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R. P. LEPPING

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INFLUENCE OF THERMAL ANISOTROPY ON BEST-FIT ESTIMATES OF SHOCK NORMALS

by

R. P. Lepping
Laboratory for Extraterrestrial Physics
NASA-Goddard Space Flight Center
Greenbelt, Maryland

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Abstract

This paper deals with the influence of thermal anisotropy on the estimates of interplanetary shock parameters and the associated normals and presents a practical theorem for quantitatively correcting for anisotropic effects by weighting the before and after magnetic fields by the same "anisotropy parameter" h . The quantity h depends only on the thermal anisotropies before and after the shock and on the angles between the magnetic fields and the shock normal. It is shown that for fast shocks and for a liberal range of realistic conditions the quantity h lies in the range $0.90 < h < 1.22$; this includes pre-shock anisotropy factors as low as $\xi_1 = 0.5$. The theorem can also be applied to most slow shocks but in those cases h usually should be lower, and sometimes markedly lower, than unity. The previously studied fast shock of January 26, 1968, from plasma and magnetic field measurements made on Explorers 33 and 35, is reexamined in light of this field-weighting theorem. Even for the extreme values of h given above little change results in the shock parameters or in the shock normal, which over the full range of h is estimated to deflect by only 1.7° for Explorer 33 and 2.0° for Explorer 35. In no case of the recalculated normal did it deflect more than 1.5° from the $h = 1$ normal, for either spacecraft. These deflections are well within the (95% certainty) error cone angle, which was 7.6° in this case.

Introduction

This study explores the question of the influence of solar wind thermal anisotropy on least squares estimates of interplanetary shock parameters and normals, using a select subset of the eight Rankine-Hugoniot conservation equations. The six equation subset is chosen to avoid using those equations which explicitly involve thermal pressure (or temperature), i.e. except for the dependence of the anisotropy factors on such quantities. In this manner the normal momentum flux equation and the least well understood of the conservation equations, the energy flux equation, are not used in the best fit analysis. Other perhaps equally important reasons may be present to justify such a truncation of the full set of equations (Lepping, 1971). To simplify matters further it is common to use this subset of equations expressed for an isotropic medium, that is for $\xi_1, \xi_2 = 1$, where

$$\xi_i = 1 - \frac{4\pi(P_{\parallel} - P_{\perp})_i}{B_i^2} \quad (1)$$

and where the subscripts refer to before (1) and after (2) the shock, and P_{\parallel} and P_{\perp} are the thermal pressures (ion plus electron) parallel and perpendicular to the magnetic field direction. Then the new subset of equations is independent of thermal pressure. This approximation was employed by Ogilvie and Burlaga (1969), Chao (1970), and Lepping and Argentiero (1971).

This paper is also restricted to the consideration of only the six equation subset but allows for non-unity anisotropy factors. These more general equations, cast in a shock frame of reference, are as follows (Jeffrey and Taniuti, 1964):

$$[\rho U_n]_1^2 = 0 \quad (2)$$

$$[\rho U_n U_t - \xi B_n B_t / 4\pi]_1^2 = 0 \quad (3, 4)$$

$$[U_n B_t - U_t B_n]_1^2 = 0 \quad (5, 6)$$

where $t = t_1$ or t_2 and

$$[B_n]_1^2 = 0 \quad (7)$$

where ρ is the plasma mass density, U_n is the plasma bulk velocity component normal to the shock surface, U_t ($t=t_1$ or t_2) are the velocity components tangential to the shock surface, B_n and B_t ($t=t_1$ or t_2) are the associated normal and tangential components of the magnetic field. The symbol $[A]_1^2 = A_2 - A_1$ refers to the difference between the pre-shock state (A_1) and post-shock state (A_2). Notice only equations (3) and (4) contain the anisotropy factors ξ_1 and ξ_2 .

Generalization of the Problem

Guided by the magnetic field coplanarity theorem, which has been shown independently by several investigators to hold for the general case of an anisotropic medium (Hudson, 1970, Neubauer, 1970 and Chao, 1970), we use the following identifications for the shock front tangential and normal unit vectors (Lepping and Argentiero, 1970):

$$\vec{t}_1 = \frac{\vec{\Delta B}}{|\vec{\Delta B}|} \quad (8)$$

$$\vec{t}_2 = \frac{\vec{B}_1 \times \vec{B}_2}{|\vec{B}_1 \times \vec{B}_2|} \quad (9)$$

$$\vec{n} = \vec{t}_1 \times \vec{t}_2 \quad (10)$$

where
$$\vec{\Delta B} \equiv \vec{B}_2 - \vec{B}_1. \quad (11)$$

These are used along with the transformation equation

$$\vec{U}_i = \vec{V}_i - \vec{V}_s \quad (i = 1,2) \quad (12)$$

in order to separate equations (2) to (7) into two convenient sets, three equations each. Notice that \vec{V}_s is the local shock velocity as measured in some "fixed" frame of reference, such as an inertial system at rest with respect to the sun. Then \vec{V}_i is the bulk plasma velocity measured in the same reference system.

These two sets of three equations each are the following:

a). the shock velocity set

$$\vec{V}_s = v_s \vec{n} \quad (13)$$

where the shock normal is given by

$$\vec{n} = \frac{\vec{\Delta B} \times (\vec{B}_1 \times \vec{B}_2)}{|\vec{\Delta B} \times (\vec{B}_1 \times \vec{B}_2)|} \quad (14)$$

from (8-10) [Note that $\vec{t}_1 \cdot \vec{t}_2 = 0$],

and the shock speed is

$$v_s = \frac{\rho_2 \vec{W} \cdot \vec{n}}{\rho_2 - \rho_1} + \vec{V}_1 \cdot \vec{n} \quad (15)$$

where
$$\vec{W} \equiv \vec{V}_2 - \vec{V}_1. \quad (16)$$

b). the general overdetermination equations

$$\vec{W} \cdot \vec{S} = 0 \quad (17)$$

$$\left(\frac{\rho_2}{\rho_1} \vec{B}_1 - \vec{B}_2 \right) \cdot (\vec{W} \times \vec{S}) = 0 \quad (18)$$

$$\left[\frac{\rho_1 \rho_2}{\rho_2 - \rho_1} \vec{W} (\vec{W} \cdot \vec{\Delta B}) + \frac{\vec{B}_1 Q}{4\pi} \right] \cdot (\vec{\Delta B} \times \vec{S}) = 0 \quad (19)$$

where $\vec{S} \equiv \vec{B}_1 \times \vec{B}_2$ (20)

and $Q \equiv \xi_1 B_1^2 + \xi_2 B_2^2 - (\xi_1 + \xi_2) (\vec{B}_1 \cdot \vec{B}_2)$ (21)

For the isotropic case, where ξ_1 and ξ_2 are equal to unity, Q becomes $|\Delta B|^2$ and equation (19) becomes

$$\left[\frac{\rho_1 \rho_2}{\rho_2 - \rho_1} \vec{W} (\vec{W} \cdot \Delta \vec{B}) + \frac{\vec{B}_1 |\Delta \vec{B}|^2}{4\pi} \right] \cdot (\Delta \vec{B} \times \vec{S}) = 0 \quad (22)$$

Equations (17), (18), and (22) are presently used to obtain estimates of the eleven shock parameters (\vec{B}_1 , \vec{B}_2 , \vec{W} , ρ_1 , ρ_2) and, in turn, the associated normal, (14), by using a least squares fit of these measured field and plasma data. Equation (15) then is used, for an average value of \vec{V}_1 over a reasonable time interval, to obtain the local shock speed.

For the general, i.e., anisotropic, problem the situation is similar except that ξ_1 , $\xi_2 \neq 1$, and this affects Q which appears in equation (19). To study this influence we proceed as follows. Let us define

$$e \equiv \xi_2 / \xi_1 - 1 \quad (23)$$

and assume that

$$(0 <) \xi_1 \leq \xi_2. \quad (24)$$

It is expected that only under very unusual circumstances will this inequality be violated and there is no observational evidence, in the case of interplanetary shocks (or in any other case of which the author is aware), for which it is violated. The condition that $\xi_1 > 0$ is equivalent to assuming that the plasma does not experience the firehose instability (Eviator and Shultz, 1970, and Burlaga, 1971).

Using (23) we see that Q can be written

$$Q = \xi_1 f |\Delta B|^2 \quad (25)$$

where $f \equiv 1 + g\epsilon$ (26)

and where $g \equiv \frac{\vec{B}_2 \cdot \vec{\Delta B}}{|\vec{\Delta B}|^2}$ (27)

With the aid of (8) g can be written

$$g = \frac{B_2 t_1}{|\vec{\Delta B}|} = \frac{1}{1 - \frac{\tan \theta_1}{\tan \theta_2}} \quad (28)$$

where (7) was used to obtain the second equality and where θ_i ($i = 1,2$) is the angle between \vec{B}_i ($i = 1,2$) and the shock normal \vec{n} . Notice that g does not depend on the magnitude of either \vec{B}_1 or \vec{B}_2 but only on their directions relative to \vec{n} . From (1) we see that ξ_1 and ξ_2 (and therefore ϵ from (23)) depend on the magnitude of \vec{B}_1 and \vec{B}_2 , respectively. Hence, Q depends on the vector quantities \vec{B}_1 and \vec{B}_2 .

It will be assumed temporarily that

$$\epsilon \ll 1, \quad (29)$$

where from (24) $\epsilon \geq 0$.

We will exclude discussion of the cases of the so-called switch-on shock (where $\theta_1=0$, $0 < \theta_2 < \pi/2$, and $g = 1$) or the switch-off shock (where $0 < \theta_1 < \pi/2$, $\theta_2 = 0$, $g = 0$), which are special cases of fast and slow shocks, respectively, because of their expected nonevolutionary characteristics (Jeffrey and Taniuti, 1964). These special cases have never been observed in interplanetary space and would be difficult to identify in any case due to the naturally occurring fluctuations around interplanetary shocks. Parallel ($\vec{B} \parallel \vec{n}$) or perpendicular ($\vec{B} \perp \vec{n}$) shocks will also not be considered here. Then the sign of g depends on whether the oblique shock under consideration is fast (+) or slow (-). Temporarily we

restrict our discussion to oblique fast shocks only. Since $0 < \theta_1 < \theta_2 < \pi/2$ in all cases of such shocks, where \vec{n} is defined here in such a way that θ_1 and θ_2 are acute angles, and since $\epsilon \geq 0$, then $g\epsilon$ is always > 0 , but it is expected to be usually $\ll 1$; i.e. $0 \leq \epsilon < g\epsilon \ll 1$ for most realistic cases. Hence f , from (26), will usually be slightly greater than unity. Since ξ_1 is usually slightly less than unity, the product $(\xi_1 f)$ in (25), which must always be positive by our assumptions, is usually closer to unity than ξ_1 . In fact, for all oblique fast shocks (since f is always > 0 in those cases) $\xi_1 f > 0$. Then for such shocks we can define a positive real parameter h to be

$$h \equiv (\xi_1 f)^{1/2} \quad (30)$$

which we will refer to as the "anisotropy parameter".

To estimate this parameter we use the results of Chao and Goldstein (1971) from their application of the R-H conservation equations to find ξ_1 and ξ_2 . [Also see Hundhausen et al., 1967] They use the entire eight equation set and retain only those values which give reasonable electron temperatures as a by-product of their analysis; this turns out to be sufficiently restrictive. Their method of finding the ξ 's from the characteristics of Alfvén waves around the shock is probably not as reliable and its results will not be considered here. Taking averages of the ξ -values that they obtain for four shocks (two fast and two slow), we find that $\xi_1 = 0.77 \pm 0.10$ and $\xi_2 = 0.80 \pm 0.11$. Hence, we arrive at characteristic ϵ 's between 0.00 and 0.19, and we retain $\epsilon \approx 0.04$ as a most probable value. Since $\tan \theta_1 / \tan \theta_2 \approx 1/3$ is a reasonable characteristic value for this ratio, we find that $f \approx 1.06$, and hence

$h \approx 0.90$ using $\xi_1 \approx 0.77$.

For the remainder of this study, restriction (29), that $\epsilon \ll 1$, will no longer hold. It was employed only to facilitate discussion of the most common cases of oblique fast shocks.

The Field Weighting Theorem

We now proceed to show, for fast and slow shocks, the significance of the anisotropy parameter, h , and the reason for choosing a 1/2-power functional relation in its definition (30). Note that (30) will hold for slow shocks, provided $f > 0$. If $f < 0$ then h becomes imaginary and (22)' below changes structure (second term assumes an explicit negative sign). Hence, the following theorem fails for slow shocks with $f < 0$. Such cases can be easily handled by a simple modification of the theorem. We begin by defining an effective magnetic field \vec{H}_i ($i=1,2$) by

$$\vec{H}_i = h \vec{B}_i \quad (i=1,2). \quad (31)$$

That is, it is the true magnetic field multiplied by the weight factor h , which as we have seen contains the effects of the pressure anisotropy and the influence of the shock normal's direction through θ_1 and θ_2 . By multiplying the general equations (17), (18), and (19) by h^2 , h^3 , and h^4 , respectively, and using the defining equations (25), (30), and (31) we obtain a new set of overdetermination equations, which are identical to the "isotropic" equations (17), (18), and (22) except now \vec{B}_i is replaced by \vec{H}_i ($i=1,2$) and the new quantities \vec{S} and $\vec{\Delta H}$ become $\vec{S} = \vec{H}_1 \times \vec{H}_2$ and $\vec{\Delta H} = \vec{H}_2 - \vec{H}_1$. Also multiplying top and bottom of the right side of equation (14) by h^3 yields, in the same manner, an identical equation for \vec{n} except in terms of \vec{H} 's instead of \vec{B} 's. Equations (13), (15), and (16) remain unchanged. Henceforth, for future reference equations (14), (17), (18),

and (22) in the \vec{H} -domain will be designated as (14)', (17)', (18)' and (22)'. So by considering the effective field \vec{H}_i instead of the real field \vec{B}_i our general six-equation set (i.e. including anisotropy) transforms into the same functional forms previously used in the isotropic case, and thus the expression for the normal, (14)', is also formally identical for the isotropic and anisotropic cases. The latter result is, of course, a trivial consequence of the fact that the magnetic field coplanarity theorem holds for the general anisotropic case. The ability to homologously transform the field by (31) and obtain "anisotropic" equations in the \vec{H} -domain corresponding identically to the "isotropic" equations in the \vec{B} -domain has important practical significance with regard to a least-squares best-fit program of estimating shock normals based on the isotropic equations (Lepping and Argentiero, 1971) for fast shocks, and for slow shocks provided $f > 0$. Without modification of such a program, it can be used for those shocks (with $f > 0$) in whose cases the isotropic approximation does not hold by applying a preconditioned weighting of the \vec{B} -field by (31) before use of the program. This we refer to as the Field Weighting Theorem.

In practice it might be necessary to carry out the scheme with h initially set equal to unity (or some other first trial) and obtain a first rough estimate of the shock parameters and the normal. The resulting normal, as well as the various shock parameters, will then reflect the inaccuracy of using an inexact value for h even though (14) and (14)' are formally identical (i.e. even though \vec{n} does not explicitly depend on h). This inaccuracy stems from the implicit dependence of the field components (as well as the plasma parameters) on the value of h through the least-squares best-fitting process; as h changes the components of

the \vec{H} 's in (14)' (or equivalently the \vec{B} 's in (14)) change with respect to each other. After this first computation with a trial h it is a simple matter to use the resulting "best-fit" estimates of the quantities \vec{B}_1 and \vec{B}_2 along with the estimate of \vec{n} to obtain first "best-fit" estimates of θ_1 and θ_2 . These combined with estimates of ξ_1 and ξ_2 (either from an educated guess or through the use of the remaining conservation equations or from reliable direct measurements) enables one to calculate a better "best-fit" estimate of h . Then all of the input \vec{B} -field data can be altered by multiplication by h , as in (31), and the effective field \vec{H} used in a repeated operation of the least-squares process to obtain an updated set of shock parameters. This process can be repeated as often as one wishes, but presently a single iteration beyond $h=h(\text{trial})$ is probably sufficient since the factors ξ_1 and ξ_2 are usually estimated only roughly. For fast shocks h is not a very sensitive function of $\frac{\tan \theta_1}{\tan \theta_2}$, ξ_1 , and ξ_2 provided $\xi_1 \leq \xi_2 \leq 1$ and $\xi_2 > 0.8$, as we will show below.

Expected Range of h for Fast Shocks

We now wish to examine for fast shocks the range of h , which from (23), (26), (28), and (30) can be expressed as

$$h = \left(\frac{\xi_2 - \xi_1 R}{1 - R} \right)^{1/2} \tag{32}$$

where $R \equiv \frac{\tan \theta_1}{\tan \theta_2}$. (33)

First we consider it reasonable to expect unity for an upper bound on ξ_2 , and retain $\xi_1 \leq \xi_2$. Table 1a gives h as a function of ξ_1 for $\xi_2 = 1.0$ and for the cases where $R = \frac{1}{10}, \frac{1}{4}, \frac{1}{3},$ and $\frac{1}{2}$. Notice that for this condition (i.e. $\xi_2=1$) $h \geq 1$. Tables 1b and 1c give h as a function of ξ_1 for ξ_2 equal to 0.9 and 0.8, respectively, for the same values of R .

TABLE 1

Values of h for $\xi_2=1.0$ (a), 0.9 (b), 0.8 (c) and for $\xi_1 \geq 0.50$

$\xi_1 \backslash R$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	
0.50	1.027	1.080	1.118	1.225	(a)
0.75	1.014	1.041	1.061	1.118	$\xi_2 = 1.0$
1.00	1.000	1.000	1.000	1.000	
0.50	0.972	1.017	1.049	1.140	(b)
0.70	0.960	0.983	1.000	1.049	$\xi_2 = 0.9$
0.90	0.949	0.949	0.949	0.949	
0.50	0.913	0.949	0.975	1.049	(c)
0.65	0.904	0.922	0.935	0.975	$\xi_2 = 0.8$
0.80	0.894	0.894	0.894	0.894	

In these tables, for intermediate values of ξ_1 and fixed R, values of h can be linearly interpolated to better than 0.2% accuracy. For the extreme case of a negligible R (or $R \equiv 0$) $h = \sqrt{\xi_2}$, independent of ξ_1 , and therefore $h = 1.000, 0.949, \text{ and } 0.894$ for $\xi_2 = 1.000, 0.900, \text{ and } 0.800$, respectively. From an examination of actual interplanetary shocks the value $R = 1/2$ appears to be a reasonable arbitrarily chosen upper limit. Many very weak fast shocks are expected to have $R > 1/2$ but these are difficult to observe. Hence, for these rather extreme ranges of ξ_1, ξ_2 , and R we see that for fast shocks h is restricted to $h = 1.06 \pm 0.17$ provided that $\xi_1 \leq \xi_2 \leq 1$ is a reasonable assumption. The value of $h = 0.90$ which we have derived from the Chao and Goldstein (1971) results lies at the lower end of this range. Similar tables can, of course, be generated for slow shocks where h would be shown in general to deviate much more markedly from unity due to the fact that $R > 1$ for such shocks. Hence, the influence of anisotropy on slow shocks is greater.

A Fast Shock Example: January 26, 1968, ≈ 1430 UT

The January 26, 1968 fast shock detected by the plasma and magnetic field sensors of Explorers 33 and 35 has been studied in terms of accurate estimates of the shock's parameters and its normal (Lepping, 1971). In that investigation the less general conservation equations, i.e., those for an isotropic medium ($\xi_1, \xi_2 = 1$), were used in a least-squares best-fit program for the estimation. We show in Table 2 the results of the calculations repeated, using the extreme values of h given above, $h=0.90$ and $h=1.22$, respectively. The previous results of $h=1.00$ are also listed for comparison. One can readily see that for this characteristic range of h, and for both spacecraft observations of the shock, the influence of thermal anisotropy is insignificant with regard to all parameters and for the normal (given by the components n_R, n_T, n_N) as well. In going

TABLE 2

January 26, 1968 Shock Parameters
Best Fit Values for an 18 Minute Analysis Interval

Parameter*	Best Fit Value for Explorer 33			Best Fit Value for Explorer 35		
	h=0.90	1.00	1.22	h=0.90	1.00	1.22
$B_{1R} (\gamma)$	-1.60	-1.59	-1.59	-0.23	-0.24	-0.24
B_{1T}	-3.16	-3.16	-3.16	-3.07	-3.07	-3.07
B_{1N}	-3.49	-3.49	-3.49	-3.83	-3.83	-3.84
B_{2R}	-5.13	-5.15	-5.20	-3.39	-3.41	-3.43
B_{2T}	-6.61	-6.60	-6.57	-6.15	-6.09	-5.94
B_{2N}	-7.56	-7.54	-7.46	-8.34	-8.26	-8.05
W_R (km/sec)	78.5	78.6	78.9	84.8	85.6	87.7
W_T	-38.1	-37.0	-34.0	-35.4	-35.0	-34.3
W_N	-28.6	-28.3	-27.7	-24.4	-24.2	-23.7
N_1 (#/cm ³)	4.18	4.19	4.19	4.43	4.45	4.50
N_2	9.69	9.67	9.61	10.60	10.52	10.34
n_R	0.828	0.826	0.821	0.854	0.850	0.838
n_T	-0.443	-0.440	-0.434	-0.413	-0.416	-0.423
n_N	-0.344	-0.352	-0.372	-0.316	-0.324	-0.345
Quality	1.05	1.05	1.05	1.04	1.04	1.03

*Vector quantities are given in the $\hat{R}-\hat{T}-\hat{N}$ coordinate system, centered at the spacecraft of interest, such that \hat{R} points away from the sun parallel to the ecliptic plane, \hat{N} is perpendicular to the plane and "northward", and $\hat{T}=\hat{N}\times\hat{R}$.

from $h = 0.9$ to $h = 1.22$ the normal deflects through 1.7° for Explorer 33 and 2.0° for Explorer 35, and neither recalculated normal deviated more than 1.5° from the $h = 1$ normal, in either spacecraft's case. These deviations are all well within the calculated (95% certainty) error-cone angle of 7.6° . Since this example is believed to be representative of oblique fast shocks in general (unless $R > 1/2$ or $\xi_1 < 0.5$ or both), we see that the influence of thermal anisotropy on estimating interplanetary fast shock normals is expected to be negligible, changing it by only a few degrees at most, and the deviations in the shock parameters due to $h \neq 1$ are equally unimportant. It is obvious, for instance, that the differences between the best-fit shock parameters for Explorers 33 and 35 are not accounted for by incorrect anisotropy considerations. More likely these differences are real and are explained by the changes in the field and plasma conditions over distances comparable to the separation of the two spacecraft (≈ 43.5 earth radii) and over the associated time delay (≈ 89 sec) of the two observations. Since the two spacecraft at the time of the shock sightings were located in that general vicinity of the earth's bow shock where upstream waves and particles fluxes have been observed, such influences could be considered as a possible specific explanation of these differences. [The spacecraft were 76.6 (Expl. 33) and 56.9 (Expl. 35) earth radii sunward of the earth during the shock passage.] If actually present, such influences could markedly change the shock parameters from one interplanetary observation point to the other and yet have a negligible effect on the shock front's ability to remain plane on the scale of interest here. However, the condition for expecting these influences to be present, i.e., that the interplanetary magnetic field line (pre-shock field in this case) passing through either spacecraft position be connected to the earth's

bow shock (Fairfield, 1969), does not hold in this case. Therefore, these differences are more likely related to true interplanetary conditions at that time rather than to phenomena associated with the earth.

In table 2 a quality index for the shock estimate is also shown. This index is defined as the square root of the ratio of the total number of points of all shock parameters used in the analysis to the standard σ -weighted least-squares loss function at convergence (Lepping and Argentiero, 1971). This index is commonly very near to unity for characteristic interplanetary shocks provided reasonable σ -weights (usually rms deviations) are used in the loss function for all parameters. The table shows that the quality of the estimates is, to three place accuracy, nearly independent of the value of h between 0.90 and 1.22. The lowest value of 1.03 for $h = 1.22$, Explorer 35, suggests (see Table 1a) that a combination such as $\xi_1 \approx 0.5$ and $\xi_2 \approx 1.0$ for the January 26, 1968 shock is not likely, where R was 0.42 for this case. All that can be safely said here is that $\Delta\xi (\equiv \xi_2 - \xi_1)$ was probably less than 0.4 or so for this shock on the weak assumption that the overall best fit analysis and model was accurate enough to resolve quality with respect to small changes in h . The quality of the estimates of the shock parameters, in most cases, must be insensitive to small changes of h from unity, and large changes are expected to occur only for slow shocks.

Discussion and Conclusions

We have shown, for most realistic fast shocks, that best-fit shock parameters and the associated normals are expected to be only weakly dependent upon thermal anisotropy in the vicinity of the shock.

[In theoretically estimating pre-shock electron temperatures by using the full Rankine-Hugoniot set of equations Chao (1971) has shown, for a sample set of 18 shocks (11 fast and 7 slow), that such estimates also are only very weakly dependent on the values of ξ_1 and ξ_2]. Conversely, the restricted subset of equations used in the best-fit procedure are therefore, for fast shocks, unable to effectively resolve the ξ_1 and ξ_2 factors even when the normal to the shock is known through other means, for instance geometrical methods. Only reliable higher moment equations, or other independent methods, can be used to ascertain the values of the anisotropy factors. From the point of view of accurately estimating shock normals, however, this is a fortunate circumstance. For slow shocks the lower moment equations should be of greater aid in obtaining ξ_1 and ξ_2 , but again conversely the normals for these shocks should be less reliably estimated due to their greater dependence on h , which is usually not very well known from independent sources.

For fast shocks, and slow shocks with $f > 0$, when either of whose value of h deviates significantly from unity and provided ξ_1 and ξ_2 are estimated with acceptable accuracy, the Field Weighting Theorem should furnish a practical means of improving the estimates of shock normals when the best-fit technique is employed.

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REFERENCES

- Burlaga, L. F., Hydromagnetic Waves and Discontinuities in the Solar Wind, NASA-X-692-71-95, March 1971. To appear in Space Sci. Rev.
- Chao, J. K., Interplanetary Collisionless Shock Waves, M.I.T. Report No. CSR TR-70-3, February, 1970.
- Chao, J. K., Evidence for Waves and/or Turbulence in the Vicinity of Shocks in Space, Proceedings of Cosmic Plasma Physics Conference, ESRIN, Frascati, Italy, September 20-24, 1971.
- Chao, J. K. and B. E. Goldstein, Thermal Anisotropies in the Vicinity of Solar Wind Shocks, M.I.T. Report No. CSR-P-71-53, February, 1971.
- Eviatar, A. and M. Schulz, Ion-Temperature Anisotropies and the Structure of the Solar Wind, Planet. Space Sci., 18, 321, 1970.
- Fairfield, D. H., Bow Shock Associated Waves Observed in the Far Upstream Interplanetary Medium, J. Geophys. Res., 74, 3541, 1969.
- Hudson, P. D., Discontinuities in an Anisotropic Plasma and Their Identification in the Solar Wind, Planet Space Sci., 18, 1611, 1970.
- Hundhausen, A. J., S. M. Bame, and N. F. Ness, Solar Wind Thermal Anisotropies: Vela 3 and IMP 3, J. Geophys. Res., 72, 5265, 1967.
- Jeffrey, A. and T. Taninti, Non-Linear Wave Propagation, Mathematics in Science and Engineering Series, Academic Press, New York, 1964.
- Lepping, R. P., A Two Spacecraft Test of a Single Spacecraft Method of Estimating Shock Normals, submitted to the Proceedings of the Solar Wind Conference, Asilomar, Pacific Grove, Calif. March 21-26, 1971 also NASA-X-692-71-138, March 1971.
- Lepping, R. P. and P. D. Argentiero, Improved Shock Normals Obtained from Combined Magnetic Field and Plasma Data from a Single Spacecraft, NASA-X-692-70-276, July 1970. See esp. Appendix A.
- Lepping, R. P. and P. D. Argentiero, Single Spacecraft Method of Estimating Shock Normals, J. Geophys. Res., 76, 4349, 1971.
- Neubauer, F. M., Jump Relations for Shock in an Anisotropic Magnetized Plasma, Z. Physik, 237, 205, 1970.
- Ogilvie, K. W. and L. F. Burlaga, Hydromagnetic Shocks in the Solar Wind, Solar Phys., 8, 422, 1969.