THE VELOCITY AND COMPOSITION OF SUPERNOVA EJECTA

Stirling A. Colgate
New Mexico Institute of Mining and Technology
Socorro, New Mexico 87801

I. Velocity Distribution

The explosion of a supernova is generally agreed upon to take place at the end of stellar evolution. Hence all models of supernova explosions depend upon a highly condensed star. In some models, especially those depending upon thermonuclear detonation, no remnant such as a neutron star is left behind. For the case in question of the Gum Nebula, a pulsar — a presumed neutron star, is believed to be a relic of the explosive event. Regardless of the mechanism of the explosion, whether a thermonuclear detonation or a neutron star-neutrino transport model, the velocity distribution and composition of the ejected matter will be roughly the same. This is because the available energy is 1 to 4 MeV per nucleon (1.4 to $3 \times 10^9$ cm/sec) depending upon the contribution from the neutron star binding energy and, in addition, all models start from at least the density of carbon synthesis, $\gtrsim 10^7$ g/cm$^3$.

If the energy were released uniformly throughout the stellar mass, if the density were uniform and gravity neglected, then a uniform spherical expansion would take place with the density independent of radius out to a surface. The non-uniform energy release, non-uniform initial density distribution and, finally, gravity all perturb this simple picture, but to a surprisingly small degree. The explosion shock wave has a strong leveling effect. The detonation in the high density central regions drives a shock wave into the lower density, lower energy regions. The expansion behind the shock reduces the energy and density at the site of origin and compresses and adds energy to the site of shock traversal. Furthermore, the specific energy behind the shock is large compared to gravitational binding and so gravity can be neglected. Therefore, to a surprising degree the expansion appears uniform.

There are two modifications to this simple picture of some importance. First, in the case of neutron star formation a fraction (25 to 50%) of the initially expanding matter falls backward and reimplodes onto the surface of the neutron star, thereby removing from observation and the universe, matter that would otherwise be offensively neutron rich. For this matter, gravity is clearly important.
The outer layers of the star are progressively lower in density, and hence for the outer 10 to 15% mass fraction, the strong shock of finite compression ratio cannot possibly compensate for the density gradient, and so a change in shock strength occurs. Again, roughly speaking, the shock wave speeds up in the outer layers in just that fashion such that the final velocity distribution is described by velocity \( \propto F^{-1/4} \), where \( F \) is the mass fraction external to the point in question. This velocity distribution results in a fair approximation to the observed, demodulated low energy cosmic rays. The combined velocity distribution is shown in Figure 1, where the reimplosion mass fraction cut-off is shown along with the modification in shock behavior due to relativity.

II. Composition

The reimploding mass fraction is presumed to be neutron rich. The bulk of the explosion is presumed to take place in matter that is predominantly either carbon, oxygen or the products of partial burning of carbon and oxygen. In either case, the product of the detonation or sudden shock heating of such material is described as quasi-equilibrium silicon burning. The final composition is presumed to be roughly one-third iron and two-thirds silicon with many and various small fractions of elements from helium to iron.

The ejected mass of these elements is reasonably consistent with supernova models, supernova frequency, past galactic history, and iron content of the galaxy. The mass fraction of iron in the sun is about \( 1.5 \times 10^{-3} \), and the additional synthesis since the formation of the sun might be at most 25%, or a mass fraction of iron in the active galaxy of \( 1.8 \times 10^{-3} \). Roughly 50% of the iron of the galaxy is tied up in white dwarfs formed early in the history of the galaxy, and containing significantly less iron than the active mixed matter, and so for \( 10^{11} M_\odot \) per galaxy, there must be \( 10^8 M_\odot \) of iron formed. One half of this was presumably formed in the first 10% of the age of the galaxy so that \( 5 \times 10^7 M_\odot \) have presumably been injected into the galaxy by supernovae during \( 10^{10} \) years. The frequency of supernovae of type I is variously estimated to be one per 40 years to one per 400 years depending upon the galaxy type. For our galaxy, which is of the large active spiral type, the higher rate is presumably more nearly correct, and so \( 2.5 \times 10^8 \) supernovae must inject \( 0.2 M_\odot \) of iron per event to make the observed iron content. In addition, \( 0.4 M_\odot \) of silicon must be injected which results in an ejected mass fraction of \( 0.6 M_\odot \). This is a reasonable estimate for type I supernovae and is self-consistent with the generation of the optical light curve by the radioactive heat of the \( ^{56}\text{Ni} \to ^{56}\text{Co} \to ^{56}\text{Fe} \) decay. 0.2 \( M_\odot \) of \(^{56}\text{Co} \) decay is also consistent with the observed large flux of low energy positrons.
Although the ejection of a relatively large mass of silicon-burning products is self-consistent for the above reasons, it definitely is not consistent with the measurements of the composition of low energy cosmic rays. If silicon-burning products were ejected with the power law of Figure 1, they would either be observed as the dominant source of low energy cosmic rays, or have to be absorbed by a relatively large factor (~1/100) since the compositions are so radically different. The cosmic ray composition corresponds to roughly a 10 times enrichment above solar composition of all elements of Z ≥ 6, relative to hydrogen. This composition could arise from the shock ejection of an outer layer of enriched helium, but not from an outer layer of carbon, oxygen, magnesium or iron. The hydrogen is produced as the surviving product of all spallation processes.

III. Mass Fraction of Helium-Burning Shell

The termination of helium shell burning occurs because the shell is expanded and cooled by radiation stress.

Let Δm = mass per unit area of helium shell. Then in hydrostatic equilibrium

\[ \Delta m g = P = \rho RT + \frac{aT^4}{3} , \]

but

\[ \Delta m = h \rho , \text{ where } h = \left( \frac{1}{\rho \frac{d\rho}{dr}} \right)^{-1} . \]

The radiation flux φ becomes

\[ \phi = \frac{c}{3 \kappa \rho} \frac{d}{dr} \left( aT^4 \right) = \frac{4c}{3 \kappa \rho} \left( aT^4 \right) \frac{1}{T} \frac{dT}{dr} . \]

For a polytrope of index 3 and/or a radiative zero solution envelope, \( \rho \sim T^3 \) and

\[ \frac{1}{\rho} \frac{d\rho}{dr} = 3 \frac{1}{T} \frac{dT}{dr} . \]
so that

$$\phi = \frac{4c(aT^4)}{9K\rho h} = \frac{4c(aT^4)}{9K\Delta m}.$$ 

Kutter, Savedoff, and Schuerman (1969) have shown that the critical luminosity \(\phi_c\) at which hydrostatic equilibrium is exceeded by the radiation stress (including \(\rho RT\)) is 1/3 the purely radiative condition so that

$$\phi_c = \frac{gc}{3\kappa}.$$ 

We apply this condition at the helium-burning zone, recognizing that mass loss will proceed further out in the envelope prior to the burning layer reversal because \(\kappa\) will be larger than the Compton opacity near the surface layer. Then termination of helium shell burning should occur when

$$\phi = \phi_c, \text{ or}$$

$$\Delta m = \frac{4(aT^4)}{3g}.$$ 

The mass fraction of the helium layer becomes

$$F = \frac{4\pi r^2 \Delta m}{M} = \frac{4}{3} \frac{aT^4 4\pi r^4}{M^2 G}.$$ 

Taking \(T = 2.4 \times 10^9\) °K at the peak of helium burning and \(r = 6.5 \times 10^8\) cm from Kutter (1971), then for \(M = M_\odot\), \(F = 3.5 \times 10^{-4}\).

This mass fraction of the star corresponds to about \(10^{-3}\) of the ejected matter and is shown in Fig. 1 as occurring at roughly \(10^{10}\) cm sec\(^{-1}\) ejection velocity or roughly 100 MeV per nucleon. The energy absorption for low energy cosmic rays due to ionization loss is roughly 100 MeV per nucleon for the path from the Vela pulsar to earth so that the composition observed at the earth at least could not be greatly enriched in silicon-burning products from the ejecta of the presumed Vela supernova.
IV. Conclusion

The high ejection velocity of the high atomic number elements silicon through iron is needed for producing the ionization of the Gum Nebula within the estimated time available without at the same time requiring too much mass (≥1 M☉) and without at the same time producing too much of the light elements lithium, beryllium and boron by spallation. In addition, a small outer mass fraction must correspond to an enriched solar abundance with or without hydrogen. These conditions are consistent with current supernova models.

References


Figure 1. The mass fractions and velocities of ejecta from three regions—the inner re-implosion, the region of products of silicon burning, and the outer shell of helium.

VELOCITY OF EJECTED MATTER (cm sec\(^{-1}\))
DISCUSSION

G. S. Kutter: Why should the planetary nebula shell still be around when the supernova explodes?

S. A. Colgate: It might not be. The ejection velocity is of the order of 20–30 km/sec. The evolution time from the ejection of the shell to the explosion of the supernova varies from $2 \times 10^8$ years in the Finzi and Wolf models down to $10^2$ years in the Arnett models.

Question: Wouldn't the core temperature be a few times $10^8 \, ^\circ$K?

Colgate: You are thinking of a core that is entirely supported by degeneracy pressure. That assumes that the mass is cut off at about 1.4 solar masses, using the Finzi and Wolf model. Arnett has discussed models up to 2.5 solar masses, that arrive in this state with a helium-burning shell – of course, the central temperature has to be higher in order to support the pressure. In other words, the star evolves along a high temperature track.