COMPUTER PROGRAM FOR CALCULATING SUPERSONIC FLOW ON THE WINDWARD SIDE OF CONICAL DELTA WINGS BY THE METHOD OF LINES

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This paper is a user's manual for a program which calculates the supersonic flow on the windward side of conical delta wings with shock attached at the sharp leading edge by the method of lines. The program also has a limited capability for computing the flow about circular and elliptic cones at incidence. It provides information including the shock shape, flow field, isentropic surface-flow properties, and force coefficients. A description of the program operation, a sample computation, and a FORTRAN IV program listing are included.
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ON THE WINDWARD SIDE OF CONICAL DELTA WINGS
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SUMMARY

This paper is a user's manual for a program which calculates the supersonic flow on the windward side of conical delta wings with shock attached at the sharp leading edge by the method of lines. The program also has a limited capability for computing the flow about circular and elliptic cones at incidence. It provides information including the shock shape, flow field, isentropic surface-flow properties, and force coefficients. A description of the program operation, a sample computation, and a FORTRAN IV program listing are included.

INTRODUCTION

In reference 1 the so-called method of lines was developed for obtaining numerical solutions of general supersonic conical flow problems, that is, those gas-dynamic problems in which the fluid properties do not vary along rays emanating from a common point in the flow. The method was applied to circular and elliptic cones and to the windward side of conical delta wings with shock attached at the sharp leading edges.

This paper describes a computer program which uses the method of reference 1 and which is primarily designed for the delta-wing problems just mentioned. The program provides information including the shock shape, flow field, isentropic surface-flow properties, and force coefficients. The program also has a limited capability for the calculation of flow about the circular and elliptic cones, but a more efficient version is available for those problems and is described in reference 2. Some remarks concerning the operation of this program for the circular and elliptic cones are relegated to appendix A.

The present program has a built-in capability for three different conical-wing cross sections: flat plate, circular arc, and modified wedge. The modified wedge is a wedge cross section with the center ridgeline rounded by a circular arc of user-specified radius; a sharp ridgeline is approximated by a very small radius. Other cross sections can be included in the program without much difficulty.
The mathematical method and coordinates are described very briefly, and details of the operation of the program are given herein. A listing of the computer program is given in appendix B together with a list of the subroutines and a flow chart. Appendix C presents a sample computation.

METHOD OF LINES

The method of lines was developed in complete detail in reference 1; hence, only a brief description is presented herein. The flow equations are initially written in a body-oriented, orthogonal, conical coordinate system \((r, \eta, \tau)\), as shown from below and behind the wing in sketch (a), where \(r\) is the distance along a conical ray, \(\eta\) is the angle measured from the body surface to the ray in a plane where \(\tau\) is constant, and \(\tau\) is a measure of the arc length along the intersection of the body surface with a sphere of radius \(r\) centered at the body apex. Specifically, \(\tau\) is determined by numerical integration on the unit sphere. The free-stream velocity vector \(V_\infty\) lies in the \(YZ\)-plane of symmetry, and the origin of the arc length \(\tau\) is taken in the windward plane of symmetry. All computations are made in the right half-plane. The integration of the system of equations is facilitated by a coordinate transformation which maps the region bounded by the shock and the body into a rectangular domain. The transformed variables are

\[
\zeta = \frac{\eta}{\eta_s} \quad \text{and} \quad \xi = \tau
\]
where $\eta = \eta_\delta(\tau)$ is the shock surface. Thus, $\zeta = 0$ on the body, and $\zeta = 1$ on the shock.

The $\xi,\zeta$-plane is divided by $N + 1$ lines parallel to the $\zeta$-axis; the line $I = 1$ is taken in the windward plane of symmetry, and the line $N + 1$ is at the wing leading edge, where the shock is assumed to be attached. The layout of the computational lines for $N = 6$ is shown in sketch (b). At each line the system of differential equations is reduced to a set of ordinary differential-difference equations by replacing the derivatives $\partial/\partial \xi$ by finite differences. The derivative of the Lagrange interpolation polynomial is used in this program with an equal number of lines on either side of the line at which $\partial/\partial \xi$ is computed; therefore, central differencing is obtained when the line spacing is equal. The initial values for the system of equations are determined from the shock relations once the shock shape is specified, and the equations are integrated numerically from the shock to the body. The system of equations is integrated simultaneously along each line $I = 1, \ldots, N$. The differential equations along any line are coupled to those along the other lines through the finite-difference approximations to the cross derivatives $\partial/\partial \xi$. The equations are integrated by a fourth-order Runge-Kutta method. The accuracy of this method allows the use of relatively large integration steps. The integration step size can generally be taken in increments of -0.1 from the shock to a value of $\zeta$ of 0.1 and in increments of -0.05 and -0.025 thereafter. An iterative process based on the Newton method is utilized to adjust the shock shape to satisfy the conditions of flow tangency on the body.

![Physical (\(\tau, \eta\)) plane](image1)

![Transformed (\(\xi, \zeta\)) plane](image2)

Sketch (b)
CONICAL-WING GEOMETRY

Three wing shapes – flat plate (M = 3), circular arc (M = 4), and modified wedge (M = 5) – are built into the program and are selected with the use of the body-selection trigger M. It is possible to substitute other wing cross-section shapes with little change in the program provided the wing shape is defined by a function with continuous first and second derivatives. For such wing shapes, the body geometry can be substituted into the logic for the circular-arc wing, and the computation for the new wing shape is found with the body-selection trigger M equal to 4. The changes required for wing shapes with discontinuities in curvature, such as the modified wedge cross section (M = 5), are more involved. No description of such changes is given herein, since each shape with curvature discontinuities must be considered individually to insure proper numerical integration for the body geometry.

The compression surface of the conical wing is defined by an equation of the form

\[ G(x_o, y_o) = 0 \]

where \( x_o \) and \( y_o \) are the rectangular coordinates of the wing and are related to the Cartesian coordinates \( X, Y, Z \) by

\[ x_o = X/Z \quad \text{and} \quad y_o = Y/Z \]

The coordinate \( X \) is in the spanwise direction, \( Z \) is along the axis of the conical wing, and the \( YZ \)-plane is the wing plane of symmetry. The windward side of the wing is in the lower half-plane; hence, it should be noted that \( Y \) and \( y_o \) have negative values there.

The functions required are \( G_x, G_{xx}, G_y, G_{yy}, \) and \( G_{xy} \), where the subscripts denote derivatives with respect to the indicated argument. These functions, with their appropriate FORTRAN name, which replace those for the circular-arc wing, are as follows:

- \( G_x \) (\( GX(I) \)), \( G_{xx} \) (\( GXX(I) \)), and \( G_y \) (\( GY(I) \)) in cards B3010, B3020, and B3030 of subroutine \( BG \), respectively.
- \( G_{yy} \) (\( GYY(I) \)) and \( G_{xy} \) (\( GXY(I) \)) in cards B3080 and B3090 of subroutine \( BG \), respectively.
- \( y_o \) (\( YOO \)), \( G_x \) (\( GXI \)), and \( G_y \) (\( GYI \)) in cards D500, D510, and D520, of subroutine \( DERIV \), respectively. At this stage \( x_o \) is the quantity \( F(1) \).
$G_x$ (GXI) and $G_y$ (GYI) in cards D910 and D920 of subroutine DERIV, respectively. At this stage $x_o$ is $F(2)$ and $y_o$ is $F(3)$.

**APPROXIMATE SHOCK SHAPE FOR CONICAL WINGS**

The approximate built-in shock shape for the conical delta wings is generally satisfactory for starting the computations. This shock shape is an even function of $\xi$ which gives both $\eta_S = 0$ and the correct value of $d\eta_S/d\xi$ at the leading edge of the wing.

The function is

$$\eta_{s,i} = \left[1 - \left(\frac{\xi_i}{\xi_{N+1}}\right)^2\right]^{2}\left[\eta_{s,1} - \left(\frac{\xi_i}{\xi_{N+1}}\right)^2\eta_{s,1} + \frac{\xi_{N+1}}{2}\left(\frac{d\eta_S}{d\xi}\right)_{N+1}\right]$$

The value of $\eta_{s,1}$ used in the program is a tangent-cone approximation increased by a factor of 1.2 to avoid the Mach wave conditions for very thin wings at small incidence. In a few computations, the approximate starting value of $\eta_{s,1}$ was so inaccurate that the required corrections $\Delta\eta_{s,i}$ were sizable; thus, a substantial "roughness" in the shock shape resulted and subsequent iterations failed. Even so, the first correction for $\eta_{s,1}$ has been quite good, so that using that value in the approximate shock shape and restarting the program have always been successful. The value of $\eta_{s,1}$, ETAS(1) in FORTRAN notation, is set in cards E710 and E720 of subroutine APPROX.

**PROGRAM OPERATION**

In general, the program operation for the conical-wing computations proceeds with little difficulty and can be made by starting from the built-in approximate shock shape. Instabilities can arise in some cases, and a change in some of the parameters involved will sometimes yield converged solutions. A number of parameters can be adjusted which can affect the convergence and computing time. Some discussion of these parameters is given subsequently with general recommendations for their values.

**Input Description**

The following list contains the program input variables, which are arranged according to order of presentation in the program.
<table>
<thead>
<tr>
<th>Input card no.</th>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STMACH</td>
<td>Free-stream Mach number, $M_{\infty}$</td>
</tr>
<tr>
<td>2</td>
<td>GAMMA</td>
<td>Ratio of specific heats, $\gamma$</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td>Number of lines. Line 1 is in the windward symmetry plane. For conical wings, line $N + 1$ is at the leading edge. For elliptic cones, line $N$ is in the leeward symmetry plane. To obtain force coefficients, $N$ must be an even number for the conical wings and an odd number for the elliptic cones.</td>
</tr>
<tr>
<td>M</td>
<td></td>
<td>Body-selection trigger</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 0$ for circular cone at zero incidence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 2$ for circular and elliptic cones</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 3$ for windward side of flat-plate wing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 4$ for windward side of circular-arc wing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 5$ for windward side of modified-wedge wing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M = 1$ and $6$ are not operational</td>
</tr>
<tr>
<td>NCASES</td>
<td></td>
<td>Number of cases to be computed in a natural sequence where the converged $\eta_{s,1}$ values for one case are used as the starting values for the next case. NCASES is equal to or greater than one.</td>
</tr>
<tr>
<td>NREAD</td>
<td></td>
<td>Trigger for reading in shock shape</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$NREAD = 0$ for built-in approximate shock shape. This option is satisfactory for elliptic cones which are nearly circular and small angles of attack.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$NREAD = 1$ when input shock shape is to be read in</td>
</tr>
<tr>
<td>Input card no.</td>
<td>FORTRAN variable</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>2</td>
<td>NSPACE</td>
<td>Line space trigger</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NSPACE = 1 for equal line spacing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NSPACE = 2 gives piecewise constant line spacing in three segments with NA lines in the first segment, (NB-NA) lines in the second segment, and (N-NB) lines in the third segment. This option is not recommended. (See input card no. 6 for further description.)</td>
</tr>
<tr>
<td></td>
<td>NPLOT</td>
<td>Trigger for punched card output used for plotting (see subsection entitled Plots)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPLOT = 0 no punched card output</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPLOT = 1 punched card output</td>
</tr>
<tr>
<td></td>
<td>NPUNCH</td>
<td>Punched card output for $\eta_{S,i}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPUNCH = 0 no punched card output</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPUNCH = 1 punched card output for each case</td>
</tr>
</tbody>
</table>

This information read in only for $M = 5$.

**PHIWD**  
Wedge angle in the plane $Z = 1$, $\phi_w$, degrees (sketch (c))

**BX**  
Parameter locating $x_0$ coordinate of junction of wedge with circular arc in the plane $Z = 1$. A sharp center ridgeline is approximated by a small value for BX, for example, $BX = 0.001$. Whenever $BX < XOBAR(2)$ (see section entitled Summary Print Block), line 1 is the only line on the curved segment. In this case, the program sets the body-curvature terms to zero in the differential equations at line 1, but not in the body-geometry equations.

![Sketch (c)](image-url)
This information read in only if $NREAD = 1$.

**ETAS(I)** Values of $\eta_{s,i}$ which are used to start the computations.

**VTESTHD** Accuracy criterion on maximum normal velocity component at body surface, in which usually a value of $10^{-3}$ suffices.

**VTEST1** Trigger for modified Newton iteration. When the maximum magnitude of the normal velocity component on the surface $V_{MAX}$ lies between $V_{TEST}$ and $V_{TEST1}$, a modified Newton method (ref. 1) is incorporated where the old Jacobian matrix of influence coefficients is used.

This information read in only if $NSPACE = 2$.

**NA** Number of lines equally spaced in first segment; must be an odd number to obtain force coefficients (sketch (d)).

**NB** Line number at end of second segment; must be an odd number to obtain force coefficients.

**ANA** For conical wings, $ANA = \frac{\Xi(NA)}{\Xi(N + 1)}$. For elliptic or circular cones, $ANA = \frac{\Xi(NA)}{\Xi(N)/2}$.

**BNB** For conical wings, $BNB = \frac{\Xi(NB)}{\Xi(N + 1)}$. For elliptic or circular cones, $BNB = \frac{\Xi(NB)}{\Xi(N)/2}$.

$\Xi$ is arc length along surface contour.

Sketch (d)
This information read in only if $M = 5$.

**THETAD**

The semiangle between the wing leading edge and the Z-axis, $\theta$, degrees

**ALPHAD**

Angle of attack, $\alpha$, degrees

This information read in only if $M \neq 5$.

**T**

For conical delta wings, $T$ is the ratio of center-line thickness to semispan.

For circular and elliptic cones, $T = b/a$, where $b$ is the semiaxis in the YZ-plane of symmetry (which contains the velocity vector) and $a$ is the semiaxis in the XZ-plane of symmetry.

**THETAD**

$\theta$, degrees

For the conical delta wings, the angle between the wing leading edge and the Z-axis (complement of sweep angle).

For the circular and elliptic cones, the cone semiangle in the XZ-plane of symmetry.

**ALPHAD**

Angle of attack, $\alpha$, degrees

Output Description

Among the first items printed are some of the input parameters (see section entitled Input Description), and in addition, some preliminary computed quantities and control parameters (quantities described in the sections entitled Auxiliary Definitions and Secondary Parameters) which are not input data but which can be readily changed within the program. A sample computation is presented in appendix C.

The following output information is printed for each pivotal (trial) shock shape: ETAS(I), ETASP(I), CP(I) (pressure coefficient) at shock, CP(I) at body, and V(I) at body for $I = 1, \ldots, N + 1$ for the conical wings and $I = 1, \ldots, N$ for the circular and elliptic cones. After the print of V(I) are the values of KCOUNT, VMAX, EPSIG, SPACER, and DETERM, followed by the corrections to the shock shape DETA(I).
After the solution has converged (that is, \( V_{MAX} < V_{TESTHD} \), where \( V_{TESTHD} \) is \( V_{TEST} \) in the sample computation), the full results are printed, starting on a new page, with the input and much of the preliminary information repeated in an orderly fashion. This arrangement allows the printing for the preliminary iterations to be separated and discarded if desired. For conical wing calculations, information relating to the wing leading-edge shock follows this printing. The following quantities, which are measured in a plane normal to the wing leading edge, are printed: \( \Delta(N+1) \), \( \beta(SONIC) \), \( \Delta(SONIC) \), \( \beta(DET) \), \( \Delta(DET) \), \( \beta(AD) \), \( \sigma(AD) \), \( VNS \), and \( AMN \). The coordinates of the tip Mach cone \( XMACH \) and \( YMACH \) are printed in the next block together with the scaled values \( XMACHB \) and \( YMACHB \) which are nondimensionalized by the wing semispan. The summary print block is then printed followed by the arc length \( X(I) \), the body coordinates \( XO(I) \) and \( YO(I) \), the shock quantities \( ETAS(I) \) and \( ETASP(I) \), and finally the zeta print blocks.

**Windward-line zeta limits.** - Following the \( ZETA = 0 \) print block, the limiting values of certain flow properties which are dependent upon the direction of approach to the nodal-point singularity, as described in reference 1, are printed. The values printed in the \( ZETA = 0 \) print block are the limits obtained by approaching the symmetry plane along the surface. The limits corresponding to an approach in the symmetry plane are tabulated in the output under the heading WINDWARD LINE ZETA LIMITS. (The entropy function \( SBAR \) is printed under the heading labeled S.)

**Force coefficients.** - After the windward-line zeta limits, the aerodynamic force and moment coefficients are printed as well as the center-of-pressure location. The quantities printed are axial force \( CZ \), normal force \( CY \), drag \( CD \), lift \( CL \), moment about X-axis \( CM \), and the coordinates of the center of pressure \( YEAR \) and \( ZBAR \). The reference area for the force and moment coefficients is the plan area.

**Plots.** - The program has a plot-option trigger, \( NPLOT \), which can be used to obtain punched card output for use in another program which, in turn, rearranges the cards in an order suitable for plotting cross-flow streamlines, cross-flow sonic lines, and cross sections of the body and shock, as well as surface and shock pressure coefficients, and surface quantities U, W, RHO, and CROSSM as functions of the nondimensional spanwise coordinate, \( \bar{x} \). This second program is not included, however, because each different computer system has its own variations in plotting routines and requirements. It is hoped that the user can adapt the punched card output for these plots to his specific requirements.

Appropriate labels identifying the inputs for the case for which the punched card output is obtained are punched in the MAIN program at cards A1310 and A1320, and the body coordinates, surface quantities, and sonic-line coordinates are punched at cards A6960 to A7070. The quantity \( NZETA \) punched at card A6950 in the MAIN program
(computed at card L750 in subroutine PRINT) is a counter of the number of integration steps from the shock and is used in the plotting program. In subroutine PRINT, $\bar{x}_s$, which is the value of $\bar{x}$ along the shock, and the shock pressure coefficient are punched at cards L540 and L550. The coordinates of the shock and cross-flow streamlines are punched at cards L800 and L810.

Auxiliary Definitions

Several quantities appear in the printout of the preliminary iterations, some of which can be useful in evaluating the sequence of iterations. Several auxiliary quantities that have not been defined elsewhere are defined in the following list:

<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAST</td>
<td>Ratio of critical speed to free-stream speed</td>
</tr>
<tr>
<td>AMN</td>
<td>Mach number based on component of stream velocity in a plane normal to wing leading edge</td>
</tr>
<tr>
<td>BETA(SONIC)</td>
<td>Shock angle for sonic velocity behind shock, degrees</td>
</tr>
<tr>
<td>BETA(DET)</td>
<td>Shock angle for shock detachment at wing leading edge, degrees</td>
</tr>
<tr>
<td>BETAD(N+1)</td>
<td>Shock angle at wing leading edge, degrees</td>
</tr>
<tr>
<td>DELTA(N+1)</td>
<td>Flow deflection across leading-edge shock measured in a plane normal to wing leading edge, degrees</td>
</tr>
<tr>
<td>DELTA(DET)</td>
<td>Flow-deflection angle across shock at detachment condition, degrees</td>
</tr>
<tr>
<td>DELTA(SONIC)</td>
<td>Flow-deflection angle across shock for sonic velocity condition, degrees</td>
</tr>
<tr>
<td>DETA</td>
<td>Correction applied to each value of $\eta_s$, $\Delta\eta$</td>
</tr>
<tr>
<td>DETERM</td>
<td>Determinant of the Jacobian, or influence coefficient, matrix (This matrix is normalized so that the sum of squares of the elements of each row is 1.)</td>
</tr>
<tr>
<td>EPSIGOM</td>
<td>Parameter for controlling step size during the numerical integration of the body geometry; used in cards C580 and C590 of subroutine RUNKUT and set in card A530 in the MAIN program</td>
</tr>
<tr>
<td>FORTRAN variable</td>
<td>Description</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>EPSIVAR</td>
<td>Perturbation parameter; changes $\eta_s$ to $(1 + \text{EPSIVAR})\eta_s$ for each perturbation integration; set in card A2690 of MAIN program and used in cards A8060, A8160, and A8210 of MAIN program</td>
</tr>
<tr>
<td>ETASP</td>
<td>Derivative of the quantity $\eta_s$ with respect to the arc length $\tau$, $\frac{d\eta_s}{d\tau}$</td>
</tr>
<tr>
<td>INTCNT</td>
<td>Number of integration steps used to determine $\xi(N)/2$ in body-geometry computation</td>
</tr>
<tr>
<td>KCOUNT</td>
<td>Total number of pivotal and variational integrations</td>
</tr>
<tr>
<td>NCYCLE</td>
<td>Number of iteration cycles. The first cycle always consists of one pivotal and $N$ variational integrations to generate the Jacobian matrix. Subsequent cycles may or may not include the $N$ perturbation integrations. No perturbation runs are made when $\text{VMAX} &lt; \text{VTEST1}$ (modified Newton computation), and the old Jacobian matrix is used to obtain the corrections $\Delta \eta$. In this case a cycle is one integration. When $\text{VMAX} &gt; \text{VTEST1}$, one cycle consists of the pivotal integration and the $N$ perturbation integrations required to generate a new Jacobian matrix.</td>
</tr>
<tr>
<td>PTINF</td>
<td>Free-stream total pressure referenced to product of free-stream density and square of free-stream velocity</td>
</tr>
<tr>
<td>RANGLE</td>
<td>Relative angle of incidence for circular and elliptic cones, $\alpha/\theta_o$, where $\theta_o$ is the cone semiapex angle in the vertical plane of symmetry and $\alpha$ is the angle of attack, degrees (not printed in delta-wing computations)</td>
</tr>
<tr>
<td>SIGMAD</td>
<td>Angle between wing and shock at leading edge, degrees</td>
</tr>
<tr>
<td>VMAX</td>
<td>Maximum magnitude of normal velocity component on body surface</td>
</tr>
<tr>
<td>VNS</td>
<td>Component of unit free-stream velocity in plane normal to wing leading edge</td>
</tr>
<tr>
<td>XINI</td>
<td>Arc length to wing leading edge on the intersection of the unit sphere with the wing</td>
</tr>
</tbody>
</table>
Secondary Parameters

A number of secondary parameters which control various computations have been built into the program. Those related to the size of the integration steps are given in the following list:

<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPSIG</td>
<td>Control parameter for triggering the extrapolation to the surface. When the ( G ) function in the denominators of the equations for the zeta derivatives becomes smaller than EPSIG, the program extrapolates to obtain values of the normal velocity component at the surface. This parameter is used in cards J960 and J980 of subroutine EQNS and set in card A2660 of the MAIN program. The normal value is ( 10^{-3} ). This parameter must be reduced in value when small integration steps are used near the body.</td>
</tr>
<tr>
<td>EPSINT, DMAX, DSMIN</td>
<td>Parameters used in variable-step integration. Variable-step integration is not recommended for either engineering computations or for computations within the entropy layer. The program uses a fixed-step mode by setting DMIN=DSMAX in cards K350 and K360 of subroutine RUNKUT2. Value of EPSINT is set in card A2700 of the MAIN program and is used in cards K1250 and K1370 of subroutine RUNKUT2.</td>
</tr>
<tr>
<td>SPACER</td>
<td>Sets the minimum distance between points to be used in the formula for extrapolation to body surface. This parameter is used in card K540 of subroutine RUNKUT2 and set in card A2670 of the MAIN program. The recommended value is SPACER=EPSIG.</td>
</tr>
</tbody>
</table>

Summary Print Block

The notation used in the summary print block is as follows:

<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td>Line number</td>
</tr>
<tr>
<td>FORTRAN variable</td>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>------------------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>PSID</td>
<td>Arc tan (y_0/x_0), degrees</td>
<td></td>
</tr>
<tr>
<td>PSISD</td>
<td>Arc tan (y_s/x_s), degrees</td>
<td></td>
</tr>
<tr>
<td>XO, YO</td>
<td>(x_o, y_o)</td>
<td>Cartesian coordinates of body in plane (Z = 1)</td>
</tr>
<tr>
<td>XOBAR, YOBAR</td>
<td>(\bar{x}_o, \bar{y}_o)</td>
<td>Cartesian coordinates of body referenced to (x_{o,\text{max}}), where (x_{o,\text{max}}) is the maximum value of (x_o)</td>
</tr>
<tr>
<td>XS, YS</td>
<td>(x_s, y_s)</td>
<td>Cartesian coordinates of shock in plane (Z = 1)</td>
</tr>
<tr>
<td>XSBAR, YSBAR</td>
<td>(\bar{x}_s, \bar{y}_s)</td>
<td>Cartesian coordinates of shock referenced to (x_{o,\text{max}})</td>
</tr>
<tr>
<td>ETAS</td>
<td>(\eta_s)</td>
<td>Value of (\eta) at shock, radians (see section entitled Zeta Print Blocks for description of (\eta))</td>
</tr>
<tr>
<td>BETAD</td>
<td>(\beta)</td>
<td>Angle between free-stream velocity vector and tangent plane to shock, degrees</td>
</tr>
<tr>
<td>XI</td>
<td>(\xi)</td>
<td>Arc length along the intersection of the body surface and the unit sphere</td>
</tr>
<tr>
<td>CPSHOCK</td>
<td>Pressure coefficient at shock wave</td>
<td></td>
</tr>
<tr>
<td>CPBODY</td>
<td>Pressure coefficient at body surface</td>
<td></td>
</tr>
</tbody>
</table>

Zeta Print Blocks

The notation used in the zeta print blocks is as follows:

<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZETA</td>
<td>Independent variable, (\eta/\eta_s); (\xi = 1) at shock and (\xi = 0) on body surface</td>
</tr>
<tr>
<td>P/ROVSQ</td>
<td>Pressure (p) referenced to product of free-stream density and square of free-stream velocity</td>
</tr>
<tr>
<td>P/ROASTSQ</td>
<td>Pressure referenced to product of free-stream density and square of critical speed</td>
</tr>
<tr>
<td>P/PTINF</td>
<td>Pressure referenced to free-stream total pressure</td>
</tr>
<tr>
<td>P/PINF</td>
<td>Pressure referenced to free-stream pressure</td>
</tr>
</tbody>
</table>

14
<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHO</td>
<td>Density $\rho$ referenced to free-stream density</td>
</tr>
<tr>
<td>U, V, W</td>
<td>$u$, $v$, $w$-components of velocity in $r$, $\eta$, $\tau$-directions, respectively, referenced to critical speed</td>
</tr>
<tr>
<td>UC, VC, WC</td>
<td>Quasi-cylindrical velocity components in $Z$-direction, and directions normal and tangential to body contour in plane $Z = 1$, respectively</td>
</tr>
<tr>
<td>VCC, WCC</td>
<td>Circular cylindrical components of velocity normal and tangential to a circle in the plane $Z = 1$, referenced to critical speed. UCC is same as UC.</td>
</tr>
<tr>
<td>VX, VY, VZ</td>
<td>Cartesian velocity components, referenced to critical speed</td>
</tr>
<tr>
<td>PSINOR</td>
<td>Arc tan ($VX/VZ$), degrees</td>
</tr>
<tr>
<td>THETNOR</td>
<td>Arc cos $\left(\sqrt{VY^2 + (VX)^2 + (VZ)^2}\right)$, degrees</td>
</tr>
<tr>
<td>XBAR, YBAR</td>
<td>Cartesian coordinates referenced to $x_{o,\text{max}}$</td>
</tr>
<tr>
<td>XBHLD, YBHLD</td>
<td>Cartesian coordinates, referenced to $x_{o,\text{max}}$, of cross-flow streamline (isentrope) that intersects the shock at line I</td>
</tr>
<tr>
<td>ETA</td>
<td>Angle measured in a plane normal to the body from ray on surface of body to ray in field, $\eta$, radians</td>
</tr>
<tr>
<td>G</td>
<td>Function that appears as a factor in the denominator of most of the equations for the $\zeta$-derivatives and which vanishes at the body ($g$ in ref. 1, p. 12)</td>
</tr>
<tr>
<td>DEQNS</td>
<td>Function which is a factor in the denominator of the equations for $\zeta$-derivatives ($D$ in ref. 1, p. 13). It vanishes when a line $\zeta = \text{Constant}$ becomes tangent to a conical characteristic. The tangency can occur only when regions of supersonic cross flow ($\text{CROSSM} \geq 1$) appear.</td>
</tr>
<tr>
<td>AM</td>
<td>Local Mach number, $\frac{\sqrt{u^2 + v^2 + w^2}}{a}$, where $a$ is the speed of sound</td>
</tr>
<tr>
<td>CROSSM</td>
<td>Cross-flow Mach number, $\frac{\sqrt{v^2 + w^2}}{a}$</td>
</tr>
<tr>
<td>FORTRAN variable</td>
<td>Description</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>SBAR</td>
<td>Entropy function, ( S = \log\left[\frac{P}{P_{INF}}\right] )</td>
</tr>
<tr>
<td>POROGAM</td>
<td>( (P/ROVSQ)/\rho^\gamma )</td>
</tr>
<tr>
<td>PT/PTINF</td>
<td>Ratio of total pressure to free-stream total pressure</td>
</tr>
<tr>
<td>PT</td>
<td>Total pressure referenced to product of free-stream density and square of free-stream velocity</td>
</tr>
<tr>
<td>BERNOUL</td>
<td>Error in Bernoulli equation, ( 1 - \left( \frac{2\gamma - 1}{\gamma - 1} P + u^2 + v^2 + w^2 \right)/\left( \gamma + 1 \right) / \gamma - 1 )</td>
</tr>
<tr>
<td>DPDZ, DUDZ, DVDZ, DWDZ, DSBDZ</td>
<td>( \frac{dp}{d\zeta}, \frac{du}{d\zeta}, \frac{dv}{d\zeta}, \frac{dw}{d\zeta}, \frac{dS}{d\zeta} ), respectively</td>
</tr>
<tr>
<td>PP, UP, VP, WP, SBARP</td>
<td>Finite-difference approximations for ( \frac{ap}{\partial \tau}, \frac{au}{\partial \tau}, \frac{av}{\partial \tau}, \frac{aw}{\partial \tau}, \frac{dS}{\partial \tau} ), respectively</td>
</tr>
<tr>
<td>XBSONIC, YBSONIC</td>
<td>Sonic-line coordinates referenced to wing semispan, ( x_0, \text{max} )</td>
</tr>
</tbody>
</table>

Accuracy Control Parameters

The accuracy of the computations improves with increasing number of lines \( N \) and with increasing number of points \( NP \) in the cross-derivative approximation formula only within certain limitations. Instabilities can arise for both the elliptic cones and the conical wings, and the instabilities are accentuated for the larger values of \( N \). The integration step size, particularly near the body surface, can also influence the accuracy of the final results. Recommended values for computations to engineering accuracy (three to four figures) are given for the principal parameters.

<table>
<thead>
<tr>
<th>FORTRAN variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td>Integration step size from shock to body; set in cards K420 to K470 of subroutine RUNKUT2. Recommended values are</td>
</tr>
<tr>
<td></td>
<td>DS = -0.1 for ( 0.1 \leq ZETA \leq 1.0 )</td>
</tr>
<tr>
<td></td>
<td>DS = -0.05 for ( 0.05 \leq ZETA \leq 0.1 )</td>
</tr>
<tr>
<td></td>
<td>DS = -0.025 for ( 0.025 \leq ZETA \leq 0.05 )</td>
</tr>
<tr>
<td></td>
<td>DS = -0.0125 for ( 0 \leq ZETA \leq 0.025 )</td>
</tr>
</tbody>
</table>
FORTRAN variable | Description
---|---
N | Number of lines
   | For conical wings, $N = 8$ to $14$.
   | For circular cones at small incidence ($\text{RANGLE} < 0.5$) or elliptic cones of moderate axis ratio ($T \geq 0.7$) and small incidence, $N = 5$ to $11$.
NP | Number of points used in computation of cross derivatives; set in cards A680 and A690 in MAIN program
   | $NP = 3$ for computations where computing time is a prime consideration.
   | $NP = 5$ for the most computations. Larger values increase the computing time substantially with little change in the overall results.
   | The accuracy with $NP = 5$ is noticeably better than with $NP = 3$.
VTESTHD | Convergence criterion on the normal velocity component at the body surface. $VTESTHD = 10^{-3}$ to $10^{-4}$ for most cases.
VTEST1 | Parameter for selection of regular or modified Newton procedure.
   | The modified Newton procedure is used after all the normal velocity components on the body are less than $VTEST1$. This value is $0.01$ to $0.05$ for most cases.

Limitations

The limitations of the method are described in detail in reference 1. For the sake of completeness, some of the important points in regard to the delta-wing calculations are repeated here; the elliptic-cone cases are discussed in references 1 and 2.

As stated earlier, this program is restricted to the supersonic flow on the windward side of conical wings without yaw and with shocks attached at the sharp leading edge. An obvious limitation in the angle-of-attack range is that $\alpha$ can not exceed the value corresponding to leading-edge shock detachment and should not be less than the value which corresponds to an expansion on the lower surface. Other limitations are in Mach number and number of lines. All delta-wing cases attempted by the authors with $M_\infty \geq 3$ have been successful. If the Mach number is too low, the shock wave may lie a considerable distance from the surface, and unacceptable error growth may occur, as described in reference 1. Similar instability arises if too many lines are used; 19 lines appear to be the practical maximum in many cases. This maximum is not a severe restriction for the delta wings, since usually the use of eight to 14 lines yields excellent accuracy.
Special System Features

The program makes use of certain features which are special to the Control Data series 6000 computer system at the Langley Research Center, and some changes may be required for other systems. The word length of this computer is 60 bits; consequently, double precision may be required on systems with 32 or 36 bit word lengths. Some seven-character variable names, which may not be acceptable to other systems, have been used in the program. The program is written in FORTRAN IV and requires a field length of 70K to compile and execute on the CDC 6000 series computer system at Langley Research Center.

Langley Research Center,
National Aeronautics and Space Administration,
APPENDIX A

REMARKS ON COMPUTATIONS FOR CIRCULAR AND ELLIPTIC CONES

As already mentioned, this program has a limited capability for computing the flow past circular and elliptic cones, but a more efficient version is available for such problems. (See ref. 2.) When the present program is used for such problems, the following remarks should be noted:

(1) Line $N$ lies in the leeward plane of symmetry. (There is no line $N + 1$ as in the wing problems.)

(2) The elliptic cone solutions can be quite sensitive to the accuracy of the shock shape; consequently, they must be developed in incremental steps. A limited provision for this type of computation is provided by the parameter NCASES, where NCASES is the total number of cases in the sequence. Its use allows the converged shock shape for one computation to be used as the initial shock shape for the next computation in the sequence.

For example, to obtain calculations for an elliptic cone with $T = 0.5$, a sequence of calculations starting with a circular cone ($T = 1.0$) and $NREAD = 0$ should be input; subsequent calculations in the sequence should have values of $T$ decreasing in small increments to $T = 0.5$. 
APPENDIX B

PROGRAM LISTING, LIST OF SUBROUTINES, AND FLOW CHART

The computational program listing is given in this appendix together with a list of the subroutines and a flow chart.

Program Listing

The computational program listing is as follows:

```fortran
PROGRAM MAIN INPUT, OUTPUT, TAPES INPUT, PUNCH
CONICAL FLOW BY THE METHOD OF LINES
COMMON FUN (20, 6), DFUN<20-6>, FUNC<20-6>, DFUNC (20, 6), B2 (20, 6)
COMMON /ERROR/ ER(18), ERR(12), BLKU2
DIMENSION IOATE(2), ETASO(20), V0(20), D(20, 20), B(20, 11), IPIVOT(20, 20)
DIMENSION DMOLD(20, 20)
DIMENSION XBSNC<50>, YBSNC<50>

... (Program continues with several hundred lines of code) ...
```

The rest of the listing continues with the program code, including subroutine declarations, flow control statements, and input/output operations.

Additional notes or comments are provided for specific sections of the code, such as

- **APPLICABLE STRENGTHS**
- **DATA**
- **NORMALIZE**
- **PLATE**
- **DECON**
- **INTERMEDIATE**
- **SIMPLIFIED**
- **TEST**
- **CALCULATION OF SIGMA**
- **DATA**
- **FURTHER**
- **SOLUTION**
- **SELECT**

The listing also includes a section titled "SELECTION OF BODY GEOMETRY" with options for various types of geometries, such as

- **ZERO-INCIDENCE CIRCULAR CONE**
- **CIRCULAR AND ELLIPTIC CONES**

These geometries are further classified with additional options for specific conditions or configurations.

---

*This is a simplified representation of the program listing. The full listing contains hundreds of lines of code and detailed comments.*
APPENDIX B

C INTEGRATION WITH XO OR YO INDEPENDENT VARIABLE - NINT=1

C INTEGRATION OVER XO - NOCHANGEP, VARIABLE STEP - NSTEP=1

C CHECK=AB/SORT1,F(1),F(2)

C CHECK=AB/SORT1,F(1),F(2)

I(K)=1200

IF (NODEBUG.EQ.1) PRINT 71,XO

IF (NODEBUG.EQ.1) PRINT 71,XO

IF (NODEBUG.EQ.1) PRINT 71,XO

CONTINUE

30 CONTINUE

30 CONTINUE

C CIRCULAR AND ELLIPTIC CONE - COMPUTATION OF XO1

C INTEGRATION OVER XO - NINT=1, VARIABLE STEP SIZE = NSTEP=1

C IF (NODEBUG.EQ.1) PRINT 71,XO

C IF (NODEBUG.EQ.1) PRINT 71,XO

C IF (NODEBUG.EQ.1) PRINT 71,XO

C IF (NODEBUG.EQ.1) PRINT 71,XO

C Go To 26

C Go To 26

C Go To 26

C Go To 26

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C Go To 26
APPENDIX B
APPENDIX B
APPENDIX B

COMMON /BLOCK6/ NEGP

IF (NDEBUG.EQ.0) GO TO 11

DO 11 I = 1,N

ANOM=-F40.0*ETA(I)

IF (I.EQ.1) GO TO 2

ANOM=-ANOM+40.0*ETA(I-1)

IF (I.EQ.N) GO TO 11

DO 10 J = 1,I-1

IF (J.EQ.1) GO TO 1

IF (J.EQ.N) GO TO 11

F40(I-1)=F40(I-1)+40.0*ETA(J)

STOP 10

GO TO 11

RETURN

C INTEGRATION OF EQUATIONS

C

COMMON /ERROR/ ER(12), EHR(12), BLK(12), THETNOR(20), PSINOR(20), PLOPT(20), PTOPT(20), PT(20), POASTSQ(20)

COMMON /BLOCK6/ NEGP

IF (NDEBUG.NE.0) GO TO 2

DO 11 I = 1,N

TEST=AX-.5.EPSINT

QS=AX(I)-QS

RETURN

C

C DMIN=0.05 MAX DSWAX GIVES A FIXED INTEGRATION STEP

C

COMMON F100(12), F200(12), F400(12), S100(12), S200(12), S400(12), S600(12), S800(12), S1000(12), S1200(12), S1400(12), S1600(12), S1800(12), S2000(12)


END

3 COMMON /ERROR/ ER(12), EHR(12), BLK(12), THETNOR(20), PSINOR(20), PLOPT(20), PTOPT(20), PT(20), POASTSQ(20)

COMMON /BLOCK6/ NEGP

IF (NDEBUG.NE.0) GO TO 2

DO 11 I = 1,N

IF (I.EQ.1) GO TO 7

IF (I.EQ.N) GO TO 11

100 CONTINUE

11 CONTINUE

RETURN

C

C FUNCTIONS AND PARAMETERS

C

COMMON /ERROR/ ER(12), EHR(12), BLK(12), THETNOR(20), PSINOR(20), PLOPT(20), PTOPT(20), PT(20), POASTSQ(20)

COMMON /BLOCK6/ NEGP

IF (NDEBUG.NE.0) GO TO 2

DO 11 I = 1,N

RETURN

C

C

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**APPENDIX B**

```
SUM=0
DO 313 M=1,N
DO 313 N=1,M+1
IF (M.EQ.N) GO TO 313
313 CONTINUE

14 FORMAT (5/$0.12H FORCE COEFFICIENTS)
SUBROUTINE SIMO (IA,IBM,DETERM,PIVOT,ANAX,SCALE)
C SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
DIMENSION (PIVOT(NP),IAMAX=1,IBM=1)
C AND Y AT WHICH DERIVATIVE IS FOUND
C
C INITIALIZATION
C
C SEARCH FOR PIVOT ELEMENT
C
C ARRAV=I
DO 7 J=1,N
IF (PIVOT(J).EQ.1) GOTO 7
3 DO 8 J=1,N+1
IF (PIVOT(J).EQ.1) GOTO 8
4 DO 14 K=1,N+1
IF (PIVOT(J).EQ.1) GOTO 14
5 TROW
ICOL=K
ARRAV=I
DO 6 J=1,N
IF (PIVOT(J).EQ.1) GOTO 6
6 CONTINUE
9 DO 15 K=1,N+1
IF (PIVOT(K).EQ.1) GOTO 15
CONTINUE
10 DETERM=DET(IBM)
30 CONTINUE
DO 29 M=1,N-1
IF (MICRO(J)).GT.1.0) GOTO 29
29 CONTINUE
GO TO 27
11 IF (ABS(PIVOT(M)).GT.1.0) GOTO 11
CONTINUE
12 FORMAT (1H C2*16D8.8X C16.8)
13 FORMAT (1H C15D8.8X C15D8.8X C15D8.8X)
```

Note: The text appears to be a portion of a FORTRAN program, possibly related to matrix operations or linear equations. The program seems to involve a loop for matrix operations, calculations for pivots and determinants, and handling of variables and arrays.
GO TO 3

DO 3

DO 3 (J=1,2,...,K)

CONTINUE

X(K) = X(J)

B = A + Y(J)/P*T

CONTINUE
APPENDIX B
Subroutines

A list of the subroutines used in this program is presented.

<table>
<thead>
<tr>
<th>FORTRAN name</th>
<th>Called by</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG</td>
<td>MAIN</td>
<td>Executive subroutine for computation of body geometry</td>
</tr>
<tr>
<td>RUNKUT</td>
<td>BG</td>
<td>Runge-Kutta integration for body geometry</td>
</tr>
<tr>
<td>DERIV</td>
<td>BG</td>
<td>Computation of derivatives for body-geometry integration</td>
</tr>
<tr>
<td></td>
<td>RUNKUT</td>
<td></td>
</tr>
<tr>
<td>APPROX</td>
<td>MAIN</td>
<td>Computation of approximate shock shape for NREAD = 0</td>
</tr>
<tr>
<td>LGRANGE</td>
<td>MAIN</td>
<td>Establishes line arrangement, accounting for symmetry where appropriate, for computation of ( \xi )-derivatives for function DIF</td>
</tr>
<tr>
<td></td>
<td>DERIV2</td>
<td></td>
</tr>
<tr>
<td>DIRCOS</td>
<td>MAIN</td>
<td>Computation of direction cosines of ( r-, \eta-, \tau )-coordinates</td>
</tr>
<tr>
<td>SHOCK</td>
<td>MAIN</td>
<td>Computation of flow quantities behind shock</td>
</tr>
<tr>
<td>DERIV2</td>
<td>MAIN</td>
<td>Executive subroutine for computation of derivatives for integration of equations</td>
</tr>
<tr>
<td></td>
<td>RUNKUT2</td>
<td></td>
</tr>
<tr>
<td>EQNS</td>
<td>DERIV2</td>
<td>Computation of ( \xi )-derivatives for integration of equations</td>
</tr>
<tr>
<td>RUNKUT2</td>
<td>MAIN</td>
<td>Runge-Kutta integration of equations</td>
</tr>
<tr>
<td>PRINT</td>
<td>MAIN</td>
<td>Print instructions</td>
</tr>
<tr>
<td>FMCOEFS</td>
<td>MAIN</td>
<td>Computation of force and moment coefficients</td>
</tr>
<tr>
<td>SIMEQ</td>
<td>MAIN</td>
<td>Solution of simultaneous linear equations to evaluate corrections ( \Delta \eta ) to the shock shape</td>
</tr>
<tr>
<td>DIF</td>
<td>LGRANGE</td>
<td>Computation of derivatives from the Lagrange formula</td>
</tr>
</tbody>
</table>
A flow chart of the computer program is given:

1. Read initial input

2. Beginning of "DO 104 L1 = 1, NCASES" Loop
   IF(M.EQ.5) Read (5, 900) THETAD, ALPHAD
   IF(M.NE.5) Read (5, 900) T, THETAD, ALPHAD

3. IF(L1.NE.1)
   yes
   no

4. IF(L1.NE.1)
   yes
   no
   THOLD = T
   THETHLD = THETAR
   EPSIHLD = EPSIGOM

5. IF(ABS(THOLD-T), GT. 1.E-05. OR. ABS(THETHLD-THETAR, GT. 1.E-05))
   yes
   no

6. Compute SIGMA on line N+1 for M = 3,4,5

7. IF(M.EQ.3.OR. M.EQ.4.OR. M.EQ.5)
   yes
   no

8. SUBROUTINE BG

9. IF(M.EQ.3.OR. M.EQ.4.OR. M.EQ.5)
   yes
   no

10. Continue with the program flow.
APPENDIX B

SUBROUTINE APPROX

Set parameters

IF(NREAD.EQ.1. AND. L1.EQ.1)

yes 17

no

IF(L1.NE.1)

yes 17

no

SUBROUTINE APPROX

17

Set parameters

18

Compute initial values for integration by
(a) Call LGRANGE (to compute ETASP)
(b) Compute ETA, H, HK, SIGMA
(c) Call DIRCOS (to compute direction cosines)
(d) Call SHOCK (to compute shock relations)
(e) IF (L.EQ.N+1) compute Mach cone from wing apex
APPENDIX B

IF(NPRINT.EQ.2.AND.(M.EQ.1.OR.
M.EQ.6.OR.(M.EQ.2.AND.T.LT.1.0)))

IF(NPRINT.EQ.1.AND.
NPIV.EQ.0)

IF(NPRINT.EQ.2.AND.
NPIV.EQ.0)

yes

31

Compute PGAM

yes

37

Print ETASP
Print CP
for intermediate runs

no

36

no

38

Dummy initialization
of extrapolation
parameters
APPENDIX B

INTEGRATION SECTION
Integrate equations from ZETA = 1. to ZETA = 0.

SUBROUTINE DERIV2 (Executive subroutine to compute derivatives)

SUBROUTINE LGRANGE (computes XI derivatives)

SUBROUTINE EQNS (computes ZETA derivatives)

Compute special quantities for print run
Compute surface conditions if ZETA = 0.

SUBROUTINE PRINT

IF(PRTNE .NE. 2)

IF(PRTNE .NE. 2)

IF(M.EQ.3 .OR. M.EQ.4 .OR. M.EQ.5)

Compute XBSECON, FBSECON

IF(ZETA.LE.0)

SUBROUTINE RUNKUT2 (Integrates equations)

A

IF(ZETA.EQ.0 .AND. NEXTRAF.EQ.1)

(This test causes the computer to go back and correct surface conditions if it is a print run.)
APPENDIX B

(IF(NEGP.EQ.0.AND. NPRINT.EQ.1)
  yes 73  (Negative pressure has been computed and results are printed.)
  no

(IF(NEGP.EQ.0.AND. NPRINT.EQ.2.AND. L1.EQ. NCASES)
  yes 104
  no

(IF(NEGP.EQ.0.AND. NPRINT.EQ.2.AND. L1.NE. NCASES)
  yes STOP 7776
  no
APPENDIX B

SUBROUTINE FMCOEFS (Compute force and moment coefficients)

TESTS FOR CONVERGENCE:
Compute VMAX
IF(NCYCLE.EQ.1.AND.VMAX.GT.VTEST) GO TO 85
IF(NCYCLE.GT.1.AND.VMAX.GT.VTEST1) GO TO 85
IF(VMAX.LE.VTEST) GO TO 80
IF(N.EQ.1) GO TO 85
IF(NCYCLE.GT.1.AND.VMAX.LT.VTEST1) NSKIP = 1
IF(NCYCLE.GT.1.AND.VMAX.LT.VTEST1) GO TO 86
APPENDIX B

NPRINT = NPRINT + 1

IF(NPRINT.EQ.3)

KCOUNT = KCOUNT + 1

IF(M.EQ.0)

Print headings for zero-incidence circular cone run

Print headings for M ≠ 0

SUBROUTINE PRINT
APPENDIX B

N_COUNT = N_COUNT + 1
K_COUNT = K_COUNT + 1
N_PIV = N_PIV + 1

IF(NCYCLE, GT, 50)
  yes
  STOP 0777
  no

IF(NCOUNT, GT, 1)
  yes
  J = J + 1
  Compute ETAS variation
  for Newton-Raphson iteration.
  Compute matrix
  elements D(I,J)

  no

Compute VMAX
Set ETAS0

IF((N_PIV, EQ, 1, AND, N, EQ, 1), OR, N_PIV, EQ, 2, OR, EPSIG, EQ, EPSIGMX)
  yes
  Begin ETAS variation
  Solve matrix \(D(I,J)\)
  for new DETAS.
  Compute new ETAS.
  Reset counters.

  no

IF(NSKIP, EQ, 1)
  yes

(This path is the modified
Newton method
(D(I,J) not recomputed))

no

18

18

End of "DO 104 LI = 1, NCASES" Loop
APPENDIX C

SAMPLE COMPUTATION

A sample computation, which required 63 seconds on the Control Data series 6600 computer system, is presented for a conical wing of circular-arc cross section. The input is

<table>
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<tr>
<th>Input card no.</th>
<th>Input</th>
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<tbody>
<tr>
<td>1</td>
<td>STMACH = 4.0, GAMMA = 1.4</td>
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<tr>
<td>2</td>
<td>N = 8, M = 4, NCASES = 1, NREAD = 0, NSPACE = 1, NPLT = 0, NPUNCH = 0</td>
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<tr>
<td>3</td>
<td>VTESTHD = 0.001, VTEST1 = 0.03</td>
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<tr>
<td>4</td>
<td>T = 0.06, THETAD = 40.0, ALPHAD = 5.0</td>
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Explanations of sections of the sample-computation output are provided at the end of this appendix. Circled numbers at the left side of each section indicate the appropriate explanatory note.

Input Cards for Sample Computation

Conical Flow by the Method of Lines

N = 3  M=4  NREAD=0  NSPACE=1  NP = 5
NA= 1  ANA = 0.
N3 = 1  BNB = 0.
VTEST = 1.0000000E-03  VTEST1 = 2.0000000E-02
# APPENDIX C

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**APPENDIX C**

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**Notes:**
- For $\eta > 0.10$, the values are extrapolated.
- $\Delta C$, $\Delta T$, $\Delta V$, and $\Delta P$ represent changes in specific heat capacity, temperature, volume, and pressure, respectively.

### References
- [Reference 1](#)
- [Reference 2](#)
### APPENDIX C

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**Table Notes:**
- Each cell represents a calculation or value based on the specified indices.
- The table likely contains a series of calculations or data points relevant to the subject matter of the document.
- The specific nature of the calculations or values is not detailed in the provided snippet.
### APPENDIX C

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### APPENDIX C

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APPENDIX C

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APPENDIX C

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APPENDIX C

Explanatory Notes

1. Some input and auxiliary parameters.

2. Initial shock shape and quantities computed from the first pivotal integration.

3. Change in the shock shape computed by regular Newton method. (See ref. 1.) Note that the results of the $N$ variational integrations are not printed.

4. Shock shape for second pivotal integration and computed quantities from this integration. Since $V_{MAX} < V_{TEST1}$ at this stage, the modified Newton method (ref. 1) is used to compute corrections to the shock shape; that is, the previously computed Jacobian matrix is used, and consequently, DETERM is equal to the value from the previously computed value. Note that a modified Newton iteration cycle requires only one integration, since the $N$ variational runs are bypassed.

5. Change in shock shape computed by modified Newton method.

6. Shock shape for third pivotal integration and computed quantities.

7. New correction to shock shape computed by modified Newton method.

8. Shock shape for fourth pivotal integration and computed quantities.

9. New correction to shock shape computed by modified Newton method.

10. Shock shape for fourth pivotal integration and computed quantities. The solution now satisfies the convergence criterion $V_{MAX} < V_{TESTHD}$.

11. Some input, auxiliary, and secondary parameters printed beginning on a new page.

12. Coordinates of Mach cone behind shock centered at wing apex.


15. Zeta print block at shock.

16. Sonic-line coordinates referenced to wing semispan.

17. Zeta print block at wing surface.

18. Windward-line zeta limit.
APPENDIX C

19) Surface-pressure coefficients and normal velocities.

20) Force coefficients and center-of-pressure location.
REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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