STUDY OF SOLID ROCKET MOTOR
FOR A
SPACE SHUTTLE BOOSTER

FINAL REPORT

APPENDIX A
SRM WATER ENTRY LOADS

Report 1917-FR1

15 March 1972
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STUDY OF SOLID ROCKET MOTOR
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SPACE SHUTTLE BOOSTER

FINAL REPORT

APPENDIX A
SRM WATER ENTRY LOADS

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Data Procurement Document 314
Data Requirement MA-02

15 March 1972

Prepared for:
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812
OBJECTIVE:

Investigate water entry loads and resultant effects on flight hardware, as outlined below.

LOADS AND INPUT DATA:

1) Initial stage weight: 117,000 lb.
2) Impact velocity, vertical: 20 ft/sec
3) Impact velocity, horizontal: 30 ft/sec (20 mph)
4) Orientation at impact:
   - Case I - Vertical, nose down
   - Case II - 45° to vertical, nose down

SPECIFIC REQUIRED RESULTS:

1) Loads on SRM structure due to impact
2) Loads on SRM structure due to bounce-back
3) Loads on SRM structure induced by side velocity combined with vertical velocity
4) Loads due to water penetration @ 20 ft/sec.

DATA SOURCES:

1) FND. OFFR. INFORMAL MEMORANDUM EM: L-5-03-MI-4 Lockheed Missle & Space Co., Date: 1/31/72
2) ENGINEERING MECHANICS, By Brooks & Wilson, 1929.
3) Row Structural Analysis of L5S note for water entry loads, by Hans Kropf, Date: 4/23/72
4) Formulas for Stress and Strain, by R.J. Roark, 4th Edition

Summary of Results and Discussion:

Refer to Pages 13 and 14.
A first approximation of penetration can be determined by separation of the problem into two parts. For relatively slow entry velocity, the penetration is conservatively equal to the equilibrium penetration plus the distance required to decelerate the SRM from initial entry conditions.

**Event Sequence:**

1) Contact water surface at 20 ft/sec
2) Decelerate from impact at 0.4 to 0.5 g's, not considering drag.
3) Decelerate from buoyant force, which increases with penetration

**Analytical Sequence:**

1) Plot water impact maximum g's versus speed, in order to determine g load at 20 ft/second contact velocity

2) Plot buoyant force versus depth of penetration
SRM WATER ENTRY LOADS

ANALYSIS: Case I - Vertical Motion

V_{(entry)}: 20 \text{ ft/sec}

\text{(Entry)}

\text{d}

\text{Water Surface}

\text{F}_{(Buoyant \ Force)}

\text{F}_{(Deceleration \ Force)}

\text{Determine Buoyant Force:}

\text{Buoyant \ Force} = \frac{\pi}{6} (\text{Volume \ Displaced}) (\text{Density})

= \frac{\pi}{6} (1728)^{2} \left( \frac{62.5}{1938} \right)

= 19,111 (0.0361) = 690 \text{ lb/in}^2

\text{Determine Deceleration after Impact:}

Applying Equation (9), p. 114, Data Source 2:

\begin{align*}
0^2 &= V_0^2 + 2a \cdot x \\
V_0 &= \text{Final \ Velocity} = 0 \\
0 &= v_{(entry)} = 20 \text{ ft/sec} \\
0 &= 20 + 2(14.14)(x) \\
x &= 0.5x \cdot 14.14 = 12.41 \text{ ft sec}^2
\end{align*}

x = \frac{400}{32.2} = 12.41 \text{ ft} = 149 \text{ inches}

\text{Determine Equilibrium Penetration:}

From flare + penetration = 170 inches

Total Penetration = (149 + 170) = 319 inches
SRM WATER ENTRY LOADS

ANALYSIS: Case I - Vertical Motion

Penetration, Inches
**SRM Entry Loads**

*Analysis: Case I - Vertical Motion*

![Graph showing force vs. impact speed.](image)
ANALYSIS: Case I - Horizontal Motion

EVENT SEQUENCE:
1) Contact water surface (90°F/sq lb)
2) SRM decelerates, imparting initial angular acceleration (ωa)
3) SRM continues to rotate about fire base
4) SRM contacts water surface in horizontal attitude
5) SRM decelerates by displacement

ANALYTICAL SEQUENCE:
1) Determine lateral deceleration characteristics (a function of penetration depth and velocity)
2) Determine initial angular velocity of SRM (resulting from deceleration of immersed section)
3) Determine angular velocity of SRM at water impact
4) Determine angular deceleration at water impact
5) Determine deceleration pressure on SRM case
6) Evaluate resultant effects on SRM case
SRM Water Entry Loads

Analysis: Case E - Horizontal Motion

Determine initial angular velocity (ω₀) resulting from 0.85 g deceleration at solid end:

Penetration: Full
Initial Velocity = 30 ft/sec
Final Velocity = 0 (at immersed end)
Deceleration Force = 0.85 g = 2.88 ft/sec (from page 2)

Note: The 0.85 g is based on the cross-sectional area of the cylinder. The deceleration due to vertical motion starts at a depth of two diameters, hence the deceleration force is about 2.88 ft/sec and goes to zero when the tangential velocity (due to angular rotation of the SRM) nears or equals the forward velocity of the SRM.

Determine time/velocity characteristics of SRM rotation:

\[ \tau = \frac{M \omega}{I} = \frac{W l^2}{I} \]

Apply Equation (2), P.256, Data Source 2:

\[ N = I \omega \]

\[ \omega = \frac{17.0 \sqrt{19,000 \times 1,000}}{1.0 \times 1200} = \frac{17.0 \times 1000 \times 19,000}{1.0 \times 1200} = \frac{17 \times 10^5}{1200} = 0.064 \text{ sec}^{-2} \]

Forward end velocity:

\[ v = 660 \left( \frac{dt}{2} \right) = \frac{660 \times (16t)}{2} = 220t \]

<table>
<thead>
<tr>
<th>t</th>
<th>t'</th>
<th>220t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>22.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>44.0</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6</td>
<td>66.0</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8</td>
<td>88.0</td>
</tr>
</tbody>
</table>

Note: The above calculation, based on this diametral penetration, indicates an increased end deceleration time of 0.4 seconds. However, the SRM cannot penetrate the water to a depth of two diameters in that short a time, therefore, another approximation based on a wider penetration is in order.
SRM WATER ENTRY LOADS

Analysis:

Case I - Horizontal Motion

Second Approximation of \( c_0 \) (see page 1):

- Penetration: One Diameter or Less
- Initial Velocity: 30 ft/sec
- Final Velocity: 0 (at impact end)
- Deceleration Force: 0.8\( g \) @ 30 ft/sec (from page 2)

\[ N = \frac{H}{R} \]

\[ \alpha = \frac{N}{H} = \frac{0.85(117,000)(8)}{117,000(320)^2} = \frac{0.85(15)(20)}{1,440,000} = 0.004 \text{ rad/ft} \]

Forward End Velocity: \( \sqrt{\frac{g t}{16}} = \sqrt{\frac{660(0.05)^2}{16}} = 13 \text{ ft/s} \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( t^2 )</th>
<th>( 13t^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>1.3 \text{ ft/s}</td>
</tr>
<tr>
<td>0.2</td>
<td>0.04</td>
<td>2.6 \text{ ft/s}</td>
</tr>
<tr>
<td>0.3</td>
<td>0.09</td>
<td>3.9 \text{ ft/s}</td>
</tr>
<tr>
<td>0.4</td>
<td>0.16</td>
<td>5.2 \text{ ft/s}</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25</td>
<td>6.5 \text{ ft/s}</td>
</tr>
</tbody>
</table>

Note: The tabulation indicates that the forward end achieves its initial rotation in a short time interval (less than 0.05 second) and then topples due to gravity attraction. This means that angular acceleration \( (\alpha) \) is very high for such a short time that this load condition can be reduced to a toppling condition with an initial angular velocity \( (\omega_0) \)
**Analysis: Case I - Horizontal Motion**

Kinematics of Toppling Condition with initial tip speed ($w_0$) of 30 ft/second:

**Determine initial angular velocity ($w_0$):**

One radian in feet = $R_0 = \frac{R}{2\pi}$ radians

\[ w_0 = \frac{\text{Radian}}{\text{Sec}} = \frac{\text{Feet/Second}}{\text{feet/radian}} \]

\[ w_0 = \frac{30 \text{ Ft/sec}}{R \text{ ft/20}} = \frac{30 \text{ Radians}}{\text{Sec}} = \frac{30 \text{ Radians}}{\text{Sec}} = \frac{30 \text{ Radians}}{5.45 \text{ Radians}} = 5.45 \text{ Radians} \]

**Apply Example 2, Page 257, Direct Form:**

Angular velocity after falling through an angle $\theta$ is based on the relationship:

\[ \frac{1}{2} \frac{d\theta}{dt} = \frac{3g}{2} (1 - \cos \theta) + \frac{1}{2} w_0^2 \]

Using polar coordinates,

\[ \frac{d}{dt} \left( \frac{d\theta}{dt} \right) = 2 \left[ \frac{3g}{2} (1 - \cos \theta) + \frac{1}{2} w_0^2 \right] \]

\[ \omega^2 = \frac{d\theta}{dt} + 3g (1 - \cos \theta) + w_0^2 \]

**Example:**\[ \omega^2 = 3g (1 - \cos \theta) + w_0^2 \]

\[ w_0 = \frac{1}{2} \text{(384) + 297} \]

\[ w_0 = 112.3 \text{ ft/sec} \]

**At End Impact Velocity at Horizontal Impact**

\[ = R \cdot w_0 = 1200 \cdot (1.123) = 1350 \text{ inches/second}, \text{ or } 112 \text{ ft/second} \]
**Aerojet Solid Propulsion Company**  
**Sacramento, California**  
**Report 1917-FR1, Appendix A**

**Subject:** SRM Water Entry Loads

**By:** WEG  
**Checked By:**

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**Analysis I: Case I**

**Determine Stress Levels and SRM Chamber Integrity:**

**Chamber:**
- **Weight:** 11,000 lb
- **I.D. (Inches):** .45
- **O.D. (Inches):** .55
- **I = 13.349, B = 2.79 in²**
- **A = 21.891 in²**

**Axial Load at 20 ft/sec Vertical Impact:**

\[ \text{Axial Load} = \text{Density} \times \text{Impulse} = \frac{25(11,700)}{2799} = 2700 \text{ lb/in}^2 \text{ Low} \]

**Axial Compression Stress at 20 ft/sec Impact:**

\[ \sigma = \frac{25(11,700)}{2799} = 2700 \text{ lb/in}^2 \text{ Low} \]

**Note:** Axial Bounding is High by comparison with data Source 3, page 1.

**Lateral Load at 30 ft/sec Horizontal Impact:**

\[ \text{Lateral Load} = 0.85 \times 2700 = 7650 \text{ lb/in}^2 \] (117,000)

**Bending Stress at 30 ft/sec Impact:**

\[ \sigma = \frac{0.85(11,700)(58)}{13,029,279} = 224 \text{ lb/in}^2 \text{ Low} \]

**M.S. = +HIGH**

**Membrane External Pressure Stress:**

\[ P_{	ext{External}} = \text{Density} \times \text{Penetration} = (20.3) \times (3.91) = 80.5 \text{ lb/in}^2 \]

**Apply Case 9.31, P.852, Data Source 4:**

\[ \frac{P_{	ext{External}}}{2} = \frac{80.5 \times 1.136}{2} = 44.2 \text{ lb/in}^2 \]

**M.S. = +HIGH**

**E = 29,000,000**

**V = 3**

**f = 0.45**

**t = 0.136**

**r = 78**

**b = 5.4**

**L = 160**

**E = 21.5 lb/in²**

\[ \frac{31.5}{11.5} = 1 = +1.74 \]

**M.S. (axial stress) = +HIGH**
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BY

SRM WATER ENTRY LOADS

BY

Analysis: Case I

Determine stress levels and SRM chamber integrity — continued:

Lateral external pressure load @ 30 ft/sec horizontal impact:

Compare cosine pressure distribution to critical buckling pressure ($P_{cr}$)

From Page 10

\[ P_{max} = \frac{W}{2A_{ref}} = \frac{55,000}{78 (152)} = 4.8 \text{ lb/in}^2 \text{ low.} \]

M.S. = \frac{3.5}{4.8} - 1 = + \text{ High}
ANALYSIS:  Case I

DETERMINE STRESS LEVELS AND SRM CHAMBER INTEGRITY—CONTINUED:

LATERAL EXTERNAL PRESSURE LOAD @ 112 F/SEC, TOPPLING IMPACT:

- Compare cosine pressure distribution to critical buckling pressure ($P_{cr}$)
- Plot of $g$ vs. length

From the above plot:
- Consider g-equipment gage acting over a 360 inch portion of SRM chamber.

Then, comparing cosine pressure distribution to critical buckling pressure ($P_{cr}$):

Calculated $P_{max} = \frac{W}{A} = \frac{600 \text{ lb}}{117,700 \text{ in}^2} = 2.02 \text{ lb/in}^2$

$M.S. = \frac{31.5}{30} - 1.0 = 0.05$

Pressure $P_{max} = \frac{45}{4.0} = 11.25 \text{ lb/in}^2$

$M.S. = \frac{31.5}{32.5} - 1.0 = 0.06$
SRM Water Entry Loads

**Vertical Entry Attitude:**

**Load Conditions:**

1. Vertical impact @ 20 fps; Vertical Vector Velocity.
2. Lateral impact @ 35 fps; Horizontal Vector Velocity.
3. Penetration after impact; Vertical Vector Velocity.
4. Toppling from vertical attitude to horizontal attitude after impact; vertical and horizontal velocity vectors.
5. Combination of 1 and 3.

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>Method of Evaluation</th>
<th>Margin of Safety</th>
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<tbody>
<tr>
<td></td>
<td>Unit Stress Level</td>
<td>Buckling Capacity</td>
</tr>
<tr>
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<td>✓</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>5</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

*Note: For discussion of results—see page 4.*
SUBJECT

SRM WATER ENTRY LOADS

SUMMARY OF RESULTS - DISCUSSION: CASE I

LOAD CONDITION 4 - TOPPLING:

This is the critical load condition, for which the margins of safety as tabulated on page 13 are considered conservative for the following reasons:

2. The SRM chamber does not topple by rotation about the forward tip, as analytically considered, but about a point located some 300 inches aft of the nose, because the chamber reaches full penetration into the water during toppling, thus reducing the effective length of rotation and reducing the extreme end impact velocity as well.

6. The SRM chamber does not impact in a true horizontal attitude at full length water contact, but at an angle of about 10° to the horizontal, thus distributing the deceleration pressure over a longer effective portion of the chamber length than the analytically considered 300 inches.
SRM Water Entry Loads

Analysis: Case II

Discussion:

The critical load on the SRM is the "slap-down" impact in the horizontal position, with the nose section in a trailing position at the time of impact.

Event Sequence:

1) Contact with water surface; initial angular velocity \( \omega_0 \) corresponding to end velocity of 20 feet/second.
2) Rotation about forward end, with increasing angular velocity due to gravity.
3) Contact water in horizontal attitude.
4) SRM decelerates due to displacement.

Analytic Sequence:

1) Establish initial angular velocity \( \omega_0 \) of SRM.
2) Determine angular velocity of SRM at "slap-down".
3) Determine deceleration pressure on SRM case.
4) Evaluate resultant effects on SRM case.
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Case II

Determine initial angular velocity ($\omega_0$):

Resultant velocity $V_E = \sqrt{V_0^2 + 30^2} = \sqrt{1300} = 36.0$ ft/sec

Note: Considering that the SRM chamber rapidly achieves a tip velocity equal to $V_E$,

$$\omega_0 = \frac{R}{v_E} = \frac{36.0}{\sqrt{1300}} = 0.655$ \text{ rad/sec}$

Angular velocity ($\omega_f$) as derived on page 9 of calculations:

$$\omega_f = \frac{3g}{2} (1 - \cos \theta) + \omega_0^2$$

$$\omega_f = \sqrt{\frac{3g}{2} (1 - \cos \theta) + \omega_0^2}$$

$$\omega_f = \sqrt{\frac{3g}{2} \frac{1}{2} \omega_0^2} + 0.418$$

$$\omega_f = \frac{1}{10} = 0.848$$ \text{ rad/sec}$

Note: By comparison, $\omega_f$ (and the corresponding impact velocity) will be less than that as calculated on page 9, therefore this load condition is non-critical.