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GENERATION OF FINITE LIFE DISTRIBUTIONAL
GOODMAN DIAGRAMS FOR RELIABILITY PREDICTION

by Dimitri Kececioglu and William Neil Guerrieri

THE UNIVERSITY OF ARIZONA
College of Engineering
Engineering Experiment Station

prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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August 31, 1971

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Technical Management

NASA Lewis Research Center
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ABSTRACT

The methodology of developing finite life distributional Goodman diagrams and surfaces is presented in this paper. The Goodman surface and diagram presents allowable combinations of alternating stress and mean stress to the design engineer. The combined stress condition presented in these surfaces and diagrams is that of an alternating bending stress and a constant shear stress. The finite life Goodman diagrams and surfaces are created from strength distributions developed at various ratios of alternating to mean stress at particular cycle life values.

The conclusions drawn in this report indicate that the Von-Mises Hencky ellipse, for cycle life values above $10^4$ cycles, is an adequate model of the finite life Goodman diagram. In addition, suggestions are made which reduce the number of experimental data points required in a fatigue data acquisition program.
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Presently in the United States and abroad fatigue data, to a large extent, is presented in the form of the conventional S-N diagram. The S-N diagram's purpose is to graphically present strength data as a function of cycle life. This is done by testing a few specimens to failure at incremented stress levels. The data is plotted on a graph of log stress versus log cycles, (see Figure 1.1). This method of presenting fatigue data does not take into account one of the fundamental and most important fatigue aspects: the variability of the fatigue mechanism; i.e., even high quality test specimens, subjected to tightly controlled test conditions will rarely, if ever, fail at precisely the same cycle of life at a given stress level.

Recently the American Society for Testing and Materials (ASTM) has suggested that a statistically significant number of specimens at each stress level be tested in order that a failure distribution be developed for each stress level (1, p. 9). The cycles-to-failure distributions which are developed can be used to construct a statistical S-N diagram. This particular diagram, as pictured in Figure 1.2, has a mean line as well as plus and minus three sigma lines.
Fig. 1.1 Conventional Alternating Stress Cycles To Failure Diagram.
Fig. 1.2 Statistical S-N Diagram
It is possible through such statistical techniques as the use of probability plotting paper, the Chi-Squared and Kolmogorov-Smirnoff goodness of fit tests, and the computation of the four statistical moments to determine the best failure distribution probability density function. The probability density functions usually considered in such an analysis are the normal, lognormal, and Weibull distributions. The Weibull, because it is a three parameter distribution can take on many different shapes, from the exponential to the lognormal, by varying the three parameters. The Weibull probability density function, because of this flexibility is the most flexible of the three distributions mentioned.

The concept of failure distributions and strength distributions should be discussed briefly in order to avoid confusion in the later sections of this report. A failure distribution is derived directly from cycle to failure data. At a given stress level specimens will fail at particular values of cycle life. Even under the tightest controlled test conditions there will be variability in the cycle life of the individual test specimens. The failure distributions represent this variability in the test specimen's cycle to failure data. It is possible to form a histogram of the test specimen's cycle to failure data from which a failure distribution can be determined which adequately described the data. An example of such a histogram and distribution is given in Figure 1.3. Specifying the type
Fig. 1.3 An example of a Cycle to Failure Histogram and Distribution.
of distribution, normal, lognormal or Weibull, and the distribution parameters, uniquely describes the cycles-to-failure data.

A strength distribution is similar to a failure distribution in that it is described by a distributional type and the corresponding distributional parameters. When a specimen fails it means that the stress has exceeded the value of the strength of the specimen. Cycles-to-failure data reveals the percentage of specimens which have a strength less than the applied stress. It is possible to transform the cycles-to-failure distributions to strength distributions. A strength distribution describes the variability of the strength of a specimen at a specific value of cycle life and may be derived from a cycles to failure distribution.

The discussion of transforming cycles-to-failure distributions to strength distributions should be preceded by a discussion of some of the basic assumptions and restrictions which govern the generation of meaningful failure distribution data. Of primary concern is that all test specimens be uniform in geometry and metallurgical properties. This is important since subsequent calculations of the strength distributions will assume that specimens tested at various stress levels came from a homogeneous population. In addition the number of test specimens should be dependent upon the variability of the data generated. The statistical significance of the desired data is dependent upon the number of test specimens which are run,
the more data points generated the more positive the experimenter can be of his theoretical distribution. It has been found that at lower stress levels the variability of the data increases which indicates that an increase in test specimens is warranted at these lower stress levels.

The strength distribution may be oriented along the stress ratio axis of a distributinal Goodman diagram. The ordinate axis of such a diagram presents values of alternating stress while the abscissa presents mean stress values. The strength distributions which are placed along axes where the ratio of mean stress to alternating stress is a constant form distributional surfaces. These surfaces are formed when the strength distributions are connected by a mean line and plus and minus three sigma limit lines. (see Figure 1.4). The Goodman surfaces are of vast importance in reliability engineering where the interference of a stress distribution with the corresponding strength distribution is used to calculate the designed-in reliability of a mechanical part.

The objective of this report is to clearly present the methodology of generating distributional Goodman diagrams. The accomplishment of this objective requires that the following subject areas be investigated: Chapters II and III explain methods of converting cycles to failure data to strength distributions; Chapters IV and V develop methods of generating finite life Goodman diagrams and surfaces; Chapter VII directs it's attention to resolving the question of static strength
Fig. 1.4 Goodman Surface Formed By Strength Distributions
distributions to be used on the finite life Goodman diagram. In Chapter VIII empirical mathematical models of the Goodman diagram are discussed. Chapter IX explains the theoretical strength theories associated with the combined stress condition of alternating bending stress and mean shear stress. Chapter X recommends an empirical mathematical model of the Goodman diagram and an associated theoretical strength theory. Chapter XII suggests two methods of reducing the amount of experimental data needed to generate finite life Goodman diagrams as well as methods of obtaining cycles to failure distributions from these Goodman diagrams with a minimum amount of actual fatigue testing.

The discussion of these subject areas requires that actual fatigue data be used in support of this effort. This investigator was extremely fortunate to have access to the complex fatigue data generated under National Aeronautical and Space Administration Grant No. 03-002-044 at The University of Arizona under the direction of Dr. Dimitri B. Kececiglu. The Combined stress condition under which this data was generated was that of an alternating bending stress and a constant, mean shear stress. Although the discussions in this report often apply themselves to this data, the concepts presented are applicable to the area of combined bending and shear stresses and in general to the broader area of any combined stresses in fatigue.
CHAPTER II
NASA METHOD OF GENERATING POLYGONS AND DISTRIBUTIONS

2.1 Theory

Mr. Richard E. Smith, Aerospace and Mechanical Engineering Department, The University of Arizona, in his master's report of August 1965, used data from Dr. H. T. Corten, Department of Theoretical and Applied Mechanics, University of Illinois, to present a method of transforming theoretical cycle to failure distributions to cumulative strength polygons. The following is a summary of the methodology of that effort (2, p. III).

The theoretical cycle to failure distribution is first determined. Because goodness of fit tests allow only for the rejection of a distribution, it is possible that more than one of the three major fatigue probability density functions, normal, lognormal, and Weibull, will be accepted. It then becomes necessary for the investigator to choose which one fits the data best, at all of the various stress levels. Once this has been determined, the failure data is uniquely described by the theoretical failure distribution probability density function rather than the failure histogram of the sample data. This probability density function is symbolized by \( f(x) \). At each
of the \( j \) stress levels pictured in Figure 2.1.1, there is a theoretical normal, lognormal, or Weibull distribution which represents the failure data.

The cumulative failure probability, up to the \( x \)'th cycle, for the \( j \) stress level is given by

\[
F_j(x) = \int_0^x f(x) \, dx
\]

(2.1.1)

Of interest to the designer is the strength distribution for a specified cycle life. Once this cycle life is specified a series of cumulative failure distributions can be calculated for each stress level. In essence, the calculation of \( F_j(x) \) is equal to calculating the area bound by the theoretical failure probability density function and the cycle line \( N \) (see Figure 2.1.2). Note the cumulative area in percent for each failure distribution to the right of the graph. Here it is important that the failure distributions are known for the full strength range in order that a cumulative failure probability of zero to one hundred percent is obtained (2, p. 23).

The physical significance of the cumulative failure distribution is the percent of specimens in which the stress has exceeded the strength. The percentage of specimens at a given stress and cycle life with a strength equal to or less than the stress is now a known quantity. A plot, as shown in Figure 2.1.3 can then be made of the cumulative failures in percent versus stress level, and is known as the cumulative strength
Fig. 2.1.1 Theoretical Cycles to Failure Distributions
Log Cycles

Fig. 2.1.2 - Cumulative Failure Technique for Strength Distribution Determination (2, p. 25).

![Cumulative Failure Technique for Strength Distribution](image)

Fig. 2.1.3 - Strength Histogram (2, p. 25).

![Strength Histogram](image)
histogram. From this cumulative strength histogram it is possible to calculate the strength frequency histogram. The strength frequency histogram for the $i$'th cycle life is given by

$$f_i(s) = (F_{j+1}(x) - F_j(x))$$

where $F_j$ and $F_{j+1}$ are given by Equation 2.1.1 and where there are $M$ different stress levels to be considered. Since the above calculation for the strength frequency histogram is based upon inferences drawn from several stress levels it is imperative that the specimens used in all stress levels be from the same statistical population and that uniform test conditions are maintained from stress level to stress level.

Upon the determination of the strength frequency histogram, $f_i(s)$, the statistical operation for goodness of fit, in this case the Chi-Squared test, can be conducted to insure that a theoretical distribution can be fitted to the histogram. For specific cycle of life values a theoretical strength distribution is specified by one of the three theoretical distributions, specifically either the normal, lognormal or Weibull distribution. It is of major importance to note that this method is capable of determining the strength distribution in either the fatigue life or infinite life portion of the S-N curve (refer to Figure 2.1.1).

This makes it unnecessary to conduct a Probit analysis or staircase test to determine the strength distribution (2, p. 31).
Beyond the endurance limit, area to the right of the knee where the S-N curve is horizontal, the failure distributions are independent of time, previously expressed in cycles. However, at some stress levels it will be impossible to achieve a cumulative failures of one hundred percent because the test will be terminated at a pre-determined time before the specimen has failed. Fortunately the cumulative failure distribution is known up until the test termination which allows one to calculate the strength distribution in an identical manner as previously discussed (2, p. 31).

The result of the strength distribution calculations allows the construction of a statistical S-N diagram, as illustrated in Figure 2.1.4.

An adequate range of stress levels is necessary so that a complete failure histogram from zero to one hundred percent can be developed. Within this range a sufficient number of failure distributions must be known at different stress levels so that there will be enough class intervals, governed by Sturges' rule, in the strength histogram. Time and economic considerations limit the number of stress levels, and consequently the number of failure distributions generated to from five to eight such levels and distributions. The use of this limited number of distributions would not give enough class intervals for an accurate strength distribution calculation. It is necessary to develop a digital computer program which will interpolate many stress-to-failure distributions from the
Fig. 2.1.4 - Strength Distribution Versus Cycles To Failure (2, p. 33).
limited number available from experimental data (2, p. 27). It is possible to have this program accomplish these interpolations and perform the methodology discussed for the transformation of cycles to failure distributions to strength distributions as well as accomplish all required statistical operations.

2.2 Computer Method

Two computer programs were developed in order that the strength distributions could be calculated. The first reduces the cycles to failure data to failure distributions at each stress level. The second computer program transforms the failure distributions to strength distributions.

Because of the large amount of fatigue data generated, a digital computer program, in Fortran language, was used to reduce the failure data at various stress levels to failure distributions. Failure data in terms of stress level and cycle life is read into the computer. The computer is capable of calculating the following parameters: mean, standard deviation, coefficients of kurtosis and skewness, as well as of performing a Chi-Squared goodness of fit test. The computer program will use the normal distribution approximation in calculating the expected frequency in the Chi-Squared test when the sample data points are greater than thirty and will use the Student-t distribution when the sample number of data points is less than thirty. A flow chart, variable definitions and computer listing of the program are given in Appendix A.
The second computer program is used to determine the strength distributions from the failure distributions and is also written in Fortran language. The main steps of the program are listed below. A flow chart, variable definitions and computer list of the program are given in Appendix B.

1. For each of the experimental stress levels read into the computer the actual cycles to failure distribution parameters are developed.

2. The computer will then calculate failure distribution parameters for interpolated stress levels at 200 increments by straight line interpolation.

3. The computer will then calculate the cumulative failure distributions for each stress level and a given series of log cycle life values.

4. Next the computer will calculate the strength frequency histogram for each cycle life.

5. The computer program then calls the computer to calculate theoretical distribution parameters from the strength frequency histograms based on a normal or lognormal distribution. These parameters include mean, standard deviation, coefficients of skewness and kurtosis. A goodness of fit test, using the Chi-Squared test will then be performed to determine which of the normal of lognormal distributions fits the data best.
6. The parameters mentioned in five are then printed out in addition to the Chi-Squared values for each cycle life. This print out allows the investigators to determine which of the two distributions is a better fit.

2.3 Results

A complete description of Dr. Corten's testing program is found in Chapter V of Richard Smith's report, together with the analysis by Smith (2). Results of his analysis are summarized herein.

The Chi-Squared goodness of fit tests indicated that for aluminum specimens the cycles to failure distributions more closely fit a lognormal distribution; whereas distributions of steel specimens fit either the normal and lognormal distributions equally well (2, p. 76). It was observed that as the sample size was increased, the lognormal distribution fit the data better than the normal.

The transformation of cycles to failure distributions to strength distributions was accomplished by assuming the failure data to be distributed lognormally. The computer program previously described was used to determine the strength polygons for various cycles of life. The program output included the cumulative strength polygons, the mean, standard deviation, coefficient of skewness and kurtosis, and the Chi-Squared goodness of fit values for the strength distributions.
Analysis of this data indicates that the normal distribution fit the strength data better than the lognormal distribution. Coefficient of skewness is generally negative indicating a normal distribution. Coefficient of kurtosis values, which should be 3.0 for the normal distribution, fluctuate about a value of 3.0. In addition the Chi-Squared values indicated that for both type specimens the normal distribution represented the data better than the lognormal distribution (2, p. 85).

2.4 Discussion as to Validity

The transformation of cycles-to-failure distributions to strength data which was proposed by Richard Smith was found to be appropriate and based on well founded principles. John Smith's methodology is identical in transforming cycles-to-failure data to strength distributions. Richard Smith applied the technique to Corten's data (3, p. III) and John Smith to the NASA data.
CHAPTER III

GENERATION OF STRENGTH DISTRIBUTIONS

3.1 Theory

Smith (3) did extensive work in developing cycles to failure and strength distributions. These distributions were generated from fatigue data of specimens which were subjected to an alternating bending stress and a constant, mean shear stress. Specimens were subjected to different ratios of alternating to mean stress, known as stress ratios, at specified alternating stress levels. The specimens were of SAE 4340 steel and were of a grooved geometry. A grooved and ungrooved test specimen are shown in Figure 3.1.1 and 3.1.2. A complete description of the test program, which was sponsored by the National Aeronautics and Space Administration under Grant Number 05-002-044 at The University of Arizona, and of the procedures and materials is given in NASA CR-120831 (3).

The methodology used by John Smith in developing the strength distribution was similar to that used by Richard E. Smith (discussed in Chapter II of this report) (2). Cycles to failure distributions were developed at specific alternating stress levels for the stress ratios of infinity, 3.5, 0.825, and 0.44. The lognormal distribution was
Fig. 3.1.1 Grooved Fatigue Test Specimen of 4340 SAE Steel
Fig. 3.1.2 Ungrooved Tensile Test Specimen of 4340 SAE Steel.
found to best describe the cycles to failure data (3, p. 47).
The decision to accept the log-normal distribution over the
normal distribution was based on the Kolmogorov-Smirnov test
(3, p. 44).

Cycles to failure distributions for particular stress
ratios at specified stress levels were plotted on S-N diagrams
of alternating stress versus log cycles. This specified the
mean line and $\pm 3\sigma$ ($\sigma$ = standard deviation) envelope. After
this was established, it was possible to interpolate many cycles-
to-failure distributions at intermediate stress levels. This
interpolation process made it possible to calculate strength
distributions at specific cycle life values.

A review of the methodology used in the calculation of
strength distributions at specific cycle life values is in ord-
er at this time. A histogram can be located along the N
cycle life line in such a way that the midpoints of the cells
are at the interpolated stress levels. (See Figure 3.1.3).
The ordinate of each of the strength histogram cells is the
area bounded by the N cycle line and the cycle to failure dis-
tributions which has been interpolated for that particular
value of alternating stress. Thus if $f(N/S_i)$ is the cycle to
failure distribution at a particular alternating stress level,
the ordinate of the strength histogram cell at that stress
level will be given by:

$$F(N/S_i) = \int_{-\infty}^{n} f(N/S_i) \, dn$$  \hspace{1cm} (3.1.1)
Fig. 3.1.3  Histogram Obtained From Cycles-to-Failure Distributions
The ordinate of each of the strength histogram cells will be:

\[ F(N/S_i) = \int_{-\infty}^{n} f(N/S_i) \, dn \quad (3.1.2) \]

The histogram which is developed in this way is the cumulative strength histogram of specimens failing by \( N \) cycles. If \( S \) is the strength variable along the \( N \) cycle life line, the probability density function can be developed in the following manner. The value of the \( i^{th} \) cell of the strength probability density histogram is given by:

\[ f(S_i) = F(N/S_i) - F(N/S_i-1) \quad (3.1.3) \]

The fact that there are many interpolated cycles to failure distributions on the \( N \) cycle life line insures an adequate number of class intervals in the strength histogram. A normal distribution is then fitted, by statistical methods, to the strength probability density histogram (3, p. 55).

3.2 **Computer Method**

John Smith developed two computer programs which were used in the strength distribution calculations. The first of which, known as CYTOPR, calculated the cycles to failure distribution parameters, mean and standard deviation as well as the coefficients of skewness and kurtosis of the normal and
lognormal distributions. It also performed the Chi-Squared and Kolmogorov-Smirnov goodness of fit tests to determine if a normal or lognormal distribution fits the data best. A flow chart, variable definition and computer listing of this modified to include a sort routine is given in Appendix C.

The program STRENG finds the normal strength distributions from the lognormal cycles to failure distribution parameters. Smith used this method because it has been found by earlier studies that the normal distribution adequately describes the strength data (2), (4). The input data of this program includes the cycles to failure data, two extrapolated lognormal distributions on either side of the experimental distributions and interpolated lognormal cycles to failure distributions between the actual experimental failure distributions. The program then calculates the mean, standard deviation, and coefficients of skewness and kurtosis for the normal strength distributions. In addition a goodness of fit test, the Kolmogorov-Smirnov test is performed on the normal strength distributions. A flow chart, variable definitions computer listing of program STRENG given in Appendix D.
3.3 Results

The statistical S-N diagrams, which present the cycles to failure distributions for stress ratios of infinity, 3.5, 0.825 and 0.44, are given in Figures 3.3.1 through 3.3.4. The mean, standard deviations, and three sigma limits of the lognormal cycles to failure distributions are presented in Table 3.3.1.

John Smith presented only one S-N diagram which had the calculated strength distribution for a stress ratio of 3.5 and cycle life values of 10,000, 50,000, and 100,000 cycles. The normal parameters of the strength distribution which was placed on this particular S-N diagram are given in Table 3.3.2 while the S-N diagram appears as Figure 3.3.5. The completion of the stress ratio tests of 0.44 allowed the calculation of strength distributions for this stress level. Table 3.3.3 presents the parameters of the normal strength distributions at three cycle life values and a stress ratio of 0.44 while these distributions were added to Figure 3.3.4.

A complete table of the normal strength distribution parameters at all stress ratios and cycle life values was unavailable and consequently recovered by use of the program STRENG. The results are presented in Table 3.3.4.
Fig. 3.3.1 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 0 (3, p. 49)
Fig. 3.1.2 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 3.5. (3, p.50)
Fig. 3.3.3 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 0.825. (3, p. 51)
Fig. 3.3.4 Cycles-to-Failure Distributions, Strength Distributions and Endurance Strength Distribution for Stress Ratio of 0.444. (3, p. 52)
### CYCLES TO FAILURE

**Table 3.3.1 LOG-NORMAL DISTRIBUTION PARAMETER ESTIMATES AND MAX D-VALUES**

(3, p.37)

<table>
<thead>
<tr>
<th>Stress Ratio</th>
<th>Average Alternating Stress Level (psi)</th>
<th>Sample Size</th>
<th>Log-Normal Dist. Parameters</th>
<th>Max. D Value*</th>
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* Maximum D-Value From K-S Test
Table 3.3.2 Strength Distribution Parameters of the Normal Distribution at Three Cycle Life Values For A Stress Ratio of 3.5. (3, p.57)

<table>
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<tr>
<th>Cycles of Life N</th>
<th>Parameter Estimate of Normal Distribution</th>
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<td>Mean (psi)</td>
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Fig. 3.3.5 Plot of Estimated Strength Distributions at Various Cycles of Life and Estimated Cycles-to-Failure Distributions at Various Stress Levels for Stress Ratio of 3.5 (3, p. 38)
Table 3.3.3. Strength Distribution Parameters of the Normal Distribution at Three Cycle Life Values for a Stress Ratio of 0.44.

<table>
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<th>Cycles of Life N</th>
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Table 3.3.4 Parameters of Normal Strength Distribution
At Specific Stress Ratios and Cycles of Life.

<table>
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<th>Cycles</th>
<th>Mean Strength psi</th>
<th>Standard Deviation psi</th>
<th>-3 Sigma Limits psi</th>
<th>+3 Sigma Limits psi</th>
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*Parameters used in Chapter V Finite Life Goodman Diagrams.
Table 3.3.4 Parameters of Normal Strength Distribution
At Specific Stress Ratios and Cycles of Life.
(Continued).

<table>
<thead>
<tr>
<th>Cycles</th>
<th>Mean Strength psi</th>
<th>Standard Deviation psi</th>
<th>-3 Sigma Limits psi</th>
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*Parameters used in Chapter V Finite Life Goodman Diagrams.
Table 3.3.4 Parameters of Normal Strength Distribution at Specific Stress Ratios and Cycles of Life. (Continued).

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<th>Cycles</th>
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*Parameters used in Chapter V Finite Life Goodman Diagrams.
Table 3.3.4 Parameters of Normal Strength Distribution At Specific Stress Ratios and Cycles of Life. (Continued).

<table>
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<tr>
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*Parameters used in Chapter V Finite Life Goodman Diagrams.
3.4 **Discussion as to Validity**

The methodology which John Smith presented was found to be well grounded and accurate. Disturbing to the original investigator was the fact that the vertical strength distributions did not completely "fill" or span the entire width of the statistical S-N diagram envelope. That is, the vertical strength distributions did not span the complete distance between the plus three sigma line and minus three sigma line of the cycles to failure distributions. This, however, was not disturbing to this investigator for the following reason.

The histogram of the vertical strength distributions was developed from the cumulative failure probabilities of a large number of cycles to failure probability density functions. Along a particular cycle life line on the statistical S-N diagram, see Figure 3.4.1, the vertical distance from the intercepted plus and minus three sigma lines is of no particular significance.

The fact that the vertical strength distribution does not span the distance between point A and B can be explained in the following manner: The cycles-to-failure distributions are lognormal while the strength distributions are normal. It must be noted that in actuality only distributions to the left of the cycle life line contribute percentile areas to the cumulative strength distribution. The two points, A and B, are then completely unrelated. The plus and minus three sigma limits, which are not necessarily points A and B, of the
Fig. 3.4.1 A Vertical Strength Distribution Placed On The ± Three Sigma Envelope of A Statistical S-N Diagram.
vertical strength distributions are derived from the cumulative failure probabilities of the individual cycle to failure probability density functions. The plus and minus three sigma limits of the vertical strength distribution are related to the variability of the individual probability density functions of the cycles to failure data at particular stress levels intercepted by the vertical cycle life line.
CHAPTER IV

GENERATION OF FINITE LIFE GOODMAN DIAGRAMS (METHOD I)

4.1 Theory

To date most of the work done with the modified Goodman Diagram and Goodman surfaces has been concerned with the objective of presenting the relationship of alternating and mean stress for infinite periods of life. For the biaxial stress condition, where the specimen is subjected to both bending and shear stress, the Goodman diagram represents combinations of these two stresses, in the cartesian plane, where the specific combinations will not cause fracture to occur over a period of infinite life. The need for a surface to describe this relation occurs where probabilistic methods are used in reliability calculations by the interference method.

It is the purpose of the method under discussion in this Chapter to break away from the traditional concepts of using a Goodman surface to graphically represent the relation between bending and shear stresses for a period of infinite life, and present, in the cartesian plane, the relation of alternating stress to mean stress where the alternating stress is a bending stress while the mean stress is a shear stress for finite periods of life. The advantage of such an
approach is that it will graphically illustrate to a designer that it is possible to have higher combinations of bending and shear stresses for a finite life design. Hence, if a part need only function for a specified finite number of hours or cycles, after which its failure is not detrimental to the success of the mission, it can be subjected to combinations of bending and shear considerably higher than if it had to function in excess of $10^6$ cycles.

The data which was used for this method was generated by the Reliability Research Laboratory of the Aerospace and Mechanical Engineering Department under NASA Grant 03-002-044 at The University of Arizona. Cycles to failure data was generated for rotating specimens which were subjected to an alternating bending and constant shear stress at specific stress ratios. Stress ratio is defined as the ratio of alternating bending stress, to mean normal stress from torque.

The cycles to failure data was generated from 1967 to 1970 for stress ratios of infinity, (pure bending), 3.5, 0.8, and 0.44. The specimens which were subjected to a bending and shear stress had a grooved geometry. The material which the specimens were made of was SAE 4340 steel. A comprehensive explanation of the test program, machines, procedures and materials used can be found in NASA CR-72839 (3). The data generated by this experimental effort, which was used to construct finite life Goodman surfaces displayed in this
Chapter, included the bending and shear stress in each of the over 300 test specimens which were run to failure. The value of the cycles life at which the specimen failed was also recorded.

In the original test program twelve to eighteen test specimens were run at each stress level for each stress ratio. The lower number of specimens, twelve, occurred at higher stress levels, where variability in the cycle life data was small. The larger sample size was used at lower stress levels where variability in the data suggested a larger number of specimens be used. Referring to Figure 4.1.1, it can be seen that if the infinity stress ratio, located along the alternating stress axis, is to be bridged to a mean stress distribution at a stress ratio of zero by a Goodman surface, for the finite periods of life, there will have to be at least two other stress ratios which contain enough data points to form two strength distributions between the alternating stress distribution and the mean stress distribution and the mean stress distribution. Because of this restriction, it was necessary to screen the cycles to failure data to determine appropriate cycle life ranges. This was accomplished with the aid of Table 4.1.1. This table records the cycles to failure data for specimens at particular stress ratios. The cycle life ranges which were determined by this method are as follows:
Figure 4.1.1. Finite Life Goodman Diagram Illustrating the Need of Two Additional Strength Distributions at $R_1$, $R_2$. 
<table>
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Stress Ratios: r = ∞, r = 3.5, r = 0.825, r = 0.44
N = 1: 1,000 - 3,500 cycles to failure
N = 2: 20,000 - 40,000 cycles to failure
N = 3: 60,000 - 90,000 cycles to failure
N = 4: 90,000 - 200,000 cycles to failure
N = 5: 6,000 - 9,000 cycles to failure

It can be seen that these groups span the cycle life spectrum from low to relatively large cycle lifes. However, all five groupings are in the cycle life range to the left of the "knee" of the S-N diagram. It would have been desirable to keep the cycle life groups as small as possible, however the restriction of having enough data points within each group to form a distribution was the governing restriction in this case. The alternating stress distribution, where the shear stress or mean stress is zero, was obtained directly from the cycle to failure data of pure bending, \( r = \infty \), specimens. After the cycle life groups were determined the distribution placed on the alternating stress axis for each group was specified by the bending stress recorded in each of the specimens which failed within the cycle life range of that particular group.

The distribution which was used on the mean stress axis in all cases was the ultimate strength distribution of an ungrooved tensile test specimen which had the same cross-sectional areas as the actual grooved fatigue test specimens (grooved...
specimen results are used in Chapter V; see Figures 3.1.1 and 3.1.2. The mean normal strength distribution was obtained directly from static tensile strength tests. In the case of such tensile tests, it is found that the grooved specimens have a higher ultimate strength. This is caused by a radial stress which is introduced into the specimen at the root of the groove. However, a grooved specimen which is subjected to static torque load would not experience such a radial stress. At this time in the investigation, it would, therefore, not have been correct to use the grooved specimens to determine the strength distribution for the mean stress axis (see Chapters V and VII).

After obtaining the cycle life groups the next step was to relate the shear stress in each of the specimens to the proper mean stress. This was done by using two predominant strength theories. These two theories are the Von-Mises Hencky theory and the maximum shear stress theory. A complete discussion of these two theories appears in Chapter VIII of this report.

The resultant stress vector, $s_{p}$, for each of the data points has a mean stress of $s_{m} = \sqrt{3} \τ$ ($\tau =$ shear stress), (3, p. 3). The resultant stress vector is a combination of mean and alternating stresses. The magnitude of the resultant vector is:
In the case where the Von Mises Hencky criterion is used to relate the shear stress to the mean stress the resultant stress vector is called stress vector I.

If the maximum shear stress theory is used to relate the mean stress to the torsional stress, the governing relation will be (13, p: 2):

\[ S_m = 2\gamma \]

The maximum shear stress theory predicts that yielding will occur when the maximum shear stress is equal to the shear stress corresponding to the shear stress produced in a simple tension test for yield strength (15, p. 152). The Mohr circle predicts that yielding will begin when (15, p. 152):

\[ \gamma_{\text{max}} = \frac{S_y}{2} \]

Hence, the mean stress is given by \( S_m = 2\gamma_{\text{max}} \).

Although both these failure theories are based on yielding as the failure criterion much experimental data indicates that they apply as well when fracture is the failure criterion as represented by the static ultimate strength.
For the maximum shear stress theory the resultant stress vector magnitude will be given by:

\[
\sigma_r^2 = (\sigma_a)^2 + (\sigma_m)^2 \quad (4.1.4)
\]

\[
\sigma_r^2 = \left\{ (\sigma_a)^2 + (2\tau)^2 \right\}^{1/2} \quad (4.1.5)
\]

If the maximum shear stress theory is used to relate the shear stress to the mean stress the resultant stress vector is referred to as stress vector II.

The resultant stress vector must be described by both a magnitude and a direction. A stress ratio was previously defined as the ratio of alternating stress to mean stress. It can be seen that the stress ratio will vary with the strength theory which relates the shear stress to mean stress. Hence, for the stress vector where the Von Mises Hencky theory was used the stress ratio becomes:

\[
r_1 = \frac{\sigma_a}{\sqrt{3}\tau} \quad (4.1.6)
\]

For the second stress vector in which the maximum shear stress theory is used to relate shear stress to mean stress:

\[
r_2 = \frac{\sigma_a}{2\tau} \quad (4.1.7)
\]
For both stress ratio cases the data had variability, described by the standard deviation of $\tau (\sigma_\tau)$, which was quite small. The mean values of $\tau$ and the corresponding standard deviation of each of the cycle life groups is given in Table 4.1.2. Because the variability of $\tau$ was small in each case the resultant stress vector was assumed to lie on the stress ratio axis, defined by the average value of $\tau$. Referring to Figure 4.1.1 it can be seen that the angle $\Theta$, along which the resultant stress vector is oriented is given by:

$$\Theta = \tan^{-1}\left(\frac{S_a}{S_m}\right)$$

(4.1.8)

The orientation of the stress vector is thus specified by the mean stress ratio, $\bar{\tau}$, for each cycle life group. For each data point in a cycle life group it was possible to use the FDP-5 Computer to perform the calculation to obtain values for $S_x$. After the $S_x$ values specified by Equations (4.1.2) and (4.1.5) were obtained for each of the data points, it was possible to compute a mean $\bar{S}_x$ and a standard deviation ($\sigma_{S_x}$) which then specified a strength distribution. These were then plotted along the mean $\bar{\tau}$-axis. After the values of $\bar{S}_x$ and $\sigma_{S_x}$ were obtained the Kolmogorov-Smirnov test was used to determine if the normal or the lognormal frequency functions could be accepted as representing the strength distributions.
Table 4.1.2 Mean and Standard Deviation of Stress Ratios.

<table>
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<th>Standard Deviation $\sigma_F$</th>
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4.2 Computer Method

Three computer programs of varying complexity were used to determine the strength distributions discussed in this chapter. The first of these, a PDP-8 computer program referred to as BAR I, used an input of bending stress and shear stress to obtain a value of $S_r$ when the Von Mises Hencky strength theory was used. For a shear stress (and alternating stress) it performs the following calculations. It first calculates a mean stress from a shear stress:

\[ S_m = \sqrt[3]{2\tau} \]  
\[ \text{(4.2.1)} \]

and then performs the operations required by Equation 4.1.2:

\[ S_r = \frac{1}{2} \left\{ \left( S_m \right)^2 + \left( \frac{1}{2} \tau \right)^2 \right\} \]  
\[ \text{(4.2.2)} \]

The second computer program referred to as BAR II which is also a PDP-8 program, performs the calculations required by the maximum shear theory to relate the shear stress to mean stress:

\[ S_m = 2\tau \]  
\[ \text{(4.2.3)} \]

BAR II then performs the calculations required by Equation 4.1.5:

\[ S_r = \left\{ \left( S_m \right)^2 + \left( 2\tau \right)^2 \right\}^{1/2} \]  
\[ \text{(4.2.4)} \]
A flow chart, variable definitions and computer listing of the PDP-8 programs BAR I and BAR II is given in Appendix E.

After the individual values for the resultant stress vector magnitudes for each specimen were computed, the data was then submitted to the CDC-6400 program ASHERRG which made the following calculations. The mean and standard deviation of $S_r$ for each cycle life group and stress ratio was calculated. These values are denoted by $\bar{S}_r$ and $\sigma_{S_r}$. In addition, the Kolmogorov-Smirnov goodness of fit test, for a normal and lognormal distribution, was conducted on each of the strength distributions. The coefficients of skewness and kurtosis were determined for each strength distribution. A flow chart, variable definitions and computer listing of program CYTOFR is given in Appendix C.

4.3 Results

For the two strength theories considered, von Mises Hensky and maximum shear stress, the results of the calculations to determine the resultant stress vector magnitudes are given in Table 4.3.1. In addition, these tables present the stress ratio mean for each cycle life group as well as the calculated value for the mean stress for each strength theory considered. Tables 4.3.2 and 4.3.3 present the mean and standard deviation of the resultant stress vector magnitude for each stress ratio by cycle life group. Table 4.3.4
presents the ultimate strength distribution of an ungrooved test specimen. Table 4.3.5 presents the Kolmogorov-Smirnov $/D_{\max}$ values for each cycle life group for both the Von Mises-Hencky strength theory as well as the maximum shear stress theory.

Based on the Kolmogorov-Smirnov test, in no case can the proposed distributions, normal or lognormal, for the resultant stress vector be rejected at the ninety percent confidence level. In ten out of fifteen resultant stress vector distributions the lognormal distribution had smaller $/D_{\max}$ values. For the normal and lognormal the $/D_{\max}$ difference was quite small, the difference occurring usually in the third decimal place. Because of this slight difference, it cannot be said that either the normal or lognormal distributions fit the data better, rather that both the normal and lognormal distributions fit the strength data equally well.

The third and fourth statistical moments, coefficients of skewness and kurtosis, give little insight into the nature of the underlying distribution. The "knowledge of the third moment gives almost no clue as to the shape of the distribution" (5, p. 109).
Table 4.3.1 Cycles to Failure Data, Bending and Shear Stress Data and the Resultant Stress Vector Magnitudes $S_r$ (I) and $S_r$ (II) for the von Mises Hencky and the Maximum Shear Stress Theory.

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Table 4.3.1  Cycles to Failure Data, Bending and Shear Stress Data and the Resultant Stress Vector Magnitudes $S_x$ (I) and $S_y$ (II) for the Von Mises Hencky and the Maximum Shear Stress Theory.

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Table 4.3.1 Cycles to Failure Data; Bending and Shear Stress Data and the Resultant Stress Vector Magnitudes $S_r$ (I) and $S_r$ (II) for the Von Mises Hencky and the Maximum Shear Stress Theory.

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Table 4.3.1  Cycles to Failure Data, Bending and Shear Stress Data and the Resultant Stress Vector Magnitudes $S_a$ (I) and $S_y$ (II) for the von Mises Hensky and the Maximum Shear Stress Theory.

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0.777 254 104,367 61,337 45,607 78,993.6 100,011 91,214 109,919
0.758 256 129,228 62,835 47,856 82,889.0 104,014 95,712 114,495
0.785 257 101,193 62,479 45,945 79,579.1 101,175 91,390 111,119
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Table 4.3.1 Cycles to Failure Data, Bending and Shear Stress Data and the Resultant Stress Vector Magnitudes $S_r$ (I) and $S_r$ (II) for the von Mises Hencky and the Maximum Shear Stress Theory.

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<td>-3σ (psi)</td>
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</tr>
<tr>
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<td>3,683</td>
<td>167,700</td>
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<td>1,954</td>
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<td>125,225</td>
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<td>177,692</td>
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<td>106,300</td>
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<td>1,062</td>
<td>89,487</td>
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<td>r = 0.729</td>
<td>146,952</td>
<td>3,883</td>
<td>158,701</td>
<td>135,203</td>
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<td></td>
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<tr>
<td>r = ∞</td>
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<td>993</td>
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<td>N=90,000-200,000</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>r = ∞</td>
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<td>3,556</td>
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<td>63,668</td>
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<td>95,557</td>
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<tr>
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<td>2,325</td>
<td>155,112</td>
<td>141,162</td>
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</tr>
</tbody>
</table>
The Goodman surfaces and diagrams generated by this method are presented in Figure 4.3.1 through Figure 4.3.10 for the various cycle life groups and strength theories.

4.4 Discussion as to Validity

The Goodman surfaces generated illustrate that for cycle life ranges which are well below infinite life the allowable combinations of bending and shear stress are much larger in magnitude than combinations of the same biaxial stress which could be sustained by a specimen for an infinite life range (Figure 4.4.1 is a Goodman Surface for a period of infinite life). As the cycle life groups mean value increases the corresponding combination of bending and shear stress continues to decrease toward the value which is presented in the infinite life diagram. A comparison of this surface to the surfaces generated for finite life periods, see Figure 4.3.9 illustrates the above conclusion.

The variability in the standard deviations of the resultant stress vector for stress ratios other than infinity and zero should be examined closely. This variability is larger in many cases than the variability of the bending stress and ultimate strength distributions used for the alternating and mean stress axis. In reviewing the methodology which was discussed, one finds that data used for the alternating stress distribution and the ultimate strength distribution
were taken from one series of tests. However, the data which was used to generate the strength distributions for ratios of 3.5, 0.825, and 0.44 were taken from a series of tests, each with their own variability. From the algebra of normal functions we see that when two standards deviations are added the resultant standard deviation becomes (16, p. 111):

\[
\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2 + 2\rho \sigma_x \sigma_y}
\]  \hspace{1cm} (4.4.1)

where \( \rho \) is the correlating coefficient. If \( \rho = 0 \), assuming independence (16, p. 111):

\[
\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}
\]  \hspace{1cm} (4.4.2)
Fig. 4.3.1 Finite Life Goodman Diagram and Surface.
Fig. 4.3.2 Finite Life Goodman Diagram and Surface.
Fig. 4.3: Finite life Goodman diagram and surface.
Fig. 4.3.4. Finite Life Goodman Diagram and Surface.
Fig. 4.3.5 Finite Life Goodman Diagram and Surface.
Fig. 4.3.9 Finite Life Goodman Diagram and Surface.
Fig. 4.3.10 Finite Life Goodman Diagram and Surface.
Table 4.3.3 Mean, Standard Deviation and ±3 Limits of Resultant Stress Vector for the Maximum Shear Stress Equation.

<table>
<thead>
<tr>
<th>Life Range - Cycles</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>+3σ</th>
<th>-3σ</th>
</tr>
</thead>
<tbody>
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<td></td>
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<td>psi</td>
<td>psi</td>
<td>psi</td>
</tr>
<tr>
<td>N=1,000-3,000</td>
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<td>158,600</td>
<td>3,641</td>
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</tr>
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</tr>
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<td>149,518</td>
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<td>2,586</td>
<td>89,994</td>
<td>71,478</td>
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<td>175,760</td>
<td>159,326</td>
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Table 4.3.4 Ultimate Strength Distribution, Mean, Standard Deviation, and ±3 Limits for Mean Stress Axis for Ungrooved Specimens.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>+3σ</th>
<th>-3σ</th>
</tr>
</thead>
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<td>psi</td>
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<td>psi</td>
<td>psi</td>
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<tr>
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<td>2,500</td>
<td>185,500</td>
<td>170,500</td>
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Table 4.3.4 Ultimate Strength Distribution, Mean, Standard Deviation, and +3 Limits for Mean Stress Axis for Ungrooved Specimens.

<table>
<thead>
<tr>
<th>r = 0</th>
<th>Mean (psi)</th>
<th>Standard Deviation (psi)</th>
<th>+3σ (psi)</th>
<th>-3σ (psi)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>178,000</td>
<td>2,500</td>
<td>185,500</td>
<td>170,500</td>
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</tbody>
</table>
Table 4.3.5 Kolmogorov-Smirnov /D_{max} Values for Stress Vectors I and II.

<table>
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<tr>
<th>Sample Size</th>
<th>Vector I</th>
<th>Vector II</th>
<th>Vector I</th>
<th>Vector II</th>
<th>D_{max}</th>
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<td>Endurance</td>
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<td>0.1390</td>
<td>-</td>
<td>0.1406</td>
<td>0.35242</td>
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<tr>
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<td>0.1597</td>
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<td>N = 2</td>
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<td></td>
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<td>Endurance</td>
<td>13</td>
<td>0.1592</td>
<td>-</td>
<td>0.1553</td>
<td>0.32549</td>
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<tr>
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<td>0.31417</td>
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<td>0.1354</td>
<td>0.35242</td>
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<td>Endurance</td>
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<td>0.2136</td>
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<td>Endurance</td>
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<td>-</td>
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<td>0.40952</td>
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<tr>
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<td>0.22802</td>
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<tr>
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<td>9</td>
<td>0.1953</td>
<td>0.1810</td>
<td>0.1929</td>
<td>0.38746</td>
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CHAPTER V

GENERATION OF FINITE LIFE GOODMAN DIAGRAM (METHOD II)

5.1 Theory

It is possible to place strength distributions which have been developed from cycles to failure distributions on statistical Goodman diagrams, as well as S-N diagrams. In Chapter III, the completed results of strength distributions developed by John Smith for the statistical S-N diagram were presented.

It is possible to place the strength distributions developed by the technique discussed in Chapter III on finite life Goodman diagrams. It is first noted that the vertical strength distributions developed in Chapter III are for a specific cycle life. This finite life Goodman diagram is then for this cycle life.

Figure 5.1.1 compares a strength distribution placed on a finite life Goodman diagram to that of the same strength distribution placed on a statistical S-N diagram. The ordinate axis of both the Goodman diagram and the statistical S-N diagram is the alternating stress level. Because of this the value of the mean of the alternating strength distribution when transformed from an S-N diagram to a finite Goodman diagram will not change. The stress ratio and alternating stress
Fig 5.1.1 Comparison of Vertical Strength Distribution to A Strength Distribution Transformed to the Finite Life Goodman Diagram.
level, which are specified by the statistical $S-N$ diagram, define the location of the mean of the strength distribution which is transformed to the finite life Goodman diagram.  

The three sigma limits of the transformed strength distribution are not the same as that of the vertical strength distribution. Figure 5.1.2 depicts a strength distribution placed on the familiar coordinate axis of the Goodman diagram. The three sigma limits of the strength distribution placed on the stress ratio axis can easily be derived from the three sigma limits of the vertical strength distribution in the following manner.

Equate the upper and lower three sigma limits of the vertical strength distribution to $S$. The stress ratio, $r$, is equal to the alternating stress divided by the mean stress which referring to Figure 5.1.2 is equal to $\tan \theta$.

\[ r = \frac{S_a}{S_m} = \tan \theta \]  \hspace{1cm} (5.1.1)

\[ \theta = \tan^{-1} (r) \]  \hspace{1cm} (5.1.2)

The upper and lower three sigma limits of the transformed strength distribution are equal to $S'$. From Figure 5.1.2 it can be seen that:

\[ \sin \theta = \frac{S_a}{S'} \]  \hspace{1cm} (5.1.3)
Fig. 5.1.2 Transformation of Vertical Strength Distribution's Upper and Lower Three Sigma Limits to the Stress Ratio Axis.
Solving equation 5.1.3 for $S'$ and substituting equation 5.1.2 yields:

$$S' = S_a / \sin \epsilon$$  \hspace{1cm} (5.1.4)$$

$$S' = S_a / \sin (\tan^{-1}(r))$$  \hspace{1cm} (5.1.5)$$

If the value of $r$ is known and the original value of the upper and lower three sigma limits are specified by the vertical strength distribution then the upper and lower three sigma limits of the transformed strength distribution can be placed along the stress ratio axis and are specified by equation 5.1.5.

Five finite life Goodman diagrams were developed by the transformation of vertical strength distribution to the stress ratio axis of the Goodman diagram. The finite life Goodman diagrams were developed for cycle lives of 3,500, 9,000, 40,000, 90,000 and 200,000 cycles. Strength distributions placed on these finite life Goodman diagrams were at stress ratios of infinity, 3.50, 0.825, and 0.44. As discussed in Chapter VII of this report the strength distribution placed on the mean stress axis was taken to be that of the ultimate strength distribution of the grooved test specimen.

A PDP-8 computer program, ROTO, was developed to perform the calculations required by the transformation of the upper and lower three sigma limits as discussed in section 5.1. A flow chart,
variable definitions and computer listing of the ROTO program is given in Appendix E.

5.2 Results

The finite life Goodman diagrams developed by this method showing the strength surfaces for discrete cycles to failure are given in Figures 5.3.1 through 5.3.5. In Table 5.3.1 the alternating stress level of the mean of the strength distributions for each stress ratio as well as the transformed upper and lower three sigma limits of the strength distributions are given. The original parameters of the vertical strength distributions appear in Table 3.1.3 as starred quantities, as they were developed by this investigator to correspond with the cycle life values of Chapter IV. The ultimate strength distribution of the grooved and ungrooved test specimen are compared in Table 5.3.2.

5.3 Discussion as to Validity

The principle of transforming vertical strength distributions to the Goodman diagram as explained in Section 5.1 is a straightforward procedure. The transformation is simply projection of a known distribution to a different plane which in this particular case is the stress ratio axis of the finite life Goodman diagram.

The vertical strength distributions have associated with them a cycle life value. It is possible because of this fact to develop finite life Goodman diagrams and surfaces from the valid vertical strength distributions. This method is consistent, in that the vertical strength distributions are all developed in the same manner, and hence, there is
no problem of differing variabilities caused by inconsistencies in the procedure of developing strength distributions at various stress levels. The elimination of the inconsistencies in procedure in developing strength distributions at various stress ratios is the principle advantage of rotating vertical strength distributions to the finite life Goodman diagram.
Fig. 5.3.1 Finite Life Goodman Diagram and Surface $N=3500$ cy.
Fig. 5.3.3 Finite Life Goodman Diagram and Surface $N=40,000$ cy.
Fig. 5.3.4 Finite Life Goodman Diagram and Surface \( N=90,000 \) cy.
Fig. 5.3.5 Finite Life Goodman Diagram and Surface $N=200,000$ cy.
Table 5.3.1 Alternating Stress Level of the Mean and Standard Deviation, and ± Three Sigma Limits of Strength Distributions Placed on Stress Ratio Axes of the Finite Life Goodman Diagram.

<table>
<thead>
<tr>
<th></th>
<th>$S_a$</th>
<th>$\sigma_S$</th>
<th>$-3\sigma$</th>
<th>$+3\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 3,500$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>135,953</td>
<td>2,628</td>
<td>128,070</td>
<td>143,837</td>
</tr>
<tr>
<td>3.5</td>
<td>128,964</td>
<td>3,643</td>
<td>118,035</td>
<td>139,894</td>
</tr>
<tr>
<td>0.825</td>
<td>126,369</td>
<td>3,739</td>
<td>115,451</td>
<td>137,587</td>
</tr>
<tr>
<td>0.44</td>
<td>105,107</td>
<td>1,260</td>
<td>101,327</td>
<td>108,837</td>
</tr>
<tr>
<td>$N = 9,000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>115,926</td>
<td>2,907</td>
<td>107,205</td>
<td>124,647</td>
</tr>
<tr>
<td>3.5</td>
<td>100,523</td>
<td>3,123</td>
<td>90,683</td>
<td>118,163</td>
</tr>
<tr>
<td>0.825</td>
<td>105,645</td>
<td>3,306</td>
<td>95,722</td>
<td>115,569</td>
</tr>
<tr>
<td>0.44</td>
<td>92,390</td>
<td>2,002</td>
<td>86,384</td>
<td>98,996</td>
</tr>
<tr>
<td>$N = 40,000$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>90,693</td>
<td>2,605</td>
<td>82,878</td>
<td>98,507</td>
</tr>
<tr>
<td>3.5</td>
<td>82,345</td>
<td>3,457</td>
<td>71,942</td>
<td>92,747</td>
</tr>
<tr>
<td>0.825</td>
<td>80,253</td>
<td>3,206</td>
<td>71,327</td>
<td>90,578</td>
</tr>
<tr>
<td>0.44</td>
<td>72,260</td>
<td>3,169</td>
<td>62,452</td>
<td>82,036</td>
</tr>
<tr>
<td>$N = 90,000$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>79,054</td>
<td>2,539</td>
<td>71,437</td>
<td>86,671</td>
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<tr>
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<td>72,967</td>
<td>2,593</td>
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<td>79,147</td>
</tr>
<tr>
<td>0.825</td>
<td>69,970</td>
<td>2,927</td>
<td>62,189</td>
<td>76,750</td>
</tr>
<tr>
<td>$N = 200,000$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>70,172</td>
<td>1,881</td>
<td>64,529</td>
<td>75,815</td>
</tr>
<tr>
<td>3.5</td>
<td>64,792</td>
<td>2,400</td>
<td>57,593</td>
<td>71,991</td>
</tr>
<tr>
<td>0.825</td>
<td>61,737</td>
<td>2,133</td>
<td>55,338</td>
<td>68,135</td>
</tr>
<tr>
<td>0.44</td>
<td>57,521</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3.2 Comparison of Grooved and Ungrooved Specimen's Ultimate Strength Distribution.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grooved</td>
<td>255,300 psi</td>
<td>2,720 psi</td>
</tr>
<tr>
<td>Ungrooved</td>
<td>178,500 psi</td>
<td>2,500 psi</td>
</tr>
</tbody>
</table>
Table 5.3.2 Comparison of Grooved and Ungrooved Specimen's Ultimate Strength Distribution.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grooved</td>
<td>255,300 psi</td>
<td>2,720 psi</td>
</tr>
<tr>
<td>Ungrooved</td>
<td>176,500 psi</td>
<td>2,500 psi</td>
</tr>
</tbody>
</table>
CHAPTER VI

EVALUATION OF GENERATION OF FINITE LIFE GOODMAN DIAGRAMS

6.1 Evaluation of Previous Techniques in Developing Goodman Diagrams and Surfaces

The methodology which was presented in Chapters IV and V for the transformation of cycles-to-failure data to strength distributions and the resulting Goodman surfaces and diagrams will now be evaluated. It appears that the method of rotating vertical strength distributions and placing them on the stress ratio axis of a Goodman diagram is the most uniform method of creating the Goodman surface. As discussed in Section 5.4 all distributions are created in the same manner which eliminates the problems of variability discussed in Section 4.4.

In viewing the general shape of both the finite life Goodman diagrams developed in Chapter IV and V the following conclusion can be drawn. By design each of the cycle life groups in both Chapters IV and V are similar. This was done for purposes of comparison. The curves shift progressively lower as cycle life value increase. The curves developed in both chapters are consistent in this respect. As cycle life design values decrease larger combinations of bending and shear stress are possible.
The two sets of finite life Goodman diagrams do have some inconsistencies. The ultimate strength distribution of the ungrooved test specimen was used as the static strength distribution in Chapter IV while the ultimate strength distribution of the grooved specimen was used in Chapter V. Both distributions were placed along the mean stress axis of the Goodman diagram. An explanation of these facts is in order at this time.

The method used in Chapter IV to generate the finite life Goodman diagrams and surfaces was completed in early February of 1971. At that time, there was doubt as to the geometry of the specimen to be used, consequently the conservative ultimate strength distribution of the ungrooved specimen was chosen. Although this doubt did exist the diagrams developed in Chapter IV did not seem to support the use of the higher ultimate strength of the grooved geometry specimens.

The finite life Goodman diagrams which were developed in Chapter V, in late March and early April of 1971, illustrated the critical importance of the geometry of specimen from which the static ultimate strength distribution was derived. All but the finite life Goodman diagram developed at 200,000 cycles showed the mean stress values at a stress ratio of 0.44 close to or above the ultimate strength distribution of the ungrooved test specimen. This fact, although quite alarming at first, made an investigation of the correct geometry of paramount importance and considerable effort, as exhibited in Chapter VII,
was made to resolve the question concerning the proper ultimate strength distribution to be placed along the mean stress axis.

6.2 Recommendations

The weaknesses and advantages of the two methods of generating Goodman surfaces and diagrams have been discussed. It is the opinion of this investigator that the method developed in Chapter V, that of forming vertical strength distributions from cycles to failure data and then transforming these to the stress ratio axis of a Goodman diagram, be considered as the appropriate method to be used when finite life Goodman diagrams are to be created from a large scale cycle to failure fatigue test program. Subsequently the results of Chapter V were weighted to a greater degree than those of Chapter IV in discussions dealing with the choice of geometry of test specimen used for the ultimate strength distribution in Chapter VII. This was also true in Chapters X and XI where the best empirical and theoretical math model of the Goodman diagram were sought.

In conclusion, the transformation of vertical strength distributions to the Goodman diagram is considered as the appropriate methodology to be used when large amounts of cycles-to-failure data are available. Chapter XIII considers a more efficient plan where it may be possible to generate Goodman surfaces with a minimum amount of actual fatigue testing.
CHAPTER VII
THE STATIC STRENGTH DISTRIBUTION TO BE PLACED ON THE MEAN STRESS AXIS OF FINITE LIFE GOODMAN DIAGRAMS

7.1 Introduction to Static Strength Distribution

In the previous chapters of this report, we discussed and developed several experimental methods of presenting fatigue data generated by relatively expensive and time consuming fatigue test programs. If however, this same information could be extracted from uniaxial static tests of materials the savings in time and effort would be tremendous. Chapters VIII and IX present several empirical and theoretical models of the Goodman diagram.

The development of relations, which take advantage of information obtained from static tests, to model fatigue data require that the following topics be investigated. For both the grooved and ungrooved geometry specimens which have been investigated, what are the equations which are used to determine the tensile yield, ultimate and breaking strengths? Secondly, is it possible to specify a theoretical strength distribution, such as the Gaussian normal or lognormal distribution, to each of these quantities? Of major concern in the development of the Goodman diagram for the grooved geometry specimen, subjected to the combined stress condition of alternating bending and constant shear stresses, is the determination...
of the strength distribution to be used along the mean stress axis. It must be determined what strength parameter should be used along the axis and if the diagram is to model the behavior of grooved specimens, on which geometry specimen, grooved or ungrooved, should this strength parameter be based.

7.2 Calculation of Yield, Ultimate and Breaking Strengths

The calculation of static tensile strengths is well documented throughout the literature. The yield strength is calculated by dividing the force initiating the yield by the cross-sectional area of the specimen. This cross-sectional area is based on the original diameter of the test specimen (6, p. 4). In the soft, ductile steels the yield strength is clearly marked by a yield point as shown in Figure 7.2.1a. In other materials where the yield point is less obvious, see Figure 7.2.1b, common practice defines the yield load as the force which is required to give a 0.2 percent plastic offset (6, p. 45).

The ultimate tensile strength is defined as the maximum load sustained by a tensile test specimen divided by the "original" cross-sectional area. However, this calculation yields a parameter which is inaccurate and artificial. The load and area on which this parameter is based do not occur simultaneously. For most ductile materials the maximum load occurs after appreciable elongation which is obviously accompanied by a reduction in area. The ultimate tensile strength,
Figure 7.2.1 a) Ductile Material With Clearly Defined Yield Point. b) Ductile Material Without Marked Yield Point and Comparison of True Stress-Strain Curve versus Engineering Stress-strain Curve. (6, p. 5)
as calculated above, is, however, the most commonly cited parameter of material strength (7, p. 152).

By definition, both the ultimate and breaking strengths are based on the original cross-sectional area of the test specimen. Hence, the breaking strength is calculated by dividing the load at fracture by the original cross-sectional area. Figure 7.2.1a indicates that the breaking strength of a ductile material may be less than the ultimate tensile strength of the material. As necking and elongation occur in the test specimen the stress in the specimen continuously decreases. A more realistic measure of material strength is the tensile fracture stress. The fracture stress is determined by dividing the load just prior to fracture by the area measured just after fracture. Although the load decreases after the ultimate tensile stress is reached, the cross-sectional area decreases more rapidly which results in an increasing "true stress." Because of this the fracture stress is equal to or greater than the ultimate tensile stress (7, p. 154).

Based on the above discussion, it can be concluded that for the grooved and ungrooved specimens the ultimate and yield strength calculations should be based on the original cross-sectional area. The calculation of the breaking strength should be based on the reduced diameter measured after fracture.
7.3 Theoretical Strength Distributions of the Strength Parameters

The calculations of the strength parameters yield, ultimate and breaking strengths have been specified. The next point to be investigated is to determine if these strength parameters exhibit a known theoretical distribution. An effort to determine if the normal or lognormal distribution was favored for the three strength parameters previously discussed was initiated. This included gathering tensile test data from the research effort carried out under NASA Grant No. 03-002-044 at The University of Arizona. Tensile test data from Phase I and Phase II of the program was gathered for tensile yield, ultimate and fracture strength of both the grooved and ungrooved test specimens. This data was statistically reduced by the computer program CYTOPR, which has the ability of performing the following statistical operations; mean, standard deviation, coefficients of skewness and kurtosis, the \( D_{\text{max}} \) value for the Kolmogorov-Smirnov goodness of fit test and the total Chi-Squared value, \( V_{n-1} \) for the Chi-Squared goodness of fit test. The AS\textsuperscript{MERG} program performs these statistical calculations for both the normal and the lognormal distributions.

Efforts to specify either the normal or lognormal distribution as favoring the yield, ultimate and breaking strengths of both the grooved and ungrooved specimens were not totally successful. The
Kolmogorov-Smirnov and Chi-Squared goodness of fit tests have the ability of only rejecting the normal and lognormal distributions as properly representing the data in question. Table 7.3.1 presents the results of the Kolmogorov-Smirnov and Chi-Squared goodness of fit test results. The mean and standard deviations of the strength parameters are given in Table 7.3.2. The breaking strength data in all but the Phase I specimens was found to be rejected as being either normally or lognormally distributed.

Previously the Kolmogorov-Smirnov test has been used as a basis in determining whether a normal or a lognormal distribution fits the data best based on which of these distributions had the smaller \( D_{\text{max}} \) value (3, p. 47). However, because of the small difference in the maximum value of \( D_{\text{max}} \) for both the normal and the lognormal distributions, the difference occurring in the second decimal place, it cannot be concluded whether the normal or the lognormal distribution gives a better fit to the experimental data. It can, however, be said that neither the normal or the lognormal distribution can be rejected as distributions representing the yield, and ultimate strength of both the grooved and ungrooved specimens.
Table 7.3.1 Kolmogorov-Smirnov and Chi-Squared Test Results for Grooved and Ungrooved Test Specimens

<table>
<thead>
<tr>
<th>Parameter</th>
<th>N</th>
<th>( D_{crit} )</th>
<th>Normal</th>
<th>LogNor</th>
<th>Normal</th>
<th>LogNor</th>
<th>( x^2 )</th>
<th>( x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNGROOVED</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>10</td>
<td>0.3686, -</td>
<td>0.2135</td>
<td>0.2104</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1970</td>
<td>35</td>
<td>0.2018, 42.3</td>
<td>0.1093</td>
<td>0.1040</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1971</td>
<td>35</td>
<td>0.2018, 42.3</td>
<td>0.1173</td>
<td>0.1187</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ultimate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>10</td>
<td>0.3686, -</td>
<td>0.2006</td>
<td>0.1979</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1970</td>
<td>34</td>
<td>0.2047, 41.4</td>
<td>0.1151</td>
<td>0.1097</td>
<td>8.099</td>
<td>8.316</td>
<td>10.172</td>
<td>10.146</td>
</tr>
<tr>
<td>1971</td>
<td>35</td>
<td>0.2018, 42.3</td>
<td>0.1239</td>
<td>0.1255</td>
<td>10.172</td>
<td>10.146</td>
<td>10.172</td>
<td>10.146</td>
</tr>
<tr>
<td>Breaking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>10</td>
<td>0.3686, -</td>
<td>0.1493</td>
<td>0.1461</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1970</td>
<td>33</td>
<td>0.2077, 40.3</td>
<td>0.2632</td>
<td>0.2710</td>
<td>5.919</td>
<td>9.139</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>33</td>
<td>0.2077, 40.3</td>
<td>0.2377</td>
<td>0.2277</td>
<td>62.579</td>
<td>48.233</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**GROOVED**

| Ultimate  |   |               |        |        |        |        |          |          |
| Phase I   | 10 | 0.3686, -     | 0.1439 | 0.1420 | -      | -      | -        | -        |
| 1970      | 33 | 0.2077, 40.3  | 0.0667 | 0.0851 | 0.788  | 0.809  | 2.222    | 2.833    |
| 1971      | 32 | 0.2108, 39.1  | 0.0907 | 0.0921 | 2.222  | 2.833  |          |          |
| Breaking  |   |               |        |        |        |        |          |          |
| Phase I   | 10 | 0.3686, -     | 0.1229 | 0.1238 | -      | -      | -        | -        |
| 1970      | *  |               |        |        | -      | -      | -        | -        |
| 1971      | *  |               |        |        | -      | -      | -        | -        |

- \( x^2 \) test could not be applied due to insufficient data points.

* Breaking load not measured with sufficient accuracy to calculate strength distribution.
Table 7.3.2 Mean Values and Standard Deviations of Tensile Strength Distributions (psi)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNGROOVED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>171,150</td>
<td>2,779</td>
</tr>
<tr>
<td>1970</td>
<td>158,285</td>
<td>5,840</td>
</tr>
<tr>
<td>1971</td>
<td>155,505</td>
<td>1,765</td>
</tr>
<tr>
<td>Ultimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>177,850</td>
<td>2,582</td>
</tr>
<tr>
<td>1970</td>
<td>167,044</td>
<td>5,273</td>
</tr>
<tr>
<td>1971</td>
<td>165,108</td>
<td>1,521</td>
</tr>
<tr>
<td>Breaking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>254,800</td>
<td>4,391</td>
</tr>
<tr>
<td>1970</td>
<td>255,904</td>
<td>12,964</td>
</tr>
<tr>
<td>1971</td>
<td>260,921</td>
<td>8,247</td>
</tr>
<tr>
<td>GROOVED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>255,300</td>
<td>2,720</td>
</tr>
<tr>
<td>1970</td>
<td>254,380</td>
<td>2,260</td>
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<tr>
<td>1971</td>
<td>269,137</td>
<td>2,832</td>
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<tr>
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<td></td>
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</tr>
<tr>
<td>Phase I</td>
<td>303,950</td>
<td>3,122</td>
</tr>
<tr>
<td>1970</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1971</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
A survey of the literature clearly indicated that strength data is usually assumed to be distributed normally. The NERVA Project Report states that, "Usually the data (strength data) will be assumed to be normally distributed, however, lognormal and Weibull distributions are acceptable and can be used much the same as normal data." (8, p. 9). Hence, it can be concluded that the normal distribution is acceptable as the theoretical distribution considering the limited experimental evidence available and the fact that there is an adequate amount of documentation in the literature to support this conclusion.

7.4. Mean Stress Axis Strength Parameter

The determination of the strength parameter, and consequently, the strength distribution to be used in any fatigue data model is dependent upon the model used and the definition of the failure mode. If it has been determined that yielding is detrimental to the proper functioning of the specimen then the distribution of yield strength should be used. If only the fracture is of concern then the ultimate strength distribution should be used (10, p. 4). Chapter X describes the Goodman line in detail. The Goodman line connects the endurance strength to the ultimate tensile strength. The failure criterion of the cycles to failure data presented in this report has been fracture. The ultimate strength distribution is concluded to be the proper distribution to be placed along the mean stress axis if the failure criterion is fracture and the fatigue model is the Goodman diagram.
7.5. Least Squares Estimate of the Ultimate Tensile Strength

The final topic which remains to be discussed concerns the geometry of the tensile test specimen whose ultimate strength distribution will be placed on the mean stress axis of the Goodman diagram. There exists the possibility of using the grooved or the ungrooved ultimate strength distribution of a tensile test specimen of equal cross-sectional area.

Initially one might conclude that if the Goodman diagram is to model the behavior of a grooved specimen then the grooved ultimate strength distribution should be chosen. Mr. Carl S. Osgood, author of Fatigue Design, in response to a letter which solicited his opinion on this subject stated, "I believe it would be rather meaningless to try for a distribution on the $S_m$ axis for both types of specimens." Robert C. Juvinall, author of Stress Strain and Strength, in a reply to the same question suggests that the static ultimate strength distribution" ... should pertain to the same notched (grooved) specimens as the $S_a-S_m$ curve itself." Juvinall concludes that the proper static strength distribution to use is that of the ultimate strength distribution along the $S_m$ axis as discussed in Section 7.4 of this report.

There are, however, logical and well presented arguments supporting the use of the ungrooved specimen's ultimate strength distribution. The mean stress axis is really at a stress ratio of zero, as the stress ratio of $S_a/S_m$ is zero at this point. Consequently, the alternating stress, $S_a$, must be zero. In the case of combined bending
and shear this means that the mean stress axis is the axis of pure shear. From tensile tests, it has been found that the grooved geometry test specimen has a much higher ultimate tensile strength. Reviewing the discussion found in Chapter IV of this report, this higher strength is caused by a radial stress which is introduced into the specimen at the root of the groove. However, a grooved specimen which is subjected to a static torque load would not experience such a radial stress and would fail at a torsional load equal to the load which causes an ungrooved specimen to fail. The mean stress axis in the particular case investigated can be thought of as representing the failure mode where pure shear is the cause of failure. Based on the above argument, it has been suggested that the ungrooved ultimate strength distribution be used on the mean stress axis of Goodman diagrams (3, p. 71).

In attempting to resolve these two differing opinions this investigator turned to an analytic evaluation of the problem. There are available two sets of Goodman diagrams, presented in Chapter IV and V of this report, on which to base such an analytic solution. Such an analytic solution was desired, as smooth curves can be drawn connecting the Goodman diagram data to both the grooved and ungrooved distribution of ultimate strength.

The technique used was based on the method of least squares. The conventional method of least squares, however, was not considered as being appropriate or even workable for the problem under consideration. A conventional least squares analysis requires that the equation of the expected line be completely specified. Using this knowledge the method of least squares will fit the "best" polynomial to the data.
Before proceeding further, let us pause and review the problem and the available information. We have developed Goodman diagrams for varying cycle lives including that of infinite life, which have mean values of the strength distributions specified at stress ratios of infinity, 3.5, 0.825, and 0.44. In addition, there are also several theoretical equations which are known to model the curve which should be drawn between the mean of these strength distributions. The equations include the von Mises-Hencky equation.
\[(S_a/S_e)^2 + (S_m/S_u)^2 = 1 \quad (7.5.1)\]

and the Gerber parabola equation

\[(S_a/S_e) + (S_m/S_u)^2 = 1 \quad (7.5.2)\]

Where:
- \(S_a\) = alternating stress
- \(S_m\) = mean stress
- \(S_e\) = endurance strength
- \(S_u\) = ultimate tensile strength.

In both of these equations the information available from the Goodman fatigue diagrams presented in Chapters IV and V specify all of the quantities except the ultimate strength.

A method was then sought which would give an estimate for the ultimate strength. It was assumed that the von Mises-Hencky equation was a valid mathematical model for the fatigue data. Graybill (9, p. 111) presents a method which can be used to calculate the least squares estimator of the ultimate strength assuming that the von Mises-Hencky equation adequately models the fatigue data. The only other assumption which needs to be made is that the fatigue data to be used is not in the low cycle fatigue range, the low cycle fatigue range being below \(10^4\) cycles. It was expected that the ultimate strength which would be predicted in this range would be quite large. This is quite a valid
assumption as low cycle life requires a completely different mathematical model than the von Mises-Hencky equation (See Chapter XI).

The first step of the estimation process for the ultimate strength using the von Mises-Hencky equation is to transform that equation into the following form:

1) \((S_a/S_e)^2 + (S_m/S_u)^2 = 1\) (7.5.3)

2) Set \(x = S_m, y = S_a\), and subtract \((S_a/S_n)^2\) from both sides yielding \((y/S_e)^2 = 1 - (x/S_u)^2\) (7.5.4)

3) Setting \(y' = S_e^2 - y^2\) (7.5.5)

4) Substitution yields \(y' = B'(x^2)\) (7.5.6)

The least squares estimate of \(B\) is given as

\[
B' = [x]^T[x]^{-1}[x]^T[y]
\]

(7.5.7)

where the brackets indicate vector quantities. Unfortunately, as Figure 7.5.1 indicates, there are only four values to be placed in the \(x\) vector. These values of the mean stress are derived from the \(y\), or alternating stress values, in the following manner. Each of the mean stress values are related to the alternating stress value by the stress ratio \(r\) where \(x\) equals
Figure 7.5.1 Goodman Diagram Illustrating Mean of Strength Distributions Used in Least Squares Estimate of Ultimate Strength. (von-Mises Hencky Equation)
It was possible to solve for the three values of mean stress by dividing the alternating stresses by \( r \) for each Goodman diagram investigated.

Thus, the \( x \) vector was specified by three values of mean stress while the \( y \) vector was specified by the corresponding values of alternating stress. The value of \( B' \) is the least squares estimator of the \( X \) axis intercept which can then be related to the ultimate strength by Equation 7.5.6.

Rewriting equation 7.5.7 in terms of the von Mises-Hencky transformed variables yields:

\[
B' = \begin{bmatrix}
  2 & 2 & 2 & 2 \\
  x_0 & x_1 & x_2 & x_3 \\
\end{bmatrix}^{-1} \begin{bmatrix}
  2 & 2 & 2 & 2 \\
  x_0 & x_1 & x_2 & x_3 \\
\end{bmatrix} \begin{bmatrix}
  y_0^2 \\
  y_1^2 \\
  y_2^2 \\
  y_3^2 \\
\end{bmatrix} \tag{7.5.8}
\]

It can readily be seen that \( x_0 \) is zero which will cause the vector equation directly above to be reduced to:
\[
B^T = \begin{bmatrix} x_1^2 & x_2^2 & x_3^2 \\ x_1 & x_2 & x_3 \\ x_2 & x_3 & y_1^2 \\ x_3 & y_2 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} y_1^2 \\ y_2 \\ y_3 \end{bmatrix} \quad (7.5.9)
\]

The first two vector multiplications \([x]\ [x]\) yield a scalar. The inverse of this scalar is simply the numerical inverse. The remaining vector multiplications are straightforward.

The vector operations, after being transformed to algebraic relationships can be programmed (LSEFD) on the PDP-8 computer for the Goodman diagram data developed in Chapters IV and V of this report. A flow chart, variable definitions and computer listing of LSEFD program is given in Appendix E.

The results of the least squares estimate is given in Tables 7.5.1 for Chapter IV Goodman diagrams and Table 7.5.2 for data extracted from the Goodman diagrams of Chapter V. Cycles to failure data and the corresponding strength distributions which this data would yield above the cycle life of 200,000 cycles is unavailable at this time. It appears from Tables 7.5.1 and 7.5.2 that as cycle life increases the least squares estimate of the ultimate strength distribution decreases. Had the least squares estimate predicted consistently a value of 255,300 psi. for the measured ultimate strength of the Phase I grooved specimens, it could have been concluded that
### Table 7.5.1  Chapter IV Goodman Diagram Data Used for a Least Squares Estimate of the Ultimate Strength.

<table>
<thead>
<tr>
<th>Cycle Life</th>
<th>Least Squares Estimate of Ultimate Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000 - 40,000</td>
<td>238,385</td>
</tr>
<tr>
<td>60,000 - 90,000</td>
<td>264,662</td>
</tr>
<tr>
<td>90,000 - 200,000</td>
<td>222,661</td>
</tr>
</tbody>
</table>

### Table 7.5.2  Chapter V Goodman Diagram Data Used for a Least Squares Estimate of the Ultimate Strength.

<table>
<thead>
<tr>
<th>Cycle Life</th>
<th>Least Squares Estimate of Ultimate Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000</td>
<td>262,533</td>
</tr>
<tr>
<td>90,000</td>
<td>234,972</td>
</tr>
<tr>
<td>200,000</td>
<td>216,190</td>
</tr>
</tbody>
</table>
the grooved geometry specimen's ultimate strength distribution was the proper distribution to use along the mean stress axis. The decrease in the least squares estimate of the ultimate strength as the cycle life increases seemingly shakes the possibility that it is indeed the grooved specimens' distributions that should be used along the mean stress axis until the limiting case is investigated. This limiting case being an infinite life Goodman diagram. When the least squares estimate technique is applied to the infinite life diagram (3, p. 75), presented in Figure 4.4.1 which was developed by John Smith for the grooved specimens under discussions, the estimate of the ultimate strength is 222,661 psi. This is well above the value of the ungrooved ultimate strength of 178,000 psi, and is 87% of the ultimate strength of the grooved specimens ultimate of 255,300 psi.

It is also important to note that there is no data available for strength distributions below a stress ratio of 0.44. Because of this the least squares estimate was based on four points with a stress ratio greater than or equal to 0.44.

It is noted that even in the Goodman diagrams of Chapter V for cycle life values of 200,000 cycles, the largest investigated alternating stress level at a stress ratio of 0.44 has only fallen 17% from the value at the stress ratio of infinity, while the mean stress value is already 76% of the ungrooved ultimate tensile strength at that particular point. The least
squares estimate would have been considerably more accurate if data below a stress ratio of 0.44 was available. It is in this region that the curve must transition to either the grooved or the ungrooved ultimate tensile strength. Perhaps if this data were available the estimates would not have fallen off to the values below the grooved ultimate strength. The analytic results, which suffered from a lack of data below a stress ratio of 0.44, show that they reach a limiting case value much greater than that of the ungrooved ultimate strength, and confirm the opinions which were solicited from noted authors on the subject of fatigue. It is the considered opinion of this investigator that the ultimate strength distribution of the grooved geometry test specimen should be used for Goodman diagrams which represent the behavior of grooved fatigue test specimens.
CHAPTER VIII

EMPIRICAL MATH MODELING OF FINITE LIFE GOODMAN DIAGRAM

8.1 Mathematical Models of the Goodman Diagram

A mathematical model of the Goodman diagram relates this alternating stress, $S_A$, to the mean stress $S_m$, by means of an algebraic equation. In most cases this equation relates a static strength parameter to the endurance strength, $S_o$, of the material in question. The safe design region of a Goodman diagram is conventionally defined as the area bounded by the ordinate and abscissa axes and the line of the mathematical fatigue model, or equation, under consideration. There are several mathematical models of the Goodman diagram. These include the modified Goodman line, the Gerber parabola, the von Mises-Hencky ellipse, the Soderberg line, the Sines line, and the Langer modification to the modified Goodman line. The objective of this chapter is to present these mathematical models, however the presentation of such models would be incomplete if not accompanied by a discussion of the strengths and shortcomings of each model. Figure 8.1 compares the six mathematical models discussed in this chapter.

8.2 Modified Goodman Line

The most widely accepted theory of combined stresses is the modified Goodman line. The modified
Fig. 8.1 Diagram Picturing The Six Mathematical Models of The Goodman Diagram.
Goodman line connects the endurance strength, $S_e$, on the ordinate axis by a straight line to the ultimate tensile strength, $S_u$, which is plotted on the abscissa axis, (10, p. 6).

It is important to note that the endurance strength must be defined in relation to a given number of cycles, beyond which the material is assumed to have an infinite life. A common cycle life value for the endurance strength of ductile steels is $10^6$ cycles. The equation of the modified Goodman line is:

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

A common criticism of the Goodman line is that it tends to be conservative where the stress ratios, $r = S_a/S_m$, is well above one or in the range $1 < r < \infty$. Even though it is conservative in this range, for lower stress ratios in the range of $r = 1/10$ and less, the modified Goodman diagram may predict safe combinations of alternating and mean stress when in actuality they could cause yielding (10, p. 7).

8.3 Gerber Parabola

The Gerber Parabola was first proposed by Gerber in 1874. At this time Gerber was attempting to fit a curve to the results of Wohler's experiments with combined stresses. The parabola which Gerber proposed, known today as the Gerber parabola, joined
the ultimate tensile strength on the abscissa to the endurance strength on the ordinate axis. These two physical properties uniquely describe the shape of the Gerber parabola. The Gerber parabola requires that a parabola be drawn so as to have a vertex on the vertical axis at the value of the endurance limit and to pass through the ultimate strength which is plotted on the horizontal axis (10, p. 8). The equation of the Gerber parabola is given by

$$\frac{S_a}{S_o} + \left(\frac{S_d}{S_u}\right)^2 = 1$$

Critics of the modified Goodman line have stated that it is too conservative. The Gerber parabola was proposed to compensate for the conservatism of the modified Goodman line. In addition, it has been found that in many cases the Gerber parabola fits the experimental data far better, in the stress ratio ranges of $1 < r < \infty$, than the modified Goodman line. Unfortunately, the Gerber parabola does not give a proper representation of the fatigue data where stress ratios of one tenth and less are encountered. At these stress ratios the Gerber parabola permits an even greater amount of yielding than the modified Goodman line.
8.4 von Mises-Hencky Ellipse

This equation, associated with the energy of distortion theory discussed in the following chapter, has been proposed for the case of combined stresses. The von Mises-Hencky equation or ellipse is given by

\[ \left( \frac{S_a}{S_e} \right)^2 + \left( \frac{S_m}{S_u} \right)^2 = 1 \]

This equation forms an ellipse in the first quadrant of the cartesian plane. Although this equation was originally proposed for static loads, it is commonly used as a model of combined stresses in fatigue (10, p. 10), and (it is not theoretically valid above the yield point of the material).

8.5 Soderberg Line

In 1930 Soderberg proposed his theory in the United States which was to eliminate the problem discussed previously concerning yielding in the safe design region. The Soderberg line eliminates the problem of the yield point of the material being exceeded at any combination of stress. If yielding does occur, the dimensions of the specimen are changed. Obviously, this change is of a permanent nature and the performance of the material is affected. Even though failure of the material may be considered as fracture, the maximum allowable stress level becomes the yield strength. The actual Soderberg line takes
these facts into account by specifying a straight line between the endurance strength on the ordinate axis and the yield strength $S_y$ on the abscissa axis (11, p. 15). The equation of the Soderberg line is given by

$$S_a = S_e \left(1 - \frac{S_m}{S_y}\right)$$

Criticism of the Soderberg line arises because it does indeed lie below the Goodman line. Because of this fact it will be even more conservative than the modified Goodman line for stress ratios in the range of $1 < r < \infty$, the Goodman line has been shown to be conservative in this region (11, p. 15, 16, p. 18).

8.6 Sines Line

The Sines line is an empirical relationship given by

$$S_a = S_e - cS_m$$

where the constant $c$ must be determined for the material under investigation. Because it is an empirical relation it can through the appropriate value of $c$, be adjusted to fit the data for a particular material. The Sines line accounts for maximum stresses up to the yield point. Consequently, the Sines line is defined only to the vertical line where the mean stress is equal to the yield strength as shown in Figure 8.1 (10, p. 9).
8.7 Langer Modification to the Modified Goodman Line

The Langer Modification to the modified Goodman line attempts to solve the problem of yielding for a different case than discussed in the Soderberg Line presentation. It will be recalled that the Soderberg line attempted to solve the problem of yield at low stress ratios. The Langer modification attempts to solve the problem of yield caused by high stress levels. At high stress levels the maximum value of the alternating and mean stress, \( S_a + S_m \), which are encountered may exceed the yield strength (11, p. 16). As discussed in the Soderberg line presentation this yielding will have an adverse effect upon the fatigue characteristics of the specimen. The Langer modification excludes the area of the safe design region where the alternating stress plus the mean stress is greater than the yield strength. Hence, the safe design region becomes the area bounded by the ordinate and abscissa axes, the Goodman line and the region which satisfies the inequality \( S_a + S_m < S_y \).

Criticism is again directed at the Langer modification because it is considered too conservative at stress ratios greater than one. The Langer modification does eliminate the criticism of the modified Goodman line where the yield strength is exceeded and at the same time does not demand the conservatism of the Soderberg line (11, p. 16).
CHAPTER IX
THEORETICAL STRENGTH THEORIES

9.1 Introduction to Strength Theories

The three principal stresses completely describe the stress state of any point in a structure. The need for a strength theory to describe the material behavior at a point arises when two or more of the principal stresses have a non-zero value. When one of the three principle stresses is non-zero the behavior of the material is described by the conventional tensile test. There are however, even in this relatively simple state of stress, differences between the true state of stress and the "engineering" stress-strain properties which have been previously discussed in Chapter VII of this report. The objective of a "theory of strength" is to relate a complex state of stress, i.e., when two or more of the principal stresses are non-zero, to the uniaxial properties which are obtained in a tensile test (13, p. 1).

The elastic portion of the total strain is related to stress by Hooke's Law. In the case of combined stresses initial yielding must be related to yielding in a tensile test by means of a flow theory. A flow theory relates the increments of plastic strain in each direction to the state of stress at the point under consideration. In the past there have been
several yield criteria, or strength theories, proposed but because of later experiments in hydrostatic stresses, which have conflicted with these theories, they seem now to be only of historical importance (13, p. 1). However, there are two such theories which do not have this particular fault. These two are the maximum shear stress criterion, and the energy of distortion or von Mises-Hencky criterion. The following chapter will discuss these two theories, and will present three additional modified strength theories proposed by Findley and Mathur (14). These discussions will be accompanied by comparison of these theories to the fatigue problem of combined stresses of bending and torque.

9.2 Energy of Distortion Theory

The octahedral shear stress, the energy of distortion, or the Von Mises-Hencky theory, as this theory is often referred to, predicts yielding to occur when the elastic energy of distortion reaches a critical value. The energy of distortion is defined as the total energy minus the energy associated with a volumetric dilation. It can be shown that the energy of distortion is proportional to the shear stress on the octahedral plane. The octahedral plane is the plane which makes equal angles with the three principle directions (13, p. 3).

Consider a cubic element of material acted upon in the three principle directions by the stresses \( s_1, s_2, \) and \( s_3 \) where \( s_1 > s_2 > s_3 \). For the unit cube, pictured in Figure 9.2.1,
Fig. 9.2.1 (a) Element with triaxial stresses; this element undergoes both volume change and angular distortion. (b) Element under hydrostatic tension undergoes only volume change. (c) Element has angular distortion without volume change.
the work done in any principle direction is given by

$$U_n = \frac{s_n \epsilon_n}{2}$$  \hspace{1cm} (9.2.1)

where $\epsilon_n$ are the three principle strains. Considering

$$\epsilon_1 = \frac{s_1}{E} - \frac{\mu s_2}{E} - \frac{\mu s_3}{E}$$  \hspace{1cm} (9.2.2a)

$$\epsilon_2 = \frac{s_2}{E} - \frac{\mu s_1}{E} - \frac{\mu s_3}{E}$$  \hspace{1cm} (9.2.2b)

$$\epsilon_3 = \frac{s_3}{E} - \frac{\mu s_1}{E} - \frac{\mu s_2}{E}$$  \hspace{1cm} (9.2.2c)

where $\mu$ = Poisson's ratio and $E$ = Modulus of Elasticity.

The total strain energy is

$$U = U_1 + U_2 + U_3 = \frac{1}{2E} \left( \frac{2}{s_1} + \frac{2}{s_2} + \frac{2}{s_3} - 2\mu(s_1s_2 + s_2s_3 + s_3s_1) \right)$$  \hspace{1cm} (9.2.3)

Defining average stress as

$$s_{avg} = \frac{s_1 + s_2 + s_3}{3}$$  \hspace{1cm} (9.2.4)

which is applied to each of the principle directions of the unit cube, the remaining stresses $s_1 - s_{avg}$, $s_2 - s_{avg}$ and $s_3 - s_{avg}$ shown in Figure 9.2.1c will only produce angular distortion. If $s_{avg}$ is substituted for $s_1$, $s_2$ and $s_3$ in Equation 9.2.3 the amount of strain energy which produces only change in volume is
If the expression for \( s_{\text{avg}}^2 \) is substituted in Equation (9.2.5) it becomes

\[
U_v = \left( \frac{3s_{\text{avg}}^2 - 2\mu(3s_{\text{avg}})}{2E} \right) \left(1 - 2\mu \right) \tag{9.2.5}
\]

\[
s_{\text{avg}}^2 = \left( \frac{s_1 + s_2 + s_3}{3} \right)^2
\]

The energy of distortion is then equal to the total energy, given by Equation 9.2.3 minus the energy of the volume change given by Equation 9.2.6. The energy of distortion is thus given by

\[
U_d = U - U_v = \left(1 + \frac{\mu}{3\epsilon} \right) \left( (s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2 \right) \frac{2}{2}
\]

When a state of pure shear exists, the shearing stress at point \( \delta_e \) is equal in magnitude to each of the principle stresses at the same point. If \( s_1 = \epsilon \) and \( s_2 = -s_1 \) the energy of distortion becomes

\[
U_d = \left(1 + \frac{\mu}{3\epsilon} \right) \left( (s - (-s))^2 + (s)^2 + (-s)^2 \right) = \left(1 + \frac{\mu s^2}{\epsilon} \right) s^2
\]

when \( s = \delta_e \), then

\[
U_d = \left(1 + \frac{\mu}{\epsilon} \right) \delta_e^2
\]

which is the energy of distortion in a torsional test specimen.
If the same unit cube is subjected to a normal stress, $s_1$, in one direction only, the other two principal stresses being zero as in a tensile test specimen subjected to an axial load, the energy of distortion, $U_d$, becomes

$$U_d = (1 + \mu/3E) \left( s_1^2 + s_2^2 \right) = (1 + \mu/3E) s_1^2$$

Equating the energy of distortion for the case of pure shear to that of the unaxial tension condition, when yielding first occurs,

$$(1 + \mu/F) \gamma_e^2 = (1 + \mu/3E) s_e^2$$

$$\gamma_e = s_e / \sqrt{3}$$

The conclusion which can be reached is that yielding and eventually a ductile fracture starts when the energy of distortion reaches a critical value. The maximum shear stress, $\gamma_e$, at a point when yielding starts is $1/\sqrt{3}$ times the maximum tensile stress, $s_e$, at the same point. Thus the von Mises-Hencky ellipse for combined bending and shear stress is given by

$$\left( \frac{S_a}{S_e} \right)^2 + \left( \frac{S_m}{S_e} \right)^2 = 1$$

$$\left( \frac{S_a}{S_e} \right)^2 + \left( \frac{1/\sqrt{3} \gamma_s}{S_e} \right)^2 = 1$$

where $\gamma$ in the above equation is the constant shear stress recorded in each test specimen studied.
9.3 Maximum Shear Stress Theory

Tresca proposed in his maximum shear stress theory that yielding occurs when the maximum shear stress reaches a critical value (13, p. 2). Yielding begins when the maximum shear stress equals the shear stress corresponding to the yield strength in the simple tension test. According to Figure 9.3.1 yielding occurs when \( \gamma_{\text{max}} = S_y/2 \) where \( S_y \) is the yield strength of the material. For a triaxial stress state three maximum shear stress may be found and are given by

\[
\gamma = \frac{(s_1 - s_2)}{2} \quad \gamma = \frac{(s_2 - s_3)}{2} \quad \gamma = \frac{(s_1 - s_3)}{2}
\]

Yielding will begin when the largest of these shearing stresses becomes equal to one-half the tensile yield strength of a simple tension test specimen (15, p. 152).

In essence the theory predicts that the shearing yield strength is equal to one-half of the tensile yield strength. The advantages of the theory is that it is easy to use, is useful for ductile materials and is conservative in describing the behavior of brittle materials (15, p. 152).

9.4 Comparison of the Maximum Shear Stress and Energy of Distortion Theories

The maximum shear stress and energy of distortion theories can be conveniently represented in Figure 9.4.1 in the two-dimensional principal stress space.
Figure 9.3.1 Mohr's Circle Showing Relation of Maximum Shearing Stress to Tensile Yield Strength (15, p. 151)
Fig. 9.41 Representation of Energy of Distortion and Maximum Shear Stress Theories in the Plane Perpendicular to the Unit Vector $\left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right)$.

(15, p. 8)
Experimental evidence indicates that the Von Mises-Hencky or energy of distortion, criterion is more accurate than the Tresca theory for predicting the yield strength of most material under biaxial stress. In addition strain hardening and creep behavior correlate much better for most materials using the Von Mises-Hencky theory. The difference between the two theories is small, the maximum difference in the two theories in any state of stress being about 16 percent. In view of this fact the Tresca theory, which is more conservative is considered satisfactory even though the von Mises-Hencky theory is more accurate (13, p. 6).

It has been stated that stress-strain properties are best correlated using the von Mises-Hencky theory. This, however, is not grounds enough to state that fatigue failures are best described by the same criterion. The energy of distortion has no directional properties and is always considered a positive quantity. This causes some serious deficiencies as a means of predicting fatigue failure. In fatigue experiments it has been found that the maximum shear stress theory will correlate results as well as the von Mises-Hencky criterion. The problem of determining which of the theories is best is difficult because of the natural scatter of fatigue data spanning the various criteria.

Referring again to directional properties of the two criteria it is noted that the maximum shear stress changes sign
When the stresses are reversed, as during a rotating beam fatigue test, whereas, it has been previously noted that the energy of distortion is always a positive quantity. In reversed bending tests these directional properties are of critical importance. As the stresses at a point are reversed the energy of distortion goes from a positive quantity to zero and then back to a positive quantity. This means that the distortion energy theory is not reflecting a reversal in loading. Reversed loading cases are very harmful to the fatigue of any structure. It is important to recognize load reversals as adding to the range of stress and strain. It would be possible to devise methods to account for load reversals in the uniaxial case, however, in the case of combined stress the problem would be quite complex (13, p. 8).

The von Mises-Hencky theory is not free of the directional property problem in fatigue. As an example one can consider the special case where the magnitudes of the principal stresses are constant but the directions are changing with time. The energy of distortion remains constant indicating that there is no fatigue loading. However, this is not the case as the shear stress theory must also be modified for the case where the directions of the principal stresses vary with time. However, in this case one would merely use the history of maximum shear stress on a fixed plane of the material to predict the fatigue life (13, p. 9).
9.5 Modified Theories of Fatigue Failure Under Combined Stresses

Findley and Hathox (14) have developed several theories of fatigue failure under combined stresses. The following section will present the three theories which were developed by them. They state that the classical theories for the initiation of yielding under combined stresses are conflicting. The investigation of Gough and Pollard (14, p. 2) concluded that the classical theories which have been proposed are inaccurate since the ratio of fatigue strength in bending, \( b = S_{0} \), to that in torsion fatigue strength, \( t = S_{0} \), is not the same for all metals as required by the classical theories of yielding.

Findley and Hathox propose that the influence of the property of anisotropy, that of having different properties in different directions, and the state of combined stress are the cause of discrepancies between proposed theories of failure and results obtained from combined stress fatigue tests (4, p. 3).

The ratio of \( b/t \) varies over a considerable range for all engineering materials. Considering only metals this ratio varies from a value of 0.9 to 2.6. If, however, the metals are grouped, within each group the ratio of \( b/t \) has a much smaller range. Cast irons and its alloys have a \( b/t \) ratio which varies from 1.3 to 2.5 with a majority of these values lying between 1.5 and 2.0. This latter range is considered to be the ductile range. The \( b/t \) ratio value of notched steels was found to be considerably less than that of unnotched steels with a majority falling between 1.0 and 1.5.
It is felt that anisotropy of the material is a contributing cause to the variation of the $b/t$ ratio. It has been suggested that fatigue failure is caused primarily by an alternating shearing stress producing repeated slip, however, the resistance of a material to this fatigue mechanism may be influenced by the magnitude and sign of the normal stress occurring on planes of maximum shearing stress. This effect may vary with the material. Differences in the effect which the normal stress may have in fatigue in a given material will cause differences in the $b/t$ ratio (14, p. 6).

9.5.1 Correction Factors

In ductile metals the cyclic principal shear stress is the quantity most closely associated with the fatigue damage. The principal shear stress theory predicts that the bending strength should be twice the shear strength (14, p. 8). However, because of the effect of anisotropy and combined stress, Findley and Lathur suggest that the principal shear stress theory be modified to the form

$$\gamma_{\text{max}} = \frac{b}{2K} = t \quad \text{or} \quad K = \frac{b}{2t} \quad (9.5.1.1)$$

where $K$ is the correction factor for anisotropy and the combined stress condition. The expression for modified principal shear stress theory becomes

$$\left[ \frac{S_c 2/S_c}{S_t} \right]^2 + \left[ \frac{S_m 2/S_u}{S_t} \right]^2 = 1 \quad (9.5.1.2)$$
Where $S_a$ and $S_m$ represent respectively the amplitudes of alternating bending stress and alternating torsional stress components of combined stress (14, p. 9).

It is important to note that when correction factors are applied to the octahedral shear stress, principal shear strain, energy of distortion, total energy of distortion, total energy of deformation and magnitude of state of stress vector theories the result is the same in all cases, that of Equation 9.5.1.2.

If correction factors are applied to the principle stress theory the governing equation for combined bending and torsional stress becomes

$$\frac{S_a}{S_E} + \frac{S_m^2}{S_u^2} = 1 \quad (9.5.1.3)$$

When the principle strain theory is modified the fatigue strength ratio $b/t$ predicted is

$$\frac{S_a}{S_E} = \frac{b}{t} = 1 + \mu \quad (9.5.1.4)$$

where $\mu$ is Poisson's ratio.

The expression for the modified principal strain theory as reported previously (17) is

$$\varepsilon_1 = 1 + \frac{\mu}{2} \frac{S_a}{S_E} \left\{ \sqrt{S_a^2 + \left(2b/t(1+\mu)\right)S_m^2 + \left(1 - \mu/1+\mu\right)S_m^2} \right\} \quad (9.5.1.5)$$

For pure bending

$$\gamma = 0, \quad S = b, \quad \text{thus} \quad \varepsilon_1 = b/2$$
Substituting for $e_1$ in equation 9.5.1.5, and simplifying the expression, the modified principal strain theory becomes

$$\mu s^2/b^2 + (1 - \mu) s/b + b^2/t^2 = 1$$

(9.5.1.6)

The fatigue strength ratio $b/t$ predicted by the principal strain theory (17) is

$$b/t = 1 + \mu$$

(9.5.1.7)

Substituting equation 9.5.1.7 in equation 9.5.1.6

$$(b/t - 1) s^2/b^2 + (2 - b/t) s/b + t^2/t^2 = 1$$

(9.5.1.8)

This equation models the combined stress state of bending and torsion for the modified principal strain theory.

A design expression has been proposed by Findley and Mathur (14) to model the fatigue failure of notched ductile metals and irons. These materials have a behavior which is intermediate between the perfectly brittle irons which have little or no slip and ductile metals which have considerable slip. The stress system which is associated with these materials may change from the principle stress theory, $b/t = 1$, for materials such as the brittle irons to the principle shear stress theory where $b/t = 2.0$ as in the ductile metals (14, p. 11).
The design expression which is suggested to model these metals under combined bending and shear is given by

\[
\left(\frac{S_a}{S_m}\right)^{b/t} + \left(\frac{S / S_u}{m_u}\right)^2 = 1
\]  

(9.5.1.9)

It is noted that the exponent \(b/t\) varies with the class of material. When \(b/t = 1\) Equation 9.5.1.9 is that of the modified principal stress theory. When \(b/t = 2\), which is the case in ductile materials the equation reduces to that of the modified shear stress theory (14, p. 11).

9.5.2 Comparison to Fatigue Data

Findley and Mahur compare their modified stress theories to actual combined bending and torsional fatigue test results. It is found that the modified shear stress theory, Equation 9.5.2.1 served as a good model for ductile metals with a \(b/t\) ratio ranging from 1.46 to 2.0. A comparison of this equation to actual fatigue tests is given in Figure 9.5.1 (14, p. 15).

The modified principles stress theory is compared to actual fatigue data of iron and iron alloys in Figure 9.5.2 and to that of notched ductile steels in the brittle range, \(b/t = 1.3\) in Figure 9.5.3. The modified principle strain theory predicts strengths which are higher than the actual data as can be seen
Figure 9.5.1  Comparison of Modified Stress Theories With Fatigue Data Generated Under Combined Bending and Torsional Stress (14, p. 24)
Figure 9.5.2 Comparison of the Modified Principal Stress (solid line) and Modified Principal Strain Theories (dashed line) To Fatigue Data of Iron and Iron Alloys. (14, p. 25)
Figure 9.5.3 Comparison of Design Expression to Fatigue Data Generated Under Bending and Torsional Stress For Notched Steels Having a b/t Ratio Greater Than 1.3. (14, p. 25)
Figure 9.5.4 Position of Modified Strength Theories as Specified By b/t Ratio. (14, p. 25)

- Curve A: Principle Stress Theory
- Curve B: Principle Strain Theory
- Curve C: Total Strain Energy Theory
- Curve D: Distortion Energy Theory
- Curve E: Principle Shear Stress Theory
In Figure 9.5.2 (14, p. 15).

It is found that the design expression, Equation 9.5.1.9 is in very good agreement with data of notched ductile steels, \( b/t \leq 1.3 \), and in good agreement with both brittle and ductile range of metals as shown in Figure 9.5.3 (9, p. 15).

Assigning the metals to regions by \( b/t \) ratio values, as discussed previously, yields the following regions:

- Region I (brittle)
- Region II (intermediate)
- Region III (ductile)

The position of each modified strength theory within the regions described in Figure 9.5.4 is specified by the value predicted by the \( b/t \) ratio of each strength theory. Table 9.5.1 presents the value predicted by each strength theory for the ratio of \( b/t \). The modified strength theories may be assigned to each of the regions described previously in the following descending order (9, p. 17).

<table>
<thead>
<tr>
<th>Region</th>
<th>Modified principal stress</th>
<th>Design expression</th>
<th>Modified principle strain</th>
<th>Modified principle shear stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region II</td>
<td>Design expression (notched ductile steels) (equation 9.5.1.9)</td>
<td>Modified principal strain</td>
<td>Modified principle stress</td>
<td>Modified principle shear stress</td>
</tr>
<tr>
<td>Region III</td>
<td>Modified principle shear stress</td>
<td>Design expression (equation 9.5.1.9)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9.5.1 Values of the Ratio $b/t$ Predicted by each Strength Theory (14, p. 17).

<table>
<thead>
<tr>
<th>Theory</th>
<th>$b/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principle stress theory</td>
<td>$b/t = 1$</td>
</tr>
<tr>
<td>Principle strain theory</td>
<td>$b/t = 1 + \mu$</td>
</tr>
<tr>
<td>Total strain energy theory</td>
<td>$b/t = \sqrt{2} + 2\mu$</td>
</tr>
<tr>
<td>Distortion energy and octahedral shear stress theory</td>
<td>$b/t = \sqrt{3}$</td>
</tr>
<tr>
<td>Principle shear stress theory</td>
<td>$b/t = 2.0$</td>
</tr>
</tbody>
</table>
CHAPTER X

RECOMMENDED EMPIRICAL MATHEMATICAL MODELS OF THE
FINITE LIFE GOODMAN DIAGRAM

In viewing each of the five finite life Goodman diagrams developed in Chapter V it can be seen that the equation of the mean line is generally of a quadratic nature. It is of interest to investigate if the mean line of the finite life Goodman diagrams can be described by one or more of the empirical mathematical models of the Goodman diagram discussed in Chapter VIII.

The von Mises-Hencky ellipse was compared to the experimental finite life Goodman diagrams presented in Figure 5.3.1 through 5.3.5. The ellipse is given by

\[(S_o/S_e)^2 + (S_m/S_u)^2 = 1\]  \hspace{1cm} (10.1)

The endurance strength and ultimate strength mean values specify the mean line of the von Mises-Hencky ellipse. The endurance strengths for each of the five von Mises-Hencky ellipses were taken from Table 5.3.1. These ellipses, superimposed upon the original experimental mean line, appear as overlays with the experimental finite life Goodman diagram in Chapter V as overlays on Figures 5.3.1 through 5.3.5. As in Chapter V the ultimate strength distribution is taken to be that of the grooved specimen. A PDP-8 program, described in Appendix E, was
used to calculate the mean stress points as specified by the
ellipse for several theoretical alternating stress levels.
These values are listed in Table 10.1 for each of the five
finite life Goodman diagrams.

The finite life Goodman diagrams for 40,000, 90,000, and
200,000 cycles are in very close agreement with the von Mises-
Hencky ellipse. The lower cycle life diagrams of 3,500 and 9,000
cycles are in very poor agreement with the von Mises-Hencky
ellipse, a fact which will be investigated later in this
Chapter.

Although the higher cycle life fatigue diagrams seemed to
be closely approximated by the von Mises-Hencky ellipse, it was
decided to determine the exponent, \( a \), where in general the
quadratic equation of interest is given by

\[
\left( \frac{S_a}{S_e} \right)^a + \left( \frac{S_m}{S_u} \right)^2 = 1
\]  \hspace{1cm} (10.2)

This was accomplished in the following manner. Equation 10.2
can be transformed to

\[
y^a + x^2 = 1
\]  \hspace{1cm} (10.3)

where

\[
y = \frac{S_a}{S_e} \\
\]

\[
x = \frac{S_m}{S_u}
\]
Table 10.1 Alternating and Mean Stress Values Predicted by the von Mises-Hencky Ellipse.

<table>
<thead>
<tr>
<th>$S_a$ - psi</th>
<th>$S_m$ - psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=3,500 cycles; $S_e=135,953$ psi; $S_u=255,300$ psi</td>
<td>134,000</td>
</tr>
<tr>
<td>128,000</td>
<td>86,038</td>
</tr>
<tr>
<td>120,000</td>
<td>119,995</td>
</tr>
<tr>
<td>100,000</td>
<td>172,959</td>
</tr>
<tr>
<td>60,000</td>
<td>229,092</td>
</tr>
<tr>
<td>20,000</td>
<td>252,522</td>
</tr>
<tr>
<td>N=9,000 cycles; $S_e=115,926$</td>
<td>112,000</td>
</tr>
<tr>
<td>168,000</td>
<td>92,779</td>
</tr>
<tr>
<td>100,000</td>
<td>129,145</td>
</tr>
<tr>
<td>80,000</td>
<td>184,145</td>
</tr>
<tr>
<td>60,000</td>
<td>218,445</td>
</tr>
<tr>
<td>20,000</td>
<td>251,472</td>
</tr>
<tr>
<td>N=40,000 cycles; $S_e=90,693$</td>
<td>88,000</td>
</tr>
<tr>
<td>84,000</td>
<td>96,000</td>
</tr>
<tr>
<td>80,000</td>
<td>120,264</td>
</tr>
<tr>
<td>60,000</td>
<td>191,445</td>
</tr>
<tr>
<td>40,000</td>
<td>229,128</td>
</tr>
<tr>
<td>20,000</td>
<td>249,015</td>
</tr>
<tr>
<td>N=90,000 cycles; $S_e=79,054$</td>
<td>68,000</td>
</tr>
<tr>
<td>64,000</td>
<td>149,866</td>
</tr>
<tr>
<td>60,000</td>
<td>166,231</td>
</tr>
<tr>
<td>40,000</td>
<td>220,207</td>
</tr>
<tr>
<td>20,000</td>
<td>246,995</td>
</tr>
<tr>
<td>N=200,000 cycles; $S_e=70,172$</td>
<td>68,000</td>
</tr>
<tr>
<td>64,000</td>
<td>104,696</td>
</tr>
<tr>
<td>60,000</td>
<td>137,388</td>
</tr>
<tr>
<td>40,000</td>
<td>209,761</td>
</tr>
<tr>
<td>20,000</td>
<td>244,711</td>
</tr>
</tbody>
</table>
Solving for $y^a$ yields

$$y^a = 1 - x^2 \quad (10.4)$$

If the natural logarithms of both sides are taken, the result is

$$a \ln y = \ln (1-x^2) \quad (10.5)$$

$$\ln y = \frac{1}{a} \ln (1-x^2) \quad (10.5)$$

If this equation is plotted on ln - ln graph paper the result will be a straight line. A least squares PDP-8 program SRFE, is available which will give the slope of the best fit equation through the data as well as the correlation coefficient. The slope of this line is equal to the value of the universe of the exponent $a$ in Equation 10.2. A description of this program is given in Appendix E. The values derived by this method for the exponent $a$ and $\rho$ are given in Table 10.2.

The finite life Goodman diagrams of 3,500 and 9,000 cycles as mentioned previously of not seem to be modeled by a quadratic equation. The various empirical models were reviewed and it was determined that the Sines Line had the greatest potential of modeling the finite life range below $10^4$ cycles. The Sines Line is given by

$$S_a = S_e - cS_m \quad (10.6)$$
Table 10.2 Values of the Exponent $a$ and Correlation Coefficient ($\rho$)

<table>
<thead>
<tr>
<th>Cycle Life $N$</th>
<th>Coeff. $a$</th>
<th>Correlation Coeff. ($\rho$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000</td>
<td>2.3821</td>
<td>.9137</td>
</tr>
<tr>
<td>90,000</td>
<td>2.3042</td>
<td>.8948</td>
</tr>
<tr>
<td>200,000</td>
<td>1.9107</td>
<td>.8915</td>
</tr>
</tbody>
</table>
The coefficient $c$ for the finite life Goodman diagrams of 3,500 and 9,000 cycles was found, by trial and error, to be 0.03. The Sines Line specified by $c = 0.03$ was placed on overlays of Figures 5.3.1 and 5.3.2. The Sines Line is valid only to the yield strength of the material. Because the yield strength of the grooved geometry specimen was not available, a dashed line was drawn beyond the yield strength of the ungrooved specimen. This is meant to signify that the yield strength of the grooved specimen is some value greater than that of the ungrooved specimen.

In conclusion, it can be said that the von Mises-Hencky ellipse quite closely approximates the mean line of the finite life Goodman diagram, where the cycle life value is above $10^4$ cycles. If the von Mises-Hencky ellipse is modified to

$$\left(\frac{S_a}{S_e} + 2\frac{S_a}{S_e}\right)^2 + \left(\frac{S_u}{S_u} + 2\frac{S_u}{S_u}\right)^2 = 1 \quad (10.7)$$

the Goodman surface is adequately described. This surface is shown in Figure 5.3.5. For life values below $10^4$ cycles the Sines Line adequately describes the mean line of the finite life Goodman diagrams. If the Sines Line is modified as shown in overlays of Figures 5.3.1 and 5.3.2 to the form

$$S_a = (S_e + 3\frac{S_e}{S_e}) - cS_m \quad , \quad (10.8)$$

then the Goodman surface is completely modeled.
The comparison of the von Mises-Hencky ellipse with the experimental finite life diagrams, and the discussion of Section 9.3 leave little doubt that the von Mises-Hencky strength theory closely predicts the fatigue failure in steel. The discussion of Section 9.3 gives adequate documentation that the von Mises-Hencky strength theory is widely considered as a strength theory for combined stress conditions. Although the von Mises-Hencky strength theory is not free of all criticisms when considered as the criterion governing fatigue failures the very close behavior of the larger cycle finite life Goodman diagrams indicate that this criterion can be continued to be accepted as governing the fatigue data generated under National Aeronautics and Space Administration Contract NGR 05-002-044 at The University of Arizona. The material which is used in this research program is SAE 4340 steel, Rockwell C 35/40.
CHAPTER XI

TWO RECOMMENDED METHODS OF REDUCING THE QUANTITY OF EXPERIMENTAL DATA NEEDED FOR A FATIGUE DATA ACQUISITION PROGRAM

The problem of reducing the number of data points in a fatigue test program is quite complex. The nature of the fatigue mechanism requires that distributions be developed to describe the cycles to failure and strength parameters of the test specimen. To date the complex fatigue test program at The University of Arizona has performed fatigue tests on well over 650 specimens. This has been expensive but the experience gained by this pioneering program has laid the foundation to develop theoretical concepts which may possibly reduce the need for expensive fatigue acquisition programs in the future. An examination of the Goodman diagrams and surfaces developed in this paper and the application of the theories proposed by W. H. Findley (14, pp. 26) seem to offer great hope for reducing the number of test specimens required in the fatigue data acquisition program.

In reviewing the finite life Goodman diagrams developed here, it becomes evident that the distributions placed along the mean and alternating stress axes are of critical importance in determining the shape and location of the finite life Goodman diagram and surface. Currently the ultimate tensile
strength distribution which is placed along the mean stress axis is developed from the results of tensile tests. These tests include the pulling of thirty-five grooved test specimens to determine the ultimate strength distribution; this method is satisfactory and should be continued.

The distribution which is placed along the alternating stress axis can be determined as it is currently being done from the stress cycles to failure data at a stress ratio of infinity. This requires the generation of cycles to failure distributions at five alternating stress levels. Currently, this would require thirty-five test specimens in each of the five stress levels tested. The methods presented in Chapters III and V could then be used to place the derived strength distributions on the finite life Goodman diagrams. Considering the conclusions reached in Chapter X the development of the finite life Goodman diagrams would be possible with two hundred test specimens. To review, the endurance strength for each cycle life would be specified by the transformation of the cycles to failure distributions at a stress ratio of infinity to the vertical strength distributions on the Goodman diagram. This procedure is outlined in Chapter V. The ultimate strength distribution is specified by the tensile test of the thirty-five grooved specimens.

The entire finite life Goodman diagram could be modeled, above $10^4$ cycles, by the von Mises-Hencky ellipse as specified by
\[ \left( \frac{S_a}{S_e} \mp 3 \sigma_c \right)^2 + \left( \frac{S_m}{S_u} \mp 3 \sigma_u \right)^2 = 1 \]  

(11.1)

The necessary data would amount to a maximum of two hundred test specimens which would include a staircase analysis of the endurance strength at $10^6$ cycles, at a stress ratio of infinity.

The required number of test specimens can be reduced significantly if the cycle life values for the finite life Goodman diagrams are determined prior to the start of the test program. If for instance, three finite life Goodman diagrams are required then the required endurance strengths can be determined by the staircase method for each cycle life.

This method would reduce the number of specimens to one hundred twenty-five to one hundred fifty depending upon the number of required diagrams. Each staircase method should have a sample size of thirty-five while the ultimate strength distribution of the grooved specimen should also be specified by thirty-five specimens.

By drawing a desired stress ratio lines on the finite life Goodman diagram, as specified by the above equation, it would be possible to develop the vertical strength distributions which would be placed on the statistical S-N diagram by following the reverse of the procedure developed in Chapter V. That is by drawing the stress ratio line on the finite life Goodman diagrams, the alternating stress level and standard deviation, in actuality the three sigma limits, of the rotated vertical
strength distributions would be obtained. It would, by the 
reverse of the method discussed in Chapter V, be possible to 
create the vertical strength distributions which would be 
placed on the statistical S-N diagram. This, at least suggests 
that it may be possible to continue one step further back to de-
termine the cycles to failure distributions. This 
method eliminates the need of generating cycles to failure dis-
tributions at the intermediate stress ratios between zero and 
infinity. Currently such investigations require slightly more 
than four hundred specimens. This number considers thirty five 
specimens to a cycles to failure distribution.

In an attempt to reduce the need for experimental data 
beyond that discussed above it appears that the theories proposed 
by Findley (14) in Chapter X offer an alternative to the expensive 
fatigue data acquisition program. Considering that the mean 
line of the finite life Goodman diagram follows the equation

\[(s_a/s_e)^a + (s_m/s_u)^2 = 1 \quad (11.2)\]

The exponent \(a\), which has previously been discussed in Chapter 
X also appeared in the design expression of Equation 9.5.1.9 
proposed by Findley for notched ductile steels.

\[(s_a/s_e)^{b/t} + s_m^2/s_u^2\]
Although the equations 9.5.1.9 and 12.2 are not exactly identical, differing in the denominators of the terms on the left hand side of the equations, they do attempt to relate quantities of bending and torsional stress. Comparing the two equations, it can be seen that \( b \) is actually \( S_e \) and \( t \) is \( S_u \). The exponent \( a \) is the ratio of bending strength to torsional strength, and could be determined experimentally. It would have to be determined if this ratio should be found from static tests or dynamic fatigue tests. In either event, facilities at The University of Arizona, including the NASA complex fatigue machines, would be adequate to determine this ratio. Findley Coleman, and Hanley cite the value of \( b/t = 1.78 \) for an SAE 4340 steel, Rockwell C 35 (13, p. 153); however, the dimensions of the test specimen are not the same as that undergoing tests at The University of Arizona. In addition the specimen which Findley, et. al., used in their studies is of the ungrooved geometry. It seems that the exponent \( a \) is in actuality Findley’s exponent \( b/t \). It would be possible to experimentally determine this value with no more than the number of specimens required by the static ultimate strength tests, or thirty five specimens.

Once the mean line is specified the standard deviation along this line could quite possibly be approximated by dividing the standard deviation of the endurance strength which is placed along the alternating stress axis by \( \sin \Theta \), where \( \Theta \) is specified.
by the ratio of alternating stress to mean stress (3, p. 73). Hence, the standard deviation along the stress ratio axis, $s_{x'}$, is given by

$$
\sigma_s = \frac{\sigma / \sin \Theta}{s_{x'}}
$$

(11.3)

This then would give an approximation of the standard deviation of the Goodman surface.

The two methods presented in this chapter, to reduce to a minimum the required number of test specimens, are not meant to be finalized proposals for a fatigue test program. They do seem to this investigator to be valid means of reducing the need of large quantities of experimental data. The later method discussed would require only the number of specimens needed to experimentally determine the ratio of b/t, quite possibly no more than 35 - 40 specimens. The method formerly discussed would require approximately 200 test specimens. Further investigation of these two methods seems prudent.
CHAPTER XII
OVERALL CONCLUSIONS

Methodologies for developing finite life Goodman diagrams and surfaces have been presented in this report. The finite life Goodman diagram presents allowable combinations of alternating and mean stress for the combined stress condition of alternating bending and constant torsional stresses for specific periods of design life. The actual Goodman surface, which is developed using two to five strength distributions at specified stress ratios, can be used to construct strength distributions at any desired stress ratio. The strength distributions are distributed normally. The technique of constructing a strength distribution at any specified stress ratio is initiated by construction of the desired stress ratio line on the Goodman diagram. The intercept of this line with the Goodman surface specifies the mean and the standard deviation, in terms of the three sigma limits, of the strength distribution at that stress ratio. The alternating stress level of the mean of the strength distribution can be read directly from the finite life Goodman diagram. This procedure is particularly valuable where the strength distribution at a specific stress ratio is required by the interference technique used in probabilistic design.
In conclusion the finite life Goodman diagrams and surfaces developed in this paper indicate that as the cycle life decreases the allowable combinations of bending and shear stress magnitudes increase. The strength distribution to be placed on the mean stress axis of the finite life Goodman diagram is concluded to be that of the same geometry test specimen as underwent fatigue tests. The grooved geometry test specimen is concluded to be the test specimen which specifies the ultimate strength distribution which is placed on the mean stress axis of the finite life Goodman diagram. The von Mises-Hencky strength theory and ellipse have been shown to adequately model the behavior of SAE 4340 steel, above the cycle life of $10^6$ cycles, under combined alternating bending and constant torsional stresses.
CHAPTER XIII
OVERALL RECOMMENDATIONS

The following recommendations are offered by this investigator:

1. As additional cycles to failure data becomes available from Phase II of the National Aeronautics and Space Administration Grant No. 03-002-044 at The University of Arizona the methods proposed in Chapter XII to reduce the required number of test specimens should be further investigated. The recovery of cycles to failure data from the finite life Goodman diagram, should be investigated as proposed in Chapter XII, using both Phase I and Phase II data.

2. A comprehensive literature search should be undertaken to obtain the complete set of papers authored by Professor W. H. Findley of Brown University. These works would be a valuable aid to the research program being conducted at The University of Arizona for the National Aeronautics and Space Administration.

3. Additional fatigue and static strength data, beyond that supplied by Phase II of the National Aeronautics and Space Administration research effort, should be acquired through computer search facilities. These
facilities utilize high speed computers to selectively retrieve and display requested information. This would be particularly helpful in determining the best theoretical distribution to be assigned to the static strength parameters. The additional data supplied by these search facilities would compliment data acquired through the experimental test programs. These search facilities include the Mechanical Properties Data Center in Traverse City, Michigan and the Defense Metals Information in Columbus, Ohio.
APPENDIX A

FORTRAN Computer Program To Reduce Cycles To Failure Data

A-1. Flow Chart
A-2. Definition of Variables
A-3. Program Listing
MAIN PROGRAM

Read CONF, T (J), AREA, CHIL (J), CHIU (J)

Read S, N, XN(I)

XNLOG(I) = LOGF(XN(I))

Calculate XNSUM, XMLOG

Calculate XM2, XM3, XM4, XSIGL, XSIG, XK3, XK4

N - 30

0

Calculate XML, XMU By student + distribution

Calculate XML, XMU By normal dist. approx.

Calculate XSRDL, XSTDU

Print N, S, XMLOG, XSIGL, XK3, XK4, XSIG, CONF, XML XMU, XSTDL, XSTDU

Figure A-1. Flow Chart for Failure Data Distribution Determination Computer Program.
SUBROUTINE PROGRAM

Subroutine KIFIT(N,)
XN, XMEAN, XSIG
CHISQ, MCL

Calculate MCL, XMAX,
XMIN, WIDTH, X(I)
XMIN(I), OFREQ(I),
EFREQ(I)

Print XN (J),
EFREQ(I), OFREQ(I)

Figure A-1 (Cont'd)
DO I = 1, MCL/2

EFREQ(I) - 4.0

0

Add EFREQ(I) & PFREQ(I) to next higher class interval values

DO I = 1, MCL/2

EFREQ(I-MCL) - 4.0

0

Add EFREQ(I-MCL) & OFREQ(I-MCL) to next lower class interval values

Print reduced histogram.

EFREQ(I), OFREQ(I), XCL

Calculate($x^2$) CHISQ

RETURN

END

Figure A-1 (Cont'd).
CONF = TWO-sided CONFIDENCE LEVEL
T = STUDENT T FOR DESIRED CONFIDENCE M AND DEGREE
OF FREEDOM N
AREA = ABSISSA VALUE OF NORMAL DISTRIBUTION FOR
SPECIFIED CONFIDENCE
CHI^2, CHI^2L = CHI SQUARE FOR CONFIDENCE M AND DEGREE
OF FREEDOM N
S = STRESS LEVEL OF TESTED SPECIMENS
N = NUMBER OF SPECIMENS
XN(I) = CYCLES TO FAILURE OF SPECIMENS
XNLOG(I) = LOG CYCLES TO FAILURE OF SPECIMENS
XNSUM = SUMMATION OF XNLOG(I)
XMLOG = MEAN LOG CYCLES AT STRENGTH, S
XSIGL = STANDARD DEVIATION IN LOG CYCLES AT STRENGTH
XSIG = SORTF WITH N-1 DEGREES OF FREEDOM
XM2, XM3, XM4 = 2ND, 3RD, 4TH MOMENTS OF LOG CYCLES
XK3 = COEFFICIENT OF SKEWNESS
XK4 = COEFFICIENT OF KURTOSIS
XM, XMU = MEAN LOWER AND UPPER CONFIDENCE LIMITS OF
MEAN LOG VALUE
XSTDL, XSTDU = STANDARD DEVIATION LOWER AND UPPER
CONFIDENCE LIMITS OF LOG VALUE
CHISQ = VALUE OF CHI SQUARE FIT OF DATA HISTOGRAM TO
NORMAL DISTRIBUTION DATA
MCL = NUMBER OF HISTOGRAM CLASSES FOR CHI SQUARE FIT
ADDITIONAL SUBROUTINE VARIABLES
N = NUMBER OF DATA POINTS
XN(I) = DATA POINT VALUES
XMEAN, XSIG = MEAN AND STANDARD DEVIATION OF DATA
VALUES
XMAX = MAXIMUM FAILURE VALUE
XMIN = MINIMUM FAILURE VALUE
WIDTH = CLASS INTERVAL WIDTH
XI(I) = CLASS BEGINNING POINT
XMID(I) = CLASS MID-POINT
OFREQ(I) = OBSERVED FREQUENCY OF OCCURRENCE IN CLASS
INTERVAL I
EFREQ(I) = EXPECTED FREQUENCY OF OCCURRENCE

Figure A-2 - Definition of Variables for Failure Data Distribution
Determination Computer Program.
Figure A-3 - 7072 Computer Program Listing for Failure Data Distribution Determination.
81 EFREQ(1) = 0.0
PRINT 46, EXP,XM1C(1)
GO TO 83
82 CONTINUE
EFREQ(1) = AN*G.3989423/XSIG*EXP((~EXP)*WIDTH
83 CONTINUE
PRINT 47,(XM(1),J=1,N)
PRINT 48, (EFREQ(1),OFREQ(1),X(1),J=1,N)
K = MCL/2
J = 1
DO 84 J=1,N
IF (EFREQ(J)-4.0) 77,84,84
77 IF (J-1) 79,84,84
79 J=J+1
IP1 = J+1
EFREQ(IP1) = EFREQ(J)*EFREQ(IP1)
OFREQ(IP1) = OFREQ(J)+OFREQ(IP1)
84 CONTINUE
K = MCL
NCIT = MCL-N+1
DO 87 J=1,K
MV1 = MCL-J+1
IF (EFREQ(MV1)-4.0) 85,87,87
85 IF (MV1=NCIT) 87,87,86
86 K = MCL-1
MV1 = MV1-1
EFREQ(MV1) = EFREQ(MV1)+EFREQ(MV1+1)
OFREQ(MV1) = OFREQ(MV1)+OFREQ(MV1+1)
87 CONTINUE
PRINT 49, (OFREQ(1),OFREQ(I),X(I),J=1,I)
MCL = K-J
CHISO = C,0
DO 88 J=1,K
CHISO = CHISO+(OFREQ(J)-EFREQ(1))*\(OFREQ(J)-EFREQ(1)\)
1 1/EFREQ(1)
88 CONTINUE
RETURN
END

C PROGRAM FOR FINDING THE MEAN, STANDARD DEVIATION
C AND CONFIDENCE LIMITS OF RANDOM VARIABLES FROM
C ASSUMED NORMAL DISTRIBUTIONS
C
C CONF = 90.
C 3 FORMAT (15)
C 5 FORMAT (1F10.5)
C 6 FORMAT (1F10.5)
C 10 FORMAT (1F10.5,15(1F10.5))
C 12 FORMAT (1C,M7,C,0,124 SPECIMENS, 7X,9D3.0,12X,12X,12X,12X,12X,12X,12X,12X,
C 1 1 HLOG MEAN,19X,1LOG SIG.DEV,7X,84K.2NESS,7X.

Figure A-3. (Cont'd)
Figure A-3 (Cont'd)
XFU = X^MLOG + AREA * XSIGL / A
IF (A=190) 37, 37, 35
35 N = 100
AK = N
37 CONTINUE
36 XSIGL = XSIGL * SORTF ((A-1.0) / CH1U(N-1))
XSIGU = XSIGL * SORTF ((A-1.0) / CH1U(N-1))
R = 1R
PRINT 12, N, S, X^MLOG, XSIGL, XX3, XX4
PRINT 15, XSIG
PRINT 13, CONF
PRINT 14, XM, XXU, XSIGL, XSIGU
IF (N=10) 40, 40, 39
39 CONTINUE
CALL KFIT(N, XMLOG, XMLOG, XSIGL, CH1SO, MCL)
PRINT 16, CH1SO, MCL
40 CONTINUE
GO TO 18
END

Figure A-3 (Cont'd)
APPENDIX B

FORTRAN Computer Program To Determine Time Dependent Strength Distribution Parameters

B-1. Flow Chart
B-2. Definition of Program Variables
B-3. Computer Program Listing
$M = 23$
$\text{INT} = 10$

Read $XJMIN, XJINT$

Read $N, SINT, ASTR(J), \ AMLOG(J), ASIGL(J)$

$L = 1$

DO $J = 1, N - 1$

Calculate $DSTR, DMEAN, DSIG$

Calculate $SLAM(J), \ SLSIG(J), M X$

DO $K = 1, M X$

$LX = L + K$

Calculate $XMLOG(LX), XSIGL(LX)$

$L = LX$

Calculation of failure distribution parameters for interpolated stress levels

Figure B-1 - Flow Chart for Time Dependent Strength Distribution Generation from Failure Distribution Parameters.
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Print STR, XMLOG (I), XSIGL (I), I, 1 = 1, LX

DO I = 1, N

XVAL = XJMIN - XJINT

DO K = 1, M

XVAL = XVAL + XJINT

Calculate z

Call for Subroutine
(z, PROB)

Calculate AREA(I,K)

DO I = 1, N, INT

Print STR, XMLOG (I), XSIGL (I)

Print K, AREA(I,K), K = 1, M

DO J = 1, M

Figure B.1 (Cont'd).

Cumulative lognormal failure distribution at N=L stress levels from failure distribution parameters.
Figure B-1 (Cont'd)

1. Calculate SNCOL (K)
2. Calculate U1, P1, P11
3. DO I = 1, M
4. DO K = 1, M
5. Print I, AREA (I, K)
6. Print K, CMC(K)
7. DO K = 1, M
8. Calculate oREP(K, M)
9. DO K = 1, M

Parameter calculations for normal distribution fit to strength distributions.

Strength frequency distributions for various log cycles of life.
Figure B-1 (Cont'd).

Calculate SM2, SM3, SM4

Calculate SSIGL(K), SK3(K), SK4(K)

Print, CYC(K), SMLOG(K), SSIGL(K), SK3(K), SK4(K), K = 1, M

DO K = 1, M

DO J = 1, N

Calculate EFREQ (J,K), CHISQ(J,K)

Print OFREQ (J,K) EFREQ (J,K), CHISQ(J,K), J=1, N

END
M = TOTAL NUMBER OF GENERATED STRENGTH DIST.
INT = NO. OF VALUES OMITTED BETWEEN PRINTOUT
XJMAX = MINIMUM LOG CYCLE VALUE FOR STRENGTH DIST.
XJINT = INTERVALS BETWEEN LOG CYCLE VALUES FOR
STRENGTH DISTRIBUTION S
N = NUMBER OF FAILURE DISTRIBUTIONS
SINT = MINIMUM STRESS INTERVAL BETWEEN INTERPOLATED
FAILURE DISTRIBUTIONS
ASTR(J) = STRESS LEVEL FOR EXPERIMENTAL FAILURE
DISTRIBUTION
AMLOG(J) = EXPERIMENTAL FAILURE DISTRIBUTION MEAN IN
LOG CYCLES AT STRESS ASTR
ASIGL(J) = EXPERIMENTAL FAILURE DISTRIBUTION
STANDARD DEVIATION IN LOG CYCLES
DSTR = DIFFERENCE BETWEEN TWO CONSECUTIVE
EXPERIMENTAL STRESS LEVELS
DMEAN = DIFFERENCE BETWEEN TWO CONSECUTIVE EXPERIMENTAL
FAILURE DISTRIBUTION MEANS
DSIG = DIFFERENCE BETWEEN TWO CONSECUTIVE EXPERIMENTAL
FAILURE DISTRIBUTION STANDARD DEVIATIONS
SLAM(J) = SLOPE BETWEEN TWO CONSECUTIVE EXPERIMENTAL
FAILURE DISTRIBUTION MEANS
SLSIGL(J) = SLOPE BETWEEN TWO CONSECUTIVE EXPERIMENTAL
FAILURE DISTRIBUTION STANDARD DEVIATIONS
L = COUNTER FOR INTERPOLATED FAILURE DISTRIBUTIONS
MX = DSTR/SINT
LX = SEQUENTIAL NUMBERING OF SAMPLE FAILURE
PARMETERS AND INTERPOLATED PARAMETERS
STR = STRESS LEVEL FOR I TH FAILURE DIST. PARAMETERS
XKLOG(I) = FAILURE MEAN IN LOG CYCLES FOR
EXPERIMENTAL AND INTERPOLATED VALUES
XSIGL(I) = FAILURE STANDARD DEVIATION IN LOG CYCLES
FOR SAMPLE AND INTERPOLATED VALUES
N = LX = NUMBER OF GENERATED FAILURE DISTRIBUTIONS
BY INTERPOLATION
XVAL = LOG CYCLE VALUES FROM XJMAX TO XJMIN + MAXJINT
FOR M STRENGTH DISTRIBUTIONS
Z = STANDARDIZED DISTANCE FOR LOG CYCLE FAILURE
DISTRIBUTION AT I TH STRESS LEVEL FOR K TH LOG CYCLES
PROB = NORMALIZED PROBABILITY DENSITY UNDER LOGNORMAL
DISTRIBUTION FROM -Z TO Z
AREA(I, K) = CUMULATIVE NORMALIZED AREA OF LOGNORMAL
FAILURE DISTRIBUTION FROM ZERO TO Z
TSSTR(I, 0) = STRESS LEVELS PRINTED OUT
NZTSTR(I, 0) = INTEGER REPRESENTATION OF TSSTR(I, 0)
CYC(K) = LOG-CYCLE VALUE CORRESPONDING TO K 5

Figure B-2. Definition of Program Variables.
FREQ(1,K) = FAILURE PROBABILITY FROM CUMULATIVE LOG-
NORMAL FAILURE DISTRIBUTION AT LOG CYCLES C YC
BETWEEN STRENGTHS I(A) AND I(B)
UI = ABSISSA VALUE OF STRENGTH HISTOGRAM CLASS
INTERVAL
FI = FREQUENCY OF OCCURRENCE IN STRENGTH HISTOGRAM
CLASS INTERVAL
FIUI = FI TIMES UI
SMLOG(K) = MEAN STRENGTH AT SET LOG CYCLES
SK2,SK3,SK4 = 2ND,3RD, AND 4TH MOMENTS OF
STRENGTH DISTRIBUTION
SS1GL(K) = STANDARD DEVIATION FOR STRENGTH AT SET
LOG CYCLES
SK3(K) = COEFFICIENT OF SKEWNESS FOR STRENGTH AT SET
LOG CYCLES
SK4(K) = COEFFICIENT OF KURTOSIS FOR STRENGTH AT SET
LOG CYCLES
TOTAL NUMBER OF I NO S = N/NINT WHERE N = LX MAX.
TOTAL NUMBER OF J S = N SAMPLE FAILURE DIS.
TOTAL NUMBER OF I S = N WHERE N = LX MAX.
TOTAL NUMBER OF K S = M

Figure B-2. (Cont'd).
**SWITCH**

COMPILE FORTRAN, EXECUTE FORTRAN

SUBROUTINE PROB(Z,PROB)

C
PROB = THE AREA UNDER NORMAL DISTRIBUTION BETWEEN
PLUS AND MINUS Z STANDARD DEVIATIONS

IF(Z<1.2) 1030,1000,1010

1000 ZSC = 2*Z

.0546943*Z*Z*Z*Z - 2*Z*Z + 1
GO TO 1070

1010 IF(Z>2.9) 1020,1060,1060

1020 ZSC = 2*Z

PIER = 1.0

FACT = 1.0

CODIN = 3.0

1030 PTERM = -PIER/ZSC/(2.0*FACT)

TERM = PTERM/CODIN

PROB = PROB + TERM

IF ((AREFTERM) - 0.00007) 1040,1040,1040

1040 FACT = FACT + 1.0

CODIN = CODIN + 2.0

GO TO 1030

1050 PROB = 0.7978656*Z*Z*ZSC

GO TO 1070

1060 REC = 1.0/(2*Z)

PROB = 1.0 - 0.7978656*EXP(-Z*Z/2.0)/2

1 (1. - REC*(1. - REC*(3. - REC*(15. - REC*105.*))))

1070 CONTINUE

RETURN

END

N = 23

INT = 10

DIMENSION ASTR(10), AMLOG(10), ASIGL(10), SLAM(9),

1 XMLOG(251), XMIGL(251), AREA(251,23), FREQ(251,23),

2 CYC(23), SMLOG(23), SSIGL(23), SM(23), SK(23),

3 NUC(15), ADFX(15), OFRE(10,23), GFRS(10,23),

4 CHISQ(10,23), ST(10), SLSIG(9), EFRE(10,23),

5 EQUIVALENCE (AREA,FREQ),

1 (XMLOG,SMLOG), (XMLOG,OFRE), (AREA,500), (FREQ),

2 (AREA,CHISQ), (AREA,250), (GFRS)

READ2,XMIN,XMAX,1

10 FORMAT (7F15.0)

C
FAILURES DISTRIBUTION VALUES ARE READ IN FROM

C
LOWEST TO HIGHEST STRESS AND STRESSES ARE

C
INTEGER VALUES OF SINT

READ 20

Figure B-3. 7072 Computer Program Listing for Time Dependent Strength Distribution Generation.
20 FORMAT (50H) BLANK
     READ 30, N, SINT, ASTR(J), AMLOG(J), ASIGL(J), J=1,N
30 FORMAT (15, F5.0/(3F10.6))
     READ 32, (NUM(J), J=1,N)
32 FORMAT ((S110))
     N+1 = N-1
     L = 1
     INDEX(J) = J
     DO 65 J=1,N
     DSTR = ASTR(J+1)-ASTR(J)
     DMEAN = AMLOG(J+1)-AMLOG(J)
     DSIG = ASIGL(J+1)-ASIGL(J)
     SLAM(J) = DSTR/DMEAN
     IF (DSIG) 40,50,60
40 SLSIG(J) = DSTR/DSIG
50 CONTINUE
     JP1 = J+1
     MX = DSTR/SINT
     INDEX(JP1) = L+MX
     SKIN = ASTR(1)
     XMLOG(1) = AMLOG(1)
     XSIGL(1) = ASIGL(1)
     DC = 0.1*K+1.1
     MX = L + K
     XMLOG(LX1) = XMLOG(LX-1) + SINT/SLAM(J)
     IF (DSIG) 70,50,70
60 XMCLG(LX) = XSIGL(LX-1)
     GO TO 80
70 XMCLG(LX) = XSIGL(LX-1) + SINT/SLSIG(J)
80 CONTINUE
     L = LX
85 CONTINUE
     PRINT 90
90 FORMAT (1H1, 8X,12HSTRENGTH-PSI,11X,CHLOG MEAN, 9X, 1 11HLOG STD DEV,9X,11HINTEGER (11)
     XINT = INT
     STR = SKIN
     DSIG = L-1,LX,1.1
     PRINT 100,STR,AMLOG(1),XSIGL(1), 1
100 FORMAT (15F10.6)
     STR = STR+SINT*XINT
80 CONTINUE
     HOLD = N
     AS = LX

CONVERTING LOGNORMAL FAILURE DIST. PARAMETERS AT N
STRESS LEVELS TO CUMULATIVE LOGNORMAL FAILURE
DISTRIBUTION
80 CONTINUE
Figure B-3 (Cont'd)
DO 180 J = 1, N
  XVAL = XVAL + XJINT
  Z = (XVAL - XLLOG(1)) / XSIGL(1)
  IF (Z) 120, 140, 140
120 IF (Z >= 3.5) 160, 150, 130
130 Z = -Z
  CALL PROB(Z, PROB)
  AREA(I, J) = (1.0 - PROB) / 2.0
  GO TO 180
140 IF (Z <= -3.5) 150, 170, 140
150 CALL PROB(Z, PROB)
  AREA(I, J) = PROB / 2.0 + 0.5
  GO TO 180
160 AREA(I, J) = 0.0
  GO TO 180
170 AREA(I, J) = 1.0
180 CONTINUE
INT2 = INT4
STR = 'MIN
DO 220 I = 1, N, INT2
  PRINT 160, STR, XVAL, XLLOG(I), XSIGL(I)
190 FORMAT (HC, 5X, 11HSTRENGTH = , F13.6, 5X, 9HLOG MEAN = , F6.3)
  1 SPCYCES = +8, (F6.3, 5X, 2HLOG STD DEVIATION = , F6.6)
  PRINT 200
200 FORMAT (HC, 3HDATA DELUXE 1S NATURAL, CUMULATIVE )
  1 'BDIST UP TO J, )
  PRINT 210, ( HC, AREA(I, J), J = 1, N)
210 FORMAT (HC, 13HSTRENGTH = , F13.6, 13HLOG MEAN = , F6.3)
  1 I = 0,
  STR = STR + 'SPLXINT4
220 CONTINUE
DO 222 J = 1, N, I, 1
  I = INDEX(J)
  XNUM = XNUM(J)
  DO 224 K = 1, M
    QFRECU(J, K) = XNUM * AREA(I, K)
224 CONTINUE
222 CONTINUE
  C. FREQUENCY DIST AND NORMAL DISTRIBUTION PARAMETERS
  XMIN = XJMIN - XJINT
  DO 245 J = 1, N
    XJ = J
    CYC(J) = XJ * XJINT + XJINT
    PRINT 240, J, CYC(J)
240 FORMAT (HC, 4HSTRENGTH = , F13.6, 13HLOG CYCLES = , F6.3)
  PRINT 250, ( HC, AREA(I, J), J = 1, N, I, 1)
250 FORMAT (HC, 13HSTRENGTH = , F13.6, 13HLOG CYCLES = , F6.3)
  PRINT 260, ( HC, AREA(I, J), J = 1, N, I, 1)
260 CONTINUE

Figure B-3 (Cont'd).
NM1 = N-1
DO 270 J=1,N
DO 270 J=1,N
P1 = 1+1
FREQ(I,J) = AREA(I,P1,J) - AREA(I,J)
270 CONTINUE
N = NM1
C
REAL AND NORMAL DISTRIBUTION PARAMETERS OF HISTOGRAM
DO 300 J=1,N
F1 = 0.
F1/J = 0.
U1 = SM1-SINT*G.5
DO 280 J=1,N
U1 = U1+SINT
F1 = F1+FREQ(I,J)
F1/J = F1/J+FREQ(I,J)*U1
280 CONTINUE
SMLOG(J) = F1/J/F1
SM2 = 0.
SM3 = 0.
SM4 = 0.
U1 = SM1-SINT*G.5
DO 290 J=1,N
U1 = U1+SINT
SG = (U1-SMLOG(J))*(U1-SMLOG(J))
SM2 = SM2 + SG*FREQ(I,J)
SM3 = SM3 + FREQ(I,J)*(U1-SMLOG(J))
SM4 = SM4 + SG*FREQ(I,J)
290 CONTINUE
SM2 = SM2/F1
SM3 = SM3/F1
SM4 = SM4/F1
SSIGL(J) = SQRT(SM2)
SK3(J) = SM3/SSIGL(J)*SM2
SK4(J) = SM4/(SM2*SM2)
300 CONTINUE
PRINT 20
PRINT 210
210 FORMAT (10H NUMBER, 10H HLOG CYCLES, 5X,
1 15H: MEAN, 15H STRENGTH, 15H STD, 15H DEVIATION, 10X,
2 10H: SKEWNESS, 10X, 10H KURTOSIS)
PRINT 220, (J, CYC(J), SMLOG(J), SSIGL(J), SK3(J),
1 SK4(J), J=1,N)
PRINT 270, (J, CYC(J), SMLOG(J), SSIGL(J), SK3(J), SK4(J),
320 FORMAT (11F,2E,2F24.6,2F20.8)
PRINT 325, (JU=J,J1,INDEX(J),J=1,JOLD)
375 FORMAT (1E, 1SHOBSERVED NUMBER, 9X, 1HIINTEGER (1)/
1 (1C,2I2)),
DO I=500 K=1,N

Figure B-3 (Cont'd)
Figure B-3. (Cont'd).
APPENDIX C

Listing of Fortran Computer Program CYTOFR Used To Determine Cycles To Failure Distributions

C-1. Flow Chart
C-2. Definition of Variables
C-3. Fortran Program Listing
Program to Calculate Parameter Estimates for the Normal and Log-Normal Distributions and Conduct Goodness-of-Fit Tests

MAIN PROGRAM (CYTOFR)

START

READ NDATA, DATA, AKURCY, STRLEV, RATIO

END OF DATA?

READ X(I)'s

READ CUMFRQ(I)'s

I = 1

PCAREA(I) = CUMFRQ(I) / NDATA

I > NDATA?

PRINT STRLV, RATIO, X(I)'s

Calculate mean and standard deviation of X(I)'s

SUBROUTINE MEAN

Do Chi-square goodness-of-fit test

SUBROUTINE CHISQA

Fig. C-1
Do Kolmogorov-Smirnov Goodness-of-Fit Test  
SUBROUTINE DTEST

Calculate Moment Coefficients of Skewness and Kurtosis  
SUBROUTINE ALPHA

AKURCY = .00001

I = 1

\[ NX(I) = \left[ \left( \frac{\log X(I)}{20} + \log_e(20) \right) \times 10000 \right] + .5 \]

\[ X(I) = NX(I) \]

\[ X(I) = \frac{X(I)}{10000} \]

YES  \rightarrow I > NDATA

NO  \rightarrow I = I + 1

PRINT STRELEV, RATIO, X(I)'s

Calculate mean and standard deviation of X(I)'s  
SUBROUTINE MEAN

Fig. C-1 (continued)
Do Chi-square goodness-of-fit test
SUBROUTINE CHISQA

Do Kolmogorov-Smirnov goodness-of-fit test
SUBROUTINE DTEST

Calculate moment coefficients of skewness and kurtosis
SUBROUTINE ALPHA

Fig. C-1 (continued)
Subroutine to Find Mean and Standard Deviation

SUBROUTINE MEAN

START

SIGMA = 0,
TOP2 = 0,

I = 1

SIGMA = SIGMA+X(I)
TOP2 = TOP2+(X(I)-MEAN)^2

YES

I > NDATA

NO

I = I+1

XMEAN = SIGMA/NDATA

DEV = SQRT(TOP2/NDATA-1.0)

PRINT XMEAN, DEV

RETURN

Fig. 5-1 (continued)
Subroutine to Find Area Under Standard Normal Curve

FUNCTION PROB(X)

START

IS (X-1.2) > 0

< 0

XSQ = (X)^2

PROB = (.79788455)(X) .99999974-XSQ[.16659433-XSQ(.024638310-XSQ(.0023974867))]

RETURN

IS X-2.9 > 0

< 0

XSQ = X^2

PROB = 1.0

PTERM = 1.0

FACTOR = 1.0

ODDINT = 3.0

RECSQ = \( \frac{1}{X^2} \)

PROB = 1.0- (.79788453) \exp[ (-X^2/2)(1.0-RECSQ(1.0-RECSQ(15.0-RECSQ(105))))]

RETURN

Fig. C-1 (continued)
Fig. C-1 (continued)
Subroutine to Conduct Chi-Square Goodness-of-Fit Test

SUBROUTINE CHISQA

START

CHISQR = 0.0

\[ K = 1.5 + (3.322 \log_{10}(DATA)) \]

REALK = K

XMAX = X(1)

XMIN = X(1)

I = 1

\[ X(i) > X\text{MAX} \] NO

YES

\[ X\text{MAX} = X(i) \]

\[ X(i) < X\text{MIN} \] NO

YES

I > NDATA

NO

I = I+1

Fig. C-1 (continued)
\[
\text{RANGE} = \text{XMAX} - \text{XMIN}
\]
\[
\text{DIVIDE} = \frac{1.0}{\text{AKURCY}}
\]
\[
\text{KW} = \left[ \frac{\text{RANGE} + \text{AKURCY}}{\text{REALK}} + (0.5)\text{AKURCY} \right] (\text{DIVIDE})
\]
\[
\text{RK1} = \text{KW}
\]
\[
\text{W} = \frac{\text{RK1}}{\text{DIVIDE}}
\]

PRINT XMAX, XMIN, W

\[
I = 1
\]

\[
A = I
\]
\[
B = (0.5)\text{AKURCY}
\]
\[
\text{CSV}(I) = \text{XMIN} + (I-1.0)(W)
\]
\[
\text{CEV}(I) = \text{CSV}(I) + W - \text{AKURCY}
\]
\[
\text{CUB}(I) = \text{CSV}(I) - B
\]
\[
\text{CUE}(I) = \text{CEV}(I) + B
\]

\[
I = I + 1
\]

\[
\text{NO} \quad \text{YES}
\]
\[
I > K
\]
\[
\text{CUB}(I) = \text{CEV}(I) + B
\]
\[
\text{CEV}(K) = \text{XMAX}
\]
\[
\text{CUB}(K) = \text{CEV}(K) + B
\]

Fig. C-1 (continued)
Fig. C-1 (continued)
\[ Z(I) = \frac{\text{CUB}(I) - \text{XHEAN}}{\text{DEV}} \]

\[ T = Z(I) \]

\[ \text{AREA}(I) = \frac{\text{PROB}(T)}{2.0} \]

\[ \text{BEGIN} \]

\[ I = 1 \]

\[ I = I + 1 \]

\[ \text{NO} \]

\[ \text{IS} \]

\[ I > K \]

\[ \text{YES} \]

\[ \text{REQAREA}(1) = .5 - \text{AREA}(1) \]

\[ \text{HANU} = K - 1 \]

\[ \text{END} \]

\[ I = 2 \]

\[ I = I + 1 \]

\[ I = I - 1 \]

\[ \text{YES} \]

\[ \text{IS} \]

\[ Z(I) \geq 0 \text{ AND } Z(K) \leq 0 \]

\[ \text{OR} \]

\[ Z(I) \leq 0 \text{ AND } Z(K) \geq 0 \]

\[ \text{NO} \]

\[ \text{REQAREA}(1) = |\text{AREA}(I) - \text{AREA}(K)| \]

\[ \text{REQAREA}(1) = \text{AREA}(I) + \text{AREA}(K) \]

\[ \text{NO} \]

\[ \text{IS} \]

\[ I > \text{HANU} \]

\[ \text{YES} \]

\[ \text{END} \]

**Fig. C-1 (continued)**
Algorithm:

1. \( \text{REQAREA} = \frac{1}{2} \text{AREA}(K-1) \)
2. \( M = 1 \)
3. \( \text{EXPFREQ}(N) = \text{DATA} \times \text{REQAREA}(M) \)
4. \( u(N) = \frac{\text{EXPFREQ}(M) - \text{FREQ}(M)}{\text{EXPFREQ}(M)}^2 \)
5. \( \text{CHISQR} = \text{CHISQR} + u(M) \)
6. If \( M > K \) then go to step 8, otherwise:
   - \( I = 1 \)
   - PRINT \( I, \text{CLB}(I), \text{CUB}(I), \text{EXPFREQ}(I), \text{FREQ}(I), \text{U}(I) \)
   - \( I = I + 1 \)
   - IF \( I > K \) then go to step 7, otherwise:
     - IF \( I > K \) then go to step 7, otherwise:
       - PRINT \( \text{CHISQR} \)
       - RETURN

Fig. C-1 (continued)
Subroutine to Conduct Kolmogorov-Smirnov Goodness-of-Fit Test

SUBROUTINE DTEST

START

I = 1

Z(I) = (X(I) - XMEAN) / DEV

T = -Z(I)

IF Z(I) < 0

DSTAT(I) = .5 - PCAREA

ARUNCH = 1 - PROB(T)

DSTAT = ARUNCH - PCAREA(I)

ARUNCP = PROB(T) / 2 + .5

DSTAT(I) = ARUNCP - PCAREA(I)

IF I > NDATA

PRINT DSTAT(I)'s

RETURN

i = I + 1

END

FIG. C-1 (continued)
Subroutine to Find the Moment Coefficients of Skewness and Kurtosis

SUBROUTINE ALPHA

START

TOP3 = 0
TOP4 = 0
VAR = 0

I = 1

VAR = VAR + (X(I) - XMEAN)^2
TOP3 = TOP3 + (X(I) - XMEAN)^3
TOP4 = TOP4 + (X(I) - XMEAN)^4

YES

I > NO DATA

NO

I = I + 1

SKEW = TOPT / DATA
STDEV = \sqrt{VAR / DATA}
ALPHA3 = SKEW / (STDEV)^3
TKURT = TOP4 / DATA
ALPHA4 = TKURT / (STDEV)^4

PRINT ALPHA3, ALPHA4

RETURN

Fig. C-1 (continued)
List of Definitions for Program to Fit Normal and Log-Normal Distributions to Cycles-to-Failure Data (PROGRAM CYTOFR)

Main Program:

NDATA = DATA = number of observations.
STRLV = stress level in psi.
AKURCY = accuracy to which cycles-to-failure data are known.
RATIO = stress ratio
X(I) = cycles-to-failure data
CUMFRQ(I) = cumulative frequency of each X(I); i.e., number of X's less than or equal to X(I).
PCAREA(I) = CUMFRQ(I)/NDATA

Subroutine to calculate the mean and standard deviation of the cycles-to-failure data (SUBROUTINE MEAN)

SIGMA = sum of the X(I)'s
XMEAN = average of the X(I)'s

TOP2 = \[ \sum_{i=1}^{n} (X(I) - XMEAN)^2 \]
DEV = standard deviation of the X(I)'s

Function subroutine to find the area under the normal curve (FUNCTION PROB(X)).

X = abscissa value for which corresponding area is desired.
PROB = desired area.

C-2. Definition of Variables
Subroutine for Chi-square goodness-of-fit test (SUBROUTINE CHISQA).

K = number of cells.
XMAX = largest value of cycles-to-failure.
XMIN = smallest value of cycles-to-failure.
CSV = cell starting value.
CEV = cell end value.
CLB = cell lower bound.
CUB = cell upper bound.
FREQ(J) = number of observations in Jth cell.
REQAREA(J) = expected value of Jth cell.
CHISQR = total Chi-square value.
U(I) = Chi-square value of Ith cell.

Subroutine for Kolmogorov-Smirnov test (SUBROUTINE DTEST).

Z(I) = abscissa value on standard normal curve for a given X(I).
ARUNCN = area under standard normal curve from - to Z(I).
DSTAT(I) = absolute difference between the data cumulative frequency and the hypothesized cumulative frequency.
XMEAN = average of the X(I)'s.
DEV = standard deviation of the X(I)'s.
PROB(T) = area under the standard normal curve from -T to +T.

C-2. (Cont'd).
Subroutine to calculate the moment coefficients of skewness and kurtosis (SUBROUTINE ALPHA).

\[
\begin{align*}
\text{ALPHA3} & = \text{moment coefficient of skewness}. \\
\text{ALPHA4} & = \text{moment coefficient of skewness}. \\
\text{VAR} & = \sum_{i=1}^{n} (X(I)-\bar{X})^2 \\
\text{TOP3} & = \sum_{i=1}^{n} (X(I)-\bar{X})^3 \\
\text{SKEW} & = \text{third moment of the data}. \\
\text{STDEV} & = \text{biased estimator for standard deviation}. \\
\text{TOP4} & = \sum_{i=1}^{n} (X(I)-\bar{X})^4 \\
\text{TKURT} & = \text{fourth moment of the data}.
\end{align*}
\]
PROGRAM CYTOF R (INPUT, OUTPUT, TAPE1=INPUT)
C----- PROGRAM TO FIT NORMAL AND LOG-NORMAL CURVE TO DATA AND CHECK
C----- GOODNESS OF FIT.

DIMENSION X(100), CSV(9), CEV(9), CLH(9), CUL(9), CMFRO(100),
  PCAREA(100), DSTAT(100), FPREQ(9), AREA(9), REQAREA(9), EXFREQ(9), U(9),
  2Z(100), X(100), RANK(100)

PROGRAM 1, 0 FIT
NORMAL AND LOG-NORMAL CURVE TO DATA
AND CHECK GOODNESS OF FIT.

6003 710 PRINT 1
6003
C----- NDATA=DATA=NUMBER OF OBSERVATIONS
C----- STRESS LEVEL IN PSI.
C----- X= NUMBER OF CYCLES TO FAILURE

60025 6 FORMAT(13, F5.1, F9.4, F10.1, F8.5)
60025 IF (EOF(1)) 56, 55
60030 55 READ 7, (X(I), I=1, NDATA)
60043 7 FORMAT(BF10.0)
C
C SORT X(I) TERMS IN ASCENDING ORDER.
C
60043 K=NDATA-1
60045 IF(K.LE.0) GO TO 30
60047 DO 20 I=1, K
60050 N=NDATA-1
60051 ISTOP=0
60053 DO 10 J=1, N
60054 IF(X(J).LE.X(J+1)) GO TO 10
60057 SAVE=X(J)
60060 X(J)=X(J+1)
60062 X(J+1)=SAVE
60063 ISTOP=ISTOP+1
60065 10 CONTINUE
60070 IF(ISTOP.EQ.0) GO TO 30
60071 20 CONTINUE
C
C SET CUMFRO(1) ARRAY

60073 30 DO 46 I=1, NDATA
60075 40 CUMFRO(1)=I
C----- PCAREA = F(I) OF OBSERVATIONS
60081 41 PCAREA(I) = CUMFRO(I)/DATA
60096 PRINT 405
60112 405 FORMAT (4X, 15H NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DAT
60112 1X, /)
60112 411 IF (RATIO.EQ.0) GO TO 414
60113 PRINT 412, STRESS, RATIO
60123 412 FORMAT (2X, 14H STRESS LEVEL = F6.1, 5H PSI, 16X
60123 11X, 14H RATIO = F6.3, /)
60123 GO TO 415
60124 414 PRINT 416, STRESS
60132 416 FORMAT (2X, 14H STRESS LEVEL = F6.1, 5H PSI, 16X
60132 11X, 15H RATIO = INFINITY, /)
60132 PRINT 404
60136 404 FORMAT (55X, 22H CYCLES TO FAILURE DATA/)
60136 PRINT 402, (X(I), I=1, NDATA)
60151 403 FORMAT (6(10X, F10.3))
151 PRINT 3
C
C-3. Fortran Program Listing.
C-3. (Cont'd).
SUBROUTINE MEAN (X, DATA, NDATA, XMEAN, DEV)

C----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.

0010 DIMENSION X(NDATA)
0011 SIGMA = 0.0
0012 DO B I=1, NDATA
0013 SIGMA = SIGMA + X(I)
0014 XMEAN = SIGMA/DATA
0015 TOP2 = 0.0
0016 DO 9 I=1,NDATA
0017 TOP2 = TOP2 + (X(I) - XMEAN)**2
0018 DEV = SQRT(TOP2/(DATA - 1.0))
0019 PRINT 14, XMEAN
0020 PRINT 15, DEV
0021 PRINT 16, MEAN
0022 PRINT 17, STD.DEVIATION
0023 RETURN
0024 END

C-3. (Cont’d).
FUNCTION PROB(X)
C-----THIS SUBROUTINE GIVES AREA UNDER NORMAL CURVE FROM -Z TO +Z
C-----Z VALUE GIVEN BY CALLING PROGRAM MUST BE A POSITIVE NUMBER.

      IF (X<1.9) 11,11,12
      XSQ=X*X
      PROB= 0.79788455*X*(0.99999774-XSQ*(6.16659433-XSQ*(0.02463831-XSQ
                     1.0*0.6023974867)))
      RETURN

      IF (X<2.9) 13,14,14
      XSQ=X*X
      PROB=1.0
      PTERM=1.0
      FACTOR=1.0
      ODDINT=3.0

      90  PTERM=-PTERM*XSQ/(2.0*FACTOR)
      TERN=PTERM/ODDINT
      PROB=PROB+TERM
      IF (ABS(TERM) < 0.00007) 80,90,90
      90  FACTOR = FACTOR*1.0
      ODDINT=ODDINT*2.0
      GO TO 97C

      83  PROB=0.79788455*X*PROB
      RETURN

      14  RECXSQ= 1.0 / (X*X)
      PROB= 1.0 - 0.79788455*EXP(-X*X/2.0)/X*(1.0-RECXSQ*(1.0-RECXSQ* (3.1-RECXSQ*(15.0-RECXSQ*105.0))))

      END

C-3. (Cont'd).
SUBROUTINE CHISQA (X, DATA, NDATA, PROB, AKURCY, XMEAN, DEV, W)

C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE CHI-SQUARE
C-----TEST FOR THE NORMAL OR LOG-NORMAL DISTRIBUTIONS.
DIMENSION X(NDATA), Z(NDATA), CSV(9), CEV(9), CUB(9)
1REQ AREA(9), APEA(9), EXFREQ(9), FREQ(9), U(9)

C-----TO DETERMINE THE NUMBER OF CLASS INTERVALS, K
K = 1.5 * 3.322 * ALOG10 (DATA)
REALK = K

C-----IN ORDER TO DETERMINE THE RANGE, FIND X(MAX) AND X(MIN)
XMAX = X(1)
XMIN = X(1)
DO 17 I = 1, NDATA
IF (X(I) .GT. XMAX) XMAX = X(I)
17 IF (X(I) .LT. XMIN) XMIN = X(I)
RANGE = XMAX - XMIN

C-----TO DETERMINE THE CLASS INTERVAL WIDTH, W
DIVIDE = 1.0 / AKURCY
KW = ((RANGE + AKURCY) / REALK) * 5 * AKURCY * DIVIDE
RK1 = KW
W = RK1 / DIVIDE
PRINT 62, XMAX
PRINT 63, XMIN

C-----ROUTINE TO ROUND OFF CLASS WIDTH TO SAME NUMBER OF PLACES AS THE ACC
DIVIDE = 1.0 / AKURCY
KW = ((RANGE + AKURCY) / REALK) * 5 * AKURCY * DIVIDE
RK1 = KW
W = RK1 / DIVIDE
PRINT 62, XMAX
PRINT 63, XMIN

C-----CHI-SQUARE TEST
PRINT 41
PRINT 406
DO 30 I = 1, K
Z(I) = (CUB(I) - XMEAN) / DEV
T = ABS (Z(I))
0 224 30 AREA(I) = PROB(I) / 2.0
REOAREA(I) = 0.5 - AREA(I)
MANU = K - 1
DO 24 I = 1, MANU
24 DO 25 J = 1, K
25 IF (X(I) .GE. CUB(J) .AND. X(I) .LE. CUB(J)) FREO(J) = FREO(J) + 1.0
CONTINUE

C----- (Cont'd)
C-----TO PRINT THE TABLE FOR CHI-SQUARE TEST

C-3. (Cont'd).
SUBROUTINE DTEST (PCAREA, NDATA, X, DEV, DSTAT, P, XMEAN, Z)

C-----SUBROUTINE TO CALCULATE THE KOLMOGOROV-SMIRNOV D-VALUES.

DIMENSION PCAREA (NDATA), X (NDATA), Z (NDATA), DSTAT (NDATA)

100 DO 100 I = 1, NDATA
200 Z (I) = (X (I) - XMEAN) / DEV
300 IF (Z (I)) 703, 764, 705

103 T = ABS (Z (I))

C-----ARUNCH=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR NEGATIVE Z.

002 ARUNCH = (1.0 - PROB (T)) / 2.0
003 DSTAT (I) = ARUNCH - PCAREA (I)
004 GO TO 706
005

704 DSTAT (I) = .5 - PCAREA (I)
006 GO TO 706
007

705 T = Z (I)

C-----ARUNCP=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR POSITIVE Z.

008 ARUNCP = PROB (T) / 2.0 + .500
009 DSTAT (I) = (ARUNCP - PCAREA (I))
010 CONTINUE

012 PRINT 708
013 PRINT 707, (DSTAT (I), I = 1, NDATA)

015 FORMAT (6(10X,F10.5))
018 FORMAT (1/40X,'53H D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST/41X,'52H (LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)/)

019 RETURN

C-3. (Cont'd).
SUBROUTINE ALPHA (X, NDATA, DATA, XMEAN, DEV, ALPHA3, ALPHA4)

DIMENSION X(NDATA)

C-----SUBROUTINE TO CALCULATE THE COEFFICIENTS OF SKEWNESS AND KURTOSIS
C-----CALCULATE THE THIRD MOMENT OF THE DATA (SKEWNESS)

TOP3 = 0.0
VAR = 0.0
DO 710 I = 1, NDATA
VAR = VAR + (X(I) - XMEAN)**2
710 TOP3 = TOP3 + (X(I) - XMEAN)**3
STDEV = SQRT(VAR/ndata)
SKEW = TOP3 / DATA
C-----ALPHA3 = MOMENT COEFFICIENT OF SKEWNESS.
ALPHA3 = SKEW/(STDEV)**3
C-----CALCULATE THE FOURTH MOMENT OF THE DATA (KURTOSIS).

TOP4 = 0.0
DO 711 I = 1, NDATA
711 TOP4 = TOP4 + (X(I) - XMEAN)**4
TKURT = TOP4 / DATA
C-----ALPHA4 = MOMENT COEFFICIENT OF KURTOSIS.
ALPHA4 = TKURT/(STDEV)**4
PRINT 712
PRINT 713
PRINT 714, ALPHA3, ALPHA4
712 FORMAT ('/19X,12MOMENT COEFFICIENT OF SKEWNESS (ALPHA3), 18X,39MOMENT COEFFICIENT OF KURTOSIS (ALPHA4)/')
713 FORMAT ('/21X,34FOR NORMAL DISTRIBUTION ALPHA3 = 0.23X,36FOR NORMAL DISTRIBUTION ALPHA4 = 3.0/')
714 FORMAT ('/26X,35FOR ABOVE DATA----ALPHA3 =F6.3,26X,25FOR ABOVE DATA----ALPHA4 =F6.3)
RETURN
END

C-3. (Cont'd).
APPENDIX D

Program STRENG (FORTRAN)

D-1. Flow Chart
D-2. Definition of Variable
D-3. Computer Listing
D-1. Flow Chart

START

INT = 1

READ XJMIN, XJMAX, STINT, M, N, RATIO

READ ASTR, AMLOG, ASIGL, NUN

J > N

IS

YES

READ XCYCLE

NO

J > M

IS

NO

YES

D-1. Flow Chart
ACYCLE (I) = EXP (AMLOG (I))

IS

I ≥ N

NO

YES

NMI = N - 1
INDEX = 1
L = 1

IS

J ≥ NMI

NO

YES

DSTR = ASTR(J+1) - ASTR (J)
DMEAN = AMLOG (J+1) - AMLOG (J)
DSIG = ASIGL (J+1) - ASIGL (J)
SLAM (J) = DSTR/DMEAN

DOES

DSIG = 0

NO

2

YES

SLSIG (J) = DSTR/DSIG

D-1. Flow Chart (Cont'd).
\[ J_{PI} = J+1, \quad M_X = \text{DSTR/SINT} \]
\[ \text{INDEX (JPI)} = L + M_X, \quad \text{SMIN} = \text{ASTR (1)} \]
\[ \text{XMLOG (1)} = \text{AMLOG (1)}, \quad \text{XSIGL (1)} = \text{ASIGL (1)} \]

**Flow Chart**

1. **IS**
   - K ≥ M_X
2. **YES**
   - L_X = L + X
   - XMLOG(L_X) = XMLOG(L_X-1) + SINT/SLAMM (J)
3. **DOES**
   - DSIG = 0
   - **NO**
   - **YES**
   - XSIGL(L_X) = XSIGL(L_X-1)
4. **XSIGL (L_X) = XSIGL (L_X-1)**
   - \[ XSIGL (L_X) = XSIGL (L_X-1) + \text{SINT/SLSIG (J)} \]
5. **L = L_X**

D-1. Flow Chart (Cont'd).

---

216
D-1. Flow Chart (Cont'd)
I \quad \text{XINT} = \text{INT} \\
\text{STR} = \text{SMIN} \\
\text{YES} \\
\text{DOES} \\
\text{LX} = \text{INT} \\
\text{NO} \\
\text{CYCLES (I)} = \\
\text{EXP} (\text{XMLOG} (I)) \\
\text{PRINT} \text{ STR, CYCLES (I)} \\
\text{XMLOG} (I), \text{ XSIGL (I), I} \\
\text{STR} = \\
\text{STR} + \text{SINT(XINT)} \\
\text{NOLD} = \text{N} \\
\text{N} = \text{LX}
DOES I = N

DOES J = M

$\text{Cyc}(J) = \text{ALOG (XCYCLE (J))}$

$z = (\text{Cyc}(J) - \text{XMLOG}(I))/\text{XSIGL}(I)$

IS $z - 35 \leq 0$

IS $z \geq 0$

YES

NO

YES

AREAS (I,J) = (PROB/2) + .5

6

AREA (I,J) = 1

NO

AREAS (I,J) = (1.0 - PROB)/2.0

6

D-1. Flow Chart (Cont’d).
INTZ = INT (4)  STR = SMIN

PRINT (HEADINGS)

DOES N = INTZ

PRINT STR, XMLOG(I) XSIGL(I)

PRINT (HEADING)

PRINT J, AREA (I,J)

DOES J=M

STR = STR + SINT(XINT) 4

D-1. Flow Chart (Cont'd).
D-1. Flow Chart (Cont'd).

7

DOES
J = OLD

YES

NO

I = INDEX (J)
XNUM = NUM(J)

YES

DOES
K = M

NO

OFREQ (J,K) = XNUM(_AREA(I,K) )

XJMIN = XJMIN - XJINT

PRINT
(HEADING)

XJ = J

NO

DOES
J = M

YES

PRINT I,
AREA(I,J)

NO

DOES
(INT)I=N

YES
D-1. Flow Chart (Cont'd).
D-1. Flow Chart (Cont'd).

\[ \text{SMLOG (J)} = \frac{\text{FIUI}}{\text{FI}} \]
\[ \text{SM2} = 0 \]
\[ \text{STMEAN (J)} = \text{ALOG (SMLOG (J))} \]
\[ \text{SM3} = 0 \]
\[ \text{SM4} = 0 \]
\[ \text{UI} = \text{SMIN} - \text{SINT}/2 \]

\[ \text{DOES I=I+N} \]

\[ \text{UI} = \text{UI} + \text{SINT} \]
\[ \text{FI} = \text{FI} + \text{FREQ (I,J)} \]
\[ \text{FIUI} = \text{FIUI} + (\text{FREQ (I,J)}) \cdot \text{UI} \]

\[ \text{DOES I=I+N} \]

\[ \text{UI} = \text{UI} + \text{SINT} \]
\[ \text{SQ} = (\text{UI} - \text{SMLOG(J)}) (\text{UI} - \text{SMLOG(J)}) \]
\[ \text{SM2} = \text{SM2} + \text{SQ} (\text{FREQ (I,J)}) \]
\[ \text{SM3} = \text{SM3} + \text{FREQ (I,J)} (\text{UI} - \text{SMLOG(J)}) \cdot \text{SQ} \]
\[ \text{SM4} = \text{SM4} + \text{SQ} (\text{SQ}) \cdot \text{FREQ (I,J)} \]
SM2 = SM2/FI,  SM3 = SM3/FI
SM4 = SM4/FI,  SSIGL(J) = SQRTF (SM2)
SIG3P(J) = SMLOG(J) + 3 (SSK-L(J))
SIG3M(J) = SMLOG(J) - 3 (SSK-L(J))
SK3(J) = SM3/((SSIGL(J)) (SM2)
SK4(J) = SM4/(SM2) (SM2)

PRINT HEADINGS
PRINT J, CYC(J), XCYCLE(J), SMLOG(J), SSIGL(J), SIG3M(J), SIG3P(J)
SK3(J), SK4(J)

NO

DOES J=M

YES

D-1. Flow Chart (Cont'd).
KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST

D-1. Flow Chart (Cont'd).

14

PRINT
(HEADINGS)

READ CYTOFR (I,J)

IF (CYTOFR(I,K) < XCYCLE(J))
THEN AN1 = AN1 + 1

IF (CYTOFR(I,K) < XCYCLE(J))
THEN AN1 = AN1 + 1

NO

YES

NO

YES

J=NDATA

I = NP - 1

ANUM(1) = 0
TOTAL = 0

K=NDATA

NDATA = NUM(J)

AN1 = 0

AN2(I) = AN1
ANUM(I) = ANUM(I-1) + AN1
TOTAL = TOTAL + AN1

DOES

NO

DOES

YES

I = NP - 1

NO
$z = ASTR(I) = SMLOG(J) / SSIGL(J)$

- If $Z > 0$, then
  - $Z + Z$ (YES)
  - $Z - 4.5 > 0$ (NO)
  - PROB = 0

- If $Z - 4.5 > 0$, then
  - PROB = $1/2(1 - PROB)$

- If $Z > 0$, then
  - PROB = 1

GFREQ(I,J) = PROB A

- Does I = NP - 1
  - Yes
  - TOTAL = 0
    - Yes
    - DAREA = 0
    - No
    - DAREA = ANUM(I)/TOTAL
- No

DMAX/(I,J) = ABS(DAREA - GFREQ(I,J))
AREA UNDER NORMAL CURVE BETWEEN + 8 - STD. Z DEVS.

ZSQ = Z x Z
PROB = 0.79788455(z)(0.99999774 - ZSQ x
(0.16659433 - ZSQ(0.02463831 - ZSQ x 0.0023974867))

ZSQ = Z(Z)

YES

IS
Z - 1.2 ≤ 0

YES

NO

IS
Z - 2.9 ≥ 0

YES

NO

REC = 1/(z x z)
PROB = 1 - .79788453 X
EXP (-z x z/z x z)
1 - REC(1 - REC(3 - REC(15 - REC x 105))

ZSQ = Z x Z
PTERM = 1
ODDIN = 3

D-1. Flow Chart (Cont'd).
PTERM = - PTERM * XZSQ / (2 * FACT)
TERM = PTERM / ODDIN
PROB = PROB + TERM

IF ABS(TERM) > 0.0007
   THEN
      PROB = 0.79788455 * Z * PROB
   ELSE
      FACT = FACT + 1
      ODDIN = ODDIN + 2
   END IF

18

D-1. Flow Chart (Cont'd).
XJMIN = log cycles of extrapolated minimum value of M
XJMAX = log cycles of extrapolated maximum value of M
M = number of cycles (XCYCLES) of XJMIN
N = number of distributions to be used at a particular stress ratio
RATIO = stress ratio
ASTR = bending stress level in psi
AMLOG = log of bending stress level
NUM = number of cycles to failure inputs (i.e., no. of XCYCLE)
XCYCLE = the actual number of cycle to failure
STINT = standard deviation of cycles to failure in log terms.
ACYCLE = number of cycles to failure or mean cycles

Interpolated stress values
  STR - ASTR
  CYCLE - ACYCLE
  XMLOG - AMLOG
  XSIGL - ASIGL

AREA (I,J) = cumulative area under log normal cycles to failure curve.

Parameters of Normal Stress Distribution at Specified Cycles-To-Failure.
  CYC = cycles in log value
  XCYCLE = cycles
  SMLOG = mean strength
  SSIGL = the standard deviation of the mean strength
  SIG3M = the minus three sigma limit
  SIG3P = the plus three sigma limit
  SK3 = the skewness value
  SK4 = the value of kurtosis

D-2. Definitions of Variables.
PROGRAM STRENG (INPUT,OUTPUT,TAPE) = INPUT

--- PROGRAM FINDS NORMAL STRENGTH DISTRIBUTIONS FROM LOGNORMAL ---

--- CYCLES TO FAILURE DISTRIBUTIONS ---

DIMENSION ASIR(1),AMLOG(1),ASIGL(1),SLAM(1),SIGM(23),
1 XLOG(301),XSIGL(301),AREA(301,23),FREQ(301,23),SIGP(23),
2 CYC(23),SMLOG(23),SSIGL(23),SK3(23),SK4(23),CYTOFR(10,25),
3 NUM(10),INDEX(10),OFREQ(10,23),DFREQ(10,23),MAX(10,23),AN2(10)
4 CHISQ(10,23),ST(10),SLSIG(9),EFREQ(10,23),INDEX(10),ANUM(10)
5 CYCLES(301),ACYCLE(25),STRO1G(10),STMEAN(23),XCYCLE(23)

EQUIVALENCE (AREA,FREQ),
1 (ASIGL,SSIGL), (XLOG,OFREQ), (AREA(500),EFREQ),
2 (AREA(Ch1S), (AREA(250),GFREQ)

IN1 = 1

5 READJXMIN,XMAX,S1N1,M,N,RATIO

10 FORMAT (2F10.6,F10.2,F10.5)

C FAILURE DISTRIBUTION VALUES ARE READ IN FROM

C LOWEST TO HIGHEST STRESS AND STRESSES ARE

C INTEGER VALUES OF SINT

C

20 READ 30,(ASIR(J),AMLOG(J),ASIGL(J),NUM(J),J=1,N)

30 FORMAT (F10.1,F10.6)

20 READ 36,(ACYCLE(J),J=1,M)

30 FORMAT (F10.9)

DO 34 I=1,N

34 ACYCLE(I)=EXP(AMLOG(I))

NM1 = N-1

L = 1

INDEX(1) = 1

DO 85 J=1,NM1

DSTR = ASIR(J+1)-ASIR(J)

DMEAN = AMLOG(J+1)-AMLOG(J)

DSIG = ASIGL(J+1)-ASIGL(J)

SLAM(J) = DSTR/DMEAN

DO 11 IF (DSIG) 40,50,40

11 XSIGL(LX) = XSIGL(LX-1)+SINT/DSIG(J)

10 CONTINUE

11 CONTINUE

MA = DSTR/SINT

INDEX(LP) = J+MA

MIN = ASIR(1)

XLOG(1) = AMLOG(1)

XSIGL(1) = ASIGL(1)

DO 33 K=1,MA

20 X = L + K

33 XLOG(LX) = XLOG(LX-1)*SINT/SLAM(J)

34 IF (USIG) 70,50,70

40 XSIGL(LX) = XSIGL(LX-1)*SINT/SLSIG(J)

50 CONTINUE

50 CONTINUE

L = LX

70 XSIGL(LX) = XSIGL(LX-1)*SINT/SLSIG(J)

80 CONTINUE

80 CONTINUE

PRINT 87

163 PRINT 87

87 F0=INAT(H)//44X,44HLOGNORMAL STRENGTH DISTRIBUTIONS FROM LOGNORMAL

2X43X43X CYCLES TO FAILURE DISTRIBUTIONS,///

10 CONTINUE

IF (RATIO,EQ.,.0) GO TO 91

91 CONTINUE

PRINT 89,RA!10
CONVERTING LOGNORMAL FAILURE DISTRIBUTION

STRESS LEVELS TO CUMULATIVE LOGNORMAL FAILURE DISTRIBUTIONS

(Cont'd)
D-3. (Cont'd)
D-3 (Cont'd)
01 056 385 PROBA = 1.0
00 069 370 GFREQ(I,J) = PROBA
01 064 370 IF (TOTAL .EQ. 0.0) GO TO 375
01 065 370 DAREA = ANUM(I)/TOTAL
01 067 370 GO TO 380
00 067 375 DAREA = 0.0
01 070 380 DMAX(I,J) = ABS(DAREA - GFREQ(I,J))
01 101 480 CONTINUE
00 103 490 PRINT 490, XCYLE(J), TOTAL, (AN2(I) + DMAX(I,J), I = 2, NP)
01 125 490 FORMAT (6X,F8.0,21H CYCLES TOTAL N = F3.0/(19(F9.0,F6.3)))
01 125 490 PRINT 322
00 131 500 CONTINUE
01 134 500 GO TO 5
01 134 510 STOP
00 136  END
SUBROUTINE NORMAL(Z,PROB)
C PROB = THE AREA UNDER NORMAL DISTRIBUTION BETWEEN
C PLUS AND MINUS Z STANDARD DEVIATIONS
00.05 IF(Z-1.2) 1000,1000,1010
00.07 1000 ZSQ = Z*Z
00.10 IF(Z=2.9) 1020,1060,1060
00.20 IF(Z=2.0) 1020,1060,1060
00.25 ZSQ = Z*Z
00.27 FACT = 1.0
00.28 PTERM = 1.0
00.29 ODDIN = 3.0
00.31 IF (ABS(TERM) > 0.00007) 1050,1040,1040
00.35 TERM = PTERM/ODDIN
00.36 PROB = PROB + TERM
00.37 IF (ABS(TERM) > 0.00007) 1050,1040,1040
00.40 FACT = FACT + 1.0
00.41 ODDIN = ODDIN + 2.0
00.43 PROB = 1.79788455*PROB
00.45 GO TO 1070
00.47 1050 PROB = 1.79788455*PROB
00.51 GO TO 1070
00.52 1060 REC = 1./Z^2
00.54 PROB = 1.0*79788455*EXP(-Z^2/2.0)/Z^2
00.58 1 (I. - REC^8(1. - REC^8(3. - REC^8(15. - REC^8(105.))))
00.62 1070 CONTINUE
00.64 1070 RETURN
00.67 END

D-3. (Cont'd)
APPENDIX E

Listing of Short PDP-8 Programs

E-1. Program BAR I
E-2. Program BAR II
E-3. Program ROTO
E-4. Program for Least Squares Estimator for Chapter IV Data
E-5. Program for Least Square Estimator for Chapter V Data
E-6. Program for Mean Stress Per von Mises-Hencky Ellipse.
E-7. Program for Slope of Best Fit Equation
01.10 A "SH" S, "SA" SA; S SH = FSQT(3) * S
02.10 S C = FSQT(SA^2 + SH + 2); T %8.1,"C" C, "SH" SH, !
02.20 GT1.1
*
*

where:  
SH = shear stress
SA = alternating stress
SM = mean stress
C = resultant stress vector magnitude Sr

E-1. Listing of PDP-8 Program BAR I.
where: SH = shear stress
SA = alternating stress
SM = mean stress
C = resultant stress vector magnitude $S_r$

E-2. Listing of PDP-8 Program Bar II.
Where \( R \) = Stress Ratio

\[ S = \text{Vertical Strength Distribution's Upper and Lower Three Sigma Limits.} \]

\[ SR = \text{Transformed Strength Distribution's Upper and Lower Three Sigma Limits.} \]

E-3. Listing of PDP-8 Program ROTO.
C-FOCAL, 1969

0.10 A "A" A, "B" B, "D" D, "E" E, "N" N
0.20 S XA=A/E
01.21 S XB=B/N
01.23 S YA=D + 2-A + 2
01.24 S YB=D + 2-B + 2

02.10 S E=XA + 2*YA+XB + 2*YB
02.11 S J=XA + 4+XB + 4
02.13 S K=E/J

0.10 S SU=FSQT(D + 2/K); T "SU" SU!

Definition of Variables

A = S\text{a}_1 = y_1
B = S\text{a}_2 = y_2
D = S_n
ya = S\text{m}_1 = x_1
yb = S\text{m}_2 = x_2
E = x_1
N = x_2

E-4. PDP-8 Program for Least Squares Estimator of the Ultimate Strength for Fatigue Data of Chapter IV.
Definition of Variables

\[ A = S_{a_1} = y_1 \]
\[ B = S_{a_2} = y_2 \]
\[ C = S_{a_3} = y_3 \]
\[ x_A = S_{a_1} = x_1 \]
\[ x_B = S_{a_2} = x_2 \]
\[ x_C = S_{a_3} = x_3 \]

E-5. PDP-8 Program for Least Squares Estimator of the Ultimate Strength for Fatigue Data (LSEFD) of Chapter V.
E-6. Listing of PDP-8 Program Which Calculated Mean Stress Specified by the Von Mises-Hencky Ellipse
C-8K MODV 11-219

02.05 A "NO. OF DATA POINTS ",ND,1,1!
02.10 T " X - AXIS    Y - AXIS",1
02.20 S XS=0 ,
02.21 S XQ=0
02.22 S YS=0
02.23 S YQ=0
02.24 S XY=0
02.40 FOR I=1,1,ND; DO 3,0
02.49 S D=(ND*XQ-XS*XS)
02.50 S A0=(YS*XQ-XS*XY)/D
02.60 S Al=(ND*XY-XS*YS)/D
02.70 S DNI=FS2?((D*ND*XY-XS*YS))
02.75 S R=(ND*XY-XS*YS)/DNI
02.80 T %B.05 I, "SLOPE ",A1," Y INTERCEPT ",AO,1,1!
02.90 T %%,04 "CORRELATION COEFFICIENT ",R,1
02.95 Q

03.10 A " ",X(I)," ",Y(I),1
03.15 S XS=XS+X(I)
03.18 S XQ=XQ+X(I)*X(I)
03.20 S YS=YS+Y(I)
03.24 S YQ=YQ+Y(I)*Y(I)
03.26 S XY=XY+X(I)*Y(I)
*GO

E-7. Listing of PDP-8 Program which Calculates Slope of Best Fit Equation (10.5).
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