SHOCK AND VIBRATION TESTS OF URANIUM MONONITRIDE FUEL PELLETS FOR A SPACE POWER NUCLEAR REACTOR

by Donald W. Adams

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**Abstract**

Shock and vibration tests were conducted on cylindrically shaped, depleted, uranium mononitride (UN) fuel pellets. The purpose of these tests was to determine the structural capabilities of the pellets when they are exposed to the shock and vibration loading that a nuclear reactor could encounter during launch into space. Various combinations of diametral and axial clearances between the pellets and their enclosing structures were tested. The results of these tests indicate that for the present state-of-the-art for fabricating UN fuel pellets, a diametral clearance of 0.254 millimeter and an axial clearance of 0.025 millimeter are tolerable when subjected to launch-induced loads.
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SUMMARY

Shock and vibration tests were conducted on cylindrically shaped, depleted, uranium mononitride (UN) fuel pellets. The purpose of these tests was to determine the structural capabilities of the pellets when they are exposed to the shock and vibration loading that a nuclear reactor may encounter during launch into space. Various combinations of diametral and axial clearances between the pellets and their enclosing structures were tested. Different pellet fabrication techniques were also used to make up the test pellets. No significant differences in structural behavior during tests were observed among pellets made by different techniques.

Preliminary vibration and shock tests on solid pellets indicated that a diametral clearance of 0.254 millimeter between the fuel pellets and their containing structure would not cause any detrimental effects on the pellet structure. For these tests the axial clearance was held constant at approximately 0.025 millimeter.

Most of these tests were then conducted with hollow pellets of a reference design concept using axial clearances of 0.025 and 0.076 millimeter. The diametral clearance for these tests was held at a constant value of 0.254 millimeter.

The results of these tests indicate that with the present state-of-the-art for fabricating UN fuel pellets, a diametral clearance of 0.254 millimeter is tolerable for the pellets to maintain their structural integrity during launch. An axial clearance of 0.025 millimeter is also tolerable, but any axial clearance greater than 0.025 millimeter can cause a failure of the pellets when they are exposed to launch loads in accordance with initial Saturn V specifications.

INTRODUCTION

Uranium mononitride (UN) ceramic pellets are fuel forms being considered for use in a compact fast reactor for space power applications. An overall description of this
reactor concept is given in reference 1. The design of the fuel element for this concept is shown in figure 1. The fuel in the element consists of several UN cylindrical pellets with approximate dimensions of 1.58-centimeter outside diameter and 0.51-centimeter inside diameter. A 0.013-millimeter tungsten barrier is placed between the pellets and the T-111 cladding to prevent a possible reaction. Collar-button-type vibration suppressors at each end of the element position the fuel pellets axially, provide a fission gas collection space (as does the hollow section of the fuel pellets), and accommodate relative thermal expansion by buckling.

In the reference design reactor core concept, the main support structure for the core consists of a cluster of tubes welded together to form a "honeycomb" structure. The fuel elements are positioned and anchored inside the tubes. Lithium coolant flows in the annular channel between the fuel element and the support tube.

In designing the fuel element, one of the ways in which diametral swelling of the fuel can be accommodated without excessive distortion of the clad is to provide clearance between the fuel and the clad to accommodate some or all of the volume change. Two limiting conditions to this approach are (1) the resulting increase in fuel operating temperature, and (2) the ability of the fuel pellets to withstand the handling and launch loads tending to bang them into the clad. Some initial diametral clearance is necessary for assembly and the accommodation of relative thermal expansion. Another area where the ability of the pellet to withstand dynamic loads has an effect on the fuel-element design is at the ends of its enclosure. The vibration suppressors, which locate the fuel axially in this design, will impact with the fuel during axial accelerations. The manufacturing tolerances necessary to maintain the required axial clearance between these surfaces and the fuel will have a decisive effect on the feasibility of the design and the cost of production.

To study the effect of clearances on the structural integrity of UN pellets, an experimental test program was conducted. In these tests UN pellets were put into containers with varying degrees of radial and axial clearances and vibrated over a range of simulated launch and shock loadings. Following these vibration and shock tests, the pellets were visually examined for structural damage which might have occurred.

Because a minimum period of 6 months was required to fabricate the pellets within the specification of the reference design, it was decided to conduct preliminary tests using a few UN pellets available from another study program being conducted at this laboratory. Tests were then continued with newly manufactured specimens.
TEST CONSIDERATIONS AND PROCEDURES

Considerations

The preliminary tests were conducted mainly to determine if any major fuel structural behavior problems could be detected from existing depleted uranium nitride pellets and to check the test procedure and equipment. Appendix A describes the pretest checkout of the test equipment.

Only three solid cylindrical pellets were available at the start of the study. It was therefore decided that in the preliminary tests only the effects of vibrations in the diametral direction would be observed for varying diametral clearances of 0.076, 0.152, and 0.254 millimeter. These tests would give an indication of limiting diametral clearances for fuel-element design studies. Heat-transfer analysis indicated that, from fuel temperature considerations, the useful clearance would be limited to less than 0.254 millimeter.

The main portion of tests using newly fabricated hollow pellets of the reference reactor design were to confirm the results of the preliminary tests and to explore the effect of axial clearance. Vibration and shock conditions used for the tests are shown in table I and figures 2 and 3. These diametral conditions are initial Saturn V specifications (ref. 2) modified to include some of the more demanding conditions of reference 3 and of a NASA unpublished internal document titled "Advanced Nuclear Space Power System Environmental Specification" (No. P2241-1, March 1, 1969).

Procedure

In table I the actual conditions run for each of the test series 1 to 7 are described. The different values of acceleration (g) loading, double amplitude (D.A.), and spectral density ((g's)²/Hz) for the various axial tests listed in the tables on pages 4 and 5 resulted because the total weight (or length) of a combination of pellets used in one test differed from other test combinations. This length difference made a change in the end spacer length necessary, for each test, to fit the existing fixture length.

The fuel section of the elements is made up of a series of stacked pellets (fig. 1). Since the length and combined weight of the pellets in the total stack is less than that of the full-length elements, the test g-loading required to simulate actual loading was increased accordingly for each test. The values of g-loading used for the tests in the axial direction were determined from the following expression:
Test g-load (axial) = Specified g-load

\[
\times \left[ \frac{\text{Total weight of fuel pellets in one complete fuel element}}{\text{Actual weight of test pellets plus end spacers}} \right]
\]

where specified g-load is equal to the g-value used in the diametral direction plus a 6-g launch acceleration load (ref. 2). This acceleration load was added to the axial tests because of the orientation of the reactor during launch. Except for test 7, where only one axial clearance was tested, the tests progressed from the smaller to the larger axial and diametral clearance (see table I). The tests were conducted in the sequence as follows:

**Shock tests.** - Shock tests are summarized in the following table:

<table>
<thead>
<tr>
<th>Number of shocks</th>
<th>Diametral tests (tests 1 to 6)</th>
<th>Axial tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diametral</td>
<td>Axial</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

All shocks are to result in a waveform of one-half sine pulse of 11 milliseconds duration.

**Sinusoidal vibration tests.** - The sinusoidal vibration tests are summarized in the following table:

<table>
<thead>
<tr>
<th>Frequency range, Hz</th>
<th>Diametral tests (tests 1 to 6)</th>
<th>Axial tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diametral</td>
<td>Axial</td>
</tr>
<tr>
<td>20 to 60</td>
<td>0.109 cm D. A.</td>
<td>8 g's peak</td>
</tr>
<tr>
<td>60 to 140</td>
<td>0.027 cm D. A.</td>
<td>15 g's peak</td>
</tr>
<tr>
<td>140 to 190</td>
<td>Reverse above</td>
<td>70.5 g's peak</td>
</tr>
<tr>
<td>190 to 2000</td>
<td>Reverse above</td>
<td>75.3 g's peak</td>
</tr>
<tr>
<td>2000 to 20</td>
<td>Reverse above</td>
<td>75.3 g's peak</td>
</tr>
</tbody>
</table>

The time rate of sweep is 1 octave per minute for all tests. D. A. denotes double amplitude of sine wave. Figure 2 shows the peak acceleration loading over the range of frequencies tested.

**Random vibration tests.** - The random vibration tests are summarized in the following table:
<table>
<thead>
<tr>
<th>Frequency range, Hz</th>
<th>Diametral tests (tests 1 to 6)</th>
<th>Axial tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 to 200</td>
<td>3-db/octave increase 0.2 (g's)$^2$/Hz</td>
<td>Tests 4 and 5</td>
</tr>
<tr>
<td>200 to 500</td>
<td>3-db/octave decrease</td>
<td>Tests 6 and 7</td>
</tr>
<tr>
<td>500 to 2000</td>
<td>3-db/octave decrease</td>
<td></td>
</tr>
</tbody>
</table>

| Overall level, g's (rms) (see appendix B) | 14.8 | 49.5 | 52.4 |
| Total test time, min | 12   | 12   | 12   |

Figure 3 shows the power spectral density ((g's)$^2$/Hz) over the range of frequencies tested. Before and after each test the pellets were measured, weighed, and visually inspected for changes. For safety reasons, smear tests were made to check the radiation levels on the test fixture, both at the test site and at the "hot lab" where the pellets were removed from the fixture. All pellets were removed under a ventilated hood to prevent inhalation of any UN dust and also because UN dust is pyrophoric.

**TEST SPECIMENS**

The three solid pellets tested separately in the preliminary test series were manufactured using two different processes. The pellet in test 1 was made by cold isostatically pressing depleted UN powder at 48 250 newtons per square centimeter into a mold. It was partially sintered at 1970 K for 2 hours, "canned" in pure iron, and hot isostatically pressed to final size and density (93 percent) at 20 680 newtons per square centimeter and 1640 K. The iron can was subsequently machined off. This method resulted in flaking plus circumferential cracking. After the flaws were machined from a number of specimens, only one pellet was usable.

The remaining two pellets for tests 2 and 3 were made by uniaxially die pressing UN powder at 13 780 newtons per square centimeter. The powder had 2 percent camphor as a binder. The pellets were then sintered at 1530 K for 1 hour to a density of 93 percent.

The oxygen content of these pellets, which is important in the overall fuel-pin material makeup, was above the level which was specified for the reference reactor design. This difference, however, was considered to be of little importance to the strength of the test pellets.

These three pellets were of random dimensions and are shown in figure 4 with the dimensions listed in table I under tests 1, 2, and 3.
The main portion of tests were conducted with hollow cylinders of depleted UN. The first batch was made by uniaxially die pressing UN powder at 13 780 newtons per square centimeter. A 2-percent-camphor binder was used. The pellets were then removed from the die and final sintered at 1530 K for 1 hour to a density of about 92.1 percent.

This die differed from that used for the pellet in tests 2 and 3 in that it had a center rod to form the inside diameter of the pellet. This method also produced numerous circumferential cracks, probably caused by pressing the soft UN pellet from the die. The cracked sections of these pellets were removed, thereby shortening the original pellet lengths. Of the 21 pellets fabricated, only 10 were of sufficient length to use. The dimension of these are shown in table I for tests 4 and 5. Each of these test groups was made up of five pellets totaling a length of approximately 4.245 centimeters. A pretest photograph of each set is shown in figures 5 and 6.

A second set of six pellets, each about 3.81 centimeters long, were received and are shown in figure 7. These pellets were made by first cold isostatically pressing the UN powder without a binder in a rubber mold at 41 350 newtons per square centimeter. The mold was removed and the pellets sintered at 2570 K for 1 hour. This eliminated the circumferential cracks and resulted in full 3.81-centimeter-long pellets, the proposed reference design length, with a density of about 93.5 percent.

Test 6 used two pellets 3.81 centimeters long, and test 7 used one pellet 3.81 centimeters long and one pellet 3.48 centimeters long. The shorter pellet length resulted from having to remove a chipped section from one end of the pellet. Dimensions of these pellets are shown in table I under tests 6 and 7.

TEST FIXTURES

A sketch of one of the test fixtures used for the preliminary tests (1 to 3) is shown in figure 8(a). Three similar fixtures were used, each having different bore diameters so that diametral clearances of 0.076, 0.152, and 0.254 millimeter existed between the pellet and the bore. End cap screws with steel spacers centered the pellet axially in the fixture and provided the desired 0.025-millimeter axial clearance for these tests.

The test fixtures were analyzed for natural frequencies by the energy method for beams (refs. 4 and 5). Various end conditions were calculated to see if there were any natural frequencies in the test operating range (see appendixes A and C). All frequencies for end conditions stiffer than fixed-free were found to be above the operating range of 2000 hertz. Fixed-free was calculated to be 1190 hertz while fixed-fixed and free-free were 7570 hertz (see appendix C).

Check runs on the fixture were also made which confirmed the fact that no natural frequency existed in the operating range of the tests. These check runs are discussed in appendix A.
Amplitudes of extraneous vibrations for different frequencies or direction were observed at various input frequencies. It was therefore concluded that reinforcing of the fixture could reduce these extraneous vibrations. At this point it was decided to use the original fixtures as shown in figure 8(a) for the preliminary horizontal tests (1 to 3). However, for the main tests (4 to 7) the fixture was stiffened as shown in figure 8(b). Checkout of the reinforced fixture showed that the extraneous vibrations are indeed reduced.

Since all the hollow pellets for the main tests had an outside diameter of 1.58 centimeters, the fixture bore was machined to 1.605 centimeters diameter. This provided the 0.254-millimeter diametral clearance limit established by the fuel temperature considerations and shown to be acceptable in the preliminary vibration tests. During the axial vibration tests, with the fixture in the vertical orientation, a T-111 spacer rests above the pellet (see fig. 8(b)). In addition to positioning the pellets, the spacer provides a weight which is used to establish the test g-loads required to simulate the actual loads which will exist on the lower pellet of a complete fuel pin (see equation in the section TEST CONSIDERATIONS AND PROCEDURES). The seal shims and the end cap screws provided the means of setting the axial clearances (<0.025 and 0.076 mm).

APPARATUS AND INSTRUMENTATION

The apparatus used for the tests was a M.B. Electronics Model-C60 shaker capable of producing sinusoidal and random vibrations from 20 to 3000 hertz with a sweep rate of 1 octave per minute. It has a sinusoidal rating of 26 700 newtons. Frequency, displacement, and g-load were programmed into the controls of the shaker.

For the shock test an AVCO drop tester was used. The fixture, encapsulating the specimen, is mounted on the drop tester table and fitted with an accelerometer. The table with the specimen and fixture is raised and dropped a predetermined height onto rubber bumpers. The combination of the height and the thickness of the rubber bumpers determine the shock load and the specified one-half sine wave rebound of 10 to 11 milliseconds. These values of height and rubber bumper thickness are precalibrated by using simulated weights. The values are within ±10 percent of specified shock and rebound values.

The shock impact and the time period of the sinusoidal impact are recorded visually on a memory oscilloscope and on tape. Later the tape is rerun and the shock wave photographed from the oscilloscope. A typical plot of shock loading against time for a fixture with specimen enclosed is shown in figure 9.
TEST RESULTS

The results of the vibration tests are summarized in table I. For the preliminary tests (1 to 3) the test pellets were solid cylinders, rather than the hollow cylinders, as established in the reference reactor fuel-pin design. Shock, sinusoidal, and random vibrations were conducted only in the diametral direction. Differences in geometry would have minimal effect on test results in this direction. Tests were conducted with diametral clearances of 0.076, 0.152, and 0.254 millimeter. All had an axial clearance of 0.025 millimeter. The only noticeable deterioration was observed after the sinusoidal tests were completed with a 0.152-millimeter diametral clearance (test 2). Some flaking was observed on the flat end of the pellet. Apparently this was caused by the number "2" etched on the face for identification (fig. 10). As the shaker does not give a true uniaxial motion, the adjacent faces of free moving pieces can pound together during a nominal diametral test. The mating faces of the pellet and the spacers were not optically flat; therefore, high loading occurred on the edges of the inscribed number. This could have caused the flaking. Following this test no further flaking or damage was noticed. Figure 11 shows pellets 1S, 2S, and 3S after testing.

Test 4 was conducted using pellets of the reference design configuration. With the diametral clearance of 0.254 millimeter and an axial clearance of 0.025 millimeter the pellets withstood the shock, sinusoidal, and random vibration loads in both the diametral and axial directions. However, when the axial clearance was increased to 0.076 millimeter, the pellets withstood the shock loads but failed during the sinusoidal test. The pellets after testing are shown in figure 12.

Test 5 was a repeat of test 4 with the same results. These pellets after testing are shown in figure 13.

In test 6 pellets having longer lengths than those previously tested were used (3.8 cm compared to 0.71 to 1.04 cm). Again these pellets withstood shock, sinusoidal, and random loads for a diametral clearance of 0.254 millimeter and with an axial clearance of less than 0.025 millimeter. However, with an axial clearance of 0.076 millimeter the pellets failed, as in tests 4 and 5. These pellets after testing are shown in figure 14.

Up to this point all tests were conducted in a sequence in which the pellet failed before a random test with 0.076-millimeter axial clearance could be conducted. For this reason, in test 7 the random vibration test was conducted first. The pellets were vibrated in the axial direction with 0.076-millimeter axial clearance and 0.254-millimeter diametral clearance. The pellets failed this test and are shown in figure 15.

CONCLUSIONS

From these tests it can be concluded that a diametral clearance of up to 0.254 millimeter and an axial clearance of approximately 0.025 millimeter are acceptable between
the UN pellets and the outer structure surrounding the pellets. With these clearances the structural integrity of the pellets will be maintained when exposed to shock, sinusoidal, and random vibrations of the initial Saturn V specifications encountered during the launching of a space power reactor.

The allowable axial clearance for withstanding these sorts of vibratory loads is quite small and will require fabrication techniques of high precision. Closure weld shrinkage is one of the critical items which will have to be controlled.

The pellets used in these tests were products for manufacturing studies as well as vibration test specimens. The results of these tests indicate little difference in pellets of various fabrication methods. All methods used produced at least one specimen with obvious external flaws which were removed prior to testing. However, unobserved internal flaws may have caused premature failures. Although the test results did not clearly indicate a best method of fabrication, the later test pellets had fewer detectable flaws. Improvements in the manufacturing techniques could result in a sounder pellet structure so that greater axial clearances might be considered.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 21, 1971,
112-27.
APPENDIX A

FIXTURE NATURAL FREQUENCY TESTS

Vibration tests were conducted to see if there were any natural or extraneous frequencies in the test fixture itself over the test range. These tests covered the range of vibrations from 20 to 2500 hertz with an input of 1 g. This range spanned the vibration frequency called for in the test specifications for the fuel pellets.

The first fixture tested was 5.07 centimeters by 5.07 centimeters by 19.3 centimeters with end caps but without fuel pellets or spacer (see fig. 8(a)). This fixture was tested in the vertical and horizontal positions. The accelerometers were mounted first near the ends and then at the middle of the fixture. Figures 16(a) to (d) show the fixture mounted on the vibration table in the vertical and horizontal positions with the accelerometers in various mounting positions. The table input control accelerometer was mounted on the shaker table next to the fixture. While the first fixture mounted horizontally was vibrated, side-motion amplitudes were observed at frequencies of 80 and 900 hertz. Figures 17(a) and (b) are graphs of the vibration g-loading against frequency for these tests and show amplitudes to be about 15 percent of the fixture vertical amplitude vibration g-loading. The vertical motion of the fixture was the same as the table input, which is shown in figures 17(c) and (d), respectively. The results of this test indicated that no further horizontal-mounted fixture tests were necessary. The fixture was then mounted and vibrated in the vertical direction (see figs. 16(c) and (d)). The vibration g-load peak amplitudes in the horizontal direction occurred at frequencies of approximately 80 and 480 hertz. Figures 18(a) to (d) show horizontal peak vibration g-loads to be as high as 88 percent of the table vertical input loads with the accelerometers at the end and about 35 percent with the accelerometers mounted at the midpoint of the fixture. Figure 18(e) shows the table vertical input amplitude of 1 g over the frequency range of from 20 to 3000 hertz. In all the tests, vibration peaks above 2000 hertz (above UN vibration specification range) were not considered.

To further reduce the side motion of the fixture, two 2.54-centimeter by 2.54-centimeter by 19.3-centimeter steel bars were added to the square fixture (see fig. 8(b)). Tests were conducted with the fixture in the vertical position only, over the frequency range of 20 to 2000 hertz. Accelerometers were mounted at the midpoint of the fixture on adjacent side faces. (Same as the first fixture in fig. 16(d).) The vibration g-load peak amplitudes were again noted, at approximately the same frequencies as in the first fixture test, at 80 and 480 hertz. Figures 19(a) and (b) show that these vibration g-load horizontal amplitudes are much smaller, being approximately 13 percent of the vertical table vibration g-load amplitudes shown in figure 19(c).

The results of these tests are shown in table II.
APPENDIX B

CALCULATION OF RANDOM VIBRATION

In calculating a typical random vibration total in $g$'s (rms) for the following power spectral density curve (fig. 3), which is for the axial tests 4 and 5, one must consider three regions of different spectral density accelerations (ref. 6):

1. Region 1, which is a nonconstant acceleration density
2. Region 2, which is a constant acceleration density
3. Region 3, which is a nonconstant acceleration density

These regions are shown in the following sketch:

The following equations apply to these areas and the nomenclature of these equations is as follows:

- $f_1$: lower frequency of region, Hz
- $f_2$: upper frequency of region, Hz
- $G$: acceleration, $g$'s (rms)
- $M$: slope of acceleration density line, dB/octave
- $W$: acceleration density (spectral density at $f_2$, $(g' s)^2/Hz$

The area under the curve (region 1) is determined by the equation

$$G^2 = \frac{3W}{3 + M} \left[ f_2 - \left( f_1 \frac{M}{f_2} \right)^{\frac{M}{3}} f_1 \right]$$

(1)
where \( f_2 > f_1 \) and \( M \) is either positive or negative (except for \( M = -3 \) db/octave, a special case). The area under constant \( W \) line (region 2) is

\[
G^2 = W(f_2 - f_1) \tag{2}
\]

where \( f_2 > f_1 \). The area under the curve (region 3) is

\[
G^2 = f_2 W \ln \frac{f_2}{f_1} \tag{3}
\]

where \( f_2 > f_1 \). This equation differs from equation (1) because the slope is -3 decibels per octave and is therefore a special case.

The total acceleration \( G \) for the regions 1, 2, 3 is the square root of the sum of the squares:

\[
G = \left( G_1^2 + G_2^2 + G_3^2 \right)^{1/2} \tag{4}
\]

Thus solving for the total \( G \) for the curve shown in the preceding sketch:

Region 1 (eq. (1)):

\[
G_1^2 = \frac{3 \times 2.25}{3 + 3} \left[ 200 - \left( \frac{20}{200} \right)^{3/3} \right]
= 1.127(200 - 2)
= 223 \text{ g's (rms)}
\]

Region 2 (eq. (2)):

\[
G_2^2 = 2.25(500 - 200)
= 668 \text{ g's (rms)}
\]

Region 3 (eq. (3)):  

12
\[ G_3^2 = 2000 \times 0.56 \ln \frac{2000}{500} \]

\[ = 1120 \times 1.386 \]

\[ = 1550 \text{ g's (rms)} \]

Therefore the total acceleration \( G \) is (eq. 4):

\[ G^2 = 223 + 668 + 1550 \]

\[ G^2 = 2441 \]

\[ G = 49.5 \text{ g's (rms)} \]

This is the overall density level and is used to determine the shaker capacity in random vibration. For example, the shaker used has a random vibration rating of 14 444 newtons. The maximum \( G \) for random is found by dividing the sum of the fixture and shaker armature weights (90.7 N + 177.7 N) into the shaker random rating of 14 444 newton; or

\[ \frac{14,444}{90.7 + 177.7} = 53.7 \text{ g's (max, rms)} \]

Thus, from the preceding calculations it can be seen that the required simulated acceleration loading value of 49.5 g's (rms) for the axial random specification is within the machine capacity of 53.7 g's (rms).
APPENDIX C

SAMPLE CALCULATION FOR NATURAL FREQUENCY OF THE FIXTURE

The natural frequency of a square bar with a center hole drilled lengthwise was calculated for varying end supports.

The mounting of the fixture (see fig. 16) was not a simple end condition. Calculations were therefore made for a uniformly loaded bar with first a fixed-free end condition; second, a fixed-fixed end; and third, a free-free end condition. The vertical fixture was probably between the fixed-fixed and the fixed-free, while the horizontal may be between the fixed-fixed and the free-free end conditions.

The equation used was the energy equation with precalculated constants for various end conditions for beams of uniform section and uniformly distributed load (ref. 5).

\[ W_n = A \sqrt[4]{\frac{EI}{\mu l^4}} \text{ rad/sec} \]

\[ f_n = \frac{A}{2\pi} \sqrt[4]{\frac{EI}{\mu l^4}} \text{ Hz} \]

where

E  modulus of elasticity
I  plane moment of inertia
\( \mu \)  mass force per unit length \( l \)
A  constant determined by the end condition

The bar had the following dimensions:

- Width, cm ........................................ 5.07
- Height, cm ....................................... 5.07
- Length, cm ....................................... 19.32
- Hole diameter, cm .............................. 1.58
- Weight, kg (N) .................................. 3.49 (34.2)
- Modulus of elasticity, N/cm² .................. 20.68×10⁶
The constants $A$ used were (see ref. 6):

- Fixed-free end: $3.52$
- Fixed-fixed end: $22.4$
- Free-free end: $22.4$

The moment of inertia $I$ of the bar is the difference between the moment of inertia for the block $I_B$ and that for the hole $I_H$. Solving for $I$,

$$I_B = \frac{D^4}{12} = \frac{(5.04)^4}{12} = 55 \text{ cm}^4$$

$$I_H = \frac{\pi d^4}{64} = \frac{\pi (1.58)^4}{64} = 0.306 \text{ cm}^4$$

$$I = 55 - 0.306 = 54.7 \text{ cm}^4$$

$$\mu = \frac{34.2 \text{ newtons}}{19.32 \text{ cm}}$$

The natural frequency of the fixture is

$$fn = \frac{A}{2\pi} \left( \frac{20.68 \times 10^6 \times 54.7 \times 980 \times 19.32}{34.2 \times 13.82 \times 10^4} \right)^{1/2}$$

$$= \frac{A}{2\pi} \left( 4.52 \times 10^6 \right)^{1/2} = \frac{A}{2\pi} \times 2125$$

$$fn = 338 \text{ A}$$

Thus, the natural frequency for each end condition is as follows:

1. Fixed-free: $fn = 3.52 \times 338 = 1190 \text{ Hz}$
2. Fixed-fixed: $fn = 22.4 \times 338 = 7570 \text{ Hz}$
3. Free-free: $fn = 22.4 \times 338 = 7570 \text{ Hz}$
REFERENCES


### TABLE I. - CONDITIONS FOR TEST SERIES 1 TO 7

<table>
<thead>
<tr>
<th>Test</th>
<th>Pellet designation&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Outside diameter, cm</th>
<th>Length, cm</th>
<th>Inside diameter, cm</th>
<th>Shock, b g's</th>
<th>Sine vibration</th>
<th>Random vibration</th>
<th>Diametral clearance, mm</th>
<th>Axial clearance, mm</th>
<th>Test results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>1.642</td>
<td>1.765</td>
<td>Solid</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.076</td>
<td>0.025</td>
<td>No failures</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>1.671</td>
<td>1.049</td>
<td>Solid</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.152</td>
<td>0.025</td>
<td>No failures</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>1.671</td>
<td>1.046</td>
<td>Solid</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.254</td>
<td>0.025</td>
<td>No failures</td>
</tr>
<tr>
<td>4</td>
<td>41</td>
<td>1.579</td>
<td>0.718</td>
<td>0.511</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.254</td>
<td>&lt;0.0254</td>
<td>No failures</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.580</td>
<td>0.912</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>51</td>
<td>1.580</td>
<td>0.933</td>
<td>0.511</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.254</td>
<td>0.025</td>
<td>No failures</td>
</tr>
<tr>
<td>6</td>
<td>61</td>
<td>1.580</td>
<td>0.931</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>10</td>
<td>1.579</td>
<td></td>
<td>0.733</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Top pellet (5) broken</td>
</tr>
<tr>
<td>11</td>
<td>1.580</td>
<td></td>
<td>0.732</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2H</td>
<td>1.580</td>
<td>3.811</td>
<td>0.511</td>
<td>15 dia.</td>
<td>(c)</td>
<td>(d)</td>
<td>0.254</td>
<td>0.025</td>
<td>No failures</td>
</tr>
<tr>
<td>4H</td>
<td>1.580</td>
<td>3.816</td>
<td>0.509</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Failed axial sinusoidal test</td>
</tr>
<tr>
<td>7</td>
<td>3H</td>
<td>1.580</td>
<td>3.813</td>
<td>0.511</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>0.254</td>
<td>0.076</td>
<td>Failed axial random test</td>
</tr>
<tr>
<td>5H</td>
<td>1.580</td>
<td>3.484</td>
<td>0.511</td>
<td></td>
<td>(e)</td>
<td>(c)</td>
<td>(d)</td>
<td>0.254</td>
<td>0.076</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>S denotes solid pellets. All others were hollow.

<sup>b</sup> Dia. denotes diametral direction; axial denotes axial direction.

<sup>c</sup> See fig. 2.

<sup>d</sup> See fig. 3.

<sup>e</sup> No test.
TABLE II. - RESULTS OF FIXTURE NATURAL FREQUENCY TESTS

<table>
<thead>
<tr>
<th>Fixture test position</th>
<th>Fixture shown in -</th>
<th>Accelerometer location&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Peak load at corresponding frequency&lt;sup&gt;b&lt;/sup&gt;. g's</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal</strong></td>
<td>Fig. 8(a)</td>
<td>End (side 2); fig. 16(a)</td>
<td>0.15 at 80 Hz; 0.13 at 900 Hz (fig. 17(a))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Midpoint (side 2); fig. 16(b)</td>
<td>0.15 at 80 Hz; 0.19 at 900 Hz (fig. 17(b))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>End (top of fixture); fig. 16(a)</td>
<td>1 at 20 to 2000 Hz (no peak) (fig. 17(c))</td>
</tr>
<tr>
<td><strong>Vertical</strong></td>
<td>Fig. 8(a)</td>
<td>End (side 1); fig. 16(c)</td>
<td>0.15 at 80 Hz; 0.42 at 450 Hz (fig. 18(a))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>End (side 2); fig. 16(c)</td>
<td>0.15 at 80 Hz; 0.88 at 450 Hz (fig. 18(b))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Midpoint (side 1); fig. 16(d)</td>
<td>0.16 at 115 Hz; 0.21 at 450 Hz (fig. 18(c))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Midpoint (side 2); fig. 16(d)</td>
<td>0.12 at 80 Hz; 0.35 at 450 Hz (fig. 18(d))</td>
</tr>
<tr>
<td><strong>Vertical</strong></td>
<td>Fig. 8(b)</td>
<td>Midpoint (side 1); fig. 16(c)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>&lt;0.1 at 80 Hz; 0.13 at 500 Hz (fig. 19(a))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Midpoint (side 2); fig. 16(c)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.15 at 80 Hz; 0.13 at 450 Hz (fig. 19(b))</td>
</tr>
</tbody>
</table>

<sup>a</sup>Side 1 - face parallel to shaker trunion bearings. Side 2 - face perpendicular to shaker trunion bearings.

<sup>b</sup>Table input (figs. 17(d), 18(e), and 19(c)) loads are 1 g over frequency range from 20 to 2500 Hz.

<sup>c</sup>Reinforced fixture mounted same as shown.
Figure 1. - Typical fuel-element cross section.
Figure 2. - Peak acceleration loading against frequency for sinusoidal vibration tests.

Figure 3. - Power spectral density against frequency for random vibration tests.
Figure 4. - Pretest, solid, uranium mononitride fuel pellets used in preliminary tests 1 to 3 to establish diametral clearance limit.

Figure 5. - Pretest, hollow, uranium mononitride fuel pellets for test 4.
Figure 6. - Pretest, hollow, uranium mononitride fuel pellets for test 5.

Figure 7. - Pretest, hollow, uranium mononitride fuel pellets for tests 6 and 7 (1H and 6H not used in these tests).
(a) Complete test fixture with specimen.

(b) Reinforced fixture with pellet positioned at end by T-111 spacers.

Figure 8. - Test fixture.
Figure 9. - Typical shock loading against time for fixture with specimen.

Figure 10. - End view of uranium mononitride pellet 25 showing flaked area. (Test 2.)
Figure 11. - Post-test, solid uranium mononitride fuel pellets used in preliminary tests 1 to 3 to establish diametral clearance limit.

Figure 12. - Post-test, hollow, uranium mononitride fuel pellets used in test 4.

Figure 13. - Post-test, hollow, uranium mononitride fuel pellets used in test 5.
Figure 14. - Post-test, hollow, uranium mononitride fuel pellets used in test 6.

Figure 15. - Post-test, hollow, uranium mononitride fuel pellets used in test 7.
Figure 16. - Preliminary test fixture and accelerometer orientation for fixture natural frequency tests.
Figure 17. - Vibration loading against frequency with square fixture (fig. 8a) in horizontal position.

(a) Accelerometer mounted on side face of fixture at one end (face parallel to shaker trunion bearing centerline).

(b) Accelerometer mounted on side face of fixture at midpoint (face parallel to shaker trunion bearing centerline).

(c) Accelerometer mounted on top of fixture at one end.

(d) Control accelerometer mounted on table.
Figure 18. - Vibration loading against frequency with square fixture (fig. 8a) in vertical position.
(a) Accelerometer mounted on front face at midpoint of fixture (face parallel to shaker trunion bearing centerline).

(b) Accelerometer mounted on side face at midpoint of fixture (face perpendicular to shaker trunion bearing centerline).

(c) Control accelerometer mounted on shaker table.

Figure 19. - Vibration loading against frequency with reinforced fixture (fig. 8b) in vertical position.