COMPARISON OF CONVENTIONAL AND MICROASPHERITY ELASTOHYDRODYNAMIC LUBRICATION OF A BALL SPINNING IN A NONCONFORMING GROOVE

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16. Abstract  
An analysis was developed for the microasperity elastohydrodynamic lubrication of a ball spinning in a nonconforming groove. This analysis was compared to the conventional elastohydrodynamic analysis of a ball spinning in a nonconforming groove. Rheological models for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether (5P4E) were constructed from spinning torque data by using both analyses. The value of the lubricant pressure-viscosity coefficient that makes the data fit the analyses of the fluids differs somewhat from published data. For all three lubricants, an exponential composite model best described the lubricant rheology. Good agreement existed with the experimental and analytical values of torque for both the conventional and microasperity elastohydrodynamic analysis for a spinning ball in a nonconforming groove.

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by Charles W. Allen, * Dennis P. Townsend, and Erwin V. Zaretsky

Lewis Research Center

SUMMARY

The microasperity elastohydrodynamic lubrication analysis for a ball spinning on a flat plate was extended to the elliptical contact case of a ball spinning in a nonconforming groove. The microasperity analysis was compared to the conventional elastohydrodynamic analysis of a spinning ball in a nonconforming groove. Rheological models for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether (5P4E) were constructed, by using both analyses, from spinning torque data obtained in a previous investigation. The value of the lubricant pressure-viscosity coefficient that makes the data fit the analyses of the fluids differs somewhat from published values. For all three lubricants, an exponential composite model best described the lubricant rheology. Good agreement existed with the experimental and analytical values of torque for both the conventional and microasperity elastohydrodynamic analysis for a spinning ball in a nonconforming groove. The polyphenyl ether lubricant exhibited a negative secondary slope in the composite exponential model, which would imply a decrease in viscosity with pressure above $117 \times 10^6$ newtons per square meter (17 000 psi).

INTRODUCTION

In recent years, a complete evolution has occurred in ball bearing design and analysis. Early work assumed that at the ball-race contact, spinning and rolling would occur at one raceway and only pure rolling at the other raceway (refs. 1 and 2). This work was based on the premise that there was a constant coefficient of friction at the ball-race contact. Work reported in reference 3 and subsequently in references 4 and 5 indicated that the friction was not dependent upon a single coefficient of friction but varied according to the contact stress, contact geometry, and lubricant type and viscosity.

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As a result, no simple coefficient of friction could be applied to a ball-race contact. Reference 6 recognizes this phenomenon and presents an analysis for the kinematics of a ball bearing considering elastohydrodynamic (EHD) effects and assumed lubricant rheological properties. However, these properties are not completely defined, resulting in some second-order inaccuracies in the analysis. Reference 7 develops a theoretical analysis based upon the elastohydrodynamic theory of lubrication for spinning torque. This model was verified experimentally. Subsequently, a microasperity analysis for elastohydrodynamic lubrication of a ball spinning on a flat surface was developed (ref. 8). By utilizing these two analyses, it is possible, based upon experimental measurements, to derive a rheological model for a test lubricant.

The work reported herein was conducted (1) to extend the microasperity analysis for elastohydrodynamic lubrication of a ball spinning in a nonconforming groove; and (2) to determine the pressure-viscosity models for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether by using a conventional EHD analysis and a microasperity EHD analysis. The spinning torque data used were obtained in the NASA spinning torque apparatus and originally reported in reference 3. These data were verified in the modified spinning torque apparatus described initially in reference 4.

APPARATUS AND SPECIMENS

Spinning Torque Apparatus

A spinning torque apparatus (see figs. 1(a) and (b)) as reported in references 3 and 4 was used for the tests reported herein. The apparatus essentially consists of a turbine drive, a pneumatic load device, an upper and lower test specimen, a lower test-housing assembly incorporating a hydrostatic air-bearing, and a torque-measuring system. In operation, the upper test specimen is pneumatically loaded against the lower test specimen through the drive shaft. As the drive shaft is rotated, the upper test specimen spins in the groove of the lower test specimen. This causes an angular deflection of the lower test-specimen housing. This angular movement is sensed optically by the torque-measuring system and is converted into a torque value. During a test, the torque is continuously recorded on a strip chart.
Turbine drive air

Drive turbine

Load chamber pressure readout

Magnetic speed pickup

Loading piston

Load air in

Ball bushing

Drive shaft

Counterweight system

Vacuum passage

Upper test specimen

Lower test specimen

Lower test-housing assembly

Foil bearing

Calibrating load pan

Damping system

Torque rod mirror

Optical torquemeter

(a) General cutaway view.

Figure 1. - Spinning-torque apparatus.
Specimens

The upper test specimen is a conventional 12.7-millimeter- (1/2-in.-) diameter bearing ball made of SAE 52100 steel having a nominal Rockwell C hardness of 61 and a surface finish of $5 \times 10^{-8}$ meters (2 μin.) rms. The lower test specimen (fig. 1(c)) is a 12.7-millimeter- (1/2-in.-) diameter ball from the same heat of material as the upper test specimen, which is modified by grinding a flat on one side and a cylindrical groove of radius $R_G$ (fig. 1(c)) on the other. The groove simulates the race groove of a bearing. The axis of the groove is parallel to the flat. The groove radius expressed as a percentage of the upper-ball diameter is defined as the ball-race conformity. The specimens used in these tests were ground to ball-race conformities of 51 percent. The surface finish of the cylindrical groove was approximately $5 \times 10^{-8}$ to $15 \times 10^{-8}$ meters (2 to 6 μin.) rms.

THEORETICAL ANALYSIS

An elastohydrodynamic analysis for the determination of the torque of a ball spinning in a nonconforming groove, shown in figure 1, was developed in reference 7. In this analysis, the area of contact is elliptical, as shown in figure 2(a). In this ellipse an elastohydrodynamic film can theoretically be predicted, due to the spinning velocity, ex-
(a) Contact ellipse for ball in nonconforming groove.

(b) Elemental roller.

Figure 2. - Ground lower ball showing contact ellipse and elemental roller.
cept within the inscribed circle of radius $b$. The frictional force over the ellipse is determined by making the system into a number of elemental rollers, as shown in figure 2(b), and determining the frictional force on each roller. By using this method the frictional force $dF$ in an elemental roller of width $dy$ sliding at a velocity of $yw$ is

$$dF = \frac{2b'\mu yw}{h} \, dy$$  \hspace{1cm} (1)$$

where the equivalent viscosity $\mu$ as given in reference 9 is

$$\mu = \frac{1}{b'} \int_{0}^{b'} \mu \, dx$$  \hspace{1cm} (2)$$

The film thickness $h$ from reference 10 is

$$h = \frac{1.6 \sigma 0.6 E'0.03 R^{0.43}}{W^{0.13}} \left( \frac{\mu yw}{2} \right)^{0.7}$$  \hspace{1cm} (3)$$

where

$$E' = \frac{E}{1 - \sigma^2}$$

For steel, $E'$ is $22.3 \times 10^{10}$ N/m$^2$ ($32.3 \times 10^6$ psi).

The assumption of a Hertzian stress distribution leads to a load distribution $W$.

$$W = \frac{0.75}{a} P \left[ 1 - \left( \frac{y}{a} \right)^2 \right]$$  \hspace{1cm} (4)$$

where $P$ is the normal load.

The lubricant viscosity on the surface at location $x$ is dependent upon the contact pressure at that point. If a Hertzian distribution is assumed, the contact pressure is given by

$$S = \frac{1.5 \, P}{\pi ab} \left( 1 - \frac{x^2}{b^2} - \frac{y^2}{a^2} \right)^{1/2}$$  \hspace{1cm} (5)$$
The actual viscosity depends upon the pressure-viscosity function.

The friction moment $M_1$ about the Z-axis due to the section of the contact ellipse for $y > +b$ and $y < -b$ is obtained by integrating the product of the friction of the elemental roller and the moment arm; thus

$$M_1 = 2 \int_b^a y \, dF$$

(6)

A numerical solution is used for this equation since a closed-form solution does not exist.

There remains the area within the inscribed circle of radius $b$ where a true elastohydrodynamic film theoretically is impossible to maintain. Therefore, the film thickness $h$ is assumed to be the same as that at $y = b$. If the inscribed circle is then divided into elemental rings of radius $r$ and width $dr$, the torque $M_2$ for the inscribed circle becomes

$$M_2 = \frac{2\pi \omega}{h_2} \int_0^b r^3 \mu_b \, dr$$

(7)

where $\mu_b$ is the equivalent viscosity at $y = b$.

$$M_s = M_1 + M_2$$

(8)

The preceding paragraphs outline the general approach used for the conventional elastohydrodynamic analysis. In using this analysis for the friction torque of a ball spinning in a nonconforming groove, it was found in reference 7 that the commonly used exponential law $\mu = \mu_o e^{\alpha p}$ for the pressure-viscosity relation gave torque values that were much higher than experimental results. However, a good correlation between theory and experiment was achieved by using a composite pressure-viscosity relation

$$\begin{cases} 
\mu = \mu_o e^{\alpha S} & \text{where } S \leq S_1 \\
\mu = \mu_o e^{\alpha S_1 + \beta(S-S_1)} & \text{where } S > S_1 
\end{cases}$$

(9)

Equation (8) in the preceding analysis cannot be used for a ball spinning on a flat plate because of the absence of a converging film; that is, two flat surfaces theoretically
cannot form a hydrodynamic film. However, the lubrication of a ball on a flat plate has been explained by a microasperity EHD theory (ref. 8). In this theory an EHD film is formed between the asperities on the two flat surfaces, as shown in figure 3. For an individual asperity, as shown in figure 4, the film thickness is given by

\[ h = 0.67 (\alpha \mu_o v) ^ {2/3} \rho ^ {1/3} \]  

(10)

Considering all the asperities contained within the contact ellipse the total area of contact is given by

\[ A = \int \frac{1.5 P}{\pi ab} \left( 1 - \frac{x^2}{b^2} - \frac{y^2}{a^2} \right)^{1/2} \frac{dx \ dy}{P_A} \]  

(11)

where \( P_A \) is the average Hertz pressure over a typical asperity. If the fluid is assumed to behave in a Newtonian manner and the velocity gradient is assumed to be linear, the shear stress \( \tau \) in the fluid at the top of the asperity is given by

\[ \tau = \frac{\mu \omega r}{h} \]  

(11)

(a) Two rough surfaces in contact.

(b) Equivalent rough surface on plane.

Figure 3. - Surfaces in contact.
Assuming (1) that the fluid between the asperities does not contribute a significant torque and (2) that the fluid viscosity is constant over each asperity peak, the total spinning torque imposed by the microasperity film over the whole contact ellipse is given by

$$M_s = \omega \int_A \frac{\mu r^2}{h} \, dA$$

(12)

This may be integrated by using numerical methods.

From the preceding discussion it can be seen that the spinning torque for a ball spinning in a nonconforming groove may be determined by either of two methods. The first is conventional EHD analysis using equation (8), which assumes a constant film thickness over the inscribed circle. The second is microasperity EHD analysis using the method developed for a ball spinning on a flat plate in reference 8 and extended to a ball spinning in a nonconforming groove.

In addition to the effect of the lubricant within the Hertzian contact region, the effect of the lubricant outside of this region must also be considered. The expression for this moment is given in reference 5 as

$$M = 4\mu \omega \int_0^{\pi/2} \int_{r_0}^{KR_B} \left( \frac{R_B + h_o - \frac{R_G}{\cos \varphi}}{\cos \varphi} + \left[ \frac{R_G}{\cos \varphi} - r \right]^2 \right)^{1/2} \, r^3 \, d\varphi \, dr$$

(13)

This may be integrated numerically over the region outside the contact ellipse.
In reality, the conventional EHD and microasperity effects are superimposed but, since the coupling between the two systems is extremely complex, the conventional EHD and microasperity EHD effects will be examined individually. The effect of the lubricant lying outside the contact zone will be added to each.

RESULTS AND DISCUSSION

The spinning torque reported in reference 3 for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether was examined by both the conventional and microasperity theories for a ball spinning in a nonconforming groove. For the di-2-ethylhexyl sebacate, good agreement was obtained between the experimental results and each theory by use of the following parameters:

\[
\alpha = 2.03 \times 10^{-8} \text{ m}^2/\text{N} \left( 1.4 \times 10^{-4} \text{ (psi)}^{-1} \right)
\]

\[
\beta = 1.45 \times 10^{-10} \text{ m}^2/\text{N} \left( 1.0 \times 10^{-6} \text{ (psi)}^{-1} \right)
\]

\[
S_1 = 275 \times 10^6 \text{ N/m}^2 \left( 40 \ 000 \text{ psi} \right)
\]

\[
\mu_0 = 16 \times 10^{-3} \text{ N-sec/m}^2 \left( 2.3 \times 10^{-6} \text{ lb-sec/in.}^2 \right)
\]

The pressure-viscosity relation given by the preceding values is portrayed in figure 5. The spinning torques for the conventional and microasperity EHD analysis are shown in figure 6. The value of \( \alpha \) used here is higher than the published values of

\[
\alpha = 0.58 \times 10^{-8} \text{ m}^2/\text{N} \left( 0.4 \times 10^{-4} \text{ (psi)}^{-1} \right) \text{ (ref. 11)}
\]

and

\[
\alpha = 1.45 \times 10^{-8} \text{ m}^2/\text{N} \left( 1.0 \times 10^{-4} \text{ (psi)}^{-1} \right) \text{ (ref. 12)}
\]

The spinning torque for the naphthenic mineral oil was similarly examined by both theories. The parameters which gave the best fit are

\[
\alpha = 4.93 \times 10^{-8} \text{ m}^2/\text{N} \left( 3.4 \times 10^{-4} \text{ (psi)}^{-1} \right)
\]
Figure 5. - Pressure-viscosity model for di-2-ethylhexyl sebacate obtained from spinning torque data taken in spinning torque apparatus. Test conditions: temperature, ambient; spinning speed, 950 rpm; ball-groove conformity, 51 percent; ambient viscosity, $\mu_0 = 1.6 \times 10^{-3}$ N·s/m² (2.3 × 10⁻⁶ lb·sec/in.²); stress at which pressure-viscosity exponent changes, $S_1 = 275 \times 10^6$ N/m² (40 000 psi).

For the conventional EHD analysis:

$$S_1 = 124 \times 10^6 \text{ N/m}^2 \text{ (18 000 psi)}$$

For the microasperity EHD analysis:

$$S_1 = 131 \times 10^6 \text{ N/m}^2 \text{ (19 000 psi)}$$
The preceding values yield the lubricant model shown in figure 7. The spinning torque for the two analyses is shown in figure 8. The value of $\alpha$ used here is considerably higher than the previously published values

$$\alpha = 0.94 \times 10^{-8} \text{ m}^2/\text{N} (0.65 \times 10^{-4} \text{ psi}^{-1}) \text{ (ref. 11)}$$

and

$$\alpha = 2.15 \times 10^{-8} \text{ m}^2/\text{N} (1.48 \times 10^{-4} \text{ psi}^{-1}) \text{ (ref. 13)}$$
Although the same value of $\alpha$ was used for the conventional and microasperity analyses, it was necessary to use different values of $S_1$ for the two analyses to obtain the best agreement.

The spinning torque for the polyphenyl ether (5P4E) was also examined by the use of both analyses. The best agreement was obtained by using the following lubricant parameters:

$$\alpha = 5.07 \times 10^{-8} \frac{m^2}{N} \left(3.5 \times 10^{-4} \text{ psi}^{-1}\right)$$

$$\mu_0 = 0.8 \text{ N-sec/m}^2 \left(1.16 \times 10^{-4} \text{ lb-sec/in.}^2\right)$$
Figure 8. - Comparison of experimental and theoretical spinning torques for a super-refined napthenic mineral oil. Pressure-viscosity coefficients: \(a = 4.93 \times 10^{-8} \text{ m}^2/\text{N} \cdot \text{m} \cdot \text{sec} \cdot \text{m}^{-2} \cdot \text{in}^{-1}; \beta = 0.\) Ambient viscosity, \(\mu_0 = 0.079 \text{ N} \cdot \text{sec} \cdot \text{m}^{-2} \cdot (1.3 \times 10^{-5} \text{ lb-sec/in}^2).\)

For the conventional EHD analysis:

\[
\beta = -1.16 \times 10^{-9} \text{ m}^2/\text{N} \ (-8.0 \times 10^{-6} \text{ (psi)}^{-1})
\]

\[
S_1 = 117 \times 10^6 \text{ N/m}^2 \ (20 \ 000 \text{ psi})
\]

For the microasperity EHD analysis:

\[
\beta = -1.4 \times 10^{-12} \text{ m}^2/\text{N} \ (-1.0 \times 10^{-8} \text{ (psi)}^{-1})
\]

\[
S_1 = 138 \times 10^6 \text{ N/m}^2 \ (20 \ 000 \text{ psi})
\]
In this case, reasonable agreement can only be obtained by using different values of $\beta$ and $S_1$ for the two different lubrication analyses. Also the value of $\beta$ is negative in both cases, indicating an apparent reduction in viscosity at high pressures. The resulting lubricant pressure-viscosity models are portrayed in figure 9. The spinning torques
against maximum Hertz stress for the two lubrication analyses are shown in figure 10.

The value of \( \alpha \) used in this analysis again differs from the previously published values

\[
\alpha = 2 \times 10^{-8} \text{ m}^2/\text{N} \ (1.38 \times 10^{-4} \text{ psi}^{-1}) \ (\text{ref. 11})
\]

and

\[
\alpha = 4.64 \times 10^{-8} \text{ m}^2/\text{N} \ (3.2 \times 10^{-4} \text{ psi}^{-1}) \ (\text{ref. 14})
\]

The explanation probably lies in the extremely large shear rates present in the spinning ball system.
For comparison purposes the spinning torque theory for the microasperity EHD analysis was used with the lubricant model for the conventional EHD analysis to calculate the spinning torque for both the super-refined naphthenic mineral oil and the polyphenyl ether. The results of these comparisons are presented in figure 11. From figure 11(a) it can be seen that the calculated spinning torque for the super-refined naphthenic mineral oil is somewhat less than the experimental spinning torque. In figure 11(b) the calculated spinning torque for the polyphenyl ether is much less than the experimental spinning torque. The basic reason for the lower calculated spinning torques when using this method is the difference in EHD film thickness between the microasperity and conventional EHD analyses. The film thickness was calculated for all three lubricants by the method of equation (3) for the conventional EHD analysis and by equation (10) for the microasperity EHD analysis. The results are plotted in figure 12, from which it is readily seen that the conventional EHD film thickness is roughly 50 times the microasperity EHD film thickness.
Figure 12. Film thickness as function of distance along major axis. Maximum Hertz stress, $850 \times 10^6$ N/m$^2$ (124,000 psi); spinning speed, 950 rpm.
CONCLUDING REMARKS

The results presented herein have demonstrated the possibility of either conventional elastohydrodynamic or microasperity elastohydrodynamic lubrication for the various lubricants tested. The values of pressure-viscosity coefficient which produce the best fit between the experimental data and the analysis differ somewhat from the published values. The published values for the initial pressure-viscosity exponent are by no means consistent. The differences between them are as great or greater than the difference between the present values and published values.

The apparent change in the pressure-viscosity exponent (from $\alpha$ to $\beta$) must occur regardless of which EHD analysis is used. Without a reduction in this exponent the torque values become impossibly high. This reduction has been rarely observed in the past, probably because most of the previous traction data were taken at much lower Hertz stresses and lower shear rates. A somewhat similar change in the exponent is, however, given in reference 15, although the transition pressure in reference 15 is much higher.

The polyphenyl ether exhibits a peculiar characteristic in that a negative secondary slope $\beta$ is required for the pressure-viscosity curve to give reasonable correlation between the theoretical and experimental results. In other rolling-element bearing studies (refs. 16 to 18), polyphenyl ether has proved to be a poor lubricant at high Hertz stresses. This may be due to the apparent reduction in viscosity observed in the current work.

The theoretical work reported herein has been on the basis of either a conventional elastohydrodynamic analysis or a microasperity elastohydrodynamic analysis with no coupling between the two. In a real system there must be an interaction of the conventional and microasperity elastohydrodynamic lubrication effects. The probable result of this would be to increase the film thickness. However, in a real system the side leakage effects would tend to reduce the film thickness. The effect of neglecting coupling between the two systems and side leakage are thus opposite.

SUMMARY OF RESULTS

The microasperity analysis for elastohydrodynamic lubrication of a ball spinning on a flat surface was extended to a ball spinning in a nonconforming groove. The analysis was compared to the conventional elastohydrodynamic analysis of a ball spinning in a nonconforming groove. Using both analyses, a rheological model was constructed for a di-2-ethylhexyl sebacate, a super-refined naphthenic mineral oil, and a polyphenyl ether.
(5P4E) based upon spinning torque data obtained in another investigation. The following results were obtained:

1. The value of the lubricant pressure-viscosity coefficient that made the data fit the analysis of the fluids differs somewhat from published values. For all three lubricants, an exponential composite model, rather than a single exponential model, best described the lubricant rheology.

2. A reasonably good agreement existed between the experimental and analytical values of torque when either the conventional or the microasperity elastohydrodynamic analysis for a ball spinning in a nonconforming groove was used.

3. The polyphenyl ether lubricant exhibited a negative secondary slope in the composite exponential model, which would imply a decrease in viscosity with pressure at pressures beyond $117 \times 10^6$ N/m$^2$ (17 000 psi).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, January 26, 1972,
132-15.
## APPENDIX - SYMBOLS

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>A</td>
<td>total asperity area, $m^2$ (in.$^2$)</td>
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<tr>
<td>a</td>
<td>major semiaxis of contact ellipse, m (in.)</td>
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<tr>
<td>b</td>
<td>minor semiaxis of contact ellipse, m (in.)</td>
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<tr>
<td>b'</td>
<td>semidiameter of contact ellipse at y, m (in.)</td>
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<td>E</td>
<td>modulus of elasticity, N/m$^2$ (psi)</td>
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<td>materials properties factor, N/m$^2$ (psi)</td>
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<td>F</td>
<td>friction force</td>
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<tr>
<td>h</td>
<td>film thickness, m (in.)</td>
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<td>minimum distance between ball and groove, m (in.)</td>
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<td>K</td>
<td>constant defining outer boundary of integration</td>
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<td>torque due to viscous drag, N-m (lb-in.)</td>
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<td>total spinning torque in Hertzian ellipse, N-m (lb-in.)</td>
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<td>spinning torque in Hertzian ellipse outside inscribed circle, N-m (lb-in.)</td>
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<tr>
<td>$M_2$</td>
<td>spinning torque in Hertzian ellipse inside inscribed circle, N-m (lb-in.)</td>
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<td>normal load, N (lb)</td>
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<td>average pressure over asperity, N/m$^2$ (psi)</td>
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<td>radius of groove, m (in.)</td>
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<td>stress at which pressure-viscosity exponent changes, N/m$^2$ (psi)</td>
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<tr>
<td>$\alpha, \beta$</td>
<td>pressure-viscosity exponents</td>
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</table>
\( \sigma \)  Poisson's ratio
\( \tau \)  shear stress, N/m\(^2\) (psi)
\( \mu \)  absolute viscosity, N-sec/m\(^2\) (lb-sec/in.\(^2\))
\( \bar{\mu} \)  equivalent viscosity, N-sec/m\(^2\) (lb-sec/in.\(^2\))
\( \mu_0 \)  ambient viscosity, N-sec/m\(^2\) (lb-sec/in.\(^2\))
\( \mu_b \)  viscosity at \( y = b \), N-sec/m (lb-sec/in.\(^2\))
\( \rho \)  radius of curvature of asperity, m (in.)
\( \omega \)  angular velocity, rad/sec
REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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