NUCLEAR-ELECTROMAGNETIC CASCADES FROM NUCLEI WITH $Z \geq 3$

W. V. Jones

Department of Physics and Astronomy
Louisiana State University
Baton Rouge, Louisiana 70803 USA

and

Max-Planck-Institut für Physik und Astrophysik
Institut für Extraterrestrische Physik
8046 Garching bei München, West Germany

A Monte Carlo simulation method has been developed for studying nuclear-electromagnetic cascades initiated by high energy nuclei with $Z \geq 3$ incident on heavy absorbers. The calculations are based on a cascade model which was first adjusted until it agreed with measurements made with protons at an accelerator. Modifications of the model used for protons include the incorporation of the probabilities for fragmentation of heavy nuclei into lighter nuclei, alpha particles and nucleons. Mean values and fluctuations of the equivalent numbers of particles in the cascades at various depths in an iron absorber are presented for protons, carbon, and iron nuclei over the 30 to 300 GeV/nucleon energy range.

1. Introduction. To be presented here are some of the first results from a Monte Carlo study of the nuclear-electromagnetic cascade development initiated by high energy nuclei with $Z \geq 3$ incident on heavy absorbers. The present results are for C and Fe nuclei incident on an iron absorber. The absorber, in keeping with the design of ionization spectrometers, consists of alternate layers of iron and plastic scintillator. The iron layers are each 2 radiation lengths (r.l.) thick, and the scintillators, used as the cascade sampling layers, are each 1/4 in. thick. The cross-sectional area used for the absorber is 50 x 50 cm$^2$.

The cascade model used in the calculations is an extension of a model originally devised for studying the cascade development initiated by protons in a small iron absorber ionization spectrometer, which was exposed both to cosmic rays in a series of balloon flights (Schmidt, et al. 1969) and to artificially accelerated protons (Jones, et al. 1969) from the Brookhaven Alternating Gradient Synchrotron (AGS). The model for protons was first adjusted until it agreed with the AGS measurements. Calculations were then carried out for higher incident energies and for absorbers other than iron (Jones, 1970). Later, the model was also extended to predict the cascade development initiated by alpha particles (Jones, 1971). The present work being done on the cascade development initiated by nuclei with $Z \geq 3$ has been undertaken because of the interest...
in using ionization spectrometers for studying the energy spectra of these nuclei in the primary cosmic rays.

2. Calculations. Space does not permit a detailed description of the modifications made in the original cascade model so that it applies to heavy nuclei. Some of the more fundamental concepts, as they apply to the calculations, should be noted. First of all, a nucleus having charge number Z and atomic mass number A consists of A nucleons originally traveling as a single particle. In the calculations it was assumed that each nucleon carried 1/A of the total energy of the nucleus.

A nucleus incident on an absorber was allowed to undergo its first interaction at a depth in the absorber selected randomly from an exponential distribution which had a mean value equal to the mean free path of that nucleus in the absorber. The mean free path for a particular projectile nucleus y incident on a target nucleus x was calculated from the familiar expression for the interaction cross-section, 

$$
\sigma_{y \rightarrow x} = \pi r_0^2 \left( A_y^{1/3} - A_x^{1/3} - b \right)^2,
$$

where b is the so-called overlap parameter. For the calculations presented here, the value b = -0.6 was used. Small changes observed in the overall cascade development by varying b are presently being studied and will be discussed elsewhere.

The various modes for generating nuclear fragments during the interaction of a heavy projectile nucleus with a heavy target nucleus are not yet well known. For the present calculations, the partial interaction cross-sections for heavy nuclei incident on hydrogen as calculated by Shapiro, et al. (1970) of the U.S. Naval Research Laboratory have been used to determine the probabilities for observing fragments with Z > 3. The probabilities for having no fragments with Z > 3 were extracted from the work of Cleghorn, et al. (1968). The number of alpha particles evolving from interactions of heavy nuclei were selected randomly from a Poisson distribution with mean value depending on the Z and A of the projectile nucleus as determined from emulsion measurements (Powell, et al. 1959). The numbers of protons and neutrons belonging to the projectile (i.e. other than recoil nucleons) which leave the interaction were determined from the conservation of nucleons after having first selected the type of fragment and the number of alpha particles which left a particular interaction.

It has been assumed that alpha particles and fragments with Z > 3 always left an interaction with an energy/nucleon equal to that of the primary nucleus at the instant of the interaction. However, the nucleons were considered as having had the opportunity either (i) to participate in the interaction by exciting the target nucleus and producing secondary particles (pions), thereby losing part of their energy in the interaction or (ii) to leave the interaction with energy equal to the energy/nucleon of the primary nucleus, i.e. not to participate in the interaction.
Each secondary heavy fragment was treated in the same manner as the incident heavy nucleus (generating additional heavy fragments, alpha particles, and nucleons). The alpha particles were allowed to initiate nuclear-electromagnetic cascades in the manner described by Jones (10/1). The nucleons were permitted to undergo successive nuclear interactions, thereby producing independent nuclear-electromagnetic cascades. In other words, the cascade development initiated by an incident heavy nucleus amounts to the superposition of the independent cascades being initiated at various depths in the absorber by the nucleons comprising the original incident nucleus.

Results and Discussion. The average number of particles in the cascades initiated by $^{12}$C and $^{56}$Fe nuclei incident on the top of the iron absorber (vertically, at the center) are shown in Fig. 1. The results are given for the number of particles per nucleon of the incident nucleus for 30, 100 and 300 GeV/nucleon total energy. For comparison, the cascade curves for primary protons are also given. As one might expect, the number of cascade particles per nucleon of the primary nucleus seems to be only weakly dependent on the atomic mass number of the nucleus. It appears that the cascade curve builds up and decays somewhat more slowly for Fe than for C, but the differences are certainly less pronounced than differences in the cascades for C and protons. The explanation for this feature is that the interaction length per nucleon is somewhat longer for heavier nuclei.

Calculations are presently being carried out to study the effects of changes in the fragmentation parameters on the overall cascade development. The two extreme cases are, of course, (i) when a heavy nucleus loses only one nucleon in each consecutive interaction and (ii) when a nucleus is completely fragmented into its constituent nucleons in the first interaction. The most probable cascade development will be somewhere between these two extremes and should not be too different from the calculated results presented here.

The vertical error bars shown in Fig. 1 should provide the reader with an idea of the fluctuations to be expected in the cascades at various stages in their development. Space does not permit a detailed presentation of these fluctuations. However, the information given in Table I for the means and standard deviations of the sums of cascade particles per nucleon of the primary nucleus can be used to estimate the accuracy with which the cascade development can be used to make primary energy.

+ Here the term "n particles" is used to represent n times the ionization deposited in a scintillator by a 4 GeV muon traversing (vertically) the scintillator. The energy loss rate of a 4 GeV muon is 17.4% greater than that of minimum-ionizing muons. In other words, to obtain the correct multiple of the minimum-ionizing muon signal, one must multiply the given particle numbers by the factor 1.174.
Fig. 1. Mean numbers of cascade particles per nucleon of the primary nucleus as a function of absorber depth for 10, 30, and 300 GeV/Nucl protons (H), carbon nuclei (C), and iron nuclei (Fe) incident on an iron absorber. The top abscissa indicates the absorber depth in units of the proton interaction length (λ) while the bottom abscissa indicates this depth in radiation lengths (r.l.).
TABLE I

Means and Standard Deviations (in parentheses) for the sums of cascade particles per nucleon of primary nucleus. Given as functions of Absorber Depth in Interaction Lengths ($\lambda$) and Primary Energy $E_0$ in GeV. Particle Sampling is each $2 \, \lambda \cdot 1$. The statistics calculated are given in the second column.

<table>
<thead>
<tr>
<th>$E_0$</th>
<th>NUCLEUS</th>
<th>2$\lambda$</th>
<th>4$\lambda$</th>
<th>6$\lambda$</th>
<th>8$\lambda$</th>
<th>10$\lambda$</th>
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<tr>
<td>750</td>
<td>H</td>
<td>221(131)</td>
<td>360(98)</td>
<td>397(76)</td>
<td>405(67)</td>
<td>407(66)</td>
</tr>
<tr>
<td>30</td>
<td>C</td>
<td>206(65)</td>
<td>348(43)</td>
<td>394(25)</td>
<td>405(22)</td>
<td>408(21)</td>
</tr>
<tr>
<td></td>
<td>Fe</td>
<td>204(41)</td>
<td>352(25)</td>
<td>399(11)</td>
<td>410(12)</td>
<td>412(13)</td>
</tr>
<tr>
<td>250</td>
<td>H</td>
<td>679(401)</td>
<td>1224(332)</td>
<td>1411(213)</td>
<td>1467(151)</td>
<td>1481(142)</td>
</tr>
<tr>
<td>100</td>
<td>C</td>
<td>805(243)</td>
<td>1175(175)</td>
<td>1400(85)</td>
<td>1465(48)</td>
<td>1481(40)</td>
</tr>
<tr>
<td></td>
<td>Fe</td>
<td>569(169)</td>
<td>1162(108)</td>
<td>1399(47)</td>
<td>1467(22)</td>
<td>1482(17)</td>
</tr>
<tr>
<td>150</td>
<td>H</td>
<td>1710(1153)</td>
<td>3630(929)</td>
<td>4355(583)</td>
<td>4582(349)</td>
<td>4636(304)</td>
</tr>
<tr>
<td>300</td>
<td>C</td>
<td>1495(711)</td>
<td>3425(482)</td>
<td>4276(279)</td>
<td>4586(143)</td>
<td>4684(84)</td>
</tr>
<tr>
<td></td>
<td>Fe</td>
<td>1379(609)</td>
<td>3322(451)</td>
<td>4237(201)</td>
<td>4563(60)</td>
<td>4655(35)</td>
</tr>
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</table>

Fig. 2. Distributions for the sums of cascade particles per nucleon (abscissa) for 100 GeV/Neucl carbon nuclei incident on an iron absorber. The distributions are for 3, 6, and 10 proton interaction lengths ($\lambda$) total absorber depth with the cascades being sampled each 2 radiation lengths ($\lambda \cdot 1$).

Measurements in an iron spectrometer having sampling layers spaced every $2 \, \lambda \cdot 1$. Figure 2 shows the types of distributions one can expect for the sums of particles for incident 100 GeV C nuclei for three different absorber depths.
It should be pointed out that the calculations presented herein are for the actual cascade particle numbers, i.e., no attempt has been made to take into account the inefficiency of the scintillators in recording the ionization of heavy Z particles. If one were to consider this inefficiency, the cascade curves for C and Fe would be separated more in the early development, where a substantial number of heavy fragments contributes to the average cascade development.

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References.


