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REDUNDANCY MANAGEMENT OF ELECTROHYDRAULIC SERVOACTUATORS BY MATHEMATICAL MODEL REFERENCING

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Redundancy Management of Electrohydraulic Servoactuators by Mathematical Model Referencing

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A description of a mathematical model reference system is presented which provides redundancy management for an electrohydraulic servoactuator. The mathematical model includes a compensation network that calculates reference parameter perturbations induced by external disturbance forces. This is accomplished by using the measured pressure differential data taken from the physical system. This technique was experimentally verified by tests performed using the H-1 engine thrust vector control system for Saturn IB. The results of these tests are included in this report. It was concluded that this technique improves the tracking accuracy of the model reference system to the extent that redundancy management of electrohydraulic servosystems may be performed using this method.

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Redundancy management
Electrohydraulic servoactuators
Thrust vector control
Aerodynamic surface control

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INTRODUCTION

As the size and complexity of space vehicles and aircraft are increasing, more and more emphasis is being placed on safety. This emphasis is being reflected in the design of electrohydraulic control systems by the use of both highly reliable components and redundant systems.

Redundancy in electrohydraulic control systems is accomplished by either the force-sharing or the detection-correction method. Force-sharing or, more precisely, force-summing is accomplished by the simultaneous use of parallel control units. The summation of control forces may occur either within the actuator or directly on the load. The simplicity and directness of this method accounts for its prevalence today. The detection-correction method, unlike force-sharing, operates with series elements. Redundancy is developed by removing any failed component from the system and replacing it with a standby. A system malfunction is detected by comparing pertinent parameters of the operational system with those of a reference system. The reference system's parameters may be generated by various methods, one of which is the use of a mathematical model of the system. The operational system is assumed to be operating correctly when the differences between the compared parameters remain below some set limit.

REDUNDANCY BY MATHEMATICAL MODELING

An example of a fail/operate redundant electrohydraulic servosystem which employs the mathematical modeling technique is shown in Figure 1.

The actuator's load position command $\beta^c$ is applied simultaneously to both the computerized model and the physical system. The reference parameter used in this example is the load's position $\beta^1$; other parameters that could have been used are the actuator's position or the servovalve's spool position.
When the difference between the measured load position \( \beta_{\text{Im}}(t) \) and the calculated load position \( \beta_{\text{lc}}(t) \) exceeds the allowable error \( \pm \delta_0 \) set in the comparator, the system is assumed to have sustained a failure. The output signal from the comparator then energizes the necessary corrective device such as a solenoid, which removes the failed component and replaces it with a standby.

The magnitude of an allowable error is determined from the dynamic performance tolerance placed on that particular control system. The computational electronics that comprise the error detection system must be capable of providing transient data of the reference parameter with sufficient accuracy to enable detection of a failure when the performance tolerances of that system are exceeded. For low-performance systems, a simple RC lag circuit may be sufficient, whereas more sophisticated systems may require models consisting of several operational amplifiers.
The method just described works very well for systems that do not encounter external disturbance forces. However, for systems that do, such as air-surface controllers and thrust-vector controllers, the above method can produce the erroneous conclusion that a component has failed since the tracking error may exceed the set threshold limits of the comparator during the time the external force is applied.

LOAD COMPENSATION

One method of correcting this problem would be to increase the threshold limits placed on the comparator. This, however, would decrease the sensitivity of the system to detect malfunctions as well as increase the time required for correction. Another means is to modify the calculated reference parameter so that it reflects the perturbations of the physical system. Theoretically, this can be accomplished if one knows the magnitude and direction of the disturbance force. This information can be obtained from the pressure differential measurement made across the actuator's piston.

The measured pressure differential has two components: that which results from command signals and that which is produced by an external disturbance force:

$$\Delta P_{\text{measured}} = \Delta P_{\text{command}} - \Delta P_{\text{disturbance}}$$  \hspace{1cm} (1)

Since the mathematical model can easily be programmed to yield the pressure differential developed by the command, the $\Delta P$ induced into the system by the disturbance force can be obtained by subtracting $\Delta P_{\text{command}}$ from $\Delta P_{\text{measured}}$:

$$\Delta P_{\text{disturbance}} = \Delta P_{\text{measured}} - \Delta P_{\text{command}}$$  \hspace{1cm} (2)

Knowing the dynamics of the control system, one can derive the transfer function which relates the disturbance force to the induced pressure differential in the actuator. This can be written in a general form as

$$\frac{\Delta P_{\text{disturbance}}}{F_{\text{disturbance}}} = \frac{A(S)}{B(S)}$$  \hspace{1cm} (3)

A second transfer function that relates the load's position perturbations caused by external disturbance forces must also be derived. It can be written as
From these two transfer functions, one can obtain the compensation network, \( K(S) \), which, when multiplied by \( \Delta P_{\text{disturbance}} \), will generate the load's position perturbations. This network is obtained by dividing equation (4) by equation (3), yielding

\[
\frac{\beta_{ld}}{\Delta P_{\text{disturbance}}} = \frac{D(S)}{A(S)} = K(S) \tag{5a}
\]

or

\[
\beta_{ld} = K(S) \Delta P_{\text{disturbance}} \tag{5b}
\]

where \( \Delta P_{\text{disturbance}} \) was calculated as indicated by equation (2). Adding these perturbations to the calculated load position, one obtains a new and more accurate value of the load position \( \beta_l \), which can now be used as the new reference parameter.

\[
\beta_l(t) = \beta_{lc}(t) + \beta_{ld}(t) \tag{6}
\]

A schematic of a fail/operate system using this external force compensation technique is shown in Figure 2.

**CONCEPT VERIFICATION TESTING**

A test program was devised to verify the disturbance load compensation concept and to establish a criterion for the magnitude of tracking accuracy which the mathematical model could produce when measured with respect to a physical system. A Saturn IB H-1 engine thrust vector control system was used for these tests. An illustration of the test configuration and the location where test data were recorded are shown in Figure 3. The measured parameters were:

- \( \beta_{am} \) - Position of the servoactuator piston rod, measured with respect to the servoactuator housing.
Figure 2. Fail/operate electrohydraulic servosystem with external load compensation.

$\beta_{em}$ - Position of the engine measured with respect to the frame of the engine simulator.

$\Delta P_m$ - Pressure differential measured across the servoactuator piston.

A block diagram of the mathematical model as it was programmed on the analog computer is shown in Figure 4. The assumptions and derivations of equations used to generate Figure 4 are presented in the Appendix. The symbols used in the block diagram are listed in this report in the Definition of Symbols.
Figure 3. System hardware simulation.

Figure 4. H-1 engine thrust vector control system.
The first test was performed to determine the computational accuracy of the mathematical model for both engine position commands and external disturbance forces. This was accomplished by developing tracking errors for various system parameters. These error signals were generated by subtracting the computed value from its measured counterpart on the physical system. Tracking errors were generated for the following system parameters: engine position $\beta_e$, actuator position $\beta_a$, and actuator pressure differential $\Delta P$.

These errors can be expressed mathematically as:

$$\delta \beta_e = \beta_{em} - \beta_{ec} \quad (7)$$

$$\delta \beta_a = \beta_{am} - \beta_{ac} \quad (8)$$

$$\delta \Delta P = \Delta P_m - \Delta P_c \quad (9)$$

The results of the test for 1/2 deg and 1 deg commands are shown in Figures 5 and 6, respectively. The maximum recorded errors were

<table>
<thead>
<tr>
<th>Command (deg)</th>
<th>$\delta \beta_a$ (deg)</th>
<th>$\delta \beta_e$ (deg)</th>
<th>$\delta \Delta P$ (N/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1/2</td>
<td>0.08</td>
<td>0.1</td>
<td>19.30</td>
</tr>
<tr>
<td>Step 1</td>
<td>0.16</td>
<td>0.2</td>
<td>35.84</td>
</tr>
</tbody>
</table>

Next, step disturbance loads of $\pm 8896$ N ($\pm 2000$ lb) were applied to the engine, while the servoactuator was being commanded to hold zero position. The data recorded during this test are shown in Figure 7. The following maximum tracking errors were measured:

Disturbance Loads | $\delta \beta_a$ | $\delta \beta_e$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Step $\pm 8896$ N ($\pm 2000$ lb)</td>
<td>0.35 deg</td>
<td>0.46 deg</td>
</tr>
</tbody>
</table>

The tracking errors determined during these tests will serve as a baseline for comparison in the evaluation of the load compensation technique.
Figure 5. One-half-degree step command with uncompensated reference system.
Figure 6. One-degree step command with uncompensated reference system.
COMPENSATION NETWORK TESTS

The mathematical model was then changed to include compensation networks for both the engine and actuator position reference parameters. The transfer functions that relate each of the chosen reference parameters to the disturbance force are given as

\[
\frac{\beta_a(S)}{F_{\text{disturbance}}(S)} = \frac{A^2\tau_f (K_s - K_{eq}) S^2 + \left[ K_{eq} K_{eq} + A^2(K_s - K_{eq}) \right] S}{D(S)}
\]

and

\[
\frac{\beta_e(S)}{F_{\text{disturbance}}(S)} = \frac{A^2K_s \tau_s S^2 + (K_s K_{eq} + K_{eq} K_{eq} + A^2 K_{eq} S + K_{eq} K_{eq}) S + K_{eq} K_{eq} A}{D(S)}
\]

where the denominator \( D(S) \) is the system characteristic equation which is given by

\[
D(S) = MA^2K_s \tau_s S^4 + \left[ A^2K_s (M + D\tau_f) + A K_{eq} K_{eq} M \tau_f + M K_{eq} K_{eq} \right] S^3
\]

\[
+ \left[ A^2K_s D + A K_{eq} K_{eq} (M + D\tau_f) + D K_{eq} K_{eq} + A^2 K_{eq} K_{eq} S \right] S^2
\]

\[
+ \left[ A K_{eq} K_{eq} K_{eq} \tau_f + A K_{eq} K_{eq} D + A^2 K_{eq} K_{eq} \right] S
\]

\[
+ A K_{eq} K_{eq} K_{eq}
\]

These transfer functions were derived from the block diagram shown in Figure 4. The other transfer function required for calculating the compensation network is

\[
\frac{\Delta P_m(S)}{F_{\text{disturbance}}(S)} = \frac{-K_{eq} K_{eq} \left[ A \tau_f S^2 + (K_v K_{fb} \tau_f + A) S + K_{eq} K_{eq} \right]}{D(S)}
\]

which was also derived from Figure 4.
Since two reference parameters were being generated for this test, two load compensation networks were necessary. The network required to yield the engine position perturbations is obtained by dividing equation (11) by equation (13), yielding

\[
K_e(s) = \frac{A^2 K \tau_f S^2 + \left( K K S \text{eq} c + K K S \text{eq} v \tau_f + A^2 K \right) S + K K A}{K S \left[ A \tau_f S^2 + \left( K K \tau_f + A \right) S + K \right]}.
\]

The compensation network for actuator perturbation is obtained by dividing equation (10) by equation (13), yielding

\[
K_a(s) = \frac{A^2 \tau_f \left( K_s - K \text{eq} \right) S^2 + \left[ K K S \text{eq} c + A^2 \left( K_s - K \text{eq} \right) \right] S}{K \text{eq} S \left[ A \tau_f S^2 + \left( K K \tau_f + A \right) S + K K \right]}.
\]

A block diagram of the mathematical model that was programmed on the AD-64 analog computer and used for the test is shown in Figure 8. This test, like the first, was divided into two parts. The first was performed to establish the tracking errors when no disturbance forces were present. The measured tracking errors for a 1-deg step command were

<table>
<thead>
<tr>
<th>Command</th>
<th>( \delta \overline{\beta_a} )</th>
<th>( \delta \overline{\beta_e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1 deg</td>
<td>0.17 deg</td>
<td>0.29 deg</td>
</tr>
</tbody>
</table>

as shown on the recorded test data in Figure 9.

Both of these errors are greater than those recorded when using the uncompensated model. This is caused by a difference which exists between the measured pressure differential and the calculated pressure differential, even though no external loads were present. Thus, the tracking error in the \( \Delta P \) parameter (shown on trace 8 of Figure 9) makes it appear to the model as if a disturbance load were present and thus produces an additional displacement in both actuator and engine positions.

For the second half of the test, a disturbance force was again applied, and the tracking errors were measured. The maximum recorded errors were
Figure 8. H-1 engine thrust vector control system including the external force compensation.

![Diagram](image)

**Disturbance Force**

<table>
<thead>
<tr>
<th>( \delta \beta_a )</th>
<th>( \delta \beta_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 8896 N (2000 lb)</td>
<td>0.10 deg</td>
</tr>
</tbody>
</table>

as shown in Figure 10. The magnitude of these errors indicates that a substantial improvement in the system tracking accuracy can be obtained with the addition of the load compensation network. The actuator tracking accuracy was improved by a factor of 3.5.

**TRACKING ERROR SENSITIVITY**

A point of academic interest is the sensitivity of the tracking errors to variations in system parameters. To illustrate these effects, the damping ratio and natural frequency will be varied on a simple second-order system. Let the transfer function for the physical system be given as

\[
\frac{X_1(S)}{X_c(S)} = \frac{\omega_1^2}{S^2 + 2\zeta_1\omega_1 S + \omega_1^2}, \quad (16)
\]

where the subscript 1 denotes a physical system parameter.
Figure 9. One-degree step command with compensated reference system.
Figure 10. 8896-N step disturbance force with compensated reference system.
The transfer function used in the mathematical model is

\[
\frac{X_p(S)}{X_c(S)} = \frac{\omega_p^2}{S^2 + 2\zeta_1 \omega_1 S + \omega_1^2}.
\]

(17)

The mathematical model parameters, denoted by the subscript 2, are considered constant and programmed into the model at their nominal system values. The tracking error \(\Delta x\) is then generated as shown in Figure 11.

![Figure 11. Tracking error sensitivity model reference system.](image)

A general expression describing the error resulting from a unit step command is

\[
\Delta x(S) = \left( \frac{\omega_1^2}{S^2 + 2\zeta_1 \omega_1 S + \omega_1^2} - \frac{\omega_2^2}{S^2 + 2\zeta_1 \omega_2 S + \omega_2^2} \right) \frac{1}{S},
\]

(18)

or, written in the time domain,

\[
\Delta x(t) = e^{-\zeta_1 \omega_1 t} \sin \left( \omega_1 \sqrt{1 - \zeta_1^2} t - \psi \right) - e^{-\zeta_2 \omega_2 t} \sin \left( \omega_2 \sqrt{1 - \zeta_2^2} t - \phi \right).
\]

(19)
The phase angles $\psi$ and $\phi$ are

$$\psi = \tan^{-1} \frac{\sqrt{1 - \xi_1^2}}{-\xi_1}$$

$$\phi = \tan^{-1} \frac{\sqrt{1 - \xi_2^2}}{-\xi_2}$$

The effects of variations in $\xi_1$ and $\omega_1$ were investigated by plotting equation (19) versus time. The first set of curves, shown in Figure 12, illustrates the effects of varying the system damping ratio ($\xi_1$) while holding all other parameters constant.

$$\omega_1 = \omega_2 = 6.00 \text{ Hz}$$

$$\xi_2 = 0.707$$

A second set of curves, shown in Figure 13, was plotted for variations in the system's natural frequency ($\omega_1$). Again, all the other parameters were held constant.

$$\omega_2 = 6.00 \text{ Hz}$$

$$\xi_1 = \xi_2 = 0.707$$

The most obvious difference between the two sets of curves is the time variation associated with the maximum tracking error when the system's natural frequency is the variable.

A better perspective of the effects of these parameter variations can be seen when one plots the data shown in Figures 12 and 13 in the form of maximum tracking error versus percentage of change in parameter. Where the percentage in parameter is given by

$$\text{percentage change} = \frac{\text{system value} - \text{model value}}{\text{system value}} \times 100$$

(20)

the results are shown in Figure 14. This curve clearly illustrates that variations in the system's natural frequency will produce greater tracking errors than the same variations in the system damping ratio.
Figure 12. Normalized tracking error vs time for second-order system with $\zeta_2$ as the variable parameter.
Figure 13. Normalized tracking error vs time for second-order system with \( \omega_2 \) as the variable parameter.
CONCLUSIONS

Two conclusions were formed as a result of this study. First, the inclusion of the external load compensation into the mathematical model improves the reference parameter's tracking accuracy during the presence of
external disturbance to the extent that mathematical modeling is a method of implementing redundancy management for electrohydraulic servoactuators. This technique is being incorporated into a device called the digital interface unit (DIU). The DIU will serve as the junction between a four-line data bus and the servoactuators. In addition to calculating the reference parameter and the tracking error, the DIU will provide the failure detection and correction logic necessary for a fail/operate-fail/operate redundant system. The DIU also converts the actuator position command from a digital to an analog signal and performs a preflight system checkout and inflight status monitoring.

The second conclusion pertains to the choice of a reference parameter. This study has shown the actuator's position to be a better reference parameter than the engine position, which is illustrated by the magnitude of the tracking errors as shown in Figure 9. These differences are the result of using a simplified mathematical model. One can hypothesize that the best reference parameter would be the first measurable state variable; that is, the one which corresponds to the highest order derivative in the system. With a simplified mathematical model, the nonlinearities and higher-order effects which occur after the first measurable parameter produce successively larger tracking errors in the parameters that follow.
APPENDIX

DERIVATION OF SYSTEM EQUATIONS

The equations that are represented in block diagram form in Figure 1 were derived based on the following assumptions:

1. Second-order and nonlinear effects in the servovalve were neglected; the servovalve is represented by a constant $K_v$, having the units cm$^3$/s/deg.

2. The geometric relationship that describes the linear motion of the actuator as a function of the rotational motion of the engine is

$$X(t) = \theta(t).$$

The ideal actuator position, $\beta_i$, can be calculated as

$$\beta_i = \int_0^t \frac{Q(t)}{A} dt,$$  \hspace{1cm} (A-2)

where $Q$ (cm$^3$/s) is servovalve flow rate and $A$ (cm$^2$) is the piston cross-sectional area.

However, to obtain the actual actuator position, $\beta_a$, one must include the effects of the oil compressibility. With the aid of Figure 3 this relationship can be shown to be

$$\beta_a = \beta_i + \frac{K_s}{K_o} (\beta_e - \beta_a).$$  \hspace{1cm} (A-3)

By lumping the structural compliance, $K_s$, with the oil compliance, $K_o$, an equivalent spring rate, $K_{eq}$, can be obtained, where

$$K_{eq} = \frac{K_o K_s}{K_o + K_s},$$  \hspace{1cm} (A-4)
Thus, equation (A-3) can be written as

\[ \beta_a = (\beta_i - \beta_e) \frac{K_{eq}}{K_s} + \beta_e \]  

(A-5)

The last equation required to describe the system is Euler's equation of motion for the engine. This can be written as

\[ K_{eq} (\beta_i - \beta_e) - F_{\text{disturbance}} - D \frac{d\beta}{dt} = M \frac{d^2\beta}{dt^2} \]  

(A-6)

where \( F_{\text{disturbance}} \) represents the external disturbance force acting on the engine and \( D \) is the engine viscous damping coefficient.

A lead-lag pressure feedback compensation network was incorporated into the servovalve to improve its stability and dynamic performance. It is represented in the block diagram as

\[ \frac{K S}{1 + \tau_f S} \]

The actuator position was feedback with a gain, \( K_{fb} \), to form the closed-loop system.
REDUNDANCY MANAGEMENT OF ELECTROHYDRAULIC SERVOACTUATORS BY MATHEMATICAL MODEL REFERENCING

By Richard A. Campbell

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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