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COMPUTER PROGRAM FOR SOLVING COMPRESSIBLE NONSIMILAR-BOUNDARY-LAYER EQUATIONS FOR LAMINAR, TRANSITIONAL, OR TURBULENT FLOWS OF A PERFECT GAS

by Joseph M. Price and Julius E. Harris

Langley Research Center
Hampton, Va. 23365

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1972

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM X-2458		

4. Title and Subtitle	5. Report Date
COMPUTER PROGRAM FOR SOLVING COMPRESSIBLE NONSIMILAR-BOUNDARY-LAYER EQUATIONS FOR LAMINAR, TRANSITIONAL, OR TURBULENT FLOWS OF A PERFECT GAS	April 1972
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Joseph M. Price and Julius E. Harris	L-8037
	10. Work Unit No.
9. Performing Organization Name and Address	136-13-05-04
NASA Langley Research Center Hampton, Va. 23365	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Memorandum
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A computer program is described which solves the compressible laminar, transitional, or turbulent boundary-layer equations for planar or axisymmetric flows. Three-point implicit difference relations are used to reduce the momentum and energy equations to finite-difference form. These equations are solved simultaneously without iteration. Turbulent flow is treated by the inclusion of either a two-layer eddy-viscosity model or a mixing-length formulation. The eddy conductivity is related to the eddy viscosity through a static turbulent Prandtl number which may be an arbitrary function of the distance from the wall boundary. The transitional boundary layer is treated by the inclusion of an intermittency function which modifies the fully turbulent model. The laminar-boundary-layer equations are recovered when the intermittency is zero, and the fully turbulent equations are solved when the intermittency is unity. This program was used to obtain the solutions presented in NASA TR R-368.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Computer program Boundary layer Compressible Laminar, transitional, or turbulent	Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	114	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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By Joseph M. Price and Julius E. Harris
Langley Research Center

SUMMARY

A computer program is described which solves the compressible laminar, transitional, or turbulent boundary-layer equations for planar or axisymmetric flows by an implicit finite-difference procedure. The program was used to obtain the solutions presented in NASA TR R-368. Turbulent flow is treated by the inclusion of either a two-layer eddy-viscosity model or a mixing-length formulation. The eddy conductivity is related to the eddy viscosity by the static turbulent Prandtl number which may be an arbitrary function of the distance from the wall boundary. The transitional boundary layer is treated by introducing an intermittency function which modifies the fully turbulent model. The intermittency function describes the probability distribution of turbulent spots and ranges from zero for laminar flow to unity for a fully turbulent flow.

INTRODUCTION

A number of finite-difference methods are currently available for computing the development of compressible turbulent boundary layers. (See, for example, refs. 1 to 8.) The numerical methods used to solve the governing equations in these references are generally different, in particular those given in references 7 and 8; however, the results are similar when common eddy-viscosity formulations are used. References 1 to 7 use implicit or Crank Nicolson type differences to reduce the governing differential equations to finite-difference form, whereas explicit-type differences are used in reference 8. A coupled solution technique is used in reference 7 (see also ref. 9) which requires no iteration procedure. However, the methods used in references 1 to 6 require iteration since the momentum and energy equations are uncoupled. Another difference, of course, among the various methods is in the formulation of the eddy-viscosity and turbulent Prandtl number formulations used to model the turbulent flux terms appearing in the mean flow equations.

This report describes a computer program developed to solve the compressible nonsimilar-boundary-layer equations for laminar, transitional, or turbulent flows of a perfect gas. The program was used to obtain the results reported in references 7 and 10. A coupled, implicit finite-difference procedure similar to that used for laminar flows in references 9 and 11 is used to solve the momentum and energy equations without iteration. The program will solve problems for two-dimensional or axisymmetric flow geometry for the flow of a perfect gas. Currently, power-law and perfect-air viscosity (Sutherland's) relations are included in the code; however, any perfect gas can be treated by inserting the correct viscosity temperature relations. Transverse curvature effects are included with the option of being neglected if desired. The equations are solved so that nonsimilar terms may also be neglected if desired.

Options are provided for either a two-layer eddy-viscosity model or an arbitrary mixing-length formulation. The turbulent Prandtl number may be either a constant or a function of distance normal to the wall boundary. The transitional region of the flow is modeled through an intermittency distribution which modifies the fully turbulent eddy-viscosity models. Transition location and extent must be specified either from experimental data or correlation relations. The laminar equations are recovered by setting the intermittency to zero.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

A damping function, $26\nu/u_\tau$

$A^+ = Au_\tau/\nu$

$\left.\begin{matrix} A1_n, B1_n, C1_n, D1_n, \\ E1_n, F1_n, G1_n \end{matrix}\right\}$ coefficients in difference equation (43a) and defined by equations (B3) to (B9)

$\left.\begin{matrix} A2_n, B2_n, C2_n, D2_n, \\ E2_n, F2_n, G2_n \end{matrix}\right\}$ coefficients in difference equation (43b) and defined by equations (B10) to (B16)

C_f skin-friction coefficient, $\dfrac{\tau_w}{\frac{1}{2}\rho u^2}$

C_{m1}, C'_{m1} defined in equations (B45) and (B46), respectively

c_p specific heat at constant pressure

$\overline{E}_{m1}, \hat{E}_{m1}$ defined in equations (B36) and (B37), respectively

$\hat{E}_Y$ defined in equation (B39)

F velocity ratio, u/u_e

F_{m1} defined in equation (B29)

F_{m2} defined in equation (B32)

F_Y defined in equation (B40)

G,H typical variables in the boundary layer (see appendix A)

$H_1, H_2, H_3, \ldots, H_{12}$ coefficients defined by equations (B17) to (B28)

h heat-transfer coefficient

i index used in grid-point notation (see eq. (41))

j flow index; $j = 0$ in planar flow, $j = 1$ in axisymmetric flow

k grid-point spacing parameter (see eq. (41))

k_l thermal conductivity

k_T eddy conductivity (see eq. (15))

k_1 constant in eddy-viscosity model (see eq. (6))

k_2 constant in eddy-viscosity model (see eq. (7))

k_3 see equation (8)

k_4 constant in intermittency function (see eq. (10))

L reference length

L_{m1}, L'_{m1} defined in equations (B34) and (B35), respectively

l defined in equation (30)

$\bar{l}$ mixing length (see eq. (13))

M Mach number

m grid-point index in X-direction (see fig. 2)

N number of grid points at each x-station (see fig. 2)

N_{Pr} Prandtl number, $c_p\mu/k_l$

$N_{Pr,t}$ static turbulent Prandtl number (see eqs. (16) and (17))

N_{St} Stanton number, $h/(c_p\rho u)$

n grid-point index in Y-direction (see fig. 2)

$P^{(1)}, P^{(2)}, P^{(3)}$ defined in equations (48)

p pressure

$Q^{(1)}, Q^{(2)}, Q^{(3)}$ defined in equations (48)

q heat-transfer rate

R,Z body axes system with origin at the stagnation point, where Z is positive downstream and R is positive radially outward (see fig. 1)

R_e unit Reynolds number, u_e/ν_e

$R_{e,x}$ Reynolds number based on x, $u_e x/\nu_e$

$R_{e,x_{t,i}}$ Reynolds number at transition, $u_e x_{t,i}/\nu_e$

R_{e,δ^*} Reynolds number based on displacement thickness, $u_e\delta^*/\nu_e$

$R_{e,\theta}$ Reynolds number based on momentum thickness, $u_e\theta/\nu_e$

R_g gas constant (see eq. (19))

r radial body coordinate measured normal to Z-axis (see fig. 1)

r_o body radius (see fig. 1)

S Sutherland's viscosity constant, 110.3° K (198.6° R)

T static temperature

T_Y defined in equation (B41)

T_{aw} adiabatic wall temperature

T_{m1}, T_{m2} defined in equations (B30) and (B33), respectively

t transverse curvature term (see eq. (23))

u velocity component in X-direction (fig. 1)

u^+ law of wall coordinate, u/u_τ

u_τ friction velocity, $\sqrt{\tau_w/\rho}$

V transformed normal-velocity component (see eq. (26))

V_{m1} defined in equation (B31)

v velocity component in Y-direction

$\tilde{v}$ velocity component, $v + \frac{\rho' v'}{\rho}$

X,Y orthogonal boundary-layer coordinate system with origin at stagnation point, where X lies along the body surface and is positive downstream and Y is normal to the body surface and positive outward (see fig. 1)

$X_1, X_2, \ldots, X_5$ functions of grid-point distribution (see eqs. (A4) to (A8))

x	boundary-layer coordinate along X-axis (see fig. 1)
$x_{t,f}$	end of transition (see fig. 1)
$x_{t,i}$	beginning of transition (see fig. 1)
$Y_1, Y_2, \ldots, Y_6$	functions of grid-point distributions (see eqs. (A12) to (A17))
y	boundary-layer coordinate along Y-axis (see fig. 1)
y^+	law of wall coordinate, yu_τ/ν
y_m	match point for two-layer eddy-viscosity model
z	axial body coordinate (see fig. 1)
α	defined in equation (30)
β	defined in equation (30)
Γ	streamwise intermittency distribution (see eq. (38))
γ	ratio of specific heats
$\bar{\gamma}$	transverse intermittency distribution (see eq. (10))
Δ^*	defined in equation (48g)
$\Delta x, \Delta y$	grid-point spacing, physical plane
Δx_t	transition extent, $x_{t,f} - x_{t,i}$
$\Delta\xi, \Delta\eta$	grid-point spacing, transformed plane (see fig. 2)
δ	boundary-layer thickness
δ^*	displacement thickness
δ^*_{inc}	incompressible displacement thickness, $\int_0^\infty (1 - F)\,dy$

$\delta_w^+ = \delta u_{\tau,w}/\nu_w$

ϵ	eddy viscosity, $-\rho\frac{\overline{u'v'}}{\partial u/\partial y}$
$\bar{\epsilon}$	eddy-viscosity function (see eq. (4))
$\bar{\epsilon}_{av}$	defined in equation (B36b)
$\tilde{\epsilon}$	eddy-viscosity function (see eq. (5))
η	transformed normal boundary-layer coordinate (see fig. 2)
Θ	static-temperature ratio, T/T_e
θ	momentum thickness (see fig. 1)
θ_s	shock-wave angle (see fig. 1)
λ	defined in equation (40)
μ	molecular viscosity
ν	kinematic viscosity
$\bar{\nu}$	average kinematic viscosity
ξ	transformed streamwise boundary-layer coordinate (see fig. 2)
$\bar{\xi}$	defined in equation (39)
ρ	density
σ	exponent in power-law viscosity relation (see eq. (21))
τ	shear stress
ϕ	local surface angle (see fig. 1)
χ	vorticity Reynolds number, $\frac{y^2}{\nu}\frac{\partial u}{\partial y}$

χ_{max} maximum local value of χ

ψ stream function

$\omega = \left(\frac{\rho_r u_r L}{\mu_r}\right)^{-1/2}$

Subscripts:

e based on boundary-layer edge conditions

i inner region of turbulent layer

m mesh point in ξ-direction (see fig. 2)

n mesh point in η-direction (see fig. 2)

o outer region of turbulent layer

r reference quantity

s shock

t total condition

w wall value

∞ free stream

Superscript:

j flow index; $j = 0$ in planar flow, $j = 1$ in axisymmetric flow

A prime on a symbol denotes a fluctuating component.

A bar over a symbol denotes the time average value.

A coordinate used as a subscript denotes the partial differential with respect to the coordinate. (See eqs. (A1).)

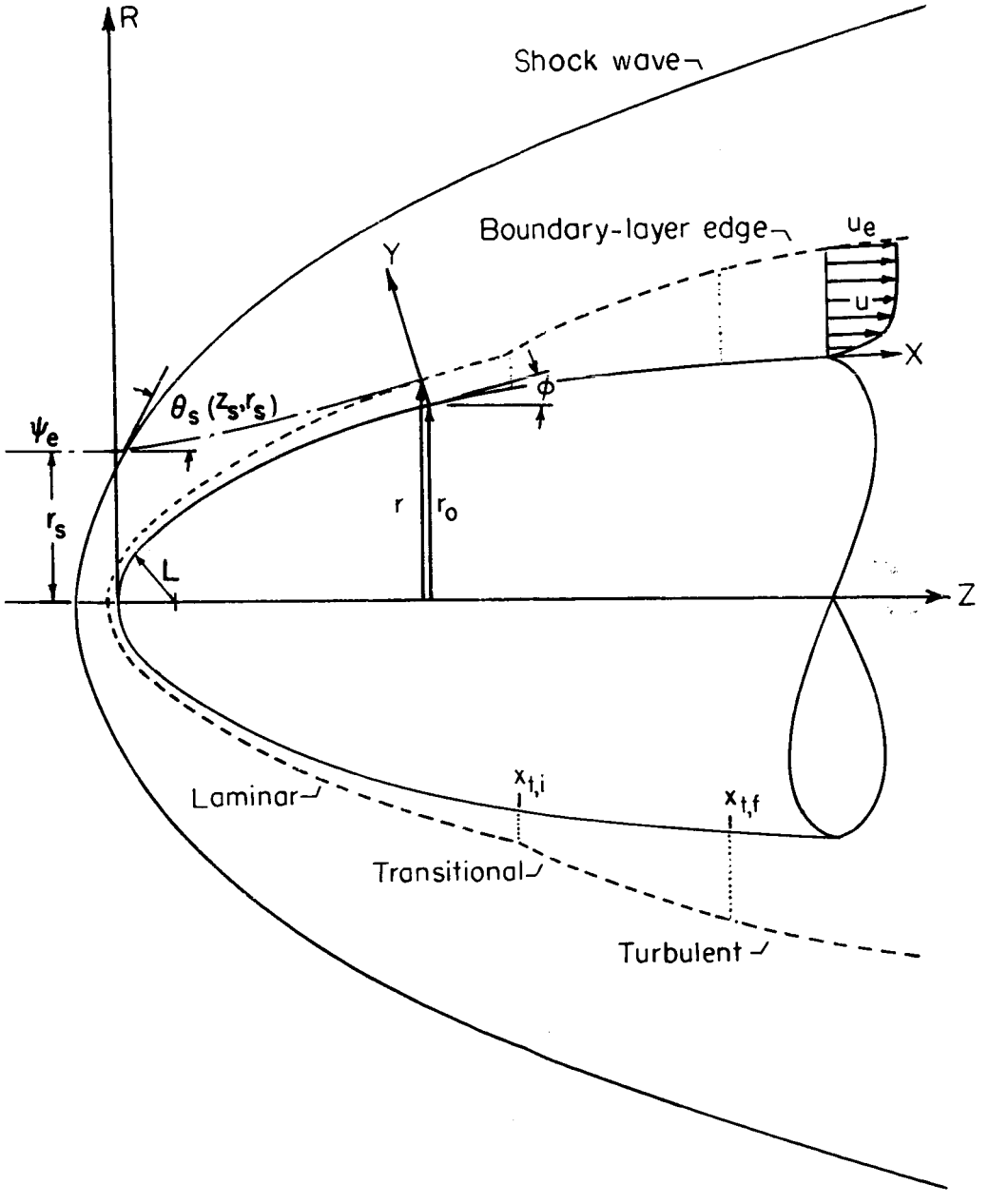


Figure 1.- Coordinate system and notation.

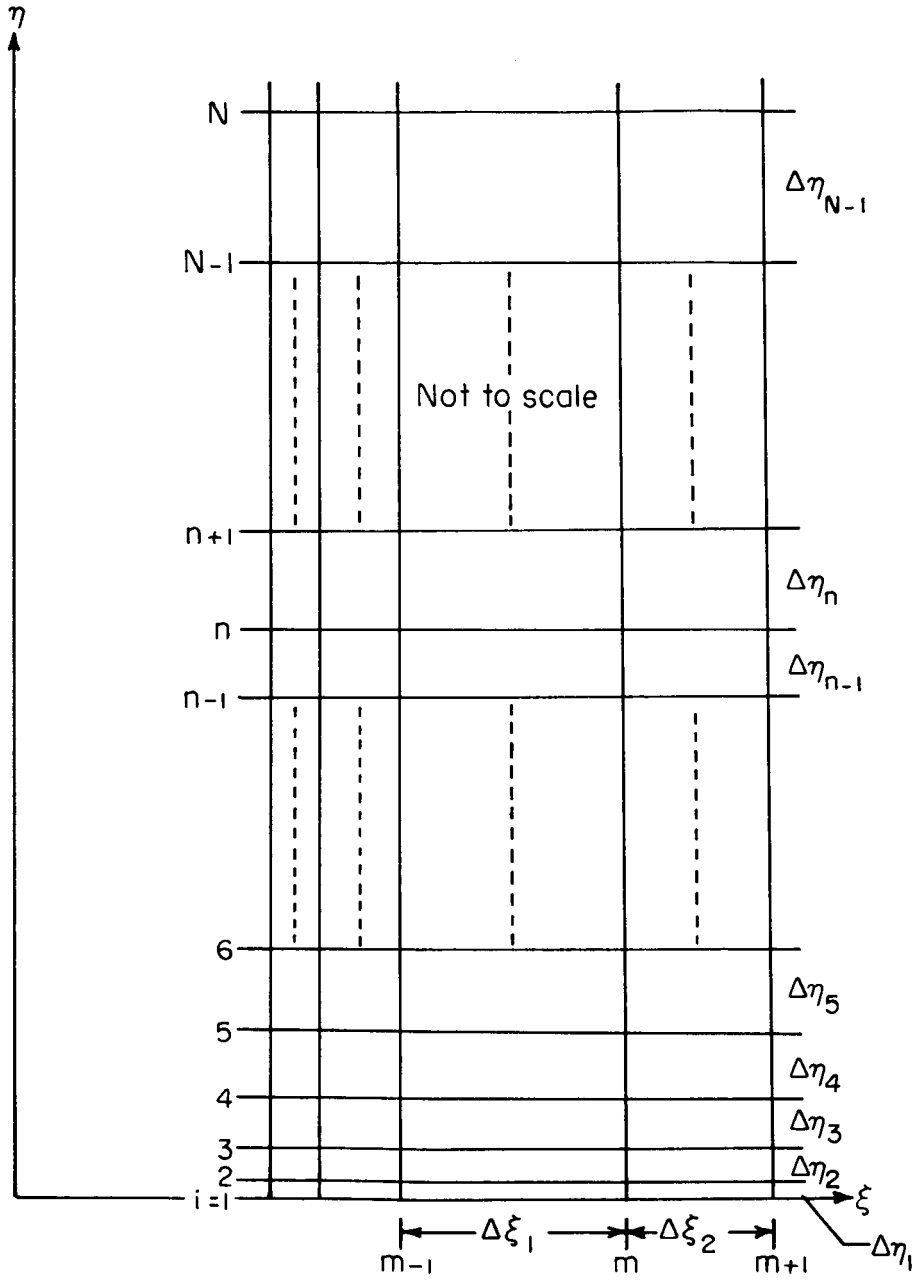


Figure 2.- Finite-difference grid model.

PROBLEM DESCRIPTION

This section presents the governing equations for compressible laminar, transitional, or turbulent boundary-layer flows together with the required boundary conditions. The eddy viscosity, eddy conductivity, transition location and extent, and transitional-flow-structure models are presented and briefly discussed; however, the reader interested in a detailed discussion of these models is referred to references 7 and 10.

Basic Partial Differential Equations

Governing equations.- The partial differential equations describing laminar, transitional, or turbulent compressible boundary-layer flows over planar or axisymmetric geometries are as follows (see refs. 7 and 12):

Continuity

$$\frac{\partial}{\partial x}\left(r^j \rho u\right) + \frac{\partial}{\partial y}\left(r^j \rho \tilde{v}\right) = 0 \tag{1}$$

Momentum

$$\rho\left(u\frac{\partial u}{\partial x} + \tilde{v}\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \frac{1}{r^j}\frac{\partial}{\partial y}\left(r^j \bar{\epsilon}\frac{\partial u}{\partial y}\right) \tag{2}$$

Energy

$$\rho\left[u\frac{\partial}{\partial x}\left(c_p T\right) + \tilde{v}\frac{\partial}{\partial y}\left(c_p T\right)\right] = u\frac{dp}{dx} + \bar{\epsilon}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{r^j}\frac{\partial}{\partial y}\left[r^j \tilde{\epsilon}\frac{\partial}{\partial y}\left(c_p T\right)\right] \tag{3}$$

where the conventional overbar notation for time mean-average variables has been dropped for brevity. The eddy-viscosity parameters $\bar{\epsilon}$ and $\tilde{\epsilon}$ are defined, respectively, as follows:

$$\bar{\epsilon} = \mu\left(1 + \frac{\epsilon}{\mu}\Gamma\right) \tag{4}$$

and

$$\tilde{\epsilon} = \frac{\mu}{N_{Pr}}\left(1 + \frac{\epsilon}{\mu}\frac{N_{Pr}}{N_{Pr,t}}\Gamma\right) \tag{5}$$

The intermittency-distribution parameter Γ is discussed in a subsequent section. (See eq. (38).)

Eddy viscosity.- Options are provided in the coded program for selecting either a two-layer eddy-viscosity model (KODVIS = 1) or a straight-forward mixing-length model (KODVIS = 2).

Two-layer model.- The equations describing the two-layer model are as follows (see ref. 7):

$$\left(\frac{\epsilon}{\mu}\right)_i = \frac{\rho}{\mu}(k_1 y D)^2 \left|\frac{\partial u}{\partial y}\right| \qquad \left(0 \leqq y \leqq y_m\right) \tag{6}$$

$$\left(\frac{\epsilon}{\mu}\right)_o = \frac{\rho}{\mu} k_2 u_e \delta^*_{inc} \bar{\gamma} \qquad \left(y_m < y\right) \tag{7}$$

where

$$D = 1 - \exp\left\{-\left[\sqrt{\frac{\nu_w}{\bar{\nu}}}\left(1 + k_3\right) - k_3\right]\frac{y}{A}\right\} \tag{8}$$

$$\delta^*_{inc} = \int_0^\infty \left(1 - \frac{u}{u_e}\right) dy \tag{9}$$

and

$$\bar{\gamma} = \frac{1 - \operatorname{erf}\left[5\left(\frac{y}{\delta} - k_4\right)\right]}{2} \tag{10}$$

The boundary-layer thickness δ appearing in equation (10) is defined as the distance normal to the wall boundary where $u/u_e = 0.995$. The empirical constants k_1, k_2, k_3, and k_4 are assigned values of 0.4, 0.0168, 0.0, and 0.78, respectively. Note that for $k_3 = -1.0$, the kinematic-viscosity term is removed from equation (8) (XT5 = -1.0). The empirical constants k_1 and k_2 can easily be treated as functions of some correlation parameter, say δ_w^+, in order to account properly for low Reynolds number effects in hypersonic flows. (See ref. 13 for discussion of low Reynolds number effects.) The location of the boundary separating the two layers y_m is determined from the continuity of eddy viscosity; that is, where

$$\left(\frac{\epsilon}{\mu}\right)_i = \left(\frac{\epsilon}{\mu}\right)_o \tag{11}$$

Mixing-length model.- A mixing-length formulation is provided (KODVIS = 2) for those interested in utilizing experimental mixing-length distributions. (See ref. 13, for example.) The eddy-viscosity distribution across the boundary layer can be written as follows:

$$\frac{\epsilon}{\mu} = \frac{\rho}{\mu} \bar{l}^2 \left|\frac{\partial u}{\partial y}\right| \tag{12}$$

where the mixing length $\bar{l}$ may be written as

$$\frac{\bar{l}}{\delta} = D\bar{\gamma}f\left(\frac{y}{\delta}\right) \tag{13}$$

Currently, the simplest possible formulation is provided in the digital code for $f\left(\frac{y}{\delta}\right)$ as follows:

$$f\left(\frac{y}{\delta}\right) = \begin{cases} 0.4\left(\frac{y}{\delta}\right) & \left(\frac{y}{\delta} \leqq 0.2\right) \\ 0.08 & \left(\frac{y}{\delta} > 0.2\right) \end{cases} \tag{14}$$

However, it should be noted that any functional variation can be utilized in the program.

Eddy conductivity and static turbulent Prandtl number.- The eddy conductivity defined as

$$k_T = -c_p\rho \frac{\overline{v'T'}}{\partial T/\partial y} \tag{15}$$

is modeled as a static turbulent Prandtl number $N_{Pr,t}$ as follows:

$$N_{Pr,t} = \frac{c_p\epsilon}{k_T} \tag{16}$$

where

$$N_{Pr,t} = \frac{\overline{u'v'}}{\overline{v'T'}}\left(\frac{\partial T/\partial y}{\partial u/\partial y}\right) \tag{17}$$

Any desired functional relation for $N_{Pr,t} = f\left(\frac{y}{\delta}\right)$ may be utilized in the digital code; three options are available. These options are (1) a constant, say $N_{Pr,t} = 0.95$, (2) an arbitrary distribution $N_{Pr,t} = f\left(\frac{y}{\delta}\right)$ supplied in tabular form, and (3) the Rotta distribution (see ref. 14) as follows:

$$N_{Pr,t} = 0.45\left[2 - \left(\frac{y}{\delta}\right)^2\right] \tag{18}$$

The system of equations is closed by the addition of the perfect-gas laws and a viscosity-temperature relation. The perfect-gas law is expressed as

$$P = \rho R_g T \tag{19}$$

Currently, the digital code is written to include the Sutherland viscosity-temperature relation for air (IGAS = 1)

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{3/2}\left(\frac{T_r + S}{T + S}\right) \tag{20}$$

as well as the power-law expression

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{\sigma} \tag{21}$$

where $\sigma = 0.647$ for helium (IGAS = 2).

Transformed plane.- The system of governing equations is singular at $x = 0$. The Probstein-Elliott (ref. 15) and Levy-Lees (ref. 16) transformation is used to remove this singularity as well as to reduce the growth of the boundary layer as the solution proceeds downstream. This transformation can be written as follows:

$$\xi(x) = \int_0^x \rho_e u_e \mu_e r_o^{2j}\, dx \tag{22a}$$

$$\eta(x,y) = \frac{\rho_e u_e r_o^j}{\sqrt{2\xi}} \int_0^y t^j \left(\frac{\rho}{\rho_e}\right) dy \tag{22b}$$

where the parameter t appearing in equation (22b) is the transverse curvature term, defined as

$$t = 1 + \frac{r}{r_o} \tag{23a}$$

or, in terms of the y-coordinate, as

$$t = 1 + \frac{y}{r_o} \cos \phi \tag{23b}$$

The relation between derivatives in the physical (x,y) and transformed (ξ,η) coordinate system is as follows:

$$\left(\frac{\partial}{\partial x}\right)_y = \rho_e u_e \mu_e r_o^{2j} \left(\frac{\partial}{\partial \xi}\right)_\eta + \left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial}{\partial \eta}\right)_\xi \tag{24a}$$

$$\left(\frac{\partial}{\partial y}\right)_x = \frac{\rho_e u_e r_o^j t^j}{\sqrt{2\xi}} \left(\frac{\rho}{\rho_e}\right)\left(\frac{\partial}{\partial \eta}\right)_\xi \tag{24b}$$

Two new parameters F and Θ are introduced and defined as

$$\left.\begin{aligned} F &= \frac{u}{u_e} \\ \Theta &= \frac{T}{T_e} \end{aligned}\right\} \tag{25}$$

as well as a transformed normal velocity

$$V = \frac{2\xi}{\rho_e u_e \mu_e r_o^{2j}}\left[F\left(\frac{\partial\eta}{\partial x}\right) + \frac{\rho\tilde{v} r_o^j t^j}{\sqrt{2\xi}}\right] \tag{26}$$

The governing equations in the transformed plane can then be expressed as follows:

Continuity

$$\frac{\partial V}{\partial\eta} + 2\xi\frac{\partial F}{\partial\xi} + F = 0 \tag{27}$$

Momentum

$$2\xi F\frac{\partial F}{\partial\xi} + V\frac{\partial F}{\partial\eta} - \frac{\partial}{\partial\eta}\left(t^{2j} l\bar{\epsilon}\frac{\partial F}{\partial\eta}\right) + \beta\left(F^2 - \Theta\right) = 0 \tag{28}$$

Energy

$$2\xi F\frac{\partial\Theta}{\partial\xi} + V\frac{\partial\Theta}{\partial\eta} - \frac{\partial}{\partial\eta}\left(t^{2j}\frac{l}{N_{Pr}}\tilde{\epsilon}\frac{\partial\Theta}{\partial\eta}\right) - \alpha l t^{2j}\bar{\epsilon}\left(\frac{\partial F}{\partial\eta}\right)^2 = 0 \tag{29}$$

where

$$\left.\begin{aligned} l &= \frac{\rho\mu}{(\rho\mu)_e} \\ \alpha &= \frac{u_e^2}{c_p T_e} \\ \beta &= \frac{2\xi}{u_e}\left(\frac{du_e}{d\xi}\right) \end{aligned}\right\} \tag{30}$$

By using the viscosity relations (eqs. (20) and (21)) and the equation of state (eq. (19)), the parameter l can be written as follows:

$$l = \sqrt{\Theta}\left(\frac{1 + \bar{S}}{\Theta + \bar{S}}\right) \qquad \text{(Air only)} \tag{31a}$$

$$l = (\Theta)^{\sigma-1.0} \qquad \text{(Power-law viscosity)} \tag{31b}$$

where $\overline{S} = S/T_e$.

The transverse-curvature term can be written in terms of the transformed variables as

$$t = \pm\left(1 + \frac{2\sqrt{2\xi}\cos\phi}{\rho_e u_e}\int_0^\eta \frac{\rho_e}{\rho}\,d\eta\right)^{1/2} \tag{32}$$

The physical coordinate normal to the wall is obtained from the inverse transformation; namely,

$$y = \frac{r_o}{\cos\phi}\left[-1 \pm \left(\frac{1 + 2\sqrt{2\xi}\cos\phi}{\rho_e u_e r_o^{2j}}\int_0^\eta \Theta\,d\eta\right)^{1/2}\right] \tag{33}$$

The positive sign is used in equations (32) and (33) for axisymmetric flow over bodies of revolution (SIGN = 1.0), and the negative sign is used for flow inside axisymmetric ducts (SIGN = -1.0).

The boundary conditions in the transformed plane are as follows:

Wall boundary

$$\left.\begin{aligned} F(\xi,0) &= 0 \\ V(\xi,0) &= V_w(\xi) \\ \Theta(\xi,0) &= \Theta_w(\xi) \end{aligned}\right\} \tag{34a}$$

or

$$\left(\frac{\partial\Theta}{\partial\eta}\right)_{\xi,0} = \left(\frac{\partial\Theta}{\partial\eta}\right)_w$$

Edge conditions

$$\left.\begin{aligned} F\left(\xi,\eta_e\right) &= 1 \\ \Theta\left(\xi,\eta_e\right) &= 1 \end{aligned}\right\} \tag{34b}$$

The boundary condition at the wall for the transformed V component can be related to the physical plane as (see ref. 7)

$$V_w = \frac{\sqrt{2\xi}}{\mu_e r_o^{2j}}\left(\frac{\rho_w v_w}{\rho_e u_e}\right) \tag{35}$$

Transition location.- Many parameters influence the location of transition. These parameters are discussed in an extensive review presented in reference 17. (See also ref. 7.)

It is not currently possible to predict with assurance the location of transition for a general geometry; however, for some particular classes of flow such as those caused by sharp cones, empirical correlations are available which can be used with confidence providing it is realized that a probable range of transition locations is being predicted and not an exact fixed point. (See ref. 7.) In the present digital code either the transition location (SST) or the stability index (SMXTR) must be specified; however, any correlation relation may be directly incorporated into the program if desired.

Transition extent.- The assumption of a universal intermittency distribution implies that the transition-zone length (transition extent) can be expressed as a function of the transition Reynolds number, $u_e x_{t,i}/\nu_e$. In reference 18 it is shown, for the transition data considered, that the data are represented on the average by the equation

$$R_{e,\Delta x_t} = 5\left(R_{e,x_{t,i}}\right)^{0.8} \tag{36}$$

where $R_{e,\Delta x_t} = \frac{u_e}{\nu_e}\left(x_{t,f} - x_{t,i}\right)$. The location of the end of transition, $x_{t,f}$ can then be obtained directly from equation (36) as follows:

$$x_{t,f} = x_{t,i} + 5R_e^{-1}\left(R_{e,x_{t,i}}\right)^{0.8} \tag{37}$$

where R_e is the local unit Reynolds number, u_e/ν_e.

In the present digital code, due to the lack of general correlations for the extent of transition, this quantity $\left(x_{t,f} - x_{t,i}\right)$ can be specified in one of two ways: (1) from equation (37) (KTCOD = 1), or (2) from the specification of $x_{t,f}/x_{t,i}$ obtained from experimental data (TLNGTH, KTCOD = 2). It should be noted that the digital code can be modified to include any desired correlation or equation in place of equation (37).

Intermittency distribution.- The parameter Γ appearing in equations (4) and (5) represents the streamwise intermittency distribution which models the turbulent spot

distribution in the transitional region; the parameter Γ is a function of the x-coordinate only and is defined as follows (see ref. 18):

$$\Gamma\left(\bar{\xi}\right) = 1 - \exp\left(-0.412\bar{\xi}^2\right) \tag{38}$$

where

$$\bar{\xi} = \frac{x - x_{t,i}}{\lambda} \tag{39}$$

and

$$\lambda = (x)_{\Gamma=\frac{3}{4}} - (x)_{\Gamma=\frac{1}{4}} \tag{40}$$

It should be noted that $\Gamma = 0$ for laminar flow, $\Gamma = 1$ for fully turbulent flow, and Γ ranges from 0 to 1 through the transitional-flow region. Equations (1) to (3) reduce to the classical laminar boundary-layer equations when Γ is set to zero. (See ref. 19.)

Numerical Solution of the Governing Equations

The system of governing equations (eqs. (19) to (21) and (27) to 29)) is parabolic and, therefore, can be numerically integrated in a step-by-step procedure in the streamwise direction. In order to cast the equations into a form in which the step-by-step procedure can be efficiently used, the derivatives with respect to ξ and η are replaced by finite-difference quotients. The method of linearization and solution used in the analysis closely parallels that of references 9 and 11.

Finite-difference mesh model.- It has been shown for laminar boundary layers that equally spaced grid points can be used in the normal coordinate direction. (See refs. 9 and 11.) However, for transitional and turbulent boundary layers the use of equally spaced grid points is not practical because the fine-mesh size required to obtain accurate results near the wall boundary is inefficient for the entire boundary layer. The grid-point spacing in the η-direction used in the program is such that the ratio of any two successive strips is a constant; that is, the successive $\Delta\eta_i$-coordinates form a geometric progression. In constructing the difference quotients, the sketch of the grid-point distribution presented in figure 2 is useful for reference. The dependent variables F, Θ, and V are assumed known at each of the N grid points along the m-1 and m stations, but are unknown at station m+1. The $\Delta\xi_1$ and $\Delta\xi_2$ values, not specified to be equal, are obtained from the specified x values $\left(x_{m-1},x_m,x_{m+1}\right)$ and from equation (22a). The relationship between the $\Delta\eta_i$-coordinates for the chosen grid-point spacing is given by the following equation (see ref. 1):

$$\Delta\eta_i = (k)^{i-1}\Delta\eta_1 \qquad (i = 1,2,3,\ldots,N) \tag{41}$$

where k is the ratio of any two successive steps (XK), $\Delta\eta_1$ is the spacing between the second grid point and the wall (note that the first grid point is at the wall boundary), and N denotes the total number of grid points across the chosen η strip. The total thickness of the η strip can then be expressed as follows:

$$\eta_N = \Delta\eta_i\left[\frac{1 - (k)^{N-1}}{1 - k}\right] \qquad (k \neq 1) \tag{42}$$

The selection of the optimum k and N values for a specified η_N-coordinate depends upon the particular problem under consideration. The main objective in the selection is to obtain the minimum number of grid points with which a convergent solution may be obtained and thereby minimize the computer-processing time for each test case. The laminar boundary layer presents no problem since a k value of unity is acceptable; however, for transitional or turbulent layers, the value of k will be a number slightly greater than unity, say between 1.02 and 1.04. If transitional or turbulent flow occurs in a given problem, the laminar portion of the boundary layer is calculated with the value of k used for the turbulent region; that is, for a given problem, k is invariant.

Difference equations.- Three-point implicit difference relations (see appendix A) are used to reduce the transformed momentum and energy equations (eqs. (28) and (29), respectively) to finite-difference form. The difference quotients produce linear difference equations when substituted into the momentum and energy equations provided truncation terms of the order $\Delta\xi_{m-1}\,\Delta\xi_m$ and $\Delta\eta_{n-1}\,\Delta\eta_n$ are neglected. (It should be noted that the truncation term for $\partial^2F/\partial\eta^2$ is of the order $\Delta\eta_{n-1} - \Delta\eta_n$.) The resulting difference equations may be written as follows:

$$A1_nF_{m+1,n-1} + B1_nF_{m+1,n} + C1_nF_{m+1,n+1} + D1_n\Theta_{m+1,n-1}$$
$$+ E1_n\Theta_{m+1,n} + F1_n\Theta_{m+1,n+1} = G1_n \tag{43a}$$

$$A2_nF_{m+1,n-1} + B2_nF_{m+1,n} + C2_nF_{m+1,n+1} + D2_n\Theta_{m+1,n-1}$$
$$+ E2_n\Theta_{m+1,n} + F2_n\Theta_{m+1,n+1} = G2_n \tag{43b}$$

The coefficients $A1_n, B1_n, \ldots, G1_n$ and $A2_n, B2_n, \ldots, G2_n$ (see appendix B) are functions of known quantities at stations m and $m-1$. It is important to note that equations (43)

are coupled through the dependent variables F and Θ; however, the dependent variable V does not appear explicitly as an unknown at station $m+1$. The variable V is uncoupled from the system because of the particular way that the nonlinear terms $V\frac{\partial F}{\partial \eta}$ and $V\frac{\partial \Theta}{\partial \eta}$ (see eqs. (28) and (29), respectively) are linearized. (See eq. (A23).)

Solution of difference equations.- The system of difference equations (eqs. (43)) represents a set of exactly $2(N - 1)$ linear algebraic equations for $2(N - 1)$ unknowns. The proper boundary conditions to be used with the difference equations are specified in equations (34). The $2(N - 1)$ linear algebraic equations may be written in tridiagonal matrix form; consequently, an efficient algorithm (Gaussian elimination) is available for simultaneous solution.

The simultaneous or coupled-solution technique is presented in appendix B of reference 9; however, because of differences between the present work and that presented in reference 9, the solution technique is discussed here in some detail.

Because of the special form of equations (43), the following relations exist (see ref. 20):

$$F_{m+1,n-1} = P^{(1)}_{m+1,n-1} + P^{(2)}_{m+1,n-1}F_{m+1,n} + P^{(3)}_{m+1,n-1}\Theta_{m+1,n} \tag{44a}$$

$$\Theta_{m+1,n-1} = Q^{(1)}_{m+1,n-1} + Q^{(2)}_{m+1,n-1}F_{m+1,n} + Q^{(3)}_{m+1,n-1}\Theta_{m+1,n} \tag{44b}$$

Next, equations (44) are substituted into equations (43) to obtain the following relations:

$$B1^{*}_{m+1,n}F_{m+1,n} + E1^{*}_{m+1,n}\Theta_{m+1,n} = G1^{*}_{m+1,n} - C1_{m+1,n}F_{m+1,n+1} - F1_{m+1,n}\Theta_{m+1,n+1} \tag{45a}$$

$$B2^{*}_{m+1,n}F_{m+1,n} + E2^{*}_{m+1,n}\Theta_{m+1,n} = G2^{*}_{m+1,n} - C2_{m+1,n}F_{m+1,n+1} - F1_{m+1,n}\Theta_{m+1,n+1} \tag{45b}$$

where

$$B1^{*}_{m+1,n} = B1_{m+1,n} + A1_{m+1,n}P^{(2)}_{m+1,n-1} + D1_{m+1,n}Q^{(2)}_{m+1,n-1} \tag{46a}$$

$$E1^{*}_{m+1,n} = E1_{m+1,n} + A1_{m+1,n}P^{(3)}_{m+1,n-1} + D1_{m+1,n}Q^{(3)}_{m+1,n-1} \tag{46b}$$

$$G1^*_{m+1,n} = G1_{m+1,n} - A1_{m+1,n}P^{(1)}_{m+1,n-1} - D1_{m+1,n}Q^{(1)}_{m+1,n-1} \tag{46c}$$

$$B2^*_{m+1,n} = B2_{m+1,n} + A2_{m+1,n}P^{(2)}_{m+1,n-1} + D2_{m+1,n}Q^{(2)}_{m+1,n-1} \tag{46d}$$

$$E2^*_{m+1,n} = E2_{m+1,n} + A2_{m+1,n}P^{(3)}_{m+1,n-1} + D2_{m+1,n}Q^{(3)}_{m+1,n-1} \tag{46e}$$

and

$$G2^*_{m+1,n} = G2_{m+1,n} - A2_{m+1,n}P^{(1)}_{m+1,n-1} - D2_{m+1,n}Q^{(1)}_{m+1,n-1} \tag{46f}$$

The unknown values of F and Θ at station $m+1,n$ are obtained from equations (45) as follows:

$$F_{m+1,n} = P^{(1)}_{m+1,n} + P^{(2)}_{m+1,n}F_{m+1,n+1} + P^{(3)}_{m+1,n}\Theta_{m+1,n+1} \tag{47a}$$

$$\Theta_{m+1,n} = Q^{(1)}_{m+1,n} + Q^{(2)}_{m+1,n}F_{m+1,n+1} + Q^{(3)}_{m+1,n}\Theta_{m+1,n+1} \tag{47b}$$

where

$$P^{(1)}_{m+1,n} = \left(E2^*_{m+1,n}G1^*_{m+1,n} - E1^*_{m+1,n}G2^*_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48a}$$

$$P^{(2)}_{m+1,n} = \left(E1^*_{m+1,n}C2_{m+1,n} - E2^*_{m+1,n}C1_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48b}$$

$$P^{(3)}_{m+1,n} = \left(E1^*_{m+1,n}F2_{m+1,n} - E2^*_{m+1,n}F1_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48c}$$

$$Q^{(1)}_{m+1,n} = \left(B1^*_{m+1,n}G2^*_{m+1,n} - B2^*_{m+1,n}G1^*_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48d}$$

$$Q^{(2)}_{m+1,n} = \left(B2^*_{m+1,n}C1_{m+1,n} - B1^*_{m+1,n}C2_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48e}$$

$$Q^{(3)}_{m+1,n} = \left(B2^*_{m+1,n}F1_{m+1,n} - B1^*_{m+1,n}F2_{m+1,n}\right)\Delta^*_{m+1,n} \tag{48f}$$

and

$$\Delta^*_{m+1,n} = \frac{1}{\left(B1^*_{m+1,n}E2^*_{m+1,n} - B2^*_{m+1,n}E1^*_{m+1,n}\right)} \tag{48g}$$

Next, equations (44) are rewritten as follows (where $n = n+1$)

$$F_{m+1,n} = P^{(1)}_{m+1,n} + P^{(2)}_{m+1,n}F_{m+1,n+1} + P^{(3)}_{m+1,n}\Theta_{m+1,n+1} \tag{49a}$$

$$\Theta_{m+1,n} = Q^{(1)}_{m+1,n} + Q^{(2)}_{m+1,n}F_{m+1,n+1} + Q^{(3)}_{m+1,n}\Theta_{m+1,n+1} \tag{49b}$$

The "no-slip" boundary condition $\left(F_{m+1,1} = 0\right)$ is applied at the wall boundary to obtain the values of $P^{(i)}_{m+1,1}$ where $i = 1,2,3$; that is,

$$P^{(1)}_{m+1,1} = P^{(2)}_{m+1,1} = P^{(3)}_{m+1,1} = 0 \tag{50}$$

The thermal condition at the wall boundary may be specified in one of two ways: (1) specified wall-temperature distribution, or (2) specified heat-transfer distribution. For a specified wall-temperature distribution it can be seen directly from equation (49b) that

$$\left.\begin{aligned} Q^{(1)}_{m+1,1} &= \Theta_{m+1,1} \\ Q^{(2)}_{m+1,1} &= Q^{(3)}_{m+1,1} = 0 \end{aligned}\right\} \tag{51}$$

The case in which a heat-transfer distribution is specified presents a more difficult problem; however, this class of flows is often of interest. (For example, consider adiabatic flows.)

The heat transfer at the wall boundary can be written in the transformed plane as follows (see ref. 7):

$$q_{m+1,1} = \frac{-\mu_r u_r^2}{\omega L}\left(\frac{\rho_e u_e T_c \mu_e r_o^j}{N_{Pr}\sqrt{2\xi}}\right)_{m+1,N} l_{m+1,1}\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} \tag{52}$$

Then, for a specified value of $q_{m+1,1}$, the gradient of Θ can be obtained directly as follows:

$$\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} = -q_{m+1,1}\left(\frac{\omega L}{\mu_r u_r^2}\right)\left(\frac{N_{Pr}\sqrt{2\xi}}{\rho_e u_e T_e \mu_e r_o^j}\right)_{m+1,N}\left(\frac{1}{l}\right)_{m+1,1} \tag{53}$$

For the grid-point spacing used in the analysis (geometric progression, see eq. (41)), the gradient of Θ evaluated at the wall, by using a 3-point relation, is as follows:

$$\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} = \frac{\left[1 - (1+k)^2\right]\Theta_{m+1,1} + (1+k)^2\Theta_{m+1,2} - \Theta_{m+1,3}}{k(1+k)\Delta\eta_1} \tag{54}$$

Equations (53) and (54) then yield the following expression for $\Theta_{m+1,1}$:

$$\Theta_{m+1,1} = \frac{k(1+k)\Delta\eta_1}{1-(1+k)^2}\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1} - \frac{(1+k)^2}{1-(1+k)^2}\Theta_{m+1,2} + \frac{1}{1-(1+k)^2}\Theta_{m+1,3} \tag{55}$$

where $\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1}$ is evaluated from equation (53). Equations (43) are next written at the $m+1,2$ point to obtain two equations in terms of $F_{m+1,n}$ and $\Theta_{m+1,n}$ where $n = 1,2,3$. (Note that $F_{m+1,1} = 0$.) The quantity $F_{m+1,3}$ is next eliminated from these two equations to obtain one equation in terms of $F_{m+1,2}$ and $\Theta_{m+1,n}$ where $n = 1,2,3$. The quantity $\Theta_{m+1,3}$ is next eliminated through use of equation (55) to obtain the relation

$$\Theta_{m+1,1} = \overline{Q}^{(1)}_{m+1,1} + \overline{Q}^{(2)}_{m+1,1}F_{m+1,2} + \overline{Q}^{(3)}_{m+1,1}\Theta_{m+1,2} \tag{56}$$

where

$$\overline{Q}^{(1)}_{m+1,1} = \frac{\left[(C2)(G1) - (C1)(G2)\right]_{m+1,2} + \left[(C2)(F1) - (C1)(F2)\right]_{m+1,2}\left[k(1+k)\Delta\eta_1\right]\left(\frac{\partial\Theta}{\partial\eta}\right)_{m+1,1}}{\Delta_{m+1,2}} \tag{57a}$$

$$\overline{Q}^{(2)}_{m+1,1} = \frac{\left[(C1)(B2) - (C2)(B1)\right]_{m+1,2}}{\Delta_{m+1,2}} \tag{57b}$$

$$\overline{Q}^{(3)}_{m+1,1} = \frac{\left[(C1)(E2) - (C2)(E1)\right]_{m+1,2} + \left[(C1)(F2) - (C2)(F1)\right]_{m+1,2}(1+k)^2}{\Delta_{m+1,2}} \tag{57c}$$

and

$$\Delta_{m+1,2} = \left\{\left[(C2)(D1) - (C1)(D2)\right] + \left[(C2)(F1) - (C1)(F2)\right]\left[1 - (1+k)^2\right]\right\}_{m+1,2} \tag{57d}$$

By comparing equations (49b) and (56) it is observed that

$$Q^{(i)}_{m+1,1} = \overline{Q}^{(i)}_{m+1,1} \qquad (i = 1,2,3) \tag{58}$$

which completes the desired boundary condition for the case of a specified heat-transfer distribution along the wall boundary. The temperature at the wall is obtained directly from equation (55) once $\Theta_{m+1,2}$ and $\Theta_{m+1,3}$ are known.

The quantities $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ where $i = 1,2,3$ (see eqs. (49)) must first be determined across the boundary layer at the $m+1$ station where $n = 1,2,\ldots,N$. These quantities are calculated by the following procedure:

(1) Perform the following steps at the first grid point away from the wall ($n = 2$):

(a) Calculate $A1_2, B1_2, \ldots, G1_2$ from equations (B3) to (B9).

(b) Calculate $A2_2, B2_2, \ldots, G2_2$ from equations (B10) to (B16).

(c) By using the results from steps (a) and (b) and the boundary conditions (eqs. (50) and (51) or (57)), calculate $B1_2^*$, $B2_2^*$, $E1_2^*$, $E2_2^*$, $G1_2^*$, and $G2_2^*$ from equations (46).

(d) By using the results from steps (a) to (c), calculate $P_2^{(i)}$ and $Q_2^{(i)}$ where $i = 1,2,3$ from equations (48).

(2) The procedure outlined in step (1) is now repeated at grid point with $n = 3$ by using the results obtained at $n = 2$. This procedure is repeated until the entire boundary layer is traversed ($n = N$) and all values of $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ are determined where $i = 1,2,3$ and $n = 2,3,4,\ldots,N$.

(3) By knowing the values of $P^{(i)}_{m+1,n}$ and $Q^{(i)}_{m+1,n}$ where $i = 1,2,3$ and $n = 2,3,4,\ldots,N$, the values of $F_{m+1,n}$ and $\Theta_{m+1,n}$ where $n = N-1,N-2,\ldots,2$ are calculated from equations (47). It should be noted that $F_{m+1,N}$ and $\Theta_{m+1,N}$ are specified edge boundary conditions (eqs. (34b)). The wall-boundary values of F and Θ are obtained from equations (34a) or equation (55) for the case of a specified wall-boundary heat-transfer distribution. Before the computations can proceed downstream, the transformed velocity $V_{m+1,n}$ must be determined across the boundary layer where $n = 2,3,\ldots,N$. This requires the solution of the continuity equation. (See eq. (27).)

Solution of continuity equation.- The continuity equation (eq. (27)) is solved numerically for the $N - 1$ unknown values of V at station $m+1$. Equation (27) is integrated once to yield the following relation for $V_{m+1,n}$:

$$V_{m+1,n} = V_{m+1,1} - \int_0^{\eta_n} \left(2\xi \frac{\partial F}{\partial \xi} + F\right)_{m+1} d\eta \tag{59}$$

where $V_{m+1,1}$ represents the boundary condition at the wall V_w. (See eq. (35).) The integral appearing in equation (59) is numerically integrated across the η-strip to obtain the $N - 1$ values of V. In the present program the trapezoidal rule of integration is used.

Initial profiles.- Initial profiles for starting the finite-difference scheme are required at two x-stations since three-point differences are utilized. The initial profiles at the stagnation point or line for blunt bodies, or near $x = 0$ for sharp-tipped bodies, are obtained by numerically solving the similar boundary-layer equations. (See eqs. (B47) to (B49).) The equations are solved by a fourth-order Runge-Kutta scheme with a Newton iteration method to modify the initial estimates of the gradients of F and Θ evaluated at the wall boundary. The $N - 1$ values of F, Θ, and V obtained at the equally spaced $N - 1$ grid points are numerically redistributed to $N - 1$ grid points whose spacing is determined from equations (41) and (42) if a variable spacing is required. (As noted previously, variable spacing is required if transitional or turbulent flow occurs.) The second initial profile located at station m is assumed identical to the one located at station $m-1$. Any errors that might be incurred because of this assumption are minimized by using an extremely small value of $\Delta\xi$; that is, an initial step size in the physical plane on the order of $\Delta x = 1 \times 10^{-5}$ is used. The solution at the unknown station $m+1$ is then obtained by the finite-difference method. Extremely small, equally spaced $\Delta\xi$-steps are used in the region of the initial profile. The step size is increased after errors due to the starting procedure have approached zero, that is, after 10 to 15 steps in $\Delta\xi$.

Evaluation of wall derivatives.- The shear stress and heat transfer at the wall are directly proportional to the gradient of F and Θ evaluated at the wall, respectively. By using G to represent a general quantity, where $G_{m+1,1}$ is the value of G evaluated at the wall, the four-point difference scheme used to evaluate derivatives at the wall is given as

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,1} = Y_7 G_{m+1,1} + Y_8 G_{m+1,2} + Y_9 G_{m+1,3} + Y_{10} G_{m+1,4} \tag{60}$$

where the coefficients $Y_7, \ldots, Y_{10}$ are defined by the following relations:

$$Y_7 = -\frac{\left(1 + k + k^2\right)^2 \left[k(1 + k) - 1\right] + (1 + k)}{(1 + k)\left(1 + k + k^2\right)k^3 \Delta\eta_1} \tag{61a}$$

$$Y_8 = \frac{\left(1 + k + k^2\right)}{k^2 \Delta\eta_1} \tag{61b}$$

$$Y_9 = -\frac{\left(1 + k + k^2\right)}{(1 + k)k^3 \Delta\eta_1} \tag{61c}$$

and

$$Y_{10} = \frac{1}{\left(1 + k + k^2\right)k^3 \Delta\eta_1} \tag{61d}$$

For the case of equally spaced grid points in the η-direction ($k = 1$), equations (61) become

$$Y_7 = -\frac{11}{6\ \Delta\eta} \tag{62a}$$

$$Y_8 = \frac{18}{6\ \Delta\eta} \tag{62b}$$

$$Y_9 = -\frac{9}{6\ \Delta\eta} \tag{62c}$$

$$Y_{10} = \frac{2}{6\ \Delta\eta} \tag{62d}$$

and equation (60) reduces to the familiar four-point relation; that is,

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,1} = -\frac{1}{6\ \Delta\eta}\left(11G_{m+1,1} - 18G_{m+1,2} + 9G_{m+1,3} - 2G_{m+1,4}\right) \tag{63}$$

PROGRAM DESCRIPTION

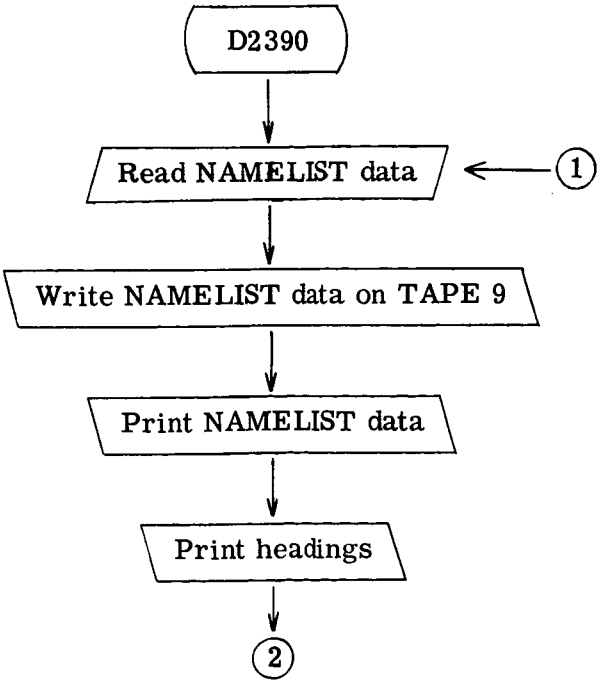
General Discussion

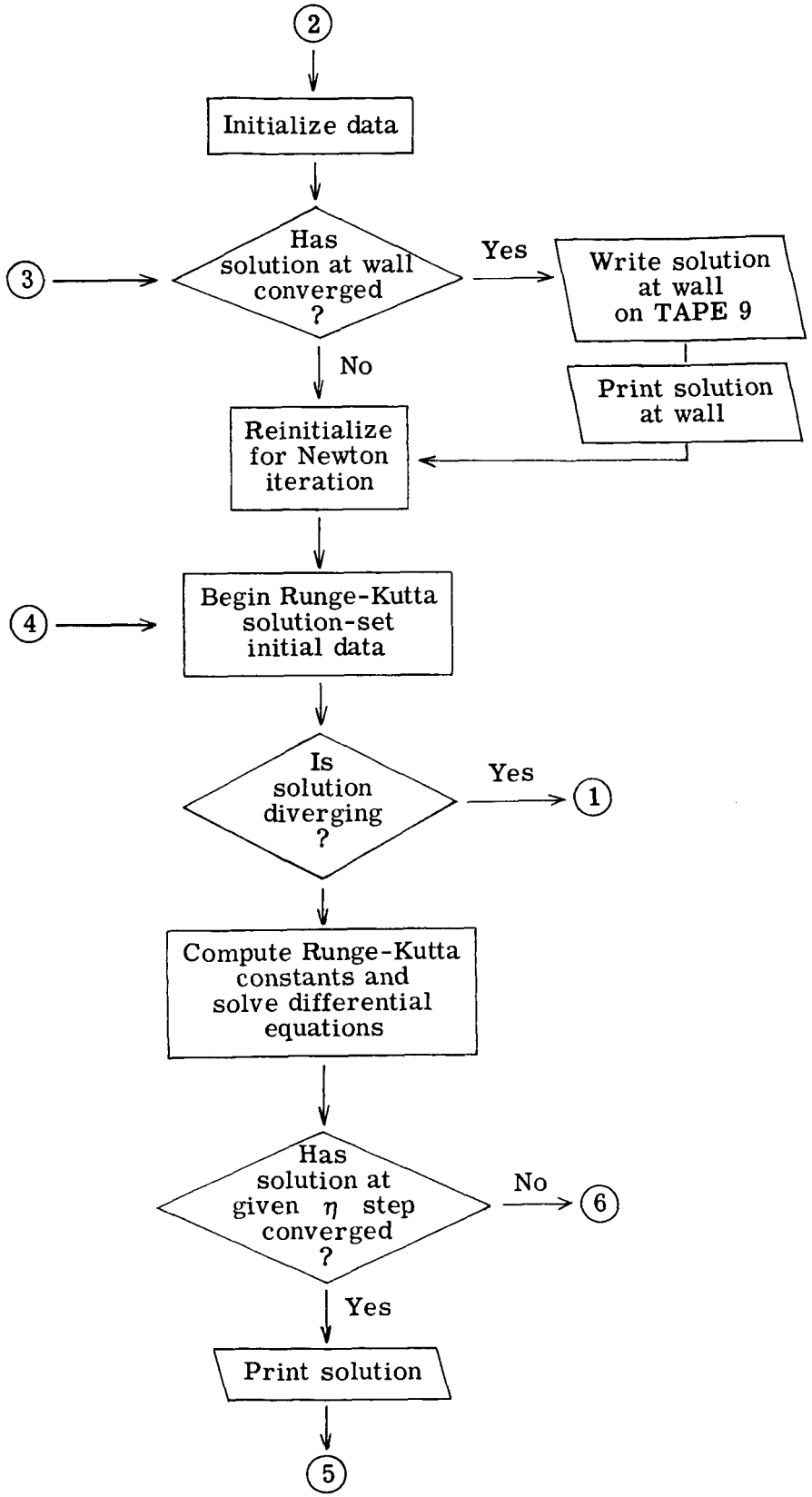
The program, written in FORTRAN IV for the CDC 6000 series computers, consists of three main programs, D2390, D23901, and D2401. Program D2390 computes the initial similarity solution to equations (B47) to (B49) with the points equally spaced in the η-direction. Program D23901 takes this solution and redistributes the values of η geometrically in subroutine GEOM, then interpolates to provide the solution over the geometrically spaced points. Program D2401 takes these data as a starting profile, reads other

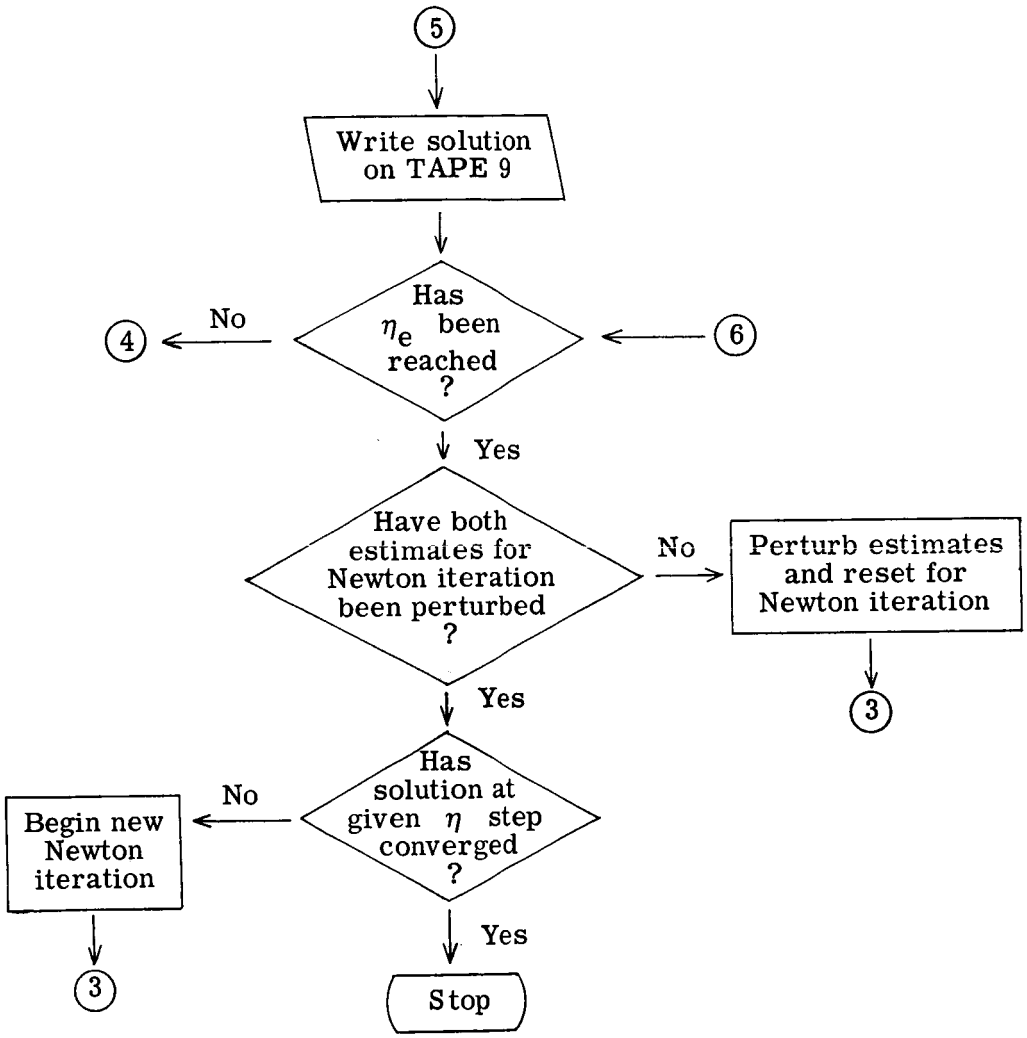
input, and computes initial conditions. Steps are then taken down the body to solve the momentum and energy equations in finite-difference form, and the continuity equation is numerically integrated. Various boundary-layer parameters such as boundary-layer thickness, displacement thickness, momentum thickness, and skin-friction and heat-transfer coefficients are then calculated and the output is printed. Program D2401 uses the following subroutines: TURBLNT calculates the eddy viscosity, its derivatives, and the intermittency distributions required for the solution of transitional and turbulent flows; VARENT reads the variable-entropy input in tabular form, computes dr_s/dz, and prints the input and the derivatives; TABLE reads the body-geometry input, nondimensionalizes it, and, if necessary, distributes the values according to specified steps and computes derivatives; SETUP determines from input where profiles and wall values are to be printed; function INTEGT integrates by using the trapezoidal rule; INUNIT converts data, if necessary, to the U.S. Customary System of Units for computation and back to the International System for printout; FTLUP performs a second-order interpolation to find intermediate values from a tabular array (see appendix C).

Descriptions, Flow Charts, and Listings of the Main Programs and Subprograms

Main program D2390.- The main program D2390 performs the locally similar solution to the continuity, momentum, and energy equations ((B47), (B48), and (B49), respectively), by using a fourth-order Runge-Kutta technique with Newton's iteration method. The flow diagram of main program D2390 is as follows:




2
Initialize data
3
Has solution at wall converged ?
Yes
Write solution at wall on TAPE 9
Print solution at wall
No
Reinitialize for Newton iteration
4
Begin Runge-Kutta solution-set initial data
Is solution diverging ?
Yes
1
Compute Runge-Kutta constants and solve differential equations
Has solution at given η step converged ?
No
6
Yes
Print solution
5


5
Write solution on TAPE 9
Has η_e been reached ?
No
4
6
Yes
Have both estimates for Newton iteration been perturbed ?
No
Perturb estimates and reset for Newton iteration
3
Yes
Has solution at given η step converged ?
No
Begin new Newton iteration
3
Yes
Stop

The program listing for the main program D2390 is as follows:

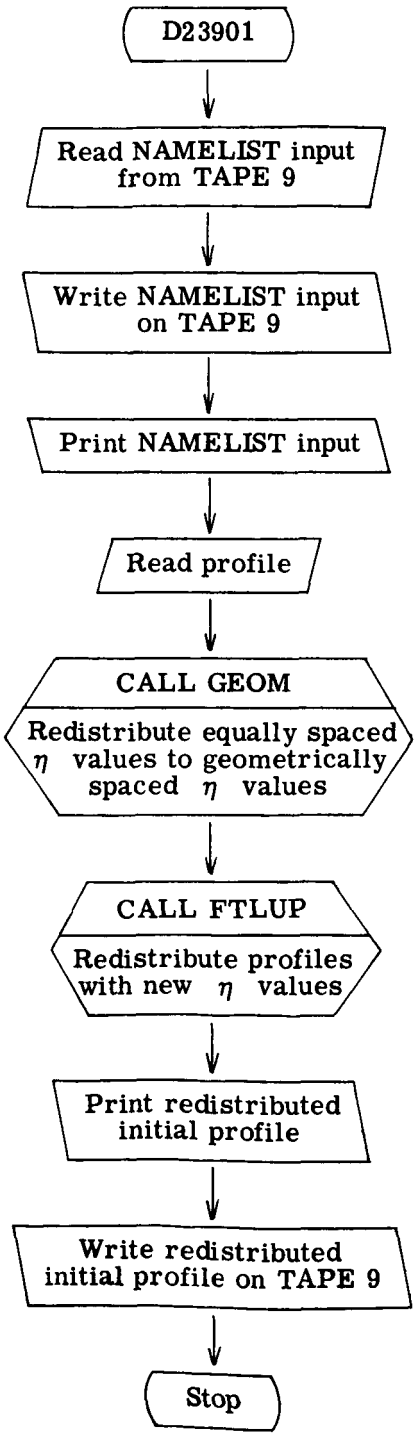
```
      PROGRAM D2390(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE9)
*        RUNGE-KUTTA METHOD FOR INITIAL VALUE PROBLEM
*        INVOLVING FIRST ORDER DIFFERENTIAL EQUATIONS
      DIMENSION YO(6), YN(6), Q(6), V(6), C(6), F(6), D(6), W(6)
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V
     1ISCON,VISPOW,KODUNIT
*        INITIALIZE DATA
      DATA HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,N/.1,10.,.01,.7,4*0.,6/
      DATA XK,IGAS/1.,1/,KODUNIT/0/,VISCON/0.0/,VISPOW/0.0/
      IGAS=1
    1 CONTINUE
      DO 2 I=1,6
    2 YO(I)=0.
*        INPUT DATA
      READ (5,NAM1)
      IF (ENDFILE 5) 3,4
    3 STOP 6
    4 CONTINUE
      IEDGE=XEND/HPR+1.5
      WRITE (6,NAM1)
      WRITE (9,NAM1)
      WRITE (6,31)
      WRITE (6,32)
      YW31=0.0
      YW33=100.0
      TFACT=100.0
      NDELT=-1
    5 IF (ABS(TFACT)-.000001) 6,7,7
*        WRITE SOLUTION AT WALL
    6 WRITE (6,34) XO,YO(1),YO(2),YO(3),YO(4),YO(5),YO(6)
      WRITE (9,34) XO,YO(1),YO(2),YO(3),YO(4),YO(5),YO(6)
    7 YW1=YO(1)
      YW2=YO(2)
      YW3=YO(3)
      YW4=YO(4)
      YW5=YO(5)
      YW6=YO(6)
      XO1=XO
      HO1=H
      XPR=HPR
```

```
*        BEGIN RUNGE-KUTTA                                          A  41   5000000
                                                                           5100000
      X=XO                                                          A  42   5200000
      DO 8 I=1,N                                                    A  43   5300000
      YN(I)=YO(I)                                                   A  44   5400000
    8 Q(I)=0.0                                                      A  45   5500000
    9 U=X                                                           A  46   5600000
      DO 10 I=1,N                                                   A  47   5700000
   10 V(I)=YN(I)                                                    A  48   5800000
      I1=1                                                          A  49   5900000
      GO TO 29                                                      A  50   6000000
                                                                           6100000
*        COMPUTE RUNGE-KUTTA CONSTANTS                              A  51   6200000
*           COMPUTE K1                                              A  52   6300000
                                                                           6400000
   11 DO 12 I=1,N                                                   A  53   6500000
      C(I)=H*F(I)                                                   A  54   6600000
      D(I)=.5*(C(I)-2.0*Q(I))                                       A  55   6700000
      W(I)=YN(I)+D(I)                                               A  56   6800000
      Q(I)=Q(I)+3.0*D(I)-.5*C(I)                                    A  57   6900000
   12 V(I)=W(I)                                                     A  58   7000000
      U=X+.5*H                                                      A  59   7100000
      I1=2                                                          A  60   7200000
      GO TO 29                                                      A  61   7300000
                                                                           7400000
*           COMPUTE K2                                              A  62   7500000
                                                                           7600000
   13 DO 14 I=1,N                                                   A  63   7700000
      C(I)=H*F(I)                                                   A  64   7800000
      D(I)=.29289325*(C(I)-Q(I))                                    A  65   7900000
      W(I)=W(I)+D(I)                                                A  66   8000000
      Q(I)=Q(I)+3.0*D(I)-.29289325*C(I)                             A  67   8100000
   14 V(I)=W(I)                                                     A  68   8200000
      I1=3                                                          A  69   8300000
      GO TO 29                                                      A  70   8400000
                                                                           8500000
*           COMPUTE K3                                              A  71   8600000
                                                                           8700000
   15 DO 16 I=1,N                                                   A  72   8800000
      C(I)=H*F(I)                                                   A  73   8900000
      D(I)=1.7071067*(C(I)-Q(I))                                    A  74   9000000
      W(I)=W(I)+D(I)                                                A  75   9100000
      Q(I)=Q(I)+3.0*D(I)-1.7071067*C(I)                             A  76   9200000
   16 V(I)=W(I)                                                     A  77   9300000
      U=X+H                                                         A  78   9400000
      I1=4                                                          A  79   9500000
      GO TO 29                                                      A  80   9600000
                                                                           9700000
```

```
*           COMPUTE K4                                                  A  81    9800000
                                                                                 9900000
   17 DO 18 I=1,N                                                       A  82   10000000
      C(I)=H*F(I)                                                       A  83   10100000
      D(I)=.16666667*(C(I)-2.0*Q(I))                                    A  84   10200000
      YN(I)=W(I)+D(I)                                                   A  85   10300000
   18 Q(I)=Q(I)+3.0*D(I)-.5*C(I)                                        A  86   10400000
      X=X+H                                                             A  87   10500000
      IF (X-XPR+.000001) 9,19,19                                        A  88   10600000
   19 CONTINUE                                                          A  89   10700000
      IF (ABS(TFACT)-.000001) 20,21,21                                  A  90   10800000
                                                                                 10900000
*        OUTPUT ONTO TAPE AND PRINT                                     A  91   11000000
                                                                                 11100000
   20 WRITE (6,34) X,YN(1),YN(2),YN(3),YN(4),YN(5),YN(6)                A  92   11200000
      WRITE (9,34) X,YN(1),YN(2),YN(3),YN(4),YN(5),YN(6)                A  93   11300000
                                                                                 11400000
*        RESET VALUES FOR NEXT NEWTONS ITERATION                        A  94   11500000
                                                                                 11600000
   21 IF (X-XEND) 28,22,22                                              A  95   11700000
   22 CONTINUE                                                          A  96   11800000
      IF (NDELT) 23,26,27                                               A  97   11900000
   23 IF (ABS(TFACT)-0.000001) 24,25,25                                 A  98   12000000
   24 CONTINUE                                                          A  99   12100000
      STOP                                                              A 100   12200000
   25 YW31=YW3                                                          A 101   12300000
      YW51=YW5                                                          A 102   12400000
      YI21=YN(2)                                                        A 103   12500000
      YI41=YN(4)                                                        A 104   12600000
      YO(3)=YW31+0.0001                                                 A 105   12700000
      YO(1)=YW1                                                         A 106   12800000
      YO(2)=YW2                                                         A 107   12900000
      YO(4)=YW4                                                         A 108   13000000
      YO(5)=YW5                                                         A 109   13100000
      YO(6)=YW6                                                         A 110   13200000
      XO=XO1                                                            A 111   13300000
      H=HO1                                                             A 112   13400000
      NDELT=0                                                           A 113   13500000
      GO TO 5                                                           A 114   13600000
   26 YI22=YN(2)                                                        A 115   13700000
      YI42=YN(4)                                                        A 116   13800000
      YO(5)=YW51+0.0001                                                 A 117   13900000
      YO(1)=YW1                                                         A 118   14000000
      YO(2)=YW2                                                         A 119   14100000
      YO(3)=YW31                                                        A 120   14200000
      YO(4)=YW4                                                         A 121   14300000
      YO(6)=YW6                                                         A 122   14400000
      XO=XO1                                                            A 123   14500000
      H=HO1                                                             A 124   14600000
      NDELT=+1                                                          A 125   14700000
      GO TO 5                                                           A 126   14800000
                                                                                 14900000
```

```
*        COMPUTE DETERMINANT
   27 DENOM=((YI22-YI21)*(YN(4)-YI41))/(0.0001**2.0)-((YN(2)-YI21)*(YI42
     1-YI41))/(0.0001**2.0)
      YO(3)=YW31+((YN(4)-YI41)*(1.0-YI21)-(YN(2)-YI21)*(1.0-YI41))/(DENO
     1M*0.0001)
      YO(5)=YW51+((YI22-YI21)*(1.0-YI41)-(YI42-YI41)*(1.0-YI21))/(DENOM*
     10.0001)
      YO(1)=YW1
      YO(2)=YW2
      YO(4)=YW4
      YO(6)=YW6
      YW33=YO(3)
      TFACT=YW33-YW31
      XO=XO1
      H=HO1
      NDELT=-1
      GO TO 5
   28 XPR=XPR+HPR
      GO TO 9
   29 CONTINUE
      IF (V(4).GE.0.) GO TO 30
      WRITE (6,33)
      GO TO 1
   30 CONTINUE
      IF (IGAS.EQ.1) VISC=SQRT(V(4))*(1.0+XKK)/(V(4)+XKK)
      IF (IGAS.EQ.2) VISC=V(4)**(VISPOW-1.)

*        SOLVE DIFFERENTIAL EQUATIONS

      F(1)=V(2)
      F(2)=V(3)/VISC
      F(3)=-V(1)*V(3)/VISC-(V(4)-V(2)*V(2))*BETA
      F(4)=V(5)/VISC
      F(5)=-PR*V(1)*V(5)/VISC-PR*ALPHA*V(3)*V(3)/VISC
      F(6)=V(4)-V(2)
      GO TO (11,13,15,17), I1

*

   31 FORMAT (1H1)
   32 FORMAT (6X,3HETA9X,5HYO(1)8X,5HYO(2)8X,5HYO(3)8X,5HYO(4)8X,5HYO(5)
     18X,5HYO(6))
   33 FORMAT (//95H SOLUTION DID NOT CONVERGE.  AN ATTEMPT WILL BE MADE
     1TO READ NEW ESTIMATES FOR YO(3) AND YO(5).)
   34 FORMAT (7F13.6)
      END
```

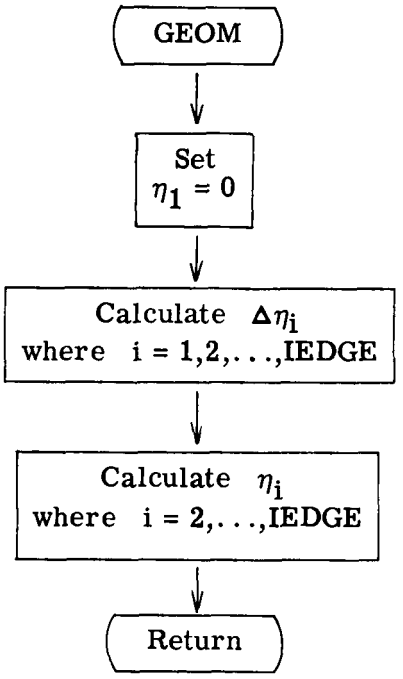
Main program D23901.- Main program D23901 takes the initial profile data from the program D2390 solution and redistributes the equally spaced η values geometrically according to input-distribution constant XK and then interpolates to redistribute the profile to correspond with the new η values. The flow diagram of the main program D23901 is as follows:



The program listing for the main program D23901 is as follows:

```
      PROGRAM D23901(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE9)
*        THIS PROGRAM READS THE ARRAYS WRITTEN BY D2390 AND CHANGES
*        THE DATA FROM EQUALLY SPACED POINTS TO A GEOMETRIC PROGRESSION

      DIMENSION A(301), AA(301), B(301), BB(301), C(301), CC(301), D(301
     1), DD(301), E(301), EE(301), F(301), FF(301), G(301), GG(301)
      DIMENSION YO(6)
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V
     1ISCON,VISPOW,KODUNIT

*       READ INITIAL VALUE INPUT FROM D2390

      REWIND 9
      READ (9,NAM1)
      IF (ENDFILE 9) 1,2
    1 STOP 7
    2 CONTINUE
      READ (9,6) (A(N),B(N),C(N),D(N),E(N),F(N),G(N),N=1,IEDGE)
      IF (ENDFILE 9) 3,4
    3 STOP 10
    4 CONTINUE

*       REDISTRIBUTE ETAS FROM D2390

      CALL GEOM (XK,XEND,IEDGE,AA)

*       REDISTRIBUTE INITIAL PROFILES WITH NEW ETAS

      DO 5 I=1,IEDGE
      CALL FTLUP (AA(I),BB(I),2,IEDGE,A,B)
      CALL FTLUP (AA(I),CC(I),2,IEDGE,A,C)
      CALL FTLUP (AA(I),DD(I),2,IEDGE,A,D)
      CALL FTLUP (AA(I),EE(I),2,IEDGE,A,E)
      CALL FTLUP (AA(I),FF(I),2,IEDGE,A,F)
    5 CALL FTLUP (AA(I),GG(I),2,IEDGE,A,G)
      REWIND 9
      WRITE (9,NAM1)

*       WRITE NEW INITIAL PROFILE

      WRITE (9,6) (AA(N),BB(N),CC(N),DD(N),EE(N),FF(N),GG(N),N=1,IEDGE)
      WRITE (6,NAM1)
      WRITE (6,7)
      WRITE (6,8)
      WRITE (6,6) (AA(N),BB(N),CC(N),DD(N),EE(N),FF(N),GG(N),N=1,IEDGE)
      STOP

*

    6 FORMAT (7E13.6)
    7 FORMAT (1H1)
    8 FORMAT (6X,3HETA9X,5HYO(1)8X,5HYO(2)8X,5HYO(3)8X,5HYO(4)8X,5HYO(5)
     18X,5HYO(6))
      END
```

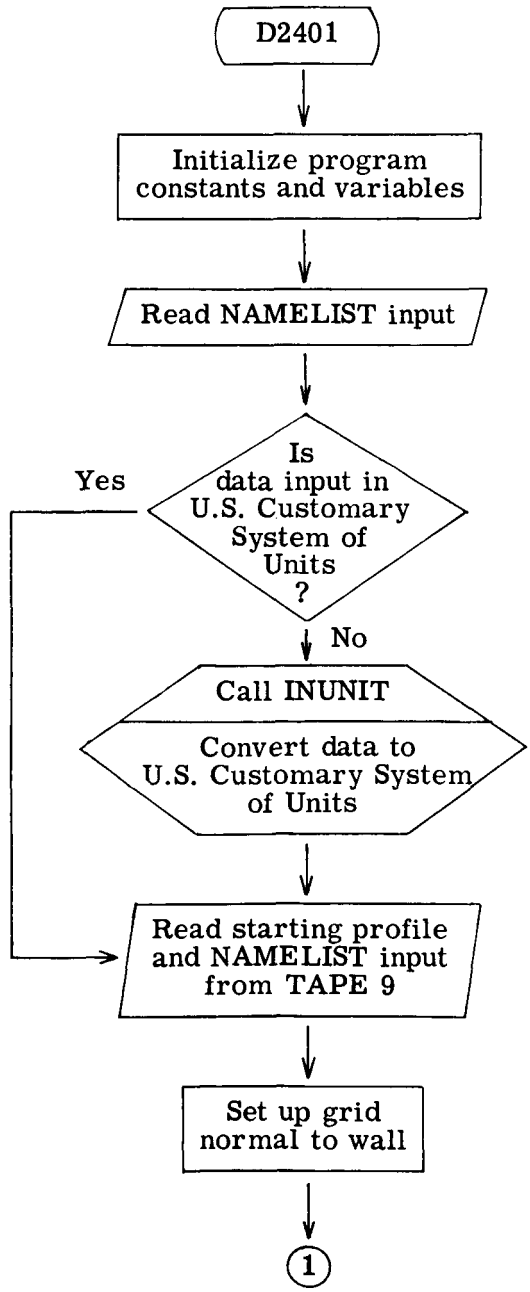
Subroutine GEOM.- Subroutine GEOM redistributes the equally spaced η values to geometrically spaced values according to input-distribution constant XK. The flow diagram for subroutine GEOM is as follows:

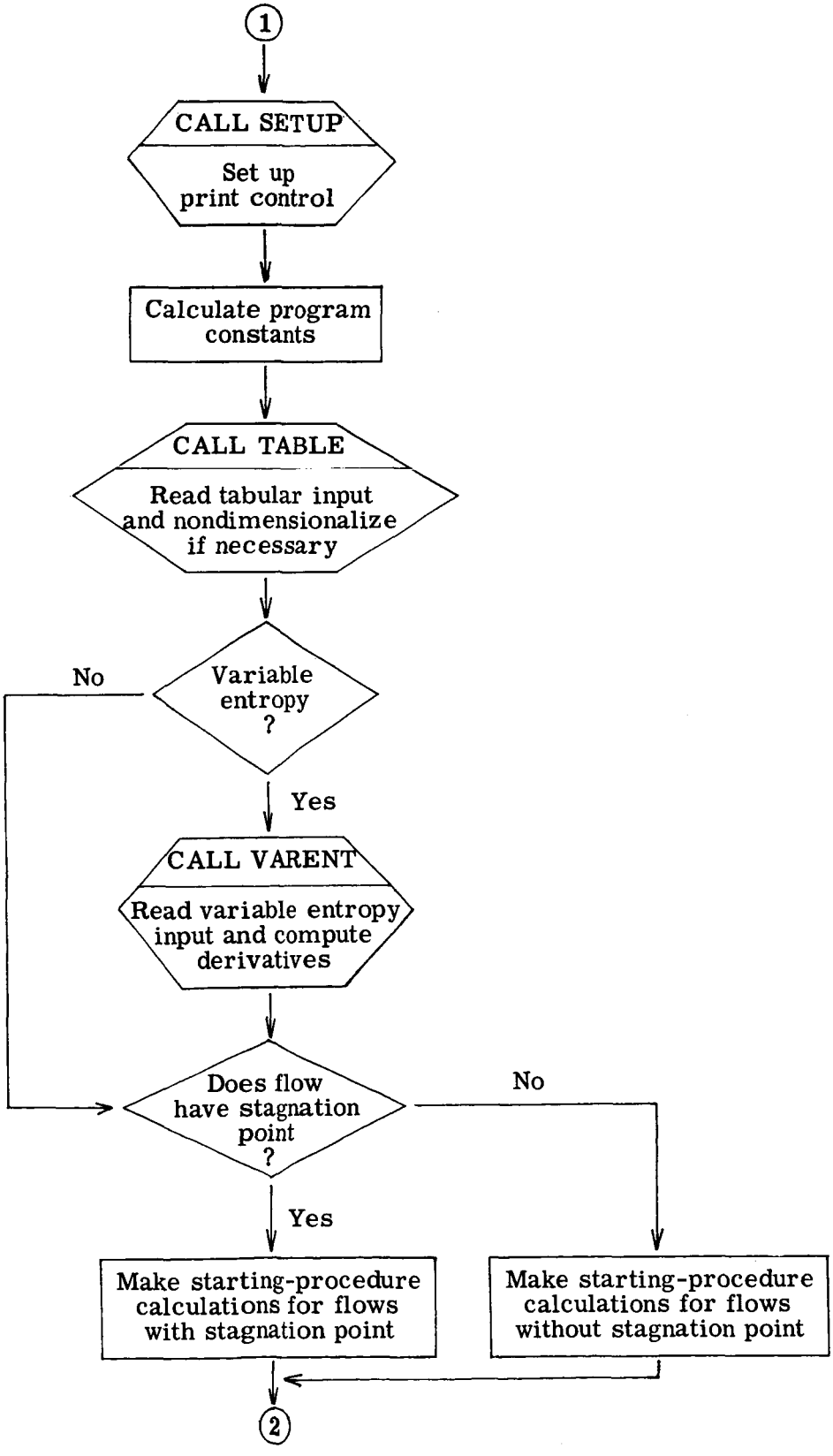


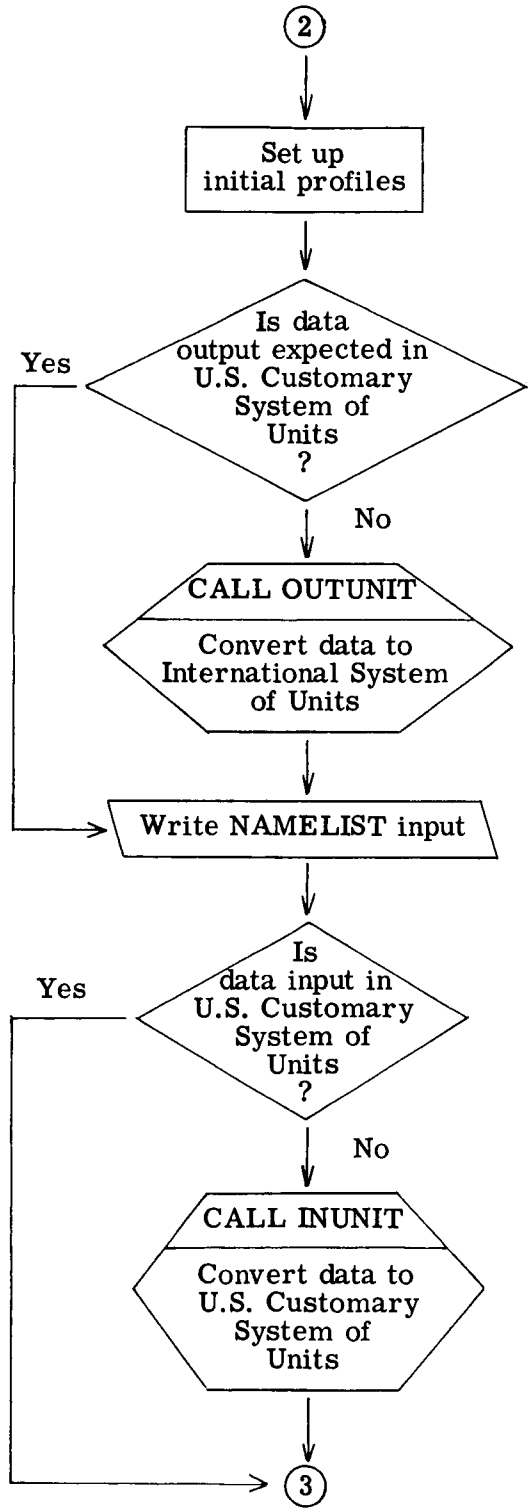
The program listing for subroutine GEOM is as follows:

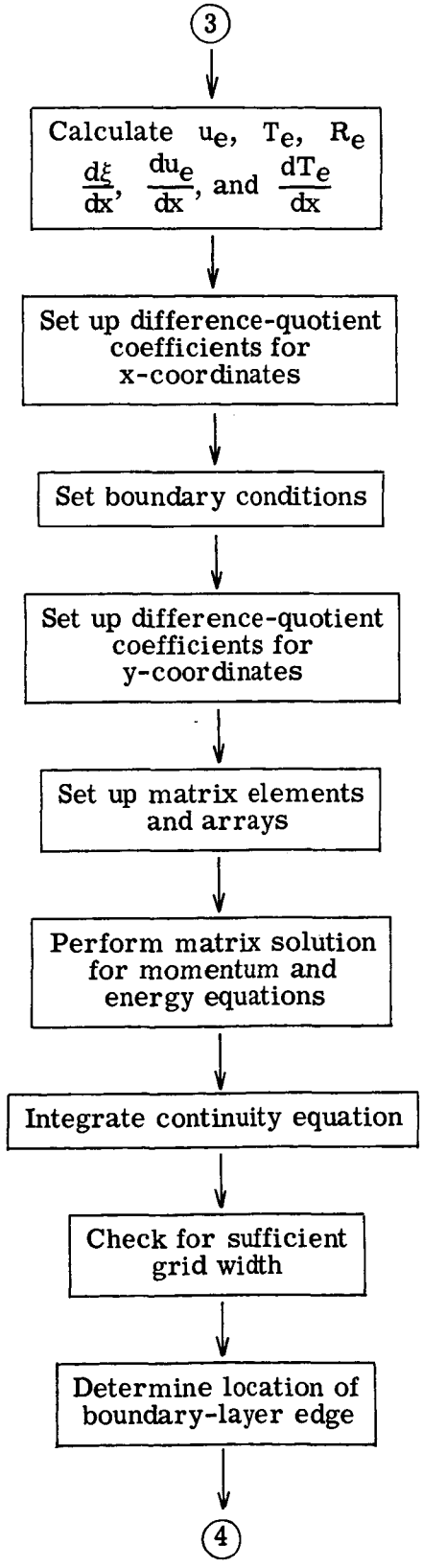
```
      SUBROUTINE GEOM (K,ETAEDGE,IEDGE,AA)                          B   1    5600000
      REAL K                                                        B   2    5700000
      DIMENSION DETA(301), AA(301)                                  B   3    5800000
      AA(1)=0.                                                      B   4    5900000
      IF (K.EQ.1.) GO TO 1                                          B   5    6000000
      DETA(1)=((1-K)/(1-K**(IEDGE-1)))*ETAEDGE                      B   6    6100000
      GO TO 2                                                       B   7    6200000
1     DETA(1)=ETAEDGE/(IEDGE-1)                                     B   8    6300000
2     AA(2)=DETA(1)                                                 B   9    6400000
      DETA(2)=K*DETA(1)                                             B  10    6500000
      DO 3 N=3,IEDGE                                                B  11    6600000
      DETA(N)=(K**(N-1))*DETA(1)                                    B  12    6700000
3     AA(N)=AA(N-1)+DETA(N-1)                                       B  13    6800000
      RETURN                                                        B  14    6900000
      END                                                           B  15-   7000000
```

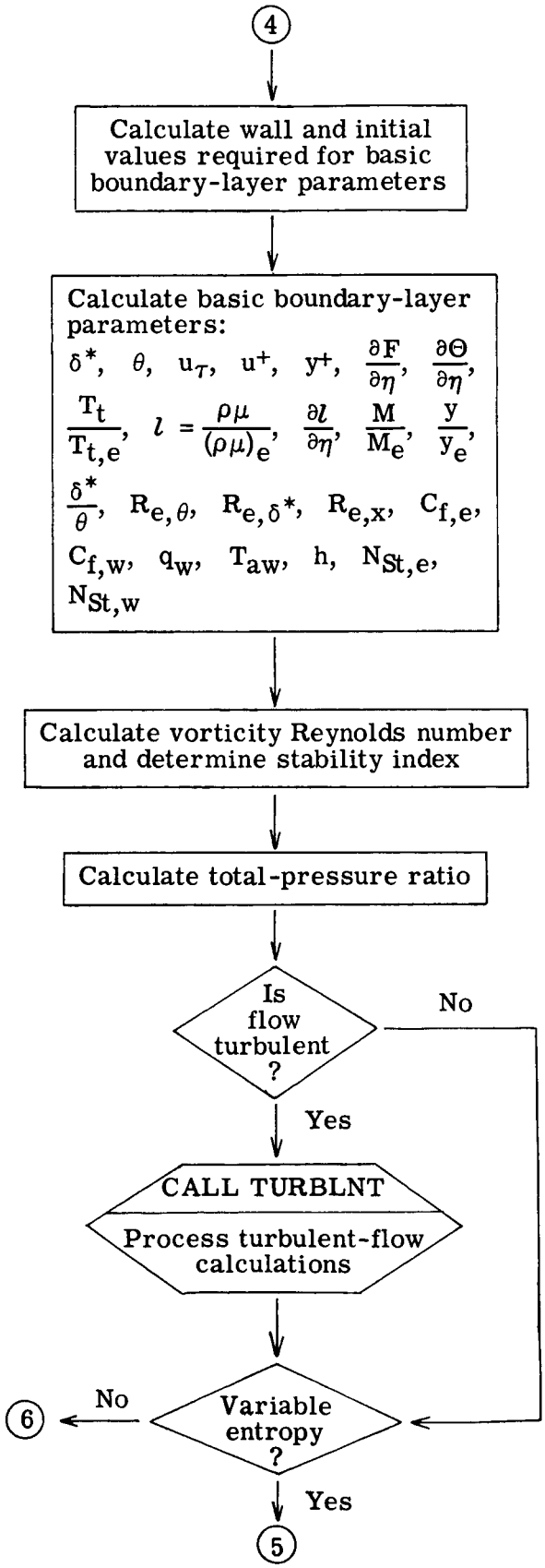
Main program D2401.- The main program D2401 controls the finite-difference solution of the boundary-layer equations. It reads the initial profile data (which may come from D2390 or D23901) and other input, computes initial conditions, solves the momentum and energy equations in finite-difference form, numerically integrates the continuity equation, calculates the boundary-layer thickness, displacement thickness, momentum thickness, and skin-friction and heat-transfer coefficients, and prints the output. The flow diagram of the main program D2401 is as follows:

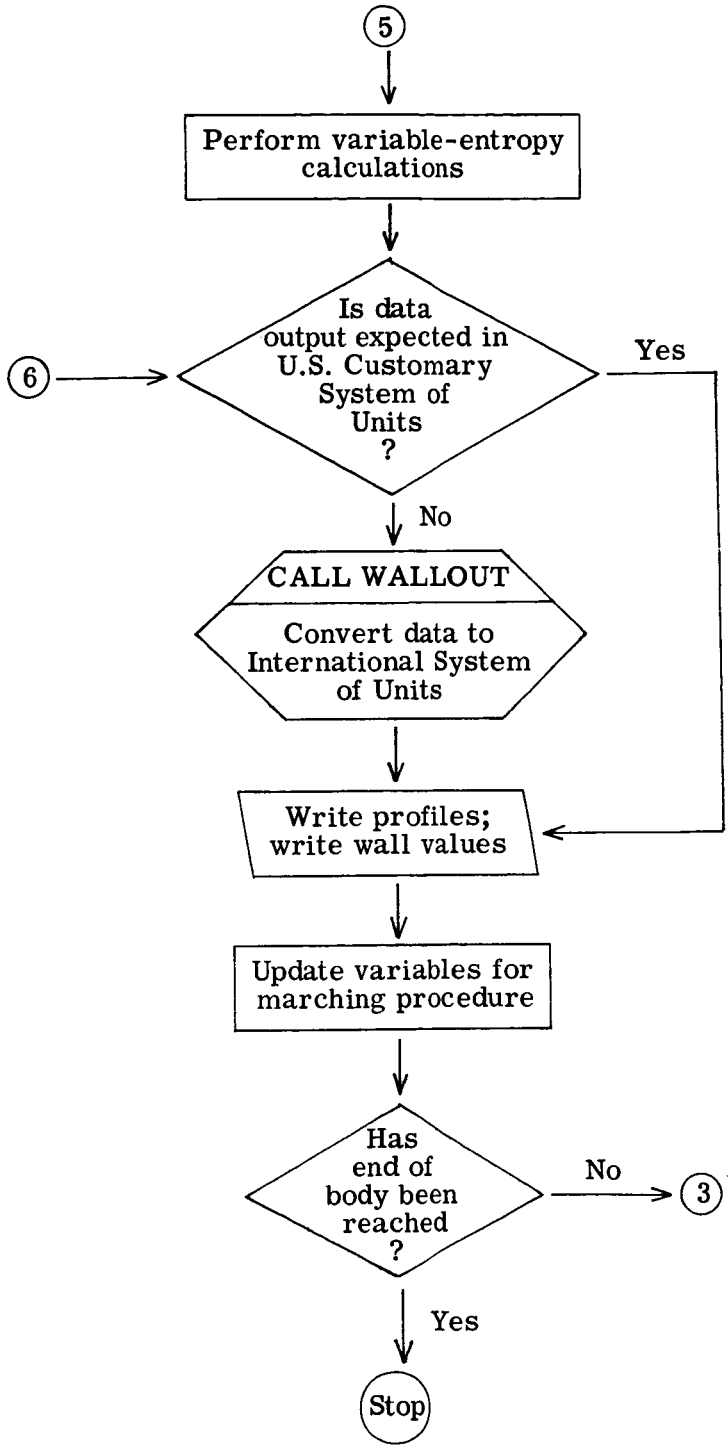



1
CALL SETUP
Set up print control
Calculate program constants
CALL TABLE
Read tabular input and nondimensionalize if necessary
Variable entropy ?
No
Yes
CALL VARENT
Read variable entropy input and compute derivatives
Does flow have stagnation point ?
No
Yes
Make starting-procedure calculations for flows with stagnation point
Make starting-procedure calculations for flows without stagnation point
2


2
Set up initial profiles
Is data output expected in U.S. Customary System of Units ?
Yes
No
CALL OUTUNIT
Convert data to International System of Units
Write NAMELIST input
Is data input in U.S. Customary System of Units ?
Yes
No
CALL INUNIT
Convert data to U.S. Customary System of Units
3


3
Calculate u_e, T_e, R_e $\frac{d\xi}{dx}$, $\frac{du_e}{dx}$, and $\frac{dT_e}{dx}$
Set up difference-quotient coefficients for x-coordinates
Set boundary conditions
Set up difference-quotient coefficients for y-coordinates
Set up matrix elements and arrays
Perform matrix solution for momentum and energy equations
Integrate continuity equation
Check for sufficient grid width
Determine location of boundary-layer edge
4


4
Calculate wall and initial values required for basic boundary-layer parameters
Calculate basic boundary-layer parameters:
δ^*, θ, u_τ, u^+, y^+, $\frac{\partial F}{\partial \eta}$, $\frac{\partial \Theta}{\partial \eta}$, $\frac{T_t}{T_{t,e}}$, $l = \frac{\rho\mu}{(\rho\mu)_e}$, $\frac{\partial l}{\partial \eta}$, $\frac{M}{M_e}$, $\frac{y}{y_e}$, $\frac{\delta^*}{\theta}$, $R_{e,\theta}$, R_{e,δ^*}, $R_{e,x}$, $C_{f,e}$, $C_{f,w}$, q_w, T_{aw}, h, $N_{St,e}$, $N_{St,w}$
Calculate vorticity Reynolds number and determine stability index
Calculate total-pressure ratio
Is flow turbulent ?
No
Yes
CALL TURBLNT
Process turbulent-flow calculations
Variable entropy ?
No
6
Yes
5


5
Perform variable-entropy calculations
Is data output expected in U.S. Customary System of Units ?
6
Yes
No
CALL WALLOUT
Convert data to International System of Units
Write profiles; write wall values
Update variables for marching procedure
Has end of body been reached ?
No
3
Yes
Stop

The program listing for the main program D2401 is as follows:

```
      PROGRAM D2401(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE21,TAPE9,T
     1APE4)
      DIMENSION EN(JK), EO(JK), EP(JK), TN(JK), TO(JK), TP(JK), VN(JK),
     1VO(JK), VP(JK), EP(JK), EPP(JK), EZ(JK), TZ(JK), PTOPT(JK), STAB2(
     2JK), MOMF(JK), XN(JK), Y(JK), DY(JK), XK1(JK), XK2(JK), XK3(JK), X
     3L1(JK), XL2(JK), XL3(JK), XLM11(JK), XLPM11(JK), RATON(JK), RATOO(
     4JK), RATOP(JK), TTOTT(JK), CROCCO(JK), UOUPL(JK), TCORD(JK), UDEF(
     5JK), NONDEL(JK), EETM(JK), EEPM(JK), UEE(JL), PROVAL(JN), PRNTVAL(
     6JM)
      DIMENSION VARA(JK), VARB(JK), VARC(JK), VARD(JK), VARE(JK)
      DIMENSION YO(6)
      DIMENSION RRS(100), ZZS(100), DRSDZS(100)
      DIMENSION PRTAR(100), GLAR(100)
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TREACT,TBAR,XLBAR,DISINC,XT1,X
     1T2,XT3,XT4,XT5,RE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT,PRT,PRTAR,GLAR,NUMB1
      COMMON /UNIT/ VISCON,PT1,TT1,WAVE,R,SU,CONE,DS,SST,RT1,P1,T1,R1,U1
     1,AA1,TREF,VISREF,PESTAR,TESTAR,RESTAR,UESTAR,MUESTAR,YESTAR,THETA,
     2TAUD,QSD,HD,UPLUS,DISP,PE,Z,TW,QW,RVWALD,PROINC,PRNTINC
      INTEGER ETAEDG,W
      REAL MOMF,NUE,NUW,KWD,KED,INTGRL,INTEGT,INTEGL,NONDEL,MUESTAR
      EXTERNAL INTEGT
      NAMELIST /NAM1/ HPR,XEND,H,PR,XKK,BETA,ALPHA,XO,YO,IEDGE,XK,IGAS,V
     1ISCON,VISPOW,KODUNIT
      NAMELIST /NAM2/ XMA,PT1,TT1,WAVE,XY1,XY2,XY3,G,R,SU,PR,PRT,IBODY,J
     1,W,ET,KODE,KODWAL,IENTRO,CONE,IEND1,A,DS,DY,KODVIS,SST,SMXTR,TLNGT
     2H,CORP,CONSTNT,XT1,XT2,XT3,XT4,XT5,PROINC,PRNTINC,IPRO,PROVAL,IPRN
     3T,PRNTVAL,NAUXPRO,BLNGTH,NPUTYPE,KODPRT,NUMB1,PRTAR,GLAR,KTCOD
      NAMELIST /NAM/ P10,T10,G,REY,RT1,P1,T1,R1,U1,AA1,TREF,VISREF,R10
      NAMELIST /DBG/ S,Y1,Y2,Y3,Y4,Y5,Y6,Z1,Z2,Z3,Z4,Z5,X,DY,DZ,ITRO,XK1
     1,XK2,XK3,XL1,XL2,XL3,EN,EO,EP,TN,TO,TP,VN,VO,VP,EP,EPP,XLM11,XLPM1
     21,RATOP

*       INITIALIZE DATA TO STANDARD INPUT

      DATA WAVE/90./,G/1.4/,R/1716./,SU/198.6/,PR/.72/,PRT/.9/,W/0/,ET/1
     1.0/,KODE/0/,KODWAL/1/,IENTRO/1/,A/1./,DS/.01/,KODVIS/1/,SST/1.E8/,
     2SMXTR/1.E8/,TLNGTH/2./,CORP/.412/,CONSTNT/0./,XT1/.4/,XT2/26./,XT3
     3/.0168/,XT4/.78/,XT5/0./,PROINC/1./,PRNTINC/.1/,NAUXPRO/0/,BLNGTH/
     40./,NPUTYPE/1/,IPRO/0/,IPRNT/0/
      DATA THATS/0./,RS/0./,P20/0./,H1,H2,H3,H4,H5,H6,H7,H8,H9,H10,H11/1
     111*0./,NSUBLY/3/,XPX/.5/,SIGN/1./,SMXN/0./,SMXO/0./,SMXP/0./,ST2MAX
     2/1000001/,TREACT/0./,KSTR/0/,ITMAX/3/,NOIT/0/,DPEDS/0./,Z/0./,KODP
     3RT/1/,KTCOD/2/,KTCO/0/,CONE/0.0/
      REWIND 9
      DO 1 I=1,JN
    1 PROVAL(I)=0.
```

```
      DO 2 I=1,JM                                                       A  46   4800000
    2 PRNTVAL(I)=0.                                                     A  47   4900000
      DO 3 I=1,JL                                                       A  48   5000000
    3 UFE(I)=0.                                                         A  49   5100000
      DO 4 I=1,JK                                                       A  50   5200000
      FEPM(I)=STAB2(I)=Y(I)=XN(I)=VN(I)=VO(I)=VP(I)=EPP(I)=XK1(I)=XK2(I)  A  51   5300000
     1=XK3(I)=XL1(I)=XL2(I)=XL3(I)=FZ(I)=TZ(I)=XLPM11(I)=0.0            A  52   5400000
      RATOP(I)=RATOO(I)=RATON(I)=TN(I)=TO(I)=TP(I)=FN(I)=FO(I)=FP(I)=EP(  A  53   5500000
     1I)=XLM11(I)=1.0                                                   A  54   5600000
    4 CONTINUE                                                          A  55   5700000
                                                                                5800000
*       READ NAMELIST INPUT                                             A  56   5900000
                                                                                6000000
      READ (9,NAM1)                                                     A  57   6100000
      IF (ENDFILE 9) 5,6                                                A  58   6200000
    5 STOP 2                                                            A  59   6300000
    6 CONTINUE                                                          A  60   6400000
      WRITE (6,NAM1)                                                    A  61   6500000
      READ (5,NAM2)                                                     A  62   6600000
      IF (ENDFILE 5) 7,8                                                A  63   6700000
    7 STOP 3                                                            A  64   6800000
    8 CONTINUE                                                          A  65   6900000
      IF (KODUNIT.EQ.1) CALL INUNIT (PROVAL,PRNTVAL,JM,JN)              A  66   7000000
      SI=DS                                                             A  67   7100000
                                                                                7200000
*       READ STARTING PROFILE                                           A  68   7300000
                                                                                7400000
      READ (9,104) Y(1),VN(1),FN(1),DUM,TN(1),X51,DUM                   A  69   7500000
      READ (9,104) (Y(N),VN(N),FN(N),DUM,TN(N),DUM,X6N,N=2,IEDGE)       A  70   7600000
      IF (ENDFILE 9) 9,10                                               A  71   7700000
    9 STOP 4                                                            A  72   7800000
   10 CONTINUE                                                          A  73   7900000
                                                                                8000000
*       SET UP GRID NORMAL TO WALL                                      A  74   8100000
                                                                                8200000
      W1=XK                                                             A  75   8300000
      W2=1+W1                                                           A  76   8400000
      W3=1+W1+W1*W1                                                     A  77   8500000
      WW1=W3*W3*(W1*W2-1.)+W2                                           A  78   8600000
      WW2=W1*W2*W3*W3                                                   A  79   8700000
      WW3=W3*W3                                                         A  80   8800000
      WW4=1.+W1                                                         A  81   8900000
      WW5=W1*W1*W1*W2*W3                                                A  82   9000000
      IF (XK.EQ.1.) GO TO 11                                            A  83   9100000
      DY(1)=((1.-XK)/(1.-XK**(IEDGE-1)))*XEND                           A  84   9200000
      GO TO 12                                                          A  85   9300000
   11 DY(1)=XEND/(IEDGE-1)                                              A  86   9400000
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   12 CONTINUE                                                          A  87    9500000
      EETM(1)=EETM(2)=1./PR                                             A  88    9600000
      DY(2)=XK*DY(1)                                                    A  89    9700000
      DO 13 N=3,JK                                                      A  90    9800000
      DY(N)=(XK**(N-1))*DY(1)                                           A  91    9900000
      EETM(N)=1./PR                                                     A  92   10000000
   13 CONTINUE                                                          A  93   10100000
                                                                                10200000
*       SET UP PRINT CONTROL                                             A  94   10300000
                                                                                10400000
      ALL=DS*IEND1                                                      A  95   10500000
      IF (BLNGTH.NE.0.) ALL=BLNGTH                                      A  96   10600000
      CALL SETUP (PROINC,PROVAL,ALL,JN,IPRO)                            A  97   10700000
      CALL SETUP (PRNTINC,PRNTVAL,ALL,JM,IPRNT)                         A  98   10800000
      WRITE (6,101)                                                     A  99   10900000
      WRITE (6,102)                                                     A 100   11000000
                                                                                11100000
*       WRITE INITIAL PROFILES                                           A 101   11200000
                                                                                11300000
      WRITE (6,103) (Y(N),VN(N),EN(N),TN(N),X51,X6N,N=1,IEDGE)          A 102   11400000
                                                                                11500000
*       PROGRAM CONSTANTS                                                A 103   11600000
                                                                                11700000
   14 CONTINUE                                                          A 104   11800000
      XMAC=1.+.5*(G-1.)*XMA**2                                          A 105   11900000
      RT1=PT1/(R*TT1)                                                   A 106   12000000
      P1=PT1/(XMAC)**(G/(G-1.))                                         A 107   12100000
      R1=RT1/(XMAC)**(1./(G-1.))                                        A 108   12200000
      T1=TT1/XMAC                                                       A 109   12300000
      AA1=SQRT(G*P1/R1)                                                 A 110   12400000
      U1=XMA*AA1                                                        A 111   12500000
      TREF=U1**2/((G/(G-1.))*R)                                         A 112   12600000
      IF (IGAS.EQ.2) GO TO 15                                           A 113   12700000
      VIS1=(2.27E-8)*(T1**1.5)/(T1+SU)                                  A 114   12800000
      VISREF=(2.27E-8)*(TREF**1.5)/(TREF+SU)                            A 115   12900000
      GO TO 16                                                          A 116   13000000
   15 VIS1=VISCON*(T1**VISPOW)                                          A 117   13100000
      VISREF=VISCON*(TREF**VISPOW)                                      A 118   13200000
   16 CONTINUE                                                          A 119   13300000
      REY=R1*U1*A/VIS1                                                  A 120   13400000
      REYREF=REY*VIS1/VISREF                                            A 121   13500000
      XCONE=.0174533*CONE                                               A 122   13600000
      EPS=1./SQRT(REYREF)                                               A 123   13700000
      XWAVE=1.745333E-02*WAVE                                           A 124   13800000
      ABC=(XMA*SIN(XWAVE))**2                                           A 125   13900000
      P10=PT1/(R1*U1*U1)                                                A 126   14000000
      IF (XWAVE.GT..0000001.OR.XWAVE.LT.-.0000001) P10=(1./(G*XMA*XMA))* A 127   14100000
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     1((XMAC*ABC*(G+1.))/(ABC*(G-1.)+2.))**(G/(G-1.))*((G+1.)/(2.*G*ABC-   A 128   14200000
     2(G-1.)))**(1./(G-1.))                                                A 129   14300000
      T10=0.5+1.0/(XMA*XMA*(G-1.0))                                        A 130   14400000
      P10=G*P10/(T10*(G-1.0))                                              A 131   14500000
      TC=SU/(T1*XMAC)                                                      A 132   14600000
      RFL=SQRT(PR)                                                         A 133   14700000
      RFT=PR**0.33333                                                      A 134   14800000
                                                                                   14900000
*        READ TABULAR DATA                                                 A 135   15000000
                                                                                   15100000
      IF (NOIT.GT.0) GO TO 21                                              A 136   15200000
      NABC=NPUTYPE+1                                                       A 137   15300000
      GO TO (21,17,18), NABC                                               A 138   15400000
   17 CALL TABLE (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)             A 139   15500000
      GO TO 19                                                             A 140   15600000
   18 CALL TABLE1 (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)            A 141   15700000
   19 GO TO (21,20), IENTRO                                                A 142   15800000
   20 CALL VARENT (RRS,ZZS,DRSDZS,NNN)                                     A 143   15900000
   21 CONTINUE                                                             A 144   16000000
      GO TO (22,24), IBODY                                                 A 145   16100000
                                                                                   16200000
*       STARTING PROCEDURE FOR FLOWS WITH STAGNATION POINT                 A 146   16300000
                                                                                   16400000
   22 CONTINUE                                                             A 147   16500000
      IF (IGAS.EQ.1) VIS10=(T10**1.5)*(1.0+T10*TC)/(T10+T10*TC)            A 148   16600000
      IF (IGAS.EQ.2) VIS10=T10**VISPOW                                     A 149   16700000
      P1PT2=P1/PT1                                                         A 150   16800000
      IF (XMA.LE.1.) GO TO 23                                              A 151   16900000
      P1PT2=((((G+1.)*XMA**2)/2.)**(-G/(G-1.)))*((G+1.)/((2.*G*XMA**2)-(   A 152   17000000
     1G-1.)))**(-1./(G-1.))                                                A 153   17100000
   23 CONTINUE                                                             A 154   17200000
      DUEDS=SQRT(2.*((G-1.)/G)*T10*(1.-P1PT2))                             A 155   17300000
      QUAN1=DUEDS                                                          A 156   17400000
      BELA=R10*VIS10*DUEDS                                                 A 157   17500000
      RMI=SI                                                               A 158   17600000
      DX1DS=BELA*(SI-DS)**(2*J+1)                                          A 159   17700000
      DXDS=BELA*SI**(2*J+1)                                                A 160   17800000
      DX1=(BELA*SI**(2*J+2))/(2*J+2)                                       A 161   17900000
      X=DX1                                                                A 162   18000000
      QS=-R10*QUAN1*SI*VIS10*T10*RMI*X51/(PR*SQRT(2.*X))                   A 163   18100000
      DISP=SQRT(2.*X)*X6N/(R10*QUAN1*SI*RMI)                               A 164   18200000
      GO TO 27                                                             A 165   18300000
                                                                                   18400000
*       STARTING PROCEDURE FOR FLOWS WITHOUT STAGNATION POINT              A 166   18500000
                                                                                   18600000
   24 CONTINUE                                                             A 167   18700000
      RE=XY1                                                               A 168   18800000
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      TESTR=XY2*T1                                                      A 169   18900000
      IF (J.EQ.0) GO TO 25                                              A 170   19000000
      RE=((G-1.0)/G)*RE*(TESTR/TREF)                                    A 171   19100000
      UE=XY3*SQRT(G*R*TESTR)/U1                                         A 172   19200000
      IF (NOIT.GT.0) UE=UEE(M)                                          A 173   19300000
      IF (IGAS.EQ.1) XNUE=((TESTR/TREF)**1.5)*((TREF+SU)/(TESTR+SU))    A 174   19400000
      IF (IGAS.EQ.2) XNUE=(TESTR/TREF)**VISPOW                          A 175   19500000
      RMI=SI*SIN(XCONE)                                                 A 176   19600000
      BELEX=RE*UE*XNUE*(SIN(XCONE)**(2*J))                              A 177   19700000
      IF (J.NE.0.AND.CONE.EQ.0.) BELEX=RE*UE*XNUE                       A 178   19800000
      DX1DS=BELEX*((SI-DS)**(2*J))                                      A 179   19900000
      GO TO 26                                                          A 180   20000000
   25 CONTINUE                                                          A 181   20100000
      RE=P1/(R1*U1*U1)                                                  A 182   20200000
      TE=T1/TREF                                                        A 183   20300000
      UE=1.                                                             A 184   20400000
      IF (NOIT.GT.0) UE=UEE(M)                                          A 185   20500000
      IF (IGAS.EQ.1) XNUE=(TE**1.5)*((1.+SU/TREF)/(TE+SU/TREF))         A 186   20600000
      IF (IGAS.EQ.2) XNUE=(T1/TREF)**.647                               A 187   20700000
      RMI=1.                                                            A 188   20800000
      BELEX=RE*UE*XNUE                                                  A 189   20900000
      DX1DS=BELEX                                                       A 190   21000000
   26 DXDS=BELEX*SI**(2*J)                                              A 191   21100000
      DX1=(BELEX*(SI**(2*J+1)))/(2*J+1)                                 A 192   21200000
      X=DX1                                                             A 193   21300000
      QS=DISP=0.                                                        A 194   21400000
   27 CONTINUE                                                          A 195   21500000
                                                                                21600000
*       SET UP INITIAL PROFILES                                          A 196   21700000
                                                                                21800000
      DO 28 N=1,IEDGE                                                   A 197   21900000
      VN(N)=-VN(N)                                                      A 198   22000000
      FO(N)=FN(N)                                                       A 199   22100000
      TO(N)=TN(N)                                                       A 200   22200000
   28 VO(N)=VN(N)                                                       A 201   22300000
      IF (KODUNIT.EQ.1) CALL OUTUNIT (PROVAL,PRNTVAL,JM,JN)             A 202   22400000
      WRITE (6,NAM2)                                                    A 203   22500000
      WRITE (6,NAM)                                                     A 204   22600000
      IF (KODUNIT.EQ.1) CALL INUNIT (PROVAL,PRNTVAL,JM,JN)              A 205   22700000
      ZS=0.                                                             A 206   22800000
      XLM1N=1.                                                          A 207   22900000
      XLM1C=XLM1N                                                       A 208   23000000
                                                                                23100000
*       BEGIN MARCHING PROCEDURE ALONG SURFACE                           A 209   23200000
                                                                                23300000
      SM1=0.                                                            A 210   23400000
      S=DS                                                              A 211   23500000
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      DO 93 M=2,IEND1                                                   A 212   23600000
      SM2=SM1                                                           A 213   23700000
      SM1=S                                                             A 214   23800000
      READ (4) S,PE,RMI,TW,Z,DPEDS,RVWALD,DRDZ,QW                       A 215   23900000
      IF (KOCWAL.NE.1) CW=QW*VISREF*U1*U1/(778.26*A)                    A 216   24000000
      PHI=ATAN(DRDZ)                                                    A 217   24100000
      COSTH=COS(PHI)                                                    A 218   24200000
      PP=DPEDS                                                          A 219   24300000
      UE=SQRT(2.0*T10*(1.0-(PE/P10)**((G-1.0)/G)))                      A 220   24400000
      IF (NOIT.GT.0) UE=UEE(M)                                          A 221   24500000
      TE=T10-0.5*UE*UE                                                  A 222   24600000
      RE=G*PE/((G-1.0)*TE)                                              A 223   24700000
      IF (IGAS.EQ.1) XNUE=(TE**1.5)*(1.0+T10*TC)/(TE+T10*TC)            A 224   24800000
      IF (IGAS.EQ.2) XNUE=TE**VISPOW                                    A 225   24900000
      DX2DS=DX1DS                                                       A 226   25000000
      DX1DS=DXDS                                                        A 227   25100000
      DXDS=RE*UE*XNUE*(RMI**(2*J))                                      A 228   25200000
                                                                                25300000
*       CHECK FOR CHANGE IN STEP INCREMENT                              A 229   25400000
                                                                                25500000
      CKK=(S-SM1)/(SM1-SM2)                                             A 230   25600000
      IF (CKK.LT..999999999) GO TO 29                                   A 231   25700000
      IF (CKK.GT.1.0000001) GO TO 29                                    A 232   25800000
      DX2=(S-SM1)*(5.*DXDS+8.*DX1DS-DX2DS)/12.                          A 233   25900000
      GO TO 30                                                          A 234   26000000
   29 DX2=(S-SM1)*(DX1DS+DXDS)/2.                                       A 235   26100000
   30 CONTINUE                                                          A 236   26200000
      X=X+DX2                                                           A 237   26300000
      DY=(2.*X)**(2.*XPX)                                               A 238   26400000
      DZ=(2.*X)**(XPX)                                                  A 239   26500000
      DUEDX=-PP/(RE*UE*DXDS)                                            A 240   26600000
      DTEDX=-UE*DUEDX                                                   A 241   26700000
      XAL=UE*UE/TE                                                      A 242   26800000
      XBE=DY*DUEDX/UE                                                   A 243   26900000
      TR=T10*TC/TE                                                      A 244   27000000
                                                                                27100000
*       SET UP DIFFERENCE-QUOTIENT COEFFICIENTS FOR X-COORDINATE        A 245   27200000
                                                                                27300000
      Z1=2.*((DX1+2.*DX2)/(DX1+DX2))                                    A 246   27400000
      Z2=2.*(DX1+DX2)/DX1                                               A 247   27500000
      Z3=2.*((DX2*DX2)/(DX1*(DX1+DX2)))                                 A 248   27600000
      Z4=(DX1+DX2)/DX1                                                  A 249   27700000
      Z5=DX2/DX1                                                        A 250   27800000
                                                                                27900000
*                       SET BOUNDARY CONDITIONS                         A 251   28000000
                                                                                28100000
      XK1(1)=0.0                                                        A 252   28200000
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      XK2(1)=0.0                                                        A 253   28300000
      XK3(1)=0.0                                                        A 254   28400000
      XL2(1)=0.0                                                        A 255   28500000
      XL3(1)=0.0                                                        A 256   28600000
      IF (KODWAL.EQ.1) GO TO 31                                         A 257   28700000
      XLM1P=Z4*XLM10-Z5*XLM1N                                           A 258   28800000
      TZ(1)=-QW*778.26*(EPS*A/(VISREF*U1*02))*(PR*OZ/(RE*UE*TE*XNUE*RMI*  A 259   28900000
     1*J))*(1.0/XLM1P)                                                  A 260   29000000
      GO TO 32                                                          A 261   29100000
   31 CONTINUE                                                          A 262   29200000
      XL1(1)=TW/TE                                                      A 263   29300000
      TP(1)=XL1(1)                                                      A 264   29400000
   32 CONTINUE                                                          A 265   29500000
      FP(1)=0.0                                                         A 266   29600000
      N=IEDGE+1                                                         A 267   29700000
      KON=N+1                                                           A 268   29800000
      FAA=OZ/(RE*UE*(RMI**J))                                           A 269   29900000
      FAB=2.*EPS*W*FAA*COSTH/(RMI**J)                                   A 270   30000000
      FAC=2.*EPS*OZ*COSTH/(RE*UE*RMI*RMI)                               A 271   30100000
      FAD=RMI/(EPS*COSTH)                                               A 272   30200000
      ITRO=0                                                            A 273   30300000
                                                                                30400000
*        SET UP DIFFERENCE COEFFICIENTS FOR Y COORDINATE                A 274   30500000
                                                                                30600000
      DY1=DY(1)                                                         A 275   30700000
      DO 39 N=2,IEDGE                                                   A 276   30800000
      DY2=XK*DY1                                                        A 277   30900000
      Y1=2./((DY1+DY2)*DY2)                                             A 278   31000000
      Y2=2./(DY1*DY2)                                                   A 279   31100000
      Y3=Y1*(DY2/DY1)                                                   A 280   31200000
      Y4=Y1*(DY1/2.)                                                    A 281   31300000
      Y5=(DY1-DY2)/(DY1*DY2)                                            A 282   31400000
      Y6=DY2/(DY1*(DY1+DY2))                                            A 283   31500000
                                                                                31600000
*       SET UP MATRIX ELEMENTS                                           A 284   31700000
                                                                                31800000
      IF (ITRO.EQ.0) GO TO 33                                           A 285   31900000
      TM1=TP(N)                                                         A 286   32000000
      RATO=RATOP(N)                                                     A 287   32100000
      DRATO=FAB*TM1                                                     A 288   32200000
      RATO2J=RATO**(2.*J)                                               A 289   32300000
      XLM1=XLM11(N)                                                     A 290   32400000
      XLPM1=XLPM11(N)                                                   A 291   32500000
      TY=TZ(N)                                                          A 292   32600000
      FY=FZ(N)                                                          A 293   32700000
      FM1=FP(N)                                                         A 294   32800000
      EM1=EP(N)                                                         A 295   32900000
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      EPM1=EPP(N)                                                       A 296   33000000
      GO TO 37                                                          A 297   33100000
   33 CONTINUE                                                          A 298   33200000
      TM1=Z4*TO(N)-Z5*TN(N)                                             A 299   33300000
      IF (TM1.GT.0.) GO TO 34                                           A 300   33400000
      TM1=(TO(N)+TN(N))/2.                                              A 301   33500000
      WRITE (6,100) S,M,N                                               A 302   33600000
   34 CONTINUE                                                          A 303   33700000
      RATO=Z4*RATOO(N)-Z5*RATON(N)                                      A 304   33800000
      DRATO=FAB*TM1                                                     A 305   33900000
      RATO2J=RATO**(2*J)                                                A 306   34000000
      IF (IGAS.EQ.2) GO TO 35                                           A 307   34100000
      XLM1=((1.+TR)*SQRT(TM1)/(TM1+TR))                                 A 308   34200000
      XLPM1=XLM1*(TR-TM1)/(2.*TM1*(TM1+TR))                             A 309   34300000
      GO TO 36                                                          A 310   34400000
   35 XLM1=TM1**(VISPOW-1.)                                             A 311   34500000
      XLPM1=(VISPOW-1.)*(TM1**(VISPOW-2.))                              A 312   34600000
   36 CONTINUE                                                          A 313   34700000
      FY=(FO(N+1)-FO(N-1))/(DY(N-1)+DY(N))                              A 314   34800000
      TY=(TO(N+1)-TO(N-1))/(DY(N-1)+DY(N))                              A 315   34900000
      FM1=Z4*FO(N)-Z5*FN(N)                                             A 316   35000000
      EM1=EP(N)                                                         A 317   35100000
      EPM1=EPP(N)                                                       A 318   35200000
   37 CONTINUE                                                          A 319   35300000
      VM1=Z4*VO(N)-Z5*VN(N)                                             A 320   35400000
      TM2=Z2*TO(N)-Z3*TN(N)                                             A 321   35500000
      FM2=Z2*FO(N)-Z3*FN(N)                                             A 322   35600000
      ETM1=EETM(N)                                                      A 323   35700000
      ETPM1=EEPM(N)                                                     A 324   35800000
      H1=DY*FM1*FT/(2.*DX2)                                             A 325   35900000
      H2=VM1-XLM1*((RATO2J*EPM1)+(EM1*DRATO))                           A 326   36000000
      H3=-RATO2J*XLM1*EM1                                               A 327   36100000
      H4=H3*XLPM1/XLM1                                                  A 328   36200000
      H5=XBE*FM1                                                        A 329   36300000
      H6=-XBE                                                           A 330   36400000
      H7=VM1-XLM1*((RATO2J*ETPM1)+(ETM1*DRATO))                         A 331   36500000
      H8=-XAL*RATO2J*XLM1*EM1                                           A 332   36600000
      H9=-RATO2J*XLPM1*ETM1                                             A 333   36700000
      H10=H9*XLM1/XLPM1                                                 A 334   36800000
      H11=H2+H4*TY                                                      A 335   36900000
      H12=H7+2.*H9*TY                                                   A 336   37000000
      A1=-Y6*H11+Y3*H3                                                  A 337   37100000
      B1=-Y5*H11-Y2*H3+H5+Z1*H1                                         A 338   37200000
      C1=Y4*H11+Y1*H3                                                   A 339   37300000
      D1=-Y6*H4*FY                                                      A 340   37400000
      E1=(D1*Y5/Y6)+H6                                                  A 341   37500000
      F1=-D1*Y4/Y6                                                      A 342   37600000
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      G1=H1*FM2+H4*TY*FY                                                A 343   37700000
      A2=-2.*H8*FY*Y6                                                   A 344   37800000
      B2=A2*Y5/Y6                                                       A 345   37900000
      C2=-A2*Y4/Y6                                                      A 346   38000000
      D2=-Y6*H12+H10*Y3                                                 A 347   38100000
      E2=-Y5*H12-H10*Y2+H1*Z1                                           A 348   38200000
      F2=Y4*H12+H10*Y1                                                  A 349   38300000
      G2=H1*TM2+H8*FY*FY+H9*TY*TY                                       A 350   38400000
      IF (KODWAL.EQ.1) GO TO 38                                         A 351   38500000
      IF (N.GT.2) GO TO 38                                              A 352   38600000
      DID=(C2*D1-C1*D2)-((C2*F1-C1*F2)*(((1.+XK)**2)-1.))               A 353   38700000
      XL1(1)=((C2*G1-C1*G2)+(C2*F1-C1*F2)*(XK*(1+XK)*DY(1))*TZ(1))/DID  A 354   38800000
      XL2(1)=-(C2*B1-C1*B2)/DID                                         A 355   38900000
      XL3(1)=-((C2*E1-C1*E2)+((C2*F1-C1*F2)*((1.+XK)**2)))/DID          A 356   39000000
   38 CONTINUE                                                          A 357   39100000
                                                                                39200000
*        SET UP MATRIX ARRAYS                                            A 358   39300000
                                                                                39400000
      B1S=B1+A1*XK2(N-1)+D1*XL2(N-1)                                    A 359   39500000
      B2S=B2+A2*XK2(N-1)+D2*XL2(N-1)                                    A 360   39600000
      E1S=E1+A1*XK3(N-1)+D1*XL3(N-1)                                    A 361   39700000
      E2S=E2+A2*XK3(N-1)+D2*XL3(N-1)                                    A 362   39800000
      G1S=G1-A1*XK1(N-1)-D1*XL1(N-1)                                    A 363   39900000
      G2S=G2-A2*XK1(N-1)-D2*XL1(N-1)                                    A 364   40000000
      D=1.0/(B1S*E2S-E1S*B2S)                                           A 365   40100000
      XK1(N)=D*(G1S*E2S-G2S*E1S)                                        A 366   40200000
      XK2(N)=D*(E1S*C2-C1*E2S)                                          A 367   40300000
      XK3(N)=D*(E1S*F2-F1*E2S)                                          A 368   40400000
      XL1(N)=D*(B1S*G2S-B2S*G1S)                                        A 369   40500000
      XL2(N)=D*(C1*B2S-B1S*C2)                                          A 370   40600000
      XL3(N)=D*(F1*B2S-B1S*F2)                                          A 371   40700000
      DY1=DY2                                                           A 372   40800000
   39 CONTINUE                                                          A 373   40900000
      N=IEDGE+1                                                         A 374   41000000
      NN=N-1                                                            A 375   41100000
      KON=NN                                                            A 376   41200000
                                                                                41300000
*        MATRIX SOLUTION FOR MOMENTUM AND ENERGY EQUATIONS               A 377   41400000
                                                                                41500000
      DO 40 N=2,NN                                                      A 378   41600000
      FP(KON)=XK1(KON)+XK2(KON)*FP(KON+1)+XK3(KON)*TP(KON+1)            A 379   41700000
      TP(KON)=XL1(KON)+XL2(KON)*FP(KON+1)+XL3(KON)*TP(KON+1)            A 380   41800000
   40 KON=KON-1                                                         A 381   41900000
      IF (KODWAL.EQ.1) GO TO 41                                         A 382   42000000
      TP(1)=(XK*(1+XK)*DY(1)*XLM1P-(1+XK)**2*TP(2)+TP(3))/(1-(1+XK)**2) A 383   42100000
      TW=TP(1)*TE                                                       A 384   42200000
      RECTOR=(TP(1)-1.)/((TT1/(TE*TREF))-1.)                            A 385   42300000
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   41 CONTINUE                                                          A 386   42400000
      TWOTT1=TP(1)*TE*TREF/TT1                                          A 387   42500000
                                                                              42600000
*        INTEGRATE CONTINUITY EQUATION                                  A 388   42700000
                                                                              42800000
      VP(1)=(RVWALD*FAA)/(R1*U1*EPS*XNUE)                               A 389   42900000
      BO=0.0                                                            A 390   43000000
      DO 42 N=2,NN                                                      A 391   43100000
      B=DY*((XPX*FP(N)/X)+FT*((Z1*FP(N)-Z2*FO(N)+Z3*FN(N))/(2.*DX2)))   A 392   43200000
      VP(N)=VP(N-1)-0.5*(B+BO)*DY(N-1)                                  A 393   43300000
   42 BO=B                                                              A 394   43400000
                                                                              43500000
*       CHECK FOR SUFFICIENT GRID WIDTH                                 A 395   43600000
                                                                              43700000
      IF (M-20) 48,48,43                                                A 396   43800000
   43 IF (ABS(FP(IEDGE-15)-FP(IEDGE-16))-.0001) 44,44,45                A 397   43900000
   44 IF (ABS(TP(IEDGE-15)-TP(IEDGE-16))-.0001) 48,48,45                A 398   44000000
   45 IF (XK.EQ.1.) GO TO 46                                            A 399   44100000
      WRITE (6,105)                                                     A 400   44200000
      STOP                                                              A 401   44300000
   46 IF (TREACT.LT.1.) GO TO 47                                        A 402   44400000
      WRITE (6,106)                                                     A 403   44500000
      STOP                                                              A 404   44600000
   47 IF (IEDGE.LE.301) GO TO 49                                        A 405   44700000
      WRITE (6,107)                                                     A 406   44800000
      STOP                                                              A 407   44900000
   48 IF (XK.NE.1..OR.TREACT.LT.1.) GO TO 50                            A 408   45000000
      WRITE (6,108)                                                     A 409   45100000
      STOP                                                              A 410   45200000
   49 IEDGE=IEDGE+1                                                     A 411   45300000
   50 CONTINUE                                                          A 412   45400000
                                                                              45500000
*       DETERMINE LOCATION OF BOUNDARY LAYER EDGE                       A 413   45600000
                                                                              45700000
      NCOUNT=0                                                          A 414   45800000
      DO 51 I=1,IEDGE                                                   A 415   45900000
      NCOUNT=NCOUNT+1                                                   A 416   46000000
      IF (FP(I).GE..9999) GO TO 52                                      A 417   46100000
   51 CONTINUE                                                          A 418   46200000
      WRITE (6,109)                                                     A 419   46300000
      WRITE (6,DBG)                                                     A 420   46400000
      CALL DUMP                                                         A 421   46500000
      STOP                                                              A 422   46600000
   52 ETAEDG=NCOUNT                                                     A 423   46700000
                                                                              46800000
*       CALCULATE WALL AND INITIAL VALUES REQUIRED                      A 424   46900000
*      FOR BASIC BOUNDARY LAYER PARAMETERS                              A 425   47000000
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                                                                        47100000
      IF (IGAS.EQ.2) GO TO 53                                   A 426   47200000
      XLM11(1)=((1.+TR)*SQRT(TP(1))/(TP(1)+TR))                 A 427   47300000
      XLM1P=XLM11(1)                                            A 428   47400000
      XLPM11(1)=XLM11(1)*(TR-TP(1))/(2.*TP(1)*(TP(1)+TR))       A 429   47500000
      GO TO 54                                                  A 430   47600000
   53 XLM11(1)=TP(1)**(VISPOW-1.)                               A 431   47700000
      XLM1P=XLM11(1)                                            A 432   47800000
      XLPM11(1)=(VISPOW-1.)*(TP(1)**(VISPOW-2.))                A 433   47900000
   54 CONTINUE                                                  A 434   48000000
      MONF(1)=FP(1)/SQRT(TP(1))                                 A 435   48100000
      IF (KODWAL.NE.1) GO TO 55                                 A 436   48200000
      TZ(1)=(-WW1*TP(1)+WW2*TP(2)-WW3*TP(3)+WW4*TP(4))/(WW5*DY(1))  A 437  48300000
   55 CONTINUE                                                  A 438   48400000
      FZ(1)=(-WW1*FP(1)+WW2*FP(2)-WW3*FP(3)+WW4*FP(4))/(WW5*DY(1))  A 439  48500000
      XMF1=2.0+XAL                                              A 440   48600000
      XMF2=TP(1)*TF/T10                                         A 441   48700000
      XMF3=1.0-XMF2                                             A 442   48800000
      TTOTT(1)=(2.0*TP(1)+XAL*FP(1)*FP(1))/XMF1                 A 443   48900000
      CROCCO(1)=(TTOTT(1)-XMF2)/XMF3                            A 444   49000000
      UOUPL(1)=0.0                                              A 445   49100000
      TCORD(1)=0.0                                              A 446   49200000
      UDEF(1)=0.0                                               A 447   49300000
      NONDEL(1)=0.0                                             A 448   49400000
      XMF4=EPS*XNUE*UE*UE*(RMI**J)*XLM11(1)*FZ(1)/DZ            A 449   49500000
      CO=0.0                                                    A 450   49600000
      DISP=0.0                                                  A 451   49700000
      THETA=0.0                                                 A 452   49800000
      DISINC=0.0                                                A 453   49900000
      TPK=0.0                                                   A 454   50000000
      KON=IEDGE+2                                               A 455   50100000
                                                                        50200000
*       CALCULATE BASIC BOUNDARY LAYER PARAMETERS               A 456   50300000
                                                                        50400000
      DO 60 N=2,KON                                             A 457   50500000
      TPK=TPK+0.5*(TP(N-1)+TP(N))*DY(N-1)                       A 458   50600000
      RATOP(N)=SIGN*SQRT(1.0+FAB*TPK)                           A 459   50700000
      XN(N)=FAD*(-1.0+SIGN*SQRT(1.0+FAC*TPK))                   A 460   50800000
      DISP=DISP+.5*((TP(N-1)-FP(N-1))+(TP(N)-FP(N)))*DY(N-1)    A 461   50900000
      C=1.0/(RATOP(N)**J)                                       A 462   51000000
      C=C*(FP(N)*(1.-FP(N)))                                    A 463   51100000
      THETA=THETA+0.5*(CO+C)*DY(N-1)                            A 464   51200000
      CO=C                                                      A 465   51300000
      IF (TREACT.EQ.0.0) GO TO 56                               A 466   51400000
      UPLUS=SQRT(XMF4*TP(N))                                    A 467   51500000
      UOUPL(N)=UE*FP(N)/UPLUS                                   A 468   51600000
      TCORD(N)=(XN(N)*UPLUS*RE)/(EPS*XNUE*((TP(N)*XLM11(N))**2))  A 469  51700000
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      UDEF(N)=UE*(1.-FP(N))/UPLUS                                       A 470   51800000
      SUBTST=ABS(UDUPL(N)-TCORD(N))/((UDUPL(N)+TCORD(N))/2.)            A 471   51900000
      IF (SUBTST.GE.0.06) GO TO 56                                      A 472   52000000
      NSUBLY=N                                                          A 473   52100000
   56 CONTINUE                                                          A 474   52200000
      DISINC=DISINC+0.5*((TP(N-1)*(1.0-FP(N-1))/RATOP(N-1))+(TP(N)*(1.0-A 475   52300000
     1FP(N))/RATOP(N)))*DY(N-1)                                         A 476   52400000
      FZ(N)=(FP(N+1)-FP(N-1))/(DY(N-1)+DY(N))                           A 477   52500000
      TZ(N)=(TP(N+1)-TP(N-1))/(DY(N-1)+DY(N))                           A 478   52600000
      TTOTT(N)=(2.0*TP(N)+XAL*FP(N)*FP(N))/XMF1                         A 479   52700000
      CROCCO(N)=(TTOTT(N)-XMF2)/XMF3                                    A 480   52800000
      IF (TP(N).GE.0.) GO TO 57                                         A 481   52900000
      WRITE (6,110) N,S,N,TP(N)                                         A 482   53000000
      WRITE (6,DBG)                                                     A 483   53100000
      STOP 2                                                            A 484   53200000
   57 CONTINUE                                                          A 485   53300000
      IF (IGAS.EQ.2) GO TO 58                                           A 486   53400000
      XLM11(N)=((1.+TR)*SQRT(TP(N))/(TP(N)+TR))                         A 487   53500000
      XLPM11(N)=XLM11(N)*(TR-TP(N))/(2.*TP(N)*(TP(N)+TR))               A 488   53600000
      GO TO 59                                                          A 489   53700000
   58 XLM11(N)=TP(N)**(VISPOW-1.)                                       A 490   53800000
      XLPM11(N)=(VISPOW-1.)*(TP(N)**(VISPOW-2.))                        A 491   53900000
   59 CONTINUE                                                          A 492   54000000
      MOME(N)=FP(N)/SQRT(TP(N))                                         A 493   54100000
   60 CONTINUE                                                          A 494   54200000
      DISINC=DISINC*EPS*FAA                                             A 495   54300000
      THETA=THETA*EPS*FAA*A                                             A 496   54400000
      DISP=DISP*EPS*FAA*A                                               A 497   54500000
      DISP=ABS(DISP)                                                    A 498   54600000
      XNDEN=XN(ETAEDG)                                                  A 499   54700000
      DO 61 N=2,KON                                                     A 500   54800000
   61 NONDEL(N)=XN(N)/XNDEN                                             A 501   54900000
      IF (J.EQ.1) GO TO 62                                              A 502   55000000
      GO TO 63                                                          A 503   55100000
   62 DISP=RMI*(-1.+SQRT(1.+(2.*DISP/RMI)))                             A 504   55200000
   63 CONTINUE                                                          A 505   55300000
      THADIS=DISP/THETA                                                 A 506   55400000
      REDELT=(RE*UE*DISP/XNUE)*REYREF                                   A 507   55500000
      RETHET=REDELT/THADIS                                              A 508   55600000
      RES=(RE*UE*S/XNUE)*REYREF                                         A 509   55700000
      XMAE=UE/SQRT(TE*(G-1.))                                           A 510   55800000
      TAUD=VISREF*U1*(XLM11(1)*RE*XNUE*UE*UE*(RMI**J)*FZ(1)/DZ)/(EPS*A) A 511   55900000
      CFE=TAUD/(.5*R1*U1*U1*RE*UE*UE)                                   A 512   56000000
      CFW=CFE*TW/TE                                                     A 513   56100000
      IF (KODWAL.NE.1) GO TO 64                                         A 514   56200000
      QS=-XLM11(1)*RE*UE*TE*XNUE*(RMI**J)*TZ(1)/(PR*DZ*EPS)             A 515   56300000
      QSD=QS*VISREF*U1*U1/(778.26*A)                                    A 516   56400000
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      GO TO 65                                                          A 517   56500000
   64 QSD=QW                                                            A 518   56600000
   65 CONTINUE                                                          A 519   56700000
      TWD=TREF*TW                                                       A 520   56800000
      TAWD=(REL+(RET-REL)*TRFACT)*(TT1-TE*TREF)+TE*TREF                 A 521   56900000
      HD=QSD/(TWD-TAWD)                                                 A 522   57000000
      CHE=778.26*HD*(G-1.)/(G*R*R1*U1*RE*UE)                            A 523   57100000
      CHW=CHE*TW/TE                                                     A 524   57200000
      HSD=HD*A*S                                                        A 525   57300000
      KED=R*VISREF*XNUE*G/(778.26*PR*(G-1.))                            A 526   57400000
      KWD=KED*XLM11(1)*TP(1)                                            A 527   57500000
      NUE=HSD/KED                                                       A 528   57600000
      NUW=HSD/KWD                                                       A 529   57700000
      TBAR=TP(1)                                                        A 530   57800000
      XLBAR=XLM11(1)                                                    A 531   57900000
      IF (TRFACT.EQ.0.0) GO TO 67                                       A 532   58000000
      SUM1=0.                                                           A 533   58100000
      SUM2=SUM1                                                         A 534   58200000
      DO 66 N=1,NSUBLY                                                  A 535   58300000
      SUM1=SUM1+TP(N)                                                   A 536   58400000
   66 SUM2=SUM2+XLM11(N)                                                A 537   58500000
      TBAR=SUM1/NSUBLY                                                  A 538   58600000
      XLBAR=SUM2/NSUBLY                                                 A 539   58700000
   67 CONTINUE                                                          A 540   58800000
                                                                                58900000
*             TOTAL PRESSURE RATIO, (PT2)BL/(PT2)REF                    A 541   59000000
                                                                                59100000
      AZ1=XMA*XMA*(G+1.)/2.                                             A 542   59200000
      AZ2=(G+1.)/(2.*G*XMA*XMA-G+1.)                                    A 543   59300000
      PTREF=PT1                                                         A 544   59400000
      IF (XMA.LE.1.) GO TO 68                                           A 545   59500000
      PTREF=(AZ1**(G/(G-1.))*(AZ2**(1./(G-1.))))/(G*XMA*XMA)            A 546   59600000
   68 CONTINUE                                                          A 547   59700000
      DO 71 I=1,IEDGE                                                   A 548   59800000
      ZEB=(MOME(I)*XMAF)**2                                             A 549   59900000
      IF (ZEB-1.) 69,69,70                                              A 550   60000000
   69 PT2=PE*((1.+(G-1.)*ZEB/2.)**(G/(G-1.)))                           A 551   60100000
      GO TO 71                                                          A 552   60200000
   70 PT2=PE*((((G+1.)*ZEB/2.)**(G/(G-1.)))*(((G+1.)/(2.*G*ZEB-G+1.))**(  A 553   60300000
     11./(G-1.))))                                                      A 554   60400000
   71 PTOPT(I)=PT2/PTREF                                                A 555   60500000
                                                                                60600000
*        CALCULATE VORTICITY REYNOLDS NUMBER                            A 556   60700000
*        AND DETERMINE STABILITY INDEX                                  A 557   60800000
                                                                                60900000
      IF (TRFACT.GT.0.0.AND.TRFACT.LT.0.9999) GO TO 74                  A 558   61000000
      DO 72 I=1,ETAEDG                                                  A 559   61100000
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      INTGRL=INTEGT(TP,I,DY)                                            A 560   61200000
      TERMB=((OZ/((RMI**J)*XNUE))*(INTGRL**2))/EPS                      A 561   61300000
   72 STAB2(I)=TERMB*FZ(I)/(XLM11(I)*TP(I)**3)                          A 562   61400000
      ST2MAX=STAB2(1)                                                   A 563   61500000
      DO 73 IX=2,IEDGE                                                  A 564   61600000
      IF (ST2MAX.GT.STAB2(IX)) GO TO 73                                 A 565   61700000
      ST2MAX=STAB2(IX)                                                  A 566   61800000
   73 CONTINUE                                                          A 567   61900000
      SMXP=ST2MAX                                                       A 568   62000000
   74 CONTINUE                                                          A 569   62100000
      DSMXO=(SMXP-SMXN)/(2.*DS)                                         A 570   62200000
      SMXN=SMXO                                                         A 571   62300000
      SMXO=SMXP                                                         A 572   62400000
      IF (KTCOD.EQ.2.OR.KTCD.EQ.1) GO TO 75                             A 573   62500000
      KTCD=1                                                            A 574   62600000
      RRRR=RE*UE*R1*U1/(XNUE*VISREF)                                    A 575   62700000
      TLNGTH=1.+(5.*RRRR**(-1)*(RRRR*A*SST)**.8)/SST                    A 576   62800000
   75 CONTINUE                                                          A 577   62900000
                                                                                63000000
*     TURBULENT PARAMETER CALCULATIONS                                  A 578   63100000
                                                                                63200000
      IF (SMXTR.LE.ST2MAX) CALL TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,E  A 579   63300000
     1PP,EETM,EEPM,VARA,VARB,VARC,VARD,VARE,JK)                         A 580   63400000
      IF (SST.LE.S*A) CALL TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,EPP,EE  A 581   63500000
     1TM,EEPM,VARA,VARB,VARC,VARD,VARE,JK)                              A 582   63600000
                                                                                63700000
*     END TURBULENT PARAMETER CALCULATIONS                              A 583   63800000
*        VARIABLE ENTROPY CALCULATIONS                                  A 584   63900000
                                                                                64000000
      GO TO (77,76), IENTRO                                             A 585   64100000
   76 INTEGL=INTEGT(FP,ETAEDG,DY)                                       A 586   64200000
      XXJ=J                                                             A 587   64300000
      RS=((J+1)*EPS*OZ*INTEGL)**((2.-XXJ)/2.)                           A 588   64400000
      CALL FTLUP (RS,ZS,2,NNN,RRS,ZZS)                                  A 589   64500000
      CALL FTLUP (RS,TANTHTS,2,NNN,RRS,DRSDZS)                          A 590   64600000
      THATS=ATAN(TANTHTS)                                               A 591   64700000
      PTTWO=(G+1.)*XMA**2*SIN(THATS)**2                                 A 592   64800000
      PTTWO=PTTWO/((G-1.)*(XMA)**2*SIN(THATS)**2+2.)                    A 593   64900000
      PTTWO=PTTWO**(G/(G-1.))*PT1                                       A 594   65000000
      PTTWO=PTTWO*((G+1.)/(2.*G*XMA**2*SIN(THATS)**2-(G-1.)))**(1./(G-1.  A 595   65100000
     1))                                                                A 596   65200000
      P20=PTTWO/(R1*U1*U1)                                              A 597   65300000
      UEE(M)=SQRT(2.*T10*(1.-(PE/P20)**((G-1.)/G)))                     A 598   65400000
      NN=IEDGE+2                                                        A 599   65500000
   77 CONTINUE                                                          A 600   65600000
      IF (IENTRO.EQ.1) P20=P10                                          A 601   65700000
                                                                                65800000
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*       PRINT PROFILES AND WALL VALUES                                  A 602   65900000
                                                                                66000000
      S=S*A                                                             A 603   66100000
      DO 78 NUMBR=1,JN                                                  A 604   66200000
      IF (S.GT.PROVAL(NUMBR)-.000001.AND.S.LT.PROVAL(NUMBR)+.000001) GO A 605   66300000
     1TO 79                                                             A 606   66400000
   78 CONTINUE                                                          A 607   66500000
      GO TO 88                                                          A 608   66600000
   79 CONTINUE                                                          A 609   66700000
      WRITE (6,111) S                                                   A 610   66800000
      IEDGEX=ETAEDG+10                                                  A 611   66900000
      IF (NAUXPRO.NE.1) GO TO 81                                        A 612   67000000
      WRITE (6,112)                                                     A 613   67100000
      DO 80 I=1,IEDGEX                                                  A 614   67200000
   80 WRITE (6,113) (Y(I),NONDEL(I),VP(I),FZ(I),TZ(I),VARA(I),VARB(I),VA A 615   67300000
     1RC(I),VARD(I),EP(I),VARE(I))                                      A 616   67400000
   81 CONTINUE                                                          A 617   67500000
      IF (KODE.EQ.1) GO TO 83                                           A 618   67600000
      IF (TREACT.GT.0.9999) GO TO 86                                    A 619   67700000
      IF (TREACT.GT.0.0.AND.TREACT.LT.0.9999) GO TO 83                  A 620   67800000
      WRITE (6,114)                                                     A 621   67900000
      DO 82 I=1,IEDGEX                                                  A 622   68000000
   82 WRITE (6,115) Y(I),NONDEL(I),EP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT( A 623   68100000
     1I),MCME(I),FZ(I),TZ(I),STAB2(I),XLM11(I)                          A 624   68200000
      GO TO 88                                                          A 625   68300000
   83 WRITE (6,114)                                                     A 626   68400000
      DO 84 I=1,IEDGEX                                                  A 627   68500000
   84 WRITE (6,115) Y(I),NONDEL(I),EP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT( A 628   68600000
     1I),MCME(I),FZ(I),TZ(I),STAB2(I),XLM11(I)                          A 629   68700000
      WRITE (6,116)                                                     A 630   68800000
      DO 85 I=1,IEDGEX                                                  A 631   68900000
   85 WRITE (6,115) Y(I),NONDEL(I),EP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT( A 632   69000000
     1I),MCME(I),TCORD(I),UOUPL(I),UDEF(I),EP(I)                        A 633   69100000
      GO TO 88                                                          A 634   69200000
   86 WRITE (6,116)                                                     A 635   69300000
      DO 87 I=1,IEDGEX                                                  A 636   69400000
   87 WRITE (6,115) Y(I),NONDEL(I),EP(I),TP(I),TTOTT(I),CROCCO(I),PTOPT( A 637   69500000
     1I),MCME(I),TCORD(I),UOUPL(I),UDEF(I),EP(I)                        A 638   69600000
   88 CONTINUE                                                          A 639   69700000
      DO 89 NUMBER=1,JM                                                 A 640   69800000
      IF (S.GT.PRNTVAL(NUMBER)-.000001.AND.S.LT.PRNTVAL(NUMBER)+.000001) A 641   69900000
     1 GO TO 90                                                         A 642   70000000
   89 CONTINUE                                                          A 643   70100000
      GO TO 91                                                          A 644   70200000
   90 CONTINUE                                                          A 645   70300000
      PESTAR=PE*R1*U1*U1                                                A 646   70400000
      TESTAR=TE*TREF                                                    A 647   70500000
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      RESTAR=RE*R1                                                      A 648   70600000
      UESTAR=UE*U1                                                      A 649   70700000
      MUESTAR=XNUE*VISREF                                               A 650   70800000
      YESTAR=XNDEN*EPS*A                                                A 651   70900000
      IF (KODUNIT.EQ.1) CALL WALLOUT (PROVAL,PRNTVAL,JM,JN)             A 652   71000000
      WRITE (6,117) S,RETHET,PP,CFW,ZS,YESTAR,X,RES,DTEDX,QSD,RS,UPLUS,R A 653   71100000
     1MI,PESTAR,DUEDX,HD,ITRO,PTREF,Z,TESTAR,DISP,CHE,TWOTT1,JPOINT,XBE, A 654   71200000
     2RESTAR,THETA,CHW,RECTOR,P20                                       A 655   71300000
      WRITE (6,118) TREACT,UESTAR,THADIS,NUE,ST2MAX,EPS,RVWALD,XMAE,TAUD A 656   71400000
     1,NUW,DSMXD,REDELT,MUESTAR,CFE,THATS,XNDEN                         A 657   71500000
      IF (KODUNIT.EQ.1) S=S*3.280839895                                 A 658   71600000
   91 CONTINUE                                                          A 659   71700000
                                                                                  71800000
*        UPDATE VARIABLES FOR MARCHING PROCEDURE                        A 660   71900000
                                                                                  72000000
      DO 92 N=1,NN                                                      A 661   72100000
      FN(N)=FO(N)                                                       A 662   72200000
      FO(N)=FP(N)                                                       A 663   72300000
      TN(N)=TO(N)                                                       A 664   72400000
      TO(N)=TP(N)                                                       A 665   72500000
      VN(N)=VO(N)                                                       A 666   72600000
      VO(N)=VP(N)                                                       A 667   72700000
      RATON(N)=RATOO(N)                                                 A 668   72800000
      RATOO(N)=RATOP(N)                                                 A 669   72900000
      XLM1N=XLM1O                                                       A 670   73000000
      XLM1O=XLM1P                                                       A 671   73100000
   92 CONTINUE                                                          A 672   73200000
   93 DX1=DX2                                                           A 673   73300000
      IF (IENTRO.EQ.1) STOP 100                                         A 674   73400000
      IF (NOIT.GE.ITMAX) STOP 77                                        A 675   73500000
      NOIT=NOIT+1                                                       A 676   73600000
      REWIND 4                                                          A 677   73700000
      REWIND 9                                                          A 678   73800000
      READ (9,NAM1)                                                     A 679   73900000
      IF (ENDFILE 9) 94,95                                              A 680   74000000
   94 STOP 12                                                           A 681   74100000
   95 CONTINUE                                                          A 682   74200000
      READ (9,104) Y(1),VN(1),FN(1),DUM,TN(1),X51,DUM                   A 683   74300000
      IF (ENDFILE 9) 96,97                                              A 684   74400000
   96 STOP 13                                                           A 685   74500000
   97 CONTINUE                                                          A 686   74600000
      READ (9,104) (Y(N),VN(N),FN(N),DUM,TN(N),DUM,X6N,N=2,IEDGE)       A 687   74700000
      IF (ENDFILE 9) 98,99                                              A 688   74800000
   98 STOP 14                                                           A 689   74900000
   99 CONTINUE                                                          A 690   75000000
      GO TO 14                                                          A 691   75100000
                                                                                  75200000
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*                                                                       A 692   75300000
                                                                                75400000
  100 FORMAT (/64H NEGATIVE TM1 REPLACED WITH ARITHMETIC MEAN OF TO AND  A 693   75500000
     1TN AT S = ,F10.4,5H M = ,I3,5H N = ,I3/)                           A 694   75600000
  101 FORMAT (1H1)                                                      A 695   75700000
  102 FORMAT (6X,3HETA9X,5HYD(1)8X,5HYD(2)8X,5HYD(4)8X,5HYD(5)8X,5HYD(6) A 696   75800000
     1)                                                                 A 697   75900000
  103 FORMAT (6E13.6)                                                   A 698   76000000
  104 FORMAT (7E13.6)                                                   A 699   76100000
  105 FORMAT (22H  GRID WIDTH TOO SMALL/35H  CANNOT ADD MORE STEPS WITH A 700   76200000
     1XK≠1.0)                                                           A 701   76300000
  106 FORMAT (22H  GRID WIDTH TOO SMALL/24HMAKE XK>1.0 AND RESUBMIT)    A 702   76400000
  107 FORMAT (22H  GRID WIDTH TOO SMALL/78HMAXIMUM LIMIT (301) FOR IEDGE A 703   76500000
     1 HAS BEEN EXCEED.   CHANGE XK AND/OR XEND AND HPR)                A 704   76600000
  108 FORMAT (53H      XK=1.0  -   XK MUST BE > 1.0 FOR TURBULENT FLOWS) A 705   76700000
  109 FORMAT (1X,26HFP(I) NEVER EQ OR GT .9999)                         A 706   76800000
  110 FORMAT (/23H YOU DID IT AGAIN - TP(,I3,13H) IS NEGATIVE/3H S=,F10. A 707   76900000
     14,4H TP(,I3,2H)=,F10.5/)                                          A 708   77000000
  111 FORMAT (//1X,2HX=F14.4,9H  PROFILE/)                              A 709   77100000
  112 FORMAT (130H        ETA         Y/YE            V    GRAD(U/UE)  GRAD A 710   77200000
     1(T/TE)        FC1         DAMP          EP1          EP2          EP A 711   77300000
     2    MIXDEL/)                                                      A 712   77400000
  113 FORMAT (11E12.3)                                                  A 713   77500000
  114 FORMAT (//4X,3HETA,8X,4HY/YE,7X,4HU/UE,7X,4HT/TE,6X,6HTT/TTE,5X,6H A 714   77600000
     1CROCCO,5X,6HPT/PTR,6X,4HM/ME,8X,2HFZ,9X,2HTZ,6X,7HVORTREY,6X,5HXLM A 715   77700000
     211/)                                                              A 716   77800000
  115 FORMAT (12E11.3)                                                  A 717   77900000
  116 FORMAT (//4X,3HETA,8X,4HY/YE,7X,4HU/UE,7X,4HT/TE,6X,6HTT/TTE,5X,6H A 718   78000000
     1CROCCO,5X,6HPT/PTR,6X,4HM/ME,7X,5HYPLUS,6X,5HUPLUS,6X,4HUDEF,6X,6H A 719   78100000
     2VISEFF/)                                                          A 720   78200000
  117 FORMAT (/2X,7HX =     ,E12.5,2X,7HRETHET=,F12.5,2X,7HDPEDX =,F12.5, A 721   78300000
     12X,7HCFW =   ,E12.5,2X,7HZSHK = ,E12.5,2X,7HYE =    ,E12.5,/2X,7HXI A 722   78400000
     2=    ,E12.5,2X,7HREX =   ,E12.5,2X,7HDTEDX =,E12.5,2X,7HQSD =   ,E12. A 723   78500000
     35,2X,7HRSHK = ,E12.5,2X,7HUTAU = ,E12.5,/2X,7HRAD =   ,E12.5,2X,7HP A 724   78600000
     4E =    ,E12.5,2X,7HDUEDX =,E12.5,2X,7HHD =    ,E12.5,2X,7HITRO = ,I1 A 725   78700000
     52,2X,7HPTR =   ,E12.5,/2X,7HZ =     ,E12.5,2X,7HTE =    ,E12.5,2X,7HD A 726   78800000
     6LTAST=,E12.5,2X,7HNSTE = ,E12.5,2X,7HTW/TT =,E12.5,2X,7HYMP =   ,I1 A 727   78900000
     72,/2X,7HBETA = ,E12.5,2X,7HRE =    ,E12.5,2X,7HTHETA =,E12.5,2X,7HN A 728   79000000
     8STW = ,E12.5,2X,7HRETRUE=,E12.5,2X,7HP20 =   ,E12.5)              A 729   79100000
  118 FORMAT (2X,7HTREFT =,E12.5,2X,7HUE =    ,E12.5,2X,7HD/T =   ,E12.5,2 A 730   79200000
     1X,7HNUE =   ,E12.5,2X,7HROUSE =,E12.5,2X,7HOMEGA =,E12.5,/2X,7HRVWA A 731   79300000
     2LD=,E12.5,2X,7HME =    ,E12.5,2X,7HTAUD = ,E12.5,2X,7HNUW =   ,E12.5 A 732   79400000
     3,2X,7HDSMXO =,E12.5/2X,7HREDELT=,E12.5,2X,7HMUE =   ,E12.5,2X,7HCFE A 733   79500000
     4 =   ,E12.5,2X,7HSWANG =,E12.5,2X,7HXD =    ,E12.5)                A 734   79600000
      END                                                               A 735-  79700000
```
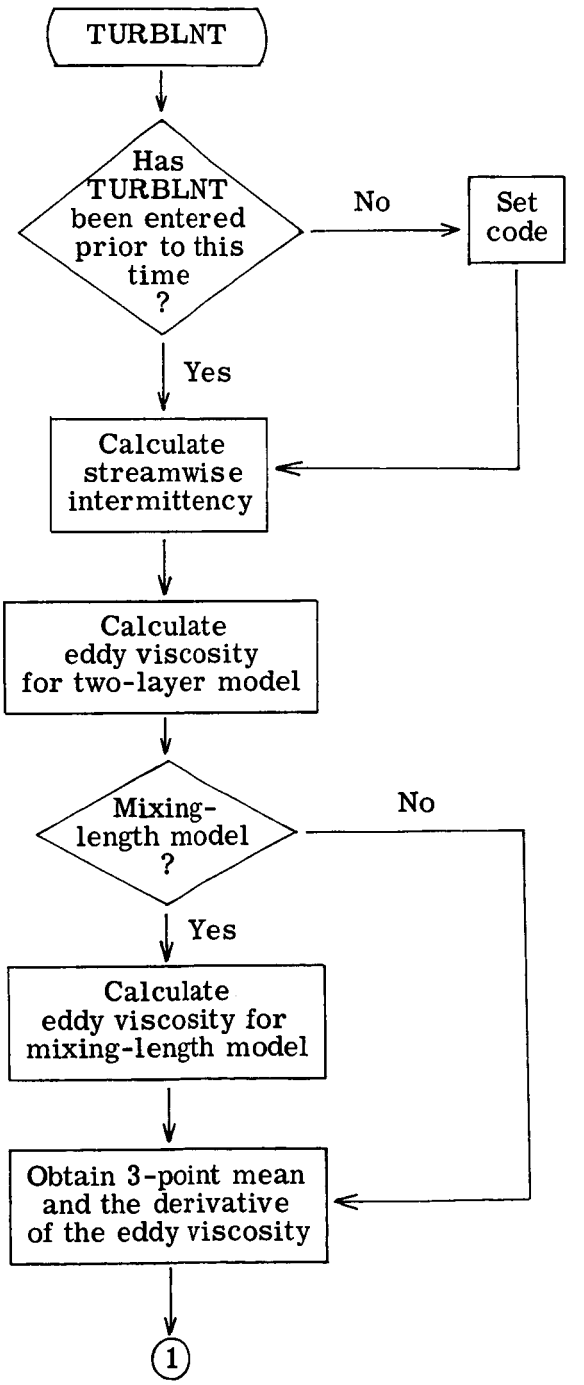
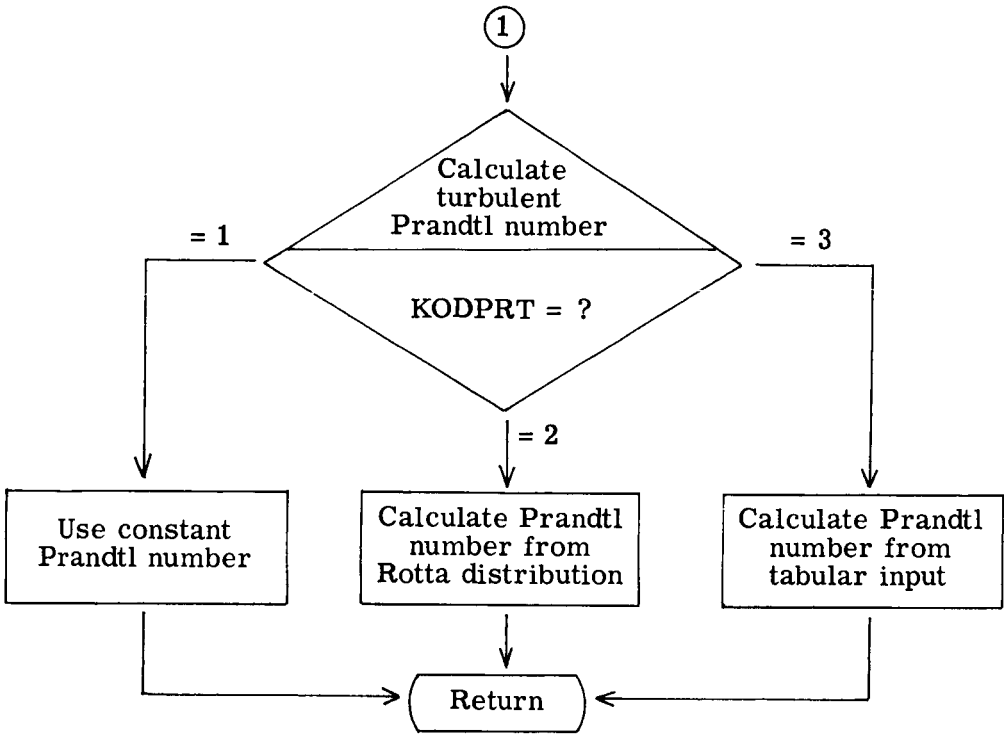

Subroutine TURBLNT.- Subroutine TURBLNT calculates the eddy viscosity, its derivatives, and the intermittency distributions required for the solution of transitional and turbulent flows. The flow diagram for subroutine TURBLNT is as follows:



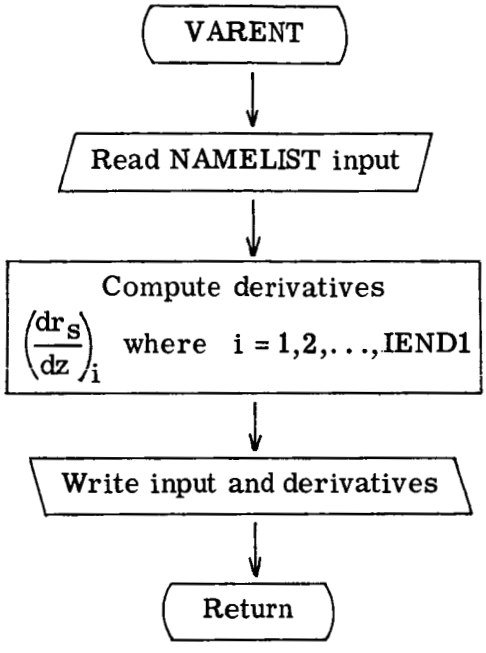

1
Calculate turbulent Prandtl number
KODPRT = ?
= 1
= 2
= 3
Use constant Prandtl number
Calculate Prandtl number from Rotta distribution
Calculate Prandtl number from tabular input
Return

The program listing for subroutine TURBLNT is as follows:

```
      SUBROUTINE TURBLNT (TP,XLM11,FZ,XN,RATOP,DY,EP,FP,EPP,EETM,EEPM,VA
     1RA,VARB,VARC,VARD,VARE,JK)
      DIMENSION VARA(JK), VARB(JK), VARC(JK), VARD(JK), VARE(JK)
      DIMENSION TP(JK), XLM11(JK), FZ(JK), XN(JK), RATOP(JK), DY(JK)
      DIMENSION EP(JK), FP(JK), EPP(JK)
      DIMENSION EETM(JK), EEPM(JK)
      DIMENSION PRTAR(100), GLAR(100)
      INTEGER ETAEDG
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TREACT,TBAR,XLBAR,DISINC,XT1,X
     1T2,XT3,XT4,XT5,RE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT,PRT,PRTAR,GLAR,NUMB1
      IF (KSTR.EQ.0) GO TO 1
      GO TO 2
    1 KSTR=1
      STR=S
      XLAMDA=STR*(TLNGTH-1.)/(SQRT((ALOG(50.))/CORP))
    2 CONTINUE

*        CALCULATE STREAMWISE INTERMITANCY

      TREACT=1.-EXP(-1.*CORP*(((S-STR)/XLAMDA)**2))

*        CALCULATE EDDY-VISCOSITY
*             2 LAYER MODEL

      IFC=0
      DO 9 N=2,IEDGE
      YINTER=.5*(1.-ERF(5.*((XN(N)/XN(ETAEDG))-XT4)))
      IF (IFC.EQ.1) GO TO 3
      XMF=RE*RE*UE*UE*((RMI*RATOP(N))**J)*ABS(FZ(N))
      XMF=XMF/(((EPS*TP(N))**3)*A*A*XNUE*SQRT(2.*X)*XLM11(N))
      FC1=XMF
      FC2=(SQRT(XLM11(1)/XLBAR)*(TP(1)/TBAR))*(1.+XT5)-XT5
      FC3=EPS*A*XN(N)
      XMF=ABS((XLM11(1)*FZ(1))-CONSTNT*(RE*UE*(RMI**J)*XBE*XN(N)/SQRT(2.
     1*X)))
      XMF=XMF*RE*RE*UE*UE*(RMI**J)/(XNUE*EPS*SQRT(2.*X))
      XMF=XMF/((XLM11(N)**2)*(TP(N)**3))
      XMF=SQRT(XMF)/(XT2*A*EPS)
      FC4=XMF
      DAMP=1.-EXP(-FC2*FC3*FC4)
      IF (KODVIS.EQ.2) GO TO 4
      VARB(N)=DAMP
      XMIXL=XT1*FC3*DAMP
      EP1=1.+TREACT*FC1*(XMIXL**2)*YINTER
      VARC(N)=EP1
    3 XMF=XT3*RE*UE*DISINC*YINTER/(((EPS*TP(N))**2)*XNUE*XLM11(N))
      FC1=RE*RE*UE*UE*((RMI*RATOP(N))**J)*ABS(FZ(N))
      FC1=FC1/(((EPS*TP(N))**3)*A*A*XNUE*SQRT(2.*X)*XLM11(N))
      VARA(N)=FC1
      EP2=1.+TREACT*XMF
```

```
      VARD(N)=EP2                                                       F  48   97500000
      IF (IFC.EQ.1) GO TO 8                                             F  49   97600000
      IF (EP1.LE.EP2.AND.IFC.EQ.0) GO TO 7                              F  50   97700000
      IFC=1                                                             F  51   97800000
      JPCINT=N                                                          F  52   97900000
      GO TO 8                                                           F  53   98000000
                                                                                98100000
*             MIXING LENGTH MODEL                                       F  54   98200000
                                                                                98300000
    4 XMIXTP=0.2*XN(ETAEDG)                                             F  55   98400000
      IF (XN(N).GT.XMIXTP) GO TO 5                                      F  56   98500000
      XMIXL=0.4*EPS*A*XN(N)                                             F  57   98600000
      GO TO 6                                                           F  58   98700000
    5 XMIXL=0.08*EPS*A*XN(ETAEDG)                                       F  59   98800000
    6 EP(N)=1.+TRFACT*FC1*((XMIXL*DAMP)**2)*YINTER                      F  60   98900000
      GO TO 9                                                           F  61   99000000
    7 EP(N)=EP1                                                         F  62   99100000
      GO TO 9                                                           F  63   99200000
    8 EP(N)=EP2                                                         F  64   99300000
    9 CONTINUE                                                          F  65   99400000
                                                                                99500000
*        OBTAIN THE THREE POINT MEAN AND                                F  66   99600000
*        THE DERIVATIVE OF EDDY-VISCOSITY                               F  67   99700000
                                                                                99800000
      DO 10 N=1,IEDGE                                                   F  68   99900000
      EP(N)=(EP(N)+EP(N+1)+EP(N+2))/3.                                  F  69  100000000
      IF (EP(N).GE..9999) EP(N)=1.                                      F  70  100100000
      IF (N.EQ.1.OR.N.EQ.2) GO TO 10                                    F  71  100200000
      EPP(N-1)=(EP(N)-EP(N-2))/(DY(N-1)+DY(N-2))                        F  72  100300000
   10 CONTINUE                                                          F  73  100400000
                                                                               100500000
*       CALCULATE TURBULENT PRANDTL NUMBER                              F  74  100600000
                                                                               100700000
      DO 14 N=2,IEDGE                                                   F  75  100800000
      GO TO (13,11,12), KODPRT                                          F  76  100900000
   11 PRT=0.95-0.45*((XN(N)/XN(ETAEDG))**2)                             F  77  101000000
      GO TO 13                                                          F  78  101100000
   12 GL=XN(N)/XN(ETAEDG)                                               F  79  101200000
      CALL FTLUP (GL,PRT,1,NUMB1,GLAR,PRTAR)                            F  80  101300000
   13 EETM(N)=(PRT+(EP(N)-1.)*PR)/(PR*PRT)                              F  81  101400000
      EETM(N)=(PRT+(EP(N)-1.)*PR)/(PR*PRT)                              F  82  101500000
   14 CONTINUE                                                          F  83  101600000
      DO 15 N=2,IEDGE                                                   F  84  101700000
   15 EEPM(N)=(EETM(N+1)-EETM(N-1))/(DY(N)+DY(N-1))                     F  85  101800000
      EEPM(1)=(-WW1*EETM(1)+WW2*EETM(2)-WW3*EETM(3)+WW4*EETM(4))        F  86  101900000
      EEPM(1)=EEPM(1)/(WW5*DY(1))                                       F  87  102000000
      XEFX=EPS*XN(ETAEDG)                                               F  88  102100000
      DO 16 N=2,IEDGE                                                   F  89  102200000
      IF (EP(N).LT.1.0) EP(N)=1.0                                       F  90  102300000
      IF (VARA(N).EQ.0.0) VARA(N)=1.0                                   F  91  102400000
   16 VARE(N)=SQRT((EP(N)-1.)/VARA(N))/XEFX                             F  92  102500000
      RETURN                                                            F  93  102600000
      END                                                               F  94- 102700000
```

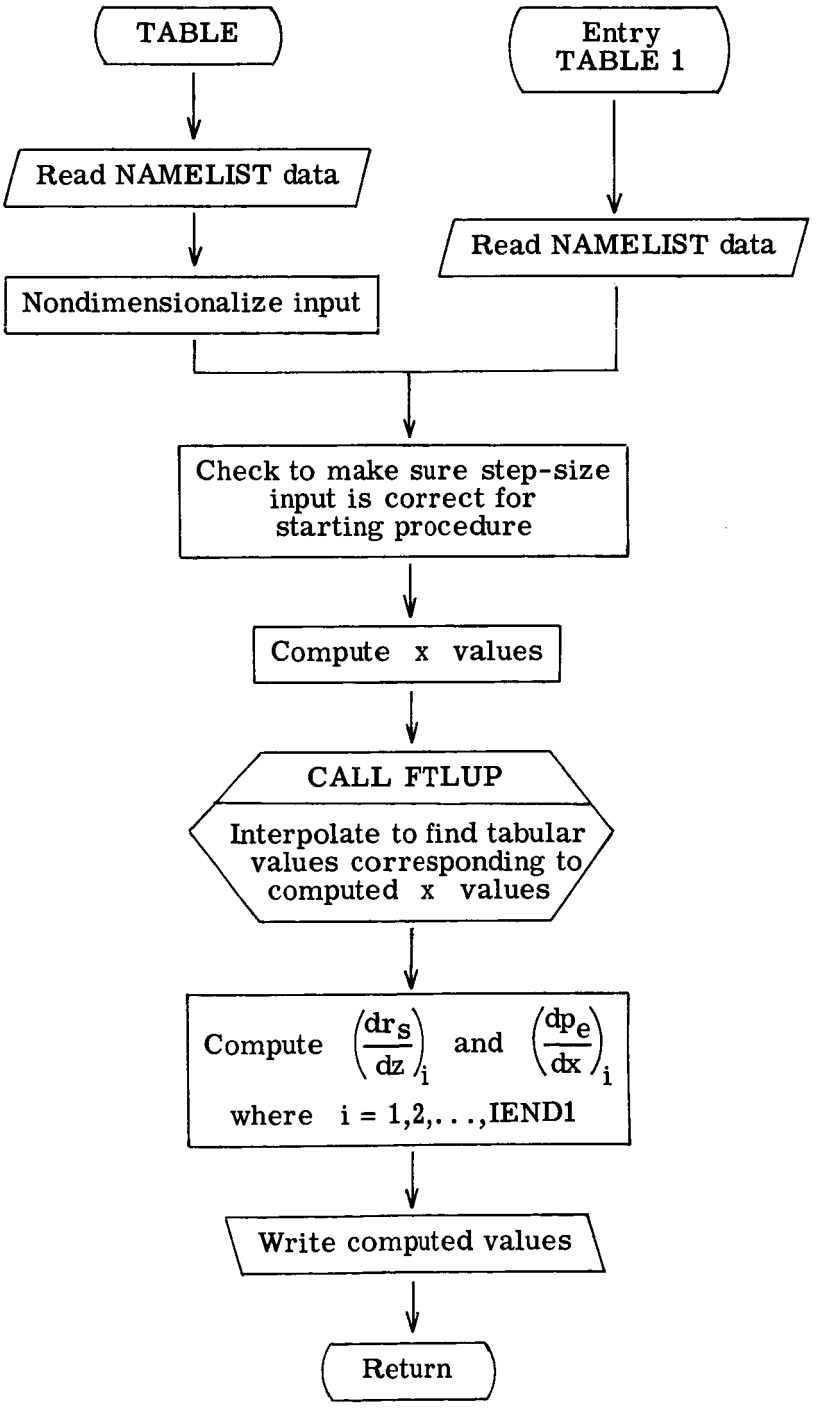
Subroutine VARENT.- Subroutine VARENT reads the variable-entropy input in tabular form, computes dr_s/dz, and then writes the input and the derivatives. The flow diagram for subroutine VARENT is as follows:



The program listing for subroutine VARENT is as follows:

```
      SUBROUTINE VARENT (RRS,ZZS,DRSDZS,NUMBER)                          E   1   91100000
      DIMENSION RRS(100), ZZS(100), DRSDZS(100)                          E   2   91200000
      NAMELIST /NAM4/ NUMBER,RRS,ZZS,DRSDZS                              E   3   91300000
      READ (5,NAM4)                                                      E   4   91400000
      NUMM1=NUMBER-1                                                     E   5   91500000
      DO 1 I=2,NUMM1                                                     E   6   91600000
    1 DRSDZS(I)=(RRS(I+1)-RRS(I-1))/(ZZS(I+1)-ZZS(I-1))                  E   7   91700000
      DRSDZS(1)=(RRS(2)-RRS(1))/(ZZS(2)-ZZS(1))                          E   8   91800000
      DRSDZS(NUMBER)=(RRS(NUMBER)-RRS(NUMBER-1))/(ZZS(NUMBER)-ZZS(NUMBER E   9   91900000
     1-1))                                                               E  10   92000000
      WRITE (6,NAM4)                                                     E  11   92100000
      RETURN                                                             E  12   92200000
      END                                                                E  13-  92300000
```

Subroutine TABLE.- Subroutine TABLE reads tabular input for body geometry, nondimensionalizes the input if necessary, distributes the values according to specified steps, and computes the derivatives. The flow diagram for subroutine TABLE is as follows:

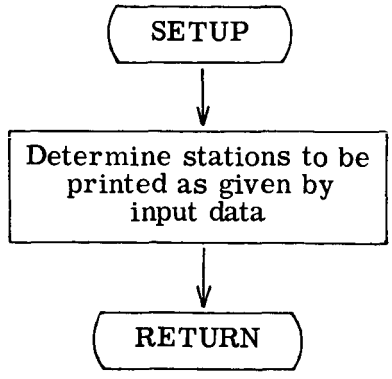


The program listing for subroutine TABLE is as follows:

```
      SUBROUTINE TABLE (IEND1,DS,R1,U1,A,TREF,KODWAL,VISREF,KODUNIT)
      DIMENSION PE(100), Z(100), RMI(100), TW(100), S(100), RVWALD(100),
     1 QW(100), PD(1000), ZED(1000), RMIDD(1000), SS(1000)
      NAMELIST /NAM3/ NUMBER,L,PE,Z,RMI,TW,S,RVWALD,SS,QW
      DATA SS(2)/0./,L/1/
      DATA (RVWALD(I),I=1,100)/100*0./
      DATA (RMI(I),I=1,100)/100*0./
      DATA C1/.0208854346/,C2/1.8/,C5/3.280839895/,C15/.0063658804/,C16/
     1.0000881/
      READ (5,NAM3)
      IF (ENDFILE 5) 1,2
    1 STOP 5
    2 WRITE (6,NAM3)
      IF (KODUNIT.NE.1) GO TO 6
*         CONVERT $NAM3  INPUT DATA TO U.S.STANDARD UNITS

      DO 3 I=1,IEND1
    3 SS(I)=SS(I)*C5
      DO 5 I=1,NUMBER
      S(I)=S(I)*C5
      Z(I)=Z(I)*C5
      RMI(I)=RMI(I)*C5
      PE(I)=PE(I)*C1
      RVWALD(I)=RVWALD(I)*C15
      IF (KODWAL.NE.1) GO TO 4
      TW(I)=TW(I)*C2
      GO TO 5
    4 QW(I)=QW(I)*C16
    5 CONTINUE
    6 DO 8 I=1,NUMBER
      PE(I)=PE(I)/(R1*U1*U1)
      S(I)=S(I)/A
      RMI(I)=RMI(I)/A
      IF (KODWAL.NE.1) GO TO 7
      TW(I)=TW(I)/TREF
      GO TO 8
    7 QW(I)=QW(I)*778.26*A/(VISREF*U1*U1)
    8 Z(I)=Z(I)/A
      GO TO 10
      ENTRY TABLE1
      READ (5,NAM3)
      WRITE (6,NAM3)
      IF (ENDFILE 5) 9,10
    9 STOP 14
   10 CONTINUE
      IF (SS(1).GT.DS+.000001.OR.SS(1).LT.DS-.000001) GO TO 11
```

```
      IF (SS(2).GT.DS+.000001.OR.SS(2).LT.DS-.000001) GO TO 11          D  46   86500000
      IF (SS(3).GT.DS+.000001.OR.SS(3).LT.DS-.000001) GO TO 11          D  47   86600000
      GO TO 12                                                          D  48   86700000
   11 WRITE (6,20) SS(1),SS(2),SS(3),DS                                 D  49   86800000
      STOP 77                                                           D  50   86900000
   12 CONTINUE                                                          D  51   87000000
      TEMP=0.                                                           D  52   87100000
      DO 13 I=1,IEND1                                                   D  53   87200000
      SS(I)=TEMP+SS(I)                                                  D  54   87300000
   13 TEMP=SS(I)                                                        D  55   87400000
      DO 14 I=1,IEND1                                                   D  56   87500000
      SD=DS*I                                                           D  57   87600000
      IF (SS(2).NE.0.) SD=SS(I)                                         D  58   87700000
      CALL FTLUP (SD,ZED(I),L,NUMBER,S,Z)                               D  59   87800000
      CALL FTLUP (SD,RMIDD(I),L,NUMBER,S,RMI)                           D  60   87900000
   14 CALL FTLUP (SD,PD(I),L,NUMBER,S,PE)                               D  61   88000000
      TWODS=2.*DS                                                       D  62   88100000
      DO 19 I=2,IEND1                                                   D  63   88200000
      SD=DS*I                                                           D  64   88300000
      IF (SS(2).NE.0.) SD=SS(I)                                         D  65   88400000
      IF (KCDWAL.EQ.1) GO TO 15                                         D  66   88500000
      CALL FTLUP (SD,QWD,L,NUMBER,S,QW)                                 D  67   88600000
      GO TO 16                                                          D  68   88700000
   15 CONTINUE                                                          D  69   88800000
      CALL FTLUP (SD,TWD,L,NUMBER,S,TW)                                 D  70   88900000
   16 CONTINUE                                                          D  71   89000000
      CALL FTLUP (SD,RVWALDD,L,NUMBER,S,RVWALD)                         D  72   89100000
      IF (I.EQ.IEND1) GO TO 17                                          D  73   89200000
      DRDZ=(RMIDD(I+1)-RMIDD(I-1))/(ZED(I+1)-ZED(I-1))                  D  74   89300000
      IF (SS(2).NE.0.) TWODS=SS(I+1)-SS(I-1)                            D  75   89400000
      DPEDSD=(PD(I+1)-PD(I-1))/TWODS                                    D  76   89500000
      GO TO 18                                                          D  77   89600000
   17 IF (SS(2).NE.0.) DDS=SS(I)-SS(I-1)                                D  78   89700000
      DPEDSD=(PD(I)-PD(I-1))/DDS                                        D  79   89800000
      DRDZ=(RMIDD(I)-RMIDD(I-1))/(ZED(I)-ZED(I-1))                      D  80   89900000
   18 WRITE (4) SD,PD(I),RMIDD(I),TWD,ZED(I),DPEDSD,RVWALDD,DRDZ,QWD    D  81   90000000
   19 CONTINUE                                                          D  82   90100000
      REWIND 4                                                          D  83   90200000
      RETURN                                                            D  84   90300000
                                                                               90400000
*                                                                       D  85   90500000
                                                                               90600000
   20 FORMAT (1X,7HSS(1) =,F9.4,/1X,7HSS(2) =,F9.4,/1X,7HSS(3) =,F9.4,/1 D  86   90700000
     1X,7HDS    =,F9.4,/1X,53HTHESE VALUES MUST BE EQUAL FOR THE STARTIN D  87   90800000
     2G PROCEDURE)                                                      D  88   90900000
      END                                                               D  89-  91000000
```

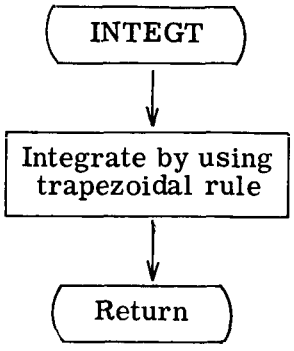
Subroutine SETUP.- Subroutine SETUP determines from the input where profiles and wall values are to be printed. The flow diagram for subroutine SETUP is as follows:



The program listing for subroutine SETUP is as follows:

```
      SUBROUTINE SETUP (A,B,C,J,K)                                      C   1   80700000
      DIMENSION B(J)                                                    C   2   80800000
      IF (A.EQ.0) RETURN                                                C   3   80900000
      KPLUS2=K+2                                                        C   4   81000000
      B(K+1)=A                                                          C   5   81100000
      DO 1 I=KPLUS2,J                                                   C   6   81200000
      B(I)=B(I-1)+A                                                     C   7   81300000
      IF (B(I).GE.C) RETURN                                             C   8   81400000
    1 CONTINUE                                                          C   9   81500000
      RETURN                                                            C  10   81600000
      END                                                               C  11-  81700000
```

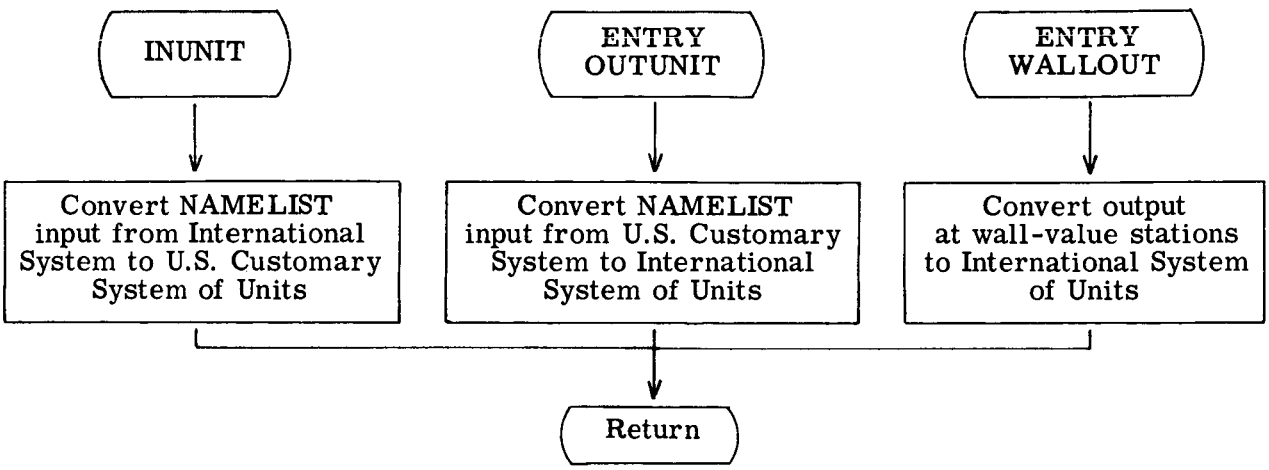
Function INTEGT.- Function subroutine INTEGT integrates by using the trapezoidal rule. The flow diagram for function INTEGT is as follows:



The program listing for function INTEGT is as follows:

```
      FUNCTION INTEGT (YY,NOPTS,DX)                                  B   1   79800000
      DIMENSION YY(201), DX(201)                                     B   2   79900000
      REAL INTEGT                                                    B   3   80000000
      INTEGT=0.                                                      B   4   80100000
      IF (NOPTS.LT.2) GO TO 2                                        B   5   80200000
      DO 1 N=2,NOPTS                                                 B   6   80300000
1     INTEGT=INTEGT+(DX(N-1)/2.)*(YY(N-1)+YY(N))                     B   7   80400000
2     RETURN                                                         B   8   80500000
      END                                                            B   9-  80600000
```

Subroutine INUNIT.- Subroutine INUNIT converts International System dimensional-input data to the U.S. Customary System of Units for calculations in the program. The subroutine then converts the data back to the International System before output. The flow diagram for subroutine INUNIT is as follows:



The program listing for subroutine INUNIT is as follows:

```
      SUBROUTINE INUNIT (PROVAL,PRNTVAL,JM,JN)                          G   1  102800000
      COMMON /TRBULNT/ S,KSTR,TLNGTH,CORP,TREACT,TBAR,XLBAR,DISINC,XT1,X G   2  102900000
     1T2,XT3,XT4,XT5,PE,UE,XNUE,J,RMI,EPS,JPOINT,IEDGE,WW1,WW2,WW3,WW4,W G   3  103000000
     2W5,N,ETAEDG,KODVIS,A,XBE,X,PR,CONSTNT,KODPRT                      G   4  103100000
      COMMON /UNIT/ VISCON,PT1,TT1,WAVE,R,SU,CONE,DS,SST,RT1,P1,T1,R1,U1 G   5  103200000
     1,AA1,TREF,VISREF,PESTAR,TESTAR,RESTAR,UESTAR,MUESTAR,YESTAR,THETA, G   6  103300000
     2TAUD,QSD,HD,UPLUS,DISP,PE,Z,TW,QW,RVWALD,PROINC,PRNTINC           G   7  103400000
      DIMENSION PROVAL(JN), PRNTVAL(JM)                                 G   8  103500000
      REAL MUESTAR                                                      G   9  103600000
      DATA RT1,P1,T1,P1,U1,AA1,TREF,VISREF/8*1.0/,UPLUS/0.0/            G  10  103700000
      DATA CC1/.0208854346/,CC2/1.8/,CC3/57.2957810375/,CC4/10.76391041/ G  11  103800000
     1,CC5/3.280839895/,CC6/.0019403196/,CC7/47.880258/,CC8/.55555555/,C G  12  103900000
     2C9/.017453292/,CC10/.09290304/,CC11/.3048/,CC12/515.379/,CC13/1134 G  13  104000000
     38.93/,CC14/20428.0758/                                            G  14  104100000
                                                                             104200000
*         CONVERT $NAM2 AND $NAM VALUES TO U.S. STANDARD UNITS          G  15  104300000
                                                                             104400000
      PROINC=PROINC*CC5                                                 G  16  104500000
      PRNTINC=PRNTINC*CC5                                               G  17  104600000
      DO 1 I=1,JN                                                       G  18  104700000
    1 PROVAL(I)=PROVAL(I)*CC5                                           G  19  104800000
      DO 2 I=1,JM                                                       G  20  104900000
    2 PRNTVAL(I)=PRNTVAL(I)*CC5                                         G  21  105000000
      A=A*CC5                                                           G  22  105100000
      U1=U1*CC5                                                         G  23  105200000
      AA1=AA1*CC5                                                       G  24  105300000
      DS=DS*CC5                                                         G  25  105400000
      SST=SST*CC5                                                       G  26  105500000
      VISCON=VISCON*CC1                                                 G  27  105600000
      PT1=PT1*CC1                                                       G  28  105700000
      P1=P1*CC1                                                         G  29  105800000
      VISREF=VISREF*CC1                                                 G  30  105900000
      TREF=TREF*CC2                                                     G  31  106000000
      T1=T1*CC2                                                         G  32  106100000
      SU=SU*CC2                                                         G  33  106200000
      TT1=TT1*CC2                                                       G  34  106300000
      CONE=CONE*CC3                                                     G  35  106400000
      WAVE=WAVE*CC3                                                     G  36  106500000
      R=R*CC4                                                           G  37  106600000
      RT1=RT1*CC6                                                       G  38  106700000
      R1=R1*CC6                                                         G  39  106800000
      GO TO 5                                                           G  40  106900000
                                                                             107000000
*         CONVERT $NAM2 AND $NAM VALUES TO INTERNATIONAL STANDARD UNITS G  41  107100000
                                                                             107200000
      ENTRY OUTUNIT                                                     G  42  107300000
      PRCINC=PROINC*CC11                                                G  43  107400000
```

```
      PRNTINC=PRNTINC*CC11                                              G  44  107500000
      DO 3 I=1,JN                                                       G  45  107600000
    3 PROVAL(I)=PROVAL(I)*CC11                                          G  46  107700000
      DO 4 I=1,JM                                                       G  47  107800000
    4 PRNTVAL(I)=PRNTVAL(I)*CC11                                        G  48  107900000
      A=A*CC11                                                          G  49  108000000
      DS=DS*CC11                                                        G  50  108100000
      SST=SST*CC11                                                      G  51  108200000
      U1=U1*CC11                                                        G  52  108300000
      AA1=AA1*CC11                                                      G  53  108400000
      VISCON=VISCON*CC7                                                 G  54  108500000
      PT1=PT1*CC7                                                       G  55  108600000
      P1=P1*CC7                                                         G  56  108700000
      VISREF=VISREF*CC7                                                 G  57  108800000
      TREF=TREF*CC8                                                     G  58  108900000
      T1=T1*CC8                                                         G  59  109000000
      TT1=TT1*CC8                                                       G  60  109100000
      SU=SU*CC9                                                         G  61  109200000
      WAVE=WAVE*CC9                                                     G  62  109300000
      CONF=CONF*CC9                                                     G  63  109400000
      R=R*CC10                                                          G  64  109500000
      RT1=RT1*CC12                                                      G  65  109600000
      R1=R1*CC12                                                        G  66  109700000
      GO TO 5                                                           G  67  109800000
                                                                               109900000
*         CONVERT WALL VALUES TO  INTERNATIONAL STANDARD UNITS          G  68  110000000
                                                                               110100000
      ENTRY WALLOUT                                                     G  69  110200000
      PESTAR=PESTAR*CC7                                                 G  70  110300000
      MUESTAR=MUESTAR*CC7                                               G  71  110400000
      TAUD=TAUD*CC7                                                     G  72  110500000
      TESTAR=TESTAR*CC8                                                 G  73  110600000
      S=S*CC11                                                          G  74  110700000
      UPLUS=UPLUS*CC11                                                  G  75  110800000
      DISP=DISP*CC11                                                    G  76  110900000
      UESTAR=UESTAR*CC11                                                G  77  111000000
      YESTAR=YESTAR*CC11                                                G  78  111100000
      THETA=THETA*CC11                                                  G  79  111200000
      RESTAR=RESTAR*CC12                                                G  80  111300000
      QSD=QSD*CC13                                                      G  81  111400000
      HD=HD*CC14                                                        G  82  111500000
    5 RETURN                                                            G  83  111600000
      END                                                               G  84- 111700000
```

USAGE

The programs are run on the Control Data 6000 Series computer under the SCOPE 3.0 operating system. The CPU time required for running all three programs is approximately 0.003 second per mesh point.

Array Dimensions

Program D2401 uses the variable-dimension capability of the preprocessor installed at the Langley Research Center to enable the user to use a minimum amount of storage for each case. If this capability is not available at the user's installation, the dimension statements at the beginning of program D2401 should be modified by inserting the following numbers in place of their equivalent designations:

JK maximum number of steps in η-direction plus 10

JL = 1, if case is considering only constant entropy
maximum number of steps in X-direction, if case is considering variable entropy

JM maximum number of wall-value stations to be printed

JN maximum number of profiles to be printed

FORTRAN statements setting these values should also be inserted immediately following the NAMELIST statements in program D2401.

Intermediate Data Storage

The output for the initial solution found in program D2390 is written on TAPE 9 as well as on the output file. Program D23901 will then read the data from TAPE 9, redistribute the points geometrically, and write the redistributed solution in place of the original distribution on TAPE 9 as well as on the output file. Program D2401 will then read the redistributed solution from TAPE 9. If redistributing the points is unnecessary, executing program D23901 may be eliminated and the solution from D2390 will be read directly by program D2401. Generally, TAPE 9 will be a disk file to be used only for a current run. However, if many cases are to be run with the same initial solution, a physical tape can be requested so that D2390 and D23901 need not be rerun for each case. When using TAPE 9 as either a disk file or a tape file, it is automatically rewound at the beginning of D23901 and again at the beginning of D2401.

Input Description

Input for all programs is standard CDC NAMELIST. Program D2390 reads input listed under $NAM1 and copies these input data as well as the output data onto TAPE 9.

Program D23901 then reads these data from TAPE 9 as input. No other input is required for D23901. Program D2401 requires the input from TAPE 9 (written by either D2390 or D23901) and the data found listed under $NAM2. Subroutine TABLE (in program D2401) requires the data listed under $NAM3. If the case being considered is using variable entropy, then subroutine VARENT (in program D2401) requires the data listed under $NAM4.

Dimensional input and output may be in either the International System of Units (KODUNIT = 1) or the U.S. Customary System of Units (KODUNIT = 0). The following listing of input and output data gives the units in the International System, followed in parentheses by the units in the U.S. Customary System. Where no units are given, the data are nondimensional.

The $NAM1 input data for program D2390 are given as follows:

HPR	η, increment for which values will be printed and stored on TAPE 9 (Default = 0.1)
XEND	η_N (see fig. 2) (Default = 10.0)
H	Runge-Kutta integration increment (Default = 0.01)
PR	N_{Pr} (Default = 0.7)
XKK	S/T_e (Default = 0.0)
BETA	β (Default = 0.0)
ALPHA	α (Default = 0.0)
XO	η_0 (Default = 0.0)
YO(1)	V_w (Default = 0.0)
YO(2)	F_w (Default = 0.0)
YO(3)	$\left(\frac{\partial F}{\partial \eta}\right)_w$ (Default = 0.0)
YO(4)	Θ_w (Default = 0.0)
YO(5)	$\left(\frac{\partial \Theta}{\partial \eta}\right)_w$ (Default = 0.0)

YO(6) $\left(\frac{\partial^2\Theta}{\partial\eta^2}\right)_w$ (Default = 0.0)

XK k (If k = 1, then program D23901 does not change data from D2390.) (Default = 1.0)

IGAS = 1 for air (Sutherland's viscosity constant)
= 2 for power-law viscosity relation (Default = 1)

VISCON constant in power-law viscosity relation, newton-sec/m^2 (lb-sec/ft^2)

VISPOW σ, exponent in power-law viscosity relation

KODUNIT = 0 if all dimensional input and output are in the U.S. Customary System of Units
= 1 if all dimensional input and output are in the International System of Units (Default = 0)

The $NAM2 input data for program D2401 are given as follows:

XMA M_∞

PT1 $p_{t,\infty}$, newton/m^2 (lb/ft^2)

TT1 $T_{t,\infty}$, ^oK (^oR)

WAVE shock-wave angle at tip of sharp body or stagnation point of blunt body (Default = 90.0), radians (degrees)

XY1 $\left(\frac{p_e}{p_\infty}\right)_{x=0}$

XY2 $\left(\frac{T_e}{T_\infty}\right)_{x=0}$

XY3 $\left(M_e\right)_{x=0}$

G γ (Default = 1.4)

R R_g, gas constant (Default = 1716.0), m^2/sec^2-^oK (ft^2/sec^2-^oR)

SU	S (Default = 198.6), ^{o}K (^{o}R)
PR	N_{Pr} (Default = 0.72)
PRT	$N_{Pr,t}$ (Default = 0.9)
IBODY	= 1 for stagnation-point flows = 2 otherwise
J	j
W	= 0 if transverse curvature is neglected = 1 if transverse curvature is included (Default = 0)
FT	= 1.0 for nonsimilar solution = 0.0 for similar solution (Default = 1.0)
KODE	= 1 if both laminar and turbulent profile values are defined for diagnostic reasons after flow is fully turbulent = 0 otherwise (Default = 0)
KODWAL	= 1 for specified temperature distribution = 2 for specified heat-transfer distribution (Default = 1)
IENTRO	= 1 for constant entropy = 2 for variable entropy (Default = 1)
CONE	cone semiapex angle (Default = 0.0), radians (degrees)
IEND1	number of steps in X-direction
A	reference length (Default = 1.0), meters (feet)
DS	initial step length in X-direction (Default = 0.01), meters (feet)
KODVIS	= 1 for two-layer eddy-viscosity model = 2 for mixing-length model (Default = 1)
SST	x-location at which transition occurs (Default = 1.0E08), meters (feet)

SMXTR critical vorticity Reynolds number (Default = 1.0E08)

TLNGTH $x_{t,f}/x_{t,i}$ (Default = 2.0)

CORP coefficient in equation (38) (Default = 0.412)

CONSTNT transition model (Default = 0.0)

XT1 k_1 (See eq. (6).) (Default = 0.4)

XT2 A^+ (Default = 26.0)

XT3 k_2 (See eq. (7).) (Default = 0.0168)

XT4 k_4 (See eq. (10).) (Default = 0.78)

XT5 k_3 (See eq. (8).) (Default = 0.0)

PROINC incremental x value for which profile printouts will be made
= 0 if only certain specified profile printouts will be made
(Default = 1.0), meters (feet)

PRNTINC incremental x value for which wall-value printouts will be made
= 0 if only certain specified wall-value stations printouts are desired
(Default = 0.1), meters (feet)

IPRO number of specified profile printouts desired (other than those determined by PROINC) (Default = 0)

PROVAL array of IPRO specific x values for which profile printouts are desired meters (feet)

IPRNT number of specified wall-value printouts desired (other than those determined by PRNTINC) (Default = 0)

PRNTVAL array of IPRNT specific x values for which printouts are desired, meters (feet)

NAUXPRO = 1 if auxiliary profile printouts are desired (see output description)
$\neq 1$ otherwise (Default = 0)

BLNGTH = 0 if using constant step size in X-direction
length of body if using variable step size in X-direction (Default = 0.0), meters (feet)

NPUTYPE = 1 for dimensional input
= 2 for nondimensional input (Default = 1)

KODPRT = 1 for constant $N_{Pr,t}$
= 2 for Rotta distribution
= 3 for tabular $N_{Pr,t} = f(y/\delta)$ (Default = 1)

NUMB1 number of values read into PRTAR and GLAR arrays if KODPRT = 3

PRTAR turbulent Prandtl number array, used only if KODPRT = 3 (NUMB1 values)

GLAR y/δ array corresponding to PRTAR, used only if KODPRT = 3 (NUMB1 values)

KTCOD = 1 if transition extent is calculated from equation (37)
= 2 if transition extent is read in as TLNGTH (Default = 2)

The $NAM3 input data for program D2401 are given as follows:

NUMBER number of values read into $NAM3 tables (Maximum = 100)

L order of interpolation to be used for $NAM3 table (Default = 1)

PE pressure-distribution array (NUMBER values), newton/m^2 (lb/ft^2)

Z axial-coordinate array (NUMBER values), meters (feet)

RMI body radial-coordinate array (NUMBER values) (Default = 1.0), meters (feet)

TW wall temperature-distribution array (NUMBER values), ^{o}K (^{o}R)

QW — wall heat-transfer-distribution array (NUMBER values), watts/m^2 (Btu/ft^2-sec)

RVWALD — mass flux at wall array, V_w (NUMBER values) (Default = 0.0), newton-sec/m^3 (lb-sec/ft^3)

S — x-station array corresponding to above table inputs (NUMBER values), meters (feet)

SS — array of incremental values between adjacent x stations for computation (Maximum = 1000), meters (feet)
The first three values for SS must equal DS for the starting procedure; that is, SS(1) = SS(2) = SS(3) = DS.

The $NAM4 input data for program D2401 are given as follows:

NUMBER — number of values read into $NAM4 tables (Maximum = 100)

RRS — array of radial coordinates of shock wave (NUMBER values)

ZZS — array of axial coordinates of shock wave (NUMBER values)

Output Description

The output for programs D2390 and D23901 consists of printing and the intermediate data on TAPE 9 as discussed earlier in this section. In program D2390, the $NAM1 input data are printed, followed by the initial profile consisting of the following values:

Initial profile

ETA = η

YO(1) = V

YO(2) = F

YO(4) = Θ

YO(5) = $\frac{\partial\Theta}{\partial\eta}$

YO(6) = $\frac{\partial^2\Theta}{\partial\eta^2}$

In program D23901 the output is printed in a form identical to that in D2390 except that the profile is redistributed. This same output is repeated in program D2401 for convenience. Next, the $NAM3 input data are printed. If the particular case is considering variable entropy, this is followed by the $NAM4 input data.

Next, the $NAM2 input data and the $NAM constants which consist of the following values are given:

P10	$\left(\frac{p_{t,e}}{\rho_r u_r^2}\right)_{x=0}$
T10	$\frac{T_{t,\infty}}{T_r}$
G	γ, ratio of specific heats
REY	$\frac{\rho_\infty u_\infty A}{\mu_\infty}$ free-stream Reynolds number
RT1	$\rho_{t,\infty}$, kilogram/m^3 (slug/ft^3)
P1	p_∞, free-stream pressure, newton/m^2 (lb/ft^2)
T1	T_∞, free-stream temperature, $^\circ$K ($^\circ$R)
R1	ρ_∞, free-stream density, kilogram/m^3 (slug/ft^3)
U1	u_∞, free-stream velocity, m/sec (ft/sec)
AA1	a_∞, free-stream speed of sound, m/sec (ft/sec)
TREF	T_r, reference temperature, $^\circ$K ($^\circ$R)
VISREF	ρ_r, reference viscosity, newton-sec/m^2 (lb-sec/ft^2)
R10	$\frac{\rho_{t,\infty}}{\rho_\infty}$

Next, the profile values are printed according to the specifications in the input. These consist of the following:

Laminar-profile values

ETA	η
Y/YE	y/y_e
U/UE	F
T/TE	Θ
TT/TTE	$T_t/T_{t,e}$
CROCCO	$\frac{T_t - T_w}{T_{t,e} - T_w}$
PT/PTR	$p_t/p_{t,r}$, total pressure ratio
M/ME	M/M_e, Mach number ratio
FZ	$\left(\frac{\partial F}{\partial \eta}\right)_{m+1,n}$
TZ	$\left(\frac{\partial \Theta}{\partial \eta}\right)_{m+1,n}$
VORTREY	$(\chi)_{m+1,n}$, vorticity Reynolds number
XLM11	$\frac{(\rho\mu)_{m+1,n}}{(\rho\mu)_e}$

Additional values for transitional and turbulent profiles

YPLUS	$\frac{yu_\tau}{\nu}$
UPLUS	$\frac{u}{u_\tau}$
UDEF	$\frac{u_e - u}{u_\tau}$
VISEFF	$1 + \frac{\epsilon}{\mu}\Gamma$, effective viscosity parameter

Auxiliary-profile values

V	V (see eq. (27))
GRAD(U/UE)	FZ
GRAD(T/TE)	TZ
FC1	$\left(\frac{\rho}{\mu}\left\lvert\frac{\partial u}{\partial y}\right\rvert\right)_{m+1,n}$
DAMP	$\left(1 - \exp\left\{-\left[\sqrt{\frac{\nu_w}{\bar{\nu}}}\left(1 + k_3\right) - k_3\right]\frac{y}{A}\right\}\right)_{m+1,n}$
EP1	$\left[\left(\frac{\epsilon}{\mu}\right)_i\right]_{m+1,n}$ (see eq. (6))
EP2	$\left[\left(\frac{\epsilon}{\mu}\right)_o\right]_{m+1,n}$ (see eq. (7))
EP	$\left(\frac{\epsilon}{\mu}\right)_{m+1,n}$
MIXDEL	$\left(\frac{l}{\delta}\right)_{m+1,n}$

Next, the wall-value stations are printed according to the specifications in the input. These consist of the following:

X	spatial coordinate, meters (feet)
XI	ξ
RAD	r_0, body radius (see fig. 1)
Z	axial coordinate of body (see fig. 1)
BETA	β, pressure-gradient parameter (see eqs. (30))
TRFCT	Γ, intermittency distribution (see eq. (38))

RVWALD	dimensional mass flux at wall (see eq. (35))
REDELT	$\frac{\rho_e u_e \delta^*}{\mu_e}$, Reynolds number based on local displacement thickness
RETHET	$\frac{\rho_e u_e \theta}{\mu_e}$
REX	$\frac{\rho_e u_e x}{\mu_e}$, local Reynolds number
PE	$\frac{p_e}{\rho_r u_r^2}$, edge pressure, newton/m^2 (lb/ft^2)
TE	$\frac{T_e}{T_r}$, edge temperature, $^{\circ}$K ($^{\circ}$R)
RE	$\frac{\rho_e}{\rho_r}$, edge density, kilogram/m^3 (slug/ft^3)
UE	$\frac{u_e}{u_r}$, edge velocity, m/sec (ft/sec)
ME	M_e, edge Mach number
MUE	$\frac{\mu_e}{\mu_r}$, newton-sec/m^2 (lb-sec/ft^2)
DPEDX	$\frac{\partial p_e}{\partial x}$, pressure gradient
DTEDX	$\frac{\partial T_e}{\partial x}$, temperature gradient
DUEDX	$\frac{\partial u_e}{\partial x}$, velocity gradient
DLTAST	δ^*, displacement thickness, meters (feet)
THETA	θ, momentum thickness, meters (feet)
D/T	$\frac{\delta^*}{\theta}$, shape factor
TAUD	τ_w, wall shear stress, newton/m^2 (lb/ft^2)
CFE	$\frac{\tau_w}{\frac{1}{2}\rho_e u_e^2}$, skin-friction coefficient based on edge condition

CFW	$\frac{\tau_w}{\frac{1}{2}\rho_w u_e^2}$, skin-friction coefficient based on wall density
QSD	heat transfer, watt/m^2 (Btu/ft^2-sec)
HD	$\frac{q_w}{T_w - T_{aw}}$, heat-transfer coefficient, watt/m^2-^{o}K (Btu/ft^2-sec-^{o}R)
NSTE	$\frac{h}{c_p(\rho u)_e}$, Stanton number based on edge condition
NSTW	$\frac{h}{c_p \rho_w u_e}$, Stanton number based on wall condition
NUE	Nusselt number based on edge condition
NUW	Nusselt number based on wall condition
SWANG	local shock-wave angle, degrees
ZSHK	axial coordinate of shock wave
RSHK	local radius of shock wave
ITRO	number of iterations performed for variable entropy
TW/TT	$\frac{\tau_w}{T_{t,\infty}}$
RFTRUE	$\frac{T_{aw} - T_e}{T_t - T_e}$, recovery factor
ROUSE	χ_{max}
DSMXO	$\left(\frac{\partial \chi}{\partial x}\right)_m$
XD	$y\sqrt{\frac{\rho_r u_r L}{\mu_r}}$
YE	δ_e, boundary-layer thickness, meters (feet)
UTAU	$u_\tau = \sqrt{\frac{T_w}{\rho}}$, m/sec (ft/sec)

PTR reference total pressure

YMP y_m

P20 $\frac{\rho_{t,e}}{\rho_r u_r^2}$

OMEGA $\left(\frac{\rho_r u_r L}{\mu_r}\right)^{-1/2}$

Sample Cases

Two sample cases are presented in order to illustrate the input and output quantities in relation to the test conditions of the particular case being considered. These cases include laminar flow over a blunt axisymmetric body and laminar, transitional, and turbulent flow over a flat plate.

Case 1.- An example of laminar flow over a blunt, axisymmetric body is given in reference 21. The body is a spherically blunted, 25° half-angle cone. The wind-tunnel test conditions are as follows:

$$M_\infty = 7.95$$

$$p_{t,\infty} = 6.31 \times 10^6 \text{ N/m}^2$$

$$T_{t,\infty} = 7.83 \times 10^2 \text{ °K}$$

$$\frac{T_w}{T_{t,\infty}} = 0.38$$

The wall-boundary pressure distribution used in the numerical solution was obtained by the technique presented in reference 22. (See ref. 7 for comparison of numerical results with experimental data.) This case requires 75000_8 storage on the CDC 6000 computers. The listing of the variable-dimension data for this particular case is as follows:

JK = 110

JL = 1

JM = 120

JN = 20

The listing of the input data for case 1 is given as follows:

```
$NAM1
HPR=0.1,
XEND=10.0,
H=0.01,
PR=0.72,
XKK=0.14085,
BETA=0.5,
ALPHA=0.0,
XO=0.0,
YO(1)=0.0,0.0,0.77,0.38,0.28,0.0,
XK=1.0,
IGAS=1,
VISCON=0.7173E-08,
VISPOW=0.647,
KODUNIT=0,
$


$NAM2
XMA=7.95,
PT1=131760.0,
TT1=1410.0,
WAVE=90.0,
G=1.4,
R=1716.0,
SU=198.6,
PR=0.72,
PRT=0.9,
IBODY=1,
J=1,
W=0,
ET=1.0,
KODE=0,
KODWAL=1,
IENTRO=1,
CONF=25.0,
IEND1=103,
A=1.0,
DS=0.0005,
KODVIS=1,
SST=0.1E+09,
SMXTR=0.1E+09,
TLNGTH=2.0,
CORP=0.412,
CONSTNT=0.0,
XT1=0.4,
XT2=26.0,
XT3=0.0168,
XT4=0.78,
XT5=0.0,
PRDINC=0.1,
PRNTINC=.005,
IPRO=1,
PROVAL(1)=0.345,
IPRNT=1,
PRNTVAL(1)=0.345,
NAUXPRO=0,
BLNGTH=0.0,
NPUTYPE=1,
KODPRT=2,
KTCOD=2,
$
```

```
$NAM3
NUMBER=77,
L=1,
PE(1)=1150.4,1149.43,1146.54,1141.72,1134.98,1126.32,1115.76,1103.29,
      1088.94,1072.74,1054.72,1034.89,1013.3,989.997,965.019,938.401,
      910.389,880.855,849.844,817.414,783.869,749.276,713.361,676.169,
      638.401,599.196,559.265,517.655,492.393,475.551,456.972,439.014,
      421.517,404.537,388.006,371.935,356.335,341.208,326.529,312.31,
      298.539,285.218,272.305,259.829,247.761,236.107,231.92,231.678,
      231.161,229.78,228.924,227.986,226.962,225.869,224.718,223.511,
      222.252,220.957,219.634,218.288,216.896,211.374,207.75,205.714,
      202.654,201.503,209.729,223.925,235.348,247.107,255.308,261.082,
      264.994,264.131,262.866,262.141,262.049,
Z(1)=0.0,0.00001853,0.00006226,0.0001354,0.0002381,0.0003708,0.0005337,
     0.0007273,0.000952,0.001208,0.001497,0.001819,0.002175,0.002565,
     0.002991,0.003455,0.003957,0.004498,0.005082,0.005708,0.00638,
     0.007101,0.007873,0.008699,0.009584,0.01053,0.01155,0.01265,
     0.01401,0.01456,0.01512,0.01568,0.01624,0.0168,0.01737,0.01794,
     0.01852,0.0191,0.01968,0.02026,0.02085,0.02144,0.02204,0.02263,
     0.02324,0.02384,0.02445,0.02505,0.02606,0.02813,0.02921,0.03031,
     0.03143,0.03259,0.03378,0.03501,0.03628,0.03759,0.03895,0.04036,
     0.04183,0.04843,0.0536, 0.05718,0.06457,0.08496,0.1067,0.1268,
     0.1474,0.1699,0.1895,0.21,0.2511,0.292,0.3347,0.3799,0.4203,
PMI(1)=0.0,0.001243,0.002277,0.003356,0.004448,0.005546,0.006648,
       0.007751,0.008856,0.009962,0.01107,0.01218,0.01329,0.01439,
       0.0155,0.01661,0.01772,0.01883,0.01994,0.02105,0.02216,0.02327,
       0.02437,0.02548,0.02659,0.02769,0.0288,0.0299,0.03116,0.03164,
       0.03211,0.03257,0.03301,0.03343,0.03385,0.03425,0.03464,0.03502,
       0.03539,0.03575,0.03609,0.03643,0.03675,0.03707,0.03737,0.03766,
       0.03795,0.03823,0.0387,0.03966,0.04016,0.04068,0.0412,0.04174,
       0.0423,0.04287,0.04346,0.04407,0.0447,0.04536,0.04605,0.04913,
       0.05154,0.05321,0.05665,0.06616,0.07631,0.08568,0.09528,0.1058,
       0.1149,0.1245,0.1436,0.1627,0.1826,0.2037,0.2225,
TW(1)=77*540.0,
RVWALD(1)=77*0.0,
S(1)=0.0,0.001243,0.002278,0.00336,0.004457,0.005563,0.006676,0.007797,
     0.008924,0.01006,0.0112,0.01236,0.01352,0.0147,0.01588,0.01709,
     0.01831,0.01954,0.02079,0.02207,0.02336,0.02469,0.02604,0.02742,
     0.02883,0.03029,0.03179,0.03335,0.0352,0.03594,0.03666,0.03738,
     0.0381,0.03881,0.03951,0.04021,0.04091,0.0416,0.04228,0.04297,
     0.04365,0.04433,0.04501,0.04568,0.04636,0.04703,0.04771,0.04836,
     0.04948,0.05177,0.05295,0.05416,0.05541,0.05668,0.058,0.05935,
     0.06075,0.0622,0.0637,0.06525,0.06688,0.07415,0.07986,0.08381,
     0.09196,0.1145,0.1385,0.1606,0.1834,0.2082,0.2298,0.2524,0.2977,
     0.3429,0.39,0.44,0.4845,
SS(1)=10*.0005,50*.001,43*.01,
$
```

The sample output for case 1 is given as follows:

ETA	YO(1)	YO(2)	YO(3)	YO(4)	YO(5)	YO(6)
0.	0.	0.	7.729421E-01	3.800000E-01	3.008175E-01	0.
1.000000E-01	2.851202E-03	5.690891E-02	7.533828E-01	4.024295E-01	3.008021E-01	3.626776E-02
2.000000E-01	1.135689E-02	1.130707E-01	7.326971E-01	4.251599E-01	3.006933E-01	6.913905E-02
3.000000E-01	2.543655E-02	1.683685E-01	7.108778E-01	4.481846E-01	3.003958E-01	9.872421E-02
4.000000E-01	4.499767E-02	2.226786E-01	6.879263E-01	4.714881E-01	2.998122E-01	1.251445E-01
5.000000E-01	6.993505E-02	2.758718E-01	6.638543E-01	4.950460E-01	2.988446E-01	1.485318E-01
6.000000E-01	1.001303E-01	3.278144E-01	6.386874E-01	5.188240E-01	2.973959E-01	1.690284E-01
7.000000E-01	1.354517E-01	3.783696E-01	6.124673E-01	5.427776E-01	2.953713E-01	1.867859E-01
8.000000E-01	1.757534E-01	4.273990E-01	5.852543E-01	5.668524E-01	2.926810E-01	2.019649E-01
9.000000E-01	2.208760E-01	4.747651E-01	5.571299E-01	5.909838E-01	2.892422E-01	2.147339E-01
1.000000E+00	2.706465E-01	5.203334E-01	5.281979E-01	6.150972E-01	2.849821E-01	2.252680E-01
1.100000E+00	3.248785E-01	5.639754E-01	4.985856E-01	6.391092E-01	2.798410E-01	2.337475E-01
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
6.100000E+00	4.999201E+00	9.999991E-01	3.801683E-06	9.999914E-01	3.271047E-05	2.117261E-01
6.200000E+00	5.099201E+00	9.999994E-01	2.515940E-06	9.999941E-01	2.274064E-05	2.117255E-01
6.300000E+00	5.199201E+00	9.999996E-01	1.654964E-06	9.999960E-01	1.569610E-05	2.117251E-01
6.400000E+00	5.299201E+00	9.999998E-01	1.082035E-06	9.999973E-01	1.075607E-05	2.117248E-01
6.500000E+00	5.399201E+00	9.999998E-01	7.031642E-07	9.999982E-01	7.317933E-06	2.117246E-01
6.600000E+00	5.499201E+00	9.999999E-01	4.541827E-07	9.999988E-01	4.943066E-06	2.117244E-01
6.700000E+00	5.599201E+00	9.999999E-01	2.915789E-07	9.999992E-01	3.314953E-06	2.117243E-01
6.800000E+00	5.699201E+00	1.000000E+00	1.860478E-07	9.999995E-01	2.207148E-06	2.117243E-01
6.900000E+00	5.799201E+00	1.000000E+00	1.179846E-07	9.999997E-01	1.459011E-06	2.117242E-01
7.000000E+00	5.899201E+00	1.000000E+00	7.436122E-08	9.999998E-01	9.575439E-07	2.117242E-01
7.100000E+00	5.999201E+00	1.000000E+00	4.657739E-08	9.999999E-01	6.239244E-07	2.117242E-01
7.200000E+00	6.099201E+00	1.000000E+00	2.899328E-08	9.999999E-01	4.036252E-07	2.117242E-01
7.300000E+00	6.199201E+00	1.000000E+00	1.793488E-08	9.999999E-01	2.592374E-07	2.117242E-01
7.400000E+00	6.299201E+00	1.000000E+00	1.102462E-08	1.000000E+00	1.653066E-07	2.117242E-01

```
$NAM

P10     =  0.924903C8449337E+00,

T10     =  0.53955539733397E+00,

G       =  0.14E+01,

REY     =  0.39738221255108E+07,

RT1     =  0.54456182115757E-01,

P1      =  0.1405651212204E+02,

T1      =  0.1033686448444E+03,

R1      =  0.79244926185209E-04,
```

```
U1      =  0.3961723846778E•04,

AA1     =  0.49833004361987E+03,

TREF    =  0.26132627103112E+04,

VISREF  =  0.10784657366036E-05,

R10     =  0.59996815820952E+01,

$END
```

X= .3450 PROFILE

ETA	Y/YE	U/UE	T/TE	TT/TTE	CROCCO	PT/PTR	M/ME	FZ	TZ	VORTREY	XLM11
0.	0.	0.	5.831E-01	3.830E-01	2.879E-15	2.296E-01	0.	3.998E-01	3.100E-01	0.	1.163E+00
1.000E-01	1.427E-02	4.021E-02	6.137E-01	4.036E-01	3.345E-02	2.307E-01	5.133E-02	4.043E-01	3.015E-01	2.286E+00	1.149E+00
2.000E-01	2.926E-02	8.086E-02	6.434E-01	4.248E-01	6.781E-02	2.339E-01	1.008E-01	4.085E-01	2.924E-01	8.525E+00	1.136E+00
3.000E-01	4.494E-02	1.219E-01	6.722E-01	4.466E-01	1.031E-01	2.390E-01	1.487E-01	4.124E-01	2.827E-01	1.801E+01	1.123E+00
4.000E-01	6.130E-02	1.633E-01	7.000E-01	4.689E-01	1.392E-01	2.460E-01	1.952E-01	4.156E-01	2.723E-01	3.022E+01	1.111E+00
5.000E-01	7.830E-02	2.050E-01	7.267E-01	4.917E-01	1.762E-01	2.548E-01	2.405E-01	4.182E-01	2.613E-01	4.480E+01	1.100E+00
6.000E-01	9.591E-02	2.470E-01	7.522E-01	5.150E-01	2.139E-01	2.654E-01	2.847E-01	4.198E-01	2.495E-01	6.144E+01	1.090E+00
7.000E-01	1.141E-01	2.890E-01	7.766E-01	5.387E-01	2.524E-01	2.780E-01	3.280E-01	4.204E-01	2.371E-01	7.990E+01	1.080E+00
8.000E-01	1.329E-01	3.310E-01	7.997E-01	5.628E-01	2.914E-01	2.924E-01	3.702E-01	4.198E-01	2.240E-01	9.994E+01	1.071E+00
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
3.900E+00	8.352E-01	9.968E-01	9.995E-01	9.975E-01	9.959E-01	8.844E-01	9.970E-01	1.183E-02	1.453E-03	6.159E+01	1.000E+00
4.000E+00	8.587E-01	9.978E-01	9.997E-01	9.982E-01	9.972E-01	8.858E-01	9.979E-01	8.752E-03	1.152E-03	4.817E+01	1.000E+00
4.100E+00	8.823E-01	9.985E-01	9.998E-01	9.988E-01	9.981E-01	8.869E-01	9.986E-01	6.344E-03	9.015E-04	3.686E+01	1.000E+00
4.200E+00	9.058E-01	9.990E-01	9.998E-01	9.992E-01	9.988E-01	8.876E-01	9.991E-01	4.495E-03	6.941E-04	2.753E+01	1.000E+00
4.300E+00	9.294E-01	9.994E-01	9.999E-01	9.995E-01	9.992E-01	8.881E-01	9.995E-01	3.101E-03	5.238E-04	2.000E+01	1.000E+00
4.400E+00	9.529E-01	9.997E-01	9.999E-01	9.997E-01	9.996E-01	8.885E-01	9.997E-01	2.073E-03	3.856E-04	1.406E+01	1.000E+00
4.500E+00	9.765E-01	9.998E-01	1.000E+00	9.999E-01	9.998E-01	8.887E-01	9.998E-01	1.332E-03	2.749E-04	9.489E+00	1.000E+00
4.600E+00	1.000E+00	9.999E-01	1.000E+00	9.999E-01	9.999E-01	8.889E-01	9.999E-01	8.128E-04	1.878E-04	6.073E+00	1.000E+00

X =	3.45000E-01	RETHET=	2.05328E+02	DPEDX =	-1.91279E-02	CFW =	1.29312E-03	ZSHK =	0.	YE =	2.49507E-03
XI =	2.24736E-03	REX =	2.43046E+05	DTEDX =	-5.06340E-01	QSD =	-3.41942E+00	RSHK =	0.	UTAU =	0.
RAD =	1.63587E-01	PE =	2.64075E+02	DUEDX =	8.31957E-01	HD =	4.29204E-03	ITRO =	0	PTR =	9.24903E-01
Z =	2.93904E-01	TE =	9.26012E+02	DLTAST=	5.57693E-04	NSTE =	1.38798E-03	TW/TT =	3.82979E-01	YMP =	R
BETA =	6.14417E-03	RE =	1.66185E-04	THETA =	2.91459E-04	NSTW =	8.09398E-04	RETRUE=	IIIII	P20 =	9.24903E-01
TRFCT =	0.	UE =	2.41116E+03	D/T =	1.91345E+00	NUE =	2.42888E+02	ROUSE =	3.62804E+02	OMEGA =	1.85343E-03
RVWALD=	0.	ME =	1.61657E+00	TAUD =	1.07121E+00	NUW =	3.58218E+02	DSMXO =	1.75918E+03		
REDELT=	3.92885E+02	MUE =	5.68785E-07	CFE =	2.21749E-03	SWANG =	0.	XD =	1.34620E+00		

Case 2.- An example of planar flow is presented in reference 23. The wind-tunnel test conditions were as follows:

$M_\infty = 2.8$

$p_{t,\infty} = 9.997 \times 10^5\ N/m^2$

$T_{t,\infty} = 3.11 \times 10^2\ {}^oK$

$q_w = 0$ (Adiabatic flow)

The location of transition was not reported in reference 23; therefore, for the present calculations it was assumed that transition occurred near the sharp leading edge of the flat-plate model. This case requires $120\,000_8$ storage on the CDC 6000 computer. The listing of the variable-dimension data for this particular case is as follows:

JK = 310

JL = 1

JM = 750

JN = 40

The listing of the input data for case 2 is given as follows:

```
$NAML
HPR=0.4,
XEND=120.0,
H=0.01,
PR=0.7,
XKK=0.911,
BETA=0.0,
ALPHA=3.135,
XO=0.0,
YO(1)=0.0,0.0,0.452,2.43,-0.04,0.0,
XK=1.02,
IGAS=1,
VISCON=0.7173E-08,
VISPOW=0.647,
KODUNIT=0,
$
```

```
$NAM2
XMA=2.8,
PT1=21024.0,
TT1=550.0,
WAVE=0.0,
XY1=1.0,
XY2=1.0,
XY3=2.8,
G=1.4,
R=1716.C,
SU=198.6,
PR=0.7,
PRT=0.95,
IBODY=2,
J=0,
W=0,
FT=1.0,
KODE=0,
KODWAL=2,
IENTRO=1,
CONE=0.0,
IEND1=1000,
A=1.0,
DS=0.01,
KODVIS=1,
SST=0.1E+09,
SMXTR=25C0.C,
TLNGTH=2.0,
CORP=0.412,
CONSTNT=0.0,
XT1=0.4,
XT2=26.0,
XT3=0.0168,
XT4=0.78,
XT5=0.0,
PROINC=2.0,
PRNTINC=0.1,
IPRO=14,
PROVAL(1)=0.1,0.15,0.2,0.25,0.3,0.35,0.4,0.45,0.02,0.05,0.08,.06,.07,1.,
IPRNT=13,
PRNTVAL(1)=2.0,4.0,C.2,0.25,0.3,0.35,0.4,0.45,0.02,0.05,0.08,.06,.07,
NAUXPRO=0,
BLNGTH=0.0,
NPUTYPE=1,
KODPRT=2,
KTCOD=1,
$


$NAM3
NUMBER=41,
L=1,
PE(1)=41*774.69861580563,
Z(1)=0.0,C.5,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,6.0,6.5,7.0,7.5,
8.0,8.5,9.0,9.5,10.0,10.5,11.0,11.5,12.0,12.5,13.0,13.5,14.0,14.5,15.0,
15.5,16.0,16.5,17.0,17.5,18.0,18.5,19.0,19.5,20.0,
RMI(1)=41*1.0,
TW(1)=41*520.4439,
QW(1)=41*0.0,
RVWALD(1)=41*0.0,
S(1)=0.0,0.5,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,6.0,6.5,7.0,7.5,
8.0,8.5,9.0,9.5,10.0,10.5,11.0,11.5,12.0,12.5,13.0,13.5,14.0,14.5,15.0,
15.5,16.0,16.5,17.0,17.5,18.0,18.5,19.0,19.5,20.0,
SS(1)=1000*.01,
$
```

Sample outputs for case 2 for regions where the flow is laminar and turbulent are given as follows:

ETA	YO(1)	YO(2)	YO(4)	YO(5)	YO(6)
0.	0.	0.	2.430000E+00	-4.938715E-02	3.643092E+00
6.328538E-03	1.883040E-05	3.246450E-03	2.429598E+00	-4.938715E-02	3.643092E+00
1.278365E-02	5.864116E-05	6.556230E-03	2.429166E+00	-4.938715E-02	3.643092E+00
1.936786E-02	1.206843E-04	9.930542E-03	2.428702E+00	-4.938715E-02	3.643092E+00
2.608375E-02	2.062704E-04	1.337061E-02	2.428206E+00	-4.938715E-02	3.643092E+00
3.293397E-02	3.167714E-04	1.687767E-02	2.427676E+00	-4.938715E-02	3.643092E+00
3.992118E-02	4.536230E-04	2.045301E-02	2.427109E+00	-4.938715E-02	3.643092E+00
4.704814E-02	6.183275E-04	2.409790E-02	2.426504E+00	-4.938715E-02	3.643092E+00
5.431765E-02	8.124567E-04	2.781366E-02	2.425860E+00	-4.938715E-02	3.643092E+00
6.173254E-02	1.037655E-03	3.160162E-02	2.425174E+00	-4.938715E-02	3.643092E+00
6.929573E-02	1.295641E-03	3.546315E-02	2.424445E+00	-4.938715E-02	3.643092E+00
7.701018E-02	1.588214E-03	3.939963E-02	2.423670E+00	-4.938715E-02	3.643092E+00
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
5.853772E+00	4.700446E+00	9.999988E-01	1.000100E+00	-4.938715E-02	3.643092E+00
5.977176E+00	4.823850E+00	9.999994E-01	1.000061E+00	-4.938715E-02	3.643092E+00
6.103048E+00	4.949722E+00	9.999998E-01	1.000033E+00	-4.938715E-02	3.643092E+00
6.231437E+00	5.078111E+00	9.999998E-01	1.000026E+00	-4.938715E-02	3.643092E+00
6.362395E+00	5.209069E+00	9.999999E-01	1.000015E+00	-4.938715E-02	3.643092E+00
6.495971E+00	5.342645E+00	1.000000E+00	1.000007E+00	-4.938715E-02	3.643092E+00
6.632219E+00	5.478893E+00	1.000000E+00	1.000006E+00	-4.938715E-02	3.643092E+00
6.771192E+00	5.617866E+00	1.000000E+00	1.000003E+00	-4.938715E-02	3.643092E+00
6.912944E+00	5.759618E+00	1.000000E+00	1.000002E+00	-4.938715E-02	3.643092E+00
7.057522E+00	5.904206E+00	1.000000E+00	1.000001E+00	-4.938715E-02	3.643092E+00
7.205011E+00	6.051685E+00	1.000000E+00	1.000000E+00	-4.938715E-02	3.643092E+00

```
$NAM

P10     =  0.24725123499179E+01,

T10     =  0.81887755102041E+00,

G       =  0.14E+01,

REY     =  0.24561099387761E+08,

RT1     =  0.22275905912269E-01,

P1      =  0.77469861580563E+03,

T1      =  0.21417445482866E+03,

R1      =  0.21078894314355E-02,
```

```
U1      =  0.20084661930434E+04,

AA1     =  0.71730935465835E+03,

TREF    =  0.67165109034267E+03,

VISREF  =  0.45404287741154E-06,

R10     =  0.10567872099961E+02,

$END
```

X= .0700 PROFILE

ETA	Y/YE	U/UE	T/TE	TT/TTE	CROCCO	PT/PTR	M/ME	FZ	TZ	VORTREY	XLM11
0.	0.	0.	2.331E+00	9.076E-01	3.843E-14	9.461E-02	0.	5.033E-01	-0.	0.	9.031E-01
6.329E-03	2.038E-03	3.185E-03	2.334E+00	9.091E-01	1.612E-02	9.462E-02	2.085E-03	5.033E-01	2.954E-01	1.771E-02	9.028E-01
1.278E-02	4.119E-03	6.434E-03	2.334E+00	9.091E-01	1.618E-02	9.462E-02	4.211E-03	5.033E-01	-7.223E-03	7.231E-02	9.028E-01
1.937E-02	6.242E-03	9.747E-03	2.334E+00	9.091E-01	1.628E-02	9.463E-02	6.380E-03	5.033E-01	-1.089E-02	1.660E-01	9.028E-01
2.608E-02	8.407E-03	1.313E-02	2.334E+00	9.091E-01	1.643E-02	9.465E-02	8.592E-03	5.033E-01	-1.462E-02	3.012E-01	9.028E-01
3.293E-02	1.061E-02	1.657E-02	2.334E+00	9.091E-01	1.664E-02	9.467E-02	1.085E-02	5.032E-01	-1.843E-02	4.803E-01	9.028E-01
3.992E-02	1.287E-02	2.009E-02	2.334E+00	9.091E-01	1.689E-02	9.470E-02	1.315E-02	5.032E-01	-2.232E-02	7.058E-01	9.028E-01
4.705E-02	1.516E-02	2.368E-02	2.334E+00	9.092E-01	1.720E-02	9.474E-02	1.550E-02	5.032E-01	-2.629E-02	9.804E-01	9.028E-01
5.432E-02	1.751E-02	2.734E-02	2.334E+00	9.092E-01	1.756E-02	9.478E-02	1.789E-02	5.032E-01	-3.033E-02	1.307E+00	9.029E-01
6.173E-02	1.990E-02	3.107E-02	2.333E+00	9.092E-01	1.799E-02	9.483E-02	2.034E-02	5.032E-01	-3.446E-02	1.689E+00	9.029E-01
6.930E-02	2.233E-02	3.487E-02	2.333E+00	9.093E-01	1.849E-02	9.488E-02	2.283E-02	5.032E-01	-3.867E-02	2.128E+00	9.029E-01
7.701E-02	2.482E-02	3.875E-02	2.333E+00	9.093E-01	1.905E-02	9.495E-02	2.537E-02	5.032E-01	-4.296E-02	2.629E+00	9.029E-01
8.488E-02	2.735E-02	4.271E-02	2.332E+00	9.094E-01	1.968E-02	9.502E-02	2.797E-02	5.031E-01	-4.734E-02	3.195E+00	9.030E-01
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4.549E+00	9.588E-01	9.997E-01	1.005E+00	1.002E+00	1.017E+00	9.949E-01	9.973E-01	9.665E-04	-1.294E-02	8.519E+01	9.999E-01
4.646E+00	9.722E-01	9.998E-01	1.004E+00	1.001E+00	1.013E+00	9.961E-01	9.979E-01	6.898E-04	-1.024E-02	6.273E+01	9.999E-01
4.745E+00	9.860E-01	9.999E-01	1.003E+00	1.001E+00	1.010E+00	9.970E-01	9.984E-01	4.841E-04	-8.001E-03	4.540E+01	9.999E-01
4.847E+00	1.000E+00	9.999E-01	1.002E+00	1.001E+00	1.008E+00	9.978E-01	9.988E-01	3.338E-04	-6.178E-03	3.227E+01	1.000E+00

```
X =      7.00000E-02  RETHET= 8.40589E+02  DPEDX = 0.          CFW =    1.14250E-03  ZSHK =  0.          YE =     5.46727E-04
XI =     2.65745E-02  REX =   1.71928E+06  DTEDX =-0.          QSD =    0.           RSHK =  0.          UTAU =   0.
RAD =    1.00000E+00  PE =    7.74699E+02  DUEDX =-0.          HD =     0.           ITRO =            0 PTR =    9.62954E-01
Z =      7.00000E-02  TE =    2.14174E+02  DLTAST= 2.68454E-04  NSTE =   0.           TW/TT = 9.07564E-01  YMP =              P
BETA =   0.           RE =    2.10789E-03  THETA = 3.42244E-05  NSTW =   0.           RETRUE= 8.48612E-01  P20 =    2.47251E+00
TREFT =  0.           UE =    2.00847E+03  D/T =   7.84394E+00  NUE =    0.           ROUSE = 2.59252E+03  OMEGA = 3.27486E-04
RVWALD= 0.            ME =    2.80000E+00  TAUD =  2.08415E+00  NUW =    0.           DSMXO = 1.99617E+04
REDELT= 6.59353E+03   MUE =   1.72371E-07  CFE =   4.90210E-04  SWANG =  0.           XO =     1.66947E+00
```

X= 2.0000 PROFILE

ETA	Y/YE	U/UE	T/TE	TT/TTE	CROCCO	PT/PTR	M/ME	YPLUS	UPLUS	UDEF	VISEFF
0.	0.	0.	2.433E+00	9.476E-01	6.780E-14	9.461E-02	0.	0.	0.	0.	1.004E+00
6.329E-03	2.884E-04	4.768E-02	2.435E+00	9.495E-01	3.668E-02	9.510E-02	3.056E-02	1.311E+00	1.171E+00	2.338E+01	1.013E+00
1.278E-02	5.822E-04	9.533E-02	2.427E+00	9.508E-01	6.082E-02	9.657E-02	6.119E-02	2.655E+00	2.344E+00	2.224E+01	1.044E+00
1.937E-02	8.807E-04	1.417E-01	2.415E+00	9.528E-01	9.826E-02	9.900E-02	9.118E-02	4.038E+00	3.493E+00	2.116E+01	1.110E+00
2.608E-02	1.183E-03	1.853E-01	2.399E+00	9.552E-01	1.442E-01	1.023E-01	1.196E-01	5.464E+00	4.583E+00	2.015E+01	1.223E+00
3.293E-02	1.490E-03	2.249E-01	2.380E+00	9.577E-01	1.927E-01	1.061E-01	1.458E-01	6.938E+00	5.584E+00	1.925E+01	1.392E+00
3.992E-02	1.800E-03	2.598E-01	2.360E+00	9.601E-01	2.387E-01	1.103E-01	1.691E-01	8.458E+00	6.480E+00	1.846E+01	1.618E+00
4.705E-02	2.113E-03	2.902E-01	2.339E+00	9.623E-01	2.797E-01	1.147E-01	1.898E-01	1.003E+01	7.270E+00	1.778E+01	1.901E+00
5.432E-02	2.430E-03	3.165E-01	2.319E+00	9.641E-01	3.149E-01	1.190E-01	2.079E-01	1.164E+01	7.964E+00	1.720E+01	2.238E+00
6.173E-02	2.751E-03	3.393E-01	2.299E+00	9.656E-01	3.444E-01	1.233E-01	2.238E-01	1.329E+01	8.572E+00	1.669E+01	2.627E+00
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3.709E+01	9.271E-01	9.939E-01	1.043E+00	1.009E+00	1.178E+00	9.494E-01	9.731E-01	1.205E+04	3.728E+01	2.298E-01	1.435E+02
3.784E+01	9.413E-01	9.959E-01	1.034E+00	1.008E+00	1.157E+00	9.610E-01	9.793E-01	1.239E+04	3.752E+01	1.559E-01	1.232E+02
3.860E+01	9.557E-01	9.975E-01	1.025E+00	1.007E+00	1.130E+00	9.716E-01	9.850E-01	1.273E+04	3.774E+01	9.601E-02	1.045E+02
3.938E+01	9.703E-01	9.987E-01	1.017E+00	1.005E+00	1.099E+00	9.811E-01	9.901E-01	1.307E+04	3.793E+01	5.084E-02	8.782E+01
4.017E+01	9.851E-01	9.994E-01	1.010E+00	1.003E+00	1.064E+00	9.892E-01	9.943E-01	1.341E+04	3.809E+01	2.099E-02	7.268E+01
4.098E+01	1.000E+00	1.000E+00	1.001E+00	1.000E+00	1.006E+00	9.993E-01	9.996E-01	1.380E+04	3.829E+01	1.042E-04	1.000E+00

X =	2.00000E+00	PETHET=	3.69327E+04	DPEDX =	6.66134E-14	CFW =	3.31613E-03	ZSHK =	0.
XI =	7.59273E-01	REX =	4.91222E+07	DTEDX =	1.75466E-13	QSD =	0.	RSHK =	0.
RAD =	1.00000E+00	PE =	7.74699E+02	DUEDX =	-1.75466E-13	HD =	0.	ITRO =	0
Z =	2.00000E+00	TE =	2.14174E+02	DLTAST=	7.14388E-03	NSTE =	0.	TW/TT =	9.47602E-01
BETA =	-2.66454E-13	RE =	2.10789E-03	THETA =	1.50371E-03	NSTW =	0.	RFTRUE=	9.14186E-01
TRFCT =	1.00000E+00	UE =	2.00847E+03	D/T =	4.75085E+00	NUE =	0.	ROUSE =	7.79506E+04
RVWALD=	0.	ME =	2.80000E+00	TAUD =	5.79372E+00	NUW =	0.	DSMXO =	2.78534E+04
REDELT=	1.75462E+05	MUE =	1.72371E-07	CFE =	1.36273E-03	SWANG =	0.	XD =	6.58196E+01

YE =	2.15550E-02
UTAU =	2.61030E-02
PTR =	9.62954E-01
YMP =	147
P20 =	2.47251E+00
OMEGA =	3.27486E-04

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 11, 1972.

APPENDIX A

DIFFERENCE RELATIONS

Three-point implicit difference relations are used to reduce the transformed momentum and energy equations (eqs. (28) and (29)) to finite-difference form. It is assumed that all data are known at the solution stations m-1 and m. (See fig. 2.) Then, it is possible to obtain the unknown quantities at the grid points for the m+1 station. In the subsequent development the notations G and H are utilized to represent any typical variable.

Taylor-series expansions are first written about the unknown grid point (m+1,n) in the ξ-direction as follows:

$$G_{m,n} = G_{m+1,n} - \Delta\xi_2\left(G_\xi\right)_{m+1,n} + \frac{\Delta\xi_2^2}{2}\left(G_{\xi\xi}\right)_{m+1,n} - \frac{\Delta\xi_2^3}{6}\left(G_{\xi\xi\xi}\right)_{m+1,n} + \ldots \tag{A1a}$$

and

$$G_{m-1,n} = G_{m+1,n} - \left(\Delta\xi_1 + \Delta\xi_2\right)\left(G_\xi\right)_{m+1,n} + \frac{\left(\Delta\xi_1 + \Delta\xi_2\right)^2}{2}\left(G_{\xi\xi}\right)_{m+1,n} - \frac{\left(\Delta\xi_1 + \Delta\xi_2\right)^3}{6}\left(G_{\xi\xi\xi}\right)_{m+1,n} + \ldots \tag{A1b}$$

where subscript notation has been utilized to denote differentiation; that is, $G_\xi \equiv \frac{\partial G}{\partial \xi}$, and so forth.

Equations (A1a) and (A1b) can be solved to yield

$$\left(\frac{\partial G}{\partial \xi}\right)_{m+1,n} = \frac{X_1 G_{m+1,n} - X_2 G_{m,n} + X_3 G_{m-1,n}}{2\,\Delta\xi_2} + \frac{\Delta\xi_2\left(\Delta\xi_1 + \Delta\xi_2\right)}{6} G_{\xi\xi\xi} + \ldots \tag{A2}$$

and

$$G_{m+1,n} = X_4 G_{m,n} - X_5 G_{m-1,n} + \frac{\Delta\xi_1\,\Delta\xi_2}{2}\left(1 + \frac{\Delta\xi_2}{\Delta\xi_1}\right) G_{\xi\xi} + \ldots \tag{A3}$$

Terms of the order of $\Delta\xi_1\,\Delta\xi_2$, or smaller, are neglected. This produces truncation errors of the order of $\Delta\xi_1\,\Delta\xi_2$ instead of $\Delta\xi_2$ as in reference 9 where two-point

difference relations are used. The $X_1, X_2, \ldots, X_5$ coefficients appearing in equations (A2) and (A3) are defined as follows:

$$X_1 = 2\frac{\Delta\xi_1 + 2\Delta\xi_2}{\Delta\xi_1 + \Delta\xi_2} \tag{A4}$$

$$X_2 = 2\frac{\Delta\xi_1 + \Delta\xi_2}{\Delta\xi_1} \tag{A5}$$

$$X_3 = 2\frac{\Delta\xi_1\Delta\xi_2}{\Delta\xi_1\left(\Delta\xi_1 + \Delta\xi_2\right)} \tag{A6}$$

$$X_4 = \frac{\Delta\xi_1 + \Delta\xi_2}{\Delta\xi_1} \tag{A7}$$

and

$$X_5 = \frac{\Delta\xi_2}{\Delta\xi_1} \tag{A8}$$

Taylor-series expansions are next written about the unknown grid point (m+1,n) in the η-direction as follows:

$$G_{m+1,n+1} = G_{m+1,n} + \Delta\eta_n\left(G_\eta\right)_{m+1,n} + \frac{\Delta\eta_n^2}{2}\left(G_{\eta\eta}\right)_{m+1,n} + \frac{\Delta\eta_n^3}{6}\left(G_{\eta\eta\eta}\right)_{m+1,n} + \ldots \tag{A9a}$$

and

$$G_{m+1,n-1} = G_{m+1,n} - \Delta\eta_{n-1}\left(G_\eta\right)_{m+1,n} + \frac{\Delta\eta_{n-1}^2}{2}\left(G_{\eta\eta}\right)_{m+1,n} - \frac{\Delta\eta_{n-1}^3}{6}\left(G_{\eta\eta\eta}\right)_{m+1,n} + \ldots \tag{A9b}$$

Equations (A9a) and (A9b) can be solved to yield

$$\left(\frac{\partial^2 G}{\partial\eta^2}\right)_{m+1,n} = Y_1 G_{m+1,n+1} - Y_2 G_{m+1,n} + Y_3 G_{m+1,n-1} + \frac{\left(\Delta\eta_{n-1} - \Delta\eta_n\right)}{3} G_{\eta\eta\eta} + \ldots \tag{A10}$$

and

$$\left(\frac{\partial G}{\partial\eta}\right)_{m+1,n} = Y_4 G_{m+1,n+1} - Y_5 G_{m+1,n} - Y_6 G_{m+1,n-1} - \frac{\Delta\eta_n\Delta\eta_{n-1}}{6} G_{\eta\eta\eta} + \ldots \tag{A11}$$

The $Y_1, Y_2, \ldots, Y_6$ coefficients appearing in equations (A10) and (A11) are defined as follows:

$$Y_1 = \frac{2}{\Delta\eta_n\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A12}$$

$$Y_2 = \frac{2}{\Delta\eta_n \Delta\eta_{n-1}} \tag{A13}$$

$$Y_3 = \frac{2}{\Delta\eta_{n-1}\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A14}$$

$$Y_4 = \frac{\Delta\eta_{n-1}}{\Delta\eta_n\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A15}$$

$$Y_5 = \frac{\Delta\eta_{n-1} - \Delta\eta_n}{\Delta\eta_n \Delta\eta_{n-1}} \tag{A16}$$

and

$$Y_6 = \frac{\Delta\eta_n}{\Delta\eta_{n-1}\left(\Delta\eta_n + \Delta\eta_{n-1}\right)} \tag{A17}$$

For the case of equally spaced grid points in the ξ- and η-coordinates, equations (A4) to (A8) and (A12) to (A17) reduce to the following relations:

$$\left.\begin{aligned} X_1 &= 3 \\ X_2 &= 4 \\ X_3 &= 1 \\ X_4 &= 2 \\ X_5 &= 1 \end{aligned}\right\} \tag{A18a}$$

and

$$\left.\begin{aligned} Y_1 &= \frac{1}{\Delta\eta^2} \\ Y_2 &= 2Y_1 \\ Y_3 &= Y_1 \\ Y_4 &= \frac{1}{2\Delta\eta} \\ Y_5 &= 0 \\ Y_6 &= Y_4 \end{aligned}\right\} \tag{A18b}$$

where $\Delta\xi$ and $\Delta\eta$ represent the spacing between the grid points in the ξ- and η-coordinates, respectively.

Equations (A2), (A3), (A10), and (A11) can then be written for constant grid-point spacing as follows:

$$\left(\frac{\partial G}{\partial \xi}\right)_{m+1,n} = \frac{3G_{m+1,n} - 4G_{m,n} + G_{m-1,n}}{2\Delta\xi} + \frac{\Delta\xi^2}{3} G_{\xi\xi\xi} + \ldots \tag{A19}$$

$$G_{m+1,n} = 2G_{m,n} - G_{m-1,n} + \Delta\xi^2 G_{\xi\xi} + \ldots \tag{A20}$$

$$\left(\frac{\partial^2 G}{\partial \eta^2}\right)_{m+1,n} = \frac{G_{m+1,n+1} - 2G_{m+1,n} + G_{m+1,n-1}}{\Delta\eta^2} - \frac{\Delta\eta^2}{12} G_{\eta\eta\eta\eta} + \ldots \tag{A21}$$

and

$$\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} = \frac{G_{m+1,n+1} - G_{m+1,n-1}}{2\Delta\eta} - \frac{\Delta\eta^2}{6} G_{\eta\eta\eta} + \ldots \tag{A22}$$

Equations (A19) to (A22) are recognized as the standard relations for equally spaced grid points. (See, for example, ref. 11.)

Quantities of the form $\left(G\frac{\partial H}{\partial \xi}\right)$ that appear in the governing equations must be linearized in order to obtain a system of linear difference equations. Quantities of this type are obtained from equations (A2) and (A3).

The procedure used to linearize nonlinear products such as $\left(\frac{\partial G}{\partial \eta}\right)\left(\frac{\partial H}{\partial \eta}\right)$ is the same as that used by Flügge-Lotz and Blottner (ref. 9) and is as follows:

$$\left[\left(\frac{\partial G}{\partial \eta}\right)\left(\frac{\partial H}{\partial \eta}\right)\right]_{m+1,n} = \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left(\frac{\partial H}{\partial \eta}\right)_{m+1,n} - \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left(\frac{\partial H}{\partial \eta}\right)_{m,n}$$

$$+ \left(\frac{\partial H}{\partial \eta}\right)_{m,n}\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} + 0\left(\Delta \xi_2\right)^2 \tag{A23}$$

where the terms $\left(\frac{\partial G}{\partial \eta}\right)_{m,n}$ and $\left(\frac{\partial H}{\partial \eta}\right)_{m,n}$ are evaluated from equation (A11), but at the known station m. By equating G to H in equation (A23), the linearized form for quantities of the type $\left(\frac{\partial G}{\partial \eta}\right)^2$ is obtained; that is,

$$\left(\frac{\partial G}{\partial \eta}\right)^2_{m+1,n} = \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\left[2\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n} - \left(\frac{\partial G}{\partial \eta}\right)_{m,n}\right] + 0\left(\Delta \xi_2\right)^2 \tag{A24}$$

where $\left(\frac{\partial G}{\partial \eta}\right)_{m+1,n}$ is obtained from equation (A22).

The preceding relations for the difference quotients produce linear-difference equations when substituted into the governing differential equations (eqs. (43)) for the conservation of momentum (eq. (28)) and energy (eq. (29)), respectively, since terms of the order of $(\Delta \xi)^2$ are neglected.

APPENDIX B

COEFFICIENTS FOR DIFFERENCE EQUATIONS

Equations (43) are the difference equations used to represent the partial differential equations for the conservation of momentum and energy, respectively. These equations are repeated for convenience as follows:

$$A1_nF_{m+1,n-1} + B1_nF_{m+1,n} + C1_nF_{m+1,n+1} + D1_n\Theta_{m+1,n-1}$$

$$+ E1_n\Theta_{m+1,n} + F1_n\Theta_{m+1,n+1} = G1_n \tag{B1}$$

$$A2_nF_{m+1,n-1} + B2_nF_{m+1,n} + C2_nF_{m+1,n+1} + D2_n\Theta_{m+1,n-1}$$

$$+ E2_n\Theta_{m+1,n} + F2_n\Theta_{m+1,n+1} = G2_n \tag{B2}$$

These equations are obtained from equations (28) and (29) and the difference quotients are presented in appendix A. The coefficients $A1_n$, $B1_n$, and so forth, in equations (B1) and (B2) are functions of known quantities evaluated at stations m and m-1. (See fig. 2.) Therefore, equations (B1) and (B2) can be solved simultaneously without iterative procedures. The coefficients $A1_n$, $B1_n$, and so forth are as follows:

$$A1_n = Y_3H_3 - Y_6H_{11} \tag{B3}$$

$$B1_n = X_1H_1 - Y_2H_3 - Y_5H_{11} + H_5 \tag{B4}$$

$$C1_n = Y_1H_3 + Y_4H_{11} \tag{B5}$$

$$D1_n = -Y_6H_4F_Y \tag{B6}$$

$$E1_n = \frac{Y_5}{Y_6}D1_n + H_6 \tag{B7}$$

$$F1_n = -\frac{Y_4}{Y_6}D1_n \tag{B8}$$

$$G1_n = H_1 F_{m2} + H_4 T_Y F_Y \tag{B9}$$

$$A2_n = -2Y_6 H_8 F_Y \tag{B10}$$

$$B2_n = \frac{Y_5}{Y_6} A2_n \tag{B11}$$

$$C2_n = -\frac{Y_4}{Y_6} A2_n \tag{B12}$$

$$D2_n = Y_3 H_{10} - Y_6 H_{12} \tag{B13}$$

$$E2_n = X_1 H_1 - Y_2 H_{10} - Y_5 H_{12} \tag{B14}$$

$$F2_n = Y_1 H_{10} + Y_4 H_{12} \tag{B15}$$

and

$$G2_n = H_1 T_{m2} + H_8 \left(F_Y\right)^2 + H_9 \left(T_Y\right)^2 \tag{B16}$$

The coefficients $Y_1, Y_2, \ldots, Y_6$ and $X_1, \ldots, X_5$ are functions of the grid-point spacing and are defined in equations (A12) to (A17) and (A4) to (A8), respectively. The coefficients $H_1, H_2, \ldots, H_{12}$ are defined as follows:

$$H_1 = \xi_{m+1} F_{m1} \frac{(FT)}{\Delta \xi_2} \tag{B17}$$

$$H_2 = V_{m1} - L_{m1}\left(\overline{E}_{m1} C'_{m1} + \overline{E}'_{m1} C_{m1}\right) \tag{B18}$$

$$H_3 = -\overline{E}_{m1} L_{m1} C_{m1} \tag{B19}$$

$$H_4 = H_3 \frac{L'_{m1}}{L_{m1}} \tag{B20}$$

$$H_5 = \beta_{m+1} F_{m1} \tag{B21}$$

$$H_6 = -\beta_{m+1} \tag{B22}$$

$$H_7 = V_{m1} - L_{m1}\left(\hat{E}_{m1}C'_{m1} + \hat{E}'_{m1}C_{m1}\right) \tag{B23}$$

$$H_8 = -\alpha_{m+1}L_{m1}\overline{E}_{m1}C_{m1} \tag{B24}$$

$$H_9 = -\hat{E}_{m1}L'_{m1}C_{m1} \tag{B25}$$

$$H_{10} = H_9\frac{L_{m1}}{L'_{m1}} \tag{B26}$$

$$H_{11} = H_2 + H_4T_Y \tag{B27}$$

and

$$H_{12} = H_7 + 2H_9T_Y \tag{B28}$$

The undefined quantities appearing in equations (B17) to (B28) are defined as follows:

$$F_{m1} = X_4F_{m,n} - X_5F_{m-1,n} \tag{B29}$$

$$T_{m1} = X_4\Theta_{m,n} - X_5\Theta_{m-1,n} \tag{B30}$$

$$V_{m1} = X_4V_{m,n} - X_5V_{m-1,n} \tag{B31}$$

$$F_{m2} = X_2F_{m,n} - X_3F_{m-1,n} \tag{B32}$$

$$T_{m2} = X_2\Theta_{m,n} - X_3\Theta_{m-1,n} \tag{B33}$$

$$L_{m1} = \sqrt{T_{m1}}\,\frac{1 + \left(\frac{S}{T_e}\right)_{m+1}}{T_{m1} + \left(\frac{S}{T_e}\right)_{m+1}} \qquad \text{(Air only)} \tag{B34a}$$

$$L_{m1} = \left(T_{m1}\right)^{\sigma-1} \qquad \text{(Power law)} \qquad \text{(B34b)}$$

$$L'_{m1} = \frac{L_{m1}}{2T_{m1}}\left[\frac{\left(\frac{S}{T_e}\right)_{m+1} - T_{m1}}{\left(\frac{S}{T_e}\right)_{m+1} + T_{m1}}\right] \qquad \text{(Air only)} \qquad \text{(B35a)}$$

$$L'_{m1} = (\sigma - 1)\left(T_{m1}\right)^{\sigma-2} \qquad \text{(Power law)} \qquad \text{(B35b)}$$

$$\overline{E}_{m1} = \left(\overline{\epsilon}_{av}\right)_{m+1,n} \qquad \text{(B36a)}$$

where

$$\left(\overline{\epsilon}_{av}\right)_{m+1,n} = \frac{\overline{\epsilon}_{m-1,n} + \overline{\epsilon}_{m,n} + \overline{\epsilon}_{m+1,n}}{3} \qquad \text{(B36b)}$$

$$\hat{E}_{m1} = \frac{\left(\tilde{\epsilon}_{av}\right)_{m+1,n}}{\sigma} \qquad \text{(B37)}$$

$$\overline{E}_Y = Y_4\overline{\epsilon}_{m,n+1} - Y_5\overline{\epsilon}_{m,n} - Y_6\overline{\epsilon}_{m,n-1} \qquad \text{(See eq. (A11))} \qquad \text{(B38)}$$

$$\hat{E}_Y = Y_4\tilde{\epsilon}_{m,n+1} - Y_5\tilde{\epsilon}_{m,n} - Y_6\tilde{\epsilon}_{m,n-1} \qquad \text{(B39)}$$

$$F_Y = Y_4F_{m,n+1} - Y_5F_{m,n} - Y_6F_{m,n-1} \qquad \text{(B40)}$$

$$T_Y = Y_4\Theta_{m,n+1} - Y_5\Theta_{m,n} - Y_6\Theta_{m,n-1} \qquad \text{(B41)}$$

$$\beta_{m+1} = \left(\frac{2\xi}{u_e}\frac{du_e}{d\xi}\right)_{m+1} \qquad \text{(See eqs. (30))} \qquad \text{(B42)}$$

and

$$\alpha_{m+1} = \left(\frac{u_e^2}{T_e}\right)_{m+1} \qquad \text{(B43)}$$

The transverse-curvature terms are contained in the quantities C_{m1} and C'_{m1} which appear explicitly in the H_2, H_3, H_7, H_8, and H_9 coefficients. The transverse-curvature term in the transformed plane (see ref. 7) may be written as follows:

$$t^{2j} = 1 + \frac{2\omega j(W)\sqrt{2\xi}\cos\phi}{\rho_e u_e}\int_0^{\eta}\Theta\, d\eta \tag{B44}$$

where t represents the ratio r/r_o and is a known quantity for the $N-1$ grid points at station $m-1$ and m. Then, the extrapolated values at $m+1,n$ are obtained as follows where the parameter C is used to represent t^{2j}:

$$C_{m1} = X_4 C_{m,n} - X_5 C_{m-1,n} \tag{B45}$$

$$C'_{m1} = Y_4 C_{m,n+1} - Y_5 C_{m,n} - Y_6 C_{m,n-1} \tag{B46}$$

Two quantities (symbols) as of now remain undefined. These are the code symbols FT and W which appear in equations (B17) and (B44), respectively. The code symbol W appearing in equation (B44) is used either to retain or neglect the transverse-curvature terms for axisymmetric flows; that is, W = 1 or 0, respectively. For planar flows, the transverse-curvature term does not appear since j equals 0. The code symbol FT (flow type) appearing in equation (B17) is used either to retain or neglect the nonsimilar terms in the governing differential equations; that is, FT = 1 or 0, respectively. If FT is assigned a value of unity, the solution to the nonsimilar equations (eqs. (27) to (29)) is obtained. If FT is assigned a value of zero, the locally similar solution is obtained; that is, the following system of equations is solved:

Continuity

$$\frac{\partial V}{\partial \eta} + F = 0 \tag{B47}$$

Momentum

$$V\frac{\partial F}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} l \bar{\epsilon}\frac{\partial F}{\partial \eta}\right) + \beta\left(F^2 - \Theta\right) = 0 \tag{B48}$$

Energy

$$V\frac{\partial \Theta}{\partial \eta} - \frac{\partial}{\partial \eta}\left(t^{2j} l \frac{\tilde{\epsilon}}{\sigma}\frac{\partial \Theta}{\partial \eta}\right) - \alpha l t^{2j}\bar{\epsilon}\left(\frac{\partial F}{\partial \eta}\right)^2 = 0 \tag{B49}$$

The governing equations for the locally similar system are obtained from equations (27) to (29) by neglecting derivatives of the dependent variables F, Θ, and V with respect to the streamwise coordinate ξ. The capability of obtaining locally similar solutions is desirable in that for a given test case the locally similar and complete nonsimilar solutions can be obtained for the identical program inputs and numerical procedures. Consequently, the effects of the nonsimilar terms on the boundary-layer characteristics for a particular case can be determined by a direct comparison of the results obtained for solutions for $FT = 1$ and 0, respectively.

APPENDIX C

LANGLEY LIBRARY SUBROUTINE FTLUP

Language: FORTRAN

Purpose: Computes $y = F(x)$ from a table of values using first- or second-order interpolation. An option to give y a constant value for any x is also provided.

Use: CALL FTLUP(X, Y, M, N, VARI, VARD)

X — The name of the independent variable x.

Y — The name of the dependent variable $y = F(x)$.

M — The order of interpolation (an integer)

$M = 0$ for y a constant. VARD(I) corresponds to VARI(I) for $I = 1, 2, \ldots, N$. For $M = 0$ or $N \leqq 1$, $y = F(VARI(1))$ for any value of x. The program extrapolates.

$M = 1$ or 2. First or second order if VARI is strictly increasing (not equal).

$M = -1$ or -2. First or second order if VARI is strictly decreasing (not equal).

N — The number of points in the table (an integer).

VARI — The name of a one-dimensional array which contains the N values of the independent variable.

VARD — The name of a one-dimensional array which contains the N values of the dependent variable.

Restrictions: All the numbers must be floating point. The values of the independent variable x in the table must be strictly increasing or strictly decreasing. The following arrays must be dimensioned by the calling program as indicated: VARI(N), VARD(N).

Accuracy: A function of the order of interpolation used.

References: (a) Nielsen, Kaj L.: Methods in Numerical Analysis. The Macmillan Co., c.1956, pp. 87-91.
(b) Milne, William Edmund: Numerical Calculus. Princeton Univ. Press, c.1949, pp. 69-73.

Storage: 430_8 locations.

Error condition: If the VARI values are not in order, the subroutine will print TABLE BELOW OUT OF ORDER FOR FTLUP AT POSITION xxx TABLE IS STORED IN LOCATION xxxxxx (absolute). It then prints the contents of VARI and VARD, and STOPS the program.

Subroutine date: September 12, 1969.

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NASA-Langley, 1972 —— 12 L-8037