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NASA Scientific and Technical Information Facility
NOISE ANALYSIS OF NUCLEATE BOILING

R. D. McKnight
K. S. Ram
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ACKNOWLEDGEMENT

This report summarizes the results of study performed by the University of Cincinnati, Cincinnati, Ohio, while under contract to NASA, Lewis Research Center, Cleveland, Ohio. This final report deals only with the aspect of noise analysis application to Two-phase Studies Contract (NGR 36-004-008). It is a pleasure to acknowledge the technical help and encouragement of Dr. B. Lubarsky and Dr. R. W. Graham, NASA and Dr. P. Harrington, University of Cincinnati.
The techniques of noise analysis have been utilized to investigate nucleate pool boiling. A simple experimental setup has been developed for obtaining the power spectrum of a nucleate boiling system. These techniques were first used to study single bubbles, and a method of relating the two-dimensional projected size and the local velocity of the bubbles to the auto-correlation functions is presented. This method is much less time-consuming than conventional methods of measurement and has no probes to disturb the system. These techniques can be used to determine the contribution of evaporation to total heat flux in nucleate boiling. Also, these techniques can be used to investigate the effect of various parameters upon the frequency response of nucleate boiling. The predominant frequencies of the power spectrum correspond to the frequencies of bubble generation. The effects of heat input, degree of subcooling, and liquid surface tension upon the power spectra of a boiling system are presented. The formation of larger bubbles with higher heat fluxes or higher bulk temperatures is evidenced by the occurrence of resonances at lower frequencies in the power spectrum. It was found that the degree of subcooling has a more pronounced effect upon bubble size than does heat flux. Also the effect of lowering surface tension can be sufficient to reduce the effect of the degree of subcooling upon the size of the bubbles.
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<td>b</td>
<td>Signal width; Duty cycle.</td>
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<tr>
<td>C(t)</td>
<td>Correlation function</td>
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<tr>
<td>D</td>
<td>Bubble Diameter</td>
</tr>
<tr>
<td>f</td>
<td>Frequency in cps.</td>
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<td>i(t), g(t)</td>
<td>Time-Varying functions</td>
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<td>( \bar{f}, \bar{g} )</td>
<td>Time averages of the functions.</td>
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<td>g</td>
<td>Gravitation constant</td>
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<tr>
<td>h(t)</td>
<td>Impulse response function</td>
</tr>
<tr>
<td>I</td>
<td>Light intensity</td>
</tr>
<tr>
<td>k</td>
<td>Thermal Conductivity</td>
</tr>
<tr>
<td>M</td>
<td>Maximum number of correlation points</td>
</tr>
<tr>
<td>N</td>
<td>Number of digitized samples</td>
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<tr>
<td>P(f)</td>
<td>Power spectral density</td>
</tr>
<tr>
<td>P'(f)</td>
<td>Smoothed power spectral density</td>
</tr>
<tr>
<td>Q</td>
<td>Heat flux, BTU/hr-ft²</td>
</tr>
<tr>
<td>R</td>
<td>Bubble radius in mm.</td>
</tr>
<tr>
<td>t</td>
<td>Time in seconds</td>
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<tr>
<td>T</td>
<td>Period (( = 1/f )); integration time</td>
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<td>Normalized correlation function</td>
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<td>Surface tension</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>Variance</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Lag time</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Attenuation coefficient</td>
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</tbody>
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### Subscripts

- \( ff \): auto-correlation
- \( fg \): cross-correlation
- \( i \): input
- \( j \): index of summation
- \( L \): Liquid
- \( m \): maximum
- \( N \): Nyquist frequency
- \( O \): Output
- \( V \): Vapor
I. INTRODUCTION

Because of the high interest in boiling as a method of heat transfer, there has been a considerable amount of research conducted to investigate certain fundamental aspects of the boiling process. As evidenced by the number of attempts, it has been exceedingly difficult to find a heat transfer correlation that will satisfactorily predict boiling heat transfer under a variety of conditions, mainly because there are so many variables which enter the boiling process. The great complexity of boiling—the number of variables that can influence boiling heat transfer, the difficulty of their measurement, and the complexity of their interaction—has severely restricted the success of the numerous investigations of this process.

Three distinct regimes of boiling can be considered; namely, nucleate, transition, and film boiling. Nucleate boiling can occur if the interface or wall attains locally a temperature above the boiling point, even though the bulk temperature of the fluid is below the boiling point. An increase in the wall temperature is accompanied by an increase in the bubble population and an increase in the heat flux. There exists a maximum wall temperature beyond which the heat flux will start to decrease. Transition boiling begins if the temperature is increased further. An unstable vapor film is formed at the wall with correspondingly higher thermal resistance. The heat flux will reach a minimum value and thereafter increase for higher wall temperatures, characterizing stable film boiling. The mechanism of thermal radiation heat transfer becomes more significant as the wall temperature is increased. However, in the film boiling regime, the wall temperature may exceed its melting point, producing "burnout". Therefore, the highest rates of heat transfer can be attained in the nucleate boiling regime, taking care to remain below the "departure from nucleate boiling" point and avoiding the...
"burnout" phenomena. Zuber has discussed the regimes of boiling and analyzed, both theoretically and experimentally, bubble growth and the hydrodynamic characteristics of nucleate pool boiling. A brief description of some of the various methods used to investigate nucleate boiling follows.

By far the most frequently employed technique in the study of boiling has been the analysis of high-speed motion pictures. A frame-by-frame analysis of the pictures provides the two-dimensional projected size and the local velocity of the bubbles. The principal disadvantages of this approach are that the analysis is very time-consuming and yields only a two-dimensional representation of the bubbles. Hsu and Graham have used high-speed motion pictures of schlieren and shadowgraph images of nucleate pool boiling to study the agitation around the bubbles and the behavior of the superheated micro-layer adjacent to the heated surface. Torikai used high-speed photography to study bubble formation and the contact area of bubbles on a heated surface. Graham and Hendricks have assessed the contributions of various heat-transfer mechanisms, including convection, transient conduction and evaporation in nucleate boiling, and have proposed an overall model which considers nucleate boiling as a series of transient heat-transfer processes at distinct regions of the surface. Their calculated values were compared with recent boiling experiments, much of which had been obtained by photographic techniques.

Moore and Mesler used a specially designed microthermocouple with an extremely short response time to study the temperature fluctuations of the boiling surface, and Rogers and Mesler combined this method of monitoring the surface temperature with simultaneous high-speed photographs of the boiling. These methods can be used to study the thermal boundary layer and also the growth time and the departure time for bubble emission. Sharp used an electrical continuity probe to study the thickness of the microlayer. Another technique which has been employed to study the microlayer involves
chemical deposition at the nucleation sites.\(^8\)

**CORRELATION TECHNIQUES**

Since nucleate boiling is a random process, the techniques of noise analysis should be useful in the study of that process. The first investigation of boiling water nuclear reactors utilizing noise analysis techniques was performed by the Argonne National Laboratory.\(^9\) Transfer functions, spectral moments and correlation functions were obtained for the BORAX-IV reactor.

Rajagopal\(^10\) reported a resonance between 10 and 20 cycles/sec. in the spectra of the Saxton reactor. His analysis showed this resonance to be related to nucleate boiling. The resonance became more pronounced at higher reactor powers. Also there was a direct relation between the estimated average core surface area in nucleate boiling and the area under the resonance.

Lummis\(^11\) developed a technique for measuring the frequency response of the boiling heat transfer process. A thin nickel film was used as a resistance thermometer, as well as a heater, by carrying out each function in a separate frequency band. The boiling fluid was regarded as a linear system whose input was a variation in heat flux and whose output was a variation in surface temperature, and the frequency response of that linear system was measured. Amplitude and phase response curves were determined for a variety of operating conditions. He concluded that (1) the response was close to that of a linear system for reasonable excursions of the heat flux density, (2) the quiescent heat flux density had only a moderate effect on the frequency response, although it had a pronounced effect upon the fluid behavior near the interface, and (3) the liquid bulk temperature had a strong effect on the frequency response.

McCurdy\(^12\) showed the relationship between the spectral density of the
spatially-averaged temperature (of a heated filament) and the auto- and cross-spectral densities of the temperature induced at individual sites. By analyzing the trace of the spatially-averaged temperature of the system, he obtained its spectral density. Using correlation techniques, he investigated nucleate pool boiling in saturated and subcooled water under normal and low gravity fields and obtained the power spectra of the temperature fluctuations at the nucleating surface. He attempted to correlate these power spectra with the bubble distributions determined by the use of motion pictures.

There has been very limited work using spectral analysis in the nucleate boiling heat transfer area. There have been no comprehensive studies of the effect of various parameters, such as temperature, pressure, viscosity, and surface tension of the liquid on the frequency spectrum of nucleate boiling. The objectives of the present work are three-fold: (1) to demonstrate that correlation techniques can be used to obtain information previously obtained using time-consuming motion picture methods; specifically, that the two-dimensional projected size of the bubble, its local velocity, and the size and frequency distributions of the bubbles can be determined; (2) to show that correlation techniques can be used to determine the frequency response (spectral density) of nucleate boiling; and (3) to determine the effect of various parameters upon the frequency spectrum; specifically, to determine the effect of the degree of subcooling and surface tension upon the frequency spectrum of nucleate boiling. The present work at least makes no pretense of treating exhaustively or definitively even the limited area of its concern. Its primary objective is rather to open new lines of useful exploration.

The present work is based on a technique using a simple optical system to obtain the frequency response of nucleate boiling. The experimental equipment and procedure are described in subsequent sections. The principal
advantages of the present method are: no probes to disturb the system; less
time-consuming than conventional techniques; and flexibility of experimental
set-up, permitting more information to be obtained. One application of the
information obtained by the present method is the determination of the void
fractions. Hsu\textsuperscript{14} has shown how the area fractions covered by
bubbling bubbles can be related with other parameters, such as bubble size and instantaneous
bubble population. The correlation functions and power spectra obtained for
the boiling system contain information about the bubble-size distribution
and population and bubble growth and departure times.

II. THEORY

In many areas of science and engineering, the accuracy of experimental
measurements has become increasingly sensitive to the point that noise has
become the limiting factor. This fact has led to the development of the
field of information and statistical communication theory.\textsuperscript{15,16,17} In this
field, techniques have been developed for the analysis of data containing
noise, that is, containing random signals. In recent years these techniques
of noise analysis have found application in many diverse fields, such as
radio astronomy, seismology, neurology, etc. The brief treatment of random
signals (noise analysis) which is included in this section follows essenti-
ally that of Thie.\textsuperscript{18}

Correlation Functions and Power Spectral Density Functions

Consider some physical process which gives rise to a time-varying
signal $f(t)$. The signal may be simple or complex periodic or have the
character of noise, that is, be random-varying. If this signal is delayed,
thereby obtaining the signal $f(t - \tau)$, which is identical to $f(t)$ but just
delayed in time, and then the product of $f(t) \cdot f(t - \tau)$ is averaged over a
sufficiently long time, the auto-correlation function, $C_{ff}(\tau)$, for the
signal $f(t)$ will be determined. Mathematically, the auto-correlation function is defined as

$$ C_{ff}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t) \cdot f(t-\tau) \, dt $$

This will have its maximum value for zero delay ($\tau=0$), and will tend to zero as $\tau$ becomes sufficiently large for signals describing realistic physical systems. Note also that $C_{ff}(\tau)$ is an even function, that is, $C_{ff}(\tau) = C_{ff}(-\tau)$, and therefore equation (1) may be written as

$$ C_{ff}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t) \cdot f(t+\tau) \, dt $$

For the case in which the signal $f(t)$ is given as $N$ discrete data points (electronically by digitizing an analog signal), the auto-correlation function may be determined by replacing the integral in equation (2) by a summation as

$$ C_{ff}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} f(t_j) \cdot f(t_j+\tau) $$

The normalized auto-correlation function is defined with respect to deviation from the mean and is given by

$$ \rho_{ff}(\tau) = \lim_{N \to \infty} \frac{1}{\sigma_f^2 N} \sum_{j=1}^{N} \left[ f(t_j) - \bar{f} \right] \cdot \left[ f(t_j+\tau) - \bar{f} \right] $$

where $\bar{f}$ is the mean value of the time-varying signal over one period of a periodic signal or over the discrete number of digitized values of a random signal, and $\sigma_f^{-2}$ is the variance of the signal $f(t)$. In practice, with $N$ data points spaced $\Delta t$ apart, $\rho_{ff}(\tau)$ is calculated for $\tau$ up to $\tau_{m} = N \Delta t < N \Delta t$. 

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The auto-correlation function and the normalized auto-correlation function are related by

\[ C_{ff}(\tau) = \sigma_f^{-2} \phi_{ff}(\tau) + (\overline{f})^2 \]  

Similarly, if two signals, \( f(t) \) and \( g(t) \), either an input and an output or two outputs to a known input, are obtained from a system, and one is delayed and the average value of the product of the signals is determined, then the result is the cross-correlation function of the two signals. This may be written mathematically as

\[ C_{fg}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)g(t+\tau) \, dt \]  

As with the auto-correlation function, one can define the normalized cross-correlation function as

\[ \phi_{fg}(\tau) = \lim_{T \to \infty} \frac{1}{2\sigma_f \sigma_g} \int_{-T}^{T} \frac{[f(t)\overline{g}](t+\tau)}{[g(t)\overline{g}](t+\tau)} \, dt \]  

and for \( N \) synchronized data points spaced \( \Delta t \) apart for both signals, the normalized cross-correlation function may be calculated for \( \tau \) up to \( \tau_m = \frac{M \Delta t}{N \Delta t} \) as
The cross-correlation function can be described as representing the degree of conformity between two signals as a function of their mutual delay.

The correlation functions are useful in describing a system's response in the time domain. The impulse response function $h(t)$ of a system is related to the input $f_1(t)$ and the output $f_o(t)$ by the following convolution integral

$$f_o(t) = \int_{-\infty}^{\infty} f_1(\tau) h(t-\tau) \, d\tau \tag{10}$$

Using this relation, the correlation functions defined in equations (2) and (7) can be written as

$$C_{10}(\tau) = \int_{-\infty}^{\infty} h(\tau) \cdot C_{11}(\tau-\tau) \, d\tau \tag{11}$$

where $C_{11}(\tau)$ and $C_{10}(\tau)$ are the auto-correlation of the input and the cross-correlation between the input and the output respectively. The frequency response of a system can be obtained by taking the Fourier transform of the correlation functions. These useful reciprocal relations are known as Weiner's theorem. Namely, the power density spectrum, $P(f)$, of a signal is
the cosine Fourier transform of the auto-correlation function, \( C(\tau) \). This may be expressed as

\[
C(\tau) = \int_{-\infty}^{\infty} P(f) \exp(\text{i} \omega \tau) \, df, \quad \omega = 2\pi f
\]

\[
= 2 \int_{0}^{\infty} P(f) \cos \omega \tau \, df
\]

where \( P(f) = \lim_{T \to \infty} \frac{1}{T} \left| \tilde{g}(f) \right|^2 \)

\[
P(f) = \lim_{T_m \to \infty} \int_{-T_m}^{T_m} C(\tau) \exp(-\text{i} \omega \tau) \, d\tau
\]

\[
= \lim_{T_m \to \infty} 2 \int_{0}^{T_m} C(\tau) \cos \omega \tau \, d\tau
\]

With \( N \) discrete values of \( C(\tau) \) spaced \( \Delta t \) apart, the power density spectrum can be expressed as

\[
P(f) = \frac{1}{N} C(0) + \frac{2}{N} \sum_{j=1}^{N} C(j \Delta t) \cos 2\pi f j \Delta t
\]

\[
= \Delta t C(0) + 2 \Delta t \sum_{j=1}^{M} C(j \Delta t) \cos 2\pi f j \Delta t
\]

**Methods of Measurement**

There are two general methods of recording and analysis: (1) Analog (or Continuous), and (2) Digital (or Discrete). Continuous recording may be accomplished by standard chart recorders or by magnetic tape recording. Having once recorded an experiment on magnetic tape, it can be rerun and reanalyzed at any time and as often as is desired. Data so recorded is easily stored and is compatible with both analog and digital methods of analysis. Also by using different recording and playback speeds, one can achieve various time transformations in the analysis. Although continuous signals can be recorded such that the degree of magnetization is proportional to the signal (direct amplitude recording), it is usually preferable to record
the frequency modulation of a carrier at constant amplitude. Auto-correlation functions can be computed from the playback-head signals from identical channels by varying the length of tape between the two heads to achieve various delays.

Of course, when digital analysis of a continuous signal is desired, an analog-to-digital conversion must be performed. If the continuous signal is available on magnetic tape this conversion can be performed automatically, whereas if the signal has been recorded on a chart or graph this conversion might be semi-automatic or manual.

When analysis is done using discrete data points, there exists a practical upper limit to the frequency. This maximum frequency, \( f_N \), is referred to as the Nyquist frequency or cut-off frequency. Frequencies above \( f_N \) cannot be detected and therefore the numerical integrations have \( f_N \) as the upper limit. In the event frequencies above the Nyquist frequency are actually present in the continuous signal, then a phenomena known as "aliasing" occurs, that is, in the digital analysis these higher frequencies will appear below the Nyquist frequency and will be indistinguishable from the lower frequencies.

The Nyquist frequency is given by

\[
f_N = \frac{1}{2\Delta t} = \frac{1}{2} \left( \frac{1}{\text{digitizing interval}} \right) = \frac{N}{2T}
\]

If the auto-correlation function is determined up to \( \tau_m = M\Delta t \leq N\Delta t \), then the highest frequency will be \( f_N = M/2\tau_m \). This means the spacing of the frequencies, \( \Delta f \), is equal to \( 1/2\tau_m \) or \( 1/2M\Delta t \). However, the spectral resolution due to a finite duration of sampling, \( T \), is equal to \( 1/T \) or \( 1/\tau_m \). This limitation results since the auto-correlation function is not known for \( \tau > \tau_m \).

Because of this, one defines a lag window which is defined for infinite \( \tau \)'s.
\[ h(\tau) \leq 1 \quad \text{for} \quad |\tau| \leq \tau_{\text{rn}} \]
\[ = 0 \quad \text{for} \quad |\tau| > \tau_{\text{rn}} \]

There are a variety of spectral windows, \( h(\tau) \), which can be used to minimize the effect upon \( P(f) \) of contributions outside the range \( f \pm \Delta f \).

(Of course, with idealized band-pass filtering, the frequencies outside \( \Delta f \) would have no effect and the frequencies within \( \Delta f \) would be unattenuated.)

Two frequently used windows \(^{19}\) are

\[
\begin{align*}
\text{Hanning} & : 0.5 + 0.5 \cos \frac{\pi \tau}{\tau_{\text{rn}}} \\
\text{Blackman} & : 0.54 + 0.46 \cos \frac{2\pi \tau}{\tau_{\text{rn}}}
\end{align*}
\]

Since the power spectra obtained by equation (14) are spaced by \( 1/2 \tau_{\text{rn}} \), it is convenient to use one of the above windows for smoothing them. These equations can be readily programmed for analysis with a digital computer.

The power spectrum, \( P(f) \), can be obtained by the following methods. First, the spectral density function can be obtained directly by using a band-pass filter and an analog squaring network. This method is a well-established technique used to obtain the spectral density of neutron fluctuations in nuclear reactors. \(^{20}\) Alternately, one can obtain the auto-correlation function and take its cosine transform to determine the power spectrum.

The block diagram for an analog circuit which can be used to obtain the power spectrum directly is given in Figure II-1. The signal, which can be pre-amplified, is passed through a band-pass filter to obtain a desired frequency \((v_o \text{ cps})\). The output of the band-pass filter is then passed through a squaring network and the squared signal is integrated with an analog computer to obtain the average power at that frequency, \( P(v_o) \).

The second method, obtaining the auto-correlation function of the signal, is a more useful method even though indirect. This is especially true if the auto-correlation function can be readily obtained, for the
operation of determining the power spectrum from the auto-correlation function is easily performed. Also, it is advantageous to have the auto-correlation function in addition to the power spectrum since the auto-correlation function is worthwhile information about the dynamic behavior of a system.

Correlation functions can be obtained from digital data from a computer solution of either equation (5) or equation (9). Correlation functions of continuous signals can be obtained directly with a signal correlator. Figure 11-2 shows a simplified block diagram of a signal correlator (Princeton Applied Research Model 100). With two signal inputs as shown in this figure, the cross-correlation function of the two signals will be computed. One hundred points of the correlation function are simultaneously computed over total delay spans ($T_m^* = 100\Delta t$) ranging from 100 microseconds to 10 seconds. This is accomplished by multiplying one input signal by one hundred separate, sequentially-delayed replicas of the second input signal. The resulting one hundred products are then individually averaged and stored in one hundred separate analog memory elements. The correlation function is continuously available in analog form as it is being computed and can be obtained by scanning the memory banks at rates suitable for display on an external oscilloscope or x-y recorder.

As stated above, the power spectrum of the system can be obtained by performing a Fourier analysis of the experimentally determined correlation functions. A computer program was written to perform the Fourier analysis of the auto-correlation function on the IBM 7040. The power spectral density and the smoothed power spectral density were calculated using the following expressions:

\[
P(f) = \Delta t C(0) + 2\Delta t \sum_{j=1}^{M} C(j\Delta t) \cos 2\pi f j \Delta t
\]

\[
P'(f) = 0.25P(f - \frac{1}{2\tau_m}) + 0.50P(f) + 0.25P(f + \frac{1}{2\tau_m})
\]
FIGURE II-2  BLOCK DIAGRAM FOR SIGNAL CORRELATOR
(input selector switch shown in position for cross-correlation)
The Hanning spectral window was used in equation (16) to obtain the smooth
power spectral density function.

The following input was used:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter or Function</th>
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<tr>
<td>T</td>
<td>Maximum correlation time, ( \tau_m )</td>
</tr>
<tr>
<td>M</td>
<td>Number of data points</td>
</tr>
<tr>
<td>( l/L )</td>
<td>Digitizing interval, ( \Delta t = T/M )</td>
</tr>
<tr>
<td>ACORR</td>
<td>Auto-correlation function, ( C(\tau) )</td>
</tr>
<tr>
<td>ACO</td>
<td>Auto-correlation function with zero delay, ( C(0) )</td>
</tr>
</tbody>
</table>

The following output was obtained:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPDEN</td>
<td>Spectral density function, ( P(f) )</td>
</tr>
<tr>
<td>SPDENS</td>
<td>Smoothed spectral density function, ( P'(f) )</td>
</tr>
</tbody>
</table>

The program used to calculate these functions is given in Figure 11-3.

One of the principal advantages of the correlation techniques is the
ability to extract signals buried in noise. Signal-to-noise ratios of 1 to
have been practical limits in quite a few engineering areas.

One such application of cross-correlation techniques was the study of
hydrodynamic turbulence. By placing two probes in a stream of fluid undergo-
ing turbulent flow and obtaining the cross-correlation function of the signal
obtained from the two probes, it is possible to determine the velocity of the
fluid. In this application, the time delay for maximum correlation is the
transit time for the turbulence to travel the known distance between the two
probes. An illustration of this two-probe cross-correlation method is the
so-called "salt velocity" technique for determining eddy velocities. These
correlation techniques are possibly the most important tools available to
investigators of turbulence and diffusion.

- 15 -
Figure 11-3  POWER SPECTRUM PROGRAM

C NOISE ANALYSIS

C THIS PROGRAM INPUTS M VALUES OF THE AUTO-CORRELATION
FUNCTION AND COMPUTES THE POWER SPECTRAL DENSITY FUNCTION,
SPDEN, AND THE SMOOTHED SPECTRAL DENSITY FUNCTION, SPDENS.

C DATA FROM 12-30-68, CURVE BB, T = 0.5 SEC, TEMP = 95.5

DIMENSION ACORR(1200), SPDEN(600), SPDENS(600)
READ (5,1) M,L
1 FORMAT (215)
PRINT 2, M,L
2 FORMAT (1HO,25X,2HM=, I5,10X,2HL=,15)
READ (5,3) ACORR(I), I=1,M
3 FORMAT (12 F6.2)
READ (3,4) ACO
4 FORMAT (F6.2)
J = L/2
DO 20 I = 1,J
SUM = 0.
DO 10 K = 1,M
SUM = SUM + ACORR(K)*COS(2.*3.14159*FLOAT(I)*
FLOAT(K)/FLOAT(L))
10 CONTINUE
SPDEN(I) = (ACO + 2.*SUM)/FLOAT(L)
20 CONTINUE
PRINT 30, ACO
30 FORMAT(1HO,25X,4HACO=,F10.6)
PRINT 40,(1,SPDEN(I),I=1,J)
40 FORMAT (1HO,10X,15,5X,9HSPDEN(I)=,F15.6)
KK = J-1
DO 50 I = 2,KK
SPDEN(I) = 0.25*(SPDEN(I+1)+SPDEN(I-1)+0.50*SPDEN(I)
50 CONTINUE
PRINT 60, (1,SPDEN(I),I = 2,KK)
60 FORMAT (1HO,10X,15,5X,10HSPDEN(I)=,F15.6)
CALL EXIT
END
**Single Bubble Expressions**

The turbulence introduced by boiling in a system contributes considerably to the efficiency of the heat transfer rate at the boiling surface. Also the size and frequency of the bubbles formed in nucleate boiling is a measure of the heat transfer due to the latent heat of vaporization. For these reasons a method of obtaining the size and velocities of bubbles using an external optical system and correlation techniques was determined. The methods were first used to study the special case of single gas bubbles in order to establish relationships between the size, velocity and frequency of generation of the bubbles and the correlation and power spectral density functions.

Experimental results are obtained by passing a parallel beam of white light through liquid media containing bubbles to a photomultiplier tube which registers the alteration of the light signals by gas bubbles of various sizes. The details of the experimental set-up and results are given in the later sections. The single bubble analysis is broken into the following general cases for a clear understanding. The mathematical expressions and the pictorial representation of the cases considered are shown in figures II-4 and II-5.

Figure II-4 represents the case when the width of the parallel beam is less than or equal to the diameter of the bubble. The frequency of bubble emission does not enter the expressions of the attenuated light intensity, but the expected signal as shown in this figure will be repeated at the same frequency as the bubble emission. When the width of the light beam is much smaller than the diameter of the bubble, the light should be attenuated during the time when any portion of the bubble is intercepting the beam. The resulting signal is a square wave as shown in the figure where the minimum intensity is related to the bubble diameter and the width of the pulse is related to the size (diameter) as well as speed of the bubble. However, if the beam width is approximately the same size as the bubble diameter, the
Case (i) $W \ll 2R$

Case (ii) $W \sim 2R$

$I_0 = \text{ORIGINAL INTENSITY}$  \hspace{1cm} $W = \text{SLIT WIDTH}$  \hspace{1cm} $R = \text{BUBBLE RADIUS}$

$I_{\text{min}} = I_0 e^{-\alpha R \mu}$

$I(t) = I_0 e^{-\left(\frac{\alpha R}{2}\right) \left(1 + \cos \frac{\pi V t}{2R}\right)}$

FIGURE II-4.
\[ f < \frac{V}{W} \]

\[ l_{\text{min}} = I_o e^{-\sqrt{RF/A}} \]

\[ f \sim \frac{1}{m} \frac{V}{W} \]

if \( w \sim n \) (ZR)

\[ I' = I_o e^{-\left(\frac{n-1}{m}\right) \sigma_{RF} / \mu} \]

\[ l_{\text{min}} = I_o e^{-\left(\frac{n_{\text{min}}-1}{m}\right) \sigma_{RF} / \mu} \]

- Low Frequency

- Medium Frequency

FIGURE II-5.
ideal signal (for a purely absorbing bubble) will be triangular. In practice
the signal is cosine in nature (rounded edges of the triangle) and is shown
in the figure II-4. Once again the minimum intensity and width of the light
signal contain information regarding the size and the speed of the bubble.

If the width of the light beam is greater than the diameter of the bubble,
then the frequency of the bubble emission would affect the nature of the
signal, as shown in figure II-5. First, for a low frequency case, when the
frequency is less than the ratio of velocity of the bubble to the width of
the beam, the beam will be attenuated by the entire bubble as it passes
through the beam. Thus the resulting signal would be a square wave. However,
as the bubble is entering and leaving the beam, only a fraction of the bubble
is intercepting the beam, and therefore the resulting signal will not be
square in the initial and final portions. At very high frequencies (i.e.,
when the frequency of bubble emission is larger compared to the ratio of
velocity of the bubble to the width of the light beam), the expected signal
will be a step function of magnitude given by the minimum light intensity.
This trivial case is not shown in the figures. However, the case of inter-
mediate frequency range is shown in figure II-5. In this case, the signal
is a step function with a superposition of individual bubble attenuations.
The ideal case shown in the figure corresponds to a frequency \( f = \frac{1}{T} \)
which is a fraction of the ratio of velocity of the bubble \( V \) to the width
of the beam \( W \). Further, the width of the light beam is assumed to be a
multiple \( n \) of the diameter \( 2R \) of the bubble. The analytical expressions
for the step and the superposition of the individual bubbles are given in the
figure.

The expected auto-correlation functions for a square wave and triangular
wave are given in figure II-6. It is important to note that the width of the
auto-correlation peak in both cases is twice the width of the pulse. Thus the
\[ f(\tau) = \begin{cases} I_m (1 + \frac{t}{b}) & ; -b \leq t \leq 0 \\ I_m (1 - \frac{t}{b}) & ; 0 < t < b \end{cases} \]

\[ \phi(\tau) = \begin{cases} \frac{I_m^2}{6b^2T_t} \left[ 3\tau^3 - 6b\tau^2 + 4b^3 \right] & ; 0 \leq \tau \leq b \\ \frac{I_m^2}{6b^2T_t} \left[ 2b - \tau \right]^3 & ; b \leq \tau \leq 2b \end{cases} \]

\[ f(\zeta) = \sum_{n \in \mathbb{Z}} \frac{I_m b}{T_t} \frac{\sin n\pi b}{n\pi b/T_t} e^{i\omega_0(t - \frac{b}{2})} \]

\[ \phi(\zeta) = \frac{I_m^2}{4} + \frac{2I_m^2}{3\pi^2} \left[ \cos \omega_0 \tau + \frac{1}{3} \cos^2 \omega_0 \tau \right] \]
accuracy is improved considerably by utilizing the correlation technique instead of the standard techniques of using an oscilloscope trace of the pulse. Sixty-cycle noise from the main voltage can hamper the accuracy for such a direct measure of the pulse width on an oscilloscope, but this error will be minimized in correlation technique. Therefore, the width of the auto-correlation peak is equal to twice the transit time for the bubble. These relationships can be used to determine the size and velocity of the bubble. Namely, if the width of the light beam is large (that is, greater than the bubble diameter), the velocity of the bubble is given by

\[
\text{Velocity of the Bubble} = \frac{\text{width of the light beam}}{\frac{1}{2} \times \text{(width of the auto-correlation peak)}}
\]

If the width of the light beam is small (that is, less than the bubble diameter), the diameter of the bubble is given by

\[
\text{Diameter of the Bubble} = \text{velocity of the bubble} \times \frac{1}{2} \times \text{(width of the auto-correlation peak)}
\]

As the width of the light beam increases, the auto-correlation curves contain larger negative signals. This nature of the correlation curves can be explained by considering the illustration shown in figure II-7. For simplicity, ten digital data points are chosen in one period of the expected light signal due to gas bubble attenuation. The relative amplitudes of the unattenuated and attenuated signals are chosen to be one and two volts respectively (the signal is shown here after inversion). The steps in computing the auto-correlation function are shown in the table. The computed auto-correlation function is also plotted in this figure, and, as can be seen from this plot, a portion of the correlation curve has negative values for its magnitude. The relative magnitudes of the positive and negative portions of this curve depend on the average of the data points, which in turn depends on the relative number of digital points in the attenuated and unattenuated portion of the light signal. This is sometimes referred to as the duty cycle of the signal
## Figure II-7: Computation of Auto-Correlation Function

The diagram illustrates the computation of the auto-correlation function $\Phi(\tau)$ for a given time series $f(t)$. The computation involves the following steps:

1. **Data Points and Amplitudes**
   - The table lists data points $i$, amplitudes $a_i$, and average values $\bar{a} = \frac{\sum a_i}{n}$.

2. **Fluctuations**
   - The fluctuation $\Delta f(t) = f(t) - \bar{f}$ is calculated for each data point.

3. **Auto-Correlation Function**
   - The auto-correlation function $\Phi(\tau) = \frac{1}{\bar{f}^2} \sum (f(t) - \bar{f})(f(t+\tau) - \bar{f})$ is computed for different lag times $\tau$.

### Table

<table>
<thead>
<tr>
<th>Data Point $i$</th>
<th>Amplitude $a_i$</th>
<th>Average Value $\bar{a} = \frac{\sum a_i}{n}$</th>
<th>Fluctuation $\Delta f(t)$</th>
<th>Auto-Correlation Function $\Phi(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td></td>
<td>-0.2</td>
<td>+0.11</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>$\frac{12}{10} = 1.2$</td>
<td>+0.3</td>
<td>+0.06</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td></td>
<td>+0.8</td>
<td>-0.015</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td></td>
<td>+0.3</td>
<td>-0.04</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td></td>
<td>-0.2</td>
<td>-0.04</td>
</tr>
<tr>
<td>6-10</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b=W/P, where W is the width of the pulse and P is the period). A general expression for the negative portion of the correlation curve can be written as

$$b W I_{\text{min}}^2 (1 - b)$$

where W is the width of the pulse and $I_{\text{min}}$ is the amplitude of the pulse.

The power spectrum can also be determined for the case of single bubbles; however, if they are being generated at a single frequency, the power spectrum will appear as a single impulse at that frequency.

**Nucleate Pool Boiling Expressions**

The auto-correlation function for a boiling system can be obtained using the same optical system as described above. Since nucleate pool boiling is a random phenomena, the auto-correlation curve will have non-zero values close to the origin and drop to zero as the delay time becomes large. If the process were completely random, this function would be a delta function or a spike at zero delay. Since it is not a delta function, the auto-correlation curve contains information about the bubble distribution in nucleate boiling. The characteristic growth and departure times for different bubble sizes can be obtained by plotting the logarithm of the amplitude of the auto-correlation function versus the delay time. Also, since the size of a bubble in nucleate boiling is dictated by the growth time ($t_g$) and the departure time ($t_d$) before the next bubble is formed, the frequency of emission is $1/(t_g + t_d)$ for that particular bubble. These characteristic frequencies for the boiling system can also be determined from the power spectral density. The power spectrum of the boiling system can be obtained by performing a Fourier analysis of the auto-correlation function. Also, since the area under the power spectral density represents the total power, the heat flux of the boiling system is physically related to the area under the spectral density curve.
III. APPARATUS AND EXPERIMENTAL PROCEDURE

The experimental set-up consists of a parallel beam of white light which passes through the boiling system and is attenuated as light is scattered by bubbles rising through the beam. A photomultiplier tube is used to detect the fluctuations in the light signal due to bubble attenuation. The signal is then analyzed by two methods to check the consistency of the results.

Optical Configuration

The optical set-up is shown in Figure III-1. A parallel light beam, obtained using a point source of light and a simple convex lens, was passed through the water tank. The transmitted light was received by a photomultiplier tube to generate the desired electrical signal. Adjustable slits were placed on both sides of the tank in order to collimate the light beam at the desired width. Since the sensitive area of the photomultiplier tube consisted of a circular area about 1-1/2 inches in diameter, the maximum width of the light beam was taken to be approximately 1-inch.

Bubbles were generated within the water tank by two methods. In the study of simple bubbles, single gas (nitrogen) bubbles were generated from an orifice in a submerged tube (as shown in Figure III-1). In the study of nucleate boiling, the bubbles were formed on a horizontal electrically-heated filament.

Power Supply and Filament Mounting

A separate submersion heater (250 watts) was mounted in the water tank with an automatic temperature controller in order to maintain a constant desired bulk temperature for cases in which the heat input to the boiling filament was not sufficient to attain that bulk temperature.

Storage batteries were used to supply power to the filament. The batteries were tapped such that a maximum of 48 volts (with 2-volt increments)
FIGURE III-1. EXPERIMENTAL SETUP
could be supplied to the filament. Batteries were used in order to minimize A.C. noise in the input power supply.

The mounting for the filament is shown in Figure III-2. The filament was clamped between aluminum blocks in order to insure good electrical contact. These were in turn mounted upon a lucite block. The lucite block reduced vibrations induced by the boiling and the agitation of the fluid, and reduced the effects of convection currents below the filament. Also it was important to eliminate the formation of bubbles from the underside of the filament. Bubbles which formed on the underside would coalesce forming larger bubbles before escaping from below the filament. This was prevented by placing a backing (of either Teflon or electrical tape) on the filament. The filament and mounting were then attached to the aluminum electrodes. The aluminum surfaces which were to be exposed to the boiling water were coated with a clear vinyl to minimize electrolysis.

Electronic Apparatus

The two methods used to obtain the power spectral density functions for the system have been described in the previous section. To obtain the power spectrum directly with the analog-filtering-squaring method, a Philbrick Operational Manifold Model RP was used to provide the various stages of amplification and integration. A Krohn-Hite Model 300 band-pass filter was used to obtain the desired frequency band, and a Philbrick Model QS-MIP Quarter Square Multiplier was used to square the signal. Alternately, the signal was input directly to a signal correlator (Princeton Applied Research Model 100). The correlation functions could then be plotted with an x-y recorder.

Experimental Procedure

The investigation proceeded in two stages as follows:

1. Measurements were first made of the power spectral density
FIGURE III-2. HEATING SURFACE CONFIGURATION
and auto-correlation functions for single bubbles. The power spectra were obtained using both of the techniques described previously. To obtain the power spectra by the analog method, the desired frequency was selected (by setting both the low cut-off and high cut-off of the band-pass filter to that frequency). This signal was then squared and integrated over a measured time interval. The power spectral density at that frequency was directly proportional to the integrated voltage and inversely proportional to the time interval and the width of the frequency band. The power spectral density was calculated at each frequency using the relationship,

\[ P(f) \propto \frac{V}{\Delta f \cdot t} = K \frac{V}{f \cdot t} \]

where \( V \) is the integrated voltage over the time interval \( t \),
\( \Delta f \) is the width of the frequency band which is directly proportional to the frequency, \( f \), and
\( K \) is a constant of proportionality (which also includes the gains of the various stages of amplification).

The power spectra were also determined by manually digitizing the auto-correlation curves obtained using a signal correlator, and performing a Fourier analysis with an IBM 7040 computer. The results of these two methods were compared in order to determine their accuracy and consistency. These functions were then analyzed to determine the bubble size, velocity and frequency of generation. During these early studies, the possible effects upon the results of various parameters of the optical system, e.g., light intensity, beam width and location, lens arrangement and magnification, were investigated. In the initial work, a cylindrical tank was used which when filled with water served as a cylindrical lens. Although this was found to have very little effect, it was eliminated by constructing a rectangular aluminum tank (capacity approximately
1700 cc.) with two opposing walls made of plate glass.

2. Having interpreted the power spectra and correlation functions obtained from single bubbles, the optical system was then used to obtain these functions for a nucleate pool boiling system. All nucleate boiling experiments were performed with water at atmospheric pressure. The bulk temperature was maintained constant with an immersion heater which was automatically controlled. Various metals were used for filaments, including nichrome, monel, alloy 875, and platinum. (The particular type of filament, as well as its dimensions, will be with the results.) The filament was maintained in a horizontal position and heated electrically to provide nucleate boiling. As in the first studies, the power spectra for the nucleate boiling system were obtained by two methods. However, the indirect method, i.e., Fourier analysis of the auto-correlation function, was found to be the more desirable method of obtaining the power spectra. In obtaining the power spectral density directly, the spectrum is obtained point-by-point, a procedure requiring as long as fifteen minutes to perform. Therefore, even though an attempt was made to maintain the properties of the nucleate boiling system constant for the duration of experimental runs, the assumption of stationarity for a period of several minutes is certainly tenuous at best. Alternatively, the auto-correlation function for the system could be obtained in a short period of the order of seconds, making valid the assumption of stationarity.

The power spectra and correlation functions for the nucleate boiling system were determined for various heat fluxes at a specific degree of subcooling. The effect of surface tension was studied by adding a surface active agent (Aerosol) to the water. The above procedure (various heat fluxes at various degrees of subcooling) was then repeated for the cases with reduced surface tension.
IV. RESULTS

The techniques of noise analysis were first used to determine the power spectra and auto-correlation functions for single gas bubbles rising in water. This portion of the investigation served two purposes: (1) a technique was developed for determining the two-dimensional size and local velocity of a bubble, and (2) the relationship between the power spectrum and auto-correlation functions for bubbles and their size, velocity and frequency of generation was established. Also, by comparing the results of the two methods of obtaining the power spectra for consistency, confidence was established in both methods.

Figure IV-1 shows a set of typical power spectra for single bubbles obtained directly by the analog method. Note that peaks occur at the frequency of generation of the bubbles and also, for the cases of low frequency of generation, at harmonics of that fundamental frequency. In this figure, for the case of 1.6 bubbles per second, the first peak occurs at 6.4 cycles per second. The power spectra for these cases were also determined indirectly from the auto-correlation functions. However, since the bubbles were being generated at a single frequency, the resulting power spectra consisted of simply an impulse or "spike" at a frequency equal to the frequency of generation. In all the cases, the results obtained by these two methods agreed very well with each other.

Figure IV-2 shows typical correlation curves for the case of single bubbles for various widths of the parallel light beam. The ordinate is the auto-correlation function and the abscissa is the delay time. In this case, the frequency was 5.7 bubbles per second and the beam width was varied from 0.58 to 1.00 inches. The first two correlation curves are for the same slit width but different gains. The remaining two curves are obtained with increased widths of the parallel beam. Note that, as explained previously, the negative
portion of the auto-correlation curve increases as the intensity of the light is increased (i.e., greater width of the light beam). The important information on these curves is the width of each peak. As described earlier, the width of the curve is related to the speed of the bubble, and the height is related to the size of the bubble. In Figure IV-3, the base width of these correlation curves is plotted as a function of the width of the light beam. Both the size and velocity of the bubble can be determined from this figure. As the figure illustrates, the diameter of the bubble is directly proportional to the base width of the auto-correlation peak for narrow beam widths. Also, the velocity is inversely proportional to the slope of this curve for large beam widths. Note that it is not necessary to obtain many points using a variety of beam widths in order to determine the size and velocity of the bubble. The velocity can be determined from a single measurement. Using a beam width approximately twice the diameter of the bubble will provide excellent results representing the average velocity of the bubble over that distance. Knowing or having determined the velocity of the bubble, its size can be determined using a narrow beam of parallel light. A velocity of 22.71 cm/sec was obtained for a bubble size of 7.8 mm diameter. The experimental values are in good agreement with published literature as well as calculated theoretical values for nitrogen gas bubbles in water at atmospheric pressure.

The techniques of noise analysis were next used to determine the power spectra and auto-correlation functions for a nucleate boiling system. This portion of the investigation served two purposes: (1) a simple technique was developed for determining the power spectra of a nucleate boiling system, and (2) the effects upon the power spectrum of nucleate boiling of the heat flux, the degree of subcooling, and the surface tension of the fluid were determined.
DETERMINATION OF SIZE AND VELOCITY OF SINGLE BUBBLES

\[ m = 2v^{-1} \]

\[ v = 22.71 \text{ cm/sec} \]

\[ d = vt \]

\[ d = 7.3 \text{ mm} \]
A set of typical power spectra for nucleate boiling is shown in Figure IV-4. These were obtained directly using the analog method. A 0.3" x 3" filament of 0.006" thick nichrome ribbon was used to produce the boiling in water at atmospheric pressure. The heat flux and liquid bulk temperature for each curve is given in the figure. The total area under each of the spectral density curves is physically related to the heat flux of the filament. This relationship is shown in Figure IV-5, which is a plot of the total area under the power spectral density curves versus the total power input to the filament. The area under these spectral density curves represents the latent heat transfer contribution to the total heat flux. For a power input below the heat flux necessary to initiate boiling, heat is transferred by free convection. Clearly then, the area under these curves will become zero for some non-zero heat flux. An estimate of the latent heat transfer can be obtained from Figure IV-5. If the total heat flux were due to evaporation, the curve of Figure IV-5 would pass through the origin. Also, it is known that the fraction of the total heat flux due to latent heat transfer increases with increasing heat flux (below the critical heat flux). Therefore, the ratio of the actual area under the spectral density curve to the "area under the spectral density curve if the only mode of heat transfer were latent heat transfer" represents the percent of the total heat flux due to evaporation. To illustrate, consider the first data point of Figure IV-5, which corresponds to approximately forty percent of the critical flux. The ratio defined above is 0.545. This indicates 54.5 percent of the total heat flux is due to evaporation. These values are in excellent agreement with the values determined by Graham and Hendricks. They have reported the contribution due to evaporation to be 57% for a heat flux equal to 40% of the critical flux.

Figure IV-6 shows a typical set of auto-correlation curves for nucleate boiling obtained using a signal correlator. These curves were obtained using
FIGURE TV-5 AREA UNDER POWER SPECTRA VERSUS POWER INPUT
FIGURE IV-6
TYPICAL CORRELATION CURVES FOR NUCLEATE BOILING

HEAT FLUX = 3.03 x 10^5 Btu/hr-ft^2
LIQUID BULK TEMPERATURE = 95.5 °C
two light beams located at different heights above the heated filament. The lower two curves are the auto-correlation functions for the two light beams and the top curve is a cross-correlation of the two light signals. The correlation time obtained from this cross-correlation function could be used to obtain the velocity of bubble propagation in nucleate boiling.

The auto-correlation functions for nucleate boiling were further analyzed in two ways. As described previously, the correlation curves were manually digitized and a Fourier analysis was performed using an IBM 7040 computer. Figure IV-7 shows the smoothed power spectral density obtained from \( \phi_{AA}(\omega) \) of Figure IV-6. The predominant peak in the spectra occurs around 25 cps, with smaller peaks around 55 cps and 125 cps. These peak frequencies correspond to the frequencies with which bubbles are emitted from the heated surface.

Alternately, the auto-correlation curves were analyzed to obtain characteristic time constants which are related to the growth and departure times for the bubbles. These time constants were determined by plotting \( \ln \phi(\tau) \) versus \( \tau \). This is illustrated in Figure IV-8 for the auto-correlation curve \( \phi_{AA}(\tau) \) of Figure IV-6. The calculated time constants shown on the curve are 6.25 msec., 2.82 msec., and 1.27 msec. These correspond to frequencies of 25.5 cps., 56.5 cps., and 125 cps., respectively. These values are seen to be the peak frequencies of Figure IV-7.

The effects of various parameters upon the power spectra of a nucleate boiling system reflect the effects of these parameters upon the frequency of bubble emission. Various investigators have found the following relationship between the frequency of bubble emission and the bubble departure diameter to exist:

\[
f_D = 0.59 \left[ \frac{a \left( \rho_v - \rho \right)}{\rho_D^2} \right]^{0.5} \]  

(16)
Figure IV-8 Determination of time constants for auto-correlation function $\phi_{AA}(\tau)$ of Figure IV-6.

$\alpha_1 = 6.25 \text{ msec.}$

$\alpha_2 = 2.82 \text{ msec.}$

$\alpha_3 = 1.27 \text{ msec.}$

Time scale: 1 sec = 2240 div.
For saturated water at one atmosphere, the constant value of this product $f_d$ is $93.0 \text{ mm/sec}$. Although the value of this product has been found to vary considerably among various individual nucleation sites, it is meaningful to consider an average value of the frequency $\bar{f}$ and an average size $D_d$. The effect of various parameters upon the power spectra of the nucleate boiling system will be discussed in terms of values of $\bar{f}$ which is inversely proportional to the bubble size, and $\bar{E} = 1/\bar{f}$ which is directly proportional to the sum of the bubble growth and departure times and to the bubble size.

The effect of the power input, the degree of subcooling, and the liquid surface tension upon the power spectrum of a nucleate boiling system is illustrated in the following figures. These data were obtained using a $3/16'' \times 3-7/32''$ filament of 0.004 Alloy-875 ribbon.

The effect of the power input to the filament upon the power spectrum is shown in Figure IV-9. These curves were obtained with the same liquid bulk temperature (18°F subcooled). Note that peaks occur in the spectrum for the lower heat flux around 40 cps. (4.0 msec.), 50 cps. (3.2 msec.), and 72 cps. (2.2 msec.), whereas for the higher heat flux the predominant frequencies occur around 32 cps. (5.0 msec.) and 60 cps. (2.6 msec.). As can be seen directly from values of the mean time $\bar{E}$, the bubble diameter increases by 25% with the higher heat flux.

In order to utilize the information contained in the power spectrum to obtain the bubble sizes, it is necessary to have some knowledge of the nucleation site density and distribution for the portion of the heated surface seen by the light beam. This is required if the width of the filament used to obtain the power spectrum was large enough to permit more than a single row of nucleation sites. That is, as the beam of light is passing through the boiling system, the bubbles rising from the surface which originate from several nucleation sites that are in the same line as the beam of light will appear as
bubbles rising from a single nucleation site at a higher frequency. Several alternatives exist for determining the correct frequency of bubble emission. The simplest method would be to use a very narrow filament (e.g., a very fine wire) which could provide only a single row of nucleation sites normal to the direction of the light beam. Also an estimate of the number of nucleation sites could be obtained by placing a substance (e.g., a dye) in the water which would preferentially deposit at the nucleation sites, permitting their number to be counted directly. Alternately, one could obtain the power spectra for heated filaments of various widths maintaining all other conditions the same in order to determine the effect of multiple rows of nucleation sites.

For the present study, a value of the departure diameter will be calculated, thereby determining the average frequency. One of the earliest and best known correlations of departure diameters presented by Fritz\textsuperscript{23} is:

\[ \alpha = 0.0119 \beta \frac{2 \gamma_c \sigma}{\sqrt{\gamma_c (\rho_L - \rho_v)}} \]

where \( \beta \) is the contact angle in degrees.

Assuming the bubble volume to be that of a sphere, the Fritz equation becomes

\[ D_d = 0.0208 \beta \frac{2 \gamma_c \sigma}{\sqrt{\gamma_c (\rho_L - \rho_v)}} \]

Zuber\textsuperscript{1} pointed out that equation (17) does not always agree with experiment and proposed the relationship:

\[ D_d = \left[ \frac{6 \gamma_c \sigma}{\epsilon (\rho_L - \rho_v)} \frac{k \Delta T}{Q} \right]^{1/3} \]

where \( \Delta T \) is the degree of subcooling, \( k \) is the thermal conductivity, and \( Q \) is the heat flux.
An average value determined by equations (17) and (18) will be used to determine the average frequency. The curve of Figure IV-9, corresponding to a heat flux of $2.89 \times 10^5$ Btu/hr-ft$^2$ ($\Delta T = 18$ F$\degree$) having a predominant peak at 32 cps, will be used for this standardization procedure. A value of $7.8 \times 10^{-3}$ ft., or 2.38 mm., is obtained using equation (17) with $\beta = 45^\circ$. Equation (18) yields a value of $2.2 \times 10^{-3}$ ft., or 0.71 mm. An average value of the departure diameter may be taken to be 1.54 mm. Using equation (16) with this calculated value for the departure diameter yields an average frequency of 9.6 cps. Therefore the ratio of the observed frequency to the calculated frequency is 3.34. Using this ratio, the predominant frequencies of the power spectra can be related to bubble sizes. Thus the peaks occurring in the spectrum for the lower heat flux of Figure IV-9 around 40 cps., 50 cps. and 72 cps. correspond to bubble diameters of 1.24 mm., 1.0 mm. and 0.70 mm., respectively. Similarly, the peaks occurring in the spectrum for the higher heat flux around 32 cps. and 60 cps. correspond to bubble diameters of 1.54 mm. and 0.82 mm., respectively. As stated above, the maximum bubble diameter increased 25% for the higher heat flux. The bubble diameters can also be calculated by assuming the bubble growth time to be approximately one-half the time constants given above. Hsu and Graham$^2$ have calculated bubble diameters as a function of growth time, and the values for the bubble diameters reported above agree quite well with their calculated values.

Figure IV-10 shows the effect of the liquid bulk temperature upon the power spectrum. These curves were obtained with the same heat flux for various degrees of subcooling. The effects are summarized in the following table which shows the appearance of lower frequencies corresponding to larger bubble sizes with higher liquid bulk temperatures.
The effect of the degree of subcooling is seen to have a more pronounced effect upon the power spectrum than does the heat flux.

The curves of Figure IV-11 represent the spectral density for a single heat flux ($1.91 \times 10^5$ Btu/hr-ft$^2$) and a single degree of subcooling ($18^\circ$F) for two different liquid surface tensions. As shown in the figure, the peaks are shifted to much lower frequencies as the surface tension is reduced. With a liquid surface tension of 27.1 dynes/cm., the peaks occur around 0-2 cps. (which is below the spectral resolution) and 11 cps. (14.5 msec.)

Figures IV-12 and IV-13 are for reduced liquid surface tension (27.1 dynes/cm.) and show the effect of the heat flux and the degree of subcooling.

The curves of Figure IV-12 are for a liquid bulk temperature of 194°F. As was shown in Figure IV-9, as the heat flux is increased, lower frequencies occur corresponding to larger bubble sizes. Figure IV-13 shows the effect of the degree of subcooling is reduced for the case of lower liquid surface tension. As shown in this figure, the lower frequencies, 0-2 cps. and 10 cps., predominate for both liquid bulk temperatures.

This portion of the investigation has demonstrated that the effects of various parameters upon the frequency response of nucleate boiling can be obtained using the techniques of noise analysis, and a comprehensive and systematic investigation of the effect of parameters which influence nucleate
HEAT FLUX = $8.05 \times 10^{-4}$ Btu/hr-ft$^2$
SURFACE TENSION = 27.1 dynes/cm

$\Delta T = 18^\circ F$
$\Delta T = 9^\circ F$

**Figure 12.3** Effect of Frequency of Shaking upon Spectrum of Refracted Liquid Surface Tension.
boiling (including heat input, degree of subcooling, liquid surface tension, liquid viscosity, pressure and nucleation site size and density distribution) should be performed.
REFERENCES


