PARAMETRIC FIT TO ELECTRON TRANSPORT PROPERTIES *

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Empirical formulations of electron transport properties are presented. Experimental data acquired by several groups over the past few years have been gathered and organized. Parametric fits have been made to these data so that the following properties of electron transport can be represented by simple functions of atomic number and energy: transmitted electron number, transmitted electron spectra, range, backscatter coefficient, angular distribution, energy deposition, bremsstrahlung intensity and energy spectra, bremsstrahlung angular distribution and photon attenuation. Though most of the data used to derive the parametric formulas were measured in experiments, some uses have been made of results generated using well-developed Monte Carlo techniques. The formulas cover the energy ranges of interest in space shielding problems (0.1 - 10.0 MeV). These formulas were developed so they could be included in an engineering handbook addressed at the problems of electron radiation in space and its effects. The intent is to provide a simple means of calculating the radiation expected aboard a spacecraft, both on its external surface and behind arbitrary amounts of effective shielding. To satisfy the requirement that the formulas be simple, mathematical rigor based on physical interaction was replaced by convenient fits to thick target data. The formulas are as accurate as these data in most cases and can be easily evaluated without a computer. Procedures for using these formulas in practical situations are suggested.

I. INTRODUCTION

In practical problems in which it is necessary to make a quick determination of the radiation dose delivered to a point partially shielded from a radiation source, it is very useful to have an inexpensive means of obtaining reasonably accurate results. In particular, in the space environment it is often necessary to establish the dose level due to both electrons and the associated secondary photons at points inside a spacecraft. Just as frequently the initial estimate becomes the burden of an engineer or scientist whose professional interest is related to the item being placed at the point of interest and who may have only a passing familiarity with the properties of electron transport and bremsstrahlung production. Simple

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methods of calculating electron transport properties and bremsstrahlung production are presented in this paper. Parametric representations of both calculated and measured data are presented.

The basic guideline imposed in finding these parametric formulas is simplicity. Many comprehensive discussions of the details of electron transport exist; in particular the work of Zerby and Keller,[1] and the earlier work of Birkhoff[2] summarize the physics and some of the mathematics that describe these phenomena. However, for the purpose stated above, these works are too detailed and demanding if a dose level is the only goal of the investigation. As a means of implementing the simplicity guideline, we have made parametric fits to shielding data. For example, to fit the bremsstrahlung production property we have sought data which correlate photon output directly to the incident electron. The processes of electrons slowing down, range straggling, backscattering and photon production, and to some extent self-shielding, are gathered into one process. That is, given an electron of a certain energy, what is the probability of observing a photon of another energy at some point around a thick piece of material ? By just fitting the thick target bremsstrahlung data all the mathematical complications inherent in a rigorous analysis incorporating all the intermediate processes are avoided. With this approach we have obtained fits either from the literature of through our own efforts for a number of properties of interest.

II. ELECTRON TRANSPORT

In the following section, formulas which describe the property specified are displayed. As pointed out in the introduction, many of these formulas are the result of the efforts of other workers. We have selected these that are presented because they best represent the experimental data we have been able to compre them with.

1. Range

There are essentially two commonly used definitions of ranges, one is the extrapolated range, the other is the continuous slowing down approximation (csda) range. The extrapolated range is more easily determined from experimental data. The following expression is due essentially to Katz and Penfold[3] with some modifications by Ebert et al.,[4]

$$R_{ex}(E,Z) = 0.565\left(\frac{125}{Z+112}\right)E - 0.423\left(\frac{175}{Z+162}\right) \qquad E > 2.5\text{ MeV} \tag{1}$$

$$= 0.38\,E^{A}\ \text{g/cm}^2 \qquad E < 2.5\text{ MeV}$$

where

$$A = (1.265 - 0.095\,\ell n\,E)\,(1.7 - .273\,\ell n\,Z)$$

Here E is the electron energy in MeV and Z is the atomic number.

The continuous slowing down approximation range is more easily calculated than is the extrapolated range and has come into rather wide use because of the very convenient tabulation of this quantity by Berger and Seltzer.[5] From the data of Ebert et al.[4] and this tabulation, we have derived the following relationship between the extrapolated and csda ranges

$$(R_{ex}/R_o) = 1.21 - 0.208\,\ell n\,Z + 0.485E \tag{2}$$

2. Transmitted Number Fraction

The number of electrons penetrating to a given depth within a material is defined as the transmitted number fraction. Ebert et al.[4] have measured this quantity and have suggested the following parametric representation

$$T(E,Z) = \exp -\alpha\left(\frac{t}{R_{ex}}\right)^{\beta} \tag{3}$$

$$\alpha = (1 - 1/\beta)^{1-\beta}$$

$$\beta = \left[\frac{387E}{Z\,(1 + 7.5 \times 10^{-5}\, Z E^2)}\right]^{1/4}$$

the quantity, t, is the penetration depth in the same units as the extrapolated range.

3. Transmitted Energy Spectra

The energy spectra of electrons penetrating to a depth (t/R_o), that is, to some fraction of the continuous slowing down range, has been fit. The experimental data measured by Lonergan et al.,[6] Costello et al.[7] and Rester et al.[8] were used as the base from which the following parametric representation was derived:

$$S(E, t, Z) = \left\{ \exp\left[- (E - E_p)^2 / W^2 \right] + CE\left[1 + \exp\left((E - E_p)/W\right)\right]^{-1} \right\} N^{-1} \qquad (4)$$

where

$$N = W\sqrt{\pi} + 1/2C\, W^2$$

$$W = 0.05\,\sqrt{Z}\,(t/R_o)$$

$$C = (t/R_o)^2\,\sqrt{Z}$$

4. Backscatter Coefficient

The backscatter coefficient is defined as the fraction of electrons incident on an infinitely thick target which are scattered in the backward direction. The following expression was derived by Tabata[9] from data he measured and is valid for $Z > 6$.

$$\eta(E, Z) = 1.28 \exp\left[- 11.9\, Z^{0.65} \times (1 + 0.103\, Z^{0.37}\, E_o^{0.65})\right] \qquad (5)$$

5. Angular Distribution

The angular distribution of electrons penetrating a slab after striking the slab normally has been fit in two regions. If the slab thickness is less than approximately 1/3 the extrapolated range, a gaussian distribution fits the data reasonably well:

$$\phi(E, Z) = \frac{\sqrt{\pi}}{K} \exp(- K^2 \theta^2) \qquad (6)$$

where

$$K^2 = E^2 / [0.121\, t\, Z^{0.88}\, \ln (t/0.00105)]$$

However, if the slab is thicker than 1/3 the extrapolated range, the distribution merges into a thickness-independent form given by

$$\phi = (0.717 + \cos\theta) \cos\theta \qquad (7)$$

where θ refers to the polar angle relative to the normal to the plane from which the electrons are emerging. It is important to note that θ is not referenced to the incident electron direction. Experimental data[7, 8] indicated that for thick targets $(t/R_{ex} > 1/3)$ the angular distribution is independent of the incident electron direction.

6. Energy Deposition

The energy deposited by electrons as they penetrate a material has been calculated for a number of cases at the Marshall Space Flight Center (MSFC). ETRAN,[10] a Monte Carlo electron transport code developed by Berger and Seltzer at NBS, was used to make these calculations. Watts and Burrell[11] of the MSFC have made parametric fits to these results for two cases. The first is for a normally incident monodirectional beam, and is given by

$$\rho_{\perp}(E, X) = \exp\left(\sum_{c=1}^{4} A_i\, X^{i-1}\right) \qquad (8)$$

where

$$A_1 = 0.913 \exp(0.963\, E) + 0.021\, E + 0.215$$

$$A_2 = 5.0 - 0.49\, E$$

$$A_3 = 57.6\,(E - 5.0) / (E + 30)$$

$$A_4 = - 1.6\, E^{0.837}$$

The second case is for an isotropic incident beam,

$$\rho_{iso}(E, X) = \exp\left(\sum_{1}^{5} A_i\, X^{i-1}\right) \qquad (9)$$

where

$$A_1 = 0.52 + 0.098\,E^{-1.47}$$
$$A_2 = \exp(-0.82\,E) - 1.0$$
$$A_3 = 2.5\,[\exp(-1.022\,E) + 1.0]$$
$$A_4 = 3.25\,\exp(-0.323\,E) + 5.8$$
$$A_5 = -15.44 + 1.55\,E - 0.0786\,E^2$$

where X is the depth of penetration.

III. PARAMETERIZATION OF THICK TARGET BREMSSTRAHLUNG DATA

When electrons slow down, they emit bremsstrahlung radiation. In applications where the electrons are stopped, this bremsstrahlung is the only radiation which penetrates through the material, and hence is important in many space applications.

The calculation of the bremsstrahlung produced in thick targets using first principles is rather complicated due to the energy loss, straggling and angular spread of the incident electrons as they penetrate through the material. Typically, rather large computer codes which either treat the problem using Monte Carlo methods[10] or by subdividing the target into many thin sections[12] are used to calculate the bremsstrahlung intensity. These calculations are quite accurate but require considerable knowledge on the part of the person running the code and also a considerable amount of computer time.

The purpose of the present effort is to provide a fairly simple analytical expression for the bremsstrahlung radiation by parametizing the available thick target bremsstrahlung data. In the past these data have been used mainly to check the accuracy of the computer codes. We will use the data itself to try to go directly to the types of expressions that will be useful to people working in the field of space physics

1. Method of Parameterization

a) Approximation for Intensity Spectrum

Most of the thick target bremsstrahlung data have been obtained with the electron beam normally incident on the target,[6, 8, 13] and we will mainly be concerned with fitting these data. Electrons encountered in space shielding situations, however, typically do not all impinge normally on the surface of the spacecraft, so one must know the bremsstrahlung dependence on the angle of incidence of the electrons. There is a limited amount of thick target bremsstrahlung data available for electrons with non-normal incidence.[14] Examples of these data are shown in Fig. 1. Here the integrated, and partially integrated intensities for 1 and 2 MeV electrons are plotted vs. the angle between the emitted radiation and the electron beam. It is fairly obvious from these data that the γ intensity is a maximum in the direction of the incident electrons. For the Al data, it is obvious that there is some symmetry about the beam direction. We will assume that the bremsstrahlung radiation can be written as a function which is symmetric about the electron direction multiplied by an attenuation factor to account for absorption in the target

$$I(E, K, \theta, \psi) = I^{o}(\theta, K, E)\ \times \tag{10}$$

$$\times \begin{cases} e^{-\mu(t - FR\cos\psi)/\cos(\psi-\theta)} : & (\psi-\theta) < \pi/2 \\ e^{-\mu FR\cos\psi / \left|\cos(\psi-\theta)\right|} : & (\psi-\theta) > \pi/2 \end{cases}$$

where the different attenuation factors correspond to forward going and backscattered radiation. In this equation, E is the energy of the electron, K is the energy of the photon, θ is the angle of the γ ray with respect to the beam direction, ψ is the angle between the beam direction and the normal to the target, t is the target thickness, μ is the mass absorption coefficient for the material, and FR is some fraction of the csda electron range.

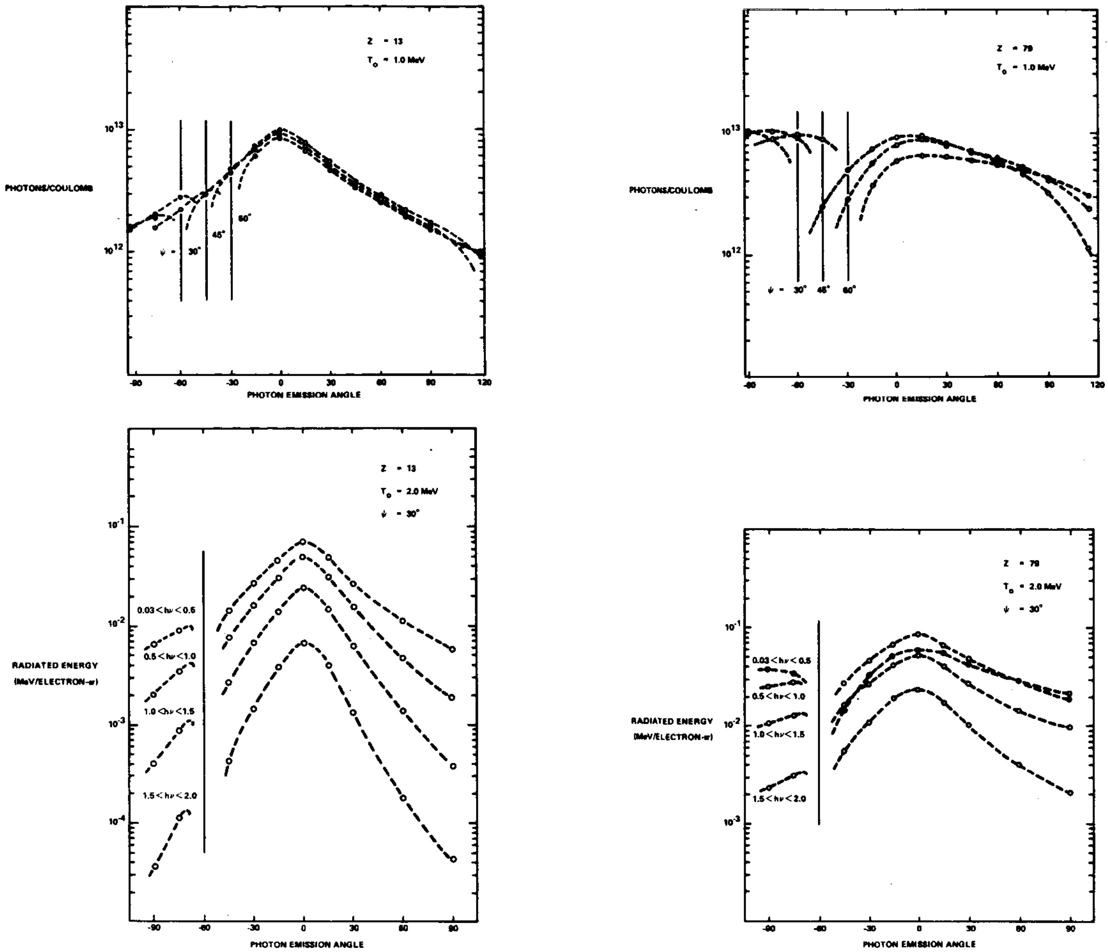

Fig. 1. Angular dependence of bremsstrahlung radiation for 1 and 2 MeV electrons incident at 30° to the normal of thick Al and Au targets.

The function $I^{o}(\theta, K, E)$ which was obtained from the data shown in the previous figure, is shown in Fig. 2. A value of F = 0.3 was used to make the curves symmetric. The curves through the points in these figures are symmetric about 0^{o} which shows that the assumption of a source term symmetric about the direction of the electrons seems to be justified, except for the low energy Au data with the γ ray emerging at angles almost parallel to the target.

2. Fits to Individual Bremsstrahlung Spectra

The energy intensity spectra of photons emitted from a thick target at an angle θ with respect to the beam were fit by the method of least squares with a 3 parameter function of the form

$$I^{o}(E, K, \theta) = A(E, \theta) e^{-K/\left(E\, B(E, \theta)\right)_x} \quad (11)$$

$$x\,(1 - K/E)^{C(E, \theta)}$$

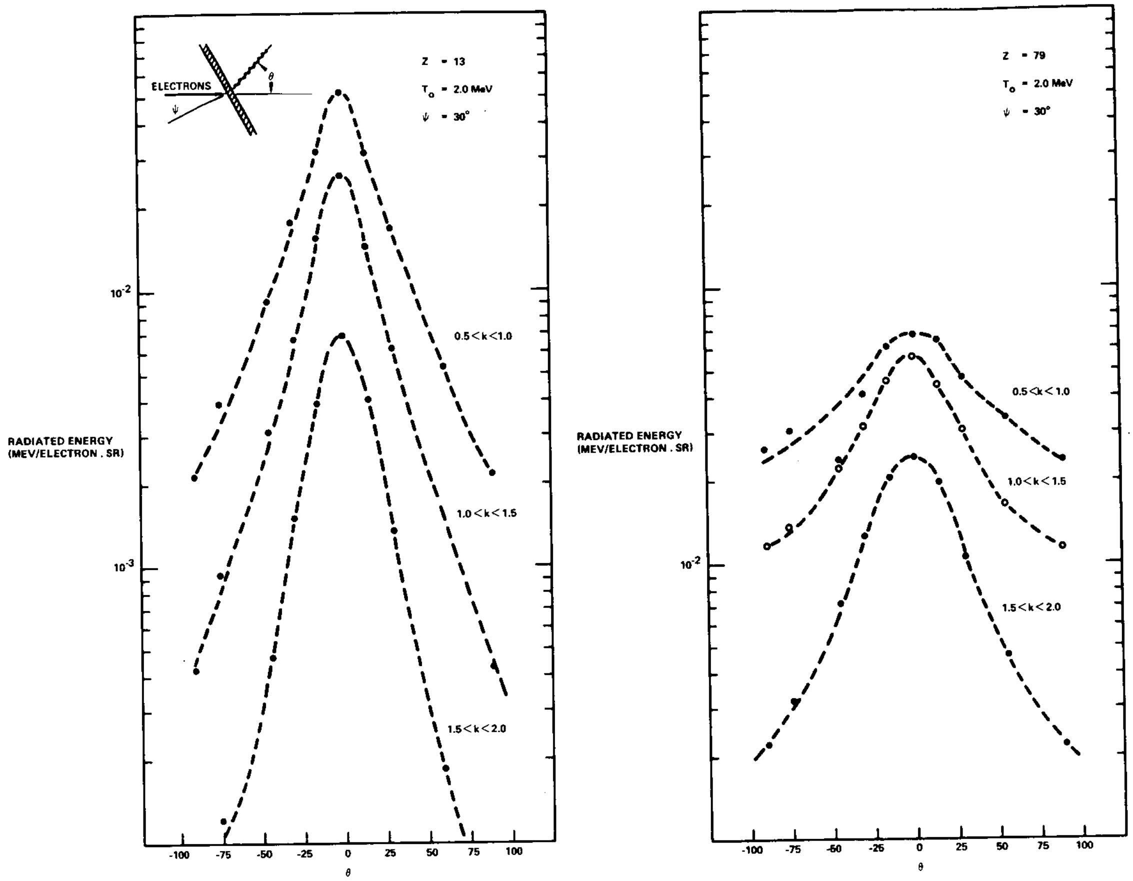

Fig. 2. Angular dependence of assumed bremsstrahlung "source" term for 2 MeV electrons incident at 30° to the normal of thick Al and Au targets.

where the parameters $A(E,\theta)$, $B(E,\theta)$ and $C(E,\theta)$ were adjusted to give a best fit for each individual spectrum. This functional form was chosen because the bremsstrahlung intensity for the elements and energies considered appear to be exponentially decreasing over much of the photon energy range and falling off faster than exponential near the end point. Samples of the fits to the LTV bremsstrahlung data[8] obtained using this 3 parameter function are shown in Fig. 3. It can be seen that the fits are all reasonable; with individual points differing from the fit by less than 10%.

The parameters $A(E,\theta)$, $B(E,\theta)$ and $C(E,\theta)$ were examined as functions of the photon angle and parameterized as

$$A(E,\theta) = a(E)\, e^{-(\theta/\psi_0(E))^2} + b(E) \exp\left[2\, e^{-(\theta/\psi_1(E))^2}\right]$$

$$B(E,\theta) = c(E)\, e^{-(\theta/\psi_2(E))} + d(E)\, e\left[-(\theta/\psi_3(E))^2\right] \qquad (12)$$

$$C(E,\theta) = 1 - (.7 - .5\, e^{-(E/E_0)^2})\, e^{-(\theta/\theta_0)^2}$$

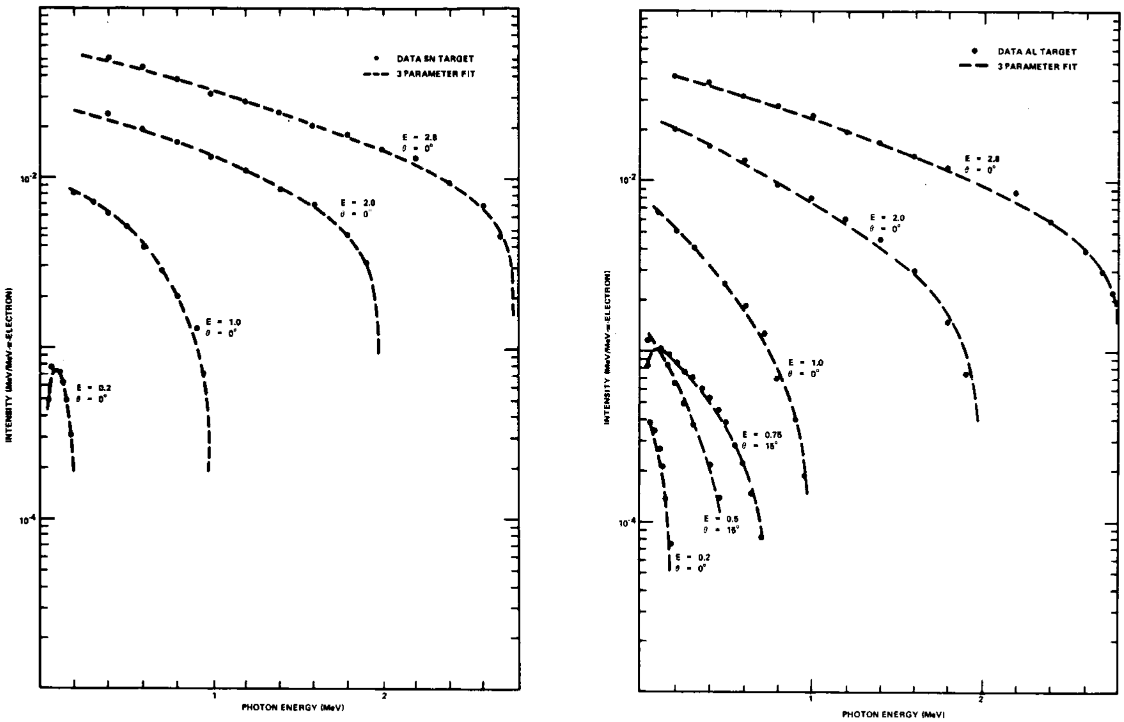

Fig. 3. Sample 3 parameter fits to thick target bremsstrahlung data on Al and Sn at .2, 1, 3, and 2.8 MeV at 0^{o}. The functional form of the curve is $F = Ae^{-K/BE}(1 - K/E)^{C}$, where A, B, and C are parameters, K is the photon energy, and E is the electron energy.

The entire distributions over both photon energy and angle were fit using the above 12 parameter function; however, only the first 8 parameters were varied for the fit.

We have at present parameterized the photon energy and angular distributions for 0.5, 0.75, 1, 2, and 2.8 MeV electrons incident on Al targets and 1, 2, 2.8, 4, and 8 MeV electrons incident on Sn targets. Samples of the fits to the photon energy and angular distributions are shown in Fig. 4. Again the fits are reasonable; with the maximum deviations of the data from the calculated curves being about 15%. It should be mentioned that the accuracy ascribed to the data by its authors is about 10% over most of the range, and 20 - 30% at the end points, so it would seem that the fits are quite adequate.

The next step in the parameterization process is to represent the 8 functions a(E), b(E), c(E), d(E), $\psi_0(E)$, $\psi_1(E)$, $\psi_2(E)$, and $\psi_3(E)$ as functions of energy for each element. Data are only available at a few values of (E), however, and the values of the 8 parameters obtained so far do not seem to follow any simple (eg. linear or exponential) pattern. The values of the parameters are given for Al and Sn in Table 1. We will generate more data at intermediate energies using the Monte Carlo calculation ETRAN-15. With this additional data we should be able to parameterize the bremsstrahlung intensity with respect to angle, photon energy and incident electron energy.

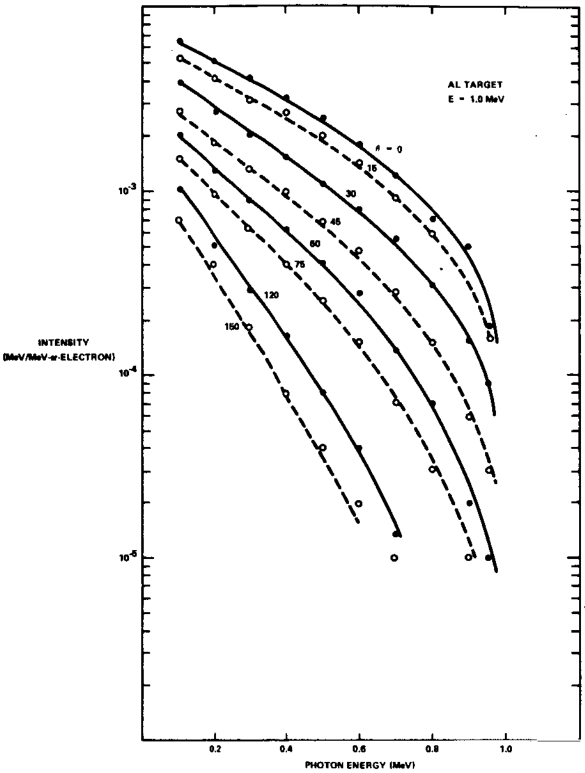

Fig. 4. Sample 8 parameter fits to photon energy and angular distribution for 1 MeV electrons on Al.

Table 1. Table of Parameters for Al and Sn Normal Electron Incidence

E (MeV)	a $\times 10^{+3}$	ψ_o (deg)	b $\times 10^{3}$	ψ_1 (deg)	c	ψ_2 (deg)	d	ψ_3 (deg)
				Al				
0.5	0.50	35.	0.17	250.	0.70	120	0.10	60
1.0	4.86	30.7	0.48	223.	0.50	120	0.088	56
2.0	14.0	18.1	1.93	85.4	0.40	140	0.075	25
2.8	23.5	10.4	4.30	57.4	0.44	100	0.093	22
				Sn				
1.0	4.10	26.3	1.26	360.	0.53	350	0.40	66
2.0	19.2	26.3	2.43	320.	0.43	310	0.27	60
2.8	27.6	14.7	6.74	118.	0.44	230	0.12	24
4.0	63.7	12.7	7.96	136.	0.44	350	0.39	29
8.0	274.0	6.5	14.3	92.4	0.38	350	0.59	34

3. Integration Over Electron Flux

Because of the interest in space shielding calculations, we have obtained energy and angular distributions for isotropic electron fluxes. Figure 5 shows the angular distributions obtained for an isotropic 1 MeV electron source incident on unit areas of Al and Sn targets whose thicknesses are the range of 1 MeV electrons. These angular distributions are somewhat simpler than the angular distributions produced by normally incident electrons.

As was mentioned earlier, the basic bremsstrahlung data have been fit using a source term and a simple attenuation factor. If one integrates only the source term $I_0(E, K, \theta)$, one obtains angular distributions as shown in Fig. 6. These angular distributions are quite regular and can be fit with a function of the form

$$I^{o}_{iso}(\theta_\gamma, E, K) = \exp\left[A_{iso} + B_{iso}\cos\theta + C_{iso}\cos(2\theta) \right] \tag{13}$$

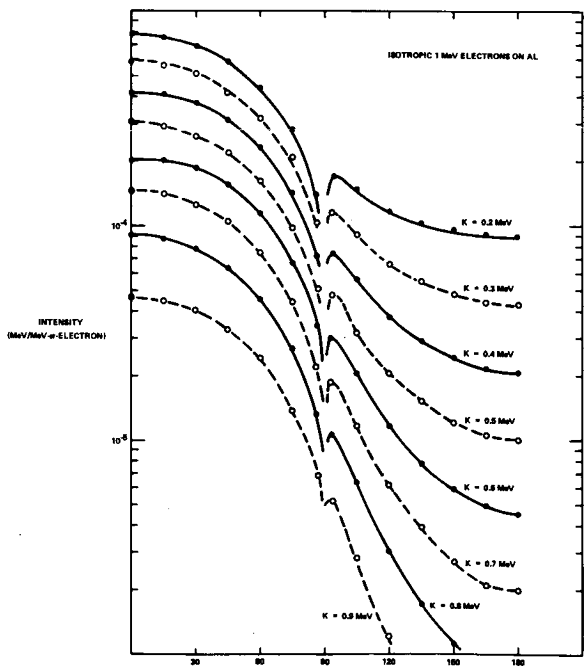

Fig. 5. Photon angular distributions for an isotropic electron source of 1 MeV electrons on one side of an Al target.

The curves through the points in Fig. 6 correspond to fits using this functional form. The functions $A_{iso}(E, K)$, $B_{iso}(E, K)$, and $C_{iso}(E, K)$ are smoothly varying functions of both K and E, but as yet, have not been fit. The values of $A(E, K)$ for electrons on Sn are shown in Fig. 7.

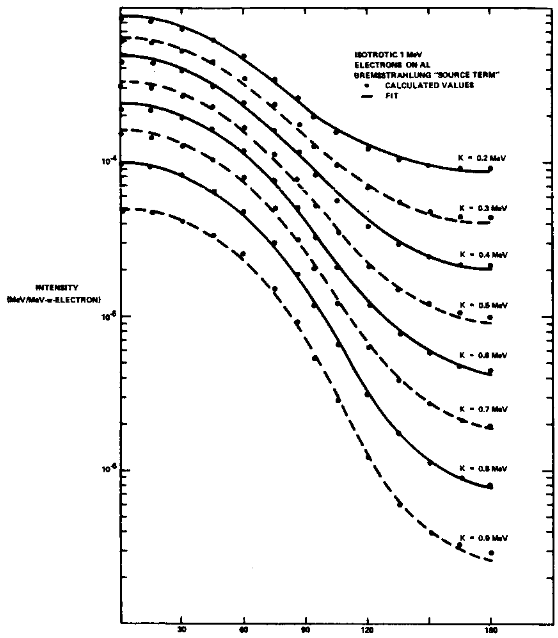

Fig. 6. Angular distribution of "unattenuated" photons for an isotropic source of 1 MeV electrons on one side of an Al target. The solid curves are least square fits using a functional form $I^{o}_{iso}(\theta, K, E,) = \exp\left[A + B\cos\theta + C\cos(2\theta)\right.$.

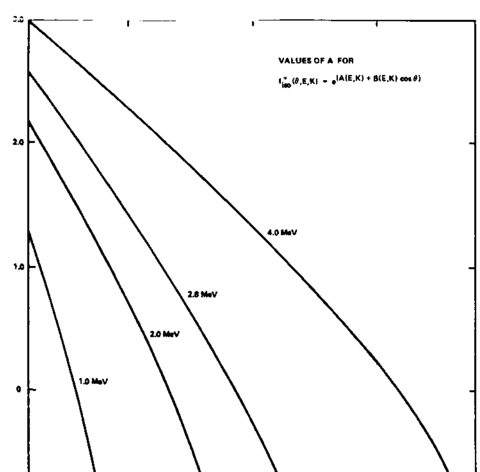

Fig. 7. Energy dependence of the parameter A used in fitting the angular distributions for the unattenuated term for an isotropic electron source on Sn at various electron energies.

4. Summary of Parametric Fits to Bremsstrahlung Data

We have obtained a parameterization of the bremsstrahlung radiation for electrons normally incident on thick targets of Al and Sn for electron energies ranging from 0.5 to 2.8 MeV for Al and 1 to 8 MeV for Sn. These bremsstrahlung distributions have been integrated over the angles of the electron incidence to obtain the photon angular distributions for isotropic electron sources. The parameterization of the radiation produced by isotropic electron sources is partially completed. These parameterizations will provide the engineer or scientist a fairly simple means of estimating the bremsstrahlung intensity inside a spacecraft for the various electron environments encountered in space.

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