STUDY OF SOLID ROCKET MOTOR FOR A SPACE SHUTTLE BOOSTER

FINAL REPORT

APPENDIX A
SRM WATER ENTRY LOADS

Report 1917-FR1
15 March 1972
Report 1917-FR1

STUDY OF SOLID ROCKET MOTOR
FOR A
SPACE SHUTTLE BOOSTER

FINAL REPORT

APPENDIX A
SRM WATER ENTRY LOADS

Contract NAS8-28428
Data Procurement Document 314
Data Requirement MA-02

15 March 1972

Prepared for:
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

aerojet solid propulsion company
**Objective:**

Investigate water entry loads and resultant effects on flight hardware, as outlined below.

**Loads and Input Data:**

1. Inert Stage Weight: 117,000 lb
2. Impact Velocity, Vertical: 20 ft/sec
3. Impact Velocity, Horizontal: 30 ft/sec (20 mph)
4. Orientation at Impact:
   - Case I - Vertical, Nose Down
   - Case II - 45° to Vertical, Nose Down

**Specific Required Results:**

1. Loads on SRM Structure due to impact
2. Loads on SRM Structure due to bounce-back (no wind)
3. Loads on SRM Structure induced by side velocity combined with vertical velocity
4. Loads due to water penetration at 20 ft/sec

**Data Sources:**

1. Engineering Memorandum, EM: L5-02-M1-4, Lockheed Missiles and Space Co., Date: 1/31/72
2. Engineering Mechanics, By: Brooke & Wilson, 1979
3. Raw Structural Analysis of SRM note for water entry loads, By: Hans Kupisch, Date: 4/20/72

**Summary of Results and Discussion:**

Refer to pages 13 and 14.
SACRAMENTO, CALIFORNIA

SUBJECT
SRM WATER-ENTRY LOADS

BY
Wieb

DISCUSSION: Case I - Vertical Motion

A FIRST APPROXIMATION OF PENE TRATION CAN BE DETERMINED BY SEPARATION OF THE PROBLEM INTO TWO PARTS. FOR RELATIVELY SLOW ENTRY VELOCITY, THE PENE TRATION IS CONSERVATIVELY EQUAL TO THE EQUILIBRIUM PENE TRATION PLUS THE DISTANCE REQUIRED TO DECELERATE THE SRM FROM ITS ENTRY CONDITIONS.

EVENT SEQUENCE:

1) CONTACT WATER SURFACE AT 20 ft/sec
2) DECELERATE FROM IMPACT TO 0.4 to 0.5g, NOT CONSIDERING DRAG.
3) DECELERATE FROM BUOYANT FORCE, WHICH INCREASES WITH PENE TRATION

ANALYTICAL SEQUENCE:

1) Plot water impact maximum g's versus speed, in order to determine load at 20 ft/second contact velocity
2) Plot buoyant force versus depth of penetration
**Analysis:** Case I - Vertical Motion

**Determine Buoyant Force:**

\[
F_{buoyant} = F = \left( \frac{\text{Volume Displaced}}{\text{Density}} \right)
\]

\[
= \frac{\pi}{4} \left( \frac{175}{2} \right)^2 \left( \frac{62.5}{1988} \right)
\]

\[
= 19.12 \times 0.0341 = 660 \text{ lb/inw}
\]

**Determine Deceleration after Impact:**

Applying Equation (7), p. 114, Data Source 2:

\[
v_0^2 = u_0^2 + 2a \cdot x
\]

\[
v_f = \text{Final Velocity} = 0
\]

\[
u_0 = v_{\text{(entry)}} = 20 \text{ ft/sec}
\]

\[
a = 0.15g = 164 \text{ ft/sec}^2
\]

\[
x = \frac{400}{31.2} = 12.41 \text{ feet} = 149 \text{ inches}
\]

**Determine Equilibrium Penetration:**

\[
\text{From Table 1: Penetration} = 170 \text{ inches}
\]

**Total Penetration:**

\[
(149 + 170) = 319 \text{ inches}
\]
SUBJECT
SRM WATER ENTRY LOADS

ANALYSIS: Case I - Vertical Motion

[Graph showing penetration in inches against load in pounds]

BY Wieg

CHECKED BY

DATE

Report 1917-FR1, Appendix A
Case I - Vertical Motion

Analysis

Force at Impact, \( F \)

Impact Speed, \( v \)

0.5g

100 150 200 50 100 150

16

14

12

10

8

6

4

2

0.5g
ANALYSIS:  Case I - Horizontal Motion

EVENT SEQUENCE:
1. Contact water surface (0.73 ft/second)
2. Dome decelerates, imparting initial angular acceleration (W)
3. SRM continues to rotate about fuselage.
4. SRM contacts water surface in horizontal attitude.
5. SRM decelerates by displacement.

ANALYTICAL SEQUENCE:
1. Determine lateral deceleration characteristics (a function of penetration depth and velocity).
2. Determine initial angular velocity of SRM (resulting from deceleration of immersed portion).
3. Determine angular velocity of SRM at water impact.
4. Determine angular deceleration at water impact.
5. Determine deceleration pressure on SRM case.
6. Evaluate resultant effects on SRM case.
**SRM Water Entry Loads**

**Analysis:**

**Case I - Horizontal Motion**

Determine initial angular velocity \( \omega_0 \) resulting from 0.85g deceleration at full end:

- **Penetration:** Full
- **Initial Velocity:** 30 ft/sec
- **Final Velocity:** 0 ft/sec (of immersed end)
- **Deceleration Force:** 0.85g, 0 ft/sec (from pages)

*Note: The 0.85g is based on the cross-sectional area of the cylinder. The penetration due to vertical motion stops when the cylinder has penetrated a depth of one diameter, hence the deceleration force is about 2.5g or 1.75 at 0.9s and goes to zero when the tangential velocity (due to angular rotation of the SRM) equals or equals the forward velocity of the SRM.*

Determine time/velocity characteristics of SRM rotation:

\[
I_{\text{endo}} = \frac{ML^2}{3} = \frac{W L^2}{3} = \frac{117,000 (1200)^2}{3 (386)} (17.5\text{ in}^3) \]

Apply equation (3), 2.35b, Data Source 2:

\[
N = \frac{I}{\omega} \quad \omega = \frac{N}{I} \]

\[
\omega = \frac{170 (117,000)(500)}{117,000 (1200)^2} = \frac{170 (386)(386)}{(1200)^2} = \frac{915,000}{1,044,900} \approx 0.889 \text{ rad/sec} \]

**Forward End Velocity:**

\[
\begin{align*}
\omega &= \frac{660 (\omega_f^2)}{t^2} = 660 \left( \frac{4\pi^2}{t^2} \right) = \frac{660 (1.64)}{t^2} = 22.7 t^2
\end{align*}
\]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \omega_f )</th>
<th>( t_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01</td>
<td>2 ft/sec</td>
</tr>
<tr>
<td>2</td>
<td>04</td>
<td>9 ft/sec</td>
</tr>
<tr>
<td>3</td>
<td>09</td>
<td>16 ft/sec</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>34 ft/sec</td>
</tr>
</tbody>
</table>

**Note:** The above calculation, based on the diameter, penetration indicates an immersed end deceleration time of 0.96 seconds. However, the SRM cannot penetrate the water to a depth of this diameter in that short a time, therefore, another approximation based on a finite penetration is in order.
SRM Water Entry Loads

Analysis: Case I - Horizontal Motion

Second Approximation of \( c_{d2} \) (see page 7):

\[
N = 1.0\%
\]

\[
\alpha = \frac{N}{H} = \frac{0.85(179,000)(18)}{119,000 (3200)} = 0.39\ \text{deg/sec}^2
\]

Forward End Velocity:

\[
\omega = \frac{\Delta t}{L} = \frac{660}{0.51} = 1310\ \text{deg/sec}
\]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \Delta t )</th>
<th>( 1310\ \text{deg/sec} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>1.3% dec</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>7% dec</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>12% dec</td>
</tr>
<tr>
<td>4</td>
<td>0.16</td>
<td>21% dec</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>37% dec</td>
</tr>
</tbody>
</table>

Note: The tabulation indicates that the forward end achieves its initial rotation in a short time interval (less than \( 10^{-4} \) second) and then topples due to gravity attraction. This means that angular acceleration \( \alpha \) is very high for such a short time that this load condition can be reduced to a towing condition with an initial angular velocity \( \omega_0 \).
SRM WATER ENTRY LOADING

ANALYSIS: Case I - Horizontal Motion

Kinematics of Toppling Condition with initial tip speed (in Radians) of 30 ft/sec:

Determine initial angular velocity ($\omega_0$):

- One radian in feet = $R_0 \times \frac{2\pi}{150}$
- $\omega_0 = \frac{\text{Radians}}{\text{Sec}} = \frac{\text{Feet/Second}}{\text{Feet/Radian}}$
- $\omega_0 = \frac{30 \text{ ft/sec}}{R_0 \times \frac{2\pi}{150}} = \frac{30 \times 2\pi}{150} = \frac{25}{\pi} \text{ rad/sec}$

Apply Example 2, Page 257, Figure 2:

Angular velocity after falling through an angle $\theta$ is based on the relationship:

$\frac{d\theta}{dt} = \frac{3\theta}{2} \left(1 - \cos \theta\right) + \frac{1}{2} \omega_0^2$

Integrating both sides to get:

$(\omega_2)^2 = \left(\frac{d\theta}{dt}\right) = \frac{3\theta}{2} \left(1 - \cos \theta\right) + \omega_0^2$

For $\theta = 90^\circ$, $\cos \theta = 0$

$\omega_2 = \left(\frac{d\theta}{dt}\right) = \sqrt{\frac{3\theta}{2} \left(1 - \cos \theta\right) + \omega_0^2}$

$\omega_2 = \sqrt{3(384) + 1.297}$

$\omega_2 = 1.123 \text{ rad/Sec}$

At end impact velocity at horizontal impact:

$= R \cdot \omega_2 = 1200 \times 1.123 = 1347 \text{ inches/second}$ or $112 \text{ ft/second}$.
SRM WATER ENTRY LOADS

BY WIEG

ANALYSIS

CASE I

DETERMINE STRESS LEVELS AND SRM CHAMBER INTEGRITY:

CHAMBER:
WEIGHT = 17,000 LB
156.0 INCHES O.D.
1.45 INCHES I.D.
I = 15,019.279 in^4
A = 21,991 in^2

AXIAL LOAD = 500000 FB/SEC VERTICAL IMPACT 0.5 g (17,000)

AXIAL COMPRESSION STRESS AT 20 FB/SEC IMPACT =\( \frac{\text{LOAD}}{\text{AREA}} = \frac{5(17,000)}{21,991} = 2.700 \text{ W/in}^2 \text{ LOW} \)

NOTE: AXIAL BUCKLING IS HIGH BY COMPARISON WITH DATA SOURCE 3, PAGE 1.

LATERAL LOAD AT 30 FB/SEC HORIZONTAL IMPACT = 0.85 g (17,000)

BENDING STRESS AT 30 FB/SEC IMPACT = \( \frac{Mc}{I} = \frac{0.85(17,000)(588)(78)}{13,529,279} = 400 \text{ W/in}^2 \text{ LOW} \)

M.S. = +HIGH

MEMBRANE EXTREMAL PRESSURE STRESS: 2 MAX PENETRATION OF 319 INCHES

\( P_{\text{extreme}} = (\text{DENSITY})(\text{Penetration}) = (0.0361)(319) = 115.2 \text{ W/in}^2 \)

\( t_c = \frac{P_c}{\frac{T}{2}} = 115.2(38) > 2000 \text{ W/in}^2 \text{ M.S. = +HIGH} \)

Any case 0.31, 0.33, Data Source 4:

\( P_{\text{crack}} = \frac{E0.475}{t_c} \sqrt{\left(\frac{l/d}{2}\right)^{2} + \frac{l}{4}} \)

\( = \frac{847 E + 13.5}{2.5} \)

\( = 847(17,000,000,000)(136) \)

\( = 31.5 \text{ W/in}^2 \)

\( t_c = \frac{P_c}{\frac{T}{2}} = \frac{31.5(480)}{11.5} = 11.5 \text{ W/in}^2 \text{ M.S. = +HIGH} \)


**SACRAMENTO, CALIFORNIA**

**SUBJECT**

SRM WATER ENTRY LOADS

**BY**

Niep

**CHECKED BY**


date

**ANALYSIS: Case I**

Determine stress levels and SRM chamber integrity - continued:

**Lateral External Pressure Load 320 Fps/sec Horizontal Impact**

Compare cosine pressure distribution to critical buckling pressure \( P_{cr} \)

\[ P_{max} = \frac{W}{xL_{max}} = \frac{50,000}{78(174)} = 4.8 \text{ Us/In}^2 \text{ low.} \]

M.I.S. \[ \frac{31.5}{4.8} - 1 = 4 \text{ High} \]
ANALYSIS: Case I

Determine stress levels and SRM chamber integrity—continued:

Lateral External Pressure Load @ 112 F/s. Toppling Impact:

Compare cosine pressure distribution to critical buckling pressure \( P_{cr} \):

\[ \text{from Page 10} \]

\[ \text{Plot of g vs. L / g} \]

From the above plot:

Consider g equipment gage acting over a 300 inch portion of SRM chamber.

Then, comparing cosine pressure distribution to critical buckling pressure \( P_{cr} \):

\[ \text{Calculated } P_{max} = \frac{W}{A_{min}} = \frac{60,000}{117,000} = 30 \text{ kips} / \text{in}^2 \]

\[ \text{M.S. } = \frac{31.5}{50} = -1.0 \pm 0.05 \]

\[ \text{Pressure } P_{max} = \frac{4.5}{4.5} = 22.5 \text{ kips} / \text{in}^2 \]

\[ \text{M.S. } = \frac{31.5}{22.5} = -1.0 \pm 0.05 \]
SRM Water Entry Loads

Summary of Results - Tabulation: Case I

Vertical Entry Attitude:

Load Conditions:
1. Vertical Impact @ 30 ft/sec; Vertical Velocity Vector.
2. Lateral Impact @ 30 ft/sec; Horizontal Velocity Vector.
3. Penetration after impact; Vertical Velocity Vector.
4. Toppling from vertical attitude to horizontal attitude after impact; Vertical and Horizontal Velocity Vectors.
5. Combination of 1 and 3.

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>Method of Evaluation</th>
<th>Margin of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit Stress Level</td>
<td>Buckling Capacity</td>
</tr>
<tr>
<td>1</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>5</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Note: For discussion of results - see Page 4.
SUBJECT

SRM WATER ENTRY LOADS

BY

Wieg

CHK BY

N

SUMMARY OF RESULTS - DISCUSSION: Case I

Load Condition 4 - Toppling:

This is the critical load condition for which the margins of safety as tabulated on page 13 are considered conservative for the following reasons:

1. The SRM chamber does not topple by rotation about the forward tip, as analytically considered, but about a point located some 300 inches aft of the nose, because the chamber reaches full penetration into the water during toppling, thus reducing the effective length of rotation and reducing the extreme end impact velocity as well.

2. The SRM chamber does not impact in a true horizontal attitude at full length water contact, but at an angle of about 10° to the horizontal, thus distributing the deceleration pressure over a longer effective portion of the chamber length than the analytically considered 300 inches.
SRM WATER ENTRY LOADS

ANALYSIS: Case II

DISCUSSION:

The critical load on the SRM is the "slap-down" impact in the horizontal position, with the nose section in a trailing position at the time of impact.

EVENT SEQUENCE:

1) Contact with water surface; initial angular velocity \( (\omega_0) \) corresponding to end velocity of 30 feet/second.
2) Rotation about forward end, with increasing angular velocity due to gravity.
3) Contact water in horizontal attitude.
4) SRM decelerates due to displacement.

ANALYTIC SEQUENCE:

1) Establish initial angular velocity \( (\omega_0) \) of SRM.
2) Determine angular velocity of SRM at "slap-down".
3) Determine deceleration pressure on SRM case.
4) Evaluate resultant effects on SRM case.
SACRAMENTO, CALIFORNIA

REPORT 1917-FRL, APPENDIX A

SUBJECT: SRM WATER ENTRY LOADS

BY: Wieg

ANALYSIS: CASE II

DETERMINE INITIAL ANGULAR VELOCITY \( \omega_0 \): 

RESULTANT VELOCITY \( V_R = \sqrt{20^2 + 30^2} = \sqrt{1300} = 36.0 \text{ ft/sec} \) 

NOTE: CONSIDERING THAT THE SRM CHAMBER RAPIDLY ACHIEVES A TIP VELOCITY EQUAL TO \( V_R \), 

\[
\begin{align*}
\omega_0 &= \frac{\text{Radius} / \text{sec}}{\text{Feet} / \text{sec}} \times \frac{\text{Feet} / \text{sec}}{\text{Radian}} \\
&= \frac{36 \text{ Feet/sec}}{10} = \frac{36 \text{ Radians/sec}}{10} = 3.6 \text{ Radians/sec} = 212.5 \text{ rad/sec}
\end{align*}
\]

APPLICATION: PAGE 257, DATA SOURCE 2.

ANGULAR VELOCITY \( \omega_f \) AS DERIVED ON PAGE 9 OR CALCULATIONS:

\[
\begin{align*}
\omega_f &= \frac{3a}{2} (1 - \cos \theta) + \omega_0^2 \\
&= \frac{3(130)}{2} (1 - \cos 45^\circ) + (36.0)^2 \\
&= \frac{390}{2} (0.707) + 1296 \\
&= 244.5 + 1296 \\
&= 1540.5 \text{ rad/sec}
\end{align*}
\]

FOR \( \theta = 45^\circ \), 

\[
\begin{align*}
\cos \theta &= 0.707 \\
1 - \cos \theta &= 0.293 \\
\end{align*}
\]

\( \theta = 45^\circ \text{ rad/dec} \)

\( \omega_f = 1540.5 \text{ rad/sec} \)

\( \omega_f = 1540.5 \times 52.36 \text{ rad/sec} \)

\( \omega_f = 82,848 \text{ rad/sec} \)

\( \omega_f = 82,848 \times 0.0174533 \)

\( \omega_f = 1.428 \text{ rad/sec} \)

NOTE: BY COMPARISON, \( \omega_f \) AND THE CORRESPONDING WORK VELOCITY WILL BE LESS THAN THAT AS CALCULATED ON PAGE 9, THEREFORE THIS LOAD CONDITION IS NON-CRITICAL.