LINEAR MODULATOR
FINAL REPORT
CONTRACT NAS8-25987
MODIFICATION 4

April 1972

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Prepared for

National Aeronautics and Space Administration
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FOREWORD

This report was prepared by Martin Marietta Corporation under Contract NAS8-25987, Exhibit A, Paragraph II.2.
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          for FDM Systems
INTRODUCTION

A study of frequency division multiplexing (FDM) systems was made for the purpose of determining the system performance that can be obtained with available state-of-the-art components. System performance was evaluated on the basis of past experience, system analysis, and component evaluation.

The system study was specifically directed to the area of FDM systems using sub-carrier channel frequencies from 4 kHz to 200 kHz and channel information bandwidths of dc to 1, 2, 4, 8, and 16 kHz. The evaluation also assumes that the demodulation will be from a tape recorder which produces frequency modulation of +1% on the signal due to the tape recorder wow and flutter. For the modulation system it is assumed that the pilot and carrier channel frequencies are stable to within +.005% and that the FM on the channel carriers is negligible. The modulator system was evaluated for the temperature range of -20° to +85° while the demodulator system was evaluated for operation at room temperature.

A large part of the study was devoted to determining the relative advantages and disadvantages of different types of modulation methods for FDM systems such as single sideband (SSB), double sideband (DSB), and quadrature double sideband (QDSB). The circuits or system requirements for certain key portions of FDM systems were determined and tested. The degree of performance of each of these circuits is reported along with expected performance of a complete FDM system. Theoretical mathematical analysis is provided to show the requirements and limiting factors on the performance of different FDM systems.

When a number of correlated sub-carrier channels are multiplexed together and spaced so as to have related frequencies and phase (that is all sub-carrier frequencies being exact multiples or sub-multiplex of each other) then one channel can be reserved as a "PILOT" frequency and all other sub-carrier channels can be reconstructed both in frequency and phase from this one pilot. This makes it possible to use synchronous detections for SSB, DSB, and QDSB modulated channels. This also makes it possible to maintain time correlation between modulation information of all of the channels at the demodulator outputs.
The following sections discuss the system requirements for each of the above systems (DSB, QDSB, and SSB) for FDM systems having sub-carrier channel spacings of 4 kHz multiples from 4 kHz to 200 kHz and channel information bandwidths of 1 kHz to 16 kHz. The degree of time correlation between channels as well as time delay of different frequencies within an information channel is evaluated for different modulation systems.
I. FREQUENCY DIVISION MULTIPLEXING SYSTEM

A FDM modulation (multiplexing) system Figure I-1 consists of a number of modulators with sub-carrier frequencies spaced at 4n kHz interval where n is the number of desired channels. Each channel is modulated with audio information frequencies \( \omega_{m_1}, \omega_{m_2}, \omega_{m_3}, \ldots, \omega_{m_n} \). The output of all of the modulators are summed as shown in Figure I-1.

The frequency synthesizer provides the sub-carrier frequencies by starting with a single "clock" frequency and then deriving all of the channel frequencies from it. Therefore, all of the channels are synchronously related both in frequency and phase allowing for synchronous detection of all of the channels in the demodulator system by reconstructing all of the reinserted demodulator carrier frequencies in a similar synthesizer in the demodulator system. A pilot frequency is provided by the modulator synthesizer which is used as the clock frequency in the demodulation system.

\( M_1, M_2, M_3 \) etc. are modulators which provide SSB, DSB, or QDSB modulation of the information data (depending on the modulation method used) at the various channel frequencies.

Summer A is used to sum all of the channels together into one composite signal which is then gain controlled by an AGC circuit. A reference tone is also summed in summer A which is used for a phase ambiguity resolver and an amplitude restorer in the demodulator system.

Summer B sums the Pilot tone and the AGC'd composite signal.

A frequency division demodulation system is shown in Figure I-2 and consists of many of the same system components as the modulator with the addition of a pilot demodulator, ambiguity resolver, and amplitude restorer.
FREQUENCY SYNTHESIZER

SUMMER A

SUMMER B

AGC

TO RF CHANNEL

FDM MODULATION SYSTEM

FIGURE 1-1
FDM DEMODULATOR SYSTEM

FIGURE-I-2
II. FDM SYSTEM EVALUATION

For the purpose of evaluating the overall system performance the operation of the individual system components will be discussed first, then the influence of each component on the overall system performance will be discussed. The overall system performance that can be expected for different system (SSB, DSB, and QDSB) with the state-of-the-art devices will be shown.

A. Single Conversion DSB Modulator - If a Multiplier such as the motorola MC1594L or MC1595L or the Analog Devices AD530 or 422K is used as modulator the output will be related to the input signals as follows:

\[ e_o = A \cos \omega_m t \cdot B \cos \omega_c t = k \frac{AB}{2} [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \]

where \( \omega_m \) is the audio modulating frequency and \( \omega_c \) is the channel carrier frequency. This shows that the output signal \( e_o \) is composed of two parts, each having an amplitude of \( k \frac{AB}{2} \) and a frequency of \( \omega_c + \omega_m \) and \( \omega_c - \omega_m \) which are the upper and lower sidebands \( \omega_m \) above and below \( \omega_c \). \( \omega_c \) and \( \omega_m \) are both balanced out in the modulation process, therefore, the signal produced is DSBSC. From this it can be seen that half of the carrier amplitude product \( AB \) is contained in each sideband. \( k \) is the gain constant of the multiplier and will be ignored in further modulator and demodulator discussion since it does not affect the modulating process.

B. Double Conversion DSB Modulation - Another method of producing DSBSC modulation is first to produce DSB at
some higher common frequency such as 256 kHz for all channels in a 4-200 kHz channel system and then translating the double sideband signals around 256 kHz down to the desired channel frequencies by mixing again in a second modulator with a translation frequency of \((2\pi 256 + \omega_c)\)kHz. The modulation system is shown below.

\[ A \cos \omega t \rightarrow M_1 \rightarrow \text{LPF}_1 \rightarrow M_2 \rightarrow \text{LPF}_2 \]

\[ B(2\pi 256)t \rightarrow (2\pi 256 + \omega_c)t \]

C. Single Conversion SSB Modulation - The block diagram below shows a method of generating SSB using a single conversion method. The desired channel frequency is fed directly into two true multipliers at 0° and 90° along with the audio frequencies shifted with a polyphase network to provide two audio components 90° out of phase with each other.

\[ \omega_c/0^\circ \]

\[ \omega_m/0^\circ \]

\[ \omega_m/90^\circ \]

\[ \omega_c/90^\circ \]
If we consider the inputs of $M_1$

$$\cos \omega_c \cdot \cos \omega_m = \cos(\omega_c + \omega_m) + \cos(\omega_c - \omega_m)$$
then the inputs to $M_2$ are:

$$\sin \omega_c \cdot \sin \omega_m = \cos(\omega_c + \omega_m) - \cos(\omega_c - \omega_m)$$

It can be seen that if these equations are added the upper sidebands are added and the lower sidebands are subtracted or cancelled leaving only the upper sideband as an SSB signal. Subtracting the equations would produce the lower sideband with the upper sideband being cancelled.

For the single conversion system the carrier frequencies must be sine waves with all harmonics down greater than 50 dB for all of the channel frequencies below 68 kHz so as not to produce interference in the higher channels. Further discussion of this will be made in the synthesizer section.

D. Double Conversion SSB Modulation - The block diagram below shows a method of generating SSB by a double conversion system. The polyphase network is the same as for the single conversion system but modulators $M_1$, $M_2$, and $M_3$ are balance modulators which can use square wave carrier inputs.
For all channels a SSB signal is generated at a carrier frequency of 256 kHz. The higher order harmonics are then removed by LPF and then the resultant SSB signal is translated down to the desired channel by mixing in M, with a \((256 + 4n)\) kHz square wave frequency where \(4n\) kHz is the final desired channel frequency. The actual output of \(M_3\) is a DSB signal with sidebands at \(4n\) kHz above and below 256 kHz. The outputs of all of the SSB channels generated by this method are then summed and filtered in one common filter to remove all of the upper sideband signals generated in \(M_3\) leaving only the 4 to 200 kHz SSB channels. The double conversion system has the advantage that only square wave carrier frequencies are required by the synthesizer but has the disadvantage that LPF for all channels must track in phase for proper demodulation. This will be further discussed in the synthesizer section.

E. Demodulator - A true multiplier can also be used as a demodulator as shown below. Here the signal inputs are the two sidebands \((\omega_c - \omega_m)\) and \((\omega_c + \omega_m)\). If the carrier \(\omega_c\) which was originally suppressed in the modulation process is reconstructed in frequency and phase and reinserted in the modulator M as shown the output audio frequency is recovered as follows:
\[ e_o = \frac{1}{2} \cos[(\omega_c + \omega_m + \omega_c)t] + \frac{1}{2} \cos[(\omega_c - \omega_m - \omega_c)t] \]
\[ + \frac{1}{2} \cos[(\omega_c + \omega_m - \omega_c)t] + \frac{1}{2} \cos[(\omega_c - \omega_m + \omega_c)t] \]
\[ = \frac{1}{2} \cos[(2\omega_c + \omega_m)t] + \frac{1}{2} \cos[(2\omega_c - \omega_m)t] + \cos\omega_m t \]

which after filtering by LPF leaves \( \cos\omega_m t \) which is the original audio signal used to modulate the sub-carrier channel.

Shown below is a double conversion demodulation system which has some advantages over a single conversion as will be shown later.

In the double conversion demodulation system the information signal is up converted to 256 kHz which is well above the highest channel frequency of 200 kHz and then down converted to base band as shown in the single conversion system.

If the input signal is \( \cos(\omega_c + \omega_m)t + \sin(\omega_c + \omega_m)t \) which is QDSBSC, the demodulation process would be the same with the exception that in the case of single conversion demodulation two demodulators would be used with the reinserted carrier to the second one being \( \sin\omega_c t \).

In the case of the double conversion demodulator the two demodulators would follow \( \text{LPF}_1 \) with the reinserted carrier to the \( M_1 \) modulator \( 90^\circ \) out of phase from the phase into modulator \( M_2 \) or \( \sin(2\pi 256)t \). The demodulation circuit would be as shown in below.
The demodulated output amplitude of each channel is proportional to the Cos of the angle between the original carrier phase and the reinserted carrier phase. Therefore, two channels at the same frequency, but 90° out of phase can be demodulated and separated by reinserting carriers 90° out of phase. The amount of feedthrough of the quadrature channel is proportional to the Cos of the phase error. LPF₂ is provided for phase tracking of the demodulation carrier phase with the signal phase error caused by LPF₁. The following analysis shows the effects of phase and amplitude error caused by filter miss-alignment of the system, jitter caused by FM, etc. The analysis will be made on a double conversion quadrature double sideband demodulation system which covers most of the areas where the effect of errors occurs. Comparisons are made for similar errors in SSB systems.
III. DOUBLE FREQUENCY TRANSLATION DEMODULATION OF QDSBSC, PHASE AND AMPLITUDE ANALYSIS

The block diagram below shows a double conversion demodulation system consisting of balanced modulation $M_1$ and $M_2$ and low pass filter $LPF_1$ and $LPF_2$.

The audio phase shift caused by $LPF_2$ and also system gain factors will be disregarded in the following analysis.

The input composite signal representing the quadrature components of one QDSBSC channel is:

$$V_1 = \cos(n\omega_c + p)t + \sin(n\omega_c + q)t$$

$n\omega_c$ = channel carrier frequency

$p = 0^\circ$ channel modulation frequency

$q = 90^\circ$ channel modulation frequency

The carrier frequency input $V_A$ to $M_1$ is the sum of the channel frequency to be demodulated and the translation frequency (selected as 256 kHz). $V_A$ is a square wave and unwanted sidebands caused by the odd harmonics will be present at the output of $M_1$. $LPF_1$ provides attenuation for these harmonic sidebands such that no inter-modulation between these sidebands and the $V_B$ fundamental and odd harmonics will be present at the output of $M_2$. 
Neglecting the odd harmonics of $V_A$, we have:

$$V_A = \cos[(2\pi (256 + n\omega_c)t + \theta_1]$$

Where $\theta_1$ = the phase of the 256 kHz carrier relative to the phase of the wanted channel and:

$$V_2 = \cos[(2\pi 256 \pm p)t + \theta_1] + \sin[(2\pi 256 \pm q)t + \theta_1]\cos[(2n\omega_c + 2\pi 256 \pm p)t + \theta_1] + \sin[(2n\omega_c + 2\pi (256 \pm q)t + \theta_1]$.$

Neglecting the modulator sum frequencies (terms containing $2n\omega_c$) since these will be outside of the pass band of LPF1 and after the down translation in M2 they will be outside of the pass band of LPF2, and using the notations from the filter characteristics below, the output from LPF1 will be:

$$V_3 = \cos (2\pi 256 + p)t + \theta_1 + e + e_2 + \cos (2\pi 256 - p)t + \theta_1 + e - e_1$$
Translating to baseband in $M_2$ and assuming LPF 2 to attenuate odd harmonics of $V_B$ and adjacent QDSBSC channels we have:

$$V_B = \cos(2 \pi 256t + \theta_2)$$

where $\theta_2$ = the phase of the 256 kHz carrier relative to the wanted channel at the $M_1$ input.

and $V_o = \cos[-pt + \theta_2 - \theta_1 - e - e_2]$  

$$+ \cos[pt + \theta_2 - \theta_1 - e + e_1]$$

$$+ \sin[-qt + \theta_2 - \theta_1 - e - e_2]$$

$$+ \sin[qt + \theta_2 - \theta_1 - e + e_1]$$

Simplifying by letting $\phi = \theta_2 - \theta_1 - e$, we have:

$$V_o = \cos pt \cos(\phi - e_2) + \sin pt \sin(\phi - e_2)$$

$$+ \cos pt \cos (\phi + e_1) - \sin pt \sin (\phi + e_1)$$

$$- \sin qt \cos (\phi e_2) + \cos qt \sin (\phi - e_2)$$

$$+ \sin qt \cos (\phi + e_1) + \cos qt \sin (\phi + e_1)$$

A. Linear Phase Slope at Operating Point in LPF - In the following analysis linear* phase slope includes all cases where $e_1 = e_2 = e$ and $e_1 = e_2 = e$

The above demodulator output equation reduces to:

$$V_o = \cos \phi \cos(pt + e_1) - \sin \phi \cos(qt + e_q)$$

* NOTE: It is not strictly necessary for the phase slope to be linear. Some curvature can be tolerated providing the requirement $e_1 = e_2$ is met.
As can be seen the output signal is dependent upon the total demodulator phase $\theta$. If the COSINE (or $0^\circ$) channel is the wanted signal then $\theta$ must be $0^\circ$, and the output is Cos-channel.

$$V_o = \cos pt \cos e_p - \sin pt \sin e_p = \cos (pt + e_p)$$

If the SINE (or $90^\circ$) channel is the wanted signal then $\theta$ must be $90^\circ$ and the output is:

**Sin - Channel:**

$$V_o = \cos qt \cos e_q - \sin qt \sin e_q = \cos (qt + e_q)$$

In a QDSBSC system quadrature feedthrough is the ratio between the wanted output signal and the unwanted signal from the quadrature channel. In dB the quadrature feedthrough is

**Cos - channel:**

$$20 \log \frac{\cos \theta}{\sin \theta} = 20 \log \cot \theta$$

**Sin - channel:**

$$20 \log \frac{\sin \theta}{\cos \theta} = 20 \log \tan \theta$$

For a required quadrature suppression of 50 dB (0.3%) the total demodulator phase must be:

**Cos - channel:**

$$\theta = 0^\circ \pm 0.18^\circ$$

**Sin - channel:**

$$\theta = 90^\circ \pm 0.18^\circ$$

The amplitude loss of the wanted signal for $\pm 0.18^\circ$ phase error will be 0.01%.

In otherwords, if LPF has a linear phase slope, the phase $\theta$ of a double conversion system must be adjusted to $90^\circ$ $+0.18^\circ$ with respect to the carrier phase of the quadrature input channel if 50 dB suppression of quadrature feedthrough is to be obtained. It should be noted that the total demodulator phase $\theta$ equals $\theta_2 - \theta_1$ so that to adjust the total phase $\theta$ only $\theta_2$ or $\theta_1$ needs to be adjusted.

In a multichannel system phase scattering should be employed to reduce the synchronous "pile-up" of the amplitude of the composite signal. In double conversion demodulation this phase scattering is easiest accomplished by scattering the 256 kHz carrier phases. Since any one fixed phase of the 256 kHz signal may be common in more than one channel, the demodulator phase adjustment should be on the CH + 256 kHz carrier. In the above discussion this means that $\theta_1$ is the fixed phase and $\theta_2$ is adjustable.
Examining the equations for the respective Cos and Sin channel output signals:

\[ V_o = \cos(\pi t + e_p) \cos - \text{channel} \]

\[ V_o = \cos(\pi t + e_q) \sin - \text{channel}. \]

We see that the output signals contain the wanted audio signals Cos \( \pi t \) and Cos \( \pi q \), but with a phase delay of \( e_p \) and \( e_q \) respectively. Referring back to the graph of the LPF filter characteristics we see that this audio delay is dependent on two factors: the slope of the filter phase; and the frequency of the sidebands with respect to the 256 kHz center frequency. For a linear phase slope in LPF, the channel delay is proportional to the filter slope and also proportional to the frequency of the channel audio signal. From a system point of view delay time is undesirable, however, only phase correlation between channels is the normal requirement in the FDM systems we are concerned with here. That is, for a given audio frequency the phase delay from channel to channel must be equal. For the double conversion system this requirement is met by having matched phase slopes on LPF in all channels.

B. Non-Linear Phase Slope in the Low Pass Filter - Consider the case when the slope of the low pass filter LPF in the 256 kHz signal line is non-linear; i.e., \( e_1 \neq e_2 \). The following analysis assumes a matching filter in the 256 kHz carrier line so that the total phase \( \phi \) is independent of the \( \pm \% \) frequency changes. It is also assumed that the magnitudes of \( e_1 \) and \( e_2 \) are independent of these carrier frequency changes.

The following examples show the effect of non-linearity on the output signal for the cos channel where \( \phi = 0^\circ \). From Section III the output equations with \( \phi = 0^\circ \) we have:

\[ V_o = \cos \pi t \cos e_1 p - \sin \pi t \sin e_1 p \]

\[ + \cos \pi t \cos e_2 p - \sin \pi t \sin e_2 p; \quad \text{audio amplitude and phase} \]

\[ - \sin \pi q \cos e_1 p - \cos \pi q \sin e_1 p \]

\[ + \sin \pi q \cos e_2 q - \cos \pi q \sin e_2 q; \quad \text{quadrature feedthrough} \]
Example 1: $e_p = e_q = 3^\circ$; $e_1 = 3.3^\circ$; $e_2 = 2.7^\circ$

Audio: $\cos pt \ (.9983 + .9989) - \sin pt \ (.05756 + .04711)$

$= 1.9972 \cos pt - .10467 \sin pt = 1.999 \angle 3^\circ$; the angle being the audio phase delay

Feedthrough: $\sin qt \ (.9983 - .9989) + \cos qt \ (-.05756 + .04711)$

$= -.0006 \sin qt - .01045 \cos qt = .007 \angle 177^\circ$

Feedthrough $\% = \frac{.007}{2} \times 100\% = 5\%$

Example 2: $e_p = e_q = 3^\circ$; $e_1 = 3.2^\circ$; $e_2 = 2.8^\circ$

Feedthrough: $\sin qt \ (.9984 - .9988) + \cos qt \ (-.05582 - .04885)$

$= -.004 \sin qt - .00694 \cos qt = .007 \angle 177^\circ$

Feedthrough $\% = \frac{.007}{2} \times 100\% = .35\%$

Example 3: $e_p = e_q = 30^\circ$; $e_1 = 30.2^\circ$; $e_2 = 29.8^\circ$

Audio: $\cos pt \ (.8643 + .8678) - \sin pt \ (.5030 + .4970)$

$= 1.7321 \cos pt - \sin pt = 2.000 \angle 30^\circ$; the angle being the audio phase delay.

Feedthrough: $-.0035 \sin qt - .006 \cos qt = .007 \angle 150^\circ$

Feedthrough $\% = \frac{.007}{2} \times 100\% = .35\%$

NOTE: The same results will be obtained for the Sin-Channel by letting $\emptyset = 90^\circ$ in the demodulator output equation.

A specification of filter slope of $\pm 0.18^\circ$ maximum deviation from an absolute straight line of phase versus frequency would obviously provide the 50 dB quadrature suppression. However, this is a stricter specification than required.

The limiting conditions for quadrature suppression of 50 dB on the filter phase slope are as follows.
1) The phase matching of LPF\textsubscript{1} versus modulation frequency above and below the operating frequency must be such that the difference $\epsilon_1 - \epsilon_2 \approx 0.36^\circ$ over the operating frequency range.

2) The frequency operating range for a 256 kHz translation frequency of 16 kHz channel bandwidth and $\pm 1\%$ frequency modulation would be 237 to 275 kHz.

3) It should be noted that LPF\textsubscript{1} need not be a low pass filter. A bandpass filter may be substituted providing the passband covers the frequency range 237 to 275 kHz.

C. Effect of Amplitude Ripple in the Filter Passband - Up to this point we have disregarded signal amplitude factors and channel gain. This was done with the assumption that the gain was independent of frequency within the passband. Because of amplitude ripple in the filters this is not necessarily true for the quadrature channel under consideration. Let us first consider the effect on the wanted channel signal. Referring to the block diagram for the double conversion demodulator we assume the output of the first multiplier to be:

$$V_2 = A \cos(2 \pi 256 + pt) + A \cos(2 \pi 256 - pt).$$

That is, the two sidebands in the Cos channel have equal amplitude. On the output of LPF\textsubscript{1} we have:

$$V_3 = A_1 \cos (2 \pi 256 + pt) + A_2 \cos 2 \pi 256 - pt;$$

where the amplitude of the two sidebands are modified so that with amplitude ripple in the filter passband $A_1 \neq A_2$, the difference being a function of ripple characteristics and modulation frequency $p$.

Assuming in-phase demodulation in the output multiplier we have:

$$V_4 = [A_1 \cos(2 \pi 256 + pt) + A_2 \cos(2 \pi 256 - pt)] \cos 2 \pi 256t.$$  

Neglecting the sum frequency terms which will be filtered out by the output low pass filter LPF\textsubscript{2} we have:

$$V_4 = A_1 \cos pt + A_2 \cos(-pt) = (A_1 + A_2) \cos pt.$$  

This equation describes the wanted signal, the two sideband amplitudes being added. For maximum amplitude ripple in LPF\textsubscript{1} of 0.1 dB peak to peak the amplitude error in the wanted signal will be 0.1 dB or 1% maximum.
Now consider the effect on quadrature feedthrough. Assuming quadrature demodulation in the output multiplier we have:

\[ V_4 = A_1 \cos(2\pi 256+p)t + A_2 \cos(2\pi 256-p)t \cos(2\pi 256t + 90^\circ). \]

Again neglecting the sum frequency terms we have:

\[ V_4 = A_1 \cos(pt-90^\circ) + A_2 \cos(-pt-90^\circ) \]

\[ = A_1 \cos pt \cos(90^\circ) - A_1 \sin pt \sin(-90) + A_2 \cos(-pt) \cos(90^\circ) - A_2 \sin(-pt) \sin(-90^\circ) \]

\[ = A_1 \sin pt - A_2 \sin pt \]

This equation describes an unwanted quadrature signal, the amplitude being the difference between the two quadrature sideband amplitudes. For maximum amplitude ripple in LPF of 0.1 dB peak to peak the amplitude of this quadrature feedthrough will be 0.5% maximum with respect to the wanted signal. This represents a quadrature suppression of 46 dB. From this it can be seen that for 50 dB suppression of the quadrature feedthrough signal, the maximum allowable amplitude ripple in LPF is approximately 0.05 dB peak to peak within the channel bandwidth.

Now considering a worst case condition for the demodulator it can be seen that for quadrature suppression of more than 50 dB using the double conversion system, signal line amplitude ripple must be less than 0.01 - 0.02 dB peak to peak. At the same time the total demodulation carrier phase must be held to within about 0.1° of the input signal quadrature carrier phase.

D. Effect of Frequency Flutter (Linear Low Pass Filter Phase Slope) - Assuming instantaneous frequency increase of the incoming composite signal by 1%. This will be automatically followed by a 1% increase in translation and demodulation carrier frequencies for a system where the synthesizer employs phase locked loops. A QDSBSC system must contain such frequency locking.
The effect of this is a 1% increase in the frequency of the sidebands under consideration at the output of LPF₁ as follows:

\[
\begin{align*}
[\cos(nω_c + p)t + \sin(nω_c + q)]1.01t + θ₁] \\
\cos[(2π256 + nω_c)t + θ₁ + e] + \sin(2π256 + q)t + θ₁ + e).
\end{align*}
\]

This frequency increase in LPF₁ will cause an increase in phase (e) of sidebands compared to previous steady state value and, therefore, the total phase θ will no longer be 0° and 90° for Cos and Sin channels respectively. For a 0.1 dB 7 pole Chebyshev filter which has the attenuation required for use in the LPF₁ application this increase corresponds to approximately 3°, causing quadrature suppression to decrease to about 25 dB.

For the linear phase slope filter used in the modified breadboard built and tested under this contract this phase changes amounts to approximately 1° with a corresponding quadrature feedthrough of approximately 35 dB.

To solve this problem, a matching filter LPF₂ is inserted at the second balanced modulator carrier input as shown below. The effect of this is to delay this carrier an equal amount and keep θ constant. For this condition the quadrature component will be reduced. With the two linear phase slope filters in the modified breadboard matched to within 0.18° the quadrature feedthrough is 50 dB below the wanted signal output level.
Another effect of the 1% change in the composite and carrier frequencies will be a corresponding 1% increase in the demodulated audio frequency. This would also be the case if a single multiplier was used as a single conversion demodulator. From the equations above it can be seen that only one adjustable phase shifter is required for each channel either on the translation frequency or the 256 kHz demodulation carrier frequency. This phase shifter must be frequency independent so as not to cause added phase shift during the 1% carrier frequency change. This phase shifter can be placed in the synthesizer VCO loop. An added feature from the filter matching should be increased phase stability versus temperature.

The advantage of this system is that all filters LPF₁ and LPF₂ for all channels are identical which greatly simplifies the filter design and manufacturing. However, the frequency synthesizer must provide frequencies from 260 kHz to 456 kHz at 4 kHz intervals. Square waves can be used with this demodulation system which simplifies the synthesizer since all carrier frequencies can be derived digitally.

```
\cos \omega_c = (2\pi 256 + n\omega_c)
```

```
\sin \omega_c = (2\pi 256)
```
E. Effect of Phase Jitter (Linear low pass filter phase slope) - The effects of phase jitter on a low pass filter with linear phase slope from the proceeding for $\phi \neq 0^\circ$ the output equation is:

$$V_o = \cos \phi \cos(pt + e_p) - \sin \phi \cos(qt + e_q)$$

$$\phi = \theta_2 - \theta_1 - e$$

where $\theta_1 =$ phase of the 256 kHz carrier

$\theta_2 =$ phase of the 256 kHz + nf carrier

$e =$ phase delay in LPF.$}\,$

Now consider a phase jitter, $A \cos \omega_a t$, on any one of the demodulation carriers. The phase of the Cos - channel is:

$$\phi = 0^\circ + A \cos \omega_a t.$$  

The output amplitude $V_o$ will be:

$$V_o = \cos(A \cos \omega_a t) \cos(pt + e_p) - \sin(A \cos \omega_a t) \cos(qt + e_q)$$

where $A =$ phase jitter amplitude (degrees)

$$\omega_a = 2 \pi \times \text{phase jitter frequency}.$$  

In the previous analysis since the total demodulator phase $\phi$ was considered a constant, the output signal was not affected by the output low pass filters LPF$_3$ and LPF$_4$. With phase jitter, however, the low phase filter will have an averaging effect on the amplitude terms dependent on the frequency $\omega_a$ of the jitter.

Consider the amplitude factor $\cos(A \cos \omega_a t)$ of the wanted signal as shown below.
The sketch shows the amplitude factor as a function of phase jitter on the filter input. The amplitude is always positive and has an average value of:

\[ \cos A + \frac{2}{\pi} (1 - \cos A) \]

which is the output amplitude value of the wanted signal for perfect filter averaging. It can be seen that for a \(10^6\) phase jitter amplitude the loss in amplitude is approximately 0.6%.

Now consider the amplitude factor \(\sin (A \cos \omega_A t)\) of the unwanted quadrature component. The sketch below shows the quadrature amplitude factor as a function of phase jitter at the filter input. The amplitude factor is periodically positive and negative and the filter averaging will reduce the amplitude effectively to zero for very high frequencies regardless of phase jitter amplitude.

For jitter frequencies below the cut-off frequency of the output low pass filters LPF\(_3\) and LPF\(_4\), the filter does not attenuate the signal and the tolerable amount of jitter amplitude is \(A = \pm 0.18^\circ\) as before, corresponding to an amplitude loss of the wanted signal \(\cos(pt + e_p)\) of less than 0.01% and quadrature suppression of the unwanted signal \(\cos(qt + e_q)\) of \(\geq 50\) dB. For jitter frequencies very much higher than the filter cut-off frequency, however, the filter will have an averaging effect which tends to remove the quadrature component while at the same time reducing the amplitude of the wanted signal.
For a 16 kHz low pass output filter sinusoidal phase jitter at 256 kHz will cause negligible quadrature feedthrough for phase jitter amplitude up to 90° and the loss in amplitude of the wanted signal will be less than 0.6% for phase jitter amplitudes below 10°.

Analysis for Sin channel (with $\phi = 90^\circ + A \cos \omega_A t$) is similar to the above.

F. True Multiplier Single Conversion QDSBSC Demodulator - A true multiplier requires a pure sinusoid for the demodulation carrier. Since the synthesizer operates on square waves a low pass filter is required in the carrier path. For $\pm1\%$ frequency modulation the phase modulation caused by this filter must be matched in the composite signal line. It will be shown below that the addition of this filter in the composite line makes this demodulation method inadequate for the lower channels. Consider block diagram below:

\[
\text{Composite Signal} \xrightarrow{\text{LPF}_1} V_3 \xrightarrow{M_1} V_4 \xrightarrow{\text{LPF}_3} V_o
\]

\[
\text{Carrier} \xrightarrow{\text{LPF}_2} V_2
\]

For: A composite signal = $\cos(n\omega_c t + p) + \sin(n\omega_c t + q)t$

\[
V_2 = \cos(n\omega_c + \theta)
\]

and using notation from the composite signal line filter LPF$_1$ characteristics shown above we have:
V_3 = \cos \left[ (n_\omega + p)t + \epsilon + e_1 \right]_p \\
+ \cos \left[ (n_\omega - p)t + \epsilon - e_2 \right]_p \\
+ \sin \left[ (n_\omega + q)t + \epsilon + e_1 \right]_q \\
+ \sin \left[ (n_\omega + q)t + \epsilon - e_2 \right]_q

Disregarding audio phase shift in LPF_3, and assuming adequate attenuation of channels other than the required channel we get:

V_o = \cos \left[ pt + \epsilon + e_1 - \theta \right]_p \\
+ \cos \left[ -pt + \epsilon - e_2 \right]_p \\
+ \sin \left[ qt + \epsilon + e_1 - \theta \right]_q \\
+ \sin \left[ -qt + \epsilon - e_2 \right]_q

Letting \epsilon - \theta = 0 and assuming e_1 = e_2 = \epsilon_p, e_1 = e_2 = \epsilon_q then:

V_o = \cos \left[ pt + \theta + \epsilon \right]_p \\
+ \cos \left[ -pt + \theta - \epsilon \right]_p \\
+ \sin \left[ qt + \theta + \epsilon \right]_q \\
+ \sin \left[ -qt + \theta - \epsilon \right]_q

The output of the Cos channel when \theta = 0^\circ is:

V_o = \cos \left( pt + \epsilon \right)_p.
The output of the Sin channel when $\theta = 90^\circ$ is:

$$V_o = \cos (qt + e_p).$$

The audio phase delay $e_p$ is unavoidable for input filter slope $\neq 0$. For single conversion demodulation the assumption that $e_1 = e_2$ is true only for the upper channels. For the lowest channel the sidebands will occupy approximately one half of the filter passband (assuming 25% modulation on channel frequency) and the phase slopes will be drastically non-linear and different for lower and higher channels. A gain loss will occur in addition to the audio phase advance and an intolerable amount of quadrature feedthrough.

For a non-linear filter phase stop ($e_1 \neq e_2$) the output of the Cos channel when $\theta = 0^\circ$ is:

$$V_o = \cos(pt + e_1_p) + \cos(pt + e_2_p) + \sin(qt + e_1_p) - \sin(qt + e_2_p)$$

$$= \cos pt \cos e_1_p - \sin pt \sin e_1_p$$

$$+ \cos pt \cos e_2_p - \sin pt \sin e_2_p$$

$$+ \sin qt \cos e_1_q - \cos qt \sin e_1_p$$

$$- \sin qt \cos e_2_p + \cos qt \sin e_2_p.$$

The first half of the equation shows that there will be an amplitude loss and phase delay of the wanted signal and the second half of the equation represents the quadrature feedthrough.

Typical values for the 4 kHz channel with $\pm 1$ kHz modulation (both $p$ and $q$) and a 7 pole, 0.1 dB Chebyshev filter is:

$$e = 210^\circ, e_1 = 90^\circ, e_2 = 70^\circ$$

Audio: $\sin pt + \cos pt (.3420) - \sin pt (.9397)$

$= -1.9397 \sin pt + .3420 \cos pt$

$= 1.97 \sqrt{80^\circ}$
Feedthrough = \(-\cos qt - \sin qt (.3420) + \cos qt (.9397)\)

\[= -.3420 \sin qt -.0603 \cos qt\]

\[= .348 \text{ } /100^\circ.\]

% Audio amplitude loss: \(\frac{3}{200}\) 100% = 1.5%

% Quadrature feedthrough: \(\frac{34.8}{200}\) 100% = 17.4%

From this numerical example it can be seen that the above method of single conversion demodulation using sin-waves is inadequate for the lower channels. Two approaches to overcome this "broadband" problem on the lower channels will now be considered.

One approach would be to use the sin-wave carrier outputs from the phase locked loops as explained in the synthesizer section. This would require the phase locked loops to operate on a "comb" type pilot signal, and with the comb frequency at 4 kHz and a loop pulling frequencies of up to 500 Hz which is not practical.

The second approach is to use phase equalizers on the carrier lines. This approach is best understood by considering the block diagram below.

The square wave carriers from the synthesizer is passed through low pass filters LPF\(_1\) and LPF\(_2\) to provide the sin-waves. The phase equalizers must be designed so that the total phase
slope on each carrier line is zero over the channel \( \pm 1\% \) frequency range. \( M_1 \) and \( M_2 \) must be true multipliers. No filter will be required in the composite signal line. The drawback with this approach is that the carrier line filter and equalizer combination is determined by the channel frequency. The filter cut-off frequency will be different from channel to channel whereas in the double conversion method the channel filters are common for all channels.

G. QDSBSC Demodulation Using Both True Multipliers and Balanced Modulators - The following block diagram shows a second approach to double conversion demodulation.

A low pass filter LPF\(_1\) transforms the square wave from the synthesizer into a sine wave. Since no carrier odd harmonics will be present at the output of the true multiplier \( M_1 \), there is no longer a requirement for a filter in the signal line. As before, LPF\(_2\) compensates for phase changes in LPF\(_1\) during the \( \pm 1\% \) frequency modulation. Since no filter is in the signal line, there is no longer a filter linearity requirement. However, LPF\(_1\) and LPF\(_2\) must have matched phase slopes over a \( \pm 1\% \) frequency range.
The drawback with this approach is that LPF₁ is determined by the channel translation frequency which ranges from 256 kHz to 460 kHz and must match the phase slope of LPF₂ which is always at 256 kHz ±1%. In the previously described double conversion method a common frequency channel filter is used for both LPF₁ and LPF₂. Also the true multiplier frequency operating range must extend to 460 kHz for a 200 kHz system. At the same time linearity must be better than 0.3% so that carrier harmonics will not demodulate other channels. This method of double conversion demodulation will not cause the audio phase delay found in the previously considered system, however, the phase shift due to LPF₂ is still present. However, the only existing multiplier capable of this performance may be Analog Devices' Model 422K.

H. QDSBSC System Phase Drift Considerations - In the previous discussion it was seen that the maximum carrier phase error that can be tolerated to provide 50 dB or better quadrature suppression was ±0.18° regardless of channel frequency. In a modulator/demodulator system it should be noted that this phase shift is the total for the system, modulator and demodulator. Phase adjustments are required in a QDSBSC system for both the 0° carrier and the 90° carrier so that for a fixed absolute phase on the modulator output signals the demodulator phase can be adjusted to match the modulator phase for maximum quadrature suppression on any one channel. If a phase drift occurs after this adjustment has been made, the drift must be limited to the ±0.18° for any one channel in a QDSB system with a 50 dB quadrature suppression requirement. The main cause of phase drift is temperature changes in the modulator portion of the system. Since all carriers are generated from a common pilot by digital multiplication and division in the phase locked loops, any drift in pilot phase will appear as phase drift in the reinserted channel carrier modified by the appropriate channel multiplication or division factor. Additionally there may be phase drift in the channel signal path itself and in the channel phase locked loops due to environmental factors and component ageing. This drift will be referred to as channel phase drift. For 50 dB quadrature suppression the channel phase drift must track the pilot phase drift according to the condition:
\[ \frac{\theta_p}{f_p} \frac{f_{ch}}{f} - \theta_{ch} \leq \pm 0.18^\circ \]

where \( \theta_p \) = pilot phase drift

\( \theta_{ch} \) = channel phase drift

\( f_p \) = pilot frequency

\( f_{ch} \) = channel frequency

In SSB the above phase tracking condition will give 50 dB sideband suppression providing the polyphase network phase shift remains within \( 90 \pm 0.18^\circ \). From the phase tracking equation it is seen that with \( \theta_{ch} = 0 \) the pilot phase drift must be limited to \( 0.115^\circ \) for a 128 kHz pilot frequency so as not to exceed a \( 0.18^\circ \) phase error in the 200 kHz channel. Pilot phase drift has negligible effect on the 4 kHz channel phase error. From this discussion it can be seen that at channel frequencies below the pilot frequency the system phase error is caused by \( \theta_{ch} \) while at channel frequencies above the pilot frequency the phase error is due to both \( \theta_p \) and \( \theta_{ch} \). The phase drift due to temperature in the modulator multipliers increases with increasing channel frequencies, and even with the pilot going through a channel multiplier for phase tracking it is extremely difficult to maintain the 50 dB suppression for channel frequencies above 100-150 kHz with a temperature range of \(-20^\circ \text{C} \) to \(+85^\circ \text{C}\). For the channels below 100 kHz \( \theta_{ch} \) will be the dominate factor. In the modulator this drift angle will be determined by the VCO phase locked loop amplifier offsets drifts. These offset drifts will limit quadrature suppression to approximately 60 dB. In the demodulator portion of the QDSB system the temperature range is only \( 25 \pm 10^\circ \text{C} \), and the phase drift will be small compared with the modulator. The major source of phase drift in the demodulator will be changes in the phase delay of the channel filters caused by filter component changes over the temperature range. For the limited temperature range the total demodulator phase drift will be of the order of \( 0.05^\circ \) or less for any one channel up to 200 kHz, corresponding to 60 dB suppression.
IV. SSB MODULATOR, PHASE SHIFT METHOD

Let us consider the block diagram below for the phase shift method of generating SSB.

The phase shift type of SSB modulator is basically a QDSBSC modulator with an added polyphase network. The polyphase network provides a 90° phase difference between the audio inputs to multipliers $M_1$ and $M_2$. The multiplier carrier input signals are in quadrature, and by summing the two multiplier outputs we get cancellation of the upper sideband and a doubling of the amplitude of the lower sideband. The polyphase network is composed of two all-pass networks with a total number of $n$ pole-zero pairs. The output phase difference has a Chebyshev type of equal ripple within the channel passband. The number of pole-zero pairs required is a function of phase error, $\Delta \theta$, and the ratio between the upper and lower frequencies, between which sideband suppression must be maintained.

The outputs from the polyphase network are:

\[
V_1 = \cos(\omega_m t + \theta + \phi_1) \\
V_2 = \cos(\omega_m t + \theta - 90^\circ)
\]
where $\omega_m = \text{modulation frequency}$

$\theta = \text{common phase delay in the two all-pass network}$

$\phi_1 = \text{network phase error}$.

The output from the multipliers are:

$V_3 = \cos(\omega_m t + \theta + \phi_1) \cos \omega t$

$= \frac{1}{2} \cos((\omega_m + \omega_m)t + \theta + \phi_1) + \frac{1}{2} \cos((\omega_m - \omega_m)t - \theta - \phi_1)$

$V_4 = \cos(\omega_m t + \theta - 90^\circ) \cos(\omega_m t - 90^\circ + \phi_2)$

$= \frac{1}{2} \cos((\omega_m + \omega_m)t + \theta + \phi_2 - 180^\circ) + \frac{1}{2} \cos((\omega_m - \omega_m)t - \theta + \phi_2)$

where $\phi_2 = \text{carrier quadrature phase error}$.

Summing the two multipliers output signals gives:

$V_0 = \frac{1}{2} \cos((\omega_m - \omega_m)t - \theta) - \phi_1$

$+ \frac{1}{2} \cos((\omega_m - \omega_m)t + \theta + \phi_2)$

$= V_A = \text{wanted sideband}$

$+ \frac{1}{2} \cos((\omega_m + \omega_m)t + \theta + \phi_1)$

$+ \frac{1}{2} \cos((\omega_m + \omega_m)t + \theta + \phi_2 - 180^\circ) = V_B = \text{unwanted sideband}$

A. **Wanted Sideband** - For $\phi_1 \leq +1^\circ$ and $\phi_2 \leq +1^\circ$ (this ensures amplitude accuracy of 0.01%) we have:

$V_A = \cos((\omega_m - \omega_m)t - \theta)$

This is the wanted output sideband, and $\theta$ is the channel phase delay.

B. **Unwanted Sideband** - The unwanted sideband components reduce to

$V_B = \frac{1}{4} \cos \phi_1 \cos((\omega_m + \omega_m)t + \theta) - \frac{1}{4} \sin \phi_1 \sin((\omega_m + \omega_m)t + \theta)$

$\quad - \cos \phi_2 \cos((\omega_m + \omega_m)t + \theta) + \frac{1}{4} \sin \phi_2 \sin((\omega_m + \omega_m)t + \theta)$
For $\phi_1 \leq +1^\circ$ and $\phi_2 \leq +1^\circ$ the two left hand terms cancel to less than 0.01% or -80 dB with respect to the wanted sideband. The two right hand terms may cancel or add depending on the signs of $\phi_1$ and $\phi_2$. With opposite signs the two terms add.

For sideband suppression $\geq 50$ dB, and with the error equally divided between the two phase error sources we have:

$$\phi_1 \leq \pm 0.18^\circ$$
$$\phi_2 \leq \pm 0.18^\circ$$

which is the maximum phase error on $\phi_1$ and $\phi_2$ that can be tolerated for a 50 dB suppression of the unwanted sideband in a SSB system.

Up to this point we have disregarded amplitude factors. For $\phi_1 = \phi_2 = 0$ we get no unwanted sideband feedthrough due to phase errors. However, if the gains are different in the two channel branches the two left hand terms of the unwanted sideband equation (V) will no longer cancel completely. It can be shown that the limiting condition for $\geq 50$ dB suppression requires channel branch gains matched to within 0.3%.

C. Sideband Selection - The foregoing block diagram and equations provide for suppression of the upper frequency sideband. If this sideband is wanted on the output, and suppression of the lower frequency sideband is required, this can be accomplished by reversing the sign of either one of the 90$^\circ$ phase shifts. Practically this can be done by either interchanging the polyphase network outputs, or by interchanging the multiplier carrier inputs.

D. Polyphase Network Characteristics - A ratio of 200 between the maximum and minimum audio passband frequencies requires a 10 pole-zero pair network. This gives a maximum phase error ripple of $\pm 0.15^\circ$. A channel phase delay is caused by the polyphase network, and for the above network the magnitude of the phase delay varies from approximately 110$^\circ$ at the minimum passband frequency to approximately 700$^\circ$ at the maximum passband frequency. For a 10 Hz to 2 kHz passband this corresponds to approximately 60 milliseconds and 10 milliseconds of time delay for 2 kHz and 10 Hz respectively. The suppression of frequencies outside the passband reduces sharply versus frequency. For the 10 Hz to 2 kHz network the suppression is 23 dB at 5 Hz and 30 dB at 3 kHz,
so that if any out-of-band signals are present on the audio input, we get a corresponding vestigial output. It is anticipated that a general temperature requirements for a SSB modulator will be approximately -20°C to +85°C. Even with extremely stable components the polyphase network branch phase delay may vary by a number of degrees over this temperature range, with the result that sideband suppression decreases and channel delay time changes. It will be shown in Section VI that in-band sideband suppression may not effect the demodulator output to a measurable extent. Phase correlation from channel to channel can only be maintained to within twice the expected polyphase network phase error. If the required phase correlation is of the order of 1° or better, temperature control of the polyphase networks will be necessary.
V. SSB DEMODULATOR, PHASE SHIFT METHOD

The SSBSC phase shift type demodulator is basically a QDSBSC demodulator with a polyphase network added.

\[ 2\cos(\omega_c - \omega_m) t \]

The QDSBSC demodulator translates the wanted channel down to baseband as follows:

\[ V_2 = 2\cos(\omega_c - \omega_m) t \cos \omega_c t \]
\[ = \cos \omega_m t + \cos(2\omega_c - \omega_m) t \]

\[ V_3 = 2\cos(\omega_c - \omega_m) t \cos(\omega_c t - 90^\circ) \]
\[ = \cos(\omega_c t - 90^\circ) + \cos((2\omega_c - \omega_m) t - 90^\circ) \]

The terms containing \( 2\omega_c \) will be filtered out by the low pass filter LPF and are neglected. On the outputs of the polyphase network we have:
\[ V_4 = \cos(\omega_m t + \theta) \]
\[ V_5 = \cos(\omega_m t - 90^\circ + \theta + 90^\circ) = \cos(\omega_m t + \theta) \]

After summing the two signals we get:
\[ V_o = 2 \cos(\omega_m t + \theta) \]

This is the audio component of the difference frequency sideband on the output. \( \theta \) is the channel phase delay caused by the polyphase network.

Let us now consider the effect on the sum frequency sideband. Assume the input signal to be:
\[ V_1 = 2 \cos(\omega_c + \omega_m) t. \]

The multiplier outputs will be:
\[ V_2 = 2 \cos(\omega_c + \omega_m) t \cos \omega_c t 
   = \cos \omega_m t + \cos(2\omega_c + \omega_m) t \]
\[ V_3 = 2 \cos(\omega_c + \omega_m) t \cos(\omega_c t - 90^\circ) 
   = \cos \omega_m t + 90^\circ + \cos(2\omega_c t + \omega_m t - 90^\circ) \]

The terms containing \( 2\omega_c \) are filtered out, and the polyphase network outputs are:
\[ V_4 = \cos \omega_m t + \theta \]
\[ V_5 = \cos \omega_m t + 90^\circ + \theta + 90^\circ = \cos(\omega_m t + 180^\circ) \]

Since \( V_4 = -V_5 \), the output from the summer is zero. Similarly to the derivations for the SSB modulator, Section IV, it can be shown that for phase errors of less than \( \pm 0.1^\circ \) in the carrier quadrature (90°) phase shift, and phase error of less than \( \pm 0.1^\circ \) in the polyphase network 90° phase shift the suppression of the sum frequency sideband will be 50 dB. Considering a possible difference in gain between the two branches of the SSB demodulator we find that the gains must be matched to within 0.3% for 50 dB sideband suppression.

If the sum frequency sideband contains the wanted audio information, this sideband can be demodulated by reversing the sign on the 90° phase shift in either the QDSBSC demodulator portion of the channel of in the polyphase network.
An interesting difference between DSB or QDSB and SSB should be noted at this point. In Section III-A it was seen that carrier phase error reduces the demodulated or wanted audio signal by a factor \( \cos \theta \), and quadrature feedthrough in a QDSB system has a multiplication factor of \( \sin \theta \), where \( \theta \) is the carrier phase error. In the above phase shift method of single sideband demodulation the output amplitude is independent of the phase of the reinserted carrier provided the quadrature condition (90° phase shift) exist between the two demodulation carriers. The effect of carrier phase error is output signal phase shift only. To see this, let us introduce a carrier phase shift \( \theta \) so that in the demodulator block diagram the carrier input signal is \( \cos (\omega t + \phi) \). The difference in the demodulator equations will be as follows for the wanted sideband

\[
\begin{align*}
V_2 &= \cos(\omega_m t + \phi_i)
\end{align*}
\]

\[
\begin{align*}
V_3 &= \cos(\omega_m t + \phi_i - 90^\circ)
\end{align*}
\]

\[
\begin{align*}
V_4 &= \cos(\omega_m t + \phi_i + \theta)
\end{align*}
\]

\[
\begin{align*}
V_5 &= \cos(\omega_m t + \phi_i - 90^\circ + \theta + 90^\circ)
\end{align*}
\]

\[
\begin{align*}
V_o &= 2 \cos(\omega_m t + \theta + \phi_i)
\end{align*}
\]

It is seen that the carrier phase error appears as an audio phase error only. Similarly it can be shown that with regard to the suppressed sideband the effect of carrier phase error appears as audio phase shift in the suppressed sideband.
VI. SSB DEMODULATOR EFFECTIVE CHANNEL FREQUENCY RESPONSE

Figures VI-1, VI-2, and VI-3 show the demodulator input frequency characteristics. The filter curve in Figure IV-1 is representative of a 7 pole Chebyshev filter. The zero frequency point on the frequency axis corresponds to the carrier frequency of the wanted channel, and channel bandwidth is 10 Hz to 2 kHz. It is evident that the output low pass filter is effectively suppressing all signals below the wanted channel. However, the filter is only partially effective for the signals in the channel immediately above the desired channel. Figure VI-2 shows the demodulated frequency response of the polyphase SSB demodulator. It provides 50 dB suppression for any vestige of the upper sideband frequencies remaining from the modulation process. Assuming 50 dB sideband suppression in the modulator the total sideband suppression of 100 dB seems excessive. For the demodulator the 50 dB requirement cannot be relaxed. This can be seen in Figure VI-1. Since the output low pass filter has no attenuation at the corner frequency (maximum audio frequency) of the N+1 channel, this frequency is attenuated by the suppression effect of the polyphase network only, and 50 dB is, therefore, required at this frequency. In the modulator however, the required sideband suppression is governed by channel to channel phase correlation and by output power considerations, and depending on these two factors the 50 dB suppression may or may not be relaxed.
OUTPUT LOW PASS FILTER FREQUENCY RESPONSE

POLYPHASE NETWORK FREQUENCY RESPONSE

EFFECTIVE CHANNEL FREQ. RESPONSE
VII. SYNTHESIZER

In order to provide channel carrier frequencies spaced at $4n$ kHz intervals which are phase coherent it is necessary to derive all frequencies both in the modulators and demodulators from a common pilot frequency. The pilot frequency in the demodulator being reconstructed from the pilot sent from the modulator. Because of the complexity of the synthesizer and the different requirements for the modulator and demodulator the synthesizer greatly influences the type of modulator and demodulator systems used.

If single conversion is used in either the modulator or demodulator true multipliers having good linearity are required so harmonics will not be generated in the frequency spectrum of a higher frequency channel. This also requires that the synthesized carrier frequencies be true sine waves with harmonics down 60-70 dB, while at the same time the desired phase must be held to within $\pm 0.18^\circ$.

If double conversion systems are used the synthesizer must provide not only a common modulation or demodulation frequency such as 256 kHz, but also translation frequencies which are $(256 + 4n)$kHz where $n$ is 1-50 for a 4 kHz to 200 kHz FDM system.

A. Modulator Synthesizer - For the FDM modulator the reference oscillator would normally be crystal controlled oscillator to provide a stable frequency which would not vary appreciably with time or temperature. All of the channel carrier frequencies would be locked to and controlled by the reference oscillator (or clock) by means of VCO-PLL's. A frequency stability of $\pm 0.01\%$ is easily obtained with a crystal oscillator and better frequency control then $\pm 0.01\%$ would not be required since the demodulated frequencies are depended on the tape recorder accuracy which cannot be controlled to better than $\pm 1\%$ anyway.

Reasonable crystal size, ruggedness, and ease of manufacturing would dictate that the crystal oscillator frequency should be above 500 kHz. Any binary frequency from 512 kHz to 8.192 MHz could be used and divided down digitally to the desired pilot frequency. A very low frequency pilot or sync frequency can be used because the frequency changes of the reference oscillator will be small and have a low rate of change allowing for a very low frequency cut-off of the VCO-PLL filters. Therefore, the reference oscillator can be divided down to 4 kHz and all channel frequencies derived directly from 4 kHz "comb" sync pulse. The comb sync signal is developed by combining all of the divider outputs from 256 kHz down to 4 kHz in
an "and" gate which produces a square wave "comb" pulse which is one half the width of a 256 kHz signal at a 4 kHz rate. Each channel frequency is then provided by a VCO-PLL or a voltage controlled crystal oscillator (VCO-PLL) which is phase controlled to the comb sync signal. The phase detector used in this type of system must be the sample and hold type. This type of phase detector was successfully used in the NASA FDM system Contract NAS8-24682. The phase between the desired frequency and the comb is compared only during the time that the comb is high providing a phase correction voltage. During the time between comb pulses the correction voltage is stored. This prevents the VCO frequency or phase from being changed between phase comparisons.

Phase scattering is desirable in a FDM system to prevent pile up or peaking of the composite signal when all channels are combined and can be provided by splitting the reference clock divider into about 8 different phases and then using these 8 phases to provide 8 combs equally spaced, therefore, 1/8 of the VCO's can be phase locked to each of the 8 combs.

Because of the low cut-off frequency of the loop filters in the modulator VCO-PLL a low pass filter can be used in the carrier frequency portion of the PLL to provide a sine wave output at the channel frequency. This makes it possible to use true multipliers in the FDM modulation system. A carrier LPF cannot be used in a loop where the loop filter cut-off frequency is high (such as 500 Hz) because the additional phase shift caused by the carriers LPF would cause too much loop phase shift, making the loop unstable.

B. Demodulator Synthesizer - The demodulator synthesizer presents a considerably different problem because of the frequency modulation of the reference and channel frequencies caused by the tape recorder wow and flutter. Since this frequency modulation can be as much as ±1% at a 500 Hz rate, it is necessary that the reinserted demodulation carrier frequency also be frequency modulated to follow the channel frequencies within ±0.18° with respect to phase. Consequently, it can be seen that a low frequency pilot such as 4 kHz cannot be used because phase correction must be made at a much higher rate in order to prevent phase error or drift to build up between phase corrections.
C. Choice of Pilot Frequency - In Section VII-B it was found that there exists a lower limit for the pilot frequency in the phase locked loop. This limitation was caused by the fact that the sum frequency component on the phase detector output produced unwanted sidebands in the loop output signal. With a required loop frequency pulling range of ±1% at a rate up to 500 Hz the loop filter could provide 50 dB suppression of these undesired sidebands only if the pilot frequency was over 100 kHz. Now consider the pilot frequency with respect to the loop sample rates. The waveforms below represent the phase detector input and output waveforms.

\[ V_1 \]

\[ V_2 \]

\[ V_3 \]

\( V_1 \) is the loop pilot input signal, \( V_2 \) is the VCO output signal and will be at 90° + θ with respect to \( V_1 \) in the locked condition, where θ is the loop phase error. \( V_3 \) is the phase detector output at a frequency of two times the pilot frequency. With no phase error in the loop \( V_3 \) is a symmetrical square wave. With phase error, however, \( V_3 \) becomes asymmetrical with a dc component proportional to θ. The dc component is the phase locked loop control or error signal. The control information is contained in each and every period of \( V_3 \), and this information cannot be obtained in a shorter time period. Therefore, with the saturated balanced modulator type phase detector in the phase locked loop the effective sampling rate of the input phase is twice the pilot frequency. If the loop pilot frequency is 128 kHz modulated ±1% (1.28 kHz) at a rate of 500 Hz, the sampling time to obtain the dc information is \( 1/256000 = 3.1 \mu \) second. Assume that a sampling period has just been concluded and that both the input pilot and the VCO frequencies are
exactly 128 kHz and let the input frequency change be as shown:

\[ f_m \ (kHz) \]

\[ \Delta f = 1.28 \ kHz \sin 2 \pi 500t \]

During the 3.1 microseconds sampling period the VCO frequency will remain at exactly 128 kHz. The input signal, however, will change 1280 Sin\(^{-1}\) \[ \frac{3.1 \cdot 360}{2000} = 12.8 \ Hz \]

During the sampling time, therefore, an average error in the VCO frequency of 6.4 Hz will occur. The VCO output phase is the quantity of interest and from Section VIII-E we have the VCO output phase, that \[ \theta_{\text{VCO}} = \int \Delta \omega_{\text{VCO}} dt \]

The accumulated VCO output phase at the end of the sampling period is therefore,

\[ \theta_{\text{VCO}} = 2 \cdot 6.4 \cdot 3.1 \cdot 10^{-6} = 124 \cdot 10^{-6} \text{ radians} \]

\[ = 0.007 \text{ degrees} \]

This is the phase locked loop phase error due to the sampling action of the phase detector.

Now consider a 4 kHz pilot frequency. The sampling frequency will be 8 kHz and the time between samples is 125 microseconds. With a \( \pm 1\% \) 40 Hz input deviation at a rate of 500 Hz the maximum error in VCO frequency due to lack of control signal during the sampling period will be

\[ \Delta f = 40 \ Sin^{-1} \]

\[ \frac{125 \cdot 360 \degree}{200} = 14.52 \ Hz \]
The average VCO frequency error is approximately 7.26 Hz and the loop phase error due to the sampling action is:

$$\theta = 2 \cdot 7.26 \cdot 125 \cdot 10^{-6} \cdot 57.6 = 0.33^\circ$$

From this it can be seen that with a ±1% input deviation at 500 Hz a loop phase error of ±0.18° corresponding to a 50 dB quadrature suppression in a QDSB system cannot be maintained regardless of the gain in the loop with a 4 kHz pilot frequency. Therefore, the pilot frequency in the demodulator system must be high enough with respect to all VCO loops so as to provide negligible phase error due to the sampling characteristics of the loop phase detector. For demodulator VCO's where maximum input frequency pulling rate is 500 Hz a pilot frequency of 128 kHz is chosen while in modulator VCO's where the frequencies are very stable the pilot frequency may be as low as 4 kHz.
VIII. VOLTAGE CONTROLLED OSCILLATOR - PHASE LOCKED LOOP

A. Phase Locked Loop (PLL) - Phase locked loops are used to provide synchronous channel carriers in an FDM system. The loop output signals are locked to the pilot, and, therefore, provide the required frequency and the phase. A basic phase locked loop is shown in below. The building blocks used are a phase detector (\( \Theta \)) and amplifier and loop filter (A), a voltage controlled oscillator (VCO), and two digital frequency dividers D1 and D2.*

The phase detector produces an output voltage (error voltage or control voltage) that is proportional to the phase difference between the reference input signal (pilot) and the divided frequency from the voltage controlled oscillator output. The phase error signal is passed through the loop amplifier and loop filter and provides the driving voltage for the VCO in such a direction as to minimize the phase error. The loop amplifier/filter provides high loop gain at low frequency to minimize loop error and good low frequency tracking response. It also provides attenuation for high frequency error signals. This prevents high frequency jitter on the output signal. Since a common pilot frequency is used to derive all channel frequencies, the division factor D1 and D2 and also the VCO frequency will vary from channel to channel.

For a double conversion system as recommended for the demodulators system, the VCO frequencies for some of the higher channels (near 200 kHz) will have to be as high as

*Two frequency dividers are needed only when the pilot frequency is not equal to, or a sub-multiple of the VCO frequency such as would be the case in a double conversion demodulator.
28 MHz for symmetrical division down by \( D_1 \) and \( D_2 \) to provide both the required translation frequency and the 128 kHz pilot frequency. This high frequency should present no problem with the high speed digital components that are now available, particularly since all of the frequencies used are square-waves and digitally derived.

**B. Phase Detector and Loop Filter** - The most commonly used phase detector is an analog multiplier. The following signals applied to a multiplier functioning as a phase detector:

\[
V_p = \sin(t + \theta_1) \\
V_v = \cos(t + \theta_2)
\]

\( V_p \) is the signal being tracked and \( V_v \) corresponds to the VCO output. Disregarding amplitude factors, the output from the phase detector is:

\[
V_o = \sin(\omega_1 - \omega_0) = \sin(\omega_1 - \omega_2) + \sin(\omega_1 + \omega_2) - \omega_0 t + (\theta_1 - \theta_2).
\]

The sum frequency sideband is filtered out leaving:

\[
V_f = \sin[(\omega_1 - \omega_2) t + \theta_1 - \theta_2].
\]

When the phase detector is used in a phase locked loop and a locked condition exists the frequency error in the loop must be reduced to zero and the filter output signal will be:

\[
V_f = \sin(\theta_1 - \theta_2) = \sin \phi
\]

Where \( \phi \) = phase error in the loop.

Allowing for phase detector gain we get:

\[
V_f = K_{pd} \sin \theta
\]

Where \( K_{pd} \) = phase detector gain in volts/degree.
This shows that the phase detector voltage is zero when
the reference signal and the VCO output are 90° apart. This
quadrature condition will always exist for a PLL in the locked
or tracking condition. If the reference signal phase (θ₁) is
positive going the error voltage will be positive, and when
negative going the error voltage will be negative thus
providing the error voltage required to drive the VCO in the
direction of reduced phase error.

C. Voltage Controlled Oscillator - There are two types
of voltage controlled oscillators that are normally used in
phase locked loops in FDM systems. One is a crystal
controlled oscillator (VCXO) and the other is an LV-VCO.
Basically there is little difference between the two except
for pulling range and pulling rate as will be explained.

The pulling range of a crystal oscillator is a function
of the ratio of the series mode capacitance C₁ and the
physical shunt capacitance C₀ of the crystal. The higher the
ratio C₀/C₁, the smaller the pulling range according to the
expression:

\[ f = \frac{C₁ fₛ}{2 (C₀ + Cₜ)} \]

Where

Typical Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>fₛ</td>
<td>series resonance frequency of crystal</td>
<td>10 MHz</td>
</tr>
<tr>
<td>C₁</td>
<td>series mode capacitance of crystal</td>
<td>0.016 pF</td>
</tr>
<tr>
<td>C₀</td>
<td>shunt capacitance of crystal</td>
<td>5 pF</td>
</tr>
<tr>
<td>Cₜ</td>
<td>external capacitance in series with crystal</td>
<td>10 to 40 pF (using a Varicap diode in series with crystal)</td>
</tr>
<tr>
<td>Δf</td>
<td>frequency shift above series resonance of crystal due to Cₜ</td>
<td></td>
</tr>
</tbody>
</table>
With the above values a 10 MHz crystal can be pulled about 4 kHz. However, this pulling range can be doubled by using an inductor as well as a capacitor in series with the crystal, which will pull the crystal frequency below series resonance as well as above it. If \( C_o \) is reduced, the pulling range \( \Delta f \) is increased. Because \( C_o \) is the physical capacitance due to the crystal plating and crystal holder, it can be tuned out by placing an indicator across the crystal. This technique may increase the pulling range by a factor of ten, or to \( \pm 0.2\% \) for a 10 MHz crystal.

If two voltage controlled crystal oscillators (VCXO) operating at 8 MHz and 14.4 MHz with complementary voltage-versus-frequency controls are mixed, the output frequency will be 6.4 MHz with a pulling range of \( \pm 2 \) to \( \pm 4\% \). Therefore, a pulling range of \( \pm 1\% \) is achievable by this method.

For an LC-VCO the oscillation frequency is given by:

\[
F_o = \frac{1}{2L(C_o + C_T)}
\]

Where

- \( f_o \) = resonant frequency
- \( L \) = resonant circuit inductance
- \( C_o \) = fixed circuit capacitance
- \( C_T \) = variable circuit capacitance

Typical Values

- \( f_o \) = 10 MHz
- \( L \) = 1 \( \mu \)H
- \( C_o \) = 100 pf
- \( C_T \) = 50 to 150 pf (using Varicap diode in parallel with \( C_o \))

The above values give a pulling range for the 10 MHz LC-VCO of approximately 10% as compared to 0.2% for a 10 MHz crystal oscillator.

Pulling Rate - The pulling rate of an oscillator is limited by its Q which does not allow rapid frequency changes. The rate \( Y \) at which an oscillator can be pulled is:

\[
Y = \frac{F_o}{Q}
\]
Where

\[ Y = \text{pulling rate} \]

\[ f_0 = \text{resonance frequency of the resonator or tuned circuit} \]

\[ Q = Q \text{ of the resonant circuit} \]

A 8 MHz crystal can be built to have a Q of 10,000 to 20,000 which would allow a pulling rate of 500 to 1000 Hz. Because the crystal Q does not go down as the frequency is lowered, it can be seen that it is necessary to go up at least 8 MHz or higher to provide a 500 Hz pulling rate as well as a 1% pulling range.

For the LC-VCO the circuit Q may be of the order of 100, and it can be seen that the pulling rate for the LC-VCO therefore, will be 100-200 times larger than for a crystal VCO.

D. Stability - There are two commonly referred to types of stability, long term and short term. Long term stability refers to slow variations in average frequency with time due to circuit ageing effects. For a crystal VCXO a typical value for long term stability is 0.005% and for an LC-VCO is about 1-2%. Short term stability refers to changes in average frequency over a time sufficiently short for long term effects to be negligible. Short term effects are due to changes in supply voltage, ambient temperature, and other circuit noise. A measure for oscillator short term noise is not easily defined, however, it can be shown that the oscillator output noise floor is a representative measure for this parameter. It is our experience that for an oscillator to be used in a high gain phase locked loop for carrier generation in an FDM system a noise floor of -70 dB measured at 1 kHz bandwidth is required for adequate system operation.

E. Gain - The gain of the VCO can be found from the deviation in frequency for a change in the input voltage as follows, the deviation of the VCO from its center frequency is:

\[ \omega_0 = KV_F \]

Where \( K = \text{VCO gain constant in degrees/sec/volt} \); and \( V_F \) is the VCO input control voltage. Since frequency is the derivative of phase the VCO operation may be described as:

\[ \frac{d\theta}{dt} = KV_F \]
That is, the VCO output phase is the integral of the input voltage.

F. Phase Locked Loop Equations - To show the loop lock-in performance we derive the nonlinear differential equation of the loop. Let \( \omega_i \) be the input frequency and \( \omega_o \) the center frequency of the VCO so that the instantaneous frequency of the VCO is:

\[
\omega_o + \Delta \omega_o = \omega_o + K V_p.
\]

The amplifier gain is \( A \) and the error voltage into the VCO is:

\[
V^f = AK^f \sin \theta.
\]

The input phase is \( \omega_i t \) and the output phase is:

\[
\theta_o = \omega_o t + t \int_0^t K V_p dt = \omega_o t + t \int_0^t AK^p_d \sin \theta dt.
\]

Letting \( \omega_i - \omega_o = \Delta \omega = \) open loop frequency difference between the input frequency and the VCO frequency, and differentiating we get:

\[
\frac{d \theta}{dt} = \Delta \omega = AK^p_d \sin \theta.
\]

This is the governing loop equation. In the steady state condition we have, by definition, no frequency error. That is:

\[
\frac{d \theta}{dt} = 0
\]

and \( \Delta \omega = AK^p_d \sin \theta. \)

G. Static Phase Error - The amount of phase error to satisfy the above equation (i.e. the steady state phase error for a given open loop frequency difference) is defined as the static phase error of the loop.

H. Loop Stability - To determine loop stability it is necessary to consider the loop phase and gain as a function of error signal frequency. It can be shown that the closed loop transfer function of the phase locked loop is:
with the corresponding open loop gain function:

$$G_0 = \frac{AKK_{pd} F_1(j\omega) F_2(j\omega)}{j\omega}$$

Where $\frac{1}{j\omega}$ represents the VCO integration

$$F_1(j\omega) = \text{low pass filter transfer function}$$

$$F_2(j\omega) = \text{lag caused by the restraining effect of Q in the VCO}$$

For a stable loop we must have a total loop phase shift of less than 180° for a loop gain $G_0 = 1$.

I. Locking Range - The locking range of the phase lock loop is defined as the maximum open loop frequency difference that the system can accommodate in the closed loop condition. In the steady state loop equation this corresponds to the maximum condition of $\sin \theta = 1$ or $\theta = 90^\circ$. We then get the locking range for the loop as:

$$\omega_L = \frac{AKK_{pd}}{\text{rad/s}}$$

The locking range can be found by setting the open loop gain $G_0 = 1$ while disregarding the frequency dependent attenuation factors, $F_1(j\omega)$ and $F_2(j\omega)$ since the error frequency equals zero in the locked condition. It should be noted that in a high gain loop, saturation of the loop amplifier will occur with only a few degrees of phase error. In such a system the locking range will be limited to the frequency difference required to saturate the system.

J. Capture Range - The capture range is the largest unlocked frequency difference at which the system will pull into lock. If the input signal frequency starts outside the locking range, the phase detector output will be an ac error signal with frequency equal to the difference between the input signal and the VCO frequency. The error signal will be attenuated by the loop filter (as opposed to the locked
condition when no frequency error exist and the loop filter does not attenuate the error signal). The loop gain is, therefore, reduced by the attenuation in the loop filter. If the input signal frequency is tuned closer to the VCO frequency the error signal frequency decreases, the filter attenuation decreases and the loop gain increases. When the open loop gain becomes greater than unity the loop feedback will exceed the input signal, and the loop will pull into lock. The open loop frequency difference at which this occurs is the capture range. Depending on the loop filter function the capture range may be limited by amplifier saturation. However, this may not be the case even in a high gain loop system. 

K. Summary of VCO-PLL Operation - To summarize the operation of the phase locked loop consider the below Bode plot of open loop gain in decibels versus logarithmic error signal frequency.

With no loop filtering the open loop gain decreases at a rate of 6 dB per octave due to the $1/j\omega$ integration term, intersecting the unity gain line at a frequency of $\omega_{c1} = ACK_{pd}$. This is the cutoff frequency of the closed loop system. That is, the transfer characteristic of the closed loop equals
unity for input variations with rates below $\omega_{cl}$ and decreases at 6 dB per octave for rates above $\omega_{cl}$. Both the locking range and capture range will equal $\omega_{cl}$ in this case.

The dotted line represents a practical loop function with $\omega_3$ being the 3 dB point of the lag effect due to the VCO Q factor and $\omega_1$ and $\omega_2$ being the 3 dB points of a lag-lead network. The closed loop cutoff frequency reduces to $\omega_{c2}$, and this will also be the new system capture range. The locking range, however, still equals $\omega_{cl}$. It should be noted that the VCO integration term causes an inherent 90° circuit lag angle so that the additional phase lag angle caused by the loop filtering must be restricted to less than 90° for all frequencies below the closed loop cutoff frequency in order to insure stable operation.

If a sin-wave loop output signal is desirable, a band-pass or low pass filter the design of which varies with carrier frequency, may be inserted on the output of the digital divider. This filter must be inside the loop so that phase variations caused by the filter during VCO frequency modulation will automatically be compensated for. The effect of such a filter on the loop operation will be equivalent to an added loop lag circuit with the break frequency dependent on the filter Q. This added lag must be considered with respect to loop stability.

L. Noise Behavior - The phase lock loop noise or jitter performance is best expressed in terms of the closed loop cut-off frequency $\omega_c$. The loop will act as a low pass filter with respect to noise on the input signal line. That is, components of noise with rates below $\omega_c$ will appear directly on the loop output while components with rates above $\omega_c$ are attenuated. This is best understood by examining the closed loop gain:

$$\frac{\theta_0}{\theta_1} = \frac{G}{1 + G}$$

Where $G = \text{open loop gain.}$
For rates below $\omega_c$ the loop gain $G$ is greater than unity and it can be seen that the closed loop gain is approximated by:

$$\frac{\theta_o}{\theta_i} \approx 1.$$ 

That is all input rates below $\omega_c$ will appear directly on the output. For rates above $\omega_c$ the closed loop gain, since $G < 1$, is approximated by:

$$\frac{\theta_o}{\theta_i} \approx G.$$ 

As $G$ decreases the output phase is less affected by input noise.

For internally generated noise, however, the loop will act as a high pass filter. This can be seen from the fact that the closed loop transfer function with respect to an internally generated noise signal $\theta_n$ is

$$\frac{\theta_o}{\theta_n} = \frac{1}{1 + G}$$

Internal noise or jitter with rates below $\omega_c$ is therefore attenuated while components with rates above $\omega_c$ will appear directly on the output.

M. VCO - Phase Locked Loop Design - In the FDM systems under consideration there are two different types of phase locked loops required; one for use in the modulation system; and one for the demodulation system. The difference is due to the fact that the composite data signal is stored on tape prior to demodulation, and during tape recorder playback and wow flutter causes frequency modulation of the composite data signal of up to ±1% at rates up to 500 Hz. The pilot, when extracted from the composite signal will contain this frequency modulation, and all the VCO's used for demodulation carrier reconstruction will, therefore, be required to track this frequency modulation while maintaining the required phase accuracy. A combination of two crystal oscillators, as
explained previously, could be used for the VCO in the phase
locked loop. However, such a design would be extremely
marginal on tracking, locking, and capture range. Also loop
stability would be critical due to the requirement for filter-
ing of the sum frequency term at the phase detector output.
Since the VCO pulling rate would be approximately 500 Hz, a
loop gain roll off would start at this frequency, with a
required sum frequency attenuation in the order of 80 dB for a
pilot frequency of 128 kHz the task of building a stable VCO
is extremely difficult using crystal oscillators. For these
reasons, the choice of oscillator for the demodulation system
VCO's is of the LC-type.

In the modulation carrier generation process, no
frequency modulation exists since the generation process or
pilot is controlled by a crystal stabilized frequency source.
The phase lock loop bandwidth requirements will be 1 to 2
orders of magnitude less than for the demodulator loops.
Therefore, either LC or crystal VCO's can be used in this
application. Crystal VCXO's are recommended.

N. Design Example - The following description pertains
to the phase locked loops designed and breadboarded under
this contract for use in a QDSBSC demodulator. The require-
ments for the loops include frequency deviations of ±1% of
the input signal at rates up to 500 Hz with a maximum phase
error of 0.18° referred to a 200 kHz channel frequency. As
will be explained the pilot frequency was chosen to be 128 kHz.

For the double conversion demodulation system as used for
this contract, and with a 64 kHz channel frequency used for
QDSBSC, the translation frequencies required from the VCO's
are 256 kHz and 320 kHz. The test results and schematics of
these VCO's are shown in an earlier report and show that
the phase tracked to better than the required ±0.18°.

The phase detector is a balanced modulator type MC1596G
operating in a saturated mode so that the output is a linear
function for phase angles from 0° to ±90°. The maximum phase
detector output voltage for a 90° signal phase difference is
approximately 2.5 volts and the phase detector gain constant
is:

\[ K_{pd} = 0.028 \text{ volts/degree} = -31 \text{ dB}. \]

The VCO gain constant for small input signals referred to the
128 kHz output in the divider chain is:
K = 1.25 \cdot 10^7 \text{ degrees/volt sec} = 142 \text{ dB.}

The maximum open loop frequency difference in the loop is \pm 1\% of the input frequency which converted to degrees/second is:

\[ \Delta \omega = 2.9 \cdot 10^6 \text{ degrees/sec} = 129 \text{ dB.} \]

The phase error at 200 kHz must be less than \pm 0.18° so that the maximum error at 128 kHz must be less than \pm 0.115° = -19 dB.

The required amplifier gain can then be found from the formula for static loop conditions:

\[ \Delta \omega = AK_{pd} \sin \theta. \]

Since the phase detector provides linear operation: \( \sin \theta = \theta \)

and

\[ A = \frac{\omega}{KK_{pd} \theta} = 129 - (142 - 31 - 19) \text{ dB} = 37 \text{ dB}. \]

The loop amplifier and filter are designed to provide a gain of 56 dB at dc and 38 dB and 500 Hz. This gives a static phase error of about 0.11° at the maximum loop pulling rate of 500 Hz and an error improvement by a factor of 10 for lower pulling rates.

The open loop gain is:

\[ G_o = AK_{pd} F(j\omega) \text{ is 167 dB at } j\omega = 1 \text{ rad/sec.} \]

This reduces to 79 dB at 500 Hz, and the closed loop cutoff frequency is 20 kHz. The theoretical maximum capture range is approximately 20 kHz and the locking range about 3 MHz; however they are both limited by amplifier saturation which occurs at VCO deviation of approximately \pm 10 kHz referred to the 128 kHz pilot frequency.

Let us now consider the 256 kHz sum frequency on the phase detector output. The effect of this signal is to cause frequency modulation of the VCO at a rate of 256 kHz, and with a deviation dependent upon the amplitude of the 256 kHz signal at the VCO input. The 256 kHz output from the phase detector is 2.5V peak as compared to a 2.5 mv peak error signal for 0.11° error at 500 Hz pulling rate. The loop
filter attenuates the 256 kHz component 71 dB with respect to the 500 Hz signal, and at the input to the VCO the 256 kHz signal is 11 dB below the 500 Hz control signal. Assuming the Q of the LV-VCO to be about 20, the VCO pulling rate is of the order of 350 kHz. Taking into account a 6 dB loss in the VCO response caused by the induction load at 256 kHz, the deviation caused by the 256 kHz signal will be 17 dB less than that caused by the 500 Hz signal. Referred to the 128 kHz divider chain output the maximum deviations are 1.28 kHz for the 500 Hz signal and 180 Hz for the 256 kHz.

Thus sidebands occur around the loop output signal frequency at ± 256 kHz with amplitude being a Bessel function of the first kind, the actual value being determined by the modulation index.

The deviations caused by the 256 kHz sum frequency signals are 360 Hz and 430 Hz for the 256 kHz and 320 kHz carrier outputs respectively. This gives modulation indexes of 0.0014 and 0.0017 respectively, which cause ±256 kHz sidebands with amplitude suppression of 63 dB on the 256 kHz carrier and 61 dB on the 320 kHz line.

From the above it can be seen that the pilot frequency must be selected in such a way that the sum frequency on the phase detector output can be suppressed adequately by the loop filter. Noting that decreased pilot frequency increases the FM modulation index and thereby also the sideband amplitudes, and that at the same time loop filter attenuation decreases, it can be seen that the pilot frequency must be in the order of 100 kHz or higher. 128 kHz was chosen because that is the highest binary frequency between 100 and 200 kHz.

0. Interference Signals - In Section VIII-L it was seen that the effect of an interference signal on the loop input with noise rates within the loop bandwidth would appear directly on the loop output. This statement refers to phase modulation noise of the input pilot, and for the preceding numerical VCO example this noise must be at frequencies 20 kHz or higher to be reduced by the loop filtering effect. Now consider an interference signal at a fixed frequency $f_1$ away from the pilot frequency $f_p$. The interference signal is multiplied by the VCO frequency, and the phase detector output will contain two beat signals, the peak amplitudes corresponding to maximum phase detector output, and the frequencies are the sum and difference of $f_1$ and $f_p$. The two signals will be
attenuated by the loop filter and cause frequency modulation of the VCO with corresponding output signal sidebands in the same manner as explained for the sum frequency component (256 kHz) in the proceeding numerical example. If the input interference signal amplitude is in the same order as the pilot amplitude and the frequency is such that \( f_1 - f_p \) lies within the loop bandwidth, loop behavior will be erratic with partial lock on either one of the two input signals. A VCO type phase locked loop, therefore, can not have interference on the input of such magnitudes and frequencies that such a condition exist, and the loop filter design must be such that any output signal sidebands caused by interference is of an acceptable magnitude. Consequently, a VCO-PLL will not function satisfactory as a pilot demodulator where adjacent channels may be 4, 8, or 12 kHz away from the pilot frequency. A method of overcoming the problem is discussed in the Pilot Demodulator section.
IX. PILOT DEMODULATOR

There are two methods* of accomplishing phase tracking both of which first require recovering the pilot frequency from the composite signal without interference from the adjacent channels. The first method simply uses a fixed tuned bandpass filter at the pilot frequency to recover the pilot and remove the adjacent as well as all other channel interference. This method is considered unsatisfactory because to provide sufficient filtering the band pass filter would require several stages and would cause at least 200° to 300° of phase shift variation for a frequency deviation of ±1% on the composite signal. This would require an equal and matching phase shift at each and every channel frequency to be demodulated. Also as shown in Section III-(A-E) matching phase is required for the information sidebands above and below the center frequency. This would require that the lower frequency channels be up converted before demodulation because of the large percentage of the channel BW occupied by the sidebands. Since the phase matching must be within ±0.18° it seems rather impractical to maintain this degree of phase matching particularly since each filter is at a different frequency depending on the channel frequency to be demodulated.

A more practical method of demodulation which does not require any phase correction of each individual channel or any special time delay matching is to provide a pilot frequency which will follow the ±1% frequency modulation of the composite signal to within ±0.18° in phase. The difficulty of this approach is to recover the pilot contained in the composite signal in such a way that it will follow both the composite signal phase and also discriminate against the adjacent and all other channels contained in the composite signal.

Such a system was built and test with the test results given in Section X-C. As shown from the test results the adjacent channel rejection was not satisfactory and more design work is recommended.

*A third method would be to use a VCO-PLL but as explained in Section VIII-0 this method would not be satisfactory because of the gain and loop filter cut-off characteristics required in a VCO-PLL.
X. TEST RESULTS

The statement of work for NASA contract NAS8-25987 Modification 4, call for testing of certain portions of an FDM system for the purpose of further evaluating the degree of performance that can be obtained in a complete FDM system.

This section gives the results of those tests along with other tests made on previous contracts which are pertinent to FDM system performance.

A. Channel Filters - New channel filters, Model No. QF-2034, were obtained from KAPPA Networks, Inc. to provide a linear phase within the channel pass band from 256 kHz ±16 kHz. Table X-1 shows the gain and phase measurements as made by KAPPA Networks. Figure X-1 show the gain measurements as measured at Martin with a wave analyzer.

The purpose of testing the new channel filters was to determine the quadrature suppression that could be obtained in a QDSBSC double conversion demodulation system if the filter phase versus frequency above and below the translation frequency (256 kHz) was linear. The effect of non-linear phase is discussed in Section III-B.

Figures X-2 and X-3 show that the quadrature suppression was about -50 dB at dc and 16 kHz reducing to about -65 dB at about 12 kHz. The variation in this suppression can be due to either non-linear phase or non-linear amplitude as shown by the analysis in Section III-B, C. As shown by Table X-1, a considerable amount of amplitude variation existed in the pass band. The output load impedance was lowered to provide a flatter gain during the quadrature suppression measurements. It is believed that with a little more careful design, improved attenuation can be obtained and quadrature suppression greater than 50 dB can be consistently maintained.

B. Test Oscillator - A test oscillator was built to provide the capability of testing the entire system under FM conditions of ±1% and a modulation rate up to 500 Hz. This test oscillator is shown in block diagram form in Figure X-4. Figures X-2 and X-3 show the quadrature suppression that was obtained with ±1% FM at 50, 250, and 500 Hz modula-
## PHASE AND ATTENUATION MEASUREMENTS

### PHASE

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### ATTENUATION

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**TABLE X-1**
Figure X-2

0° CHANNEL QUADRATURE SUPP.
±19° FM
Figure 3-3
90° CHANNEL QUADRATURE SUPP
± 1% FM
TEST OSCILLATOR
CH. FREQ. 64 KHZ @ 0°, 90°
Pilot Freq. 128 KHz @ 0°

FIGURE-X-4
tion rates. As shown by the curves the quadrature suppression was greater than 50 dB at 50 Hz but increased to about 45 dB at 500 Hz. This shows that the ±1% modulation does not present a problem but because the suppression was worst at 500 Hz it indicates that the loop gain in the two automatic phase loops in the test oscillator were not high enough at 500 Hz. The actual gain at 500 Hz is about 12 dB. This can easily be increased by 20-30 dB if the loop amplifiers were replaced by high frequency amplifiers. HA741's were used and HA-2500 series should be used. Since it was demonstrated that the system will function satisfactorily with ±1% FM, and because of time and parts availability the loop filter amplifiers were not replaced. Theoretically there should be no problem in rebuilding the loop filters to provide satisfactory operation with ±1% FM at 500 Hz.

C. Pilot Demodulator - Figure X-5 shows a pilot demodulator system which was designed to follow the input pilot frequency contained in the composite signal and also reject or discriminate against the adjacent channels.

The system is made up of a number of LC tuned circuits isolated by op-amps to provide higher Q and good selectivity. The output of the first stage is fed into a AD530 multiplier M1 used as a phase detector. The output of the phase detector is filtered by LPF1 and then fed to LC1 through a varactor to tune the band pass filter to make it follow the input frequency. This reduces the phase to near 0° through LC1. The correction voltage is also "forward" fed to LC2 and LC3 to cause them to track LC1. As a result the total phase error of the three band pass filters is held to within about ±5° or less. The second multiplier M2 compares the composite signal input to the output phase of LC3 providing an output phase error voltage which is filtered by LPF2. This output voltage is then fed to LC2 and LC3 through varactors which reduces the overall phase error to less then ±0.18°. The reason for using the composite signal for one input of M2 is to compare and correct the output phase directly to the pilot phase contained in the composite signal. This prevents pile-up or additions of phase errors through a number of circuits.
As mentioned earlier one of the problems associated with the pilot demodulator is to prevent interference from channels adjacent to the pilot. For this reason it is recommended that the pilot be 128 kHz (so as to be high enough in frequency to provide nearly continuous phase correction in the synthesizer) and also that the reference tone be 132 kHz and additionally it is recommended that the 124 kHz channel not be used as an information channel but that it have a 124 kHz tone equal in amplitude but opposite in phase to the 132 kHz reference channel. The purpose of this is so that the 124 and 132 kHz channel will look like "out-of-phase" double sidebands to the 128 kHz pilot. This will cause them to cancel in the modulators (phase detectors) \( M_1 \) and \( M_2 \) which greatly reduces the phase loop filter requirements and allows for more loop gain and, therefore, tighter phase control. The amount of reduction in the 4 kHz interference signal using this method should be 35-40 dB.

The advantage of a pilot detector of this kind (one that closely follows the input signal phase) is that all other channels can be demodulated without any time delay or other time or phase correction circuits.

The test results show that this pilot demodulator did follow the input phase to less than 0.18° but the adjacent channel rejection was not satisfactory. The adjacent channel (8 kHz) from 128 kHz, was down only 20 dB. The problem is that the interference feeds into the phase detector \( M_2 \) causing a modulating signal which rides on the control signal and, therefore, modulates the tuned circuits causing FM sidebands at 8 kHz intervals.

It is believed that if a pilot demodulator of this type is followed by one or two more sections, the adjacent channels can be eliminated. This method of course would require much tighter phase control on each section because the phase errors would add up.

D. Breadboard Tests - The linear modulator-demodulator breadboard was tested with the modification listed in items 1 and 2 above. The pilot demodulator was not sufficiently developed at the time to be used. Besides modifications 1 and 2 the modulator system true multipliers were replaced by Analog Devices 422K. The results of the test are shown in Section A, C, and E of this section.
E. Multiplier Evaluation - Several true multipliers were tested as replacements for the Intronics 502 used in the original breadboard. It was hoped that the new Motorola MC1594L would be a satisfactory replacement because of its small size, weight and price. However, the non-linearities in the device produce spurious responses of the audio and carrier that were down only 40, 43, 42, 46, etc. dB as examples, even at room temperature. Therefore, no more tests were made on this multiplier.

The Analog Devices AD530J which is a true multiplier completely contained in a TO-5 can, and with deposited internal resistors instead of defused resistors look very attractive. Tests made at room temperature showed that the linearity was good and all spurious responses were down greater than 50 dB. However, the phase shift at 200 kHz was 14° making it unusable for QDSB or SSB where phase tracking of 0.1° over the temperature range of -20 to +85° is required.

The Analog Devices 422K which is a discrete component type true multiplier was tested and found to perform better than the Intronics 502. The phase shift was about 1.5° at 200 kHz with a phase change of about ±0.09° for one unit and about ±0.04° for the other units. Since only two units were tested this is not a very good sample, but if the phase tracking holds for most of this type of multipliers it should be possible to hold 50 dB quadrature suppression as far as the modulator system is concerned.
XI. MODULATOR SYSTEM RECOMMENDATION

A. Recommendation for Single Conversion - On the basis of the synthesizer analysis, performance of available true multipliers, temperature requirements, phase tolerance analysis, etc., it is recommended that the modulation system be of the single conversion type. The factors influencing this recommendation are listed below.

1. Because of the wide temperature range (-20 to +85°C), voltage controlled crystal oscillators (VCXO's) should be used in PLL to synthesize the channel carriers. This makes it possible to provide stable frequencies which can be phase-locked to a low frequency (4 kHz) comb as a pilot for all channels. Also because of the stability of the VCXO's, a very low frequency loop filter cut-off can be used allowing for a channel frequency LPF to be inserted in the loop to provide a good clean sine wave output without affecting the loop stability. This makes it possible to use true multipliers for single conversion.

2. Since the true multipliers have very low harmonic outputs, channel filters are not required following the modulator as would be the case if square waves or balance modulators were used. This eliminates the channel carrier phase shift that would be caused by the temperature effects of the filters.

3. As shown by the tests on the Analog Devices 422K true multiplier, the state-of-the-art of true multipliers has advanced to the point where it is now feasible to use them in an FDM system. The performance is still somewhat limited at the high frequency range because of the phase tracking requirements with temperature. When used in a QDSB system this would cause the demodulated quadrature suppression to increase to about -45 dB at 200 kHz. The quadrature suppression would increase with decreasing frequency, crossing the -50 dB point between 100 to 150 kHz. Suppression of at least 50 dB should be possible for all of the lower channels.

A single conversion modulation system using two Analog Devices' 422K true multipliers was installed in the breadboard. This modulation system is part of the modified breadboard and produces QDSBSC at a channel frequency of 64 kHz.
The output frequency spectrum showed that all spurious responses were down greater than 50 dB over the temperature range of -20° to +85°C.

DEMODULATOR SYSTEM RECOMMENDATIONS

B. Recommendations for Double Conversion - On the basis of the synthesizer analysis, phase tolerance requirements, the FM on the signal due to tape recorder wow and flutter, temperature consideration, and test data; it is recommended that a double conversion demodulation system be used. The reasons for this recommendation are listed below:

1. Because of the ±1% FM on the composite signal it is not possible to provide sine wave channel demodulation frequencies that will follow the composite signal phase. This is particularly true for the lower channels as explained in the filter effects Section III and the synthesizer Section VII.

2. Because of the lower temperature range requirement and because the demodulator is ground based equipment, it is feasible to use matching filters as required in the double translation system. Also, for the design recommended, all of the filters in the demodulation portion of the system are at the same frequency which reduces the cost of design and manufacture and greatly simplifies the testing of the phase matching.

3. While more frequencies are required from the synthesizer all of the outputs are square waves which can be derived digitally from LC-VCO-PLL.

4. If a two or three channel receiver is desired it would be relatively easy to switch the divider chains and change the LC-VCO's to provide any desired channel. If single conversion demodulation was used a different LPF for each channel frequency would be required. However, as explained in the synthesizer section, because of the ±1% FM on the signal, sine waves which follow in phase cannot be produced.
XII. FDM SYSTEM PERFORMANCE COMPARISON

A. Bandwidth Occupancy - In the preceding sections emphasis has been placed on the analysis of the phase shift method of single sideband generation (PMSSB) and QDSB systems. Other methods of SSB generation require filtering for sideband suppression. The disadvantage of the filtering methods compared to PMSSB is a required reduction of 50% of channel bandwidths so that the number of usable channels for a given system bandwidth is reduced by a factor of two. DSB system has the same channel reduction factor. Also the phase shift due to the filters would be different from channel to channel and time correlation would be very difficult. Therefore, we have given this method no consideration because straight DSB is much simpler and provides the same bandwidth utilization.

B. Amplitude Error - An advantage of the DSB over QDSB and PMSSB is a greatly reduced requirement on the phase of the reinserted channel carriers. Carrier phase error in DSB causes a decrease in the output amplitude. Since the amplitude is a Cosine function of the phase error, 4.5° of phase error can be tolerated for a 0.3% amplitude error. The comparable phase error in a QDSB system for a 0.3% amplitude error is 0.18° corresponding to 50 dB of suppression of the quadrature signal in the channel, the error now being caused by quadrature feedthrough, rather than amplitude loss. In PMSSB no amplitude loss will be caused by carrier phase error, however, in the demodulator feedthrough from the neighboring channel will appear as amplitude error. To limit this feedthrough to 0.3%, the demodulator carrier phase error must be less than 0.18°.

C. Frequency Response - Comparing frequency response, the advantage is on the side of DSB/QDSB systems where dc response is feasible while SSB systems have an inherent audio low frequency cut-off.

D. Time Delay - Assuming the system configuration to include a single conversion modulator and a double conversion demodulator, the channel time delay in DSB and QDSB systems is independent of audio frequency with the total channel delay in the order of 2 microseconds. This audio time delay is caused by the signal line filter in the double conversion demodulation technique as explained in Section III, and the
time delay will be common for all system channels. Carrier phase error will not affect the channel delay. For PMSSB a carrier phase error will appear directly as an equivalent audio phase error in the demodulated audio frequency. Therefore, any carrier phase error causes a time delay in the audio frequency. In Section IV-D it was seen that the polyphase network delay time constitutes the major part of the channel delay, and the delay time for each polyphase network varied from 60 milliseconds to 10 milliseconds for audio frequencies of 10 Hz and 2 kHz respectively. Over the temperature range of -20°C to +85°C the modulator polyphase network phase delay may vary by a number of degrees (without temperature control of the networks) corresponding to delay time variations of 1 of 2 milliseconds at low audio frequencies. Non-calibrated system time correlation can, therefore, be maintained to the order of 2 microseconds for QDSB while the comparable figure for PMSSB is of the order of 2 milliseconds.

E. Noise Behavior - If equal average powers are assumed for SSB and DSB/QDSB, the additional noise involved from the reception of two sidebands is exactly compensated for by the coherent addition of these sidebands so that the effective signal to noise ratio will be identical. If man-made noise rather than white noise is considered there exists an advantage to two-to-one in average power in favor of DSB/QDSB.

F. System Complexity - In Section IV it was seen that PMSSB basically uses the same components as QDSB with the addition of the polyphase network in both the modulator and the demodulator. With the added parts, the time consuming alignment procedure and the temperature dependency of the polyphase networks to be considered it is clearly seen that cost, size, and power consumption of PMSSB exceed that of QDSB. For DSB and QDSB and a given number of channels the cost and complexity is comparable.
APPENDIX A
DEFINITIONS

FREQUENCY DIVISION MULTIPLEXING (FDM) - A system for multiplexing
together a number of different sub-carrier channel frequencies which are
each modulated with information data.

MODULATOR - A system component such as a true multiplier or modulator
used in an FDM System.

DEMODULATOR - A system component such as a true multiplier or modulator
used in an FDM system.

MODULATOR SYSTEM - A complete FDM multiplexing system from channel
inputs to output driver.

DEMODULATOR SYSTEM - A complete FDM demodulation system from input
to channel outputs.

FEEDTHROUGH - Unwanted output signal at the same frequency as that
of the input signal (e.g. audio, carrier feedthrough).

QUADRATURE FEEDTHROUGH - Unwanted output signal in a QDSB demodulator
at the same frequency as that of the wanted signal in the quadrature
(90°) channel.

HARMONIC DISTORTION - Unwanted output signal at frequency n times
input frequency (n = 2,3,4 ...).

DISTORTION PRODUCT - Unwanted output signal caused by nonlinearities
in a multiplication process, the frequency being the sum or difference
of any combination of the two input signals, fundamental, and harmonics.

DC OFFSET - Unwanted output DC voltage.

NULL VOLTAGE - Unwanted output signal that may be adjusted for minimum.

SPURIOUS RESPONSE - Any unwanted output signal.

CHANNEL BANDWIDTH - Data frequency passband of individual channels.

LINEARITY - Refers to straight line and includes all points within
+X% of full scale from best straight line.

CHANNEL TIME/P HASE CORRELATION - Comparison of the time delay/phase
delay through any two channels at a given data frequency.
STABILITY (GAIN, PHASE, FREQUENCY) - Refers to maximum deviation from initial setting as a function of time, temperature, power supply, etc.

PHASE SCATTERING - All carrier phases are synchronized with respect to the pilot phase (or comb phase). Scattering refers to positioning of individual channel carrier phases so as to prevent excessive amplitude peaking in the composite data signal.

AGC RESPONSE TIME - Referring to the figure, the Attack time is the time required for the regulator output level to decrease from 90% A to 10% A when the input level has undergone a step increase of 6 dB. The Recovery time is the time required for regulator output level to rise from 10% A to 90% A when the input level has undergone a step decrease at 6 dB.

STATIC PHASE ERROR - Steady state output phase error in a phase locked loop due to open loop difference in loop input signal frequency and the frequency of the loop oscillator.

PHASE JITTER - Any AC phase error in a phase locked loop.

NOISE FLOOR - Inherent circuit output noise amplitude with respect to the wanted signal. (Bandwidth specified)

PHASE LOCKED LOOP (PLL) PULLING RATE - Assuming the PLL input signal to be frequency modulated, the pulling rate is the modulation frequency.
PLL TRACKING RESPONSE - Phase error as a function of input signal frequency pulling rate.

PLL LOCKING RANGE - The locking range is the maximum unlocked frequency difference that the system can accommodate in the closed loop condition.

PLL CAPTURE RANGE - The capture range is the largest unlocked frequency difference at which the system will pull into lock.
APPENDIX B

SUGGESTED GENERAL SPECIFICATIONS FOR FDM SYSTEMS

The listed specifications are intended to cover general requirements for SSB, DSB and QDSB suppressed carrier systems. TBD is an abbreviation of: to be determined.

1. MODULATOR:

   INPUT LEVEL - Each channel shall be capable of accepting TBD data levels.

   INPUT IMPEDANCE - The input impedance of each data input terminal shall be TBD over the input data frequency range.

   OUTPUT LEVEL - The full scale output data level shall be TBD.

   OUTPUT IMPEDANCE - The data output terminal output impedance shall be TBD over the output frequency range.

   BANDWIDTH - The channel bandwidth shall be TBD.

   FREQUENCY RESPONSE - Each channel shall be flat within TBD over the channel bandwidth at full scale input.

   SPURIOUS RESPONSE - All output spurious responses shall be suppressed TBD with respect to full scale wanted signals.

   AMPLITUDE LINEARITY - The output amplitude shall be linear to within TBD for a TBD input level range.

   CHANNEL TIME/PHASE CORRELATION - The time correlation from channel to channel shall vary by no more than TBD for data frequencies between TBD; and the phase correlation from channel to channel shall vary by no more than TBD for data frequencies between TBD.

   GAIN STABILITY - The channel gain shall be stable to within TBD.

   REFERENCE FREQUENCY STABILITY - The reference frequency shall be stable to within TBD.

   CARRIER PHASE SCATTERING - The carrier phase scattering shall be TBD.
CARRIER PHASE STABILITY - The phase of the carriers shall be stable to within TBD with respect to the pilot phase.

AGC RESPONSE TIME - The AGC attack time and recovery time shall be TBD.

AGC OUTPUT LEVEL - The AGC steady state output level shall remain constant to within TBD for a TBD change in input signal level.

2. DEMODULATOR:

INPUT LEVEL - Each channel shall be capable of accepting TBD data levels.

INPUT IMPEDANCE - The input impedance of each data input terminal shall be TBD over the input data frequency range.

OUTPUT LEVEL - The full scale output data level shall be TBD.

OUTPUT IMPEDANCE - The data output terminal output impedance shall be TBD over the output frequency range.

BANDWIDTH - The channel bandwidth shall be TBD.

FREQUENCY RESPONSE - Each channel shall be flat within TBD over the channel bandwidth at full scale input.

SPURIOUS RESPONSE - All output spurious responses shall be suppressed TBD with respect to full scale wanted signal.

AMPLITUDE LINEARITY - The output amplitude shall be linear to within TBD for a TBD input level range.

CHANNEL TIME/PHASE CORRELATION - The time correlation from channel to channel shall vary by no more than TBD for data frequencies between TBD, and the phase correlation from channel to channel shall vary by no more than TBD for data frequencies between TBD.

GAIN STABILITY - The channel gain shall be stable to within TBD.

PREREGULATOR (IF REQUIRED) - The preregulator shall regulate the composite input signal to a TBD level with attack and recovery time of TBD.
AGC RESPONSE TIME - the AGC attack time and recovery time shall be TBD.

AGC OUTPUT LEVEL RESTORER - The output level of the composite signal shall be restored so that the AGC'd demodulated reference tone is within TBD% of the Non-AGC'd pilot tone.

PILOT DEMODULATOR INPUT SIGNAL - The pilot demodulator shall operate as specified for pilot input signal deviation of TBD at rates of TBD, and with interference signals at TBD frequencies and at TBD levels.

PILOT DEMODULATOR STATIC PHASE ERROR - Pilot static phase error shall not exceed TBD with respect to the input pilot signal.

PILOT DEMODULATOR PHASE JITTER - Pilot phase jitter shall not exceed TBD with respect to the input pilot signal.

PILOT DEMODULATOR SPURIOUS RESPONSE - Spurious responses on pilot signal shall be suppressed TBD with respect to the signal amplitude.

PILOT LOCKING RANGE - The pilot locking range shall exceed TBD.

PILOT CAPTURE RANGE - The pilot capture range shall exceed TBD.

CARRIER PHASE SCATTERING - The carrier phase scattering shall be TBD, and the carrier phases shall be adjustable over a range of TBD.

CARRIER STATIC PHASE ERROR - The carrier static phase error shall not exceed TBD.

CARRIER PHASE JITTER - The carrier phase jitter shall not exceed TBD.

CARRIER SPURIOUS RESPONSE - Spurious responses on the carrier signal lines shall be suppressed TBD with respect to the carrier amplitude.

CARRIER LOCKING RANGE - The carrier locking range shall exceed TBD.

CARRIER CAPTURE RANGE - The carrier capture range shall exceed TBD.