A METHOD FOR NONLINEAR EXPONENTIAL
REGRESSION ANALYSIS

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October 29, 1971

NASA

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This report presents a computer-oriented technique for performing a nonlinear exponential regression analysis on decay-type experimental data. The technique involves the least squares procedure wherein the nonlinear problem is linearized by expansion in a Taylor series. A linear curve-fit procedure for determining the initial nominal estimates for the unknown exponential model parameters is included as an integral part of this technique. A correction matrix is derived and then applied to the nominal estimate to produce an improved set of model parameters. The solution cycle is repeated until some predetermined criterion is satisfied.
ACKNOWLEDGMENT

The author acknowledges the assistance of Mr. M. C. Davidson of the Space Sciences Laboratory who provided the physical process data used in the analysis herein.
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INTRODUCTION

The investigation of physical processes frequently requires the use of models that simulate or describe the processes. A model is often chosen so that certain variables interact in the model according to physical theories associated with the particular process. Formulation of a model often results in the form referred to as mathematical models. This is the familiar representation of the physical process by one or more equations that encompass the physical theory. A model equation contains identified independent variables and unknown parameters. Regression analysis is the statistical tool used to determine these unknown parameters, thereby providing an analytical representation of the experimental data.

The general procedure in regression analysis is to take partial derivatives of a specific model-dependent minimizing function. These partial derivatives are taken with respect to each of the unknown model parameters. If the set of equations obtained by setting these partial derivatives equal to zero can be solved by the usual algebraic methods, the fitting or analytical representation is accomplished. However, if these equations are transcendental in one or more of the unknown parameters, they cannot be solved by the usual algebraic methods.

The processes of particular interest in this report are those that can be described by decaying exponential forms. A mathematical model that contains more than one exponential term results in a set of transcendental normal equations if conventional forms of regression analysis are used. Thus, one usually resorts to iterative methods that require initial estimates for the parameters. The method described herein involves the least squares procedure, whereby the nonlinear problem is linearized by expanding in a Taylor series. In this iterative method, we first develop a starting nominal guess for the model parameters. A correction matrix is derived and then applied to the nominal guess to produce an improved set of model parameters. This procedure is continued until some predetermined criterion is satisfied. The number of iterations necessary for convergence is closely related to this criterion, the initial estimates, and the form of the exponential model.
Additional information on the various methods of curve-fitting decay-type data to a sum of exponentials is given in References 1 through 5. Procedures for obtaining the initial parameter estimates are discussed in References 6 through 8. It is noted that the initial estimate procedure herein is not restricted to equally spaced data.

Application of the procedure is illustrated with data obtained from a particular process concerning the anodic oxidation of metals. In this process one expects an exponential or logarithmic behavior. From an analysis of the results, it is concluded that an adequate two-term exponential representation of the data is obtained. Thus, the analytical representation of the physical process data is accomplished using an exponential decay-type model.

**MATHEMATICAL THEORY FOR EXPONENTIAL REGRESSION ANALYSIS**

In general, we are given the set of observed values \( \{(t_1, f_1^0), (t_2, f_2^0), \ldots, (t_n, f_n^0)\} \). We assume that the function to be fitted to these data is of the following form:

\[
f^c = A_1 e^{-B_1 t} + A_2 e^{-B_2 t} + \ldots + A_m e^{-B_m t} + K = f(A, B, K, t) ,
\]

where \( f^c \) represents the calculated value of the response and \( K, A_i, B_i (i = 1, 2, \ldots, m) \) are the \( 2m + 1 \) parameters to be estimated. The independent variable is time \( t \). We first consider the simple case \( m = 1 \) and \( K = 0 \) which results in a conventional least squares solution for the unknowns \( A_1 \) and \( B_1 \). We have the following model:

\[
f^c = A_1 e^{-B_1 t} ,
\]

By taking the natural log of both sides,

\[
\ln f^c = \ln A_1 - B_1 t .
\]
Let

\[ C = \ln A_i \]

and

\[ Y = \ln f^o \]  \quad (4)

Then,

\[ Y = C - B_1 t \]  \quad (5)

The least squares solutions for \( C \) and \( B_1 \) then become

\[
B_1 = - \frac{m \sum_{i=1}^{n} t_i Y_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} Y_i}{m \sum_{i=1}^{n} t_i^2 - \left( \sum_{i=1}^{n} t_i \right)^2} \]  \quad (6)

and

\[
C = \left[ \frac{\sum_{i=1}^{n} t_i \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i Y_i}{m \sum_{i=1}^{n} t_i^2 - \left( \sum_{i=1}^{n} t_i \right)^2} \right] \]  \quad (7)

For \( m > 1 \) and \( K \neq 0 \) we proceed as follows. We write the parameters as initial approximations or nominal values plus unknown corrections; that is,
\[ A_1 = \tilde{A}_1 + \Delta A_1 \]
\[ B_1 = \tilde{B}_1 + \Delta B_1 \]
\[ A_2 = \tilde{A}_2 + \Delta A_2 \]
\[ B_2 = \tilde{B}_2 + \Delta B_2 \]
\[ \vdots \]
\[ A_m = \tilde{A}_m + \Delta A_m \]
\[ B_m = \tilde{B}_m + \Delta B_m \]
\[ K = \tilde{K} + \Delta K \]  

(8)

For any assumed functional form, the following condition equation can be written:

\[ f_i^o - f_i^c - V_{fi} = 0 \]  

(9)

where

\[ f_i^o \] = observed values of the response variable,
\[ f_i^c \] = calculated values of the response variable,

and

\[ V_{fi} \] = residuals associated with the response variable.

If we substitute equation (8) into equation (1) and the result into equation (9), we obtain

\[ f_i^o - f_i \left( \tilde{A}_1 + \Delta A_1, \tilde{B}_1 + \Delta B_1, \ldots, \tilde{A}_m + \Delta A_m, \tilde{B}_m + \Delta B_m, \tilde{K} + \Delta K, t \right) - V_{fi} = 0 \]  

(10)
Expanding the left side of equation (11) in a Taylor series about the estimates \( \tilde{A}_1, \tilde{B}_1, \ldots, \tilde{A}_m, \tilde{B}_m, \tilde{K} \) and neglecting higher order terms than the first, we have (i = 1, 2, \ldots, n)

\[
V_{fi} = N_i - F_{1i} \Delta A_1 - F_{2i} \Delta B_1 - F_{3i} \Delta A_2 - F_{4i} \Delta B_2 - \ldots \\
- F_{2m-1,i} \Delta A_m - F_{2m,i} \Delta B_m - F_{2m+1,i} \Delta K
\]

(12)

where

\[
F_{1i} = \frac{\partial f_i}{\partial A_1} \bigg|_0 \\
F_{4i} = \frac{\partial f_i}{\partial B_2} \bigg|_0 \\
F_{2i} = \frac{\partial f_i}{\partial B_1} \bigg|_0 \\
F_{2m-1,i} = \frac{\partial f_i}{\partial A_m} \bigg|_0 \\
F_{2m,i} = \frac{\partial f_i}{\partial B_m} \bigg|_0 \\
F_{2m+1,i} = \frac{\partial f_i}{\partial K} \bigg|_0
\]

(13)
The \( n \) equations given by equation (12) are the linearized condition or residual equations. According to the Gauss least squares principle, the best representation of the data is that which makes the weighted sum of the squares of the residuals a minimum. Thus, the minimizing function is

\[
S = f \left( \Delta A_1, \Delta B_1, \ldots, \Delta A_m, \Delta B_m, \Delta K \right)
\]

\[
= W_1 V_1^2 + W_2 V_2^2 + \ldots + W_n V_n^2
\]  

(15)

The \( 2m + 1 \) linear algebraic equations for determining the \( \Delta \) increments to the initial estimates are now obtained by taking the partial derivative of \( S \) with respect to each of the unknown corrections and setting the result equal to zero. These equations, frequently referred to as the normal equations, are given by

\[
\frac{\partial S}{\partial \Delta A_1} = 2W_1 V_1 \frac{\partial V_1}{\partial \Delta A_1} + 2W_2 V_2 \frac{\partial V_2}{\partial \Delta A_1} + \ldots + 2W_n V_n \frac{\partial V_n}{\partial \Delta A_1} = 0
\]

\[
\frac{\partial S}{\partial \Delta B_1} = 2W_1 V_1 \frac{\partial V_1}{\partial \Delta B_1} + 2W_2 V_2 \frac{\partial V_2}{\partial \Delta B_1} + \ldots + 2W_n V_n \frac{\partial V_n}{\partial \Delta B_1} = 0
\]

\[
\vdots
\]

\[
\frac{\partial S}{\partial \Delta K} = 2W_1 V_1 \frac{\partial V_1}{\partial \Delta K} + 2W_2 V_2 \frac{\partial V_2}{\partial \Delta K} + \ldots + 2W_n V_n \frac{\partial V_n}{\partial \Delta K} = 0
\]

(16)
or

\[
\begin{align*}
-W_{f11}V_{f1} & - W_{f21}V_{f2} - \cdots - W_{fn1}V_{fn} = 0 \\
-W_{f12}V_{f1} & - W_{f22}V_{f2} - \cdots - W_{fn2}V_{fn} = 0 \\
& \vdots \\
-W_{f1n}V_{f1} & - W_{f2n}V_{f2} - \cdots - W_{fnn}V_{fnn} = 0
\end{align*}
\]  \hspace{1cm} (17)

We now express equation (12) in the following more convenient matrix expression

\[
\bar{V} + \bar{B} \bar{\Delta} - \bar{N} = 0
\]  \hspace{1cm} (18)

where

\[
\bar{V} = \begin{bmatrix} V_{f1} \\ V_{f2} \\ \vdots \\ V_{fn} \end{bmatrix} \hspace{1cm} \text{[nx1]}
\]

\[
\bar{B} = \begin{bmatrix} F_{f11}F_{21} & \cdots & F_{f2m+1,1} \\ F_{f12}F_{22} & \cdots & F_{f2m+1,2} \\ \vdots & \vdots & \vdots \\ F_{f1n}F_{2n} & \cdots & F_{f2m+1,n} \end{bmatrix} \hspace{1cm} \text{[nx(2m+1)]}
\]  \hspace{1cm} (20)
By using the denotations for $\mathbf{V}$ and $\mathbf{B}$ as given by equations (19) and (20), respectively, we can also rewrite equation (17) as

$$\mathbf{B}^T \overline{\mathbf{W}} \mathbf{V} = 0 \quad , \quad (23)$$

where

$$\overline{\mathbf{W}} = \begin{bmatrix} \frac{\sigma_o^2}{\sigma_{f1}^2} & 0 & \cdots & 0 \\ 0 & \frac{\sigma_o^2}{\sigma_{f2}^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\sigma_o^2}{\sigma_{fn}^2} \end{bmatrix} \quad , \quad (24)$$
Solving equation (18) for $\overline{V}$, 

$$\overline{V} = \overline{N} - \overline{B} \overline{\Delta} \quad .$$  \hspace{1cm} (25)

Substituting equation (25) into equation (23),

$$\overline{B}^T \overline{W} (\overline{N} - \overline{B} \overline{\Delta}) = 0 \quad .$$  \hspace{1cm} (26)

Solving for $\overline{\Delta}$,

$$\overline{B}^T \overline{W} \overline{N} - \overline{B}^T \overline{W} \overline{B} \overline{\Delta} = 0$$

(27)

or

$$\overline{\Delta} = (\overline{B}^T \overline{W} \overline{B})^{-1} (\overline{B}^T \overline{W} \overline{N}) \quad .$$  \hspace{1cm} (28)

An improved set of values for the parameters is then given by equation (8). The process or cycle is repeated to produce the corrections resulting from the second cycle. These corrections are then added to the estimates from the first cycle:

$$\begin{align*}
A_1^I &= \Delta A_1^I + A_1 \\
B_1^I &= \Delta B_1^I + B_1 \\
A_2^I &= \Delta A_2^I + A_2 \\
B_2^I &= \Delta B_2^I + B_2 \\
&\vdots \\
A_m^I &= \Delta A_m^I + A_m \\
B_m^I &= \Delta B_m^I + B_m \\
K^I &= \Delta K^I + K
\end{align*}$$

(29)
These values represent an improved set of estimates to use for the third cycle. An iterative procedure is thus set up for improving the parameter estimates to any prescribed degree of accuracy consistent with the accuracy of the observed data.

The standard algorithm is based on obtaining a nominal solution that, hopefully, converges to the correct solution. The algorithm is summarized as follows:

1. Let $\hat{A}^k$ denote the kth nominal; linearize about $\hat{A}^k$.
2. Solve the resulting linear least squares problem.
3. Use the new solution as the new nominal.
4. Check for convergence. If convergence has not occurred, repeat steps 1 through 3.

The standard deviation of each of the converged parameters is calculated from

$$\bar{\sigma} = \sigma \bar{c},$$

where

$$\sigma_f = \left[ \frac{\sum_{i=1}^{n} \left( f_i^o - f_i^c \right)^2}{n - 2m - 1} \right]^{1/2}$$

and

$$\bar{c} = \left[ \begin{array}{c} \sqrt{c_{11}} \\ \sqrt{c_{22}} \\ \vdots \\ \sqrt{c_{2m+1,2m+1}} \end{array} \right]$$

(30)

(31)

(32)
The \( c \) elements in equation (32) refer to diagonal elements in the inverse matrix \((\overline{B}^T \overline{W} \overline{B})^{-1}\).

**PROCEDURE FOR OBTAINING THE INITIAL PARAMETER ESTIMATES**

"Peeling-Off" Approach

An iterative method for nonlinear exponential regression analysis was developed in the previous section. Inherent in this method is a requirement for initial estimates of the parameters. This section presents a least squares "peeling-off" procedure for arriving at these initial estimates.

Our assumed exponential model is of the form given by equation (1). Generally speaking, if we plot decay-type data in the form \( \ln f^0 \) against \( t \) where \( f^0 \) is the observed response and \( t \) is the independent variable, then for large \( t \) the curve is approximately a straight line. Consider the following Figure 1.

![Figure 1. Logarithmic time decay illustration.](image)

If we fit a straight line to the last three data points by the method of least squares, the assumed form is

\[
\ln f^0 = -\overline{B}_m t + D
\]

(33)

However, this is equivalent to the equation

\[
f^0 = \tilde{A}_m e^{-\tilde{B}_m t}
\]
where

\[ D = \ln \tilde{A}_m \]  \hspace{1cm} (34)

The coefficients \( \tilde{B}_m \) and \( D \) are given by

\[
D = \frac{\sum_{i=n}^{n-2} t_i^2 \sum_{i=n}^{n-2} \ln f_i^o - \sum_{i=n}^{n-2} t_i \sum_{i=n}^{n-2} t_i \ln f_i^o}{3 \sum_{i=n}^{n-2} t_i^2 - \left( \sum_{i=n}^{n-2} t_i \right)^2} \]  \hspace{1cm} (35)

and

\[
\tilde{B}_m = - \left[ \frac{3 \sum_{i=n}^{n-2} t_i \ln f_i^o - \sum_{i=n}^{n-2} t_i \sum_{i=n}^{n-2} \ln f_i^o}{3 \sum_{i=n}^{n-2} t_i^2 - \left( \sum_{i=n}^{n-2} t_i \right)^2} \right] \]  \hspace{1cm} (36)

Thus, we have determined the least squares values for \( \tilde{A}_m \) and \( \tilde{B}_m \). We can then obtain values for the residuals from \( i = 0, 1, 2 \)

\[
R_{n-i} = f_{n-i}^o - \tilde{B}_m t_{n-i} \]  \hspace{1cm} (37)

We now take the next three data points at \( t_{n-3}, t_{n-4}, \) and \( t_{n-5} \) and subtract the corresponding term \( \tilde{A}_m e^{-\tilde{B}_m t} \) and the arbitrary constant \( \tilde{K} \) from \( f^o \) to obtain the following residuals
Next, a straight line is fitted to the data for $\ln |R_{n-i}|$ against $t_{n-i}$, $i = 3, 4, 5$. This determines $\tilde{A}_{m-1}$ and $\tilde{B}_{m-1}$. The residuals for the next three data points are determined from

$$
R_{n-6} = f^0_{n-6} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-6}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-6}} - K
$$

$$
R_{n-7} = f^0_{n-7} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-7}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-7}} - K
$$

$$
R_{n-8} = f^0_{n-8} - \tilde{A}_{m-1} e^{-\tilde{B}_{m-1} t_{n-8}} - \tilde{A}_m e^{-\tilde{B}_m t_{n-8}} - K
$$

We now fit a straight line to $\ln |R_{n-i}|$ against $t_{n-i}$, $i = 6, 7, 8$. This determines $\tilde{A}_{m-2}$ and $\tilde{B}_{m-2}$. We continue this process until all the $\tilde{A}$'s and $\tilde{B}$'s are determined. In general, $\tilde{A}_1$ and $\tilde{B}_1$ are determined from a set that contains more than three data points. That is, the points remaining after $\tilde{A}_2$ and $\tilde{B}_2$ have been determined are used for determining $\tilde{A}_1$ and $\tilde{B}_1$. It is noted that since three points are chosen as a minimum for calculating a particular $\tilde{A}$ and $\tilde{B}$, we must have $n \geq 3m$ where $n$ is the number of data points. At this point we calculate the weighted sum of squares of the deviations using the initial parameter estimates.
where

\[ W_i = \frac{1}{\sigma_{f_i}^2} \]

\( f_i^o \) = observed response ,

and

\( f_i^c \) = fitted response.

**Iteration Philosophy**

The iteration logic can be summarized by the following steps:

1. Repeat the process, but use the last four data points to obtain the initial estimates of \( \tilde{A}_m \) and \( \tilde{B}_m \). The next three data points are used to obtain \( \tilde{A}_{m-1} \), \( \tilde{B}_{m-1} \), etc., for other \( \tilde{A} \)'s and \( \tilde{B} \)'s. This yields a second set of parameter estimates from which we can calculate another \( \tilde{F} \); call it \( \tilde{F}_2 \).

2. Use the last five data points and obtain a third set of parameters which yield \( \tilde{F}_3 \).

3. We continue this, always keeping three points as a minimum in determining \( \tilde{A}_i \) and \( \tilde{B}_i \).

4. Increase the constant to \( \tilde{K} + 0.05 \tilde{K} \) and repeat steps 1 through 3. Terminate when the constant reaches a predetermined value.

5. Parameter estimates yielding the minimum \( \tilde{F} \) are chosen as the initial estimates.
COMPUTER PROGRAM DEVELOPMENT

The computer programs to implement the previously developed theory for exponential regression analysis were organized and developed according to two general types of exponential models. One concerns a single exponential and the sum of exponentials without a constant, and the other concerns the sum of exponentials with a constant included. Two highly flexible computer programs were thus developed for the MSFC UNIVAC 1108 digital computer. Each program contains double-precision capability and SC-4020 plotting procedures. In addition, the "peeling-off" procedure for obtaining initial parameter estimates is an integral part of each program segment.

The logic flow for Programs I and II is depicted in Figures 2 and 3. The parameter NCASES is the number of cases of data processed in each program. The models that can be investigated in Program I are

Model I: \[ f^C = A_1e^{-B_1t} + A_2e^{-B_2t} + A_3e^{-B_3t} + K \]

and

Model II: \[ f^C = A_1e^{-B_1t} + A_2e^{-B_2t} + K \]

Those models that can be investigated in Program II are

Model III: \[ f^C = A_1e^{-B_1t} + A_2e^{-B_2t} + A_3e^{-B_3t} \]

Model IV: \[ f^C = A_1e^{-B_1t} + A_2e^{-B_2t} \]

and

Model V: \[ f^C = A_1e^{-B_1t} \]

The characteristics of both programs are summarized in Table 1. It should be noted that both programs can be easily extended to include additional exponential terms if desired.
Figure 2. Program I block diagram summary.
TABLE 1. COMPUTER PROGRAM SUMMARY FOR EXPONENTIAL CURVE FITTING

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<thead>
<tr>
<th>Program Number&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Exponential Model Fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(1) $A_{i}, B_{i}, K : i = 1, 2, 3$</td>
</tr>
<tr>
<td></td>
<td>(2) $A_{i}, B_{i}, K : i = 1, 2$</td>
</tr>
<tr>
<td>II</td>
<td>(1) $A_{i}, B_{i} : i = 1, 2, 3$</td>
</tr>
<tr>
<td></td>
<td>(2) $A_{i}, B_{i} : i = 1, 2$</td>
</tr>
<tr>
<td></td>
<td>(3) $A_{i}, B_{i}$</td>
</tr>
</tbody>
</table>

<sup>a</sup> Each program has a built-in capability for obtaining initial parameter estimates.

The parameter ITERM is used to determine control transfer in each program. For example, if ITERM has the value one, parameters for the particular model associated with it are determined. A value of zero indicates that parameters for the associated model are not determined. A double-precision matrix inverse routine using the Gaussian elimination procedure is used in each program.

RESULTS AND CONCLUSIONS

Discussion of the Physical Process

In an open circuit transient analysis of the anodic oxidation of metals, one expects an exponential or logarithmic behavior. This fact is evident from the experimental data when the layer is passive; that is, normal growth is taking place. In some cases, however, growth is truncated by the onset of oxygen evolution. In this region we have electronic conduction in addition to a small amount of anodic or growth conduction. During the open circuit break, therefore, we expect the conduction to be initially dominated by electrons and
as the voltage decreases, the conduction becomes primarily anodic conduction. Thus, the transient analysis has to have the capability to consider two or more conduction mechanisms, each with different relaxation times.

Basically, the voltage data from the anodic oxidation process are classified into four sets. Each set of data represents a different time held in oxygen evolution. When plotted as a function of time, these voltage data exhibit an exponential or decay-type behavior. The response data are represented as an observed voltage, which is designated as $f^0$.

**Discussion of Results**

The observed decay data were processed through both Programs I and II to assess the validity of the various exponential models. A summary of the initial estimates for the various assumed models is presented in Tables 2, 3, and 4. The minimum value for the $\tilde{F}$ quantity associated with the selected initial estimates is given in these tables.

**TABLE 2. INITIAL ESTIMATE SUMMARY FOR TWO-TERM EXPONENTIAL MODEL PLUS A CONSTANT**

<table>
<thead>
<tr>
<th>Groups of Initial Estimates</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group Number with Smallest $\tilde{F}$</td>
<td>441</td>
<td>441</td>
<td>441</td>
<td>441</td>
</tr>
<tr>
<td>Smallest $\tilde{F}$ Value</td>
<td>0.2083</td>
<td>0.3113</td>
<td>2.9021</td>
<td>5.986</td>
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</tbody>
</table>
### Table 3. Initial Estimate Summary for Two-Term Exponential Model

<table>
<thead>
<tr>
<th>Groups of Initial Estimates</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group Number with Smallest $\widetilde{F}$</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>Smallest $\widetilde{F}$ Value</td>
<td>0.0827</td>
<td>0.0455</td>
<td>0.01185</td>
<td>0.02952</td>
</tr>
</tbody>
</table>

### Table 4. Initial Estimate Summary for Three-Term Exponential Model Plus a Constant

<table>
<thead>
<tr>
<th>Groups of Initial Estimates</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group Number with Smallest $\widetilde{F}$</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>—</td>
</tr>
<tr>
<td>Smallest $\widetilde{F}$ Value</td>
<td>0.2132</td>
<td>0.3532</td>
<td>57.45</td>
<td>—</td>
</tr>
</tbody>
</table>

Numerical problems were encountered for both the three-term model and the three-term plus a constant model. As indicated in Table 5, divergence occurred for the set 2 and set 3 data. It is noted that the determinant of the coefficients for solving for the correction matrix was $0.911 \times 10^{-18}$ in one case and $0.162 \times 10^{-2}$ in the other case. Convergence failed to occur for the set 2 data even with relatively good initial estimates.
<table>
<thead>
<tr>
<th>Data Set Number</th>
<th>Parameter and Error Estimate</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
</tr>
<tr>
<td>A1</td>
<td>0.506027</td>
<td>0.550958</td>
<td>0.524024</td>
<td></td>
<td>0.289545</td>
</tr>
<tr>
<td>σ</td>
<td>0.0319</td>
<td></td>
<td></td>
<td></td>
<td>0.006907</td>
</tr>
<tr>
<td>B1</td>
<td>1.144510</td>
<td>1.003510</td>
<td>0.121633</td>
<td></td>
<td>0.000905</td>
</tr>
<tr>
<td>σ</td>
<td>0.3064</td>
<td></td>
<td></td>
<td></td>
<td>0.006935</td>
</tr>
<tr>
<td>A2</td>
<td>0.523248</td>
<td>0.571808</td>
<td>0.036133</td>
<td></td>
<td>0.237985</td>
</tr>
<tr>
<td>σ</td>
<td>0.0200</td>
<td></td>
<td></td>
<td></td>
<td>0.006907</td>
</tr>
<tr>
<td>B2</td>
<td>0.117757</td>
<td>0.063911</td>
<td>0.065537</td>
<td></td>
<td>0.000905</td>
</tr>
<tr>
<td>σ</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td>0.006935</td>
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<tr>
<td>A3</td>
<td>0.231458</td>
<td>0.108318</td>
<td>0.473872</td>
<td></td>
<td>0.777985</td>
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<tr>
<td>σ</td>
<td>0.0066</td>
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<td>0.006907</td>
</tr>
<tr>
<td>B3</td>
<td>0.0100038</td>
<td>0.012974</td>
<td>0.011831</td>
<td></td>
<td>0.000905</td>
</tr>
<tr>
<td>σ</td>
<td>0.0005</td>
<td></td>
<td></td>
<td></td>
<td>0.006935</td>
</tr>
<tr>
<td>K</td>
<td>0.0525</td>
<td>0.0689</td>
<td>0.0770</td>
<td></td>
<td>0.21350</td>
</tr>
<tr>
<td>σ</td>
<td>0.0039</td>
<td></td>
<td></td>
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<td>0.006907</td>
</tr>
<tr>
<td>σf</td>
<td>0.0591</td>
<td>0.0321</td>
<td>0.0761</td>
<td></td>
<td>0.9705</td>
</tr>
<tr>
<td>Determinant Value</td>
<td>7.90</td>
<td></td>
<td>0.0016</td>
<td></td>
<td>0.911 × 10^{-18}</td>
</tr>
</tbody>
</table>

a. Diverged.
b. Numerical problems for assumed model.
The data in Table 6 summarize the results for the two-term plus a constant model. As shown in this table convergence failed to occur for the set 3 and set 4 data. The initial estimates for the set 1 and set 2 data showed some disagreement with the cycle 1 estimates. The initial estimate values for set 1 and set 2 are significantly smaller than the values at the end of cycle 1. Rather high error estimates for the $A_1$ and $B_1$ parameters are also evident in this table.

Results obtained for the two-term exponential model are perhaps the most encouraging from the standpoint of adequately describing the data. As indicated in Table 7, highly accurate parameter estimates were obtained. The converged estimates represent an improvement over the initial estimates with the exception of the set 4 data. Here it is noted that $\sigma_f = 0.0215$ for the initial estimates as compared to $\sigma_f = 0.0600$ for the improved estimates. The initial estimates were thus chosen as the representation for the set 4 data. The models that appear to adequately describe the observation data are

Set 1: $f^C = 0.924798e^{-0.160783t} + 0.230238e^{-0.010751t}$,

Set 2: $f^C = 0.629470e^{-0.23023t} + 0.676240e^{-0.01642t}$,

Set 3: $f^C = 0.598702e^{-0.126326t} + 0.707297e^{-0.006132t}$,

and

Set 4: $f^C = 0.424970e^{-0.109368t} + 0.897037e^{-0.005876t}$

The observation data and the models evaluated at the corresponding time point are presented in graphical form in Figures 4 through 7. Residual data are presented in Figures 8 through 11. These figures indicate an adequate model representation of the data. It is concluded that changing the exponential functional form for these data to one other than a two-term model is not warranted in view of the problems encountered with the other models.

The coefficients for each model are plotted in Figure 12 as a function of the time to oxygen evolution associated with each set. These data enable one to simulate the physical process using a two-term exponential model at conditions other than those tested.
<table>
<thead>
<tr>
<th>Data Set Number</th>
<th>Parameter and Error Estimate</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Estimate</td>
<td>Cycle 1 Estimate</td>
<td>Initial Estimate</td>
<td>Cycle 1 Estimate</td>
<td>Initial Estimate</td>
</tr>
<tr>
<td>A_1</td>
<td>$0.912635$</td>
<td>$0.826217$</td>
<td>$0.673531$</td>
<td>$0.315643$</td>
<td>$0.332689$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>—</td>
<td>$0.4746$</td>
<td>—</td>
<td>$0.2146$</td>
<td>—</td>
</tr>
<tr>
<td>B_1</td>
<td>$0.158482$</td>
<td>$0.206468$</td>
<td>$0.113233$</td>
<td>$0.218638$</td>
<td>$0.287924$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>—</td>
<td>$0.1355$</td>
<td>—</td>
<td>$0.0569$</td>
<td>—</td>
</tr>
<tr>
<td>A_2</td>
<td>$0.169134$</td>
<td>$0.472633$</td>
<td>$0.437571$</td>
<td>$0.414676$</td>
<td>$0.746388$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>—</td>
<td>$0.1237$</td>
<td>—</td>
<td>$0.0759$</td>
<td>—</td>
</tr>
<tr>
<td>B_2</td>
<td>$0.006810$</td>
<td>$0.067656$</td>
<td>$0.011059$</td>
<td>$0.049739$</td>
<td>$0.006666$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>—</td>
<td>$0.0123$</td>
<td>—</td>
<td>$0.0035$</td>
<td>—</td>
</tr>
<tr>
<td>K</td>
<td>$0.05250$</td>
<td>$0.834446$</td>
<td>$0.0770$</td>
<td>$0.55022$</td>
<td>$0.21350$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>—</td>
<td>$0.0540$</td>
<td>—</td>
<td>$0.0431$</td>
<td>—</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>$0.0575$</td>
<td>$0.6926$</td>
<td>$0.0703$</td>
<td>$0.3556$</td>
<td>$0.21463$</td>
</tr>
<tr>
<td>Determinant Value</td>
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<td>$0.9687 \cdot 10^4$</td>
<td>$0.7801 \cdot 10^5$</td>
<td>$0.8811 \cdot 10^4$</td>
<td>—</td>
</tr>
</tbody>
</table>

a. Diverged.
TABLE 7. RESULTS FOR TWO-TERM EXPONENTIAL MODEL

<table>
<thead>
<tr>
<th>Data Set Number</th>
<th>Parameter and Error Estimate</th>
<th>Set 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td>Initial Estimate</td>
<td>Improved Estimate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A_1 _σ</td>
<td>0.951721</td>
<td>0.924798</td>
<td>0.696720</td>
<td>0.629470</td>
<td>0.593509</td>
<td>0.598702</td>
<td>0.424970</td>
<td>0.488538</td>
<td>0.0224</td>
<td>0.0161</td>
</tr>
<tr>
<td></td>
<td>B_1 _σ</td>
<td>0.140500</td>
<td>0.160783</td>
<td>0.256310</td>
<td>0.230230</td>
<td>0.182970</td>
<td>0.126326</td>
<td>0.109368</td>
<td>0.036581</td>
<td>0.0054</td>
<td>0.0109</td>
</tr>
<tr>
<td></td>
<td>A_2 _σ</td>
<td>0.181307</td>
<td>0.230238</td>
<td>0.629470</td>
<td>0.676240</td>
<td>0.746388</td>
<td>0.707297</td>
<td>0.897037</td>
<td>0.504932</td>
<td>0.0063</td>
<td>0.0048</td>
</tr>
<tr>
<td></td>
<td>B_2 _σ</td>
<td>0.007505</td>
<td>0.010751</td>
<td>0.014747</td>
<td>0.01642</td>
<td>0.006686</td>
<td>0.006132</td>
<td>0.005876</td>
<td>0.004847</td>
<td>0.0006</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>σ_f Determinant Value</td>
<td>0.0359</td>
<td>0.0342</td>
<td>0.0267</td>
<td>0.0201</td>
<td>0.0136</td>
<td>0.0089</td>
<td>0.0215</td>
<td>0.0600</td>
<td>0.5229·10^6</td>
<td>0.1292·10^6</td>
</tr>
</tbody>
</table>
Figure 4. Observed and computed response for Model IV analysis on set 1 data.
Figure 5. Observed and computed response for Model IV analysis on set 2 data.
Figure 6. Observed and computed response for Model IV analysis on set 3 data.
Figure 7. Observed and computed response for Model IV analysis on set 4 data.
Figure 8. Residuals for Model IV analysis on set 1 data.
Figure 9. Residuals for Model IV analysis on set 2 data.
Figure 10. Residuals for Model IV analysis on set 3 data.
Figure 11. Residuals for Model IV analysis on set 4 data.
Figure 12. Model IV coefficients versus oxygen evolution time.
This appendix presents operational information on the UNIVAC 1108 computer programs concerning the regression analysis application of exponential models to decay-type data. The general organization of the operational version of the two developed programs is depicted in Figure A-1. Since both programs are basically similar, only information concerning Program II is presented. A complete program listing, job card example (Fig. A-2), input preparation, and sample output are included.

Figure A-1. Program organization.
Set 1 Data: 2 exponentials

Figure A-2. Job card example.
Description of Data Deck Input Parameters

The card immediately following the @XQT card is the first card of the input data deck. This card specifies the number of cases (NCASES) of data that are to be processed. The format is of the form +XX(13) and appears in columns 1 through 3. The information between the $INPUT card and the $ card is associated with a specific set of data and is input under the non-executable NAMELIST statement. For example, the input statement in the program is

```
NAMELIST/INPUT/T,Y,NN,TL,TR,YB,YT,YB1,YT1,VARY,TOLER,
ITERM4,ITERM3,ITERM2.
```

The forms that the input data take include variable name and subscripted variable. In the usage above, T and Y are subscripted arrays and the remaining variables are simple variable names. The specific format of the data can be either integer constants (i.e., +218) or real constants (i.e., 1.85921E+00, with or without the E notation). The description of the variables in the NAMELIST statement follows.

- **T** — array containing the values for the independent variable
- **Y** — array containing the values for the dependent variable
- **NN** — number of data points
- **TL** — left plot limit for the horizontal T axis
- **TR** — right plot limit for the horizontal T axis
- **YB** — bottom plot limit for the vertical Y axis
- **YT** — top plot limit for the vertical Y axis
- **YB1** — bottom plot limit for vertical residual axis
- **YT1** — top plot limit for vertical residual axis
- **VARY** — variance for dependent variable
- **TOLER** — iteration parameter
ITERM4 — control parameter for Model III
ITERM3 — control parameter for Model IV
ITERM2 — control parameter for Model V
Computer Listing of Program II
### MAIN PROGRAM

**STORAGE USED:** CODES:  GSRT7I: DATA:  023731: BLANK: COMMON2: GOOGO

**EXTERNAL REFERENCES (BLOCK, NAME):**

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**STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME):**

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43
FORMAT (1H1)

FORMAT (1H1)

FORMAT (83H NON-LINEAR EXponential REGRESSION ANALYSIS USING AN IT
ERATIVE CORRECTION PROCEDURE)

FORMAT (13H ASSUMED MODEL 5 THREE EXPONENTIALS)

FORMAT (13H PARAMETER ESTIMATES)

FORMAT (13H ASSUMED MODEL IS TWO EXPONENTIALS)

FORMAT (7H OBSERVED RESPONSE, COMPUTED RESPONSE, AND RESIDUALS)

FORMAT (7H DETERMINANT VALUE)

FORMAT (5H ASSUMED MODEL IS ONE EXPONENTIAL)

FORMA T (76H OBSERVED RESPONSE, COMPUTED RESPONSE, AND RESIDUALS USING INITIAL ESTIMATES)

FORMAT (3X,1DH RESIDUALS)

FORMAT (3AX,1DH RESIDUALS)

FORMAT (9H INVERSE OF BT*W)

FORMAT (9H INVERSE TIMES ORIGINAL MATRIX)

FORMAT (21H END OF CYCLE NUMBER 13)

FORMAT (7H OBSERVED RESPONSE, COMPUTED RESPONSE, AND RESIDUALS USING INITIAL ESTIMATES)

FORMAT (3H ASSUMED MODEL IS ONE EXPONENTIAL)

FORMAT (2H PARAMETER ESTIMATES)

FORMAT (3X,1DH RESIDUALS)
46172 72* DATA *(BCDI(1,1)=1,1,1,2) /HTIME 6*H
46174 73* DATA *(BCDI(1,1)=1,1,1,2) /HRESPON,6HSE 1G*6H
46176 74* DATA *(BCDI(1,1)=1,1,1,2) /HRESIDU,6HAL 1G*6H
46200 75* READ (S,118) NCASES
46203 76* WRITE (6,11C)
46205 77* READ (S,INPUT)
46210 78* WRITE (6,INPUT)
46213 79* IF (ITERM=6) GO TO 700
46215 80* IF (ITERM=2) GO TO 4000
46215 81* C
46215 82* C INITIAL ESTIMATES FOR THREE EXPONENTIALS
46215 83* C STRAIGHT LINE FIT TO LOG Y VS. TIME
46215 84* C
46217 85* IJ=5;
46222 86* J1=2
46223 87* BJ1=3.
46224 88* 3UC0 IJ=1+1
46225 89* SM1=U.
46226 90* SM2=C.
46233 91* SM3=U.
46236 92* SM4=C.
46237 93* JJ=NN-J1
46238 94* DO 30G 1=JJJNN
46239 95* SM1=TI(I)=2*SM1
46239 96* SM2=DLOG(Y(I))=SM2
46239 97* SM3=T(I)+SM3
46239 98* 3UC0 SM4=TI(I)=DLOG(Y(I))=SM4
46240 99* CAA3=BJ1*SM1=SM3*2
46241 100* CAA3*(SM1+SM2+SM3+SM4)/CAA3
46241 101* C
46241 102* C ESTIMATES FOR A3,63
46241 103* C
46242 104* B3=-((BJ1*SM4-SM3*SM2)/CAA3)
46243 105* A3=EXP(CA3)
46244 106* J2=J1+3
46244 107* C
46244 108* C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME
46244 109* C
46245 110* JJ=NN-J2
46246 111* JJ=NN-J1-J1
46247 112* SM1=C.
46248 113* SM2=0.
46250 114* SM3=U.
46252 115* SM4=U.
46253 116* DO 30J 1=JJ+JN
46254 117* X3(I)=B3*T(I),
46257 118* HES(I)=Y(I)-A3*EXP(X3(I))
46260 119* HES(I)=DABS(HES(I))
46261 120* SM1=T(I)**2*SM1
46262 121* SM2=DLOG(HES(I))=SM2
46263 122* SM3=T(I)+SM3
46264 123* 3U1 SM4=T(I)=DLOG(HES(I)+SM4
46266 124* CAA2=3.*SM1=SM3*2
46267 125* CAA2*(SM1+SM2+SM3+SM4)/CAA2
46267 126* C
46267 127* C ESTIMATES FOR A2,62

46
C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME

DO 300 J=1, JN
G0300 142* X3(I)=-B3*T(I)
G0301 142* X2(I)=-B2*T(I)
G0302 143* RES(I)=Y(I)-A2*DELXP(X2(I))-A3*DELXP(X3(I))
G0303 144* RES(I)=DABS(RES(I))
G0304 145* SM1=T(I)*SM1
G0305 146* SM2=LOG(RES(I))*SM2
G0306 147* SM3=T(I)*SM3
G0307 148* SUM2 =SM1*SM1+SM2+SM3
G0308 149* CAA1=BJ2*SM1-SM3*2
G0309 150* CAA1=(SM1+SM2+SM3)/CAA1
G0310 151* C
G0311 152* C ESTIMATES FOR A1,B1
G0312 153* C
G0313 154* BJ1=-(BJ2*SM4-SM3*SM2)/CAA1
G0314 155* A1=EXP(CA1)
G0315 156* C
G0316 157* C WEIGHTED SUM OF SQUARES OF DEVIATIONS, W=1.
G0317 158* C
G0318 159* DO 3U3 I=1 , Nf
G0319 160* X1(I)=-BJ1*T(I)
G0320 161* X2(I)=-B2*T(I)
G0321 162* X3(I)=-B3*T(I)
G0322 163* SUMDEV=J+
G0323 164* DO J=1 , Nf
G0324 165* B(I,I)=DELXP(XI(I))
G0325 166* B(I,1)=DELXP(X2(I))
G0326 167* B(I,5)=DELXP(X3(I))
G0327 168* SUMDEV=(Y(I)-A1*B(I,1)-A2*B(I,3)-A3*B(I,5))*2+SUMDEV
G0328 169* J=J+1
G0329 170* BJ1=BJ1+1.
G0330 171* KTEM=N1-N1-6
G0331 172* C
G0332 173* AR(1,I,I)=A1
G0333 174* AR(2,I,I)=B1
G0334 175* AR(3,I,I)=A2
G0335 176* AR(4,I,I)=B2
G0336 177* AR(5,I,I)=A3
G0337 178* AR(6,I,I)=B3
G0338 179* NKOT=1
G0339 180* IF (KTEM+EQ+0) GO TO 305
G0340 181* GO TO 330U
G0341 182* JU5 CONTINUE
C SELECTION OF SMALLEST F AND CORRESPONDING PARAMETERS

DO 307 J=1,J
DO 306 J=1,J

CONTINUE
IF (F(I) .LT. 0.) GO TO 306
GO TO 307
F(I) = F(I)

CONTINUE
J=1
NKOT=NKOT-1
IF (NKOT.EQ.0) GO TO 310
GO TO 306

J=1
K=K+1
NKOT=NKOT-1
GO TO 308

IF (F(I) .LT. 0.) GO TO 306
GO TO 307
F(I) = F(I)

CONTINUE
A1=AR(1,J)
A2=AR(2,J)
A3=AR(3,J)
JSEL3=J

WRITE (6,101)
WRITE (6,120)
WRITE (6,119)
WRITE (6,120)
DO 311 J=1,J
WRITE (6,121) (I,F(I),(AR(J,J),J=1,6))
WRITE (6,119)
WRITE (6,120)
JSEL3=J

WRITE (6,123) JSEL3
CONTINUE
IF (ITERM3.EQ.0) GO TO 5000

C INITIAL ESTIMATES FOR TWO EXPONENTIALS
C STRAIGHT LINE FIT TO LOG Y VS. TIME

CONTINUE
C

I=4
J=2
K=3

CONTINUE
SM1=C
SM2=D
SM3=D

CONTINUE
C & M 73

---

DO 460 I = JJ, NN

SM1 = T(I) * 2 * SM1

SM2 = DLOG(Y(I)) * SM2

SM3 = T(I) * SM3

SM4 = T(I) * DLOG(Y(I)) * SM4

CA2 = BJ1 * SM1 - SM3 * 2

CA2 = (SM1 * SM2 - SM3 * SM4) / CA2

C

C ESTIMATES FOR A2, B2

B2 = ((BJ1 * SM4 - SM3 * SM2) / CA2)

A2 = EXP(CA2)

J2 = J1 + 3

C

C STRAIGHT LINE FIT TO LOG RESIDUAL VS. TIME

C

---

DO 462 I = 1, JN

X2(I) = -B2 * T(I)

RES(I) = Y(I) - A2 * DEXP(X2(I))

RES(I) = DABS(RES(I))

SM1 = T(I) * 2 + SM1

SM2 = DLOG(RES(I)) * SM2

SM3 = T(I) * SM3

SM4 = T(I) * DLOG(RES(I)) + SM4

CA1 = BJ2 * SM1 - SM3 * 2

CA1 = (SM1 * SM2 - SM3 * SM4) / CA1

SUMDEV = 0.

SUMDEV = Y(I) - A1 * B(I, I) - A2 * B(I, 3) * 2 * SUMDEV

J1 = J1 + 1

BJ1 = BJ1 + 1.

JTER = NN = J1 - 3

F(I, J) = SUMDEV

ARK(I, I) = A1

ARK(2, J) = B1
C STATEMENTS NUMBERED 500-599 REFER TO PROGRAM FOR THREE EXPONENTIALS

IF (ITERM4.EQ.1) GO To 600

CONTINUE
3675 352. C THREE EXPONENTIALS
3675 353. C IIENM=1
3675 354. C
3676 355. WRITE (6,140)
3676 356. WRITE (6,142)
3702 357. WRITE (6,140)
3704 358. WRITE (6,140)
3706 359. WRITE (6,149)
3710 360. WRITE (6,140) A1,B1,A2,B2,A3,B3
3720 361. KCYCLE=0
3721 362. AI=A1
3722 363. BI=B1
3723 364. A2=A2
3725 366. A3=A3
3726 367. B3=B3
3727 368. BJ=NN
3730 369. SUM=$2$
3731 370. WRITE (6,142)
3733 371. WRITE (6,141)
3735 372. WRITE (6,141)
3737 373. DO 50 SUM=1$1$,NN
3742 374. X1(1)=-B1*T(1)
3743 375. X2(1)=-B2*T(1)
3744 376. X3(1)=-B3*T(1)
3745 377. B(1,1)=DEXP(X1(1))
3746 378. B(1,3)=DEXP(X2(1))
3747 379. B(1,5)=DEXP(X3(1))
3750 380. YC(1)=AI*B(1,1)+A2*B(1,3)+A3*B(1,5)
3751 381. RES(1)=Y(1)-YC(1)
3752 382. RES(1)=RES(1)
3753 383. TS(1)=T(1)
3754 384. YS(1)=Y(1)
3755 385. YCS(1)=YC(1)
3756 386. SUM=$2$*RES(1)*$2$+SUM$2$
3757 387. SUM$2$ WRITE (6,128) Y(1),YC(1),RES(1)
3765 388. SIGYN=SQR(SUM$2$/B(1-6,N))
3766 389. WRITE (6,143) SIGYN
3771 390. KK=NN
3772 391. CALL QUIK3L (-1 TL,TR,XY,35,BCDT,BCDY,KK,TS,YS)
3773 392. CALL QUIK3L (0 TL,TR,XY,35,BCDT,BCDY=-KK,TS,YCS)
3774 393. CALL QUIK3L (-1 TL,TR,XY,35,BCDT,BCDR=-KK,TS,HSS)
3774 394. C
3774 395. C & MATRIX
3774 396. C
3775 397. DO SUM=1,NN
3796 398. SUM=1. VARY
3799 399. C
3799 400. C & MATRIX
3799 401. C
3802 402. SUM=1. CONTINUE
3803 403. KCYCLE=KCYCLE+1
3804 404. WRITE (6,104)
3805 405. WRITE (6,106) KCYCLE
3806 406. DO SUM=1,NN
3807 407. X1(1)=-B1*T(1)
C1015 438  \quad X2(I)=B22*T(1)
C1016 439  \quad X3(I)=B33*T(1)
C1020 439  \quad WRITE (6,138)
C1022 439  \quad DO 533 I=1,NH
C1025 439  \quad B(I,1)=DEXP(X1(I))
C1026 439  \quad B(I,2)=A11*T(1)*B(I,1)
C1027 439  \quad B(I,3)=DEXP(X2(I))
C1030 439  \quad B(I,4)=A22*T(1)*B(I,3)
C1031 439  \quad B(I,5)=DEXP(X3(I))
C1032 439  \quad B(I,6)=A33*T(1)*B(I,5)
C1033 439  \quad WRITE (6,128) (B(I,JB),JB=1,6)
C1033 449  \quad C
C1033 420  \quad C TRANSPOSE OF B MATRIX
C1042 422  \quad DO 544 K=1,6
C1045 422  \quad DO 544 I=1,NN
C1050 424  \quad S4(K,I)=B(I,K)
C1050 425  \quad C
C1050 426  \quad C N MATRIX
C1050 427  \quad C
C1053 428  \quad DO 555 I=1,NN
C1056 429  \quad S5 (I,1) = Y(I)-A11*B(I,1)-A22*B(I,3)-A33*B(I,5)
C1056 430  \quad C
C1056 431  \quad C B-TRANSPOSE*WN
C1056 432  \quad C
C1106 433  \quad DO 566 I=1,NN
C1106 434  \quad S6(I,1)=D*I*DJO
C1106 435  \quad DO 577 I=1,NN
C1106 436  \quad S7(I,1)=Y(I)*WN(I,1)
C1106 437  \quad DO 588 I=1,6
C1107 438  \quad S8(D,1)=DJO
C1107 439  \quad DO 599 M=1,6
C1109 441  \quad S9(D,M) = WT(M,N)*D1(N,1)+D2(M,1)
C1109 442  \quad C
C1105 443  \quad C u-TRANSPOSE*u*B
C1105 444  \quad C
C1115 445  \quad DO 510 J=1,6
C1113 446  \quad DO 510 J=1,6
C1116 447  \quad S10(D1,J)=D*JO
C1121 448  \quad DO 511 P=1,6
C1124 449  \quad DO 511 N=1,NN
C1127 450  \quad S11(D1(N,P)*WN(N,P)*D(N,P)
C1132 451  \quad DO 512 J=1,6
C1135 452  \quad DO 512 J=1,6
C1140 453  \quad S12(D1(J),J)=DJO
C1143 454  \quad DO 513 N=1,6
C1146 455  \quad DO 513 P=1,6
C1151 456  \quad DO 513 N=1,NN
C1154 457  \quad S13(D3(M,N),D(N,P)*D3(M,P)
C1160 458  \quad WRITE (6,144)
C1162 459  \quad DO 5130 J=1,6
C1165 460  \quad S13(D3(M,N),D3(M,P)
C1165 461  \quad C
C1165 462  \quad C INVERSE OF B-TRANSPOSE*u*B
C1165 463  \quad C
C1174 464* 12=0
C1175 465* DO 514 I=1,6
C1176 466* DO 514 J=1,6
C1177 467* 12=12+1
C1178 468* D(12)*D3(J,1)
C1179 469* 514 DD(12)=D3(J,1)
C1180 470* N=6
C1181 471* M=0
C1182 472* CALL INVRT(D,N,M,DETER)
C1183 473* C
C1184 474* C INVERSE MATRIX
C1185 475* C
C1186 476* K1=1
C1187 477* K2=6
C1188 478* K3=6
C1189 479* K4=6
C1190 480* WRITE (6,137)
C1191 481* 5140 WRITE (6,128) (D(I),1=K1,K2)
C1192 482* K1=K2+1
C1193 483* K2=K2+K3
C1194 484* K4=K4-1
C1195 485* IF (K4.EQ.0) GO TO 5141
C1196 486* GO TO 5140
C1197 487* 5141 CONTINUE
C1198 488* C
C1199 489* C INVERSE*ORIGINAL MATRIX
C1200 490* C
C1201 491* 12=3
C1202 492* DO 515 I=1,6
C1203 493* DO 515 J=1,6
C1204 494* DO 12=12+1
C1205 495* 515 D33(J,1)=D(12)
C1206 496* DO 515G I=1,6
C1207 497* DO 515G J=1,6
C1208 498* 515G DIDEN(1,J)=D+D6
C1209 499* DO 515I M=1,6
C1210 500* DO 515J N=1,6
C1211 501* 515 DIDEN(M,P)=D33(M,N)*D3(N,P)*DIDEN(M,P)
C1212 502* 515C INVERSE*ORIGINAL MATRIX
C1213 503* 12=1
C1214 504* WRITE (6,127)
C1215 505* 5152 WRITE (6,128) (DIDEN(I,J),1=1,6)
C1216 506* 507* IF (1M.EQ.0) GO TO 5153
C1217 508* 1M=1+1
C1218 509* 1=1+1
C1219 510* GO TO 5152
C1220 511* 5153 CONTINUE
C1221 512* WRITE (6,101)
C1222 513* WRITE (6,129)
C1223 514* WRITE (6,128) DETER
C1224 515* C
C1225 516* C DELTA MATRIX
C1226 517* C
C1227 518* DO 516 I=1,6
C1228 519* 516 DEL(1,1)=6+D6
C1229 53
DO 517 M=1,4
DO 517 N=1,6
DEL(M,1)=DEL(M,1)+02(M,N)*DEL(1,1)+DEL(M,1)
C
C IMPROVED PARAMETER ESTIMATES
C
DO 524 AT=AT+DEL(T,I)
B11=B11*DEL(2,1)
A22*A22*DEL(3,1)
B22*B22*DEL(4,1)
A33*A33*DEL(5,1)
B33*B33*DEL(6,1)
C
C RESIDUALS USING IMPROVED PARAMETER ESTIMATES
C
DO 533 DO 519 I=1,NN
DO 520 I=1,NN
B(I,1)=DEXP(X(I,1))
B(I,3)=DEXP(X2(I))
B(I,5)=DEXP(X3(I))
WRITE (6,101)
WRITE (6,111) Y(I), YC(I), RES(I)
DO 524 SUM1=SUM1+SUM1
C PARAMETER STANDARD DEVIATIONS
C
SIG1=SIG1+SUM1/(BJ-6,1))
SIG2=SIG2+SUM2/(BJ-6,1))
SIG3=SIG3+SUM3/(BJ-6,1))
C
C STANDARD DEVIATION OF RESPONSE VARIABLE USING IMPROVED ESTIMATES
C
SIG1=SIG1+SUM1/(BJ-6,1))
SIG2=SIG2+SUM2/(BJ-6,1))
SIG3=SIG3+SUM3/(BJ-6,1))
C
C PARAMETER STANDARD DEVIATIONS
C
SIG1=SIG1+SUM1/(BJ-6,1))
SIG2=SIG2+SUM2/(BJ-6,1))
SIG3=SIG3+SUM3/(BJ-6,1))
\begin{verbatim}
G1441 576* SGB3=SIGY2/SKRT(D3(6,6))
G1442 577* WRITE (6,161)
G1444 578* WRITE (6,167)
G1446 579* WRITE (6,166) (DEL(1,1),L=1,6)
G1454 580* WRITE (6,161)
G1456 581* WRITE (6,169)
G1460 582* WRITE (6,169) A11,B11,A22,B22,A33,B33
G1470 583* WRITE (6,161)
G1472 584* WRITE (6,166)
G1474 585* WRITE (6,161) SIGY1,SIGY2
G1500 586* WRITE (6,161)
G1502 587* WRITE (6,162)
G1504 588* WRITE (6,161) SGA1,SGB1,SGA2,SGB2,SGA3,SGB3
G1504 589* C ITERATION LOGIC CYCLE
G1504 590* C TOL1=SIGY2**2-SIGY1**2
G1515 591* DTOL=TOLER-TOL1
G1536 592* DTOL=ABS(DTOL)
G1557 593* IF ((DTOL-.001)**2) GT .001) GO TO 524
G1551 594* GO TO 525
G1522 595* TOLER=TOL1
G1525 596* WRITE (6,161)
G1525 597* WRITE (6,164) TOLER
G1525 598* WRITE (6,161) KCYCLE
G1533 601* S25 WRITE (6,161)
G1535 603* WRITE (6,160)
G1537 604* WRITE (6,165) DTOL
G1544 605* WRITE (6,1617)
G1546 606* KK=KK
G1547 607* CALL QUIK3L (-1,L,TR,YB,YT,43,BCDT,BCDY,KK,TS,YS)
G1550 608* CALL QUIK3L (L,TL,TR,YB,YT,35,BCDT,BCDY,KK,TS,YCS)
G1551 609* CALL QUIK3L (-1,TL,TR,YB1,YT1,35,BCDT,BCDR,KK,TS,YCS)
G1551 610* NCASES=NCASES+1
G1553 611* IF (NCASES.EQ.0) GO TO 800
G1553 612* C C STATEMENTS NUMBERED 611-699 REFER TO PROGRAM FOR TWO
G1553 613* C EXPONENTIALS
G1553 614* C EXPONENTIALS
G1553 615* C
G1555 616* IF (ITERM3.EQ.0) GO TO 260
G1557 617* 606 CONTINUE
G1557 618* C C TIII EXPONENTIALS
G1557 619* C ITERM3=1
G1557 620* C
G1557 621* C
G156C 622* WRITE (6,160)
G1562 623* WRITE (6,162)
G1564 624* WRITE (6,161)
G1566 625* WRITE (6,130)
G157C 626* WRITE (6,109)
G157C 627* WRITE (6,131) A1,B1,A2,B2
G157C 628* KCYCLE=0
G1601 629* A11=A1
G1602 630* B11=B1
G1603 631* A22=E2
\end{verbatim}
SUMW(,' = 0,
WRITE (6,142)
WRITE (6,141)
DO 6CS1
X1 = B11*T(1)
X2(1) = B22*B(1,1)
B(1,1) = DEXP(X1(1))
B(1,3) = DEXP(X2(1))
Y(1) = A11*6(1,1) + A22*B(1,3)
RES(1) = Y(1) - YC(1)
RES(1) = RES(1)
TS(1) = T(1)
YS(I) = Y(I)
YCS(I) = YCS(I)
SUMW = RES(1) * 2 * SUMW
WRITE (6,128) Y(I), YC(I), RES(I)
SIGYIN = SQRT(SUMW/(BJ-4))
WRITE (6,143) SIGYIN
CALL QUIK3L (-1, TL, TR, Ye, YT, 43, BCDT, BCDY, K, TS, YS)
CALL QUIK3L (0, TL, TR, Ye, YT, 35, BCDT, BCDY, K, TS, YS)
CALL QUIK3L (-1, TL, TR, Ye, YT, 35, BCDT, BCDY, K, TS, YS)
DO 6Ul1=1, NN
6Ul W(I) = ./VARY
DO 6U1 1=1, NN
WRITE (6,128) B(I, JB), JB=1, 4
C TRANSPOSE OF B MATRIX
C N MATRIX
C
DA = 6 - i _ N N ,

$B_{-1} = \text{TRANSPOSE} \ast W \ast b$

DO 645 I = 1, NN

$D_{21} = B_{-1} \ast C_{-1} \ast B_{-1}$

DO 646 J = 1, M

$D_{31} = B_{-1} \ast C_{-1} \ast B_{-1}$

CALL INVR1 (D, N, M, DET)

WRITE (6, 137)

WRITE (6, 128) (D (I), I = K1, K2)

DO 647 I = 1, K2

DO 648 J = 1, K2

CALL INVR1 (D, N, M, DET)

WRITE (6, 137)

WRITE (6, 128) (D (I), I = K1, K2)

DO 647 I = 1, K2

DO 648 J = 1, K2

CALL INVR1 (D, N, M, DET)

WRITE (6, 137)

WRITE (6, 128) (D (I), I = K1, K2)
GO TO 6140

GO TO 6141

CONTINUE

12=0

DO 615 I=1,4

DO 615 J=1,4

12=12+1

63(I,J)=U+UD

WRITE (6,127)

WRITE (6,128) (DJDEN(I,J),J=1,J)

WRITE (6,129)

WRITE (6,130) DETER

DEL < I > =U.DC

DO 617 M=1,4

DEL(M,J)=D33(M,N)*D3(N,J)+DEL(M,J)

DEL(M,1)=D33(M,N)*D3(N,1)+DEL(M,1)

A(I)=A(I)+DEL(1,1)

B(Il)=B(1)+DEL(2,1)

A22=A22+DEL(3,1)

B22=B22+DEL(4,1)

X(I)=B(I)+DEL(I,1)

X2(I)=B22+DEL(2,1)

DO 619 I=1,NN

X1(1)=-B11*T(1)

X2(1)=-B22*T(1)

DO 620 I=1,NN

B(I)=B(I)*T(I)

WRITE (6,131) (DJDEN(I,J),J=1,J)

WRITE (6,132) DETER

WRITE (6,133)

WRITE (6,134) (DJDEN(I,J),J=1,J)

WRITE (6,135) DETER

WRITE (6,136)

WRITE (6,137) (DJDEN(I,J),J=1,J)

WRITE (6,138) DETER
2240 0640 WRITE (6,101)
2242 0641 WRITE (6,146)
2244 0642 DO 621 I = 1, NN
2247 0643 YC(I) = A1*B(I,1) + A2*B(I,2) + B(I,3)
2250 0644 RES(I) = Y(I) - YC(I)
2251 0645 RSS = RSS + RES(I)**2
2252 0646 YCST = YST(YC(I))
2253 0647 WRITE (6,128) Y(I), YC(I), RES(I)
2253 0648 C STANDARD DEVIATION OF RESPONSE VARIABLE USING INITIAL ESTIMATES
2253 0649 C
2256 0650 B = NN
2257 0651 SUM2 = 0
2259 0652 DO 622 I = 1, NN
2261 0653 SUM2 = SUM2 + SUM1 + RES(I)**2
2266 0654 SIGY1 = SQRT(SUM1/(B - 4))
2267 0655 SIGY2 = SQRT(SUM2/(B - 5))
2269 0656 C STANDARD DEVIATION OF RESPONSE VARIABLE USING IMPROVED ESTIMATES
2270 0657 C
2271 0658 B = NN
2272 0659 SUM2 = 0
2273 0660 DO 622 I = 1, NN
2274 0661 SUM2 = SUM2 + RES(I)**2
2276 0662 SIGY1 = SQRT(SUM1/(B - 4))
2277 0663 SIGY2 = SQRT(SUM2/(B - 5))
2279 0664 C PARAMETER STANDARD DEVIATIONS
2280 0665 C
2280 0666 SGA1 = SIGY2/SQRT(D3(1,1))
2282 0667 SGB1 = SIGY2/SQRT(D3(2,2))
2283 0668 SGA2 = SIGY2/SQRT(D3(3,3))
2284 0669 SGB2 = SIGY2/SQRT(D3(4,4))
2286 0670 WRITE (6,121)
2287 0671 WRITE (6,107)
2288 0672 (DEL(1,1), I = 1, 4)
2290 0673 WRITE (6,101)
2291 0674 WRITE (6,133) A1, B1, A2, B2
2293 0675 WRITE (6,101)
2294 0676 WRITE (6,111) SIGY1, SIGY2
2295 0677 WRITE (6,101)
2296 0678 WRITE (6,112)
2297 0679 WRITE (6,134) SGA1, SGB1, SGA2, SGB2
2299 0680 WRITE (6,101)
2300 0681 WRITE (6,101)
2301 0682 C ITERATION LOGIC CYCLE
2304 0683 C
2304 0684 TOL1 = SIGY1**2 - SIGY1**2
2305 0685 DTOL = TOLER - TOL1
2306 0686 DTOL = ABS(DTOL)
2307 0687 IF (TDTOL = 0.0) THEN TOL1 GO TO 629
2308 0688 WRITE (6,101)
2309 0689 GO TO 625
2310 0690 TOLER = TOL1
2311 0691 WRITE (6,101)
2312 0692 WRITE (6,114) TOLER
2313 0693 WRITE (6,141) KCYCLE
2315 0694 WRITE (6,101)
2316 0695 WRITE (6,101)
2317 0696 WRITE (6,101)
2318 0697 WRITE (6,114)
2319 0698 WRITE (6,141)
2320 0699 WRITE (6,101)
2321 0699 WRITE (6,101)
2322 0699 WRITE (6,101)
2323 0699 WRITE (6,101)
2324 0699 WRITE (6,101)
2325 0699 WRITE (6,101)
2326 0699 WRITE (6,101)
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2392 0699 WRITE (6,101)
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2396 0699 WRITE (6,101)
2397 0699 WRITE (6,101)
2398 0699 WRITE (6,101)
2399 0699 WRITE (6,101)
2400 0699 WRITE (6,101)
2401 0699 WRITE (6,101)
IF (NCASES.EQ.0) GO TO 800

CALL QUIK3L (-1, TL, TR, YB, YT, TS, YS)
CALL QUIK3L (1, TL, TR, YB, YT, TS, YS)
CALL QUIK3L (1, TL, TR, YB, YT, TS, YS)
CALL QUIK3L (1, TL, TR, YB, YT, TS, YS)
CALL QUIK3L (1, TL, TR, YB, YT, TS, YS)

C STATEMENTS NUMBERED 760-799 REFER TO PROGRAM FOR ONE EXPONENTIAL
END OF COMPILATION: NO DIAGNOSTICS.
EXTERNAL REFERENCES (BLOCK, NAME)

GO03 INVT
GO04 NAME

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

GO01 GO02 1316 VENT WUT15 1116 GO01 GO027 1116 GO01 GO027 1236 GO01 GO027 1236
GO01 GO021 1976 JU01 JU255 1976 JU01 JU255 2016 JU01 JU255 2076 JU01 JU255 2206
GO01 GO023 2376 JU01 JU237 2376 JU01 JU237 2956 JU01 JU237 2596 JU01 JU237 2596
GO01 GO027 756 JU01 JU256 756 JU01 JU256 APPA JU01 JU256 APPA JU01 JU256 1003 JU01 JU256 1003
GO01 GO021 190 JU01 JU250 INDEX JU01 JU250 INDEX JU01 JU250 INDEX JU01 JU250 INDEX
GO01 GO027 18 JU01 JU256 JU01 JU256 257 K JU01 JU256 257 K JU01 JU256 257 K
GO01 GO027 SIGN

GO011 1* SUBROUTINE INVCT(A,M,N,DERT)
GO012 2X PARAMETER IDIM=20
GO013 3X C MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLV
GO014 4X C A INPUT MATRIX FOR INVERSION OR AUGMENTED MATRIX FOR SIM. EOS.
GO015 5X C E ORDER OF COEFFICIENT MATRIX
GO016 6X C MAT FOR INVERSION ONLY
GO017 7X C NUMBER OF CONSTANT VECTORS
GO018 8X C DETERM DETERMINANT OF COEFFICIENT MATRIX
GO019 9X C DOUBLE PRECISION A11,DETER,SIGN,AMAX
GO020 10X DIMENSION (PIV101IM),IDETR10121
GO021 11X DETER بغ D0
GO022 12X SIGMA=1.000
GO023 13X UU 23 JPEG
GO024 14X ZC IFIV12345
GO025 15X UU12
GO026 16X UU12 ZK
GO027 17X ANP32456
GO028 18X 45 DD 78 RGB
GO029 19X IF IFIV12345678956
GO030 20X 52 DD 75 RGB
GO031 21X IF IFIV12345678956
GO032 22X 55 INDEX=34789
GO033 23X IFDAX=567891111 56,75,75
GO034 24X AO IPI
GO035 25X IN
GO036 26X AANA=DABA111ID1
GO037 27X 75 CONTINUE

62
00147  28*  76 CONTINUE
00151  29*  IPIV(IC) = IPIV(IC) + 1
00152  30*  IF (IR = IC) 190, 115, 90
00155  31*  90 SIGN = SIGN
00156  32*  DO 110 L = 1, NN
00161  33*  IND = (L-1) * N + IR
00162  34*  IND2 = (L-1) * N + IC
00163  35*  AMAX = A(IND)
00164  36*  A(IND) = A(IND2)
00165  37*  110 A(IND2) = AMAX
00167  38*  115 INDEX(K, 1) = IR
00170  39*  INDEX(K, 2) = IC
00171  40*  IND = (IC-1) * N + IC
00172  41*  AMAX = A(IND)
00173  42*  DETER = DETER * AMAX
00174  43*  IF (DETER) 140, 255, 140
00177  44*  140 A(IND) = 1.000
00200  45*  DO 150 L = 1, NN
00203  46*  IND = (L-1) * N + IC
00204  47*  150 A(IND) = A(IND) / AMAX
00206  48*  DO 181 L = 1, N
00211  49*  IF (L-IC) 165, 181, 165
00214  50*  165 IND = (IC-1) * N + L
00215  51*  165 AMAX = A(IND)
00216  52*  A(IND) = 0.000
00217  53*  DO 180 I = 1, NN
00222  54*  IND = (I-1) * N + L
00223  55*  IND2 = (I-1) * N + IC
00224  56*  A(IND) = A(IND) - A(IND2) * AMAX
00225  57*  180 CONTINUE
00227  58*  181 CONTINUE
00231  59*  182 CONTINUE
00233  60*  DO 235 I = 1, N
00236  61*  L = N + 1 - I
00237  62*  IR = INDEX(L, 1)
00240  63*  IC = INDEX(L, 2)
00241  64*  IF (IR = IC) 210, 235, 210
00244  65*  210 DO 230 K = 1, N
00247  66*  IND = (IR + 1) * N + K
00250  67*  IND2 = (IC-1) * N + K
00251  68*  AMAX = A(IND)
00252  69*  A(IND) = A(IND2)
00253  70*  230 A(IND2) = AMAX
00255  71*  235 CONTINUE
00257  72*  DETER = SIGN * DETER
00260  73*  RETURN
00261  74*  250 M = 1
00262  75*  255 RETURN
00263  76*  END

END OF COMPILATION! NO DIAGNOSTICS.
SUBROUTINE DMATM
ENTRY POINT LCMAT

STORAGE USE: CODE(1) CODE(2) DATA(1) DATA(2) BLANK COMMON(2) CODE(3)

EXTERNAL REFERENCES (BLOCK, NAME)
CODE TRAEX

CODE NAME

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)
CODE 24601 24602 24603 24604 24605 24606 24607 24608 24609 24610 24611 24612 24613 24614 24615

J3416 14 SUBROUTINE DMATM(AB,M,N,K)
J3417 24 ABSTRACT
J3418 34 C GENERAL MATRIX MULTIPLICATION ROUTINE WITH TRANSPOSE OPTIONS
J3420 44 C WHERE, M IS THE NUMBER OF ROWS OF A
J3422 54 C N IS THE NUMBER OF COLUMNS OF A
J3424 64 C M N IS THE NUMBER OF COLUMNS OF B OR I
J3426 74 C TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
J3428 84 C THE FOLLOWING PRODUCTS MAY BE OBTAINED
J3430 94 C (C)M(A)(M, A AND N POSITIVE
J3432 104 C (C)M(A)(M, A AND N NEGATIVE
J3434 114 C (C)N(A)(M, A AND N POSITIVE
J3436 124 C (C)N(A)(M, A AND N NEGATIVE
J3438 134 C WHERE T INDICATES TRANSPOSE
J3440 144 C IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF A
J3442 154 C IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF B
J3444 164 C
J3446 174 C ABSTRACT
J3448 184 C DIMENSION C
J3450 194 C ABSTRACT
J3452 204 C ABSTRACT
J3454 214 C DIMENSION AB
J3456 224 C

******************************************************************************

J3458 244 DOUBLE PRECISION CD, M, N
J3460 254 M, N
J3462 264 M, N
J3464 274 M, N
J3466 284 M, N
J3468 294 M, N

64
| GO114 | 36* | IA2=1 |
| GO115 | 31* | GO TO 2 |
| GO116 | 32* | IA1 = 1 |
| GO117 | 33* | IA2=IN |
| GO120 | 34* | 2 IF(N < LT. D) GO TO 3 |
| GO122 | 35* | IB1=1 |
| GO123 | 36* | IB2=IN |
| GO124 | 37* | GO TO 4 |
| GO125 | 38* | 3 IB1=K |
| GO126 | 39* | IB2=1 |
| GO127 | 40* | 4 DO 7 LM=1,1M |
| GO132 | 41* | LC=LM |
| GO133 | 42* | IB3=1 |
| GO134 | 43* | DO 6 LP=1,K |
| GO137 | 44* | CD = D+DO |
| GO140 | 45* | LA=IA3 |
| GO141 | 46* | LB=IB3 |
| GO142 | 47* | DO 5 LN=1,IN |
| GO145 | 48* | CD = CD + A(LA)*B(LB) |
| GO146 | 49* | LA=LA+IA1 |
| GO147 | 50* | LB=LB+IB3 |
| GO151 | 51* | C(LC) = CD |
| GO152 | 52* | LC = LC + IM |
| GO153 | 53* | 6 IB3=IB3+IB2 |
| GO155 | 54* | 7 IA3=IA3+IA2 |
| GO157 | 55* | RETURN |
| GO160 | 56* | END |

END OF COMPILATION; NO DIAGNOSTICS.
LIB SYSMSFCS.

ENTRY POINT TRACL ALREADY DEFINED

ADDRESS LIMITS 031000 037532 040000 076226
STARTING ADDRESS 031714
WORDS DECIMAL 15707 1800 15511 2000

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**SYSS*RLIBS* LEVEL MS7-0**

END OF COLLECTION - TIME 3.972 SECONDS
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NON-LINEAR EXPONENTIAL REGRESSION ANALYSIS USING AN ITERATIVE CORRECTION PROCEDURE

ASSUMED MODEL IS TWO EXPONENTIALS

INITIAL ESTIMATES FOR PARAMETERS

\[ A_1 = 0.951712 \quad B_1 = 1.4656 + DG \quad A_2 = 1.8137 + DG \quad B_2 = 756509 - 02 \]

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80
DETERMINANT VALUE

\[ +522963 \times 10^6 \]
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PARAMETER CORRECTIONS
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IMPROVED PARAMETER ESTIMATES
A1 = +924979 + 00   B1 = +163743 + 00   A2 = +230230 + 00   B2 = +107515 + 01

RESPONSE VARIABLE STANDARD DEVIATION
SIGY1 = +349525 + 01   SIGY2 = +341563 + 01

COEFFICIENT STANDARD DEVIATION
SGA1 = +223941 + 01   SGB1 = +54864 - 02   SGA2 = +629989 - 02   SGB2 = +602567 - 03

TOLEN = +123492 + 03
END OF CYCLE NUMBER 1
REFERENCES


A METHOD FOR NONLINEAR EXPONENTIAL REGRESSION ANALYSIS

By Bobby G. Junkin

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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Director, Computation Laboratory
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