ARCHED-OUTER-RACE
BALL-BEARING ANALYSIS
CONSIDERING CENTRIFUGAL FORCES

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A first-order thrust load analysis that considers centrifugal forces but which neglects gyroscopics, elastohydrodynamics, and thermal effects was performed. The analysis was applied to a 150-mm-bore angular-contact ball bearing. Fatigue life, contact loads, and contact angles are shown for conventional and arched bearings. The results indicate that an arched bearing is highly desirable for high-speed applications. In particular, at an applied load of 4448 N (1000 lb) and a DN value of 3 million (20 000 rpm), the arched bearing shows an improvement in life of 306 percent over that of a conventional bearing.
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SUMMARY

A thrust load analysis of an arched-outer-race ball bearing that considers centrifugal forces but that neglects gyroscopics, elastohydrodynamics, and thermal effects was performed. Elliptic integrals were evaluated by using the Landen transformation. A one-point iteration method was used in evaluating the load-deflection constant. A Newton-Raphson method of iteration was used in evaluating the axial displacement and the radial and axial projection of the distance between the ball center and the outer-raceway groove curvature center. Fatigue life evaluations were made. The similar analysis of a conventional bearing can be directly obtained from the arched-bearing analysis by simply letting the amount of arching be zero ($g = 0$) and not considering equations related to the unloaded half of the outer race.

The analysis was applied to a 150-millimeter-bore, angular-contact ball bearing. Results for life, contact loads, and contact angles are shown for a conventional bearing ($g = 0$) and several arched bearings ($g = 0.127$ mm (0.005 in.), 0.254 mm (0.010 in.), . . . , and 0.762 mm (0.030 in.)). The results indicate that an arched bearing is highly desirable for high-speed applications. In particular, for a DN value of 3 million (20 000 rpm) and an applied axial load of 4448 newtons (1000 lb), an arched bearing shows an improvement in life of 306 percent over that of a conventional bearing. At 4.2 million DN (28 000 rpm), the corresponding improvement is 340 percent. It was also found that the arched bearing does not offer the advantages at low speeds that it does at high speeds.

INTRODUCTION

Aircraft gas turbine engine rotor bearings currently operate in the speed range from 1.5 to 2 million DN (bearing bore in mm times shaft speed in rpm). It is esti-
mated that engine designs of the next decade will require bearings to operate at DN values of 3 million or more (ref. 1). In this DN range, analyses (refs. 2 and 3) predict a prohibitive reduction in bearing fatigue life due to the high centrifugal forces developed between the rolling elements and the outer race.

Several approaches to the high-speed-bearing problem have been suggested and are being developed. One approach is to reduce ball mass through the use of thin-wall spherically hollow balls (ref. 4) or drilled balls (refs. 5 to 7). Theory indicates that significant improvements in bearing fatigue life can be obtained at DN values of 3 million and above with a 50 percent or greater weight removal from the balls. Flexure failures have occurred with both hollow and drilled balls after short running times, however, so that both of these concepts must still be considered highly experimental.

Hybrid bearings consisting of a combination ball and fluid film bearing constitute a second approach. The parallel hybrid bearing (ref. 8), in which the fluid film bearing and ball bearing share the system load with both operating at full speed, can be used to improve high-speed-ball-bearing life. However, the effectiveness of the parallel hybrid bearing diminishes at high speeds because it does not attenuate centrifugal effects in the ball bearing. The series hybrid bearing (refs. 9 and 10), in which a fluid film bearing and a ball bearing both carry full system load while each operates at part speed, is theoretically the most effective approach to extending high-speed-ball-bearing life. This concept, too, is still quite experimental. Mechanical complexity is a problem, and effects on shaft stiffness and rotor dynamics must be evaluated.

Initial experiments with an arched-outter-race ball bearing (ref. 11) indicated that this design operated with lower torque than a conventional angular-contact bearing. The experiments of reference 11 were conducted at DN values up to about 1 million. In light of the successful experiments of reference 11, the arched-outter-race ball bearing seemed to be a promising high-speed-bearing concept because of its ability to share the centrifugal loading at two outer-race contacts per ball.

The objective of the work described in this report was to conduct a fatigue life analysis of the arched-outter-race ball bearing and to compare the fatigue life of this bearing with that of a conventional bearing at various combinations of thrust load and speed. A first-order thrust load analysis, in which gyroscopic, elastohydrodynamic and thermal effects are neglected, is reported.

**SYMBOLS**

A distance between raceway groove curvature centers

[math]
\alpha
[/math] right-side-outter-race curvature center

assemimajor axis of projected contact ellipse

2
B \quad f_o + f_1 - 1 = A/D

\( B \) ball center initially

b \quad \text{seminor axis of projected contact ellipse}

c \quad \text{initial position, inner-raceway groove curvature center}

c \quad \text{function of axial displacement defined in eq. (53)}

d \quad \text{ball diameter}

d \quad \text{left-side-outer-race curvature center}

d_m \quad \text{pitch diameter}

E \quad \text{percent improvement of arch bearing (g = 0.005) over that of conventional bearing (g = 0), } \left( \frac{L_{|g=0.005} - L_{|g=0}}{L_{|g=0}} \right) \times 100

\( E \) elliptical integral of second kind

F \quad \text{curvature difference}

F_a \quad \text{axially applied load}

F_c \quad \text{centrifugal force}

F \quad \text{elliptic integral of first kind}

f \quad r/D

G, H \quad \text{functions of V and W defined by eqs. (27) and (28)}

G \quad \text{inner-race contact, initially}

h \quad \text{amount of arching, or width of material removed from outer race of conventional bearing}

H \quad \text{outer-race contact, initially}

h \quad \text{distance from top of arch to top of ball when bearing is in radial contact position}

J \quad \text{inner-race contact, finally}

J \quad \text{function of k defined by eq. (45)}

J \quad \text{right-outer-race contact, finally}

K \quad \text{load-deflection constant}

\( K \) left-outer-race contact, finally

k \quad a/b
life, hr
ball center, finally
defined by eq. (63)
final position, inner-raceway groove curvature center
ball mass
defined by eq. (64)
tip of arch
rotational speed
basic dynamic capacity of raceway contact
bearing diametral clearance
free end play
ball normal load
raceway groove curvature radius
distance between inner- and outer-raceway groove curvature center loci
diametral play
$\tau_0/\sigma_{\text{max}}$
$\left(\tau_0/\sigma_{\text{max}}\right)_{k=1}$
surface velocity
number of stress cycles per revolution
radial projection of distance between ball center and outer-raceway groove curvature center
axial projection of distance between ball center and outer-raceway groove curvature center
defined in eq. (31)
diameter of ball track
defined in eq. (32)
number of balls
depth of maximum shear stress
radial contact angle
axial contact angle
\[\gamma = D \cos \beta / \bar{d}_m\]
\[\Delta \text{ distance between raceway groove curvature center and final position of ball center}\]
\[\delta \text{ contact deformation}\]
\[\delta_a \text{ axial displacement}\]
\[\delta^* \text{ defined by eq. (50)}\]
\[\zeta \text{ ratio of depth of maximum shear stress to semiminor axis, } z_0/b\]
\[\zeta_1 \left(\zeta\right)_{k=1}\]
\[\eta \text{ defined by eq. (9)}\]
\[\lambda \text{ modulus of elasticity}\]
\[\xi \text{ Poisson's ratio}\]
\[\rho \text{ curvature sum}\]
\[\sigma_{\text{max}} \text{ maximum normal stress}\]
\[\tau_0 \text{ maximum orthogonal subsurface shear stress}\]
\[\varphi \text{ auxiliary angle}\]
\[\Omega \text{ defined in eq. (74)}\]
\[\omega_m \text{ orbital speed of ball}\]

Subscripts:

\[B \text{ ball}\]
\[i \text{ inner raceway}\]
\[n \text{ iteration}\]
\[o \text{ outer raceway}\]
\[ol \text{ left outer raceway}\]
\[or \text{ right outer raceway}\]
\[x \text{ x-direction}\]
\[y \text{ y-direction}\]
\[z \text{ z-direction}\]

Superscript:

\[-\text{ final position}\]
Figure 1 shows how the arched outer race is made. A conventional outer race is shown in figure 1(a) with a race radius of $r_0$. Also shown in figure 1(a) is the portion of the conventional outer race that is removed in forming an arched outer race. Figure 1(b) shows the arched outer race with the portion of length $g$ removed. Note that there are now two outer-race radius centers separated by a distance $g$.

Figure 2 shows the arched bearing while in a noncontacting position. Here the pitch diameter $d_m$, diametral clearance $P_d$, diametral play $s_d$, and raceway diameters $d_i$ and $d_o$ are defined. The diametral play is the total amount of radial movement allowed in the bearing. Furthermore, the diametral clearance is the diametral play plus two times the distance from the bottom of the ball to the tip of the arch when the bearing is in a radial contact position.

Figure 3 shows the arched bearing in a radial contact position. Instead of contacting at one point at the bottom of the outer raceway, the ball contacts at two points separated by an angle $2\alpha$. From figure 3 the radial contact angle $\alpha$ can be written as

$$\alpha = \sin^{-1}\left(\frac{g}{2r_0 - D}\right)$$

A distance which needs to be formulated is the distance from the tip of the arch to the bottom of the ball when the ball and raceway are in the radial contact position as shown in figure 3. This distance is defined as $h$. From figure 3(b) and the Pythagorean theorem the following can be written:

$$r_o^2 = \left(r_o - \frac{D}{2}\right)^2 \sin^2 \alpha + \left[h + \frac{D}{2} + \left(r_o - \frac{D}{2}\right) \cos \alpha \right]^2$$

Solving for $h$ results in

$$h = -\frac{D}{2} - \left(r_o - \frac{D}{2}\right) \cos \alpha + \frac{1}{2} \left[D(4r_o - D) + (2r_o - D)^2 \cos^2 \alpha \right]^{1/2}$$

Note that as one might expect as $\alpha \rightarrow 0^\circ$, $h \rightarrow 0$. With $h$ known, a number of conventional bearing parameters can be formulated from figures 2 and 3. The outer-raceway diameter may be written as

$$d_o = d_i + P_d + 2D$$
where

\[ P_d = S_d + 2h \]  

(4)

From equations (3) and (4) the diametral play can be written as

\[ S_d = d_o - d_i - 2D - 2h \]  

(5)

The pitch diameter \( d_m \) from figure 2 can be expressed as

\[ d_m = d_i + \frac{S_d}{2} + D \]  

(6)

Figure 4 shows the arched ball bearing while in the axial contact position. Note that the ball is in the top position. From this figure the distance between the center of curvature of the inner and right outer race can be written as

\[ A = r_o + r_i - D \]

With \( f_o = r_o / D \) and \( f_i = r_i / D \) this equation becomes

\[ A = BD \]  

(7)

where

\[ B = f_o + f_i - 1 \]  

(8)

From figure 4(b) the following equation can be written:

\[ r_o^2 = \left( \frac{a}{2} \right)^2 + (r_o - \eta)^2 \]

Solving for \( \eta \) gives

\[ \eta = r_o - \sqrt{r_o^2 - \left( \frac{a}{2} \right)^2} \]  

(9)
With \( \eta \) known, the contact angle can be expressed as

\[
\cos \beta = \frac{A - \frac{P_d}{2} - \eta}{A}
\]

or

\[
\beta = \cos^{-1} \left( \frac{A - \frac{P_d}{2} - \eta}{A} \right)
\]  

(10)

The end play of an arched bearing is

\[
P_e = 2A \sin \beta - g
\]  

(11)

**ANALYSIS**

**Contact Geometry**

From the experimental work of Haines and Edmonds (ref. 11) it is observed that the arched bearing will initially operate with two-point contact at the lower speeds and then with three-point contact at higher speeds when the centrifugal forces become significant. When centrifugal force acts on the ball, the inner- and outer-raceway contact angles are dissimilar; therefore, the lines of action between raceway groove curvature radius centers become discontinuous, as shown in figure 5. In this figure the right- and left-outer-raceway groove curvature centers \( \mathcal{O}_r \) and \( \mathcal{O}_l \) are fixed in space, and the inner-raceway groove curvature center \( \mathcal{C} \) moves axially relative to these fixed centers.

When the general approach of reference 12 is followed and figure 5 is used, the distance between the fixed right- and left-outer-raceway groove curvature centers \( \mathcal{O}_r \) and \( \mathcal{O}_l \) and the final position of the ball center \( \mathcal{L} \) can be written as

\[
\Delta_{ol} = r_o - \frac{D}{2} + \delta_{ol} = (f_o - 0.5)D + \delta_{ol}
\]  

(12)

\[
\Delta_{or} = (f_o - 0.5)D + \delta_{or}
\]  

(13)

where \( \delta_{ol} \) is the normal contact deformation at the left-outer-raceway center, and \( \delta_{or} \)
is the normal contact deformation at the right-outer-raceway center. Similarly, the distance between the final inner-raceway groove curvature center \( y \) and the final position of the ball center \( z \) is

\[
\Delta_i = (f_i - 0.5)D + \delta_i
\]

(14)

where \( \delta_i \) is the normal contact deformation at the inner-raceway center.

The axial distance between the final position of the inner- and right-outer-raceway groove curvature center is

\[
S_x = A \sin \beta + \delta_a
\]

(15)

where \( \delta_a \) is the axial displacement. The radial distance between the final position of the inner-raceway groove curvature center and the right- or left-outer-raceway groove curvature center is

\[
S_z = A \cos \beta
\]

(16)

From figure 5 and equations (12) to (16) the following can be written:

\[
\cos \beta_{ol} = \frac{V}{(f_o - 0.5)D + \delta_{ol}}
\]

(17)

\[
\sin \beta_{ol} = \frac{g - W}{(f_o - 0.5)D + \delta_{ol}}
\]

(18)

\[
\cos \beta_{or} = \frac{V}{(f_o - 0.5)D + \delta_{or}}
\]

(19)

\[
\sin \beta_{or} = \frac{W}{(f_o - 0.5)D + \delta_{or}}
\]

(20)

\[
\cos \beta_1 = \frac{A \cos \beta - V}{(f_i - 0.5)D + \delta_i}
\]

(21)

\[
\sin \beta_1 = \frac{A \sin \beta + \delta_a - W}{(f_i - 0.5)D + \delta_i}
\]

(22)
Using the Pythagorean theorem and regrouping terms result in

\[ \delta_{ol} = \sqrt{V^2 + (g - W)^2 - D(f_0 - 0.5)} \]  

(23)

\[ \delta_{or} = \sqrt{V^2 + W^2 - D(f_0 - 0.5)} \]  

(24)

\[ \delta_i = \sqrt{(A \cos \beta - V)^2 + (A \sin \beta + \delta_a - W)^2 - D(f_1 - 0.5)} \]  

(25)

The normal loads shown in figure 6 are related to the normal contact deformation in the following way:

\[ Q = K \delta^{1.5} \]  

(26)

With proper subscripting of i, ol, and or this equation could represent the normal loads of the inner ring \( Q_i \), left outer ring \( Q_{ol} \), or right outer ring \( Q_{or} \). From figure 6 the equilibrium equations of the forces in the horizontal and vertical directions are

\[ Q_i \sin \beta_i + Q_{ol} \sin \beta_{ol} - Q_{or} \sin \beta_{or} = 0 \]

\[ Q_i \cos \beta_i - Q_{ol} \cos \beta_{ol} - Q_{or} \cos \beta_{or} + F_c = 0 \]

Substituting equations (15) to (22) and (26) into these equations gives

\[ \frac{K_i \delta_i^{1.5}(S_x - W)}{(f_1 - 0.5)D + \delta_i} - \frac{K_{or} \delta_{or}^{1.5}W}{(f_0 - 0.5)D + \delta_{or}} + \frac{K_{ol} \delta_{ol}^{1.5}(g - W)}{(f_0 - 0.5)D + \delta_{ol}} = 0 = G(V, W) \]  

(27)

\[ \frac{K_i \delta_i^{1.5}(S_z - V)}{(f_1 - 0.5)D + \delta_i} - \frac{K_{or} \delta_{or}^{1.5}V}{(f_0 - 0.5)D + \delta_{or}} - \frac{K_{ol} \delta_{ol}^{1.5}V}{(f_0 - 0.5)D + \delta_{ol}} + F_c = 0 = H(V, W) \]  

(28)

Before equations (27) and (28) are solved for \( V \) and \( W \), the expressions for the load-deflection constants, the centrifugal force, and the axial contact displacement must be developed.
Centrifugal Force

From figure 7 the following two right triangles can be drawn:

Solving for \(x\) and \(y\), we get

\[
x = \left( r_o - \frac{D}{2} \right) \cos \beta
\]

\[
y = \left( r_o - \frac{D + \delta_{or}}{2} \right) \cos \beta_{or} - \left( r_o - \frac{D}{2} \right) \cos \beta
\]

Note that in figure 7 the unbarred values represent initial location, and the barred values represent final location when the centrifugal forces have acted on the ball. From figure 7 the pitch diameter when the centrifugal force acts on the ball is

\[
\overline{d_m} = d_m + 2y
\]

or

\[
\overline{d_m} = d_m + 2 \left( r_o - \frac{D + \delta_{or}}{2} \right) \cos \beta_{or} - 2 \left( r_o - \frac{D}{2} \right) \cos \beta
\]

Also from figure 7 the contact diameters can be written

\[
\overline{Y}_1 = (1 - \gamma_1) \overline{d_m}
\]

\[
\overline{Y}_{o\ell} = (1 + \gamma_{o\ell}) \overline{d_m}
\]

\[
\overline{Y}_{or} = (1 + \gamma_{or}) \overline{d_m}
\]
where

\[
\begin{align*}
\gamma_i &= \frac{D \cos \beta_i}{d_m} \\
\gamma_{ol} &= \frac{D \cos \beta_{ol}}{d_m} \\
\gamma_{or} &= \frac{D \cos \beta_{or}}{d_m}
\end{align*}
\]

(35)

The surface velocities at the contacts and contact diameters are represented vectorially by

From this representation the following equation which neglects spin, can be written:

\[
\frac{U_i - U_{or}}{U_m - U_{or}} = \frac{\bar{V}_{or} - \bar{V}_i}{2} \cdot \frac{\bar{V}_{or} - d_m}{\bar{V}_{or} - \bar{V}_i} \]

or

\[
U_m = U_{or} + (U_i - U_{or}) \left( \frac{\bar{V}_{or} - d_m}{\bar{V}_{or} - \bar{V}_i} \right)
\]

(36)
But

\[ U = \omega \frac{\mathbf{Y}}{2} = \frac{\pi n \mathbf{Y}}{60} \]

Therefore,

\[ \omega_m = \frac{\pi}{30 \overline{d}_m} \left[ n_{or} \overline{Y}_{or} + (n_1 \overline{Y}_1 - n_{or} \overline{Y}_{or}) \left( \frac{\overline{Y}_{or} - \overline{d}_m}{\overline{Y}_{or} - \overline{Y}_i} \right) \right] \quad (37) \]

Finally, the equation for the centrifugal force is

\[ F_c = \frac{1}{2} m \overline{d}_m \omega_m^2 \quad (38) \]

Where \( m \) is the ball mass.

Load-Deflection Constants

The equations for the curvature sum and differences can be obtained from reference 12:

\[ \rho_1 = \frac{1}{D} \left( 4 - \frac{1}{f_i} + \frac{2\gamma_i}{1 - \gamma_i} \right) \quad (39) \]

\[ \rho_{ol} = \frac{1}{D} \left( 4 - \frac{1}{f_o} - \frac{2\gamma_{ol}}{1 + \gamma_{ol}} \right) \quad (40) \]

\[ \rho_{or} = \frac{1}{D} \left( 4 - \frac{1}{f_o} - \frac{2\gamma_{or}}{1 + \gamma_{or}} \right) \quad (41) \]

\[ F_i = \frac{\frac{1}{f_i} + \frac{2\gamma_i}{1 - \gamma_i}}{4 - \frac{1}{f_i} + \frac{2\gamma_i}{1 - \gamma_i}} \quad (42) \]
From reference 12 auxiliary equations (45) to (47), relating the curvature difference and the elliptic integrals of the first and second kind, can be written:

\[ k = \sqrt{\frac{2\mathcal{F} - \mathcal{E}(1 + F)}{\mathcal{E}(1 - F)}} = J(k) \]  

\[ \mathcal{F} = \int_0^{\pi/2} \left[ 1 - \left(1 - \frac{1}{k^2}\right) \sin^2 \varphi \right]^{-1/2} \, d\varphi \]  

\[ \mathcal{E} = \int_0^{\pi/2} \left[ 1 - \left(1 - \frac{1}{k^2}\right) \sin^2 \varphi \right]^{1/2} \, d\varphi \]  

where

\[ k = \frac{a}{b} \]

in which \( a \) and \( b \) are the semimajor and semiminor axes, respectively, of the projected elliptical area of contact. A one-point iteration method will be used in evaluating equation (45), where

\[ k_{n+1} = J(k_n) \]
From reference 12 the following can be written:

\[
\delta = \delta^* \left[ \frac{3Q}{2\rho} \left( \frac{1}{\lambda_B} + \frac{1}{\lambda} \right) \right]^{2/3} \tag{49}
\]

where

\[
\delta^* = \frac{2\mathcal{F}\left( \frac{\pi}{2k^2\varepsilon} \right)}{\pi} \tag{50}
\]

From equations (26) and (49) the following can be written:

\[
K = \frac{(2)^{5/2}(\delta^*)^{-3/2}(\rho)^{-1/2}}{3} \left( \frac{1 - \xi_B^2}{\lambda_B} + \frac{1 - \xi^2}{\lambda} \right) \tag{51}
\]

With proper subscripting of \( i, o_l, \) and \( o_r \) in equations (45) to (51) three sets of corresponding equations can be obtained for the inner race, the outer left race, and the outer right race.

The elliptic integrals (eqs. (46) and (47)) were evaluated by using a method described by Bulirsch (ref. 13). In this reference a short algorithm is described which makes it possible to compute these integrals very rapidly. The method of evaluation is the Landen transformation. This method, coupled with using a one-point iteration method in evaluating \( k_i, k_{ol}, \) and \( k_{or} \) proved to be a fast method as compared to presently used formulations. For example, compared to a Ralston integration scheme the method used was 26 times faster.

**Axial Displacement**

Thrust ball bearings subjected to centric thrust load have the load distributed equally among the balls. Hence,

\[
Q_i = \frac{F_a}{Z \sin \beta_i} \tag{52}
\]
where $Z$ is the number of balls. Substituting equation (26) into equation (52) gives

$$0 = -\frac{F_a}{ZK_i} + \delta_i^{3/2} \sin \beta_i = C(\delta_a)$$  \hspace{1cm} (53)

A Newton-Raphson iteration method will be used to evaluate $\delta_a$ in equation (53). The derivative of $C(\delta_a)$ with respect to $\delta_a$ when equation (22) and (25) are used, gives

$$\frac{\partial C(\delta_a)}{\partial \delta_a} = \frac{\delta_i^{1/2}}{D(f_i - 0.5) + \delta_i} \left\{ \frac{\sin^2 \beta_i}{2} \left[ \delta_i + 3D(f_i - 0.5) \right] \right\}$$  \hspace{1cm} (54)

Therefore, the Newton-Raphson iteration equation is written as

$$\delta_a, n+1 = \delta_a, n - \frac{C(\delta_a, n)}{\frac{\partial C(\delta_a, n)}{\partial \delta_a}}$$  \hspace{1cm} (55)

Therefore, for a given value of $V$ and $W$ and using equations developed thus far, one is able to use equation (55) to find $\delta_a$ while satisfying equation (53).

**Computer Evaluation**

Having derived expressions for the centrifugal force $F_c$, the load deflection constants $K_i$, $K_{oL}$, and $K_{or}$, and the axial displacement $\delta_a$ in equations (38), (51), and (55), we can now return to solving for $V$ and $W$ in equations (27) and (28). The method to be used to solve this system of nonlinear equations is the Newton-Raphson iteration method, which can be found in most numerical analysis books (e.g., ref. 14). When this method is used, the following equations can be written:

$$V_{n+1} = V_n - \frac{1}{\Omega} \left( G \frac{\partial H}{\partial W} - H \frac{\partial G}{\partial W} \right)_{V=V_n, W=W_n}$$  \hspace{1cm} (56)

$$W_{n+1} = W_n - \frac{1}{\Omega} \left( H \frac{\partial G}{\partial V} - G \frac{\partial H}{\partial V} \right)_{V=V_n, W=W_n}$$  \hspace{1cm} (57)
where

$$\Omega = \left( \frac{\partial G}{\partial W} \frac{\partial H}{\partial V} - \frac{\partial G}{\partial V} \frac{\partial H}{\partial W} \right)_{V=V_n, W=W_n}$$

(58)

$$\frac{\partial G}{\partial V} = -N_1(A \cos \beta - V)(A \sin \beta + \delta_a - W) + N_{ol}V(g - W) - N_{or}VW$$

(59)

$$\frac{\partial G}{\partial W} = -N_1(A \sin \beta + \delta_a - W)^2 - N_{ol}(g - W)^2 - N_{or}W^2 - M_1 - M_{ol} - M_{or}$$

(60)

$$\frac{\partial H}{\partial V} = -N_1(A \cos \beta - V)^2 - V^2(N_{ol} + N_{or}) - M_1 - M_{ol} - M_{or}$$

(61)

$$\frac{\partial H}{\partial W} = -N_1(A \cos \beta - V)(A \sin \beta + \delta_a - W) + N_{ol}V(g - W) - N_{or}VW$$

(62)

where

$$M = \frac{K\delta^{3/2}}{\delta + D(f - 0.5)}$$

(63)

$$N = \frac{K\delta^{1/2}[\delta + 3D(f - 0.5)]}{2[\delta + D(f - 0.5)]^3}$$

(64)

Therefore, with equations (56) and (57) we are able to find values of \( V \) and \( W \) which satisfy the system of equations. With \( V \) and \( W \) known and given equations (17) to (26), the contact loads \( Q_1 \), \( Q_{ol} \), and \( Q_{or} \) and angles \( \beta_1 \), \( \beta_{ol} \), and \( \beta_{or} \) can be evaluated.

**Derivation of Fatigue Life**

From the weakest link theory, on which the Weibull equation is based, we get the relation between life of an assembly (the bearing) and its components (the inner and outer rings):
\[ \frac{1}{L} = \frac{12n_i}{1 \times 10^6} \left[ \left( \frac{1}{L_i} \right)^{10/9} + \left( \frac{1}{L_{o1}} \right)^{10/9} + \left( \frac{1}{L_{or}} \right)^{10/9} \right]^{9/10} \]  

(65)

In this equation life \( L \) is expressed in hours. A material improvement factor of 5 has been assumed; however, no adjustment factors for reliability or operating conditions have been added. For point contact

\[ L = \left( \frac{P}{Q} \right)^3 \]  

(66)

Therefore, equation (65) becomes

\[ L = \frac{1 \times 10^6}{12n_i} \left[ \left( \frac{Q_i}{P_i} \right)^{10/3} + \left( \frac{Q_{o1}}{P_{o1}} \right)^{10/3} + \left( \frac{Q_{or}}{P_{or}} \right)^{10/3} \right]^{0.9} \]  

(67)

The contact loads are defined by equation (26). From Lundberg and Palmgren (ref. 15) the following can be written:

\[ P = 84,000 \cdot D^{1.8} \left( \frac{T_1}{T} \right)^{3.1} \left( \frac{\xi}{\xi_1} \right)^{0.4} \left( \frac{\partial \varepsilon}{\pi D \rho} \right)^{2.1} (k)^{0.7} \left( \frac{D}{d} \right)^{0.3} u^{-1/3} \]  

(68)

With proper subscripting of \( i, o1, \) and \( or \) this equation can represent the dynamic loads of the inner ring \( P_i \), the left outer ring \( P_{o1} \), and the right outer ring \( P_{or} \).

Variations of the \( T \) and \( \xi \) functions with curvature over the range from 0.52 to 0.54 are 2.3 and 0.8 percent, respectively. The variation of the product of these functions over the curvature range from 0.52 to 0.54 is less than 2 percent. Therefore, for the range just described, the products of the \( T \) and \( \xi \) functions can be considered a constant in equation (68), or

\[ \left( \frac{T_1}{T} \right)^{3.1} \left( \frac{\xi}{\xi_1} \right)^{0.4} = 0.718 \]  

(69)

The number of stress cycles per revolution for each contact is, to a good approximation,

\[ u_i = \frac{Z}{2} \left( 1 + \gamma_i \right) \]  

(70)
Substituting equations (69) to (72) into equation (68), one can obtain the dynamic capacity at the inner ring and left and right outer rings as

\[
\begin{align*}
    u_{ol} &= \frac{Z}{2} (1 - \gamma_{ol}) \quad \text{(71)} \\
    u_{or} &= \frac{Z}{2} (1 - \gamma_{or}) \quad \text{(72)}
\end{align*}
\]

Therefore, from equations (26), (73) to (75), and (67) the life in hours of the bearing can be obtained. The equations for a conventional bearing can be directly obtained from the arched-bearing analysis by simply letting the amount of arcing be zero \((g = 0)\) and not considering equations related to the left outer race.

**DISCUSSION OF RESULTS**

**Comparison of Arched and Conventional Bearings**

A conventional 150-millimeter ball thrust bearing was used for the computer evaluation. Bearing parameters and results such as life, contact loads, and contact angles for various speeds and axial loads are shown in table I for the conventional bearing. Then calculations were made for the arched bearing (fig. 1(b)). The diametral play \(S_d\) (fig. 2) was set fixed at 0.2499 millimeter (0.0098 in.) for all the results presented. In an arched bearing the free contact angle becomes larger than that of the conventional bearing even though the diametral play is held constant. The greater the amount of...
arching (the larger the \( g \)), the higher the free contact angle. Also some of the other bearing geometry parameters were changed by changing the amount of arching. Tables II to VII show the effect of the amount of arching \( (g = 0.127 \text{ mm (0.005 in.)}, 0.254 \text{ mm (0.010 in.)}, \ldots, 0.762 \text{ mm (0.030 in.)}) \) on life, contact loads, and contact angles for various speeds and axial loads.

The following observations can be made from the results in tables I to VII:

1. For high speeds \( (n_1 \geq 20,000 \text{ rpm}) \) there is a substantial increase in life for an arched bearing compared to a conventional bearing.

2. At high speeds and light loads the longest life is obtained with a \( g \) of 0.127 millimeter (0.005 in.).

3. There is less advantage in using an arched bearing at high loads \( (F_a \geq 13,345 \text{ N (3000 lb)}) \).

4. For low speeds the arched bearing does not offer the advantages that it does for high speeds.

5. At low speeds and a small amount of arching the arched bearing operates at a two-point contact.

6. As the applied load \( F_a \) increases, the speed for initial three-point contact decreases.

7. As the amount of arching is increased, the speed where the arched bearing has initial three-point contact decreases. The arched bearing operates with three-point contact whenever \( Q_{0L} \) is not zero.

8. Even small contact loads \( (Q_{0L} \approx 100 \text{ N (22.48 lb)}) \) at the left outer race help to improve the life significantly over a conventional bearing.

Figure 8 shows the percent improvement in fatigue life for an arched bearing \( (g = 0.127 \text{ mm (0.005 in.)}) \) over that of a conventional bearing for axial loads of 4448, 13 345, and 22 241 newtons (1000, 3000, and 5000 lb). The ordinate \( E \) of figure 8 is defined by the following relation:

\[
E = \left( \frac{L|g=0.005 - L|g=0}{L|g=0} \right) \times 100
\]  

(76)

The figure shows that the improvement over the conventional bearing is significant for high-speed applications. For example, at \( n_1 = 28,000 \text{ rpm} \) and \( F_a = 4448 \text{ newtons (1000 lb)} \), the improvement in life for an arched bearing \( (g = 0.127 \text{ mm (0.005 in.)}) \) is 340 percent. As the applied load \( F_a \) increases, the advantage of the arched bearing becomes less significant.

20
Bearing Size Effects

The use of bearing sizes other than the 150-millimeter-bore diameter for comparing the performance of arched and conventional bearings would yield somewhat different but qualitatively similar results. The relative importance of centrifugal effects in bearings of different sizes can be determined by comparing the ratio of $D^3n^2$ to the dynamic capacity. The factor $D^3n^2$ is proportional to centrifugal force, and dynamic capacity is a measure of the load capacity of the bearing. For extra-light series angular contact ball bearings operating at 3 million DN the following data are obtained:

<table>
<thead>
<tr>
<th>Bore diameter, mm</th>
<th>$D^3n^2$</th>
<th>Dynamic capacity</th>
<th>$D^3n^2$ dynamic capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>$1.46\times10^8$</td>
<td>5 010</td>
<td>$2.91\times10^4$</td>
</tr>
<tr>
<td>100</td>
<td>1.6</td>
<td>14 400</td>
<td>1.11</td>
</tr>
<tr>
<td>150</td>
<td>2.68</td>
<td>31 210</td>
<td>.86</td>
</tr>
<tr>
<td>200</td>
<td>4.38</td>
<td>54 790</td>
<td>.80</td>
</tr>
</tbody>
</table>

It is seen that centrifugal effects are relatively more severe in small bearings when DN is kept constant. Thus, life improvement of the arched bearing will be greater than that shown for bearings smaller than 150-millimeter bore and somewhat less for larger bearings.

SUMMARY OF RESULTS

A first-order thrust load analysis of an arched bearing which considers centrifugal forces but which neglects gyroscopics, elastohydrodynamics, and thermal effects was performed. Elliptic integrals were evaluated by using the Landen transformation. A one-point iteration method was used in evaluating the load-deflection constant. A Newton-Raphson method of iteration was used in evaluating the axial displacement and the radial and axial projection of the distance between the ball center and the outer-raceway groove curvature center. Fatigue life evaluations were made. The similar analysis of a conventional bearing can be obtained directly from the arched-bearing analysis by simply letting the amount of arching be zero ($g = 0$) and not considering equations related to the unloaded half of the outer race.
Computer solutions were obtained for a 150-millimeter-bore ball bearing. The amount of arching investigated was from zero to \( g \) equal to 0.76 millimeter (0.030 in.). The following results were obtained:

1. The arched bearing shows significant improvements in fatigue life over a conventional bearing, especially at high speeds. In particular, at an axial load of 4448 newtons (1000 lb) the life improvement is 306 percent at 3 million DN and 340 percent at 4.2 million DN.

2. There is an optimal value of \( g \) to produce maximum life for a given value of diametral play and thrust load. For the particular bearing investigated life improvement was greatest at a thrust load of 4448 newtons (1000 lb) when \( g \) was 0.127 millimeter (0.005 in.).

3. For low speeds the arched bearing does not offer the advantages that it does for high speeds.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 10, 1972,
114-03.

REFERENCES


TABLE I. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR CONVENTIONAL BEARING (ZERO ARCHING) AT CONTACT ANGLE OF 25° AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

[Inner-raceway groove curvature, 0.54; outer-raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls]

<table>
<thead>
<tr>
<th>Rotational speed of inner raceway, ( n_i ), rpm</th>
<th>4000</th>
<th>8000</th>
<th>12 000</th>
<th>16 000</th>
<th>20 000</th>
<th>24 000</th>
<th>28 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axially applied load, ( F_a ), 4448 N (1000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, ( L ), hr</td>
<td>342 291</td>
<td>84 727</td>
<td>11 478</td>
<td>1955</td>
<td>450</td>
<td>130</td>
<td>45</td>
</tr>
<tr>
<td>Inner-raceway load, ( Q_i ), N</td>
<td>426.3</td>
<td>389.5</td>
<td>376.0</td>
<td>368.1</td>
<td>361.2</td>
<td>354.5</td>
<td>347.9</td>
</tr>
<tr>
<td>Right-outer-raceway load, ( Q_{OR} ), N</td>
<td>570.1</td>
<td>1037</td>
<td>1911</td>
<td>3167</td>
<td>4809</td>
<td>6849</td>
<td>9302</td>
</tr>
<tr>
<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>28.31</td>
<td>31.28</td>
<td>32.53</td>
<td>33.32</td>
<td>34.04</td>
<td>34.77</td>
<td>35.53</td>
</tr>
<tr>
<td>Right-outer-raceway contact angle, ( \beta_{OR} ), deg</td>
<td>20.77</td>
<td>11.25</td>
<td>6.075</td>
<td>3.662</td>
<td>2.411</td>
<td>1.693</td>
<td>1.246</td>
</tr>
</tbody>
</table>

| Axially applied load, \( F_a \), 13 345 N (3000 lb) |      |      |        |        |        |        |        |
| Life, \( L \), hr                              | 13 432 | 6996 | 3163  | 1007  | 302   | 99    | 37     |
| Inner-raceway load, \( Q_i \), N               | 1297  | 1208 | 1138  | 1098  | 1072  | 1050  | 1030   |
| Right-outer-raceway load, \( Q_{OR} \), N      | 1433  | 1799 | 2579  | 3796  | 5427  | 7466  | 9924   |
| Inner-raceway contact angle, \( \beta_i \), deg | 27.88 | 30.14 | 32.20 | 33.53 | 34.48 | 35.29 | 36.08  |
| Right-outer-raceway contact angle, \( \beta_{OR} \), deg | 25.03 | 19.70 | 13.60 | 9.192 | 6.417 | 4.659 | 3.503  |

| Axially applied load, \( F_a \), 22 241 N (5000 lb) |      |      |        |        |        |        |        |
| Life, \( L \), hr                              | 3060  | 1649 | 1016  | 500   | 199   | 76    | 30     |
| Inner-raceway load, \( Q_i \), N               | 2137  | 2030 | 1915  | 1833  | 1778  | 1737  | 1702   |
| Right-outer-raceway load, \( Q_{OR} \), N      | 2271  | 2598 | 3293  | 4440  | 6035  | 8060  | 10 515 |
| Inner-raceway contact angle, \( \beta_i \), deg | 28.23 | 29.87 | 31.86 | 33.47 | 34.65 | 35.60 | 36.44  |
| Right-outer-raceway contact angle, \( \beta_{OR} \), deg | 26.43 | 22.90 | 17.88 | 13.16 | 9.642 | 7.204 | 5.516  |
TABLE II. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH 0.127-MILLIMETER (0.005-IN.) ARCHING AT CONTACT ANGLE OF 25.46° AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls.

<table>
<thead>
<tr>
<th>Rotational speed of inner raceway, ( n_i ), rpm</th>
<th>4000</th>
<th>8000</th>
<th>12 000</th>
<th>16 000</th>
<th>20 000</th>
<th>24 000</th>
<th>28 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axially applied load, ( F_a ), 4448 N (1000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, ( L ), hr</td>
<td>358 866</td>
<td>89 881</td>
<td>26 299</td>
<td>6804</td>
<td>1825</td>
<td>560</td>
<td>198</td>
</tr>
<tr>
<td>Inner-raceway load, ( Q_i ), N</td>
<td>419.5</td>
<td>384.2</td>
<td>379.3</td>
<td>375.0</td>
<td>370.5</td>
<td>366.0</td>
<td>361.4</td>
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<td>0</td>
<td>20.01</td>
<td>503.4</td>
<td>1178</td>
<td>2031</td>
<td>3068</td>
<td>4292</td>
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<tr>
<td>Right-outer-raceway load, ( Q_{OR} ), N</td>
<td>563.4</td>
<td>1015</td>
<td>1404</td>
<td>1969</td>
<td>2727</td>
<td>3682</td>
<td>4837</td>
</tr>
<tr>
<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>31.41</td>
<td>31.76</td>
<td>32.21</td>
<td>32.63</td>
<td>33.07</td>
<td>33.53</td>
<td>34.01</td>
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<td>0</td>
<td>4.645</td>
<td>5.761</td>
<td>6.318</td>
<td>6.655</td>
<td>6.867</td>
<td>6.996</td>
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<tr>
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<td>10.62</td>
<td>11.59</td>
<td>10.37</td>
<td>9.698</td>
<td>9.236</td>
<td>8.892</td>
<td>8.621</td>
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<td>Axially applied load, ( F_a ), 13 345 N (3000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, ( L ), hr</td>
<td>14 036</td>
<td>7288</td>
<td>3411</td>
<td>1752</td>
<td>817</td>
<td>348</td>
<td>146</td>
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<td>1279</td>
<td>1180</td>
<td>1127</td>
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<td>1095</td>
<td>1082</td>
<td>1069</td>
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<td>0</td>
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<td>1701</td>
<td>2802</td>
<td>4087</td>
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<td>1783</td>
<td>2492</td>
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<td>3691</td>
<td>4578</td>
<td>5675</td>
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<td>28.31</td>
<td>30.63</td>
<td>32.57</td>
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<td>33.63</td>
<td>34.10</td>
<td>34.58</td>
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<td>0</td>
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<td>4.339</td>
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<td>5.467</td>
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<td>25.38</td>
<td>19.89</td>
<td>14.15</td>
<td>12.54</td>
<td>11.49</td>
<td>10.71</td>
<td>10.11</td>
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<tr>
<td>Axially applied load, ( F_a ), 22 241 N (5000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, ( L ), hr</td>
<td>3188</td>
<td>1717</td>
<td>1051</td>
<td>622</td>
<td>372</td>
<td>204</td>
<td>103</td>
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<td>Inner-raceway load, ( Q_i ), N</td>
<td>2109</td>
<td>2003</td>
<td>1890</td>
<td>1836</td>
<td>1809</td>
<td>1785</td>
<td>1763</td>
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<tr>
<td>Left-outer-raceway load, ( Q_{OL} ), N</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48.37</td>
<td>1423</td>
<td>2567</td>
<td>3898</td>
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<td>Right-outer-raceway load, ( Q_{OR} ), N</td>
<td>2243</td>
<td>2571</td>
<td>3272</td>
<td>3955</td>
<td>4595</td>
<td>5428</td>
<td>6474</td>
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<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>28.64</td>
<td>30.32</td>
<td>32.34</td>
<td>33.41</td>
<td>33.98</td>
<td>34.49</td>
<td>34.99</td>
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<td>0</td>
<td>0.9138</td>
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<td>3.315</td>
<td>4.060</td>
<td></td>
</tr>
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<td>Right-outer-raceway contact angle, ( \beta_{OR} ), deg</td>
<td>26.79</td>
<td>23.15</td>
<td>18.00</td>
<td>14.93</td>
<td>13.44</td>
<td>12.33</td>
<td>11.46</td>
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</table>
TABLE III. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH 0.254-MILLIMETER (0.010-IN.) ARCHING AT CONTACT ANGLE OF 26.82° AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls.

| Rotational speed of inner raceway, \( n_i \), rpm |
|---------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 4000                            | 8000              | 12 000            | 16 000            | 20 000            | 24 000            | 28 000            |
| Axially applied load, \( F_a \), 4448 N (1000 lb) |
| Life, L, hr                     | 411 173           | 128 403           | 31 701            | 7084              | 1803              | 545               | 192               |
| Inner-raceway load, \( Q_i \), N | 400.5             | 382.2             | 378.4             | 374.2             | 369.8             | 365.2             | 360.6             |
| Left-outter-raceway load, \( Q_{OL} \), N | 0                 | 194.2             | 644.8             | 1279              | 2101              | 3113              | 4320              |
| Right-outter-raceway load, \( Q_{OR} \), N | 545.1            | 847.7             | 1276              | 1886              | 2681              | 3667              | 4848              |
| Inner-raceway contact angle, \( \beta_i \), deg | 31.46             | 31.94             | 32.30             | 32.70             | 33.14             | 33.61             | 34.11             |
| Left-outter-raceway contact angle, \( \beta_{OL} \), deg | 0                 | 15.40             | 15.53             | 15.55             | 15.53             | 15.47             | 15.38             |
| Right-outter-raceway contact angle, \( \beta_{OR} \), deg | 17.71             | 17.41             | 17.07             | 16.81             | 16.57             | 16.36             | 16.15             |
| Axially applied load, \( F_a \), 13 345 N (3000 lb) |
| Life, L, hr                     | 15 944            | 8191              | 4210              | 2011              | 855               | 345               | 142               |
| Inner-raceway load, \( Q_i \), N | 1228              | 1141              | 1115              | 1103              | 1091              | 1078              | 1065              |
| Left-outter-raceway load, \( Q_{OL} \), N | 0                 | 371.4             | 1020              | 1861              | 2896              | 4128              |
| Right-outter-raceway load, \( Q_{OR} \), N | 1363            | 1738              | 2212              | 2800              | 3571              | 4533              | 5691              |
| Inner-raceway contact angle, \( \beta_i \), deg | 29.60             | 32.10             | 32.96             | 33.36             | 33.78             | 34.23             | 34.71             |
| Left-outter-raceway contact angle, \( \beta_{OL} \), deg | 0                 | 14.12             | 14.38             | 14.52             | 14.60             | 14.62             |
| Right-outter-raceway contact angle, \( \beta_{OR} \), deg | 26.42             | 20.42             | 18.37             | 17.88             | 17.49             | 17.14             | 16.84             |
| Axially applied load, \( F_a \), 22 241 N (5000 lb) |
| Life, L, hr                     | 3591              | 1931              | 1180              | 707               | 396               | 204               | 100               |
| Inner-raceway load, \( Q_i \), N | 2031              | 1926              | 1837              | 1816              | 1796              | 1776              | 1755              |
| Left-outter-raceway load, \( Q_{OL} \), N | 0                 | 0                 | 132.2             | 778.2             | 1631              | 2682              | 3934              |
| Right-outter-raceway load, \( Q_{OR} \), N | 2164            | 2496              | 3106              | 3683              | 4434              | 5373              | 6509              |
| Inner-raceway contact angle, \( \beta_i \), deg | 29.86             | 31.67             | 33.40             | 33.84             | 34.26             | 34.70             | 35.17             |
| Left-outter-raceway contact angle, \( \beta_{OL} \), deg | 0                 | 12.84             | 13.31             | 13.60             | 13.78             | 13.90             |
| Right-outter-raceway contact angle, \( \beta_{OR} \), deg | 27.85             | 23.89             | 19.56             | 18.85             | 18.33             | 17.88             | 17.49             |
TABLE IV. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH 0.381-MILLIMETER (0.015-IN.) ARCHING AT CONTACT ANGLE OF 29.06° AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

[Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls]

<table>
<thead>
<tr>
<th>Rotational speed of inner raceway, n_i, rpm</th>
<th>4000</th>
<th>8000</th>
<th>12 000</th>
<th>16 000</th>
<th>20 000</th>
<th>24 000</th>
<th>28 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axially applied load, F_a, 4448 N (1000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, L, hr</td>
<td>479 904</td>
<td>149 602</td>
<td>33 736</td>
<td>7027</td>
<td>1739</td>
<td>521</td>
<td>183</td>
</tr>
<tr>
<td>Inner-raceway load, Q_i, N</td>
<td>384.5</td>
<td>381.4</td>
<td>377.5</td>
<td>373.2</td>
<td>368.6</td>
<td>363.9</td>
<td>359.0</td>
</tr>
<tr>
<td>Left-outer-raceway load, Q_oL_i, N</td>
<td>37.71</td>
<td>301.3</td>
<td>745.3</td>
<td>1373</td>
<td>2189</td>
<td>3197</td>
<td>4404</td>
</tr>
<tr>
<td>Right-outer-raceway load, Q_or_i, N</td>
<td>501.5</td>
<td>761.1</td>
<td>1201</td>
<td>1822</td>
<td>2631</td>
<td>3632</td>
<td>4830</td>
</tr>
<tr>
<td>Inner-raceway contact angle, β_i, deg</td>
<td>31.72</td>
<td>32.02</td>
<td>32.38</td>
<td>32.80</td>
<td>33.26</td>
<td>33.75</td>
<td>34.28</td>
</tr>
<tr>
<td>Left-outer-raceway contact angle, β_oL_i, deg</td>
<td>24.60</td>
<td>24.62</td>
<td>24.54</td>
<td>24.41</td>
<td>24.25</td>
<td>24.06</td>
<td>23.85</td>
</tr>
<tr>
<td>Right-outer-raceway contact angle, β_or_i, deg</td>
<td>25.77</td>
<td>25.49</td>
<td>25.24</td>
<td>24.99</td>
<td>24.75</td>
<td>24.50</td>
<td>24.24</td>
</tr>
</tbody>
</table>

| Axially applied load, F_a, 13 345 N (3000 lb) |      |      |        |        |        |        |        |
| Life, L, hr                               | 19 476 | 9317 | 4829 | 2218 | 888 | 343 | 138 |
| Inner-raceway load, Q_i, N                | 1152 | 1121 | 1110 | 1099 | 1087 | 1074 | 1060 |
| Left-outer-raceway load, Q_oL_i, N        | 0 | 198.1 | 642.1 | 1273 | 2094 | 3109 | 4324 |
| Right-outer-raceway load, Q_or_i, N       | 1287 | 1561 | 1993 | 2608 | 3409 | 4403 | 5594 |
| Inner-raceway contact angle, β_i, deg     | 31.77 | 32.78 | 33.12 | 33.50 | 33.93 | 34.40 | 34.90 |
| Left-outer-raceway contact angle, β_oL_i, deg | 0 | 23.81 | 23.84 | 23.80 | 23.71 | 23.58 | 23.42 |
| Right-outer-raceway contact angle, β_or_i, deg | 28.13 | 26.09 | 25.75 | 25.44 | 25.14 | 24.84 | 24.55 |

| Axially applied load, F_a, 22 241 N (5000 lb) |      |      |        |        |        |        |        |
| Life, L, hr                               | 4335 | 2254 | 1342 | 796 | 431 | 212 | 100 |
| Inner-raceway load, Q_i, N                | 1913 | 1840 | 1824 | 1806 | 1787 | 1767 | 1746 |
| Left-outer-raceway load, Q_oL_i, N        | 0 | 96.79 | 536.2 | 1167 | 1991 | 3011 | 4232 |
| Right-outer-raceway load, Q_or_i, N       | 2045 | 2339 | 2768 | 3376 | 4170 | 5155 | 6340 |
| Inner-raceway contact angle, β_i, deg     | 31.90 | 33.32 | 33.67 | 34.04 | 34.45 | 34.90 | 35.38 |
| Left-outer-raceway contact angle, β_oL_i, deg | 0 | 23.12 | 23.23 | 23.25 | 23.21 | 23.13 | 23.01 |
| Right-outer-raceway contact angle, β_or_i, deg | 29.63 | 26.63 | 26.21 | 25.84 | 25.50 | 25.18 | 24.85 |
TABLE V. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH 0.508-MILLIMETER (0.020-IN.) ARCHING AT CONTACT ANGLE OF 32.16° AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls

<table>
<thead>
<tr>
<th>Rotational speed of inner raceway, ( n_i ), rpm</th>
<th>4000</th>
<th>8000</th>
<th>12 000</th>
<th>16 000</th>
<th>20 000</th>
<th>24 000</th>
<th>28 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axially applied load, ( F_a ), 4448 N (1000 lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life, L, hr</td>
<td>501.596</td>
<td>153.552</td>
<td>32.663</td>
<td>6613</td>
<td>1623</td>
<td>485</td>
<td>170</td>
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<td>Inner-raceway load, ( Q_i ), N</td>
<td>383.5</td>
<td>380.3</td>
<td>376.2</td>
<td>371.6</td>
<td>366.6</td>
<td>361.5</td>
<td>356.3</td>
</tr>
<tr>
<td>Left-outer-raceway load, ( Q_{Ol} ), N</td>
<td>110.1</td>
<td>372.6</td>
<td>815.4</td>
<td>1443</td>
<td>2260</td>
<td>3272</td>
<td>4486</td>
</tr>
<tr>
<td>Right-outer-raceway load, ( Q_{Or} ), N</td>
<td>462.8</td>
<td>725.2</td>
<td>1167</td>
<td>1793</td>
<td>2609</td>
<td>3620</td>
<td>4831</td>
</tr>
<tr>
<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>31.81</td>
<td>32.11</td>
<td>32.50</td>
<td>32.96</td>
<td>33.46</td>
<td>34.00</td>
<td>34.57</td>
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<tr>
<td>Left-outer-raceway contact angle, ( \beta_{Ol} ), deg</td>
<td>34.26</td>
<td>34.14</td>
<td>33.94</td>
<td>33.70</td>
<td>33.42</td>
<td>33.11</td>
<td>32.79</td>
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<tr>
<td>Right-outer-raceway contact angle, ( \beta_{Or} ), deg</td>
<td>34.83</td>
<td>34.59</td>
<td>34.31</td>
<td>34.01</td>
<td>33.69</td>
<td>33.35</td>
<td>33.00</td>
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<table>
<thead>
<tr>
<th>Axially applied load, ( F_a ), 13 345 N (3000 lb)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Life, L, hr</td>
<td>21.476</td>
<td>9806</td>
<td>5032</td>
<td>2233</td>
<td>860</td>
<td>324</td>
<td>129</td>
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<tr>
<td>Inner-raceway load, ( Q_i ), N</td>
<td>1123</td>
<td>1115</td>
<td>1105</td>
<td>1093</td>
<td>1080</td>
<td>1066</td>
<td>1052</td>
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<tr>
<td>Left-outer-raceway load, ( Q_{Ol} ), N</td>
<td>138.3</td>
<td>398.5</td>
<td>839.4</td>
<td>1466</td>
<td>2283</td>
<td>3296</td>
<td>4511</td>
</tr>
<tr>
<td>Right-outer-raceway load, ( Q_{Or} ), N</td>
<td>1191</td>
<td>1450</td>
<td>1889</td>
<td>2512</td>
<td>3326</td>
<td>4334</td>
<td>5544</td>
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<tr>
<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>32.70</td>
<td>32.95</td>
<td>33.29</td>
<td>33.70</td>
<td>34.16</td>
<td>34.67</td>
<td>35.20</td>
</tr>
<tr>
<td>Left-outer-raceway contact angle, ( \beta_{Ol} ), deg</td>
<td>33.67</td>
<td>33.61</td>
<td>33.47</td>
<td>33.28</td>
<td>33.05</td>
<td>32.78</td>
<td>32.49</td>
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<tr>
<td>Right-outer-raceway contact angle, ( \beta_{Or} ), deg</td>
<td>35.00</td>
<td>34.76</td>
<td>34.47</td>
<td>34.15</td>
<td>33.82</td>
<td>33.48</td>
<td>33.12</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Axially applied load, ( F_a ), 22 241 N (5000 lb)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life, L, hr</td>
<td>4966</td>
<td>2381</td>
<td>1415</td>
<td>827</td>
<td>434</td>
<td>207</td>
<td>95</td>
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<tr>
<td>Inner-raceway load, ( Q_i ), N</td>
<td>1839</td>
<td>1828</td>
<td>1813</td>
<td>1795</td>
<td>1775</td>
<td>1754</td>
<td>1732</td>
</tr>
<tr>
<td>Left-outer-raceway load, ( Q_{Ol} ), N</td>
<td>153.1</td>
<td>410.9</td>
<td>849.6</td>
<td>1474</td>
<td>2290</td>
<td>3303</td>
<td>4520</td>
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<tr>
<td>Right-outer-raceway load, ( Q_{Or} ), N</td>
<td>1901</td>
<td>2159</td>
<td>2595</td>
<td>3216</td>
<td>4025</td>
<td>5031</td>
<td>6239</td>
</tr>
<tr>
<td>Inner-raceway contact angle, ( \beta_i ), deg</td>
<td>33.35</td>
<td>33.58</td>
<td>33.90</td>
<td>34.28</td>
<td>34.71</td>
<td>35.19</td>
<td>35.70</td>
</tr>
<tr>
<td>Left-outer-raceway contact angle, ( \beta_{Ol} ), deg</td>
<td>33.21</td>
<td>33.17</td>
<td>33.07</td>
<td>32.91</td>
<td>32.71</td>
<td>32.47</td>
<td>32.20</td>
</tr>
<tr>
<td>Right-outer-raceway contact angle, ( \beta_{Or} ), deg</td>
<td>35.15</td>
<td>34.91</td>
<td>34.61</td>
<td>34.29</td>
<td>33.95</td>
<td>33.60</td>
<td>33.23</td>
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</tbody>
</table>
TABLE VI - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH
0.635-MILLIMETER (0.025-IN.) ARCHING AT CONTACT ANGLE OF 36.22°
AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.875 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls.

| Rotational speed of inner raceway, \( n_i \), rpm |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 4000            | 8000            | 12 000          | 16 000          | 20 000          | 24 000          | 28 000          |
| Axially applied load, \( F_a \), 4448 N (1000 lb) |
| Life, \( L \), hr | 498 260         | 144 853         | 29 572          | 5971            | 1472            | 441             | 154             |
| Inner-raceway load, \( Q_i \), N | 381.9           | 378.5           | 373.9           | 368.8           | 363.2           | 357.6           | 351.9           |
| Left-outerraceway load, \( Q_{ol} \), N | 175.4           | 435.0           | 874.3           | 1499            | 2317            | 3333            | 4556            |
| Right-outerraceway load, \( Q_{or} \), N | 459.5           | 719.4           | 1159            | 1785            | 2603            | 3620            | 4842            |
| Inner-raceway contact angle, \( \beta_i \), deg | 31.96           | 32.28           | 32.73           | 33.25           | 33.82           | 34.43           | 35.08           |
| Left-outerraceway contact angle, \( \beta_{ol} \), deg | 44.94           | 44.72           | 44.41           | 44.03           | 43.61           | 43.16           | 42.68           |
| Right-outerraceway contact angle, \( \beta_{or} \), deg | 45.24           | 44.97           | 44.61           | 44.21           | 43.77           | 43.30           | 42.80           |
| Axially applied load, \( F_a \), 13 345 N (3000 lb) |
| Life, \( L \), hr | 21 776          | 9812            | 4880            | 2081            | 782             | 292             | 116             |
| Inner-raceway load, \( Q_i \), N | 1115            | 1107            | 1096            | 1083            | 1069            | 1054            | 1038            |
| Left-outerraceway load, \( Q_{ol} \), N | 322.5           | 579.7           | 1016            | 1637            | 2450            | 3464            | 4685            |
| Right-outerraceway load, \( Q_{or} \), N | 1175            | 1433            | 1870            | 2493            | 3308            | 4323            | 5545            |
| Inner-raceway contact angle, \( \beta_i \), deg | 32.97           | 33.23           | 33.60           | 34.05           | 34.57           | 35.14           | 35.75           |
| Left-outerraceway contact angle, \( \beta_{ol} \), deg | 44.35           | 44.23           | 43.98           | 43.65           | 43.27           | 42.86           | 42.40           |
| Right-outerraceway contact angle, \( \beta_{or} \), deg | 45.08           | 44.85           | 44.52           | 44.14           | 43.71           | 43.26           | 42.77           |
| Axially applied load, \( F_a \), 22 241 N (5000 lb) |
| Life, \( L \), hr | 5083            | 2420            | 1415            | 803             | 408             | 189             | 86              |
| Inner-raceway load, \( Q_i \), N | 1821            | 1811            | 1796            | 1777            | 1755            | 1733            | 1709            |
| Left-outerraceway load, \( Q_{ol} \), N | 445.2           | 700.6           | 1134            | 1752            | 2562            | 3573            | 4792            |
| Right-outerraceway load, \( Q_{or} \), N | 1868            | 2124            | 2560            | 3180            | 3992            | 5006            | 6225            |
| Inner-raceway contact angle, \( \beta_i \), deg | 33.72           | 33.94           | 34.27           | 34.68           | 35.16           | 35.70           | 36.28           |
| Left-outerraceway contact angle, \( \beta_{ol} \), deg | 43.97           | 43.83           | 43.61           | 43.32           | 42.97           | 42.58           | 42.15           |
| Right-outerraceway contact angle, \( \beta_{or} \), deg | 44.97           | 44.76           | 44.45           | 44.09           | 43.67           | 43.23           | 42.75           |

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TABLE VII. - LIFE, CONTACT LOADS, AND CONTACT ANGLES FOR BEARING WITH
0.762-MILLIMETER (0.030-IN.) ARCHING AT CONTACT ANGLE OF 41.87°
AND VARIOUS SPEEDS AND AXIAL APPLIED FORCES

| Inner-raceway groove curvature, 0.54; outer raceway groove curvature, 0.52; pitch diameter, 187.55 mm (7.3838 in.); ball diameter, 22.23 mm (0.8750 in.); diametral play, 0.2499 mm (0.0098 in.); 22 balls. |
|---|---|---|---|---|---|---|---|---|
| Rotational speed of inner raceway, \( n_i \), rpm |
| 4000 | 8000 | 12 000 | 16 000 | 20 000 | 24 000 | 28 000 |
| Axially applied load, \( F_a \), 4448 N (1000 lb) |
| Life, \( L \), hr | 451 172 | 120 028 | 24 682 | 5205 | 1315 | 397 | 139 |
| Inner-raceway load, \( Q_i \), N | 378.2 | 374.2 | 366.8 | 362.6 | 355.9 | 349.2 | 342.5 |
| Left-outer-raceway load, \( Q_{OL} \), N | 267.5 | 514.0 | 935.2 | 1542 | 2345 | 3355 | 4584 |
| Right-outer-raceway load, \( Q_{OR} \), N | 505.3 | 752.5 | 1175 | 1782 | 2586 | 3598 | 4828 |
| Inner-raceway contact angle, \( \beta_i \), deg | 32.31 | 32.70 | 33.25 | 33.89 | 34.61 | 35.38 | 36.19 |
| Left-outer-raceway contact angle, \( \beta_{OL} \), deg | 58.01 | 57.66 | 57.17 | 56.59 | 55.93 | 55.23 | 54.48 |
| Right-outer-raceway contact angle, \( \beta_{OR} \), deg | 58.15 | 57.79 | 57.28 | 56.68 | 56.02 | 55.30 | 54.55 |
| Axially applied load, \( F_a \), 13 345 N (3000 lb) |
| Life, \( L \), hr | 20 985 | 9073 | 4257 | 1754 | 660 | 249 | 100 |
| Inner-raceway load, \( Q_i \), N | 1096 | 1088 | 1076 | 1061 | 1045 | 1028 | 1010 |
| Left-outer-raceway load, \( Q_{OL} \), N | 575.7 | 818.8 | 1234 | 1832 | 2626 | 3627 | 4848 |
| Right-outer-raceway load, \( Q_{OR} \), N | 1292 | 1537 | 1954 | 2555 | 3352 | 4357 | 5582 |
| Inner-raceway contact angle, \( \beta_i \), deg | 33.60 | 33.88 | 34.32 | 34.86 | 35.49 | 36.18 | 36.93 |
| Left-outer-raceway contact angle, \( \beta_{OL} \), deg | 57.22 | 56.96 | 56.57 | 56.06 | 55.48 | 54.83 | 54.13 |
| Right-outer-raceway contact angle, \( \beta_{OR} \), deg | 57.55 | 57.27 | 56.85 | 56.32 | 55.72 | 55.05 | 54.34 |
| Axially applied load, \( F_a \), 22 241 N (5000 lb) |
| Life, \( L \), hr | 5023 | 2335 | 1310 | 710 | 349 | 160 | 73 |
| Inner-raceway load, \( Q_i \), N | 1784 | 1774 | 1757 | 1736 | 1713 | 1687 | 1660 |
| Left-outer-raceway load, \( Q_{OL} \), N | 836.7 | 1078 | 1490 | 2083 | 2871 | 3864 | 5078 |
| Right-outer-raceway load, \( Q_{OR} \), N | 2035 | 2279 | 2694 | 3292 | 4084 | 5083 | 6303 |
| Inner-raceway contact angle, \( \beta_i \), deg | 34.51 | 34.75 | 35.13 | 35.61 | 36.18 | 36.82 | 37.52 |
| Left-outer-raceway contact angle, \( \beta_{OL} \), deg | 56.64 | 56.42 | 56.07 | 55.62 | 55.08 | 54.48 | 53.82 |
| Right-outer-raceway contact angle, \( \beta_{OR} \), deg | 57.13 | 56.89 | 56.51 | 56.03 | 55.47 | 54.84 | 54.16 |
Figure 1. - Bearing outer race geometries.

Figure 2. - Arched ball bearing in noncontacting position.

Figure 3. - Arched ball bearing radially loaded.
Figure 4. - Arched ball bearing axially loaded.

(a) Details of contact. (b) Axial contact position of top ball.

Figure 6. - Normal ball loading. Ball in top position; bearing axially loaded.

Figure 5. - Position of ball center and raceway groove curvature centers with and without centrifugal force acting on the ball. Points shown for ball in top position, with bearing loaded axially.

Figure 7. - Position of ball and raceway groove curvature centers and contacts with and without centrifugal force acting on ball.
Axially applied load, $F_N$ (lb)

- 4448 (1000)
- 13 485 (3000)
- 22 241 (5000)

Figure 8. Effect of speed on percent improvement of arched bearing (arching, 0.127 mm; 0.005 in.) compared to that of conventional bearing (zero arching) for various axially applied loads.
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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