}_w = \frac{\sqrt{2}(\rho_w \mu_w) \beta^{1/2}/(\partial h/\partial \eta)_{w}}{N_{Pr}(\rho e \mu)_{e}^{1/2}/(\partial \eta)_{w}} \left[ 1 - \frac{\beta_{2,w}}{M_{w}(N_{Le,w})^{1/3}} \sum \frac{F_i h_i,w(z_{i,e} - z_{i,w})}{h_s - h_w} \right] \]

\[ + (N_{Le,w})^{2/3} \sum \frac{h_i,w(z_{i,e} - z_{i,w})}{h_s - h_w} \] (25)

At the stagnation point, the velocity gradient for Newtonian flow is

\[ \beta = \frac{1}{R} \sqrt{2p_e} \left( 1 - \frac{p_\infty}{p_e} \right) \] (26)

For

\[ p_e >> p_\infty \]
the velocity gradient can be approximated by

$$\beta = \frac{1}{R} \left| \frac{2p_e}{\rho_e} \right|$$

(27)

With the use of the ideal gas equation

$$p = \frac{\rho R T}{M}$$

(28)

and the boundary-layer approximation at the stagnation point

$$p_e = p_w = p_s$$

(29)

equation (25) becomes

$$\frac{R}{\rho_s} = 8^{1/4} \left( \frac{M_e}{N_{Pr,w}} \right)^{1/2} \frac{\rho_w \mu_w}{\rho_e \mu_e} \left[ \frac{(\rho h)}{1/2 (\rho h)_{w}} \right] \left[ 1 - \frac{\beta_{2,w}}{M_w (N_{Le,w})^{1/3}} \sum \frac{F_i h_i, w (z_i, e - z_i, w)}{h_s - h_w} \right]$$

$$+ \left( \frac{N_{Le,w}}{h_s - h_w} \right)^{2/3} \sum \frac{h_i, w (z_i, e - z_i, w)}{h_s - h_w}$$

(30)

An expression is now needed which includes the enthalpy gradient at the wall, and it was necessary to use a correlation of the results from the computer study. A discussion of the enthalpy gradient at the wall is given in appendix A. A correlation which is reasonably valid for all the base gases and gas mixtures is presented (eq. (A3)) as follows:

$$\frac{1}{h_s - h_w} \left( \frac{\partial h}{\partial \eta} \right)_w = 0.58 \left( \frac{N_{Pr,w}}{\rho_w \mu_w} \right)^{0.4} \frac{\rho_e \mu_e}{\rho_w \mu_w} \left( \frac{\mu_o, e}{\mu_e} \right)^{1/2} \left( \frac{M_o}{M_e} \right)^{1/8}$$

(31)

Special note should be taken of the parameter \( \mu_o, e \), which is the viscosity evaluated at the temperature at the outer edge of the boundary layer (\( T_e \)) for a mixture based on the initial composition of the cold mixture or the ambient composition of the atmosphere surrounding a planet. For example, in determining the heat transfer in a gas mixture of 0.60 N₂-0.20 Ar-0.20 CO₂, the viscosity \( \mu_o, e \) is based on this composition rather than the chemical equilibrium composition at the outer edge of the boundary layer.

From the enthalpy-gradient correlation of equation (31), the heat transfer from equation (30) becomes
The stagnation-point convective heating to a blunt axisymmetric body is often expressed in terms of a heat-transfer coefficient. This coefficient is considered to be essentially constant over a range of conditions for a particular gas or gas mixture. As presented in references 16, 25, 28, and 44, the heat-transfer coefficient $K$ is defined as

$$K = \dot{q}_w \sqrt{\frac{R}{p_s}} (h_s - h_w)$$

When equations (32) and (33) are compared, it is seen that the heat-transfer coefficient can be written

$$K = 0.58(8)^{1/4} \left( M_e \right)^{1/4} \left( \frac{M_O}{M_e} \right)^{1/8} \left( \mu_o,e \right)^{1/2} \left( h_s - h_w \right) \left[ 1 - \frac{\beta_{2,w}}{M_w(N_{Le,w})^{1/3}} \right]$$

$$\times \left[ \sum h_i,w(z_i,e - z_i,w) \right] + \left( N_{Le,w} \right)^{2/3} \left[ \sum h_i,w(z_i,e - z_i,w) \right]$$

(32)

The viscosity is a dominant factor for the convective heat transfer of gases; thus, in order to find a general expression for all gases, the viscosity must be expressed in terms of basic parameters. For gas mixtures, the exact calculation of the viscosity is too complex to be used in any simple expression for the heat transfer. For example, see references 45 to 48. In the present analysis, a simple summation is proposed for mixture viscosity which agrees adequately with experimental results and theoretical models. The viscosity of the pure components is based on the analysis presented in reference 47. A discussion of the present viscosity relations is presented in appendix B.

The relation proposed for the mixture viscosity is (eq. (B6)) as follows:

$$\frac{1}{\mu} = M^{1/2} \sum \frac{c_i}{M_1^{1/2} \mu_1}$$

(35)
For a pure gas component

\[ \mu_i = \dot{\alpha} \gamma_i M_i^{1/2} T^{0.65} \]  

where the transport parameter \( \gamma_i \) is

\[ \gamma_i = \frac{1}{\sigma_i^{2} \frac{1}{k}} \]  

and the constant \( \alpha \) is

\[ \alpha = 2.2621 \times 10^{-6} \]  

With the use of the above viscosity relations, equation (34) becomes

\[ K = \frac{0.58(8)^{1/4}}{(N_{Pr,w})^{0.6}} \left[ \frac{\alpha^{1/2} \left( \frac{M_e}{M_0} \right)^{1/8}}{N_{Le,w}} \right]^{0.075} \left( \sum \left( \frac{c_{0,i}}{M_{0,i} \gamma_{0,i}} \right) \right)^{-1/2} \left[ 1 - \frac{\beta_{2,w}}{M_{w}(N_{Le,w})^{1/3}} \right] \]

\[ \times \left[ \left( \frac{F_i h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} \right) + \left( \frac{N_{Le,w}}{2/3} \right) \left( \frac{h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} \right) \right] \]  

As previously mentioned, the heat-transfer coefficient is used as a constant over a range of conditions. In the present analysis, the Lewis number augmentation factor (appendix C),

\[ 1 - \frac{\beta_{2,w}}{M_{w}(N_{Le,w})^{1/3}} \left( \frac{\sum F_i h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} \right) + \left( \frac{N_{Le,w}}{2/3} \right) \left( \frac{h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} \right) \]

is assumed to be approximately equal to 1. This approximation is based on a comparison with the equivalent expression for equal binary diffusion coefficients. (See refs. 8 and 49.) Furthermore, the following approximation is made

\[ \left( \frac{M_e}{M_0} \right)^{1/8} T_e^{0.075} \approx 1.81 \]
The terms in equation (40) have small exponents and will have a small variation over a range of conditions. Also, $M_e$ will decrease with increasing $T_e$ due to dissociation and ionization. The value used in the approximation was determined from the computer results. The effect of the parameters in equation (40) on the heat-transfer coefficient will be discussed further in the next section.

With the above approximations and evaluation of the constants, equation (39) for the heat-transfer coefficient can be expressed as

$$K = \frac{0.0885}{(N_{Pr,w})^{0.6}} \left( \sum \frac{c_{O,i}}{M_{O,i} \gamma_{O,i}} \right)^{-1/2}$$  \hspace{1cm} (41)$$

The Prandtl number at the wall varied from 0.39 to 0.71 for the various base gases and gas mixtures considered in the present study. However, the Prandtl number at the wall only varied from 0.67 to 0.71 for those base gases and gas mixtures which are presently considered for planetary entry. For $N_{Pr,w} = 0.69$,

$$K = 0.1106 \left( \sum \frac{c_{O,i}}{M_{O,i} \gamma_{O,i}} \right)^{-1/2}$$  \hspace{1cm} (42)$$

The present analysis has shown that the approximate stagnation-point convective heating for gases has a general relation involving the mass fraction $(c_{O,i})$, molecular weight $(M_{O,i})$, and transport parameter $(\gamma_{O,i})$ of the base gas or the base gases of a mixture. The necessary values for calculating $\gamma_{O,i}$ from equation (37) are given in reference 50 for numerous gases. Listed in table I are the values for the base gases considered in the present study. The approximate convective heating to the stagnation point of a blunt axisymmetric body can be calculated by the use of equation (33) and either equation (41) or (42) for base gases and gas mixtures in chemical equilibrium.

The heat-transfer coefficient for gas mixtures can also be calculated if the heat-transfer coefficients $(K_i)$ of the base gases are known. By manipulating equation (41), it can be shown that

$$\frac{1}{K^2} = (N_{Pr,w})^{1.2} \sum \frac{c_{O,i}}{K_i(N_{Pr,w,i})^{0.6} 2}$$  \hspace{1cm} (43)$$
The present analysis is for the stagnation-point convective heating to an axisymmetric blunt body and the developed relation is only applicable to this problem. However, the basic technique used in the present development could be applied to other heat-transfer problems as well.

RESULTS AND DISCUSSION

The present analysis has shown that the convective heating for gases can be expressed by a general relation involving the mass fraction \( c_{o,i} \), molecular weight \( M_{o,i} \), and transport parameter \( \gamma_{o,i} \). These parameters are known for most gases. The results of this general expression are compared with the results of the numerical solution to the boundary-layer equations from the present computer study and with the results of analytical and experimental studies presented in the literature.

Results from the present computer study are presented in figure 1 in the form of

\[ \frac{\dot{q}_w}{R \sqrt{P_S}} \] as a function of \( h_S - h_w \). The linear curve shown in figure 1 for a particular gas or gas mixture is based on a single heat-transfer coefficient for the gas as determined by a least-square fit of the values of \( K \) from each data point. The heat-transfer coefficient for each gas could be correlated with good accuracy by a single coefficient for a range of enthalpies, pressures, and wall temperatures. The heat-transfer coefficients for the gases and the average error and maximum error for the correlation are listed in table II.

The present analysis showed that the heat-transfer coefficient for gases can be expressed as (see eq. (41))

\[
K = \frac{0.0885}{\left( N_{Pr,w} \right)^{0.6}} \left( \sum \frac{c_{o,i}}{M_{o,i} \gamma_{o,i}} \right)^{-1/2}
\]

and for \( N_{Pr,w} = 0.69 \) (see eq. (42))
The analytical expressions (eqs. (41) and (42)) for the heat-transfer coefficient are compared with the present computer results in figure 2. As shown by the comparison, the general expression is in good agreement with the present computer results. Furthermore, the data presented in figure 2(b) show that equation (41), which includes the wall Prandtl number, is in better agreement with the computer results. Heat-transfer coefficients were determined for various gases from experimental and analytical studies presented in the literature; and, as shown by the comparison in figure 3, there is good agreement between the general relation and the results from other studies (refs. 13 to 22, 24 to 26, and 28). In references 14 and 15 the convective heating for several base gases and gas mixtures was calculated by the same method. The heat-transfer coefficients determined from these references are also compared separately with the general relation in figure 4. As shown by the data, the heat-transfer coefficients from references 14 and 15 are linear functions of the parameter \( \left( \sum \frac{c_{O_i}}{M_{O_i} \gamma_{O_i}} \right)^{-1/2} \); however, the slope is slightly greater than given by the present results.

The previous discussions have been concerned with the validity of the final, overall expression from the present analysis. The trend of the variations of the heat-transfer coefficient can also be explained from the analysis. From the computer results presented in figure 1, the reduced-heating parameter \( \left( \frac{\dot{q}_w}{\sqrt{R \frac{P}{P_S}}} \right) \) for some gases (for example, ammonia) appears to have an "apparent" wall temperature effect. This apparent temperature effect is actually a wall Prandtl number effect. The analytical analysis, see equation (41), shows such an effect and it can be substantiated from analysis of the computer results. For the gases which showed a wall temperature effect, there was a variation in the wall Prandtl number for the different wall temperature values.

The computer results in figure 1 also show a decrease in reduced-heating parameter with decreased pressure for a given enthalpy. This effect can be explained by examining equation (40). The value of 1.81 was based on an average over a range of conditions. For a given enthalpy, the degree of dissociation and ionization will be greatest at the lower pressures, and the molecular weight \( M_e \) will be decreased. Also, this effect will cause
the temperature $T_e$ to be lower at the lower pressures. Without the approximation of equation (40), equation (42) can be expressed as follows for $N_{Pr,w} = 0.69$:

$$K = 0.0632 \left( \frac{M_e}{M_0} \right)^{1/8} T_e^{0.075} \left( \sum \frac{c_{o,i}}{M_{o,i} \gamma_{o,i}} \right)^{-1/2}$$  \quad (45)

The trend of $M_e$ and $T_e$ with pressure, from the computer results, is shown in figure 5. Also, in figure 5, the heat-transfer coefficient as calculated by equation (45), with the value of $M_e$ and $T_e$ taken from the computer solution, is compared with the heat-transfer coefficient from the computer results. As shown by the comparison, the present analysis predicts correctly the effect of pressure.

Equation (45) also shows the trend of the heat-transfer coefficient with enthalpy. The heat-transfer coefficients from the computer study and from equation (45) are compared in figure 6 for several cases. The term $\left( \frac{M_e}{M_0} \right)^{1/8} T_e^{0.075}$ gives the predominant trend of the coefficient with enthalpy and, as shown in figure 6, good agreement generally exists between the present analysis and the computer results. The present analysis gives good final results and also gives the correct trend of intermediate steps.

The present results of the analytical and computer studies were compared with three other methods for calculating the convective heating of gases. These methods are presented in references 14, 16, and 28 and are different from each other and from the present analysis. These methods were based on a smaller number of gases than the present study.

A simple method for determining the convective, stagnation-point heating of gas mixtures is presented in reference 28. In this method the heat-transfer coefficient of gas mixtures is calculated by knowing the coefficient of the base gases by the relation

$$\frac{1}{K} = \sum \frac{c_{o,i}}{K_i}$$  \quad (46)

The relation was derived by empirical means and was based on mixtures of CO$_2$, N$_2$, and Ar. Similar expressions (see eqs. (43) and (44)) were derived for a wider range of gas mixtures in the present analysis:
A comparison between equations (43), (44), and (46) and the present computer results is given in figure 7 for some binary gas mixtures and in table III for all the gas mixtures considered in the present study. In comparing the summation relations, the heat-transfer coefficients for the base gases are taken from the computer results. The summation relations from the present studies, equations (43) and (44), have good agreement with the computer results for their respective conditions. Equation (43) has the best agreement for all the gases; however, equations (44) is equally valid for the gases when the Prandtl numbers at the wall are approximately equal for both the mixtures and the component base gases. The empirical method of reference 28, equation (46), has good agreement with the computer results for the mixtures where the Prandtl numbers are approximately equal for the mixtures and component base gases. However, this method is not as accurate as equation (43) when the Prandtl numbers for the mixtures are different from those for the component base gases. The method of reference 28 was based on empirical means, whereas the present relations were derived from analytical considerations. From equations (44) and (46) and analysis of the computer results, it can be seen that differentiation between a first and second power for the heat-transfer coefficient would be difficult by empirical means.

The method presented in reference 14 and later extended in reference 15 is based on knowing the low-temperature properties of the gas. In using the method, a correlation would have to be made for each particular gas, and, for air, N₂, CO₂, Ar, H₂, and several N₂-CO₂-Ar mixtures, the necessary correlative parameter are listed in reference 15. Basically the method of reference 14 and the present theoretical analysis are in agreement concerning low-temperature properties. In the present analysis, the heat-transfer coefficient is a function of the parameters \( N_{Pr,w} \), \( \gamma_o \), \( M_o \), and \( c_o \). These parameters are low-temperature properties of the gases. The low-temperature properties give the major component of the convective heating, and the boundary-layer-edge properties (high-temperature properties) give minor variations to the heating. As previously shown in
figure 4, heat-transfer coefficients can be determined from the results of references 14 and 15 and were shown to be linear functions of the parameter \( \left( \sum \frac{c_{o,i}}{M_{o,i} \gamma_{o,i}} \right)^{-1/2} \). This is in agreement with the present analysis. The present summation relation of equation (44) was applied to the data of references 14 and 15. As shown in table IV, the heat-transfer coefficients determined by the present relation is in good agreement with those determined with the use of references 14 and 15. The Prandtl number at the wall for gases considered for planetary entry is approximately 0.69; therefore, the present method is easier to use than the method presented in reference 14.

The results of reference 16 are presented as a correlation of the heat-transfer coefficient with molecular weight in the form

\[
K = 0.0323 + 0.00233M_o
\]  

(47)

or

\[
K = 0.0323 + 0.00233 \left( \sum \frac{c_{o,i}}{M_{o,i}} \right)^{-1}
\]  

(48)

A comparison of the results presented in reference 16 and the present computer results is shown in figure 8. The molecular weight correlation of reference 16 was based on a limited number of gases and for these gases the correlation is valid. However, as shown by the present results, this method is not valid for arbitrary gases.

CONCLUSIONS

The present analysis has resulted in the development of an approximate method for calculating the convective stagnation-point heating to an axisymmetric blunt body for gases in chemical equilibrium. The heating is expressed by a general relation of the mass fractions, molecular weights, and transport parameters of the base gases. Comparisons with the more exact computer study and with other analytical and experimental studies have shown the method to be valid. Also, it has been shown that a summation relation derived from the analysis is valid for calculating the heating of gas mixtures from the data of the base gases comprising the mixtures. For the gases presently considered for planetary entry, the approximate convective heating can be easily calculated. Since the analysis was based on a wide range of base gases and gas mixtures, the results should be valid.
for most gases. The basic technique developed for the prediction of stagnation-point heating to an axisymmetric blunt body could be applied to other heat-transfer problems as well.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., October 8, 1971.
APPENDIX A

ENTHALPY GRADIENT AT THE WALL

The results presented in references 51 and 52 have shown that the enthalpy gradient at the wall for the stagnation point of a blunt axisymmetric body in high-enthalpy air can be expressed by

\[ \frac{1}{h_S - h_w} \left( \frac{\partial h}{\partial \eta} \right)_w = a \left( \frac{\rho e \mu e}{\rho_w \mu_w} \right)^b \]  

(A1)

where \( a \) and \( b \) are given as follows:

\[
\begin{array}{ccc}
\text{a} & \text{b} & \text{Determined from reference} - \\
0.51 & 0.975 & 51 \\
0.49 & 0.938 & 52
\end{array}
\]

Also, from analysis of the heat-transfer relations presented in references 8 and 53,

\[ \frac{1}{h_S - h_w} \left( \frac{\partial h}{\partial \eta} \right)_w = a \left( \frac{\rho e \mu e}{\rho_w \mu_w} \right)^b \left( N_{Pr,w} \right)^c \]  

(A2)

where \( a \), \( b \), and \( c \) are given as follows:

\[
\begin{array}{ccc}
\text{a} & \text{b} & \text{c} & \text{Determined from reference} - \\
0.54 & 0.900 & 0.4 & 8 \\
0.54 & 0.980 & 0.4 & 53
\end{array}
\]

The interest in the present study was whether a single expression similar to equation (A2) could be determined and whether it would be reasonably valid for all gases. If the exponent for the density-viscosity product was 1, then the heat-transfer equation (eq. (30)) could be made simpler by the elimination of terms. A similar approach was attempted in reference 14, but a single correlation could not be determined for air, nitrogen, hydrogen, carbon dioxide, and argon. Preliminary work in the present study showed that such a correlation was feasible and additional studies were performed.
APPENDIX A – Continued

The data used to correlate the enthalpy gradient were taken from the results of the present computer study. The initial correlation is shown in figure A1 in the format of equation (A2) with \( b = 1 \) for the primary base gases and air. The data of the various gases are in good agreement with each other; however, a straight line could not be fitted through the zero point and there is a slight curvature to the data. Also, the relation was not in good agreement for all gases considered in the present study. Variations to the basic form of equation (A2) were tried and the best correlation obtained to the data was

\[
\frac{1}{h_S - h_W} \left( \frac{dh}{d\eta} \right)_W = 0.58 \left( N_{Pr, w} \right)^{0.4} \frac{\rho e u_e}{\rho_W h_W \left( \mu_{O_e} \right)}^{1/2} \left( \frac{M_O}{M_e} \right)^{1/8}
\]

(A3)

As shown in figure A2(a), the final correlation, equation (A3), has good agreement for the gas mixtures which are presently considered in planetary entry and for the base gases which comprise these mixtures. Representative data for all the gases considered in the present study are also compared with equation (A3) in figure A2(b). The greatest deviation from equation (A3) is for the gases with a significant variation of the molecular weight at the wall for a range of conditions, especially with stagnation enthalpy. The variation of molecular weight is affected by chemical reaction and diffusion.
APPENDIX A – Continued

(a) Gases considered for planetary entry.

(b) All gases considered in present study.

Figure A2. - Final correlation of the enthalpy gradient.
APPENDIX A – Concluded

The correlation of equation (A3) for the enthalpy gradient at the wall was based on boundary-layer results for a wide range of flow conditions and a wide range of base gases and gas mixtures. The base gases included monatomic, diatomic, and polyatomic gases. The gas mixtures were composed of base gases of only high molecular weights, of only low molecular weights, of high and low molecular weights, of basically inert gases, and of reactive gases. Since the correlation was in agreement for a wide range of base gases and gas mixtures, the correlation should be equally valid for most gases.
APPENDIX B

VISCOSITY RELATIONS

The viscosity equation for a pure component from reference 47, is

\[
\mu_i = 2.6693 \times 10^{-6} \frac{\sqrt{M_i T}}{\sigma_i^{2} \Omega^{(2,2)*}}
\]  \hspace{1cm} (B1)

where

- \( \mu_i \) viscosity, N-s/m²
- \( T \) temperature, K
- \( M_i \) molecular weight
- \( \sigma_i \) collision diameter, Å
- \( \Omega^{(2,2)*} \) reduced collision integral for viscosity

The reduced collision integral \( \Omega^{(2,2)*} \) is based on the Lennard-Jones (6-12) potential and is a slowly varying function of the nondimensional temperature \( T^* \) defined as

\[
T^* = \frac{T}{\epsilon/k}
\]  \hspace{1cm} (B2)

where \( \epsilon \) is the maximum energy of attraction and \( k \) is the Boltzmann constant. The values of \( \Omega^{(2,2)*} \) as a function of \( T^* \) are presented in reference 47 and are shown in figure B1. For the species and temperatures of interest in the present study, the reduced collision integral can be expressed as

\[
\Omega^{(2,2)*} = 1.18(T^*)^{-0.15}
\]  \hspace{1cm} (B3)

By combining equations (B1), (B2), and (B3), the viscosity for a pure species can be expressed as

\[
\mu_i = (2.2621 \times 10^{-6}) \gamma_i M_i^{1/2} T^{0.65}
\]  \hspace{1cm} (B4)
where the transport parameter is given as

\[ \gamma_i = \frac{1}{\sigma_i 2(\epsilon_i/k)^{0.15}} \]  \hspace{1cm} (B5)

A tabulation of the collision diameter \( \sigma_i \) and the maximum energy of attraction \( \epsilon_i/k \) for numerous species are given in reference 50.

A simple summation expression is proposed for the mixture viscosity. This expression is given by

\[ \frac{1}{\mu} = M^{1/2} \sum \frac{c_i}{M_i^{1/2} \mu_i} \]  \hspace{1cm} (B6)

The exact treatment of mixture viscosity involves more complicated expressions. (For example, see refs. 45 to 48.) The simple relation given by equation (B6) has been compared with experimental results and with more exact calculations. Some results of the comparison are shown in table BI and figure B2. (Also see refs. 54 to 60.) As shown by these results, equation (B6) is in good agreement with experimental and calculated values.
### APPENDIX B – Continued

**TABLE BI. COMPARISON OF MIXTURE VISCOSITIES**

<table>
<thead>
<tr>
<th>Gas composition (composition by mole fraction)</th>
<th>Temp., K</th>
<th>Reference</th>
<th>μ, N-s/m² From reference</th>
<th>Computed (eq. (B6)) with $\mu_1$ from reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1754 He - 0.5556 Ne - 0.2670 Ar</td>
<td>293</td>
<td>54</td>
<td>$27.34 \times 10^{-6}$</td>
<td>$27.05 \times 10^{-6}$</td>
</tr>
<tr>
<td>0.5175 He - 0.3210 Ne - 0.1615 Ar</td>
<td>293</td>
<td>54</td>
<td>25.86</td>
<td>26.42</td>
</tr>
<tr>
<td>0.3594 He - 0.3193 Ne - 0.3213 Ar</td>
<td>473</td>
<td>46</td>
<td>36.00</td>
<td>36.65</td>
</tr>
<tr>
<td>0.3333 H₂ - 0.3333 Ne - 0.3333 CO₂</td>
<td>298</td>
<td>46</td>
<td>19.02</td>
<td>18.72</td>
</tr>
<tr>
<td>0.2500 H₂ - 0.2500 Ne - 0.2500 CO₂ - 0.2500 CC₁₂F₂</td>
<td>298</td>
<td>46</td>
<td>15.93</td>
<td>16.69</td>
</tr>
<tr>
<td>0.5429 He - 0.2189 Ne - 0.2382 Ar</td>
<td>473</td>
<td>55 and 57</td>
<td>37.90</td>
<td>38.08</td>
</tr>
<tr>
<td>0.3594 He - 0.3193 Ne - 0.3213 Ar</td>
<td>473</td>
<td>55 and 57</td>
<td>35.74</td>
<td>36.77</td>
</tr>
<tr>
<td>0.5000 H₂ - 0.5000 Ar</td>
<td>5000</td>
<td>58</td>
<td>316.00</td>
<td>320.30</td>
</tr>
<tr>
<td>0.5000 N₂ - 0.5000 Ar</td>
<td>10000</td>
<td>59</td>
<td>244.00</td>
<td>241.80</td>
</tr>
<tr>
<td>0.4000 N₂ - 0.6000 N₂</td>
<td>10000</td>
<td>59</td>
<td>234.00</td>
<td>241.80</td>
</tr>
<tr>
<td>0.6000 O₂ - 0.4000 O₂</td>
<td>10000</td>
<td>59</td>
<td>25.04</td>
<td>25.90</td>
</tr>
<tr>
<td>0.4828 C₂H₄ - 0.5172 NH₃</td>
<td>523</td>
<td>56</td>
<td>17.64</td>
<td>17.43</td>
</tr>
<tr>
<td>0.6719 H₂ - 0.3281 N₂</td>
<td>523</td>
<td>60</td>
<td>22.02</td>
<td>23.25</td>
</tr>
</tbody>
</table>

![Graph 1](image1)

(a) Comparison with reference 45.

![Graph 2](image2)

(b) Comparison with references 55 and 56.

![Graph 3](image3)

(c) Comparison with reference 54.

Figure B2. - A comparison of the present proposed relation for mixture viscosity with other studies.
When equations (B4), (B5), and (B6) are combined, the mixture viscosity can be expressed as

\[
\mu = \frac{(2.2621 \times 10^{-6}) T^{0.65}}{M^{1/2} \sum \frac{C_i}{M_i \nu_i}}
\]  

(B7)
APPENDIX C

LEWIS NUMBER AUGMENTATION FACTOR

In the present analysis, the Lewis number augmentation factor

\[ 1 - \frac{\beta_{2,w}}{M_w(N_{Le,w})^{1/3}} \sum \frac{F_i h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} + \left(\frac{N_{Le,w}}{2/3} \right) \sum \frac{h_{i,w}(z_{i,e} - z_{i,w})}{h_s - h_w} \]

is assumed to have a value of 1. This approximation is based on a comparison with the equivalent expression for equal diffusion coefficients. The equations for the bifurcation approximation of multicomponent diffusion will reduce to the usual equation for binary diffusion when the binary diffusion coefficients are assumed equal.

When the binary diffusion coefficients are assumed equal, then \( F_i = 1, \ \beta_2 = M, \) and \( z_i = c_i, \) and the approximation for the Lewis number augmentation factor is

\[ 1 + \frac{N_{Le,w}}{(N_{Le,w})^{1/3}} \sum \frac{h_{i,w}(c_{i,e} - c_{i,w})}{h_s - h_w} \approx 1 \]  

(C1)

The approximation is exact for a Lewis number of 1 or if the species concentrations do not vary through the boundary layer. Also, the approximation is reasonably valid over a range of other conditions.

Let an energy factor \( E \) be defined as

\[ E = \frac{\sum h_{i,w}(c_{i,e} - c_{i,w})}{h_s - h_w} \]  

(C2)

or

\[ E = \left( \sum h_{i,w}c_{i,e} \right) - h_w \]  

(C3)

Now,

\[ h_{i,e} \geq h_{i,w} \]  

(C4)

From equations (C3) and (C4), the maximum value of \( E \) will be

\[ E_{\text{max}} = \frac{h_s - h_w}{h_s - h_w} = 1 \]  

(C5)
whereas the minimum value of $E$ will be zero. Therefore,

$$0 \leq E \leq 1$$ \quad (C6)

Expressions for the Lewis number augmentation factor are presented in references 8 and 49 for binary diffusion as follows:

<table>
<thead>
<tr>
<th>Reference</th>
<th>Lewis number augmentation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>$1 + \frac{N_{Le} - 1}{(N_{Le})^{1/3}} E$</td>
</tr>
<tr>
<td>49</td>
<td>$\left[1 + (N_{Le} - 1)E\right]^{0.6}$</td>
</tr>
<tr>
<td>8</td>
<td>$1 + \left[(N_{Le})^{0.52} - 1\right]E$</td>
</tr>
</tbody>
</table>

The Lewis number augmentation factors are presented in table CI for a range of Lewis numbers and energy factors. As shown by the data, the Lewis number augmentation factors...
APPENDIX C – Concluded

factor is approximately 1 except for the cases where the Lewis number is significantly different from 1 and the energy factor approaches the maximum value. The energy factor will approach the minimum for high enthalpies at the outer edge of the boundary layer unless the wall enthalpies of the species present at the boundary-layer edge are significantly greater than the wall enthalpies of the species present at the wall.

From analysis of the present computer results, it was noted that the Lewis number at the wall did not vary significantly from 1 for the base gases and gas mixtures which are presently considered for planetary entry. Significant variation from 1 was noted for the gases in which significant dissociation or reactions occurred at the wall. Thus, it is concluded that the approximation of 1 for the Lewis number augmentation factor is reasonably valid for a wide range of conditions and gases.
REFERENCES


TABLE I.- MOLECULAR WEIGHTS AND TRANSPORT
PARAMETERS FOR BASE GASES

<table>
<thead>
<tr>
<th>Base gas</th>
<th>M</th>
<th>( \sigma, \text{Å} ) (a)</th>
<th>( \epsilon/k, \text{K} ) (a)</th>
<th>( \gamma ) (b)</th>
<th>( M\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(_2)</td>
<td>28.014</td>
<td>3.798</td>
<td>71.4</td>
<td>0.03654</td>
<td>1.024</td>
</tr>
<tr>
<td>O(_2)</td>
<td>32.000</td>
<td>3.467</td>
<td>106.7</td>
<td>0.04129</td>
<td>1.321</td>
</tr>
<tr>
<td>H(_2)</td>
<td>2.016</td>
<td>2.827</td>
<td>59.7</td>
<td>0.06775</td>
<td>0.137</td>
</tr>
<tr>
<td>He</td>
<td>4.003</td>
<td>2.551</td>
<td>10.22</td>
<td>0.10845</td>
<td>0.434</td>
</tr>
<tr>
<td>Ne</td>
<td>20.183</td>
<td>2.820</td>
<td>32.8</td>
<td>0.07449</td>
<td>1.504</td>
</tr>
<tr>
<td>Ar</td>
<td>39.948</td>
<td>3.542</td>
<td>93.3</td>
<td>0.04036</td>
<td>1.612</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>44.011</td>
<td>3.941</td>
<td>195.2</td>
<td>0.02919</td>
<td>1.285</td>
</tr>
<tr>
<td>NH(_3)</td>
<td>17.031</td>
<td>2.900</td>
<td>558.3</td>
<td>0.04605</td>
<td>0.784</td>
</tr>
<tr>
<td>CH(_4)</td>
<td>16.043</td>
<td>3.758</td>
<td>148.6</td>
<td>0.03345</td>
<td>0.537</td>
</tr>
</tbody>
</table>

\(^{a}\)From reference 50.

\(^{b}\)Calculated from equation (37).
<table>
<thead>
<tr>
<th>Gas composition (composition by mass fraction)</th>
<th>( K_r ) \text{ kg/s-m}^{3/2}-\text{atm}^{1/2}</th>
<th>Error for correlation, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{N}_2 )</td>
<td>0.1112</td>
<td></td>
</tr>
<tr>
<td>( \text{O}_2 )</td>
<td>0.1201</td>
<td></td>
</tr>
<tr>
<td>( \text{H}_2 )</td>
<td>0.0395</td>
<td></td>
</tr>
<tr>
<td>( \text{He} )</td>
<td>0.0797</td>
<td></td>
</tr>
<tr>
<td>( \text{Ne} )</td>
<td>0.1474</td>
<td></td>
</tr>
<tr>
<td>( \text{Ar} )</td>
<td>0.1495</td>
<td></td>
</tr>
<tr>
<td>( \text{CO}_2 )</td>
<td>0.1210</td>
<td></td>
</tr>
<tr>
<td>( \text{NH}_3 )</td>
<td>0.0990</td>
<td></td>
</tr>
<tr>
<td>( \text{CH}_4 )</td>
<td>0.0807</td>
<td></td>
</tr>
<tr>
<td>( 0.2326 \text{O}_2-0.7680 \text{N}_2 ) (air)</td>
<td>0.1113</td>
<td>3.6</td>
</tr>
<tr>
<td>( 0.6714 \text{CO}_2-0.3286 \text{Ar} )</td>
<td>0.1262</td>
<td>3.4</td>
</tr>
<tr>
<td>( 0.1345 \text{CO}_2-0.8655 \text{N}_2 )</td>
<td>0.1113</td>
<td>3.6</td>
</tr>
<tr>
<td>( 0.5000 \text{CO}_2-0.5000 \text{N}_2 )</td>
<td>0.1131</td>
<td>3.4</td>
</tr>
<tr>
<td>( 0.8500 \text{CO}_2-0.1500 \text{N}_2 )</td>
<td>0.1173</td>
<td>3.0</td>
</tr>
<tr>
<td>( 0.7500 \text{O}_2-0.2500 \text{N}_2 )</td>
<td>0.1164</td>
<td>3.0</td>
</tr>
<tr>
<td>( 0.5000 \text{O}_2-0.5000 \text{N}_2 )</td>
<td>0.1132</td>
<td>3.1</td>
</tr>
<tr>
<td>( 0.1500 \text{H}_2-0.8500 \text{He} )</td>
<td>0.0657</td>
<td>4.3</td>
</tr>
<tr>
<td>( 0.3500 \text{H}_2-0.6500 \text{He} )</td>
<td>0.0547</td>
<td>3.3</td>
</tr>
<tr>
<td>( 0.6500 \text{H}_2-0.3500 \text{He} )</td>
<td>0.0459</td>
<td>3.4</td>
</tr>
<tr>
<td>( 0.5000 \text{H}_2-0.5000 \text{Ar} )</td>
<td>0.0622</td>
<td>4.9</td>
</tr>
<tr>
<td>( 0.1500 \text{N}_2-0.8500 \text{H}_2 )</td>
<td>0.0473</td>
<td>5.7</td>
</tr>
<tr>
<td>( 0.5000 \text{N}_2-0.5000 \text{H}_2 )</td>
<td>0.0636</td>
<td>6.4</td>
</tr>
<tr>
<td>( 0.2000 \text{CO}_2-0.8000 \text{H}_2 )</td>
<td>0.0537</td>
<td>7.6</td>
</tr>
<tr>
<td>( 0.4000 \text{CO}_2-0.6000 \text{H}_2 )</td>
<td>0.0629</td>
<td>7.6</td>
</tr>
<tr>
<td>( 0.6000 \text{CO}_2-0.4000 \text{H}_2 )</td>
<td>0.0771</td>
<td>8.6</td>
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<tr>
<td>( 0.8000 \text{CO}_2-0.2000 \text{H}_2 )</td>
<td>0.1002</td>
<td>10.2</td>
</tr>
<tr>
<td>( 0.3626 \text{CO}_2-0.3077 \text{N}_2-0.3297 \text{Ar} )</td>
<td>0.1218</td>
<td>2.7</td>
</tr>
<tr>
<td>( 0.1339 \text{CO}_2-0.8525 \text{N}_2-0.0136 \text{Ar} )</td>
<td>0.1118</td>
<td>2.9</td>
</tr>
<tr>
<td>( 0.3000 \text{Ne}-0.3000 \text{Ar}-0.4000 \text{He} )</td>
<td>0.1143</td>
<td>6.3</td>
</tr>
<tr>
<td>( 0.3500 \text{H}_2-0.3500 \text{CO}_2-0.3000 \text{N}_2 )</td>
<td>0.0809</td>
<td>9.0</td>
</tr>
<tr>
<td>( 0.352 \text{H}_2-0.423 \text{He}-0.176 \text{Ne}-0.024 \text{CH}_4-0.025 \text{NH}_3 )</td>
<td>0.0662</td>
<td>6.2</td>
</tr>
</tbody>
</table>
# Table III: Comparison of Summation Relations for Gas Mixtures

<table>
<thead>
<tr>
<th>Gas composition (composition by mass fraction)</th>
<th>$N_{Pr,w}$ (a)</th>
<th>Computer results</th>
<th>K, kg/s-$m^{3/2}.atm^{1/2}$</th>
<th>Present, eq. (43)</th>
<th>Present, eq. (44)</th>
<th>Ref. 28, eq. (46)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$_2$</td>
<td>0.680</td>
<td>0.1112</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O$_2$</td>
<td>0.685</td>
<td>0.1201</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H$_2$</td>
<td>0.675</td>
<td>0.0395</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>He</td>
<td>0.677</td>
<td>0.0797</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>0.667</td>
<td>0.1474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>0.667</td>
<td>0.1495</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO$_2$</td>
<td>0.685</td>
<td>0.1210</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH$_3$</td>
<td>0.520</td>
<td>0.0990</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH$_4$</td>
<td>0.608</td>
<td>0.0807</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2320 O$_2$-0.7680 N$_2$ (air)</td>
<td>0.695</td>
<td>0.1113</td>
<td>0.1117</td>
<td>0.1131</td>
<td>0.1131</td>
<td></td>
</tr>
<tr>
<td>0.6714 CO$_2$-0.3286 Ar</td>
<td>0.711</td>
<td>0.1262</td>
<td>0.1252</td>
<td>0.1285</td>
<td>0.1291</td>
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</tr>
<tr>
<td>0.1345 CO$_2$-0.8655 N$_2$</td>
<td>0.692</td>
<td>0.1113</td>
<td>0.1113</td>
<td>0.1124</td>
<td>0.1124</td>
<td></td>
</tr>
<tr>
<td>0.5000 CO$_2$-0.5000 N$_2$</td>
<td>0.695</td>
<td>0.1131</td>
<td>0.1145</td>
<td>0.1158</td>
<td>0.1159</td>
<td></td>
</tr>
<tr>
<td>0.8500 CO$_2$-0.1500 N$_2$</td>
<td>0.691</td>
<td>0.1173</td>
<td>0.1187</td>
<td>0.1194</td>
<td>0.1194</td>
<td></td>
</tr>
<tr>
<td>0.7500 O$_2$-0.2500 N$_2$</td>
<td>0.698</td>
<td>0.1164</td>
<td>0.1162</td>
<td>0.1177</td>
<td>0.1177</td>
<td></td>
</tr>
<tr>
<td>0.5000 O$_2$-0.5000 N$_2$</td>
<td>0.702</td>
<td>0.1132</td>
<td>0.1134</td>
<td>0.1154</td>
<td>0.1155</td>
<td></td>
</tr>
<tr>
<td>0.1500 H$_2$-0.8500 He</td>
<td>0.697</td>
<td>0.0657</td>
<td>0.0644</td>
<td>0.0659</td>
<td>0.0691</td>
<td></td>
</tr>
<tr>
<td>0.3500 H$_2$-0.6500 He</td>
<td>0.710</td>
<td>0.0547</td>
<td>0.0536</td>
<td>0.0553</td>
<td>0.0588</td>
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</tr>
<tr>
<td>0.6500 H$_2$-0.3500 He</td>
<td>0.705</td>
<td>0.0545</td>
<td>0.0448</td>
<td>0.0460</td>
<td>0.0480</td>
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</tr>
<tr>
<td>0.5000 H$_2$-0.5000 Ar</td>
<td>0.496</td>
<td>0.0622</td>
<td>0.0649</td>
<td>0.0540</td>
<td>0.0625</td>
<td></td>
</tr>
<tr>
<td>0.1500 N$_2$-0.8500 H$_2$</td>
<td>0.570</td>
<td>0.0473</td>
<td>0.0469</td>
<td>0.0424</td>
<td>0.0437</td>
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<tr>
<td>0.5000 N$_2$-0.5000 H$_2$</td>
<td>0.477</td>
<td>0.0636</td>
<td>0.0649</td>
<td>0.0526</td>
<td>0.0583</td>
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<tr>
<td>0.2000 CO$_2$-0.8000 H$_2$</td>
<td>0.540</td>
<td>0.0537</td>
<td>0.0498</td>
<td>0.0436</td>
<td>0.0456</td>
<td></td>
</tr>
<tr>
<td>0.4000 CO$_2$-0.6000 H$_2$</td>
<td>0.479</td>
<td>0.0629</td>
<td>0.0606</td>
<td>0.0493</td>
<td>0.0541</td>
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</tr>
<tr>
<td>0.6000 CO$_2$-0.4000 H$_2$</td>
<td>0.439</td>
<td>0.0771</td>
<td>0.0752</td>
<td>0.0580</td>
<td>0.0662</td>
<td></td>
</tr>
<tr>
<td>0.8000 CO$_2$-0.2000 H$_2$</td>
<td>0.457</td>
<td>0.1002</td>
<td>0.0937</td>
<td>0.0740</td>
<td>0.0857</td>
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</tr>
<tr>
<td>0.3636 CO$_2$-0.3077 N$_2$-0.3297 Ar</td>
<td>0.710</td>
<td>0.1218</td>
<td>0.1213</td>
<td>0.1246</td>
<td>0.1255</td>
<td></td>
</tr>
<tr>
<td>0.1339 CO$_2$-0.8525 N$_2$-0.0136 Ar</td>
<td>0.693</td>
<td>0.1118</td>
<td>0.1115</td>
<td>0.1127</td>
<td>0.1128</td>
<td></td>
</tr>
<tr>
<td>0.3000 Ne-0.3000 Ar-0.4000 He</td>
<td>0.485</td>
<td>0.1143</td>
<td>0.1275</td>
<td>0.1053</td>
<td>0.1104</td>
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</tr>
<tr>
<td>0.3500 H$_2$-0.3500 CO$_2$-0.3000 N$_2$</td>
<td>0.430</td>
<td>0.0809</td>
<td>0.0795</td>
<td>0.0606</td>
<td>0.0692</td>
<td></td>
</tr>
<tr>
<td>0.352 H$_2$-0.423 He-0.176 Ne-0.024 CH$_4$-0.025 NH$_3$</td>
<td>0.515</td>
<td>0.0662</td>
<td>0.0669</td>
<td>0.0571</td>
<td>0.0626</td>
<td>0.0626</td>
</tr>
</tbody>
</table>

\*Average value from computer results for a range of conditions.
TABLE IV.- COMPARISON OF SUMMATION RELATION
WITH REFERENCES 14 AND 15

<table>
<thead>
<tr>
<th>Gas composition (composition by mass fraction)</th>
<th>K, kg/s-m^{3/2}-atm^{1/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>.1477</td>
</tr>
<tr>
<td>N₂</td>
<td>.1259</td>
</tr>
<tr>
<td>Ar</td>
<td>.1596</td>
</tr>
<tr>
<td>0.8655 N₂-0.1345 CO₂</td>
<td>.1278</td>
</tr>
<tr>
<td>0.3889 N₂-0.6111 CO₂</td>
<td>.1369</td>
</tr>
<tr>
<td>0.6717 CO₂-0.3283 Ar</td>
<td>.1513</td>
</tr>
<tr>
<td>0.5614 Ar-0.3149 N₂-0.1237 CO₂</td>
<td>.1459</td>
</tr>
</tbody>
</table>

Refs. 14 and 15                              Present, eq. (44)

0.1278                                        0.1283
0.1369                                        0.1379
0.1513                                        0.1513
0.1459                                        0.1450
(a) Key for all parts of figure 1.

(b) Nitrogen.

Figure 1.- Heat-transfer results from present computer study.
Figure 1. - Continued.

(c) Oxygen.

(d) Hydrogen.

Figure 1.- Continued.
(e) Helium.

(f) Neon.

Figure 1.- Continued.
(g) Argon.

(h) Carbon dioxide.

Figure 1.- Continued.
(i) Ammonia.

(j) Methane.

Figure 1.- Continued.
(k) 0.2320 O\textsubscript{2} - 0.7680 N\textsubscript{2} (air).

(l) 0.6714 CO\textsubscript{2} - 0.3286 Ar.

Figure 1. - Continued.
(m) $0.1345 \text{CO}_2 - 0.8655 \text{N}_2$.

(n) $0.50 \text{CO}_2 - 0.50 \text{N}_2$.

Figure 1. - Continued.
(o) \(0.85 \text{CO}_2 - 0.15 \text{N}_2\).

(p) \(0.75 \text{O}_2 - 0.25 \text{N}_2\).

Figure 1.- Continued.
(q) 0.50 O₂-0.50 N₂.

(r) 0.15 H₂-0.85 He.

Figure 1.- Continued.
Figure 1. - Continued.

(s) 0.35 H₂ - 0.65 He.

(t) 0.65 H₂ - 0.35 He.
(u) $0.50 \text{H}_2 - 0.50 \text{Ar}$.

(v) $0.15 \text{N}_2 - 0.85 \text{H}_2$.

Figure 1.- Continued.
(w) 0.50 N₂ - 0.50 H₂.

(x) 0.20 CO₂ - 0.80 H₂.

Figure 1.—Continued.
(z) 0.60 CO$_2$-0.40 H$_2$.

Figure 1.- Continued.
(aa) $0.80 \text{CO}_2 - 0.20 \text{H}_2$.

(bb) $0.3826 \text{CO}_2 - 0.3077 \text{N}_2 - 0.3297 \text{Ar}$.

Figure 1.- Continued.
(cc) 0.1339 CO$_2$-0.8525 N$_2$-0.0136 Ar.

(dd) 0.30 Ne-0.30 Ar-0.40 He.

Figure 1.- Continued.
(ee) \(0.35 \text{H}_2-0.35 \text{CO}_2-0.30 \text{N}_2\).

(ff) \(0.352 \text{H}_2-0.423 \text{He}-0.176 \text{Ne}-0.024 \text{CH}_4-0.025 \text{NH}_3\).

Figure 1.- Concluded.
Figure 2.- A comparison between the present analytical analysis and the present computer results for the heat-transfer coefficient. (Units for $K$ are kg/s-m$^3$/atm$^{1/2}$.)
Figure 3.- A comparison of the present analytical analysis with other studies.
Figure 4.- A comparison of the present results with the results of references 14 and 15. (Units for $K$ are $kg/s-m^{3/2}-atm^{1/2}$.)

\[ \left( \sum \frac{c_{0,i}}{R_0^i \eta_{0,i}} \right)^{1/2} \]
(a) \(0.2320 \text{ O}_2 - 0.7680 \text{ N}_2\) (air). \(T_w = 1111\) K; 
\(h_s = 81.354\) MJ/kg.

(b) \(0.85 \text{ CO}_2 - 0.15 \text{ N}_2\). \(T_w = 300\) K; 
\(h_s = 104.596\) MJ/kg.

Figure 5. - The trend of the heat-transfer coefficient with pressure. (Units for \(K\) are \(\text{kg/s-m}^{3/2}\text{-atm}^{1/2}\).)
Figure 6.- The trend of the heat-transfer coefficient with enthalpy. (Units for $K$ are kg/s-m$^3$-atm$^{1/2}$.)
Figure 7. - A comparison of the summation relations with the present computer results for some binary gas mixtures. (Units for $K$ are $kg/s\cdot m^{3/2}\cdot atm^{1/2}$.)
Figure 8. - A comparison of the present computer results with the molecular weight correlation of reference 16. (Units for $K$ are $\text{kg/s-m}^{3/2-\text{atm}^{1/2}}$.)
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—National Aeronautics and Space Act of 1958

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