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FOR REFRACTION IN THE IONOSPHERE

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## GEOS Satellite Tracking Corrections for Refraction in the Ionosphere

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### ABSTRACT

This document compares the analytic formulations at different elevation angles and at a frequency of 2-GHz for the ionospheric refraction corrections used on the GEOS satellite tracking data. The formulas and ray-trace results for elevations greater than  $10^\circ$ , where most satellite tracking is done, differ in  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  by less than 0.4 millidegrees (1.4 arc-seconds), 12 meters, and 12 cm/sec, respectively. In comparison to most operational requirements, this is insignificant. However, for the GEOS Observation Systems Intercomparison Investigation, these differences are equivalent in size to observed differences in system biases for some of the best electronic geodetic tracking systems and are probably contributing to the observed biases.

For all the GEOS ionospheric refraction correction formulas examined in this report, the range rate refraction correction for  $\Delta \dot{R}$  is the time derivative of the formula for  $\Delta R$ , as might be expected. It is much easier to derive  $\Delta \dot{R}$  from  $\Delta R$  in this manner, rather than independently from basic principles as was usually done in the earlier documents.

The ray-trace results and most of the more detailed analytic correction formulas show that the ionospheric refraction correction for range rate on an overhead pass is a maximum for elevation angles between  $15^\circ$  and  $30^\circ$  and falls off rapidly for both higher and lower elevation angles, contrary to the effect of the troposphere and to some reports in the literature.

## GLOSSARY

E	measured elevation
$E_c$	elevation corrected for refraction
R	measured range
$R_c$	range corrected for refraction
$\dot{R}$	measured range rate
$\dot{R}_c$	range rate corrected for refraction
$\dot{E}$	rate of change of elevation (refer to Appendix B)
f	radio frequency
$h_t$	height of the satellite = $\frac{4}{3} \times 10^6$ meters
$h_m$	height of maximum electron density
H	ionospheric scale height for the Chapman model ionosphere
$H_d$	ionospheric scale height for the DC program
$N_e$	electron density in electrons/m <sup>3</sup>
$N_T$	total electron content in a vertical column of area 1 m <sup>2</sup> , $N_T = \int N_e dh$
$N_{em}$	electron density in electrons/m <sup>3</sup> at $h_m$
$N_i$	refractivity at height $h_i$
$N_{im}$	refractivity at height $h_m$ , ( $N_{im} < 0$ for the ionosphere)
$\mu$	refractive index
e or $e^1$	Napierian base = 2.71828183
$M_o$	$N_{im} He^1$
$\Delta E$	elevation correction = $E - E_c$
$\Delta R$	range correction = $R - R_c$
$\Delta \dot{R}$	range rate correction = $\dot{R} - \dot{R}_c$
$R_e$	earth radius = 6,378,166 meters
$H_g$	ionospheric scale height factor for GPRO arising from parabolic model ionosphere. Nominally GPRO uses $H_g = 50$ km
$y_m$	thickness parameter for F layer bipolar profile in Bent model ionosphere
k	$k = 1/H_b$ = decay constant for topside exponential profile in Bent model ionosphere
$R_h$	$R_h = h_m - y_m$ in Bent model ionosphere

# GEOS SATELLITE TRACKING CORRECTIONS FOR REFRACTION IN THE IONOSPHERE

## INTRODUCTION

The refraction correction formulas for correcting the elevation ( $E$ ), range ( $R$ ), and range rate ( $\dot{R}$ ) tracking data from the GEOS geodetic tracking systems are not standardized. Different refraction correction formulas are applied to the same data by different users. The different formulas arise from the use of different mathematical models of the atmosphere or from different approximations to the same model.

This document presents some of the different sets of ionospheric refraction correction formulas now being used on GEOS data and compares the magnitude, at a frequency of 2 GHz, of the corrections to  $E$ ,  $R$ , and  $\dot{R}$  for different elevation angles. The 2-GHz frequency is close to the frequencies used by the GRARR, the Apollo USB, and various other S-band radar tracking systems. This is a continuation of a study of the comparison of atmospheric refraction correction formulas for GEOS data, in support of the GEOS Observation Systems Intercomparison Investigation. The tropospheric refraction correction comparisons are given in reference 1. Not all of the tracking systems represented in reference 1 are included here because the ionospheric refraction correction is not always applied as an analytic correction formula. In particular, the Army Secor ranging system and the Navy TRANET Doppler system use multiple frequency refraction correction techniques. For C-band radar tracking data, at a frequency of 5.7 GHz, the ionospheric correction is very small and is therefore often neglected or else lumped together with tropospheric parameters. For example, for C-band at  $10^\circ$ ,  $\Delta R = 0.99$  meters,  $\Delta \dot{R} = 0.13$  cm/sec, and  $\Delta E = 0.076$  millidegrees (0.27 seconds of arc).

The sets of refraction correction equations for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  being used by the Goddard Space Flight Center (GSFC) on GEOS data are compared with each other in this document. The formulas for  $\Delta E$  and  $\Delta R$  are also compared with the results of ray-tracings through a modified Chapman ionosphere using the Eastern Test Range (ETR) REEK program. In addition, the  $\Delta \dot{R}$  refraction correction formulas are compared with values of  $\frac{d(\Delta R)}{dt}$  obtained by differentiating the polynomial fitted to the curve of the REEK  $\Delta R$  vs time values. Results of ray-tracing through the Bent model ionosphere (reference 23) are plotted for comparison.

In Appendix A, the ionospheric refraction correction equations are first given as they appear in the referenced source documents and then are converted to a common notation and functional form to simplify the comparison of these equations.

## CORRECTION FORMULAS FOR $\Delta E$ , $\Delta R$ , AND $\Delta \dot{R}$

Table 1 lists the GSFC GEOS-2 tracking data processing programs and indicates which programs apply  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  ionospheric refraction corrections.

Table 1  
GSFC GEOS-2 Tracking Data Processing Programs

Processing Programs	$\Delta E$	$\Delta R$	$\Delta \dot{R}$
Differential Correction, DC, used with GSFC operational Orbit Determination Programs.(reference 2)	Yes	Yes	Yes
Dr. J. J. Freeman Formulations (reference 5)	Yes	Yes	Yes
GEOS Preprocessor Program, GPRO, used with the Geodetic Data Adjustment Program, GDAP, (reference 4) and with the Network Adjustment Programs, NAP-1 and NAP-2 (reference 19)	No	Yes	Yes
NONAME, all versions prior to MONSTER (8/20/70) which corrects for $\Delta R$ and $\Delta \dot{R}$ using a table look-up for two GRARR VHF stations	No	No	No
GEOS-2 GRARR Validation Program, GEOVAP (reference 26)	No	Yes	Yes
Definitive Orbit Determination System, DODS (reference 24). Same formulation as DC Program, reference 2, but implementation differs (reference 25)	Yes	Yes	Yes
NAP-3 (reference 23)	Yes	Yes	Yes
Goddard Trajectory Determination System, GTDS, Techniques not yet finalized	Yes	Yes	Yes

The analytic corrections in these programs assume that the proper correction to the observations  $E$ ,  $R$ , and  $\dot{R}$  for the net effect of the ionosphere is to decrease  $E$  by the positive quantity  $\Delta E$ , to decrease  $R$  by the positive quantity  $\Delta R$ , and to decrease  $\dot{R}$  by the quantity  $\Delta \dot{R}$ . Both  $\dot{R}$  and  $\Delta \dot{R}$  may be positive or negative, but both always have the same sign. Thus, the correction  $\Delta \dot{R}$  has the effect of always reducing the magnitude of  $\dot{R}$ . To be compatible with these conventions for the analytic corrections, the group option in the REEK ray-trace program was always used. A discussion of the meaning of the group option in REEK is given in reference 28, and an analysis of the ionospheric correction for the GRARR system defining when to use group and when to use phase corrections, is given in reference 29. The  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  ionospheric refraction correction formulas used with GEOS-1 and GEOS-2 data are listed in Tables 2, 3, and 4, respectively.

Table 2  
Elevation Refraction Correction Formulas

Refraction Formulation	Equation	Elevation Angle Refraction Correction $\Delta E$ $\Delta E$ (radians) = $E - E_c$
GSFC DC & DODS	A1	$\frac{4}{5} \Delta E_{io}$
GSFC Freeman	A4	$\Delta E_{io}$
NAP-3	A26	$\cos^{-1} \left[ \frac{X_1 \cos \alpha - X_2}{(X_1^2 + X_2^2 - 2X_1 X_2 \cos \alpha)^{1/2}} \right]$
GTDS		under development
$E_{io} = \frac{-N_{im} H e^1}{h_t} \text{ctn } E = -\frac{M_o}{h_t} \text{ctn } E$		

Table 3.  
Range Refraction Correction Formulas

Refraction Formulation	Equation	Range Refraction Correction $\Delta R$ : $\Delta R$ (meters) = $R - R_c$
GSFC DC & DODS ( $E < 10^0$ )	A2a	$\frac{4}{5} \Delta R_{i0} \left\{ \frac{\sin E}{1 - \left( \frac{R_e \cos E}{R_e + h_m} \right)^2} \right\}^{1/2}$
GSFC DC & DODS ( $E > 10^0$ )	A2b	$\frac{4}{5} \Delta R_{i0}$
GSFC Freeman	A5	$\Delta R_{i0} \left[ 1 - \left( \frac{H + h_m}{R_e} \right) \text{ctn}^2 E \right]$
GSFC GPRO (GDAP)	A7	$\frac{-8 H_g N_{im} \text{csc} E_1}{1 + \left[ 1 + \frac{25 H_g \text{ctn}^2 E_1}{3 (R_e + h_m - 3 H_g)} \right]^{1/2}}$
NAP-3	A28	$-N_{im} \left[ \frac{9}{16K} \left( 1 - e^{-\left( h_s - h_m - \frac{y_m}{2} \right) K} \right) + \frac{459}{480} y_m \right]^{1/2} \left[ 1 - \left( \frac{R_e}{R_e + R_h} \right) \cos^2 E \right]$
GEOVAP	2	$\Delta R_{i0}$
GTDS		under development
$\Delta R_{i0} = -N_{im} \text{He}^1 \text{csc} E = -M_0 \text{csc} E$		

Table 4  
Range Rate Refraction Correction Formulas

Refraction Formulation	Equation	Range Rate Refraction Correction $\Delta \dot{R}$ $\Delta \dot{R}$ (meters/sec) = $\dot{R} - \dot{R}_c$
GSFC DC & DODS (E < 10 <sup>0</sup> )	A3a	$\frac{4}{5} \Delta \dot{R}_{i0} \left( \frac{R_e}{R+h_m} \right)^2 \sin^3 E \left[ 1 - \left( \frac{R_e \cos E}{R+h_m} \right)^2 \right]^{-3/2}$
GSFC DC & DODS (E > 10 <sup>0</sup> )	A3b	$\frac{4}{5} \Delta \dot{R}_{i0}$
GSFC Freeman	A6	$\Delta \dot{R}_{i0} \left[ 1 + \left( 1 - \frac{3}{2 \sin^2 E} \right) \left( \frac{H+h_m}{R_e} \right) \right]$
GSFC GPRO (GDAP)	A8	$\frac{-8 N_{im} H_g \dot{E}_1 \text{ctn } E_1 \csc E_1 \left[ 1 + \frac{1 - 4\beta_2^2}{(1 + 4\beta_2^2 \text{ctn}^2 E_1)^{1/2}} \right]}{\left[ 1 + (1 + 4\beta_2^2 \text{ctn}^2 E_1)^{1/2} \right]^2}$
NAP-3	A29	$\frac{\dot{E}(\Delta R) \left( \frac{R_e}{R_e + R_h} \right)^2 \sin E \cos E}{1 - \left( \frac{R_e \cos E}{R_e + R_h} \right)^2}$
GEOVAP	3	$\Delta \dot{R}_{i0}$
GTDS		under development
$\Delta R_{i0} = N_{im} \dot{E}_1 \text{ctn } E \csc E = M_o \frac{d(\csc E)}{dt}$		

The DC, DODS, FREEMAN, and GEOVAP formulas listed in Table 1 employ as a factor nominal or zero-order correction functions for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$ , which are defined as

$$E - E_c = \Delta E_{io} = \frac{-N_{im} H e^1}{h_t} \text{ctn } E = \frac{-M_o}{h_t} \text{ctn } E \quad (1)$$

$$R - R_c = \Delta R_{io} = -N_{im} H e^1 \text{csc } E = -M_o \text{csc } E \quad (2)$$

and the derivative of  $\Delta R_{io}$

$$\dot{R} - \dot{R}_c = \Delta \dot{R}_{io} = +N_{im} H e^1 \dot{E} \text{ctn } E \text{csc } E = M_o \dot{E} \text{ctn } E \text{csc } E \quad (3)$$

where  $M_o = N_{im} H e^1$ . Note that  $M_o$  is a multiplicative factor in all these zero-order correction functions. It can be shown that  $M_o = N_{im} H e^1 \approx \int_0^{h_t} N_i dh$ , when using the Chapman model ionosphere.

These  $\Delta E_{io}$  and  $\Delta R_{io}$  corrections result from the following assumptions:

- (1) Modified Chapman ionosphere
- (2) Flat layered, horizontally homogeneous ionosphere
- (3) Satellite height,  $h_t$ , large compared to scale height,  $H$
- (4) Negligible ray path bending

The GPRO and NAP-3 formulas are different in appearance from the DC, DODS, Freeman, and GEOVAP formulas due to the choice of a parabolic ionospheric model in GPRO by Willmann (references 4 and 22) and a joined topside exponential, and bottomsides biparabolic, ionospheric model in NAP-3 by Bent (reference 23) compared to the Chapman model used by DC, DODS, Freeman, and GEOVAP. The GPRO and NAP-3 models enable the correction formulas, which involve integration along the ray path, to be integrated in closed form, but at the resulting expense of more complicated formulas.

In order to compare the analytic refraction correction equations, we need typical values for  $N_{im}$ ,  $h_m$ , and  $H$ . These are obtained as follows:

The refraction index, to first order approximation, is given by (reference 11, eq 2.78)

$$\mu_i = \sqrt{1 - \frac{80.5 N_e}{f^2}} \approx 1 - \frac{40.25 N_e}{f^2} \quad (4)$$

therefore, the refractivity at any height is

$$N_i = \mu_i - 1 \approx - \frac{40.25 N_e}{f^2}$$

and the refractivity at the height of maximum electron density is

$$N_{im} = \mu_{im} - 1 = - \frac{40.25 N_{em}}{f^2} \quad (5)$$

which provides one of the required multiplicative factors if  $N_{em}$  and  $f$  are known. The scale height in kilometers is determined in this study from

$$H = \frac{5}{3} \left[ 30 + 0.2 (h_m - 200) \right] \quad (6a)$$

which provides the other required multiplicative factor if  $h_m$  is known.

The origin and validity of this type of linear relationship between  $H$  and  $h_m$  is unknown to us, but the first mention we found of it was by Charnow (1959), reference 27, who quotes the scale height as

$$H = \frac{4}{3} \left[ 30 + 0.2 (h_m - 200) \right] \quad (6b)$$

This formula for scale height was also quoted in reference 2 (DC program), reference 24 (DODS program), and reference 25 (DODS program).

A similar linear relationship was proposed by Wright (1960, 1961), references 30 and 31, based on ionograms from 11 stations collected during the period January 1959 to February 1960.

Freeman (1965), reference 5, originally quotes the scale height as

$$H = \frac{8}{3} \left[ 30 + 0.2 (h_m - 200) \right]$$

but, based on 292 ionograms at Pennsylvania State University, he reduces this by a factor of 1.61 to give equation (6a).

It is not known how closely other ionospheres, removed in time or distance from Freeman's sample set, adhere to equation (6a). However, since equation (6a) does represent real ionospheres for this region, and according to Wright, a similar relationship is valid for other regions, it was used to generate values of scale height,  $H$ , for these comparisons. The factor of  $\frac{4}{5}$  in the DC and DODS corrections for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  in Tables 2, 3, and 4 is due to our use of equation (6a) rather than (6b) in the definition of scale height,  $H$ .

Values of  $N_{em}$  and  $h_m$  were obtained from each of the midnight and noon mean  $N_e(h)$  profiles given in reference 11, pages 128 and 129, for Newfoundland, Grand Bahama, and Huancayo over a period of 1 year (1959 to 1960) near a solar cycle maximum. The lowest, average, and highest of the  $N_{em}$  and  $h_m$  values obtained in this manner were then used with equations (5) and (6a), with a frequency of 2 GHz, to obtain the low, average, and high values of  $N_{im}$  and  $H$  which are listed in Table 5.

The refraction correction is proportional to  $N_{im}$  in all of the formulas (except NAP-3 for  $\Delta E$ ), and hence is inversely proportional to the square of the operating frequency. By scaling, the results in this document can be applied at any operating frequency; in particular, the  $N_{im}$  scaling factors given in Table 6 should be used for the GEOS 1 and 2 tracking frequencies.

The refraction correction is also proportional to  $H$  in the DC, DODS, Freeman, and GEOVAP formulas. The scale height,  $H$ , which arises from the use of the Chapman model in the derivation of these formulas, is determined by means of a semiempirical function of the maximum refractivity height,  $h_m$ . The range of  $H$  from Table 5 is from 76.667 meters to 150.000 meters; that is, the maximum is about twice the minimum value.  $N_{im}$  varies by a factor of 10, which means that the product  $N_{im} H$  can vary by a factor of 20, as shown in Table 5. It follows that for a given elevation angle, the values of  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  can vary by a factor of about 20 due to possible variations in the ionospheric parameters.

#### CORRECTIONS FOR $\Delta E$ , $\Delta R$ , AND $\Delta \dot{R}$ FROM REEK RAY-TRACES

The REEK ray-trace program, for a given elevation angle, requires as input the true range to the target and a refractivity profile. The true range to the satellite was obtained for each of the specified true elevation angles from the simplified orbit program described in Appendix B. The refractivity profiles used as input to

the REEK ray-trace program were generated from all nine combinations of the specified values of the  $N_{im}$  and  $H$  listed in Table 5 using the Chapman model and a compatible Bent model.

The Chapman profile is defined as follows:

$$N_i = N_{im} e^{(1 - Z - e^{-Z})} \quad (7)$$

$$Z = \frac{h_i - h_m}{H} \quad (8)$$

$h_i$  = height of each profile point going from 112.5 km to 1325 km in 12.5 km steps

The refractivity was set to zero from  $h_i = 0$  to  $h_i = 112$  km.

The three Chapman and compatible Bent refractivity profiles generated from the low, average, and high pairs of  $N_{im}$  and  $H$ , listed in Table 5 and shown in Figure 1, illustrate the shape and possible variation in these profiles.

The ray-trace values for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  from the Chapman model are listed in Table 7 for all nine of the  $N_{im}$ ,  $H$  combinations given in Table 5. These values are plotted in Figures 2, 3, and 4. Several features shown in these graphs are of interest and are as follows:

- a. They verify the prior conclusion that the observed variation in  $N_{im}$  has more effect on  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  than the observed variation in  $H$ , as inferred from equation (6a) and the observed variation in  $h_m$ . Because of this, only the low, average, and high combinations of  $N_{im}$  and  $H$  given in Table 5 are used in the subsequent analytic and ray-trace refraction correction graphs in Figures 5 through 13.
- b. The ratio of maximum to minimum refraction correction due to a change in elevation angle for a given ionosphere is about 100:1 for  $\Delta E$ , only about 3:1 for  $\Delta R$ , and about 10:1 for  $\Delta \dot{R}$ . These generalizations exclude the low probability elevation angles within a degree or so of zenith where  $\Delta E$  and  $\Delta \dot{R}$  both approach zero. Normally  $\Delta \dot{R}$  approaches zero at the point of closest approach where  $\dot{E} = 0$ , but for an overhead pass (used here and described in appendix B) this occurs at zenith. For the chosen 2-GHz frequency and for our average ionosphere, the refraction corrections fall between 0.7 and 0.007 millidegrees (2.5 and 0.025 arc-seconds) for  $\Delta E$ , between 9 and 3 meters for  $\Delta R$ , and between 1.3 and 0.13 cm/sec for  $\Delta \dot{R}$ .

- c. At an elevation of about  $10^\circ$ , for both  $\Delta E$  and  $\Delta R$ , the correction curves cross over so that the thicker ionospheres with larger values of scale height, H, require a larger refraction correction for  $\Delta E$  and  $\Delta R$  above  $10^\circ$  and less correction below  $10^\circ$  than do thinner ionospheres with the same  $N_{im}$  but smaller H.

Table 5  
Low, Average, and High  $N_{im}$ ,  $h_m$ , and H Values (Near Solar Cycle Maximum)

	$N_{em}$ ( $10^{17}$ el/m <sup>3</sup> )	$-N_{im}$ ( $10^{-6}$ )	$h_m$ (km)	H (km)	$-N_{im} H$ (meters)
Low (3,3)	2.19	2.21	280	76.667	0.169
Average (2,2)	10.60	10.67	364	104.667	1.117
High (1,1)	23.99	24.14	500	150.000	3.621

Table 6  
Correction Factors for Other Tracking Systems

System	Frequency			Multiplication Factor for $N_{im}$
	Up (MHz)	Down (MHz)	Up/Down Equivalent (MHz)	
C-band TRANET	5690	5765	5727	0.122
			108	342.936
			(NSSDC data)	
			162 } *	152.416
			324 } *	38.104
Secor	420.9	224.5 449.0	280.18	50.972
			434.27	21.210
			434.27	21.210
1. GRARR*	2271.9328 2270.1328	1705.000 1705.000	1928.6	1.075
			1928.0	1.076
2. GRARR**	1799.2 1801.0	2253 2253	1988.3	1.012
			1989.5	1.011
3. GRARR (GEOS-C) Proposed	2074.6375	2253	2158.3	0.859
*For GEOS-1 and GEOS-2 **Between GEOS-2 and GEOS-C				

Table 7  
REEK Ray-Trace Values From Chapman Model

H (km)	76.667	76.667	76.667	76.667	104.667	104.667	104.667	150	150	150
Nkm (10 <sup>-5</sup> )	2.21	10.67	24.14	24.14	10.67	24.14	24.14	2.21	10.67	10.67
Elevation (deg)	$\Delta E$ (milli deg)									
0.15	0.1503	0.7284	1.6479	0.1398	0.6773	1.5324	0.1257	0.6089	0.6089	1.3774
1.50	0.1538	0.7454	1.6864	0.1435	0.6950	1.5723	0.1293	0.6263	0.6263	1.4168
15.00	0.0966	0.4678	1.0580	0.1045	0.5060	1.1446	0.1097	0.5311	0.5311	1.2013
30.00	0.0441	0.2136	0.4833	0.0538	0.2607	0.5899	0.0653	0.3167	0.3167	0.7164
45.00	0.0234	0.1134	0.2565	0.0300	0.1455	0.3291	0.0390	0.1891	0.1891	0.4378
60.00	0.0128	0.0623	0.1409	0.0168	0.0816	0.1847	0.0226	0.1095	0.1095	0.2477
90.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Elevation (deg)	$\Delta R$ (m)									
0.15	1.538	7.459	16.878	1.860	9.021	20.411	2.305	11.181	11.181	25.299
1.50	1.532	7.432	16.816	1.855	8.995	20.353	2.301	11.158	11.158	25.247
15.00	1.176	5.705	12.308	1.496	7.257	16.419	1.951	9.464	9.464	21.412
30.00	0.811	3.933	8.899	1.076	5.217	11.803	1.475	7.154	7.154	16.185
45.00	0.620	3.007	6.803	0.837	4.058	9.181	1.175	5.697	5.697	12.888
60.00	0.521	2.528	5.720	0.709	3.437	7.776	1.006	4.877	4.877	11.034
90.00	0.458	2.223	5.030	0.626	3.035	6.867	0.894	4.334	4.334	9.805
Elevation (deg)	$\Delta \dot{R}$ (cm/sec)									
0.15	—	0.0322	0.0080	0.0154	0.2398	0.0080	0.0145	0.1360	0.1360	—
1.50	0.0476	0.2466	0.5275	0.0420	0.2247	0.4850	0.0395	0.1954	0.1954	0.4207
15.00	0.2552	1.2372	2.8002	0.2760	1.3392	3.0300	0.2905	1.4098	1.4098	3.1884
30.00	0.2287	1.1098	2.5107	0.2786	1.3512	3.0570	0.3378	1.6390	1.6390	3.7078
45.00	0.1719	0.8341	1.8868	0.2203	1.1714	2.4186	0.2859	1.3867	1.3867	3.1378
60.00	0.1156	0.5606	1.2685	0.1514	0.7343	1.6512	0.2027	0.9834	0.9834	2.2258
90.00	0.0001	0.0003	0.0011	0.0001	0.0006	0.0019	0.0001	0.0011	0.0011	0.0017

- d. The  $\Delta \dot{R}$  corrections for the ionosphere are maximum for an overhead pass at elevation angles between  $15^\circ$ , for the thinner ionospheres (low H), and  $30^\circ$  for the thicker ionospheres (high H). The  $\Delta \dot{R}$  values decrease for both lower and higher elevation angles in sharp contrast to their behavior in the troposphere. The decrease in  $\Delta \dot{R}$  for lower elevation angles contradicts the results by Millman (reference 32). Weisbrod (reference 33) predicts that the  $\Delta E$  ionospheric refraction correction also decreases for lower elevation angles. This is also supported by our curves.

#### DERIVATION OF COMPATIBLE BENT PROFILES FROM THE CHAPMAN PROFILES

The Bent profile (reference 23) is given in Appendix A, page A-10 as an exponential function of the parameters  $N_{im}$  and  $k$  for the topside ionosphere and as a biparabolic function of the parameters  $N_{im}$  and  $y_m$  for the F-layer and bottomside ionosphere. The NAP-3 formulas assume the Bent model and therefore incorporate the Bent model parameters. The NAP-3 formulas were compared with REEK ray-traces through some selected Bent ionospheric profiles as well as the Chapman profiles to determine whether the use of the different models has a significant effect. The different model profiles were made compatible by constraining the total electron content or total refractivity integral or zenith  $\Delta R$  corrections to be the same.

$$\text{Thus } \int_0^{h_t} N_i \text{ (Bent) } dh = \int_0^{h_t} N_i \text{ (Chapman) } dh \approx N_{im} H_e^1$$

Since the total electron content is the dominant feature of the ionosphere with regard to refraction corrections, this constraint reduces the differences between the models to second order effects due to shape differences.

It is also assumed that the parameters  $N_{im}$  and  $h_m$  are the same for both models.

The Bent model topside profile is represented by an exponential decay function with scaling coefficient  $\frac{1}{k} = H_b$ . The Chapman model profile rapidly approaches an exponential decay function with scaling coefficient  $H$  at heights of several  $H$  above  $h_m$ . Therefore, we somewhat arbitrarily chose  $H_b = H$ .

Due to the above assumptions, we have defined the parameters

$$\text{Bent } [N_{im}, h_m, H_b] = \text{Chapman } [N_{im}, h_m, H].$$

The only remaining undefined parameter is  $y_m$ , the thickness parameter for the Bent F-layer and bottomside model. Values of  $y_m$  were chosen to satisfy the total electron content equality constraint above. The effects of this method for defining the compatible Bent refractivity profiles are shown in Figure 1, where a slight downward shift of the compatible Bent profile, relative to the reference Chapman profile, is observed.

The values for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  obtained by REEK ray-traces through the Bent profiles are plotted in Figures 5 through 13 for comparison with the NAP-3 formulas and with the ray-trace through the Chapman profiles.

#### COMPARISON OF ANALYTIC FORMULAS AND REEK RAY-TRACES

Values  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  (calculated from the analytic formulas given in Tables 2, 3, and 4) are plotted in Figures 5 through 13. The values of the Chapman ionospheric parameters  $N_{im}$ ,  $h_m$ , and  $H$  given in Table 5 are used in the DC, DODS, Freeman, and GEOVAP formulas. For GPRO the same  $N_{im}$  and  $h_m$  parameter values are used, but the thickness parameter for the GPRO parabolic ionosphere is kept constant at  $H_g = 50$  km, as in the normal usage of GPRO. For NAP-3, the same  $N_{im}$ ,  $h_m$ , and  $H = H_b$  parameter values are used and, in addition,  $y_m$  is determined as described above so that  $\Delta R (E = 90^\circ)$ , calculated from Bent's analytic formula (A27 or A28), is the same  $\Delta R (E = 90^\circ)$  determined from the REEK ray-traces. The same is true for  $\Delta \dot{R} (E = 90^\circ)$ . The parameter values used are given below:

	$-N_{im}$ ( $10^{-6}$ )	$h_m$ (km)	$H$ (km)	$H_g$ (km)	$H_b$ (km)	$y_m$ (km)
Low	2.21	280	76.667	50	76.667	172.833
Average	10.67	364	104.667	50	104.667	235.944
High	24.14	500	150.000	50	150.000	337.603

These same parameters are used in generating the Chapman and Bent profiles for input into the REEK ray-tracing program. The results of these ray-traces are also shown in Figures 5 through 13. There is only a slight difference between the REEK ray-trace through the Chapman and compatible Bent profiles at low elevation angles, as can be seen from Figures 5 through 13.

The DC/DODS correction values are all 20% below the zero order correction  $\Delta E_{i0}$ ,  $\Delta R_{i0}$ ,  $\Delta \dot{R}_{i0}$  for elevation angles above  $10^\circ$ , and about 20% below the ray-trace corrections for angles below  $10^\circ$ . This is due to the  $\frac{4}{5}$  factor explained

earlier in connection with the semiempirical equation for H. If equation (6b) rather than (6a) had been used to define H for the Chapman profiles in the REEK ray-traces, the DC/DODS corrections for  $E < 10^\circ$  would have closely agreed with these ray-traces. Furthermore, if the DC/DODS formula for  $E < 10^\circ$  had also been used for  $E > 10^\circ$ , the DC/DODS corrections for  $E > 10^\circ$  would have better agreed with the ray-traces. This can be seen from the dotted extensions of the DC ( $E < 10^\circ$ ) graphs. Multiplication of the DC/DODS curves by the factor  $\frac{5}{4}$  is equivalent to assuming the same total electron content for the DC/DODS and REEK ray-trace ionospheres. This was done for the selected elevation angles  $E = 1.5^\circ, 6^\circ, 15^\circ, \text{ and } 90^\circ$  to show the resulting good agreement between the DC/DODS and the ray-trace results in Figure 5-13.

The Freeman correction for  $\Delta R$  deteriorates rapidly for  $E < 30^\circ$ , as Freeman acknowledges in reference 5. His correction for  $\Delta \dot{R}$  deteriorates for  $E < 50^\circ$ .

The GPRO results are quite good for our low values of the ionospheric parameters  $N_{im}$  and  $h_m$ , but, for our average and high values of  $N_{im}$  and  $h_m$ , the GPRO results are consistently too low. This is most likely due to the use of a constant value of  $H_g = 50$  km for the parabolic model thickness parameter. If the  $H_g$  parameter were allowed to adjust, to keep the total electron content in the Willmann GPRO model equivalent to that in the Chapman profile used in REEK, the GPRO results would match the ray-trace results more closely for all cases. This can be accomplished by setting:

$$4N_{im} H_g = e^1 N_{im} H$$

then

$$H_g = \left(\frac{e}{4}\right) H \quad \text{rather than } 50 \text{ km}$$

For the chosen values of H, the new values of  $H_g$  are calculated below along with the proper multiplicative factors to adjust  $\Delta R$  and  $\Delta \dot{R}$ .

	H (km)	$H_g = (e/4) H$ (km)	Multiplicative Factor for $\Delta R$ and $\Delta \dot{R}$ $m = \frac{H_g}{50}$
Low	76.667	52.101	1.0420
Average	104.667	71.129	1.4226
High	150.000	101.936	2.0387

Applying these multiplicative factors to the GPRO graphs for  $\Delta R$  and  $\Delta \dot{R}$  for the selected values of  $E = 1.5^\circ, 6^\circ, 15^\circ,$  and  $90^\circ$  yields the improved values indicated in Figures 8 through 13.

The NAP-3 values for  $\Delta E$  are consistently lower than the REEK ray-traces for all elevations. The NAP-3 values for  $\Delta R$  and  $\Delta \dot{R}$  were forced to agree with ray-trace values at  $E = 90^\circ$ , but are consistently higher than the ray-traces for all other elevation angles. Although the magnitudes of the NAP-3 corrections do not agree with the REEK ray-traces, the NAP-3 correction curves, in general, match the shapes of the ray-trace curves. A modified version of the NAP-3 ionospheric refraction formulas will soon be available and may improve these results.

Assuming the same total electron content, it can be shown that  $N_{im} H e^1 = 40.3 N_T / f^2$ , and the only difference between NAP-3 (equation A27) and DC ( $E < 10^\circ$ ) (equation A2a) formulas for  $\Delta R$  is in the use of  $R_h = h_m - y_m$  in NAP-3 compared to  $h_m$  alone in DC. In Figure A-3, if the line from the center of the earth were drawn to intersect the ray-line at  $h_m$  within the ionosphere rather than at  $h_m - y_m$  at the bottom of the ionosphere, the resulting NAP-3 formula for  $\Delta R$  would be identical to the DC ( $E < 10^\circ$ ) formula for  $\Delta R$  and would match the REEK ray-trace better, assuming the NAP-3, DC/DODS, and REEK ionosphere all have the same total electron content. The same conclusions apply to the NAP-3 formulas for  $\Delta \dot{R}$ .

We have also tested the preliminary versions of the GTDS formulas. However, since these formulas are still under development, the results are not reported here.

#### DEVIATION AMONG REFRACTION CORRECTIONS

For elevation angles above about  $10^\circ$ , where most satellite tracking is done, the refraction error curves in Figures 5 through 13, deviate in general, from each other less than at lower angles.

The maximum deviations among the corrections for the average ionospheres in Figures 6, 9, and 12 are given in Table 8 for selected angles. The Freeman corrections are not included below  $E = 30^\circ$ , since Freeman specifically warns against it in reference 5. As explained earlier, some of the spread among the computed refraction corrections could be reduced by using different values of the thickness parameters  $H, H_g, H_b, y_m$  or by a different choice of effective refraction height of the ionosphere.

Table 8  
Maximum Deviation Among Refraction Corrections

E (degrees)	$\delta\Delta E$ (millidegrees)	$\delta\Delta R$ (meters)	$\delta\dot{\Delta R}$ (cm/sec)
6	0.828	22.34	29.79
10	0.390	11.30	11.29
15	0.223	6.27	5.16
20	0.168	4.09	2.94
30	0.128	2.30	1.95
40	0.103	1.60	0.79
60	0.058	1.07	0.35
90	0.000	0.90	0.00

RELATIONSHIPS BETWEEN REFRACTION CORRECTIONS FOR  $\Delta E$ ,  $\Delta R$ , AND  $\dot{\Delta R}$

NOMINAL ZERO ORDER CORRECTIONS

Several relations connecting the nominal refraction correction formulas for  $\Delta E$ ,  $\Delta R$ , and  $\dot{\Delta R}$  are apparent by inspection of these formulas. They are

$$\Delta E_{io} = \frac{-N_{im} H e^1}{h_t} \quad \text{ctn } E = \frac{\Delta R_{io}}{h_t} \cos E$$

$$\Delta R_{io} = -N_{im} H e^1 \csc E = h_t \Delta E_{io} \sec E$$

$$\dot{\Delta R}_{io} = +N_{im} H e^1 \dot{E} \csc E \text{ ctn } E = -\Delta R_{io} \dot{E} \text{ ctn } E =$$

$$-h_t \dot{E} \Delta E_{io} \sec E \text{ ctn } E = \frac{d(\Delta R_{io})}{dt} = \dot{E} \frac{d(\Delta R_{io})}{dE}$$

HIGHER, ORDER CORRECTIONS

It is of interest to determine whether relationships similar to the above also hold for the higher order GEOS ionospheric refraction correction formulas for  $\Delta E$ ,  $\Delta R$ , and  $\dot{\Delta R}$  in Tables 2, 3, and 4. The only formula for  $\Delta E$  which is more complicated than the nominal formula is the NAP-3 formula. There seems to be no simple relation between  $\Delta E$  and  $\Delta R$  for NAP-3.

It can easily be shown that for all the GEOS ionospheric refraction correction formulas in Tables 3 and 4, the range rate refraction correction formula for  $\dot{\Delta R}$  is simply the time derivative of the formula for  $\Delta R$ , as might be expected for consistent levels of approximation for  $\Delta R$  and  $\dot{\Delta R}$ .

Thus, 
$$\dot{\Delta R} = \frac{d(\Delta R)}{dt} = \dot{E} \frac{d(\Delta R)}{dE}$$

holds for all formulas. This is not generally stated in the earlier source documents where the formulas for  $\dot{\Delta R}$  are derived more laboriously from basic principles.

The GEOS range rate measurements are all derived by determining the number of doppler cycles in a short time interval. The number of doppler cycles in this time interval corresponds to the range shift in satellite position relative to the station during the same time interval, where range shift is measured in wavelengths of the transmitter frequency. The measured range shift includes the differential effect of the atmosphere on the doppler cycles at the beginning and end points of spacecraft position corresponding to the beginning and end points of the shift measurement time interval. Therefore, it is reasonable that the range rate refraction correction should be expressed as the difference in range refraction correction divided by the corresponding difference in time, or as a derivative, with respect to time, of the range refraction correction formula.

### SUMMARY

The purpose of this study was to compare the different ionospheric refraction correction techniques used with GEOS E, R, and  $\dot{R}$  data to determine whether significant deviations exist among these techniques. If deviations exist, it is difficult to know which technique gives the best results, since no perfect model of the ionosphere is available. However, REEK ray-trace results through typical Chapman and Bent model ionosphere profiles were included as references for comparison of the results from the various formulas and should indicate which formulas are best.

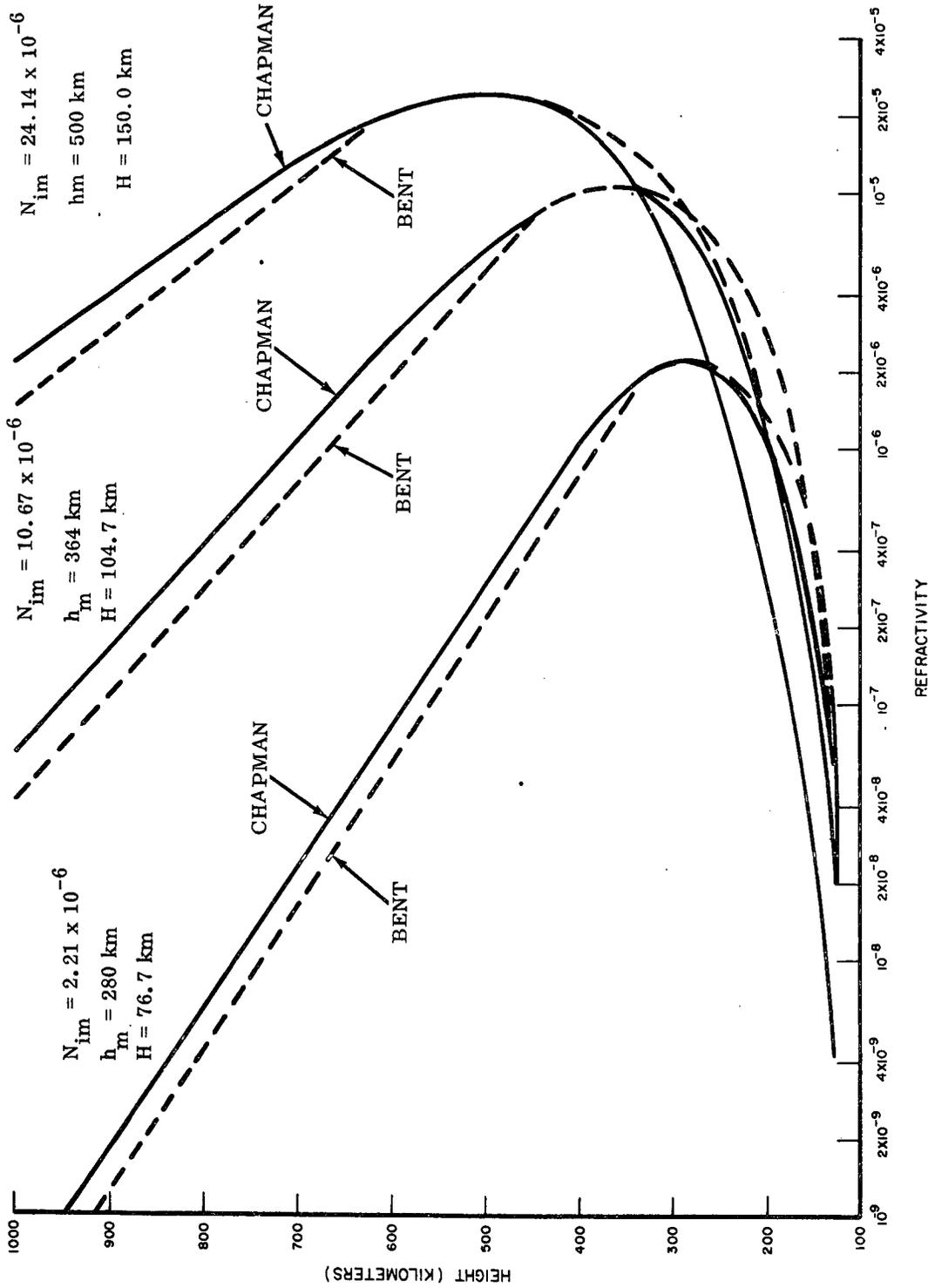
The following statements summarize the results of this study:

1. Some of the GEOS ionospheric refraction correction formulas and the REEK ray-trace results for  $\Delta E$ ,  $\Delta R$ , and  $\dot{\Delta R}$  are in poor agreement for  $E < 30^\circ$ . The zero order corrections  $\Delta R_{10}$  and  $\dot{\Delta R}_{10}$  tend to overcorrect at all elevation angles, becoming worse at the lower elevation angles. At  $E = 10^\circ$ ,  $\Delta R_{10}$  is twice the REEK ray-trace value and  $\dot{\Delta R}_{10}$  is ten times the REEK value, so that no correction at all is

preferable to the zero order corrections.  $\Delta E_{10}$  also tends to over-correct for  $E < 10^\circ$ , but under-corrects for  $E > 10^\circ$  and matches REEK at about  $10^\circ$ .

2. For  $E > 10^\circ$ , where most satellite tracking is done, the differences in  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  are insignificant in comparison to most operational requirements. However, for the GEOS Observation Systems Inter-comparison Investigation (GOSII), these differences (refer to Table 7) are larger in size than observed differences in system biases for some of the best electronic geodetic tracking systems. Therefore, the systematic differences in refraction correction techniques are affecting the GOSII results.
3. For all the GEOS ionospheric refraction correction formulas examined in this report, the range rate refraction correction for  $\Delta \dot{R}$  is the time derivative of the formula for  $\Delta R$ . In the earlier source documents, the  $\Delta \dot{R}$  and  $\Delta R$  formulas are derived independently.
4. Assuming the same total electron content, the NAP-3 and DC/DODS corrections for  $\Delta R$  and  $\Delta \dot{R}$  are identical except for the use of an effective ionosphere height of  $R_h = h_m - y_m$  in NAP-3 compared to  $h_m$  alone in DC/DODS. This may account for the differences between NAP-3 and REEK.
5. The REEK ray-traces show that the ionospheric correction for  $\Delta \dot{R}$  has a strong maximum between  $15^\circ$  and  $30^\circ$  elevation, contrary to the tropospheric correction and contrary to some reports in the literature.
6. All of the ionospheric correction formulas for  $\Delta E$ ,  $\Delta R$ , and  $\Delta \dot{R}$  depend on the integral of the refractivity or electron content. In all the ionospheric models discussed here (Chapman, Willmann, and Bent) this integral depends on the product of  $N_{im}$  and some thickness parameter. Reasonably accurate values of  $N_{im}$  and  $h_m$  based on bottomside ionograms, are available. The thickness parameter  $H$  used in the formulas based on the Chapman model is not well known. In fact, the DC/DODS semiempirical linear relationship used to derive  $H$  from  $h_m$  differs from the Freeman version by a factor of 20%. The uncertainties in specifying the thickness parameters can theoretically be reduced by means of topside ionograms or Faraday rotation measurements, or multiple frequency techniques such as those employed by the Secor and TRANET systems.

7. The DC/DODS ( $E < 10^0$ ) corrections programmed for  $\Delta R$  and  $\Delta \dot{R}$  should also be used for  $E > 10^0$ . The currently programmed corrections for  $E > 10^0$  are flat earth approximations which lead to significant errors above  $10^0$ .
8. The DC/DODS elevation angle correction has the proper magnitude for a single path angle correction, not the dual path interferometer correction intended. The interferometer correction is smaller in magnitude, and opposite in sign, and may be obtained from any of the range rate ( $\Delta \dot{R}$ ) correction equations, as explained in reference 1.
9. All of the formulas compared in this report assume the satellite is well above the bulk of the ionosphere. This is a good assumption for GEOS-1 and 2 whose perigees are both above 1080 km. However, many satellites operate within the ionosphere with electrons above as well as below the orbit. In these cases, the ionospheric refraction correction formulas based on the total electron content will in general be in error to the extent that it may be preferable not to make corrections. Some programs, such as NAP-3, provide a separate formulation for this case.



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Figure 1. Chapman and Compatible Bent Refractivity Profiles

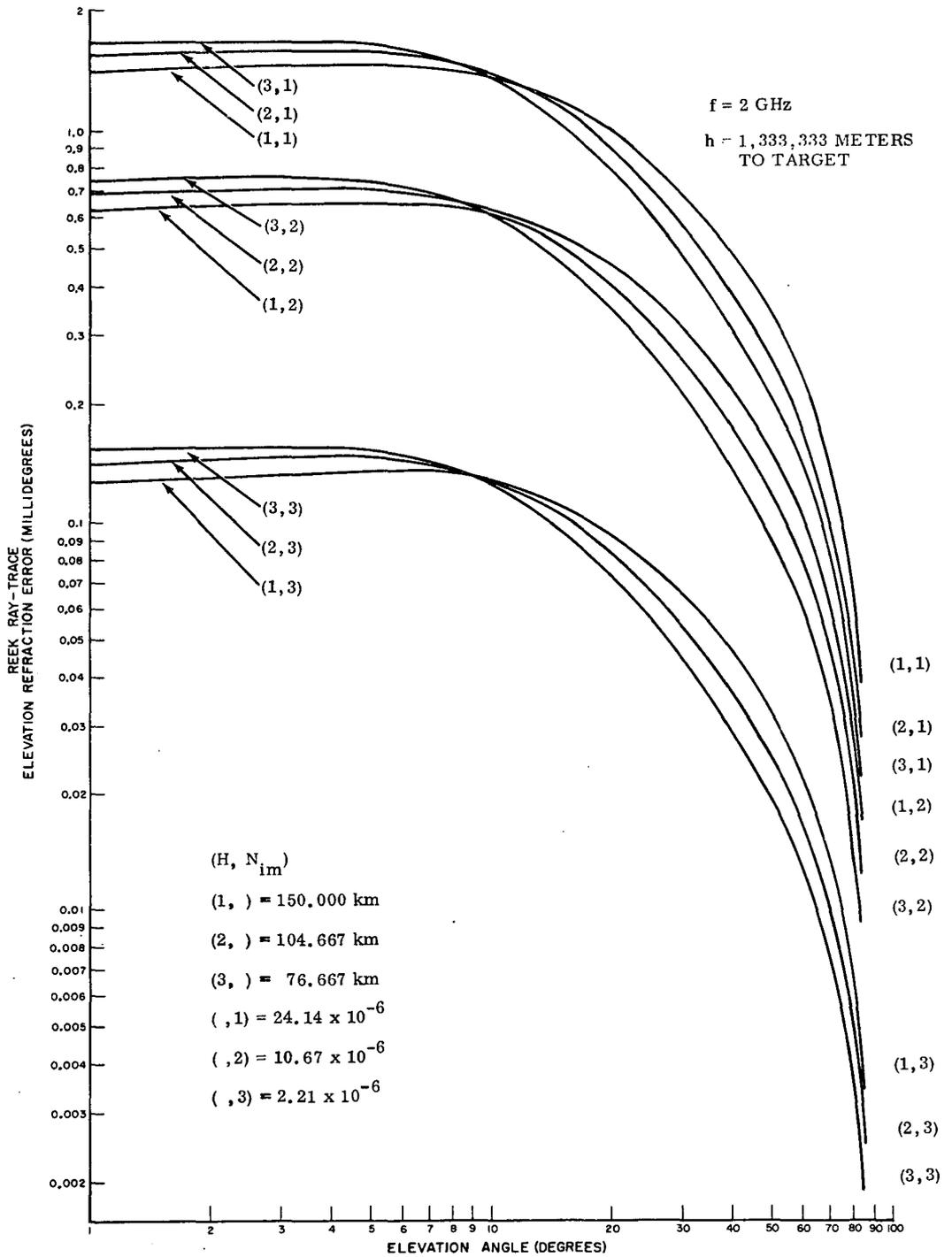


Figure 2. Elevation Error Due to Ionospheric Refraction

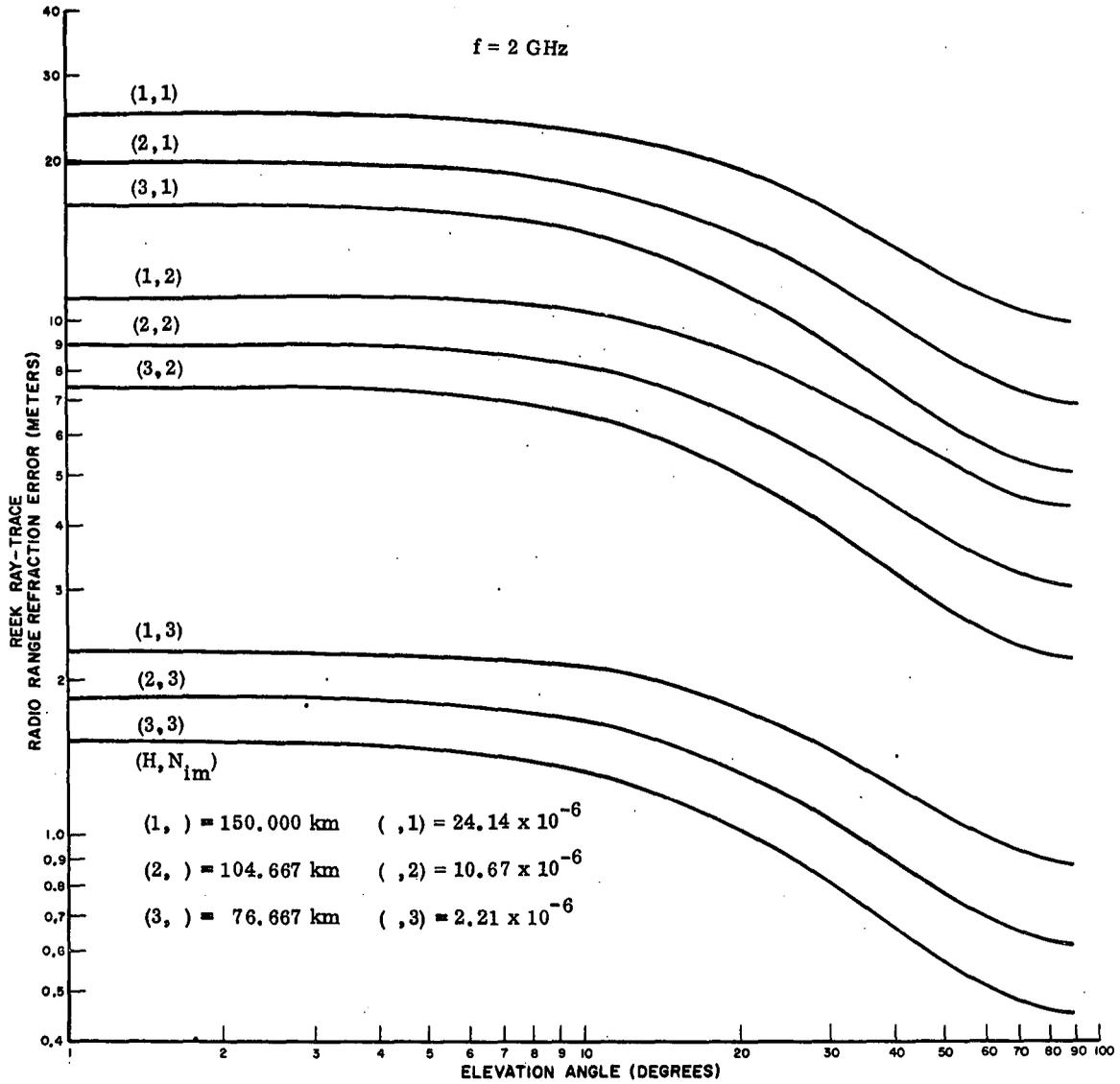


Figure 3. Range Error Due to Ionospheric Refraction

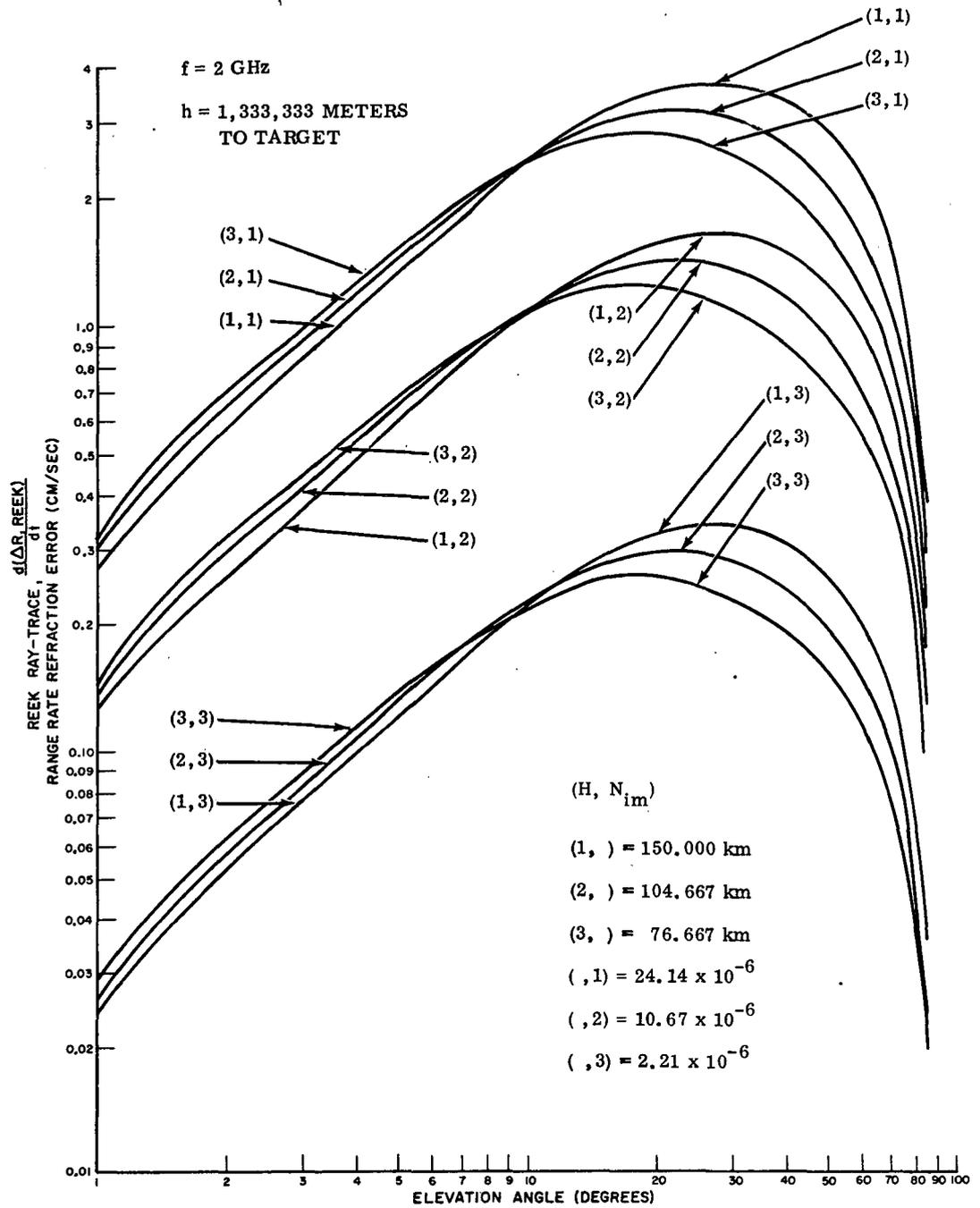


Figure 4. Range Rate Error Due to Ionospheric Refraction

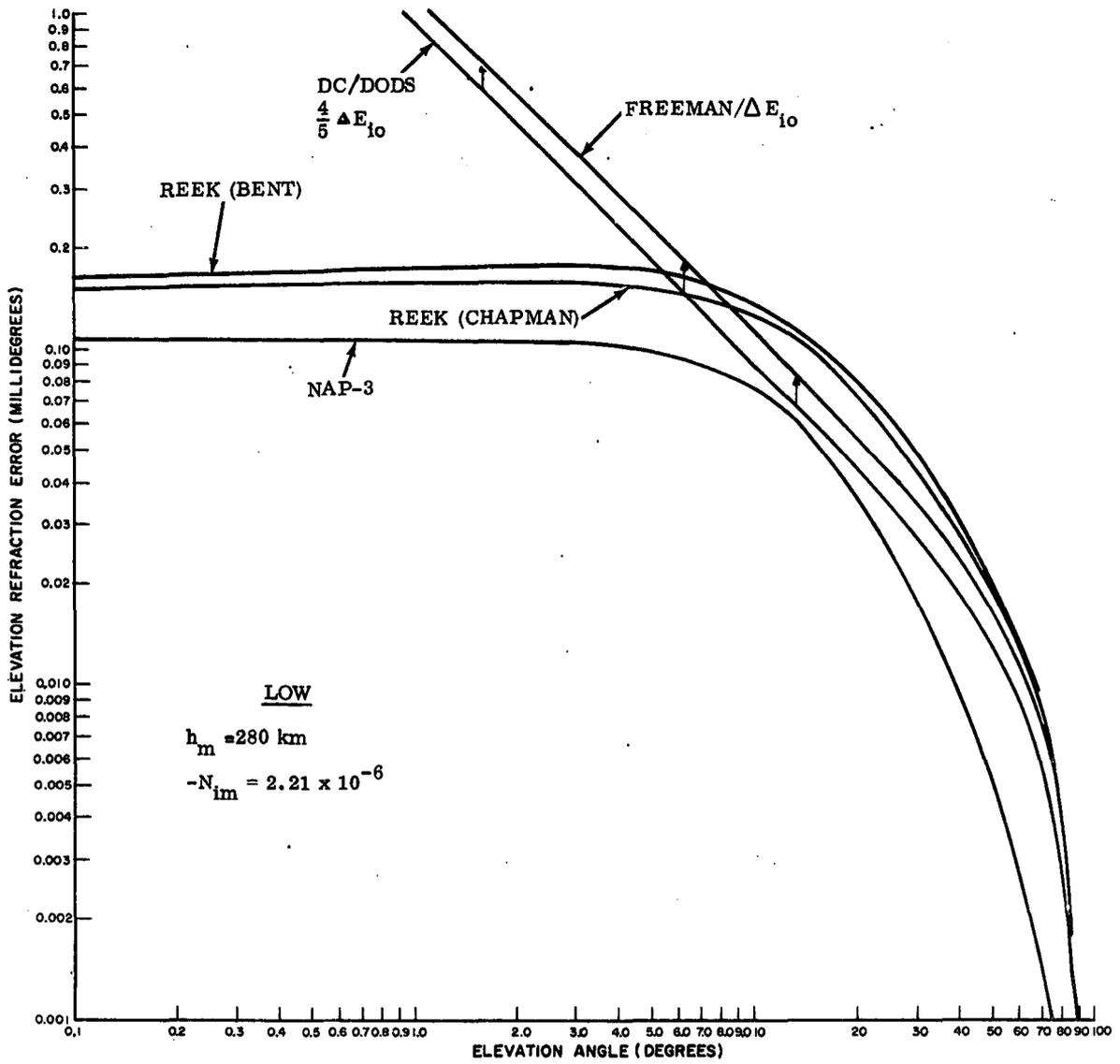


Figure 5. Elevation Error Due to Ionospheric Refraction

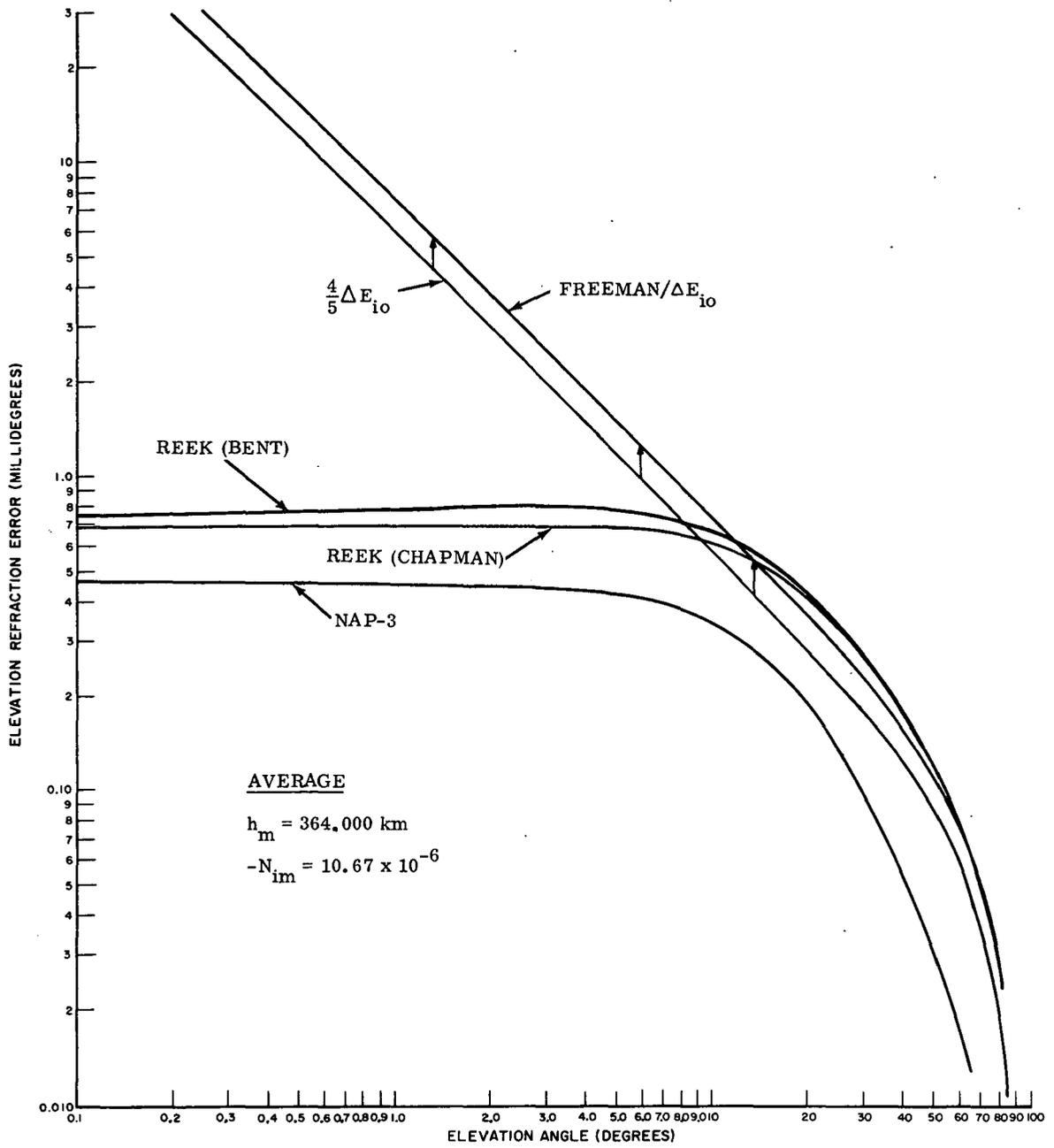


Figure 6. Elevation Error Due to Ionospheric Refraction

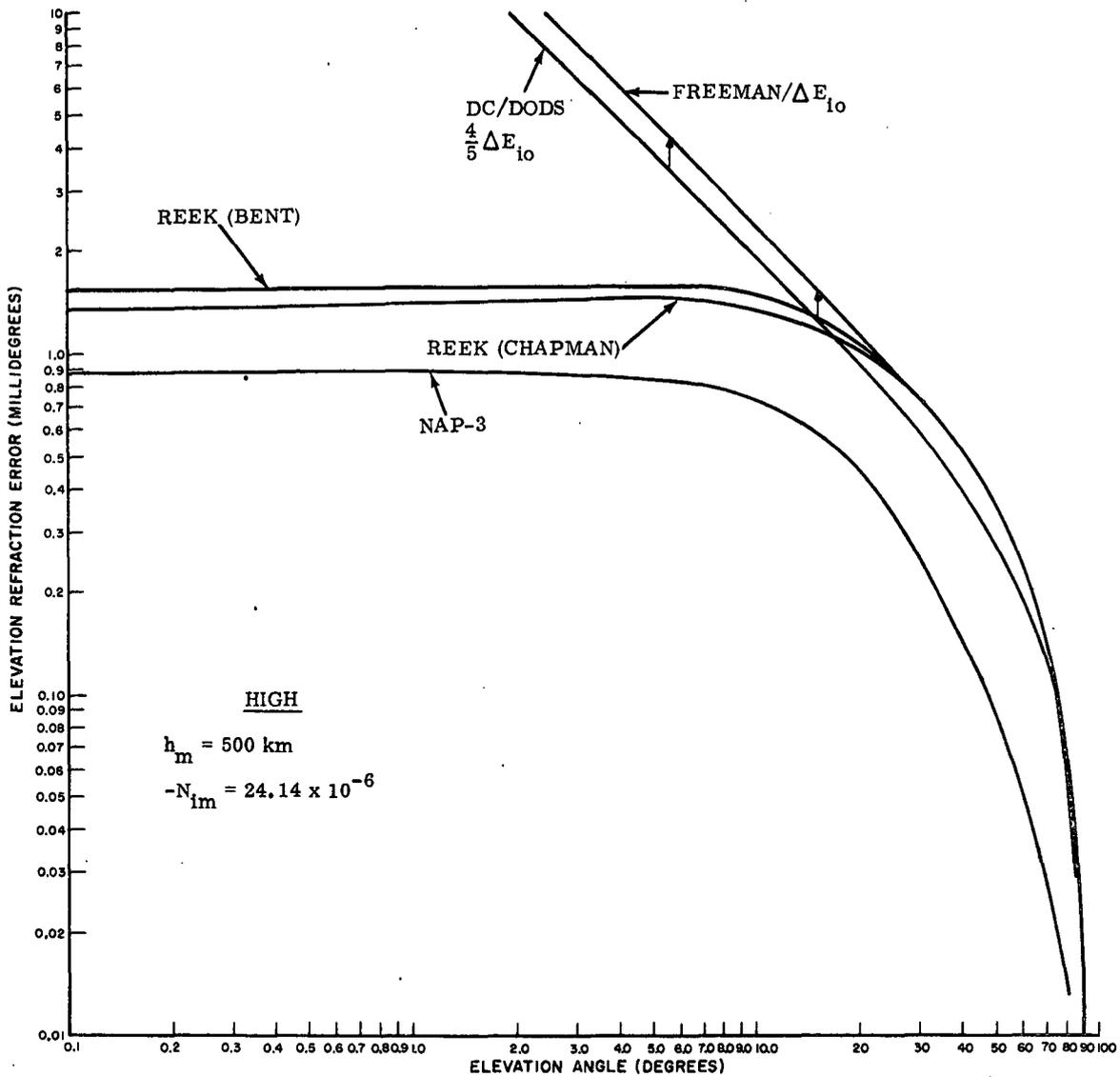


Figure 7. Elevation Error Due to Ionospheric Refraction

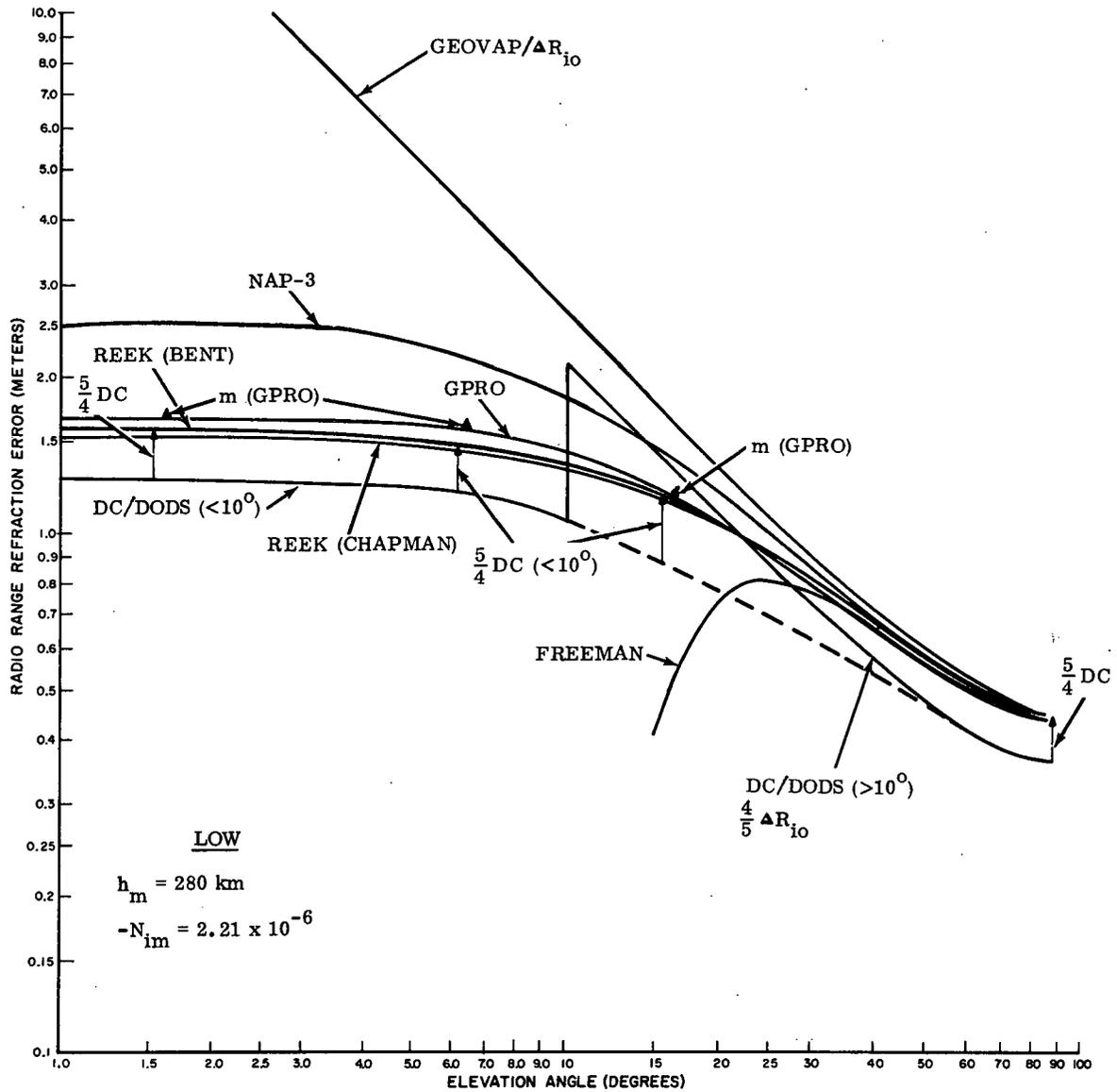


Figure 8. Range Error Due to Ionospheric Refraction

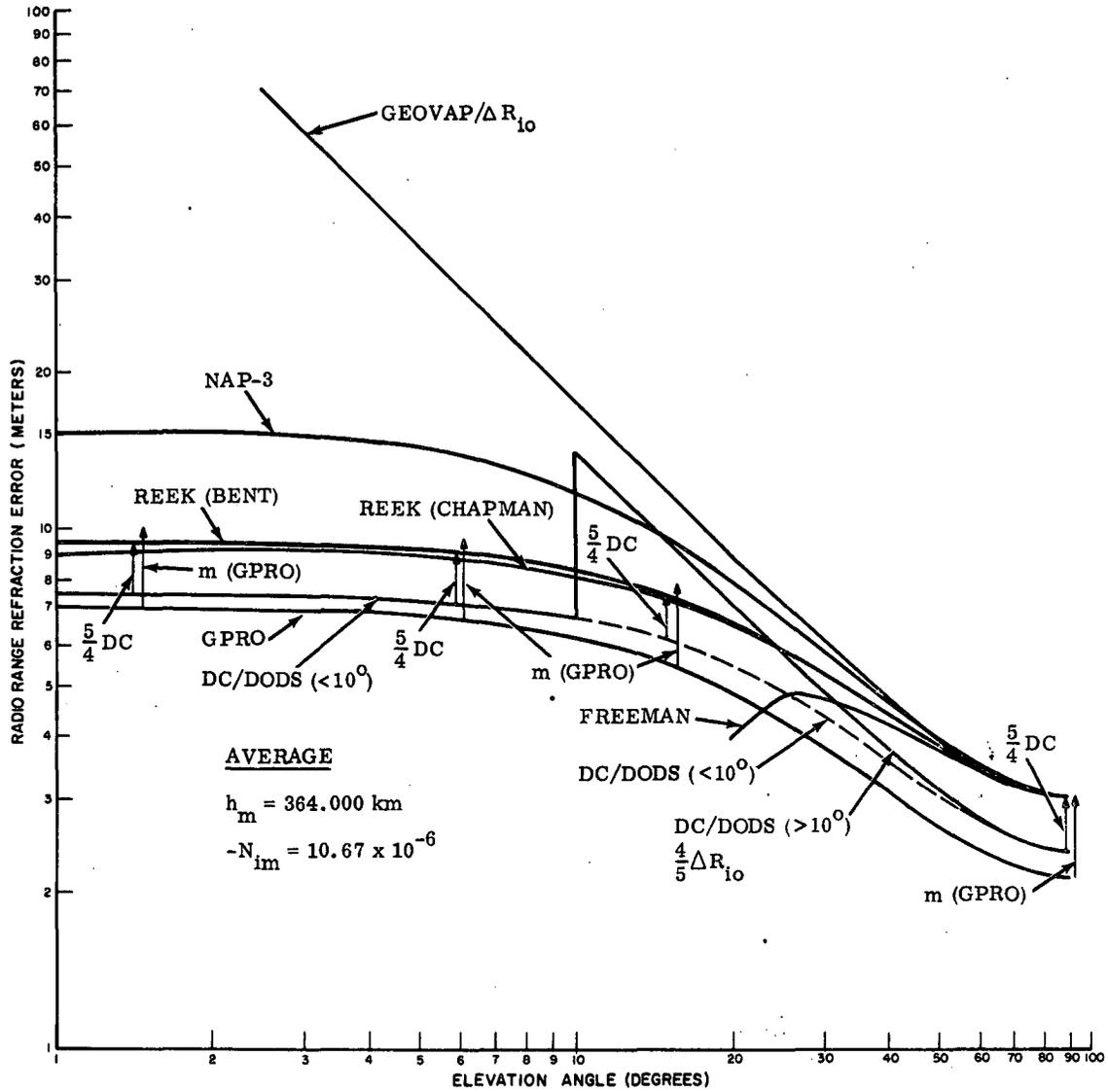


Figure 9. Range Error Due to Ionospheric Refraction

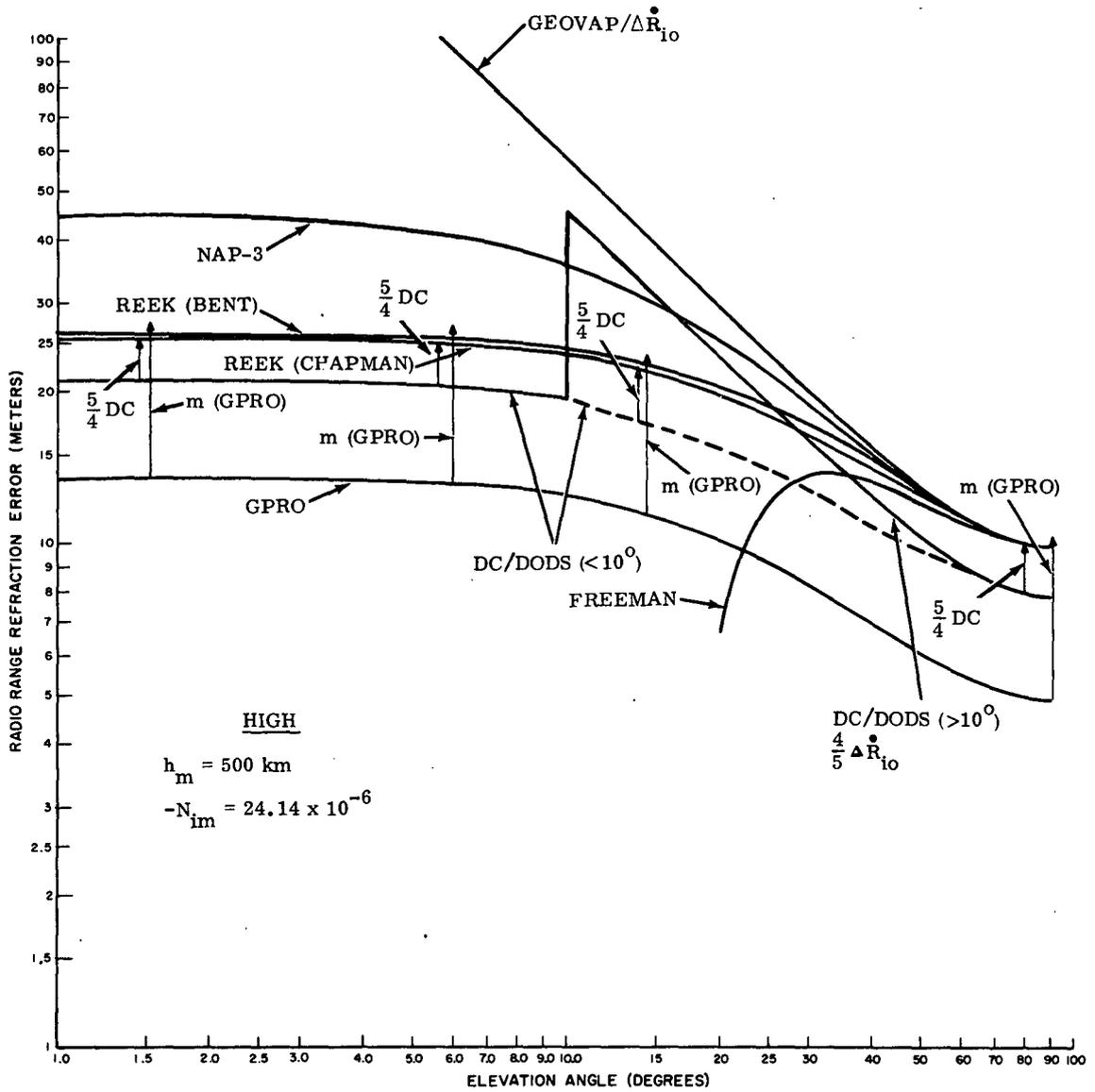


Figure 10. Range Error Due to Ionospheric Refraction

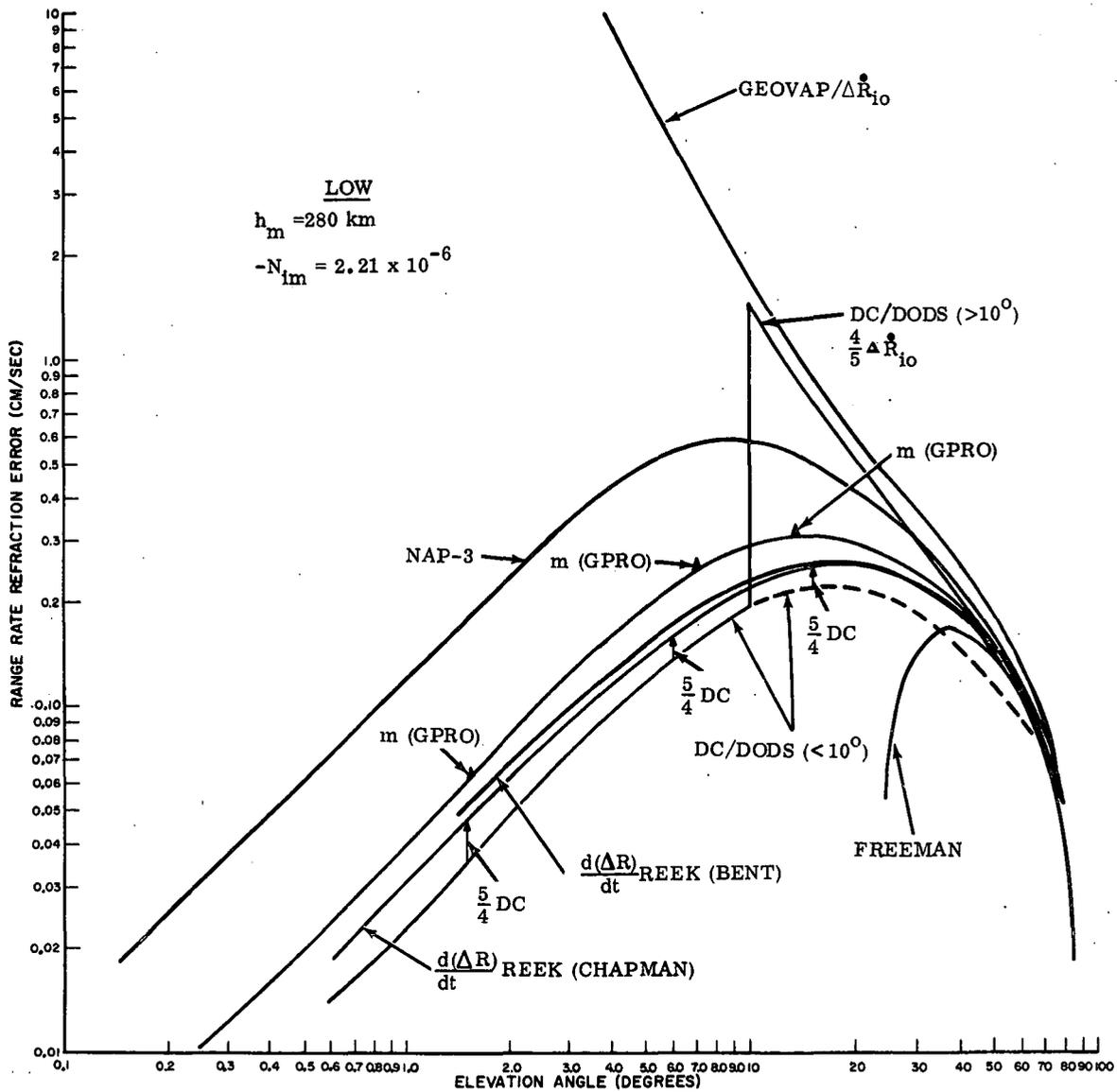


Figure 11. Range Rate Error Due to Ionospheric Refraction

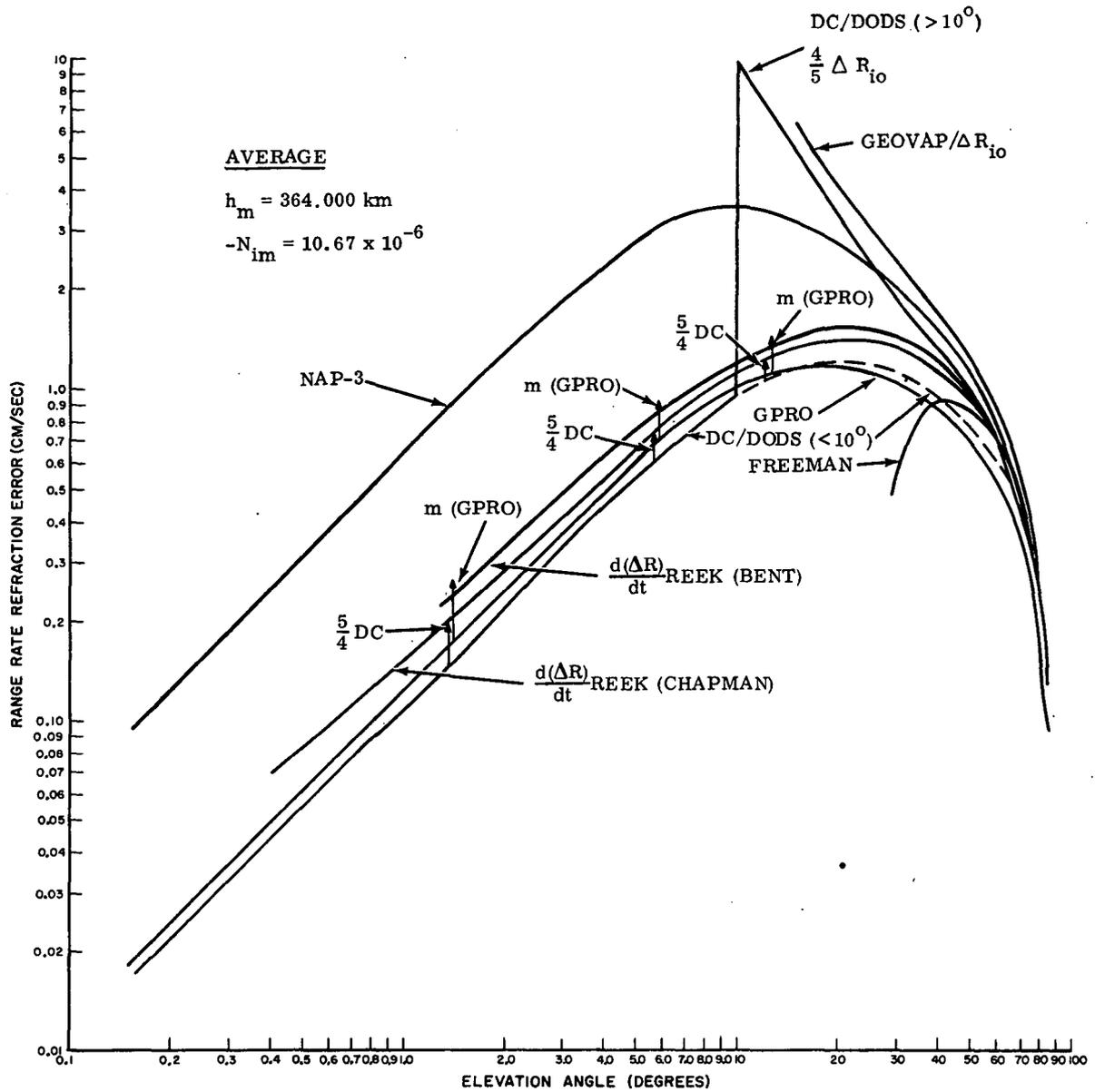


Figure 12. Range Rate Error Due to Ionospheric Refraction

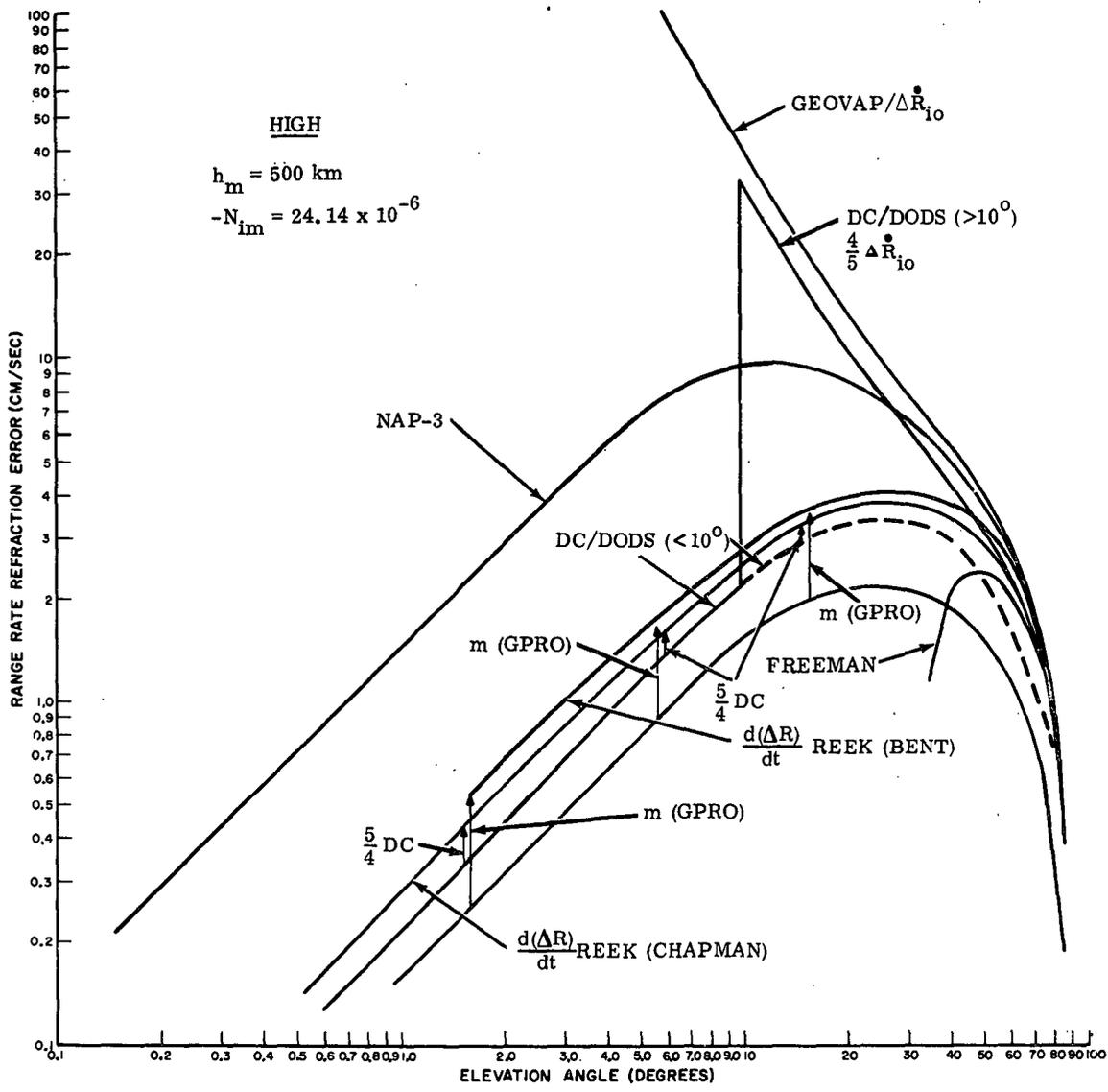


Figure 13. Range Rate Error Due to Ionospheric Refraction

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**APPENDIX A**  
**REFRACTION CORRECTION FORMULATIONS**

APPENDIX A  
REFRACTION CORRECTION FORMULATIONS

A.1 Different groups using data from the GEOS geodetic tracking systems employ different refraction correction equations. This appendix lists some of the different refraction correction equations, and describes how they are being used. For comparison of the equations, the original forms and notations are converted into a common form and notation.

A.2 GSFC ORBIT DIFFERENTIAL CORRECTION PROGRAM (DC)

The GSFC Orbit Differential Correction Program (DC) was developed by the Advanced Orbit Programming Branch at GSFC. It is used for operational orbit updating and contains optional refraction correction formulas for interferometer direction cosines and for range and range rate. It is documented in reference 2, program No. F116.

A.2.1 DC MINITRACK DIRECTION COSINE CORRECTION OR ELEVATION ANGLE CORRECTION

The direction cosine corrections are

$$l_c = \frac{l}{q} \text{ and } m_c = \frac{m}{q} \quad \text{equation (2), F116}$$

where

$l, m$  = measured direction cosines

$l_c, m_c$  = refraction corrected direction cosines

$$q = \frac{1 + N_o}{1 + A_s - \frac{N_1(h, t)}{h_t}} \quad \text{equation (4), F116}$$

$A_s = N_s = (\mu - 1)$  = tropospheric refractivity at the station; a constant for a given station for a given month. For example, a typical  $N_s = 0.000313$ .

$$1 + N_o = 1$$

Keeping the ionospheric part

$$\Delta \ell = \ell - \ell_c = \frac{+ \ell N_1 (h, t)}{h_t} \quad \Delta m = m - m_c = \frac{+ m N_1 (h, t)}{h_t}$$

are the changes in observed direction cosines due to refraction in the DC program.

As shown below, these equations are equivalent to a single refraction correction equation for elevation angle.

By definition  $\ell = \cos E \sin A$   $m = \cos E \cos A$

where E = apparent elevation angle of incident ray

A = apparent azimuth angle of incident ray

Refraction causes small changes in elevation angle  $\Delta E$ , but in azimuth,  $\Delta A = 0$ .

Therefore, taking the derivatives

$$\Delta \ell = -\Delta E \sin E \sin A \quad \Delta m = -\Delta E \sin E \cos A,$$

$$\Delta E = \frac{-\Delta \ell}{\sin E \sin A} = \frac{-\ell N_1 (h, t)}{\sin E \sin A h_t} \quad \Delta E = \frac{-\Delta m}{\sin E \cos A} = \frac{-m N_1 (h, t)}{\sin E \cos A h_t}$$

$$= -\frac{N_1 (h, t)}{h_t} \text{ctn } E \quad = -\frac{N_1 (h, t)}{h_t} \text{ctn } E$$

$$N_1 (h, t) = N_d H_d e^V \quad \text{equation (19), F116}$$

$$H_d = kH = \frac{4}{3} [30 + 0.2 (h_m - 200)] \quad \text{equation (8), F116}$$

$$k = \frac{4}{5} \quad \text{equation (6a) and (6b) of this document}$$

$$V = 1 - e^u \quad \text{equation (5), F116}$$

$$u = \frac{h_m - h}{H_d} \quad \text{equation (6), F116}$$

$$N_d = -N_{im} > 0$$

Average GEOS-2 height,  $h = \frac{4}{3} \times 10^6$  meters, and  $h_m = 364$  km; then

$$H_d = 83.73 \text{ km, and } u \cong -11.58$$

$$V = 1 - 9 \times 10^{-6} = 0.99999 \sim 1$$

$$e^V = 2.718255 = \sim e$$

$$N_1(h, t) = N_d H_d e^V \approx -N_{im} \left( \frac{4}{5} H \right) e^1$$

$$\Delta E = E - E_c = \frac{N_1(h, t)}{h_t} \text{ ctn } E = + \left( \frac{4}{5} \right) \frac{N_{im} H e^1}{h_t} \text{ ctn } E$$

$$\Delta E = - \frac{4}{5} \Delta E_{io} \quad (A1)$$

Thus, both of the equations (2, F116) lead to a single refraction correction equation for the elevation angle. Note that the sign is incorrect, that the correction is for a single ray and not an interferometer ray pair as intended, and that the  $\frac{4}{5}$  factor arises from assuming the equation (6b) rather than equation (6a) relation between scale height, H, and the F-layer height,  $h_m$ .

#### A.2.2 DC RANGE CORRECTION

From equation (12), F116, for  $E \leq 10^\circ$ ,

$$\Delta R_1 = R - R_c = C_1 A_s \text{ csc } \phi_1 + B_s N_1(h, t) \text{ csc } \phi_2$$

From equation (14), F116, for  $E > 10^\circ$ ,

$$\Delta R_2 = R - R_c = C_1 A_s \text{ csc } E + B_s N_1(h, t) \text{ csc } E$$

where  $R$  = measured range, and  $R_c$  = refraction corrected range

For the ionosphere alone with  $E \leq 10^\circ$

$$\Delta R_1 = B_s N_1(h, t) \text{ csc } \phi_2$$

where

$$\phi_2 = \cos^{-1} \left( \frac{R_e \cos E}{R_e + h_m} \right) \quad \text{equation (17), F116}$$

$$R_e = \text{earth radius} = 6,378,166 \text{ meters}$$

$$B_s = 1$$

$$\csc \phi_2 = \left[ 1 - \left( \frac{R_e \cos E}{R_e + h_m} \right)^2 \right]^{-1/2}$$

$$\Delta R_1 = -N_{im} H_d e^1 \left[ 1 - \left( \frac{R_e \cos E}{R_e + h_m} \right)^2 \right]^{-1/2}$$

Since  $H_d = \frac{4}{5} H$  and  $\Delta R_{io} = -N_{im} H e^1 \csc E$

For  $E \leq 10^\circ$

$$\Delta R_1 = -\frac{4}{5} N_i H e^1 \left[ 1 - \left( \frac{R_e \cos E}{R_e + h_m} \right)^2 \right]^{-1/2} = \frac{\Delta R_{io} \frac{4}{5} \sin E}{\left[ 1 - \left( \frac{R_e \cos E}{R_e + h_m} \right)^2 \right]^{1/2}} \quad (A2a)$$

For  $E > 10^\circ$

$$\Delta R_2 = -N_{im} H_d e^1 \csc E = + \frac{4}{5} \Delta R_{io} \quad (A2b)$$

### A.2.3 DC RANGE RATE CORRECTION

From equation (13) F116, for  $E \leq 10^\circ$

$$\Delta \dot{R}_1 = \dot{R} - \dot{R}_{c1} = \left\{ \left[ B_s \dot{\underline{r}}(t) \cdot \underline{r}^*(t) \right] \frac{dN_1(h, t)}{dh} \csc \phi_2 \right. \\ \left. - C_1 A_s \csc^2 \phi_1 \operatorname{ctn} \phi_1 \cos \phi_{im} \dot{E} \sin E \right. \\ \left. - B_s N_1(h, t) \csc^2 \phi_2 \operatorname{ctn} \phi_2 \left( \frac{R_e}{R_e + h_{ms}(t)} \right) \dot{E} \sin E \right\}$$

From equation (15) F116, for  $E > 10^\circ$

$$\Delta \dot{R}_2 = \dot{R} - \dot{R}_{c2} = \left\{ \left[ B_s \dot{\underline{r}}(t) \cdot \underline{r}^*(t) \right] \frac{dN_1(h, t)}{dh} \right. \\ \left. - \left[ C_1 A_s + B_s N_1(h, t) \right] E \operatorname{ctn} E \right\} \csc E$$

where

$\dot{R}$  = measured range rate

$\dot{R}_c$  = refraction corrected range rate

The ionospheric part for  $E \leq 10^\circ$  is

$$\Delta \dot{R}_1 = B_s \left[ \dot{\underline{r}}(t) \cdot \underline{r}^*(t) \right] \frac{dN_1(h, t)}{dh} \csc \phi_2$$

$$-B_s N_1(h, t) \csc^2 \phi_2 \operatorname{ctn} \phi_2 \left( \frac{R_e}{R_e + h_m} \right) \dot{E} \sin E$$

$$\frac{d}{dh} (N_1(h, t)) = \frac{N_{im}(h, t) e^u}{H_d} = -N_{im} e^v e^u \quad \text{equation (20), F116}$$

for GEOS height;  $N_{im} e^u \cong 0$

For  $E \leq 10^\circ$

$$\Delta \dot{R}_1 = +N_{im} H_d e^1 \left( \frac{R_e}{R_e + h_m} \right)^2 \dot{E} \cos E \sin E \left[ 1 - \left( \frac{R_e}{R_e + h_m} \right)^2 \cos^2 E \right]^{-3/2}$$

$$= +\frac{4}{5} \Delta \dot{R}_{10} \left( \frac{R_e}{R_e + h_m} \right)^2 \sin^3 E \left[ 1 - \left( \frac{R_e}{R_e + h_m} \right)^2 \cos^2 E \right]^{-3/2} \quad (\text{A3a})$$

For  $E > 10^\circ$

$$\Delta \dot{R}_2 = -N_1(h, t) \dot{E} \operatorname{ctn} E \csc E = N_{im} H_d e^1 \dot{E} \operatorname{ctn} E \csc E$$

$$= +\frac{4}{5} \Delta \dot{R}_{10} \quad (\text{A3b})$$

### A.3 FREEMAN CORRECTION PROGRAMS

The following formulas were formulated by J. J. Freeman Associates, Inc., under Contract NAS5-9782 for the Operations Evaluation Branch (OEB) at Goddard and are documented in reference 5. The purpose of the contract was to develop better refraction correction equations for use in the calibration of angle, range, and range rate systems.

### A.3.1 FREEMAN ELEVATION ANGLE CORRECTION

The Freeman elevation angle correction (flat-earth approximation) is

$$\Delta E = E - E_c = \text{ctn } E \left[ N_s \left( 1 - \frac{1}{dh_t} \right) - \frac{1}{h_t} \int_0^h N_i dh \right] \quad \begin{array}{l} \text{(A13)} \\ \text{ref. 5} \end{array}$$

where

$E$  = observed elevation angle

$E_c$  = refraction correction elevation angle

$N_s = (\mu - 1)$  = tropospheric refractivity at the station

Keeping the ionospheric part

$$\Delta E = - \frac{\text{ctn } E}{h_t} \int_0^{ht} N_i dh$$

$$\Delta E = -N_{im} \frac{H}{h_t} e^1 \text{ctn } E = \Delta E_{io} \quad \text{(A4)}$$

### A.3.2 FREEMAN RANGE CORRECTION

The Freeman range correction (quasi-flat-earth approximation) is

$$\Delta R = R - R_c = \frac{1}{\cos \alpha} \left[ \int_0^h |N| dh - \frac{\tan^2 \alpha}{R_e} \int_0^h |N| h' dh' \right] \quad \begin{array}{l} \text{(A25)} \\ \text{ref. 5} \end{array}$$

where

$R$  = electromagnetically determined range

$R_c$  = true refraction corrected range (meters)

$h$  = satellite height  $\approx 10^3$  km for GEOS

$h'$  = distance from earth surface to ray path

$R_e$  = distance from earth center to surface

$\alpha = (\pi/2 - E)$  = apparent zenith angle

$|N|$  = absolute value of tropospheric or ionospheric refractivity =  $|u - 1|$

$$\Delta R = \frac{1}{\sin E} \left[ \int_0^h |N| dh - \frac{\text{ctn}^2 E}{R_e} \int_0^h |N| h dh \right]$$

In the previous equation, if  $|N|$  assumes the value  $-N_i$

then the ionospheric formula is

$$\Delta R = -N_{im} H e^1 \left[ 1 - \left( \frac{H + h}{R_e} \right) \text{ctn}^2 E \right] \text{csc } E \quad \text{(A5)}$$

$$= \Delta R_{io} \left[ 1 - \left( \frac{H + h_m}{R_e} \right) \text{ctn}^2 E \right]$$

### A. 3.3 FREEMAN RANGE RATE CORRECTION

From equation (A51), reference 5

$$\begin{aligned} \Delta \dot{R} &= N_p \left[ V_R + V_\theta \tan \alpha \left( 1 - \frac{h}{R_s \cos \alpha} \right) \right] \\ &\quad - \frac{V_\theta \tan \alpha}{R \cos \alpha} \int_0^h |N| \left[ 1 + \frac{h'}{R_s} \left( 1 - \frac{3}{\cos^2 \alpha} \right) \right] dh' \end{aligned}$$

where

$$V_\theta = R \dot{E}$$

As  $h'$  approaches the GEOS satellite height of approximately  $10^3$  km, the refractivity at the target,  $N_p$ , approaches zero. Therefore the range rate correction equation reduces to

$$\Delta \dot{R} = - \frac{\dot{E} \cos E}{\sin^2 E} \left[ \int_0^h |N| dh + \left( \frac{1 - \frac{3}{\sin^2 E}}{R_e} \right) \int_0^{h_t} |N| h dh \right]$$

In the previous equation, if  $|N|$  assumes the value  $-N_i$

$$\begin{aligned} \Delta \dot{R} &= +N_{im} H e^1 \frac{\dot{E} \cos E}{\sin^2 E} \left[ 1 + \left( 1 - \frac{3}{\sin^2 E} \right) \left( \frac{H + h_m}{R_e} \right) \right] \\ &= +\Delta \dot{R}_{io} \left[ 1 + \left( 1 - \frac{3}{\sin^2 E} \right) \left( \frac{H + h_m}{R_e} \right) \right] \end{aligned} \quad (A6)$$

### A.5 GSFC GPRO PROGRAM

The GEOS Preprocessor Program (GPRO) was developed for the OEB at GSFC by DBA Systems, Inc., under contract NAS 5-9860, and is documented in references 4 and 22. It is used with the Geodetic Data Adjustment Program (GDAP), and with the Network Adjustment Programs (NAP-1 and NAP-2). The GPRO program utilizes a parabolic model for the ionosphere.

### A.5.1 GPRO RANGE REFRACTION CORRECTION

The GPRO range refraction correction, from reference 4, appendix C, section 1a, is

$$\Delta R_1 = \frac{2\beta_1}{D_1} = \frac{2\beta_1}{\sin E_1 + \left[ \sin^2 E_1 + 4\beta_2^2 \cos^2 E_1 \right]^{1/2}}$$

$$= \frac{2\beta_1 \csc E_1}{1 + \left[ 1 + 4\beta_2^2 \cot^2 E_1 \right]^{1/2}}$$

where  $\Delta R_1$  = correction for ionospheric refraction

$E_1$  = local elevation angle of ray upon entry to ionosphere

The angle  $E_1$  can be computed from

$$\cos E_1 = \frac{R_e \cos E}{R_e + h_m - 3H_g}$$

where

$E$  = apparent elevation angle at observer

$h_m - 3H_g$  = height of the base of the ionosphere

$\beta_1, \beta_2$  are given by the following functions of ionospheric parameters

$$\beta_1 = -2H_g \left( \frac{f_m}{f} \right)^2 = -2H_g \frac{80.5 N_m}{f^2} = -4 H_g N_{im}$$

$$4\beta_2^2 = \frac{25 H_g^2}{3 (R_e + h_m - 3H_g)}$$

where

$f$  = ranging frequency (or modulation frequencies)

$f_m$  = critical frequency ( $f_m^2 = 80.5 N_m$ )

$N_m$  = maximum electron density

$N_{im}$  = maximum refractivity,  $\left[ N_{im} = -\frac{1}{2} \left( \frac{f_m}{f} \right)^2 = \frac{-40.25 N_{im}}{f^2} \right]$

$h_m$  = height corresponding to maximum electron density

$H_g$  = thickness factor of parabolic ionosphere = 50 km

$R_e$  = radius of the earth

$$\Delta R_i = \frac{-8 H_g N_{im} \csc E_1}{1 + \left[ 1 + \frac{25 H_g \operatorname{ctn}^2 E_1}{3(R_e + h_m - 3H_g)} \right]^{1/2}} \quad (\text{A7})$$

#### A.5.2 GPRO RANGE RATE CORRECTION

The GPRO range rate refraction correction, from reference 22 is.

$$\Delta \dot{R} = \frac{-2\beta_1 \dot{D}_1}{D_1^2}$$

where

$$D_1 = \sin E_1 + (\sin^2 E_1 + 4\beta_2^2 \cos^2 E_1)^{1/2}$$

$$\dot{D}_1 = \left[ \cos E_1 + (\sin^2 E_1 + 4\beta_2^2 \cos^2 E_1)^{-1/2} (\sin E_1 \cos E_1 - 4\beta_2^2 \cos E_1 \sin E_1) \right] \dot{E}_1$$

$$\cos E_1 = \frac{R_e \cos E}{R_e + h_m - 3H_g}$$

$$\dot{E}_1 = \frac{(R_e \sin E) \dot{E}}{(R_e + h_m - 3H_g) \sin E_1}$$

$$\Delta \dot{R} = \frac{-2\beta_1 \left[ \cos E_1 + \frac{\sin E_1 \cos E_1 - 4\beta_2^2 \cos E_1 \sin E_1}{(\sin^2 E_1 + 4\beta_2^2 \cos^2 E_1)^{1/2}} \right] \dot{E}_1}{\left[ \sin E_1 + (\sin^2 E_1 + 4\beta_2^2 \cos^2 E_1)^{1/2} \right]^2}$$

$$\Delta \dot{R} = \frac{-2\beta_1 \dot{E}_1 \cos E_1 \left[ 1 + \frac{1 - 4\beta_2^2}{(1 + 4\beta_2^2 \operatorname{ctn}^2 E_1)^{1/2}} \right]}{\sin^2 E_1 \left[ 1 + (4\beta_2^2 \operatorname{ctn}^2 E_1)^{1/2} \right]^2}$$

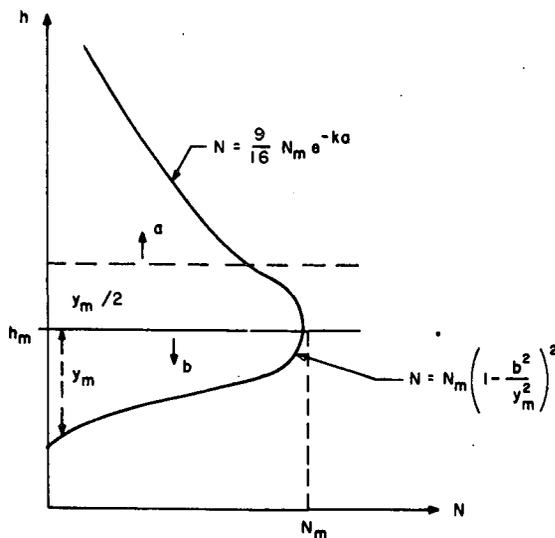
$$\Delta R = \frac{-8 N_{im} H_g \dot{E}_1 \operatorname{ctn} E_1 \operatorname{csc} E_1 \left[ 1 + \frac{1 - 4\beta_2^2}{(1 + 4\beta_2^2 \operatorname{ctn}^2 E_1)^{1/2}} \right]}{\left[ 1 + (1 + 4\beta_2^2 \operatorname{ctn}^2 E_1)^{1/2} \right]^2} \quad (\text{A8})$$

## A.6 GSFC NAP-3 PROGRAM

The Network Adjustment Program (NAP-3) formulas were formulated by DBA System, Inc. (Contract No. NAS5-17730), and are documented in reference 23. Further explanation of the Bent profile, NAP-3 Elevation Angle Correction, NAP-3 Range Correction, and NAP-3 Range Rate Correction is presented in the following paragraphs (reference 23 and supplementary notes by Dr. R. Bent).

### A.6.1 BENT PROFILE

The Bent profile was obtained from the analysis of many tens-of-thousands of topside and bottomside profiles and it fits closely to actual profiles. It contains a bipolarabolic lower portion and an exponential upper portion and is used in all NAP-3 computations on satellite refraction effects. The Bent profile is illustrated in Figure A-1.



529-A-1

Figure A-1. Bent Profile

where

$k$  = exponential decay constant  
 $N$  = electron density at height  $h$

$N_m$  = maximum electron density  
 $y_m$  = thickness factor for bipolarabolic model ionosphere

### A.6.2 NAP-3 ELEVATION ANGLE CORRECTION

The passage of a ray path through the ionosphere is illustrated in Figure A-2 from whence it can be shown that the deviation angle  $\alpha_1 = \alpha_2 = \alpha_3 =$ , say,  $\alpha$ .

The main angles under consideration are the angle of elevation  $E$ , the deviation angle  $\alpha_1$  (or  $\alpha$ ) and the error in the elevation angle  $\beta$ . The angle at the satellite between the line of sight and actual path is  $(\beta - \alpha)$ .

First, obtain the deviation angle  $\alpha$  and from this compute  $\beta$ .

The deviation angle  $\alpha_1 = \theta_T - \theta_1$ , but as  $\alpha_1 = \alpha_2 = \alpha_3$ , let them all be called  $\alpha$ :

$$\therefore \alpha = \theta_T - \theta_1 \quad (A9)$$

where

$\theta_T$  = the angle subtended at the center of the earth by the limits of the true ray path in the ionosphere

$\theta_1$  = the angle subtended from the same height limits by the apparent ray path.

Snell's law gives:

$$\rho \mu \sin i = \rho \sin \varphi = \text{constant} \quad (A10)$$

where

$i$  and  $\varphi$  = the angles of incidence of the true and apparent ray paths respectively at height  $\rho$ , the following equation is derived:

$$d\theta = \frac{\tan i}{\rho} d\rho \quad (A11)$$

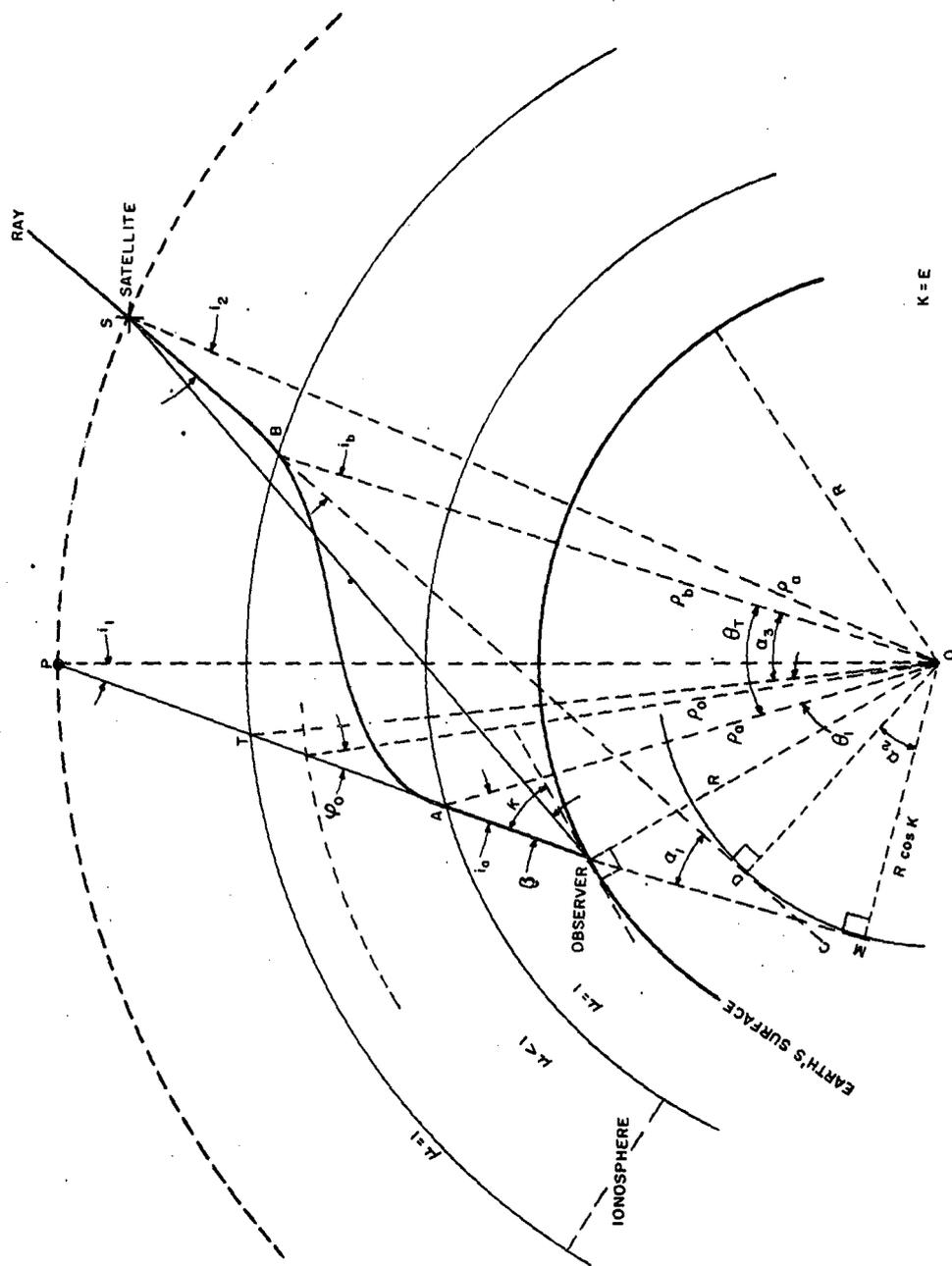
which, when combined with equation (A10) yields the following equation:

$$d\theta = \frac{\sin \varphi}{\rho \sqrt{\mu^2 - \sin^2 \varphi}} d\rho \quad (A12)$$

Simplifying the Appleton-Hartree equation and neglecting magnetic field effects (which can safely be done at very high frequencies) the equation for the index of refraction is given by

$$\mu^2 = 1 - \left( \frac{f_N}{f} \right)^2 \quad (A13)$$

where  $f$  = the frequency of transmission  $f_N$  = the plasma frequency at the height under investigation.



529-A-2

Figure A-2. Derivation of Elevation Correction

However, as the electron density  $N$  is proportional to  $f^2$ , equation (A13) can be rewritten including the critical frequency as:

$$\mu^2 = 1 - \left( \frac{f_c}{f} \right)^2 \cdot \frac{N}{N_m} \quad (\text{A14})$$

where

$f_c$  = the critical frequency of the F layer corresponding to the electron density  $N_m$

$N$  = the electron density at the height under investigation

Rewrite equation (A14) as follows:

$$\mu^2 = 1 - x \quad (\text{A15})$$

Combining equations (A12) and (A15) yields:

$$\theta_T = \int_a^b \frac{\sin \varphi}{\rho \sqrt{\cos^2 \varphi - x}} d\rho \quad (\text{A16})$$

where

$a$  and  $b$  = the lower and upper heights of the ionosphere.

To obtain  $\theta_1$  of equation (A9), assume that the ray travels in a straight line with  $\mu = 1$  and  $x = 0$ . Equation (A16) then yields:

$$\theta_1 = \int_a^b \frac{\tan \varphi}{\rho} d\rho. \quad (\text{A17})$$

Now the frequencies of vertical and oblique incidence radio waves are related by the expression:

$$f = f_v \sec \varphi$$

where

$f$  = the frequency of the oblique incidence wave,

$f_v$  = the equivalent vertical incidence frequency, and

$\varphi$  = the angle of incidence of the apparent oblique incidence ray path.

Now at the point of reflection  $\frac{f_v}{f} \sec \varphi = 1$  and if this function is less than unity the ray proceeds through the ionosphere and therefore the value of  $\frac{f_N}{f} \sec \varphi$  is a measure of the deviation of the penetrating ray. Its maximum value will occur at or just below the height of maximum electron density. An increase in this deviation factor indicates an increase in the deviation angle of the penetrating and apparent ray path. The maximum deviation occurring when the value of  $\frac{f_c}{f} \sec \varphi$  is just below unity. Let this deviation factor be  $u$  where:

$$u = \left( \frac{f_N}{f} \sec \varphi \right)^2 \quad (\text{A18})$$

which, from equations (A13) and (A15) gives

$$u = x \sec^2 \varphi.$$

Therefore equation (A16) can be rewritten as:

$$\theta_T = \int_a^b \frac{\tan \varphi}{\rho (1-u)^{\frac{1}{2}}} d\rho$$

hence

$$\alpha = \theta_T - \theta_1 = \int_a^b \frac{\tan \varphi}{\rho} \left[ (1-u)^{-\frac{1}{2}} - 1 \right] d\rho. \quad (\text{A19})$$

At the height of refraction  $u = 1$  and at the height of maximum electron density  $u$  is simply the square of the deviation factor and for all our purposes  $u < 1$ .

For all rays that penetrate the ionosphere we can apply the binomial theorem to equation (A19), to give:

$$\alpha = \frac{1}{2} \int_a^b \frac{u \tan \varphi}{\rho} \left[ 1 + \frac{3}{4} u + \frac{5}{8} u^2 + \frac{35}{64} u^3 + \dots \right] d\rho$$

hence

$$\alpha = \frac{1}{2} \int_a^b \frac{x \tan \varphi \sec^2 \varphi}{\rho} \left[ 1 + \frac{3}{4} u + \frac{5}{8} u^2 + \frac{35}{64} u^3 + \dots \right] d\rho.$$

Applying the second law of the mean for integrals:

$$\alpha = \frac{1}{2} \xi \int_a^b \frac{x \tan \varphi \sec^2 \varphi}{\rho} d\rho \quad (\text{A20})$$

where

$\xi$  = a number between the greatest and least values of the series over the interval (a, b).

$u = 0$  at the limits (a, b) and has a maximum value, less than unity, near the height of maximum electron density.

The value of  $\xi$  must therefore lie between unity and the value of the series at maximum electron density and is defined as the mean of these two values.

Applying the second law of the mean for integrals once more to equation (A20) yields:

$$a = \frac{1}{2} \xi \frac{\tan \varphi_0 \sec^2 \varphi_0}{\rho_0} \int_a^b x d\rho \quad (\text{A21})$$

where

$$\rho_0 = \rho_m \left(1 + 0.5333 \frac{y_m}{\rho_m}\right) = \text{some radius between limits a and b}$$

$$\rho_m = R_e + h_m$$

$\varphi_0$  = the value of  $\varphi$  at the radius  $\rho_0$

From Figure A2

$$\sin \varphi_0 = \frac{R_e}{\rho_0} \cos E \quad (\text{A22})$$

then

$$u_{\max} = \frac{\left(\frac{f_c}{f}\right)^2}{\left[1 - \left(\frac{R_e}{\rho_0}\right)^2 \cos^2 E\right]} \quad (\text{A23})$$

$\xi_{\text{mean}}$  is obtained from a table of  $u_{\max}$  vs  $\xi_{\text{mean}}$  values.  $\xi_{\text{mean}} \simeq 1$  for  $f = 2$  GHz.

Now,

$$x = \left(\frac{f_c}{f}\right)^2 \cdot \frac{N}{N_m} \quad (\text{A24})$$

Therefore, the deviation angle is given by

$$a = \frac{1}{2} \left(\frac{f_c}{f}\right)^2 \xi \frac{\tan \varphi_0 \sec^2 \varphi_0}{\rho_0} \cdot \frac{N_T}{N_m} \quad (\text{A25})$$

where  $N_T$  is given by  $\int_a^b N d\rho$  and is the total electron content of a vertical column of unit cross-section of the ionosphere.

It can now be shown from Figure A-2 that

$$\beta = \Delta E = \cos^{-1} \left[ \frac{X_1 \cos \alpha - X_2}{(X_1^2 + X_2^2 - 2X_1 X_2 \cos \alpha)^{1/2}} \right] \quad (\text{A26})$$

where

$$X_1 = (R_s^2 - R_e^2 \cos^2 E)^{1/2} + R_e \cos E \tan \left( \frac{\alpha}{2} \right)$$

$$X_2 = R_e \sin E - R_e \cos E \tan \frac{\alpha}{2}$$

$$R_s = R_e + h$$

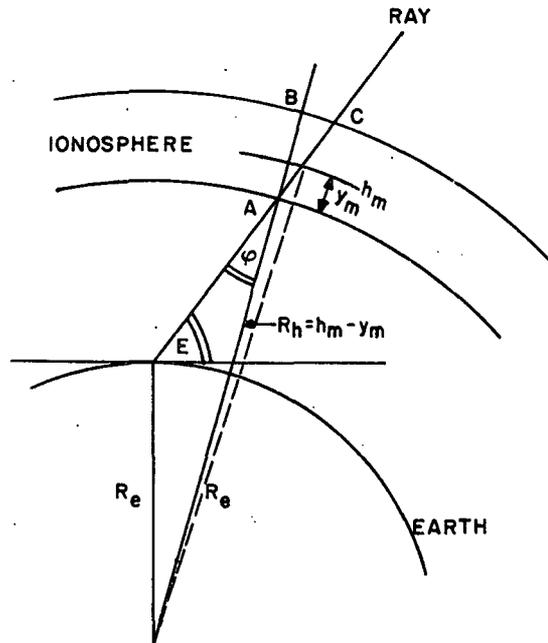
$R_e$  = radius of the earth

$h_t$  = height of the satellite

### A.6.3 NAP-3 RANGE CORRECTION (REFERENCE 23)

Assuming the total electron content ( $N_T$ ) has been computed in the vertical direction through the ionosphere along AB (Figure A-3), then the one-way value of

$$\Delta R = \frac{40.3}{f^2} \cdot \frac{N_T}{\cos \varphi} \text{ for the ray along AC.}$$



529-A-3

Figure A-3. Range Correction Diagram

If  $E$  is the satellite elevation angle:

$$\cos \varphi = \sqrt{1 - \left( \frac{R_e}{R_e + R_h} \right)^2 \cos^2 E},$$

where  $R_e$  is the radius of the earth and  $R_h$  is the height of the bottom of the ionosphere above the surface, ( $R_h = h_m - y_m$ ),

$$\therefore \Delta R = \frac{40.3 N_T}{f^2 \sqrt{1 - \left( \frac{R_e}{R_e + R_h} \right)^2 \cos^2 E}} \quad (A27)$$

When the satellite height,  $h_s$ , is above the top of the bipolar layer ( $h_m + \frac{y_m}{2}$ ), the equation  $N_T = \int_0^h N dh$  gives:

$$N_T = \int_0^{(h_s - h_m - \frac{y_m}{2})} \frac{9}{16} N_m e^{-ka} da + \int_{-\frac{y_m}{2}}^{\frac{y_m}{2}} N_m \left( 1 - \frac{b^2}{y_m^2} \right) db,$$

$$\therefore N_T = N_m \left\{ \frac{9}{16K} \left( 1 - e^{-(h_s - h_m - \frac{y_m}{2})K} \right) + \frac{459}{480} y_m \right\}.$$

From reference 23, equations 1, 3, and 4:

$$\Delta R = \left( \frac{f_o F_2}{f} \right)^2 \frac{40.3 \times 1.24 \times 10^{-2} \left[ \frac{9}{16K} \left( 1 - e^{-(h_s - h_m - \frac{y_m}{2})K} \right) + \frac{459}{480} y_m \right]}{\left[ 1 - \left( \frac{R_e}{R_e + R_h} \right)^2 \cos^2 E \right]^{1/2}}$$

$$\text{using } N_{im} \left( \frac{\text{electrons}}{m^3} \right) = 1.24 \times 10^{-2} (f_o F_2 \text{ cps})^2$$

$$\text{and } N_{im} = \frac{-40.3 N_m}{f^2}$$

we have

$$\Delta R = \frac{-N_{im} \left[ \frac{9}{16K} \left( 1 - e^{-(h_s - h_m - \frac{y_m}{2})K} \right) + \frac{459}{480} y_m \right]}{\left[ 1 - \left( \frac{R_e}{R_e + R_h} \right)^2 \cos^2 E \right]^{1/2}} \quad (A28)$$

#### A.6.4 NAP-3 RANGE-RATE CORRECTION

Equation (A27), when differentiated with respect to time, provides the range-rate correction.

$$\dot{\Delta R} = \frac{\partial(\Delta R)}{\partial E} \dot{E} + \frac{\partial(\Delta R)}{\partial h_s} \dot{h}_s = \frac{\partial(\Delta R)}{\partial E} \dot{E}$$

since

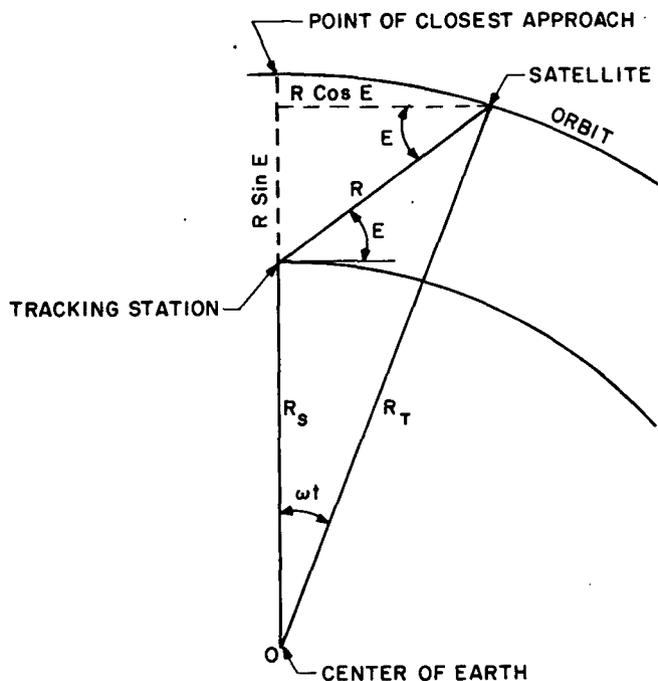
$$\dot{h}_s = 0 \text{ for our circular orbit,}$$

$$\dot{\Delta R} = - \frac{\dot{E} (\Delta R) \left( \frac{R_e}{R_e + R_h} \right)^2 \sin E \cos E}{1 - \left( \frac{R_e \cos E}{R_e + R_h} \right)^2} \quad (A29)$$

**APPENDIX B**  
**ELEVATION RATE DETERMINATION**

APPENDIX B  
ELEVATION RATE DETERMINATION

The required expression for elevation rate is derived from the geometry shown in Figure B-1. In this figure a circular overhead orbit around a spherical, stationary earth is assumed.



529-B-1

Figure B-1. Plane Containing Satellite, Tracking Station, and Center of Earth

$$\sin \omega t = \frac{R \cos E}{R_T}; \quad \cos E = \frac{R_T \sin \omega t}{R} \quad (B1)$$

$$\cos \omega t = \frac{(R_s + R \sin E)}{R_T}; \quad \sin E = \frac{(R_T \cos \omega t - R_s)}{R} \quad (B2)$$

$$R = (R_s^2 + R_T^2 - 2R_T R_s \cos \omega t)^{1/2} \quad (B3)$$

substitute (B3) into (B1).

$$\cos E = R_T \sin \omega t (R_S^2 + R_T^2 - 2R_T R_S \cos \omega t)^{-1/2}$$

then by differentiation,

$$-\sin E \frac{dE}{dt} = \omega R_T R^{-1} \cos \omega t - \omega R_T R^{-3} (R_T R_S \sin^2 \omega t) \quad (B4)$$

substitute (B2) into (B4)

$$-R^{-1} (R_T \cos \omega t - R_S) \frac{dE}{dt} = \omega R_T R^{-1} (R^2 \cos \omega t - R_T R_S \sin^2 \omega t) (R^{-2})$$

$$\frac{dE}{dt} = \left[ -\omega R_T (R_T^2 \cos \omega t + R_S^2 \cos \omega t - 2R_T R_S \cos^2 \omega t - R_T R_S + R_T R_S \cos^2 \omega t) \right] \left[ R^2 (R_T \cos \omega t - R_S) \right]^{-1}$$

$$\frac{dE}{dt} = \frac{\left[ -\omega R_T (R_T - R_S \cos \omega t) (R_T \cos \omega t - R_S) \right]}{R^2 (R_T \cos \omega t - R_S)}$$

$$\frac{dE}{dt} = -\omega R_T (R_T - R_S \cos \omega t) (R_T^2 + R_S^2 - 2R_T R_S \cos \omega t)^{-1} \quad (B5)$$

$R_T$  = distance from the center of earth to the satellite (7,711,499 meters)

$R_S$  = distance from the center of earth to the station (6,378,166 meters)

$R$  = distance from the station to the satellite

$\omega$  = angular velocity of the satellite =  $\frac{2\pi}{T}$

$T$  = period of the satellite =  $T_c \left( \frac{R_T}{R_S} \right)^{3/2}$  (page 96 of reference 18)

$T_c$  = 84.347 minutes, the period of a circular orbit at the earth's surface