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Gravity Flow of Powder in a Lunar Environment

(In Two Parts)

2. Analysis of Flow Initiation

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Report of Investigations 7577

Gravity Flow of Powder in a Lunar Environment

(In Two Parts)

2. Analysis of Flow Initiation

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UNITED STATES DEPARTMENT OF THE INTERIOR
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GRAVITY FLOW OF POWDER IN A LUNAR ENVIRONMENT

(In Two Parts)

2. Analysis of Flow Initiation

by

William G. Pariseau [1]

ABSTRACT

A small displacement-small strain finite element technique utilizing the constant strain triangle and incremental constitutive equations for elastic-plastic media that are nonhardening and obey a Coulomb yield condition has been applied to the analysis of gravity flow initiation in a V-shaped hopper containing a powder under lunar environmental conditions. Three methods of loading were examined. Of the three, the method of computing the initial state of stress in a filled hopper prior to drawdown by adding material to the hopper layer-by-layer is superior (as expected). Results of the analysis of a typical hopper problem show that the initial state of stress, the elastic moduli, and the strength parameters have an important influence on material response subsequent to the opening of the hopper outlet.

INTRODUCTION

The purpose of this Bureau of Mines paper is to present the results of a preliminary investigation concerning the applicability of finite element techniques to the analysis of the initiation of gravity flows of powders in a lunar environment.

On the moon as on the earth, efficient storage and transfer of particulate materials may, under favorable conditions, be accomplished by gravity bins and hoppers. The lunar environment with respect to materials handling is characterized by ultrahigh vacuum, absence of moisture, and low gravitational acceleration, and thus may offer distinct materials-handling advantages in comparison to the terrestrial environment, where moisture, gas counterflows, and consolidation are frequent causes of binhopper malfunction. The same physical principles apply in either case so that in theory the design of a gravity flow binhopper for handling powder in a lunar environment is less difficult than in a terrestrial environment. However, the primitive state of the predictive art as regards the functional design of binhoppers, in contrast to structural design, limits the confidence one is willing to invest in present

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design procedures. Experimental duplication of lunar conditions (such as equivalent lunar gravity) on the earth presents even greater obstacles, economic as well as technical, to the design approach. The inevitable compromise that circumstances dictate consists of experimental determination of material properties under lunar conditions and the use of the computer as a laboratory for the investigation of the mechanics of gravity flow of powder in a lunar environment.

The determination of the requisite material properties has been described in part 1 (30)[2] of this two-part paper. In this part, the results of a preliminary investigation of the suitability of a finite element computer code for functional binhopper design are described. Theoretical emphasis is upon the broad aspects of particulate media mechanics, whereas a case study approach is utilized to illustrate specific features of a typical finite element binhopper analysis.

ACKNOWLEDGMENTS

Grateful acknowledgment is made to H. D. Dahl, former graduate student, Pennsylvania State University, whose computer program with but slight modification was used for this research. This work has been carried out with joint National Aeronautics and Space Administration and Bureau of Mines sponsorship under NASA Contract R 09-040-001.

STATEMENT OF THE PROBLEM

The problem of functional binhopper design has general features common to all design situations as well as specific features particular to a given set of circumstances. Common to all design situations is the necessity of selecting an appropriate mathematical model of material behavior. Features particular to a given set of circumstances include formulation and solution of a specific boundary value problem.

General Features of the Mathematical Model

Although powders and similar particulate materials are indeed discrete, the lack of a useful theory for the description of the mechanical behavior of discontinua necessitates recourse to continuum models of material response. An order of magnitude argument is helpful here. If a characteristic linear dimension of a representative particle (for example, average particle diameter) is an order of magnitude less than a characteristic linear dimension of the hopper outlet (for example, width), then a continuum model may be reasonable.

In this investigation, the actual material was replaced mathematically by a homogeneous, isotropic, nonhardening elastic-plastic medium. Temperature and time effects were neglected, so that material response was tacitly assumed to be isothermal and time-independent. These assumptions may be relaxed, but

[2]Underlined numbers in parentheses refer to items in the list of references at the end of this report.

only at the expense of a considerable increase in experimental difficulties in determining the requisite material properties. If, for example, one wishes to relax the assumption of nonhardening, then one must be prepared to quantitatively specify an applicable hardening rule. This trade-off between increased realism in material description and added difficulty in material properties testing is present regardless of the binhopper environment. However, the point of diminishing returns can only be decided on the basis of a given design situation. The same accuracy, precision, and reliability will not be required in all cases, since the cost of a malfunction and, conversely, the benefit of reliable design will be different in each instance.

A nonhardening, elastic-plastic material deforms without permanent volume or shape change up to some state of stress at which large, permanent deformations become possible with little additional increase in stress (9). The essential elements of theory in addition to the equations of equilibrium and the geometry of deformation include the yield function and constitutive equations. Inasmuch as gravitational stresses must be of the same magnitude as the strength of the material, it is essential that body forces be incorporated into the analysis. In this study, a Mohr-Coulomb yield condition and associated flow rule were utilized (4, 39). The constitutive equations were taken in incremental form under the assumption that the total strain increment was composed of elastic and plastic components in the yielding material elements, and were elastic otherwise. The elasticity is assumed to be linear over small increments of load, but different moduli may be used for loading and unloading. The plasticity is also incremental. Body forces are applied in a manner appropriate to accreted bodies (2, 14) that are subsequently "cut." This means that the state of stress just prior to drawdown is obtained by first adding material layer by layer to the hopper and simultaneously accumulating the stress, strain, and displacement increments in each material element due to the added load of the new layer. Hopper drawdown is then initiated by incrementally removing the loads exerted on the outlet by the prestressed material within, that is, by a "cut" across the hopper outlet. Thus, the computer simulates binhopper fill-flow initiation processes. Arbitrary fill-flow (cut) sequences are accomplished without difficulty. If a fill sequence is not specified, drawdown will proceed from an initial state of stress due to gravity loads applied under complete lateral constraint. Additionally, body forces may be applied in the classical "turn-on" manner.

The binhopper fill-flow simulation is based upon the finite element technique of analysis (17, 35, 42). This technique is a numerical method for solving boundary value problems in mathematical physics and is eminently suited for coping with the nonlinearities of elastic-plastic analyses (27-28). Nonhomogeneity and anisotropy are readily introduced into the computations. The end result of a finite element formulation of an elastic-plastic problem is a rather large system of algebraic equations of matrix form $F = KU$, where F and U are forces and displacements, and the elements of K reflect the material response to load entirely analogous to the force-displacement equation for a spring of stiffness K. Once the displacements are obtained, it is an easy matter to compute the associated strains and stresses.

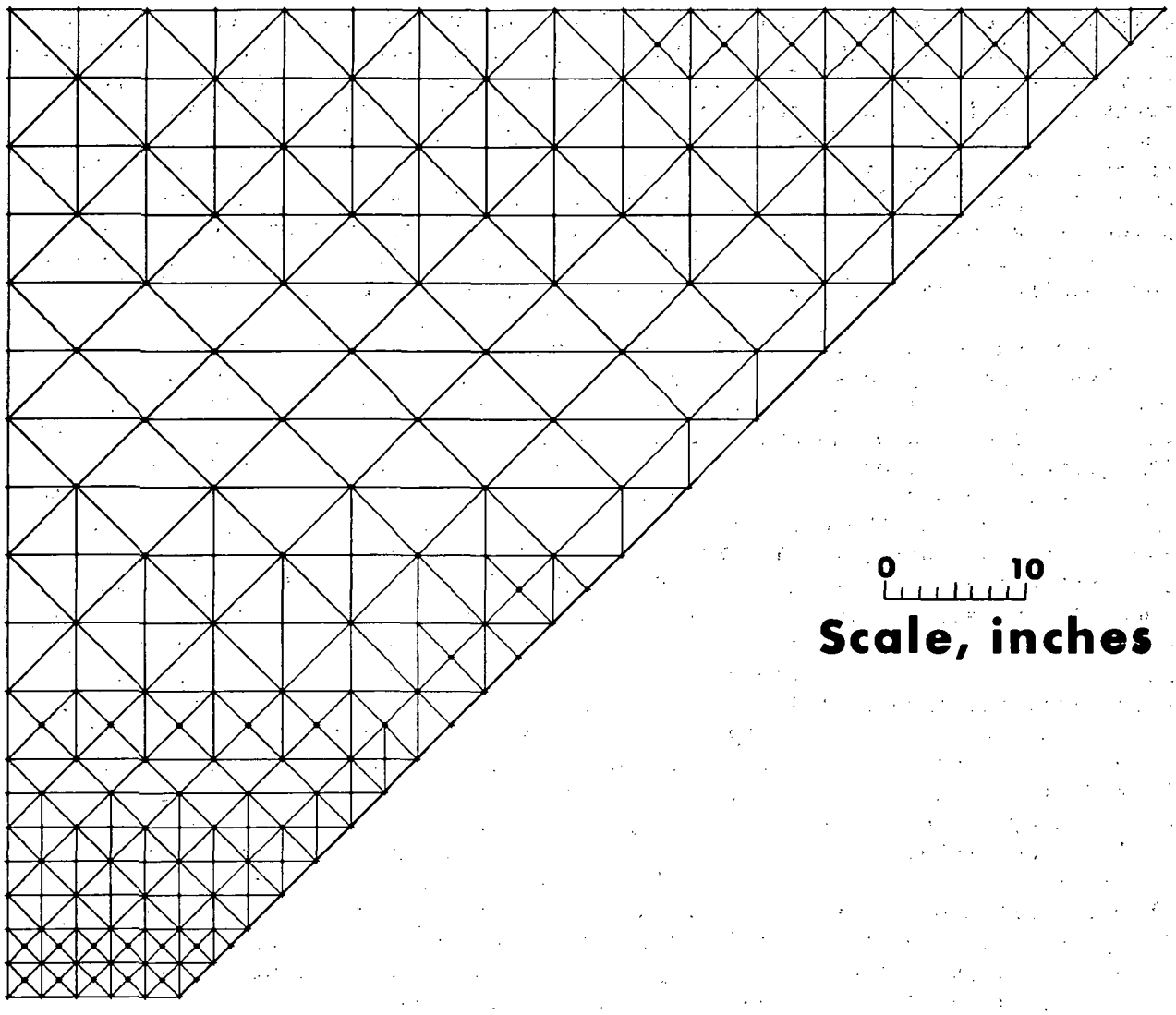

FIGURE 1. - Typical Element Mesh Used in Analysis.

Implementation of the finite element technique proceeds through four major stages. First the material is partitioned into a number of discrete or finite elements. Triangular elements of unit thickness were used in the present analysis. Other shapes can be employed, however. Partitioning is accomplished by simply drawing the binhopper to scale and then dividing the material into relatively small triangles where stress gradients are expected to be large, as at the outlet edge, and into larger elements elsewhere. Figure 1 shows a typical hopper element mesh used in this investigation. Next the properties of the individual elements are specified. An application of the principle of virtual work suffices to define the nodal force-displacement relationships once the mathematical material model has been selected. The individual elements are then assembled into a mechanically consistent whole, and finally the resulting system of equations is solved for nodal point displacements. Computation of strains and stresses follows.

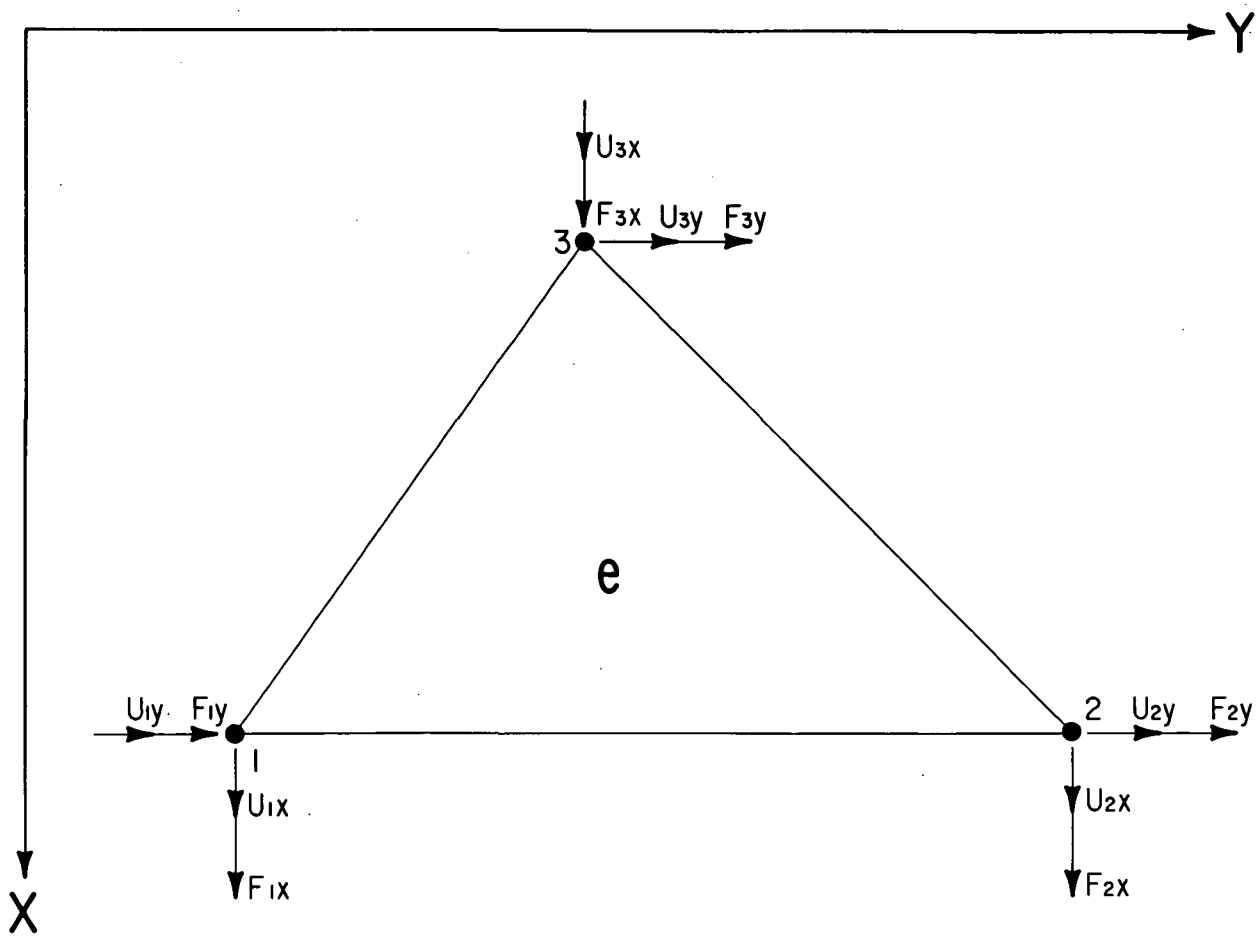

FIGURE 2. - Typical Triangular Element.

The equations of interest are the displacement relationships assumed for the elements, the geometry of strain, the constitutive equations, and the final system of equations for the entire body. Constant strain triangles were employed, so that the displacements throughout an element are linear functions of the nodal point displacements; thus

$$\{u\}_e = [N]\,\{u_i\}_e\,, \tag{1}$$

where { } indicates a column matrix; $\{u\}$ is a listing of the displacement components at a point within the element e; $[N]$ is a matrix linear in position, and $\{u_i\}$ is a listing of displacements at the nodal points as shown for a typical element in figure 2. From the geometry of strain and equation 1

$$\{\varepsilon\} = [B]\,\{u_i\}, \tag{2}$$

where $\{\varepsilon\}$ are the components of strain and $[B]$ is a matrix of constant terms. The elastic-plastic constitutive equations relating increments of stress $\{d\sigma\}$ to increments of strain are

$$\{d\sigma\} = \left([E] - \frac{[E]\{\partial Y/\partial\sigma\}\ \{\partial Y/\partial\sigma\}^T\ [E]}{\{\partial Y/\partial\sigma\}^T\ [E]\ \{\partial Y/\partial\sigma\}} \right) \{d\varepsilon\}, \tag{3}$$

or

$$\{d\sigma\} = ([E] - [E^p]) \{d\varepsilon\},$$

where $[E]$ is a matrix of elastic moduli and $\{\partial Y/\partial\sigma\}$ is a column matrix whose elements are derivatives of the yield function Y with respect to the components of stress. The superscript T means transpose. Equations 3 are the matrix form of an inverted stress-strain relationship for elastic-ideally plastic media. A similar relationship can also be obtained for hardening materials, provided an appropriate hardening rule is available (27-28). If an element remains elastic during a load increment, then only the first term on the right of equation 3 applies. Elements that undergo transition from the elastic to the elastic-plastic regime or the reverse during a load increment are strained elastically for only a portion of the increment. Otherwise equation 3 applies during the entire increment of load.

The nodal point forces $\{F\}$ are made work equivalent to the surface tractions, body forces, and initial stresses acting on an element through an application of the virtual work identity. One thus obtains for the system

$$\{\Delta F_i\} + \{\Delta F_i^g\} + \{\Delta F_i^s\} = [K]\ \{\Delta U_i\}, \tag{4}$$

where the terms on the left are column listings of increments in externally applied nodal forces, nodal forces due to gravity, and nodal forces due to initial stresses, respectively. The matrix $[K]$ reflects the material response to load, and $\{\Delta U_i\}$ is a listing of nodal point displacement increments. One has for the nodal forces

$$\{F_i^g\} = \int_V [N]^T\ \{X\}\ dV \tag{5a}$$

and

$$\{F_i^s\} = -\int_V [B]^T \{\sigma^\circ\}\ dV, \tag{5b}$$

where $\{X\}$ and $\{\sigma^\circ\}$ are listings of body force components and initial stresses, respectively, and V signifies volume of an element. Also

$$[K] = \left(\sum_{i=1}^{M}\ [L]_i^T\ [k]_i [L]_i\right), \tag{6}$$

where M is the number of elements in the mesh, the $[L]_i$ are partition matrices of a master location matrix $[L]$ that enables one to assemble the individual elements into an equilibrated whole, and

$$[k] = \int_V [B]^T\ ([E] - [E^p])\ [B] dV. \tag{7}$$

Equation 7 refers to an individual element, whereas equation 6 applies to the system. Again for a purely elastic deformation $[E^p] = [0]$ following the notation in equation 3.

Once the system equation 4 has been solved for the unknown displacement increments $\{\Delta U_i\}$, applications of equations 2 and 3 yield the corresponding strain and stress changes. Increments of nodal forces are then applied once again and the corresponding displacement, strain, and stress changes are added to the previously computed values. The process continues until the nodal forces, in effect, obtain their prescribed values. During each increment of load, all elements are tested as to whether they undergo transition or not; that is, whether they are to be included in the elastic-plastic domain or not at the end of the loading increment. The final result is a complete approximate solution to a problem in time-independent, elastic-plastic theory.

Alternative procedures, some of only historical interest, are available (1, 12, 18, 42 - 43). However, the one presented here seems to be in current favor amongst the competing small displacement-small strain formulations. A more complicated large displacement-large strain formulation has been described (16).

Specific Features of the Plane Strain Model

A plane strain formulation was followed in the present investigation. Hoppers analyzed were assumed to be V-shaped and fitted with slot outlets. The outlet is assumed to be long in comparison to its width, so that variations in stress, strain, and displacement are negligible in the length or z direction. An order of magnitude argument suffices here also, although in practice length-to-width ratios as low as four have been found satisfactory (31, 33). The small displacement (small strain) formulation restricts the analysis to small increments of load during the hopper fill stage and to the early or initial phases of hopper drawdown. Thus, in contrast to previous binhopper investigations utilizing plasticity theory (13, 20-21, 23-24, 29-34, 37-38), the elastic components of strain are not considered negligible in comparison to the plastic components, nor is steady-state flow assumed. However, inertia forces are considered negligible during the application of load increments. The loads are supposed to be applied slowly. As before, the analysis is time- and temperature-independent, and the material is considered to be homogeneous, isotropic, and to obey a linear yield condition in plane strain.

Hopper fill sequences are simulated by adding horizontal layers of elements having weight to existing layers of prestressed material lacking weight. Displacement, strain, and stress changes are computed and then added to the initial displacement, strains, and stresses. The material body within the hopper is thus formed by accretion; the body forces are treated accordingly (2, 14). During the fill sequence, zero displacements normal to the hopper centerline, outlet, and walls are specified. Additionally, a sliding friction condition is applied to any unbalanced nodal force along the hopper wall. Thus, if a nodal point on the wall moves, it must move tangential to the wall; and nodal point reaction forces, if present, obey a sliding-friction condition.

Hopper drawdown is initiated by considering the material to be in an initially stressed but unstrained state. Zero displacements normal to the

hopper centerline and walls are again specified. The reaction forces at the outlet nodes due to the presence of initial stresses are then diminished incrementally to zero after zeroing all other nodal forces. Changes in displacement, strain, and stress are computed for each increment and accumulated until the end of the last load increment (actually a decrement). Application of the last load increment corresponds to a fully open outlet and the establishment of the final stress, strain, and displacement fields.

The material properties required for the analysis are the elastic and plastic moduli, the coefficient of wall-powder friction, and the unit weight of material (lunar gravity is approximately one-sixth earth gravity). Young's modulus and Poisson's ratio are the elastic moduli utilized in the analysis and are obtained from the one-dimensional compression test data described in part 1 of this paper (30). The uniaxial tensile and compressive strengths are the plastic moduli that were employed and were computed from the cohesion and angle of internal friction. The latter strength parameters were determined from direct and torsional shear data also described in part 1 (30). The coefficient of wall-powder friction was determined from a direct-shear test.

Solution of the system of equation 4 was obtained through a Gauss-Seidel iterative scheme (10-11) using an overrelaxation factor between 1.8 and 1.9. Local inversion of the stiffness matrix for nodes along the inclined hopper walls and a simultaneous iteration for both displacement components, utilizing effective flexibility coefficients, was employed (26, 41). Iteration on all other nodes is direct and separate. The program exits from the "solve" subroutine whenever the norm of the residual matrix is reduced below a specified amount, exceeds a specified number, or a specified number of iterations has occurred. Accordingly, convergence is obtained, lost or in-between. The in-between situation may be converging or diverging at exit time, which is apparent from program output information. Well-posed problems will always converge, but unrestrained displacements, which may arise in different situations, will lead to loss of convergence. Run times vary with the number of elements that yield, but drawdown runs seldom exceed 1-1/2 minutes for a purely elastic analysis and 5 minutes for a typical elastic-plastic computation, with 30 pct of the several hundred elements in the mesh failing.

The program with slight modification was written by H. D. Dahl (3). It is clear that such a program is not restricted to the solution of binhopper problems, although it is believed that this investigation is the first application of finite element techniques to such problems. Additional features in the program, but not pertinent to the present discussion, enable one to solve a great variety of elastic-plastic boundary value problems. These features include provisions for possible anisotropy, nonhomogeneity, nonzero displacement and force boundary conditions, and purely elastic analyses.

CASE STUDY PROCEDURES AND RESULTS

Features of importance to the present case study include the hopper geometry, material properties and fields of stress, strain, and displacement. Hopper geometry was specified for convenience; material properties were determined experimentally. Of dominant interest were the procedural

features and results of the finite element analysis. Three procedures were utilized: (1) drawdown from an initial state of stress due to a layer-by-layer fill sequence (computed initial stress procedure) (2) drawdown from an initial stress due to gravity applied to a laterally constrained material (estimated initial stress procedure), and (3) drawdown as a result of gravity turn-on (no initial stress procedure). The three procedures lead to significantly different final states of stress, strain, and displacement.

Computed Initial Stress Procedure and Results

In this procedure, horizontal layers of material having weight are added sequentially to the material previously contained in the hopper. The load exerted by the new layer of material on the old is brought to its final value in 10 increments. After each increment of load, the displacement, strain, and stress changes are computed; the stress changes are then added to the previous stresses, and a test for yielding is made. If yield occurs, the appropriate changes in element properties are made. Subsequent behavior of a failed element may be elastic-plastic if loading continues, or elastic if unloading occurs. The process is repeated for each layer until the hopper is filled. In this way, the state of stress in a filled hopper prior to drawdown is computed. Strains and displacements may be computed in the same manner.

Elastic moduli obtained from the loading portion of the first load-unload cycle of the one-dimensional compression tests are used during the fill sequence. Usually, such tests show a nonlinear first cycle response over an extended range of loading. However, over a relatively small range of stress, as encountered in the hopper analysis (generally less than 1 psi), the moduli may reasonably be considered as constant. Thus, in this preliminary investigation the same elastic moduli were used for all elements during the fill sequence.

The results of the computed initial stress procedure are shown in figures 3 and 4. Figure 3 shows the orientation and magnitudes of the major and minor principal stresses in rosette form. All stresses are compressive, and the horizontal stress is always less than the vertical stress. The major principal stress (largest compression) is generally within 15° of the vertical. A small zone of failed elements develops and grows during the filling operation and is outlined in final form in figure 3. Figure 4 shows the distribution of vertical and horizontal stresses. The similarity between the form of the distribution of vertical centerline stress and reported measurements is striking (19).

Results of drawdown from the computed initial state of stress are shown in figures 5, 6, and 7. Figure 5 shows the principal stresses and extent of the failure zone after opening the hopper outlet. Opening of the hopper outlet is achieved by application of 10 increments of load removal at the nodes along the outlet. The removal of load by increments corresponds to a slow opening of the hopper gate. The first increment of load removal initiates drawdown. The last increment opens the gate fully. During the drawdown process, the elastic moduli are those obtained from the unloading portion of the first cycle of the one-dimensional compression tests. Figure 6 shows the

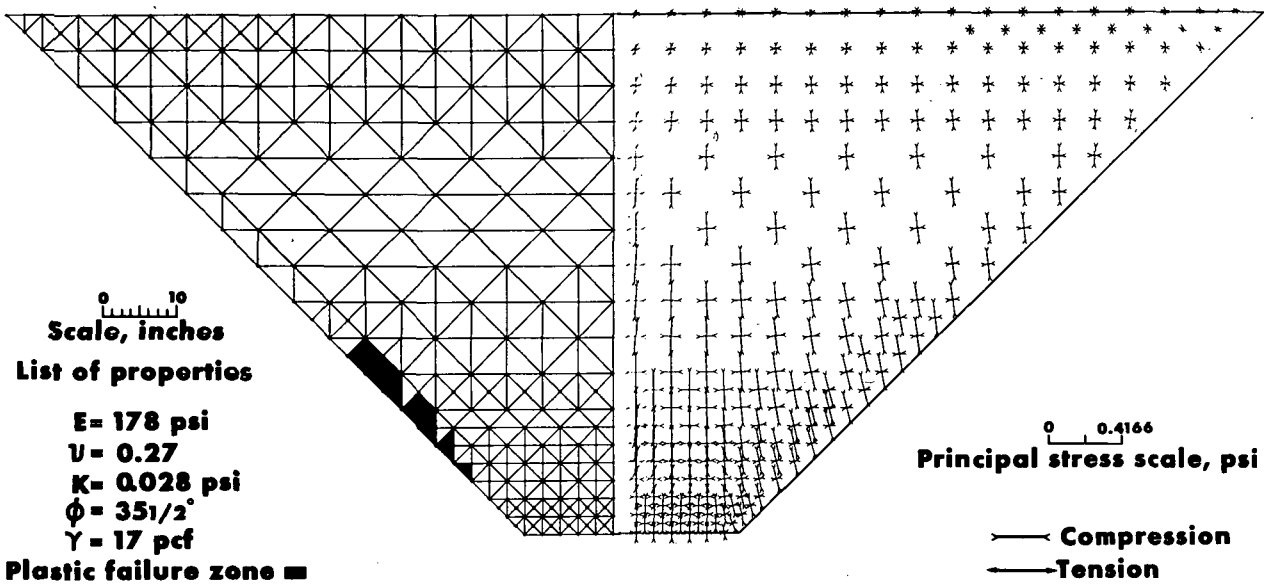

FIGURE 3. - Principal Stresses in a Hopper Filled Sequentially Layer by Horizontal Layer and Extent of Failure Zone After Filling (Computed Initial Stress Procedure).

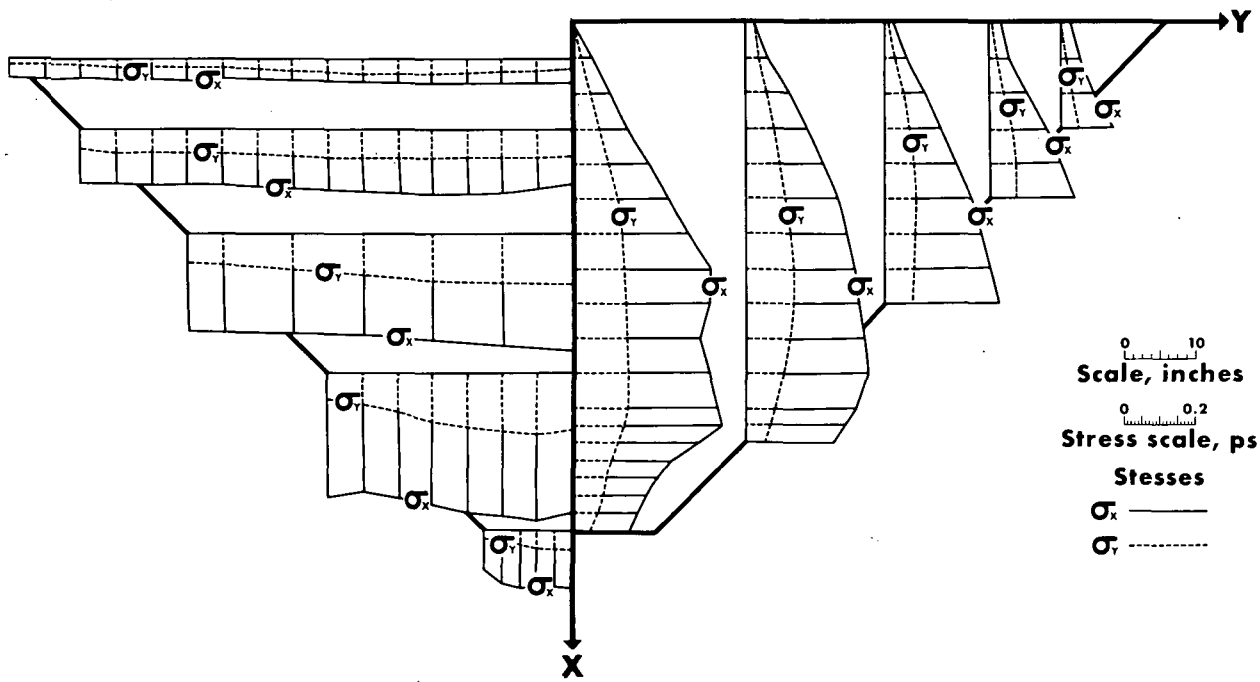

FIGURE 4. - Distribution of Vertical (σ_{xx}) and Horizontal (σ_{yy}) Stresses After Hopper Filling (Computed Initial Stress Procedure).

distribution of horizontal and vertical stresses after the hopper gate is fully open. The redistribution of stress relative to the initial state (fig. 4) is qualitatively as one might anticipate; that is, the vertical stress (σ_{xx}) is greatly reduced in the vicinity of the outlet, whereas the horizontal stress (σ_{yy}) tends to increase. Figure 7 shows the principal strains and displacement field obtained after opening the hopper outlet.

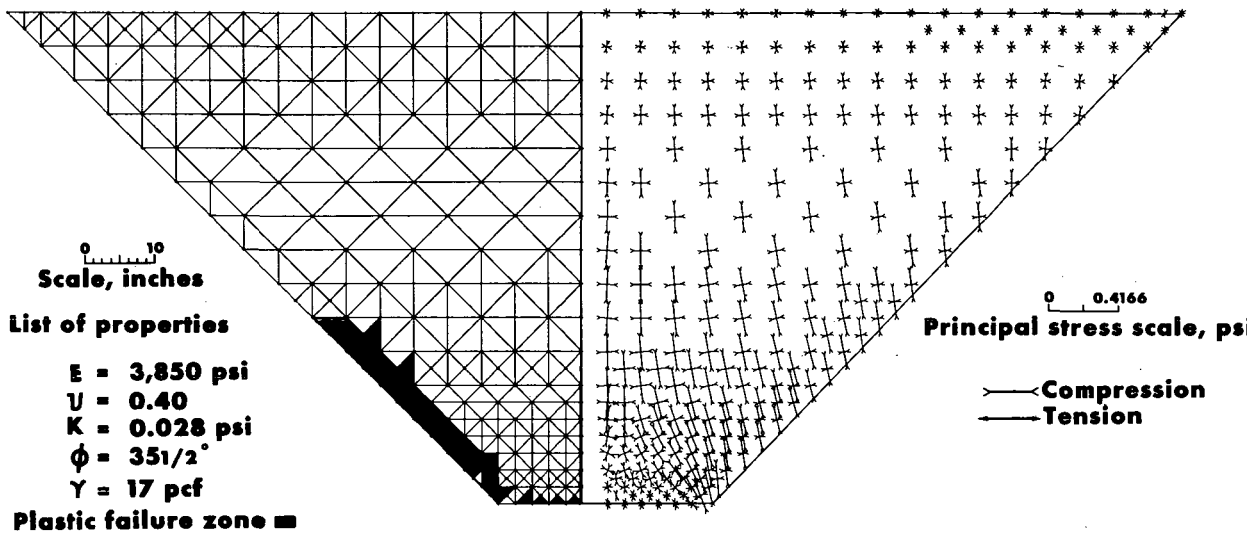

FIGURE 5. - Principal Stresses and Extent of Failure Zone After Opening Hopper Outlet (Computed Initial Stress Procedure).

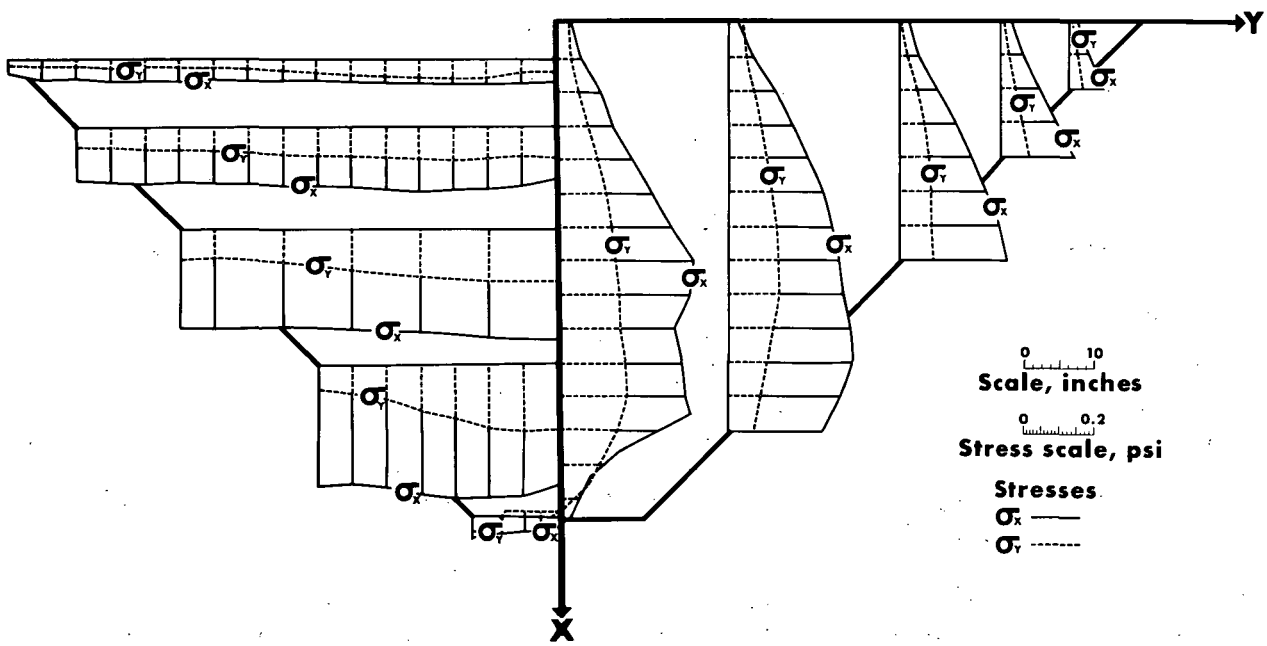

FIGURE 6. - Distribution of Vertical (σ_{xx}) and Horizontal (σ_{yy}) Stresses After Hopper Drawdown (Computed Initial Stress Procedure).

Estimated Initial Stress Procedure and Results

In this procedure, the state of stress prior to drawdown is estimated on the basis of complete lateral restraint of the material. Accordingly, the vertical stress at a point is computed as the product of unit weight and depth, and the horizontal stress is computed as the product of a constant and the vertical stress. This procedure, although unrealistic for binhopper

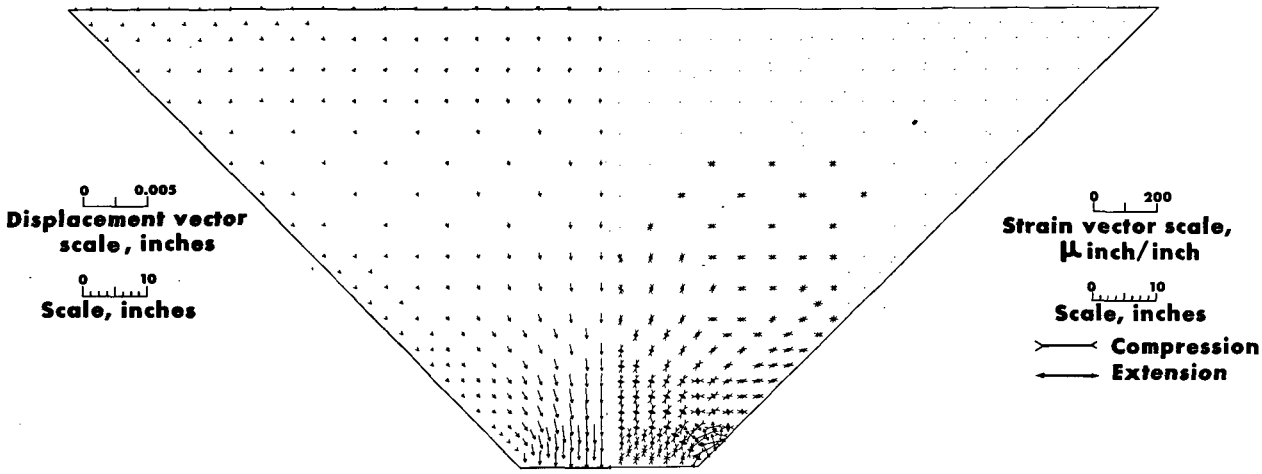

FIGURE 7. - Principal Strains and Displacements After Opening the Hopper Outlet (Computed Initial Stress Procedure).

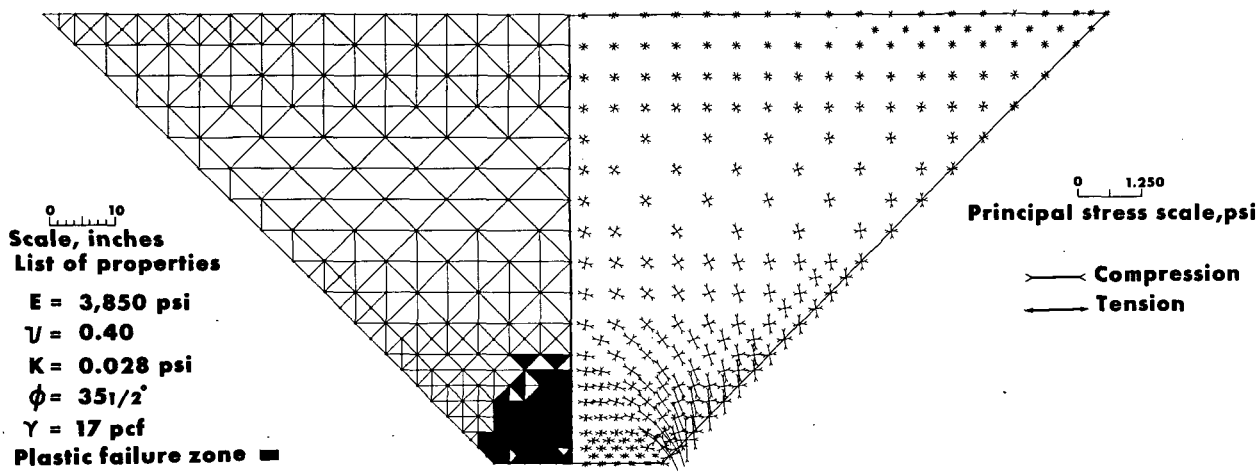

FIGURE 8. - Principal Stresses and Extent of Failure Zone After Opening the Hopper Outlet (Estimated Initial Stress Procedure).

analyses, is a common assumption in the analysis of soil and rock mechanics problems that involve cuts. The material properties used were the same as those in the computed initial stress drawdown procedure.

Results of the estimated initial stress procedure are shown in figures 8 and 9. Figure 8 shows the principal stresses and the extent of the failure zone, and figure 9 shows the principal strains and displacement field predicted by the estimated initial stress procedure.

No Initial Stress Procedure and Results

In this procedure, gravity is simply turned on or applied to the material in the hopper, much as a magnetic field may be applied to a piece of iron by turning on an electric circuit. This is the classical method of treating body forces in continuum mechanics. The turn-on here, however, is perforce

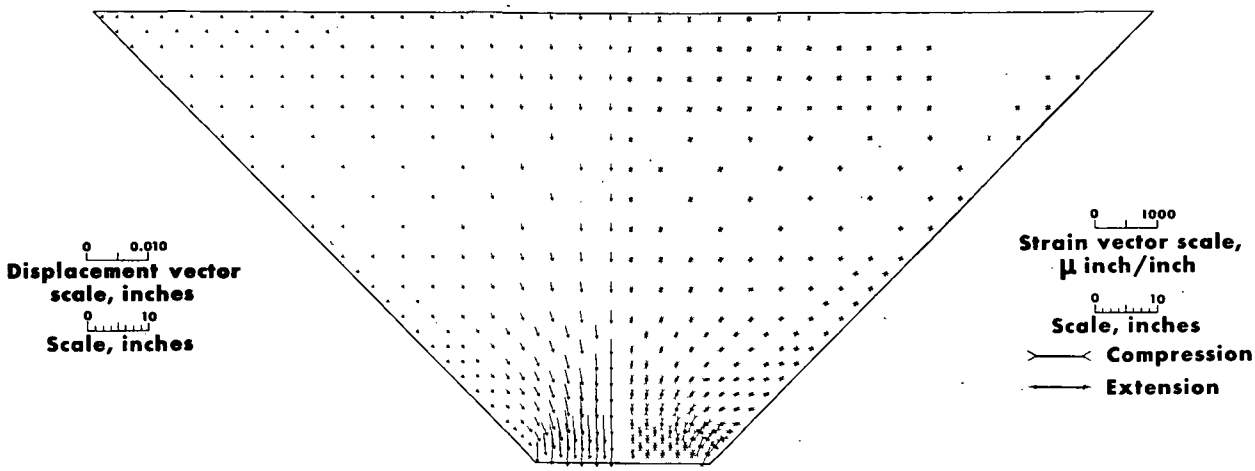

FIGURE 9. - Principal Strains and Displacements After Opening Hopper Outlet (Estimated Initial Stress Procedure).

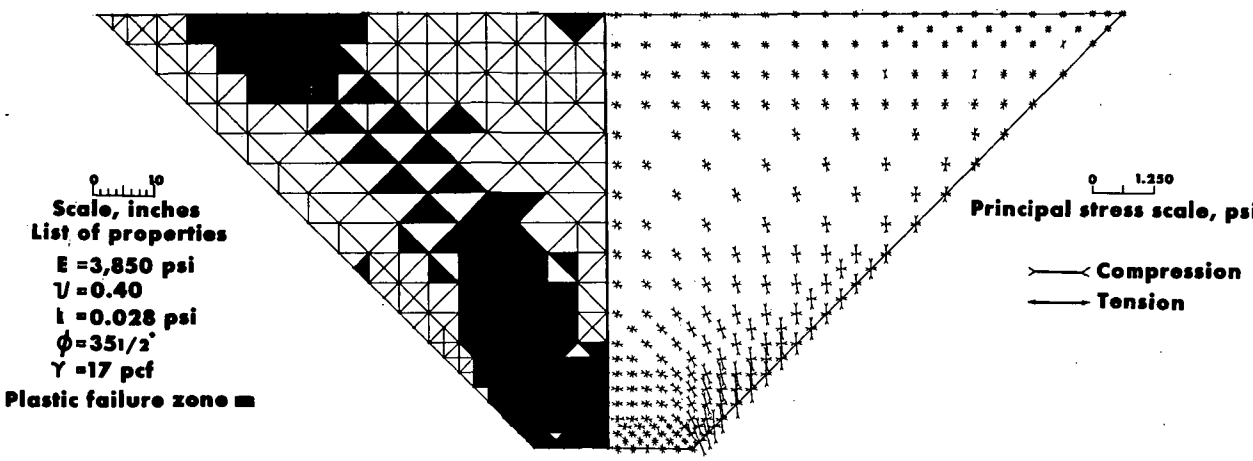

FIGURE 10. - Principal Stresses and Extent of Failure Zone After Opening Hopper Outlet (No Initial Stress Procedure).

incremental in order to account for possible element failures and consequent change in material response. Material properties during drawdown were the same as in the previous drawdown analyses.

Results of the no initial stress procedure are shown in figures 10 and 11. Figure 10 shows the principal stresses and extent of the failure zones, and figure 11 shows the principal strains and displacement field.

DISCUSSION OF RESULTS

The results of this investigation indicate that finite element techniques are indeed applicable to the analysis of binhopper problems. Although a quantitative comparison between the results of physical and computer experimentation is not possible at this time, qualitative results are encouraging. Program runs using actual material properties of a simulated lunar powder,

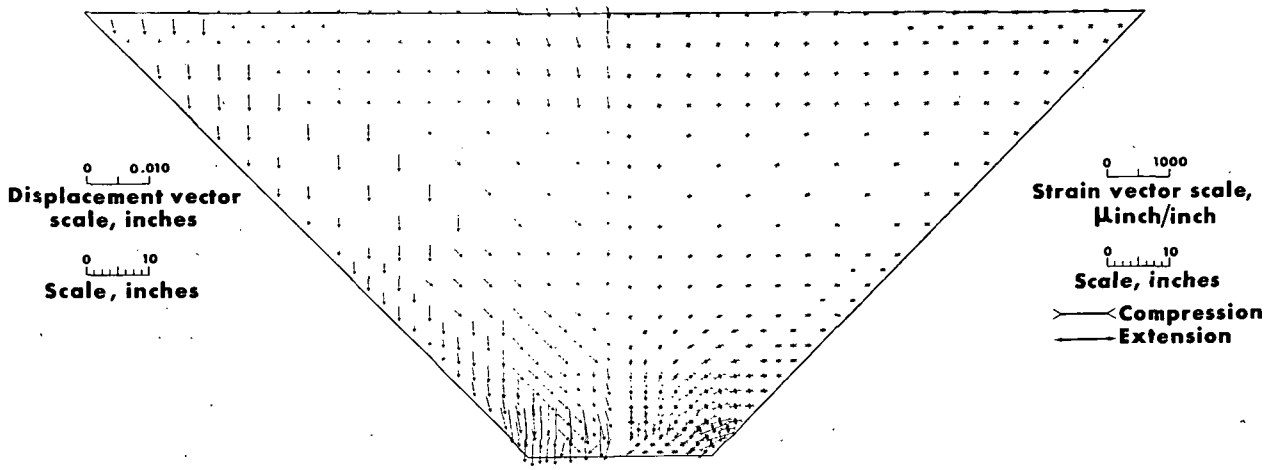

FIGURE 11. - Principal Strains and Displacements After Opening Hopper Outlet (No Initial Stress Procedure).

a realistic hopper geometry, and three distinct loading procedures resulted in convergent solutions in every case. An extensive discussion of qualitative results is prohibited by the lack of experimental data. However, computed initial stresses along the hopper centerline show reasonable agreement with measured values (19), and there seems to be no reason to doubt that additional support for the method will be obtained as the appropriate experimental data are accumulated.

Within the context of the method itself, two important results emerge: (1) the state of stress in a hopper prior to drawdown has a decisive influence upon the response of the material to the opening of the hopper outlet, and (2) the stress changes during drawdown initiation are highly dependent upon differences in the elastic moduli obtained from the virgin compression and subsequent unloading-loading curves.

The final stresses obtained by adding the stress changes due to drawdown initiation (opening of the outlet) to the initial stresses determine whether or not failure (and thus flow) ensues. If the final stresses satisfy the yield condition, then at least local failure occurs. If the zones of local failure grow and coalesce to span the outlet as the hopper gate is opened, then free gravity flow can reasonably be expected to follow. For a hopper of specified geometry containing a material with prescribed properties, the stress changes will be the same in every case, provided the support reactions of the hopper gate are the same in every case. However, the support reactions of a closed outlet are determined by factors affecting the hopper filling operation, including the elastic moduli obtained from the virgin compression curve. Differences between these moduli and the moduli used during drawdown will determine the stress changes during drawdown initiation, other factors being equal.

Even if the material strength is not significantly affected by compaction during hopper filling (as is the present case), the method of filling and the differences in elastic moduli during filling and drawdown may have a

drastic effect on the initial state of stress and the subsequent stress changes. Thus, the initial state of stress and the elastic properties of the material, in addition to the strength properties, will govern the flow-no flow conditions pertaining to drawdown from an at-rest state,[3] as demonstrated through the three procedures described in this paper.

Of the three procedures described, the computed initial stress procedure is preferable. In fact, there would appear to be no substantial reason for using either the estimated initial stress procedure or the no initial stress procedure when the computed initial stress option is available. The computed initial stress procedure is certainly more in the spirit of actual hopper operation than the other two. It is not necessary to use horizontal layers and the same material properties in every layer. Chevron-shaped layers would do just as well, and material nonlinearity, nonhomogeneity, and anisotropy (elastic and plastic) can be incorporated in the analysis without difficulty. In principle, geometric nonlinearities can also be treated.

It is worthwhile to note explicitly that the computed initial stress procedure, although developed primarily for analysis of flow initiation, does provide one with a much improved technique for computing bin wall loads during both filling and emptying operations. It is no longer necessary to resort to one of the numerous approximate models for estimating bin wall loads (22).

Two additional features of interest but not direct objects of the present investigation are the wall boundary conditions and the appropriateness of the underlying mathematical model.

It is common practice in binhopper mechanics to prescribe a Coulomb friction boundary condition over the hopper walls, and in some cases Coulomb yield is prescribed for the material within. These conditions imply that the material is sliding and possibly yielding at the binhopper wall. Neither may be the case. In fact, these conditions are precisely the events one wishes to predict. The writer also suspects that the prescription of sliding friction over an interface between a particulate medium and solid wall may be an oversimplication of the actual phenomena. What is really known a priori at the hopper walls is that material does not flow across the wall. The appropriate boundary condition is therefore one of displacement rather than stress. In any event, an incremental solution will be required.

Strictly speaking, the results obtained in the present investigation apply only to the idealized material. The reasonableness of the underlying mathematical model will determine the utility of the results in engineering practice. There seems to be general agreement that particulate media behave elastic-plastically in some sense (8), although agreement is not unanimous

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[3]These results may be compared with prevailing analysis (20, 36, 40) based on a hypothetical arch that is assumed to span the outlet and to be in a state of limiting equilibrium. The difficulty, of course, is that one has no assurance that such a state of stress will ever be obtained. This is precisely the problem: to compute the state of stress as one attempts to initiate drawdown by opening the hopper gate.

(25). However, existing experimental evidence is based mainly upon observations of large strain phenomena, and hence no decision is possible concerning the small displacement-small strain (10^{-4} or less) analyses described here. It is known that particulate materials are frictional to a degree, and are therefore not stable in the sense of Drucker (5-6). Associated flow rules in conjunction with Coulomb yield do not appear to lead to useful results in the analysis of large strain phenomena (15, 20, 23, 32-33). Nevertheless, small-strain problems may perhaps be profitably analyzed using an associated flow rule and a properly determined yield condition. Simple hardening rules may also prove useful (7). What is obviously required is fundamental research concerning constitutive equations for particulate media.

CONCLUSION

Simulation of actual hopper filling and emptying sequences is possible by the finite element method. Detailed predictions of stress, strain, and displacement are readily obtained. In particular, bin wall loads during filling, at rest, and during drawdown initiation, are easily calculated.

Although complicated material behavior can be handled with relative ease at the present time, the full potential of the method remains to be exploited. One possibility is that of attaching structural elements to the solid elements at the hopper boundaries, introducing suitable design criteria, and proceeding to a fully automated binhopper design program. However, the utility of the results of such a computation will be restricted by the physical appropriateness of the underlying mathematical model, and by the reliability of the material properties that constitute an important program input. The perennial problems of engineering thus remain, but at a considerably increased level of sophistication.

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