ATMOSPHERIC SOUND PROPAGATION

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ABSTRACT

The propagation of sound waves at infrasonic frequencies (oscillation periods 1.0 - 1000 seconds) in the atmosphere is being studied by a network of seven stations separated geographically by distances of the order of thousands of kilometers. One of the typical stations, in Washington, D. C., has an array of five microphones separated by distances of about 7 kilometers. Each microphone is at ground level and is connected to the central station by means of a leased telephone line. In effect the array is "steered" to look for sound waves in a programmed sequence of search directions. The station measures the following characteristics of infrasonic waves passing through Washington: (1) the amplitude and waveform of the incident sound pressure, (2) the direction of propagation of the wave, (3) the horizontal phase velocity, and (4) the distribution of sound wave energy at various frequencies of oscillation. Some infrasonic sources which have been identified and studied include the aurora borealis, tornadoes, volcanos, gravity waves on the oceans, earthquakes, and atmospheric instability waves caused by winds at the tropopause. Waves of unknown origin seem to radiate from several geographical locations, including one in the Argentine.
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1. INTRODUCTION

Sound waves have two principal uses to which acoustical researches have been directed intensively during the last fifty years. The first use, a very ancient one, is in the art of communication between men and the handling of everyday affairs, by hearing and speech. Audio-frequency waves, those audible to man in the frequency range from 15 to 17000 Hz, are necessary for this purpose. They are propagated through the atmosphere for relatively short distances. But unwanted sounds—noise—of the same frequencies interfere with communication and indeed with other aspects of man's activity. The importance of audible sound to man has led to an extensive technology for the quantitative measurement, analysis, and display of such sound waves. Researchers have developed equipment and methods for producing controlled amounts of sound and vibration, and for reducing noise.

A more recent use for sound waves is for researches into the structure and properties of matter—solids, liquids, and gases. Generally speaking, high-frequency sound (at ultrasonic frequencies) is used for studying solids and liquids. Industry has found ultrasonic waves valuable for location of flaws in thick pieces of metal, by measurement and display of the scattered sound from the flaws. Sound waves offer the only practical method for exploring the contours of the sea bottoms, and for locating underwater objects such as submarines at distances more than a few feet away. But for researches on gases, including the atmosphere, sound waves have been found useful over a wide range of frequencies. The microstructure of gases is studied in the laboratory with ultrasound. Waves at audible and infrasonic frequencies, some as low as \(10^{-4}\) Hz, are studied in the free atmosphere. When the details of propagation through a medium are known, then properties of the source itself can be deduced from the sound it radiates, measured at distant points.

Researches on the two principal uses for sound are dependent upon, and unified by, (a) modern electroacoustics which makes the controlled generation of sound waves and accurate measurements of them possible, and (b) the theory of sound propagation and its mathematical-physical formulation.

We are considering in particular propagation through the atmosphere. Some studies aim at measuring the influence of unwanted sound, e.g., aircraft noise on man. Others look towards measurement of atmospheric properties by probing—remote sensing—with sound waves. Still others seek to determine the properties of distant sources of sound. But all researches on atmospheric
sound must depend on theoretical analysis of sound propagation, coupled with measurements of sound at only a relatively few available locations, to arrive at useful results.

We concentrate our attention on infrasound in the atmosphere—sound waves whose frequencies of oscillation are less than the lowest frequency, about 15 Hz, that can be heard. Of particular interest are those waves whose oscillation periods lie in the range of 1.0 to 1000 sec, because such waves propagate for distances of thousands of kilometers without substantial loss of energy. Sounds at these frequencies are almost always present at measurable intensities. Those of natural origin have many causes, including tornados, volcanic explosions, earthquakes, the aurora borealis, waves on the seas, and large meteorites. Man-made sources include powerful explosions and the shock waves from vehicles moving through the atmosphere at supersonic speeds, at altitudes below about 125 km.

2. PRINCIPAL FEATURES OF SOUND PROPAGATION

2.1 Sound Pressure

The passage of an infrasonic wave causes pressure oscillations as it traverses the atmosphere. For infrasound of natural origin, the amplitude \( p \) of the sound pressure is often in the range of 0.1 to 100 dyn/cm\(^2\), and infrasonic microphones are usually designed to respond to such pressures. The atmospheric pressure \( B \approx 10^6 \) dyn/cm\(^2\).

A microphone converts the sound pressure at a particular point into electric current variations having the same waveform. The passage of a sound wave is also accompanied by small vibratory displacements and small variations in temperature of the atmosphere. Microphones have been designed which respond to one or the other of these parameters of the sound wave. For example, the hot-wire microphone responds to the vibratory particle velocity of the air. But any microphone must be located in principle out-of-doors, and it therefore responds to the variable pressure effects of the turbulent eddies associated with the wind, in addition to the effects of the sound wave. An examination of the generation of noise pressure variations caused by turbulent flow shows that the ratio of the desired acoustical signal to unwanted flow noise is greater (by at least an order of magnitude) for the sound pressure than is the ratio for the particle velocity. Therefore measurements of atmospheric sound are always made with pressure microphones.
2.2 Speed of Sound

The square of the phase velocity $c^2$ for sound in a gas at a uniform temperature is the ratio of the gas's modulus of elasticity to its density:

$$c^2 = \frac{\gamma B}{\rho}$$

where $B$ (see above) is the atmospheric pressure in dynes per square centimeter, and $\rho$ is the density in grams per cubic centimeter. For air, the adiabatic gas constant $\gamma = 1.402$ (dimensionless). $\gamma B$ is the adiabatic modulus of elasticity for the atmosphere. But the equation of state for air is $B = \rho RK$, where $K$ is the absolute temperature and $R$ is a constant. Therefore $c^2 = \frac{\gamma B}{\rho} = \gamma \rho RK / \rho = \gamma RK$, and so finally

$$c = \text{constant} \times \sqrt{K}$$ (1)

Equation (1) shows that the speed of sound is independent of the density of the atmosphere, but directly proportional to the square root of the absolute temperature. For air at a temperature of 20 °C = 293 °K, the speed $c$ is about 344 m/sec. From this, the sound velocity can be found at other temperatures by means of Eq. (1). The formula is applicable for all sound waves from the low infrasonic frequency of $f = 0.01$ Hz (wavelength $\lambda = 34$ km) through audible frequencies, $f \approx 1000$ Hz, to ultrasonic frequencies, $f > 20,000$ Hz.

Although the sound speed $c$ is an important parameter, the propagation of sound waves through the atmosphere cannot be characterized by a single unique speed. It is useful to distinguish between four velocities of sound. The phase velocity $c$ (see above) is the speed at which a surface of constant phase travels through the atmosphere for a sinusoidal oscillation having $\omega = 2\pi f$ ($f$ is the frequency). The local phase velocity of sound is fixed only by the temperature of the atmosphere in the vicinity of the region of interest. For long waves extending vertically in the atmosphere with its substantial differences in temperature, the phase velocity is a function of the entire temperature distribution (see the next sub-section 2.3). In such cases, the phase velocity depends upon the wavelength $\lambda$, and hence upon $k = 2\pi / \lambda$. The group velocity is defined in acoustics as $c_g = d\omega / dk$. The signal velocity is defined by $c_s = D/T$, where $D$ is the distance from the source of sound to the microphone, and $T$ is the time between the radiation from the source and the emergence (at the microphone) of the signal from noise. In general, the signal velocity differs from both the phase velocity and the group velocity; the relationships between them depend on the structure of the entire atmospheric path, including winds.
The fourth velocity is directly measured at an infrasonic station. Infra-
sonic microphones are usually on the earth's surface and therefore approx-
imately in the same plane. The speed of a line of constant phase for a sound
wave traveling over the earth's surface can be determined from the output
of the several microphones. This speed $c_h$ is usually called the horizontal
trace velocity. It depends on the elevation above the horizon of the ray
direction for incident plane waves. When the angle of incidence is $\theta$ (eleva-
tion angle $= \pi/2 - \theta$), then $c_h = c/\sin \theta$.

2.3 Effect of Temperature Distribution

The atmosphere is never in an isothermal state, but is approximately
horizontally stratified with the variation in temperature being a function
principally of altitude above the surface. In fact the first evidence for a
region of warm air at an altitude of about 50 km, at the same or a slightly
higher temperature than that at ground level, came from early observations
on the anomalous audibility of sounds from large explosions heard at dis-
tances greater than about 100 km from the explosive source. The audible
waves were in effect used to remotely sense the temperature at the 50 km
altitude of the mesosphere.

The phase velocity therefore varies with altitude since the temperature
does, see Eq. (1). This variation is a gross feature of the atmosphere and
substantially affects the propagation to great distances. The data on the
atmosphere obtained from sound propagation measurements, and from instru-
mented rockets and satellites, show that the temperature, and hence the local
phase velocity, depend on location on the earth's surface as well as on
altitude, and also vary with time. The average properties have been incor-
porated into various "standard atmospheres." The distribution of tempera-
ture and sound speed with altitude for the 1962 U. S. Standard Atmosphere
is shown in Figure 1. The curves should be regarded as averages over all
seasons of the year for northern temperate latitudes. The standard atmo-
sphere is useful for analytical investigations into sound propagation.

A detailed mathematical analysis for the propagation of sound shows
that the speed minimum in the stratosphere results in waves emitted at low
altitudes being "channeled" between the ground and the layer of relatively
high sound speed at 50 km altitude. Loosely speaking, the layer serves as
a reflector, albeit a poor one. For the shorter waves, $T < 15$ sec (approx-
imately), sound-ray trajectories are useful for studying propagation.
Temperature and sound velocity in the 1962 U. S. Standard Atmosphere. Details of the real atmosphere vary with location on the earth's surface and with the seasons.

In general, the rays emitted at low elevation angles $\theta$ from a source at ground level are alternately reflected between the layer at 50 km altitude and the surface of the ground.

2.4 Influence of Gravity

Since the atmosphere is in the gravitational field of the earth, its density decreases approximately exponentially with altitude $z$ above the surface. For an isothermal atmosphere with a sound velocity $c = 333 \text{ m/sec}$, the density will decrease as $\exp(-z/H)$, where $H = \text{scale height of the atmosphere} \approx 8.1 \text{ km}$. The sound pressure for a plane wave of sound sent vertically upward will decrease as $\exp(-z/2H)$, but the particle velocity will increase as $\exp(+z/2H)$, so that the sound intensity would remain constant. If the frequency of oscillation is decreased until $T_R = 4\pi c/\gamma g = 305 \text{ sec}$, ($\gamma = 1.40$, $g = 9.8 \text{ m/sec}^2$), then the phase velocity of the upward traveling wave becomes infinite. But for waves of period shorter than about 100 sec (frequencies greater than about $10^{-2} \text{ Hz}$), gravity effects on sound speed
Figure 2. Phase velocities for sound waves in the atmosphere at very low frequencies. \( \theta = \text{angle of incidence for plane waves.} \)

are not significant.

\( T_R \) is the resonant period for vertical oscillations of the atmosphere. Another important resonant period is the Väisälä period \( T_V = 337 \text{ sec} \), for the isothermal atmosphere \( (c = 333 \text{ m/sec}) \) under consideration. \( T_V \) is the natural period of oscillation for a small parcel of air which is displaced adiabatically in a vertical direction, in an isothermal atmosphere horizontally stratified by gravity. Plane sinusoidal waves of periods \( T > 337 \text{ sec} \) are usually called "acoustic-gravity" waves, although the atmospheric motions satisfy the same equations of motion as do the sound waves of shorter period \( T < T_R \). Because the phase velocity is substantially less than the high-frequency velocity \( c = 333 \text{ m/sec} \), the waves might be called subsonic oscillations. The phase velocity is furthermore, for a particular period \( T \), a function of the angle \( \theta \) between the direction of propagation and the horizontal plane. See Figure 2 for curves showing how the speeds of plane acoustic-gravity waves vary with frequency and angle \( \theta \).

In general the speeds are low enough so that wind and temperature gradients can have a strong effect on the propagation. In fact, it appears that acoustic-gravity waves have never been detected with certainty more than a hundred kilometers or so away from the source. For example, vertical
oscillations of the jet stream at an altitude of about 10 km produce strong subsonic pressure oscillations, at periods $T > 300$ sec, at ground level over a large area of the eastern seaboard of the United States. But these occur only when the jet stream is vertically overhead. Acoustic-gravity waves are probably scattered and absorbed strongly by wind and temperature gradients in the atmosphere, and so are not propagated with measurable intensities over global distances away from the source area.

2.5 Atmospheric Absorption

The absorption of infrasound in the atmosphere, due to viscosity and heat conduction, is considerably less than the absorption for audible sounds because of the low frequency of oscillation. The absorption coefficient $\alpha$, defined by the spatial variation of $p$, $|p(x)| = p_o \exp (-\alpha x)$, is about $1.6 \times 10^{-4}/T^2$ decibels (dB/m). $B$ is the barometric pressure in dyn/cm$^2$. For a plane wave of sound in the lower atmosphere at $T = 10$ sec, the absorption is, therefore, less than $2 \times 10^{-9}$ dB/km. Hence the loss due to this absorption mechanism is totally insignificant, even for propagation over distances of thousands of kilometers. The absorption in the upper atmosphere is substantially greater because of the lower barometric pressure. At an altitude of 90 km, where the barometric pressure $\approx 1$ dyn/cm$^2$, the absorption $\approx 2 \times 10^{-3}$ dB/km for waves of 10-sec periods.

Up to altitudes of about 10 km in the troposphere, the absorption due to water vapor should be considered. The exact variation of this absorption with barometric pressure is not accurately known for infrasonic frequencies. We estimate that at sea level (altitude = 0 km) the absorption coefficient might be as large as $5 \times 10^{-9}/T^2$ dB/m, which is about 30 times greater than the absorption for viscosity and heat conduction, as indicated previously. But the absorption due to water vapor is still insignificant for infrasound at $T = 10$ sec, being only $5 \times 10^{-8}$ dB/km. This corresponds to an energy loss of less than one percent after propagation half-way around the earth, a distance of 20,000 km.

At very low frequencies, there is an absorption due to relaxation of the thermal energy stored in vibrations of the diatomic molecules in air. We estimate the absorption coefficient to be almost 1000 times greater than that of the viscosity-heat-conduction loss. Therefore, waves in the lower atmosphere at $T = 10$ sec have $\alpha \approx 10^{-6}$ dB/km. Again, this is an insignificant loss.
The atmosphere has inhomogeneities in temperature and density arising, for example, from solar heating of the ground. Inhomogeneities in density and motion are associated with turbulence in the wind as it passes over trees, buildings, hills, etc. Furthermore, sound waves are scattered by such obstacles as well as by the atmospheric inhomogeneities. All of these effects cause attenuation of sound-wave energy. But, the attenuation is estimated to be quite small when the wavelength is greater than about 1 km.

The net result is that the total attenuation for infrasound in the atmosphere is small enough so that propagation can occur over thousands of kilometers without substantial loss of energy. An example of this is the sound from the tremendous explosion of the volcano Krakatoa in the East Indies in 1883. The absorption of infrasound from the explosion was low enough so that the waves were still detectable after having traveled around the earth several times. Even though electroacoustic equipment suitable for measurement of infrasound did not then exist, the inaudible sound waves from this disturbance had sound pressures so great that readable deflections were produced on barographs all over the world.

2.6 Influence of Winds

The wind speed \( w \) near the surface of the earth is rarely a substantial fraction of the speed of sound \( c \). The wind Mach number \( \beta = w/c \) is usually less than 0.05. But when the jet stream blows at the tropopause at an altitude of about 12 km, then \( \beta \) is at least 0.1 and sometimes as great as 0.25.

Winds near the stratopause at an altitude of 50 km can be even stronger, and \( \beta = 0.35 \) occasionally. At the stratopause the phase velocity of sound has a maximum, since there is a temperature maximum at that level (see Figure 1). Therefore the wind speed and its direction with respect to the direction of sound propagation have important influences on the channeled propagation of sound between the surface of the earth and the stratopause.

In temperate latitudes in the northern hemisphere, there are strong westerly winds for a long regime (about five months) during the winter. There is a shorter regime of strong easterlies during the summer, about two and one-half months. The following is a short table of winds at the stratopause over the continental United States.
Table 1

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Mean values of wind speed in m/sec.</th>
<th>16 Oct. to 31 March</th>
<th>1 April to 31 May</th>
<th>1 June to 15 Aug.</th>
<th>16 Aug. to 15 Oct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>(+) = towards east, (-) = towards west</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>+52</td>
<td>+52</td>
<td>+2</td>
<td>-42</td>
<td>0</td>
</tr>
<tr>
<td>55</td>
<td>+58</td>
<td>+58</td>
<td>-5</td>
<td>-47</td>
<td>+2</td>
</tr>
<tr>
<td>60</td>
<td>+61</td>
<td>+61</td>
<td>-8</td>
<td>-51</td>
<td>+5</td>
</tr>
</tbody>
</table>

The mean north-south (meridional) wind components are less than 10 m/sec at any time of the year, and have an annual average speed of 6 m/sec toward the north. Fuller details on the winds at various altitudes have been published by the Air Force Cambridge Research Laboratories (1965).

We see that the effective speed of sound at the stratopause for propagation to the east during the winter is \(330 + 57 = 387\) m/sec (see Figure 1 and Table 1), whereas at the surface the speed is much less, \(c = 335\) m/sec. But for propagation to the west, the effective speed at the stratopause is \(330 - 57 = 273\) m/sec, and the speed at the surface is substantially greater. In brief, the conclusion is that over the continental United States the 50-km thick atmospheric layer between the stratopause and the surface of the earth serves as a waveguide for eastward propagation of sound energy during the winter, but not for westward propagation. In summer the opposite is true; the waveguide effect is only for westward propagation.

For sound waves generated by a source near the earth's surface, the signal velocity \(c_{\text{S}}\) for propagation eastward will generally differ from that westward. The amount of the difference will of course vary with the seasons, because of the seasonal changes in the stratopause winds. In addition because of the difference in waveguide properties, the attenuation of infrasonic waves for eastward propagation will differ from westward propagation. For example, the westward attenuation in winter will be much greater.

Analytical expressions for the effect of stratopause winds on infrasonic signal velocity and attenuation need to be developed. We suggest that such expressions, applied properly to data on propagation of infrasound, could be used to remotely sense winds at the stratopause.
3. MEASUREMENTS OF INFRASOUND

3.1 Measurement System

The electroacoustic system used at each of the infrasonic stations in the ESSA network consists of an array of at least four microphones, associated electronic filter-amplifiers, and recorders. The system is designed for determining four characteristics of infrasonic waves passing through the station area: (1) the amplitude and waveform of the incident sound pressure, (2) the direction of propagation of the wave, (3) the horizontal phase velocity, and (4) the distribution of sound wave energy at various frequencies of oscillation.

The microphones are located at ground level, approximately in the same plane, and about 7 km apart. See Figure 3 for the station at Washington, D. C. Effects on each microphone of pressure fluctuations caused by local

![Diagram](image) 

Figure 3. Location of line-microphones at the infrasonics station in Washington, D.C. Recordings are made at the Buro site.
turbulent wind eddies are minimized by noise-reducing lines of pipe which are about 300 m long, have capillary inlets, and are connected to the inlet to the microphone. The theory of this noise-reducing line has been described by Daniels (1959). For sound waves of wavelength greater than about 3 km, the line microphone is essentially nondirectional and does not attenuate the sound pressure appreciably. However, noise due to random pressure fluctuations in the period range of 1.0 to 30 sec, such as that caused by wind turbulence, is reduced considerably.

The microphones are of the electrostatic condenser type, and produce frequency-modulated voltages, on a carrier frequency of about 1500 Hz, proportional to the incident sound pressure. These voltages are transmitted by telephone wires to a central location where they are demodulated, amplified, and recorded by several means that will be described below. Band-pass filters are introduced into the amplifiers when a higher signal-to-noise ratio is desired for the sound under study. Earthquake waves, for example, are best studied with a band-pass filter passing sounds having periods of oscillation between 0.4 and 20 sec, as in Figure 4.

![Figure 4. Response curves of some filters used with an infrasonic microphone system. The ordinate scale applies to ink-on-translucent-paper chart recordings.](image-url)
Calibration of each microphone is done by connecting its inlet through a short hose to a calibrating barrel with a volume of about 0.19 m$^3$. An oscillating piston on top of the barrel produces accurately known sinusoidal sound pressures within it at various low frequencies.

3.2 Recording on Paper Charts

A convenient recording scheme has been in use for many years. It is in analog form, in real time, as ink-on-translucent-paper traces. Each line-microphone is recorded on its own paper chart. The waveform of each recording is that of the sound pressure as modified by the gain, as a function of frequency, of the microphone and electronic filter combination. An important feature of the scheme is the accurate timing trace for each recording.

The four characteristics of the sound wave (see the preceding subsection 3.1) are obtained from the ink-on-paper traces by a procedure based on visual congruence (correlation) between pairs of recordings, which are matched by superposition on a transparent table top illuminated from below, for example. At "best correlation," time differences are obtained for arrival of the same sound waveform at the several pairs of microphones. The direction of propagation and the horizontal phase velocity are found from these differences by a simple geometrical procedure described by Matheson (1966). The sound pressure amplitude is obtained from a calibration of the microphone-recording system with an oscillating piston source, and the dominant periods are found by inspection of the recorded waveform. The success of the correlation scheme depends on the fact that almost all sound waves coming from distant sources have approximately plane wavefront surfaces of constant phase.

3.3 Magnetic Tape Recording

With the magnetic tape scheme, sound pressures at the several line-microphones are recorded in analog form on parallel channels (tracks) on the tape along with time. When a sound wave is present, its direction of propagation and horizontal phase velocity are obtained from the magnetic tape recording by means of an automatic multichannel correlator. This is essentially an analog computing instrument which receives the magnetic tape recording and produces an output trace (on paper tape) proportional to the average of the cross-correlations between pairs of microphone voltages.
Variable time delays are mechanically introduced into each microphone channel, with the delays corresponding to a systematic search for correlation at all azimuths and over a range of horizontal phase velocities between \( c = \) the speed of sound and \( c\sqrt{2} \). Details on the automatic correlator have been given by Brown (1963). The direction of propagation and the horizontal phase velocity are read from the output trace of the correlator.

4. RESULTS OF OBSERVATIONS ON INFRASONIC WAVES

We proceed to describe infrasound caused by the following geophysical disturbances: volcanic explosions, the aurora borealis, earthquakes, microbaroms due to ocean waves, subsonic oscillations of the jet stream, and shock waves from the entry of meteorites and satellites into the atmosphere.

There are other natural sources of infrasound not yet fully studied. In particular severe storms such as tornadoes (Cook and Young, 1962), and the passage of winds over certain mountainous areas, give rise to infrasonic waves in the atmosphere. Two areas of "mountain" waves seem to be the Pacific coast of North America between north latitudes 40° and 60°, and the region of Argentina east of the Andes Mountains between south latitudes 25° and 35°.

4.1 Volcanic Explosions

Sufficiently strong volcanic explosions occur frequently enough to provide many useful data on the propagation of infrasound over global distances through the atmosphere. We mentioned earlier the tremendous explosion of Krakatoa in the East Indies in 1883, whose infrasonic waves were still detectable after having traveled around the earth several times. Following are a few of the volcanos whose explosions radiated substantial amount of infrasound: Bezymyanny in eastern Siberia in 1956, reported by Passechnik (1959); Mt. Agung on the island of Bali in 1963, reported by Goerke et al (1965); Mt. Redoubt in southern Alaska in 1966, reported by Wilson (1966); the caldera on Isla Fernandina in the Galapagos Islands in the spring of 1968, still under study.

The sound waves from Mt. Agung were observed at three infrasonic stations in the continental U.S.A. The essential features of the data are shown in Table 2 and Figure 5.
Table 2
Sound Waves from the Explosion of Mt. Agung
8.3° south lat., 115.5° east long.,
at 0855 h, May 16, 1963 (UT).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>40.1°N</td>
<td>42.5°N</td>
<td>39.0°N</td>
</tr>
<tr>
<td></td>
<td>105.2°W</td>
<td>71.2°W</td>
<td>77.1°W</td>
</tr>
<tr>
<td>Short great-circle</td>
<td>14,700</td>
<td>16,200</td>
<td>16,300</td>
</tr>
<tr>
<td>distance from</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mt. Agung, km</td>
<td>25,300*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed infrasonic</td>
<td>2301 h, May 16</td>
<td>0028 h, May 17</td>
<td>0150 h, May 17</td>
</tr>
<tr>
<td>arrival, UT</td>
<td>0757 h,*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>May 17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal velocity</td>
<td>288</td>
<td>289</td>
<td>268 †</td>
</tr>
<tr>
<td>m/sec</td>
<td>305*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum amplitude,</td>
<td>&gt;6.6</td>
<td>10.6</td>
<td>9.0</td>
</tr>
<tr>
<td>dyn/cm²</td>
<td>2.4*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured azimuth of</td>
<td>304°</td>
<td>350°</td>
<td>347° †</td>
</tr>
<tr>
<td>sound wave arrival</td>
<td>111° *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Azimuth of great-</td>
<td>300.5°</td>
<td>348.3°</td>
<td>336.4°</td>
</tr>
<tr>
<td>-circle to Mt. Agung</td>
<td>120.5° *</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Via great-circle path through antipode
† Uncertain because start of received signal was obscured by noise.
‡ Observed about 2 1/2 hr. after start of signal.

The sound pressure at each station emerged slowly from noise, and so the measured transit times might be somewhat greater than the "true" (least) time. In particular, the rather low signal velocity deduced from the Washington data is due to masking of the early part of the infrasound by wind noise. The low signal velocity of 288 m/sec deduced from the Boulder and Boston data (short great-circle paths) might be due in part to easterly winds at the stratopause over the Pacific Ocean and the continental United
Figure 5. Variations of azimuth for arrival of infrasound from Mt. Agung in Boulder on May 16 - 17, 1963. The three points are for sporadic appearances of sound waves in noise at about 0800 - 0900 h.

About two-thirds of the antipodal (long) great-circle path to Boulder is in the southern hemisphere, where much less is known of the upper atmosphere winds. The infrasonic signal velocity of 305 m/sec is a little higher than the average of 300 m/sec for the wind-free atmosphere. We therefore estimate that the stratopause winds at the 50 km altitude in the southern hemisphere, during the infrasound transit in May, must have been mainly toward the west, at an average speed no greater than $-10$ m/sec.

But this conclusion is at variance with the deduction from Webb's (1964) hypothesis that the southern hemisphere winds can be deduced by symmetry from the northern hemisphere data. The hypothesis would lead to strong winds, at the southern hemisphere stratopause, toward the east at $+50$ m/sec over about half of the infrasonic propagation path. Furthermore, such strong adverse winds, if present, would have reduced the antipodal sound intensity by a much greater amount than was actually observed.
The reader should note that the wind data published by the AFCRL (1965) for the stratopause are based on measurements made with vertically ascending rockets at a number of geographical locations. In other words, the wind measurements were made at isolated points on the earth's surface, separated by thousands of kilometers. Measurements of sound propagation offer the potentiality (not yet realized in full) of obtaining average winds over long paths in the atmosphere. These propagation data should be useful supplements to the rocket data.

4.2 Auroral Infrasonic Waves

Two types of infrasonic waves caused by the auroral borealis are found in the atmosphere of the northern hemisphere at temperate and high latitudes. The first type is found at mid-latitudes during sufficiently strong magnetic storms even in the absence of a visible aurora at the geographical location of the infrasonic station. The second type of infrasonic wave, found near the auroral oval at high latitudes when visible sharply-defined auroral forms travel overhead across the station location at supersonic speeds, has directions of propagation and horizontal trace velocities very nearly the same as those of the visible auroral form. Before discussing these two types of waves, we digress to present a short description of a magnetic storm and related phenomena.

With the advent of a solar flare or a sun storm, electromagnetic radiation reaches the earth almost immediately. An ionized-gas cloud sometimes arrives one or two days later. This plasma cloud perturbs the magnetic field of the earth. Mid-latitude observatories see a rise in the horizontal component of the magnetic field, followed by a larger decrease and a recovery lasting several days. The strong and erratic variations that result are known as magnetic storms, magnetic activity, or disturbance variations. A measure of this solar-particle radiation effect is furnished by the planetary magnetic index Kp which is derived from data from a number of participating magnetic observatories. One of a series of numbers from 0 to 9 is given to each three-hour interval of each day, a larger number indicating a greater departure from undisturbed conditions. During large magnetic storms, magnetic fluctuations with periods from a few seconds to several minutes occur, radio communications are disturbed, x-rays are observed with instruments carried in balloons, and the aurora is observed in mid-latitudes.

Waves recorded during the magnetic storm of February 11, 1958, at
Washington, exhibited a more-or-less typical behavior pattern. The storm began on February 11 at 0126 UT. It was accompanied by an intense red aurora visible in Washington. The first distinguishable sound waves arrived about 0642 UT from a north-northwest direction and had a trace velocity of 775 m/sec. Measurements at 0905 UT indicated a direction slightly more from the west and a trace velocity of 750 m/sec. The sound waves decreased in amplitude and disappeared between 1100 and 1200 UT. Comparison of the large trace velocity, usually greater than 400 m/sec at the mid-latitudes of Washington and Boulder, with the local speed of sound is often enough to distinguish these waves from other infrasound. There are variations with time in the apparent direction and trace velocity of the waves. There is an apparent short-period cutoff near $T = 15$ sec.

Figure 6. Azimuth of auroral infrasound arriving during magnetic storms, as a function of local time in Washington, D.C. Bearing (= azimuth) = angle, in degrees east of north, to the direction from which the sound comes. The open circles are measured azimuths during 1956 - 1962.
The remarkably consistent changes in direction of arrival with time of day are shown in Figure 6. Direction changes from the northeast in the evening, through north about midnight, then northwest in the morning, and to the northeast again somewhat suddenly after local noon. The data in this figure were restricted to signals with trace velocities above 390 m/sec to help prevent possible confusion with sound from other sources.

Sound waves usually arrive at Washington, D. C., within 5 or 6 hr of rise of \( K \) to a value of 5 or higher. Predominant periods range from 20 to several hundred seconds. The pressure amplitude is usually less than 3 dyn/cm\(^2\), but is sometimes 7 or greater. Durations range from 1 or 2 hr to more than 24 hr, with a mean of about 6 hr. During the active solar years 1960 and 1961, auroral infrasonic waves were observed for more than 200 hr each year at Washington, D. C. Additional details are given by Chrzanowski et al (1961, 1962).

A very simple hypothesis may serve to explain qualitatively the experimental observations on these sound waves at mid-latitudes. Imagine a somewhat diffuse source in the lower ionosphere and fixed in geomagnetic latitude on the side of the earth opposite the sun. Let the magnetic latitude of the center of the source be that of the auroral zone, or about 66°. Waves from the source spread out through the ionospheric plasma at supersonic speeds. Sound waves leak out from the lower surface of the plasma and therefore have the supersonic horizontal trace velocities observed at mid-latitudes. The earth will turn underneath the source once each day. This qualitatively explains the diurnal change of direction of infrasound observed at Washington, D. C. The relative absence of short periods and the large trace velocity suggest a high-altitude source where the mean free path of the molecules is long and the modes excited in the atmospheric wave guide have wave normals with a vertical component. This picture is, of course, oversimplified. Since the aurora moves south with increasing geomagnetic activity, it is possible that the sound source may vary in geomagnetic latitude with strength of the disturbance. Fluctuations in longitude of the source may also occur.

On the basis of the above hypothesis and the observations of duration and amplitude at Washington, D. C., it seems reasonable to assume that perhaps one quarter of the earth's surface is simultaneously bathed in acoustic radiation with an average pressure of about 1 dyn/cm\(^2\). This suggests an acoustic source of roughly \( 10^9 \) W during a typical magnetic storm.
The other type of infrasonic waves, caused by visible aurorae, has been observed by C. R. Wilson and his colleagues (1967a, 1967b) at the infrasonic station at College, Alaska. The basic observations are: (a) the horizontal trace velocity is supersonic, $c_h > 450$ m/sec; (b) the transient pulses of sound have about the same direction of propagation and velocity as fast-moving auroral arcs overhead at Alaska, measured with an all-sky camera; (c) the dominant period of oscillation is about 20 sec; (d) peak sound pressure is typically $5$ dyn/cm$^2$; and (e) each pulse is of only a few minutes duration.

\[\text{Figure 7. Acoustical shock wave caused by supersonic motion of the leading edge of an auroral form.}\]

The observations can be very well explained by means of Wilson's shock wave model, in which the supersonic motion of the leading edge of an auroral wave gives rise to an acoustical shock wave (see Figure 7). The lower edge of the aurora serves as a line source (perpendicular to the plane of the paper). A particular pulse arrived at the ground station microphones 420 sec after an auroral arc passed overhead. The 420-sec delay corresponds to a source altitude of 140 km for an assumed average $c = 300$ m/sec and measured $c_h = 680$ m/sec. This altitude is to be compared with the known heights of visible auroral arcs, which in most instances have streamers extending from 110-km to 145-km altitudes.
Auroral infrasound is apparently not propagated into the equatorial zone. The infrasonic stations at Huancayo, Peru (12° S lat.) and La Paz, Bolivia (17° S lat.) have not yet detected infrasound from either the aurora borealis or the aurora australis. The station at Tel Aviv, Israel (32° N lat.) has observed auroral infrasound on only two or three occasions. These observations are consistent with the leakage of acoustic waves from an ionospheric disturbance originating in the auroral zone.

4.3 Earthquakes

After a strong earthquake, traveling waves spread from it over the earth's surface and radiate sound into the atmosphere as well. The vertical component of the earth's surface motions gives rise to the sound radiation. There are several different types of earthquake waves, and they all travel with speeds much greater than the velocity of sound in air. As a consequence, the sound radiations are propagated upward in a direction almost perpendicular to the earth's surface. The strongest surface motions at locations away from the epicenter of the earthquake are those caused by Rayleigh waves. These travel entirely on the surface of the earth and have periods of oscillation \( T \) between about 10 and 50 sec. The phase velocities \( c_0 \) of these waves when traveling over continental surfaces are about 3.5 km/sec. The sound from Rayleigh waves is occasionally strong enough to reach the ionosphere and cause substantial motions there.

The sound radiated by strong earthquakes can be measured at infrasonic stations. Usually the detected sound is that locally radiated by earthquake waves passing through the geographical area of the station. But from a very strong earthquake, sound radiated directly from the epicentral area into the atmosphere can be measured at an infrasonic station several thousand kilometers away. This occurred at the time of the great Alaskan earthquake in 1964, whose epicentral sound was readily measured at the Washington, D. C. infrasonic station.

Let us look into the characteristics of a few of the waves which spread out from the focus of an earthquake. The focus is the location on or near the surface at which the earthquake occurs (Figure 8). The epicenter is the point on the surface where a radius vector terminates on passing from the center of the earth through the focus. There are three waves of principal interest to our discussion. The first wave to arrive at a distant location is a longitudinal wave which has passed through the body of the earth; this
Figure 8. Seismic waves from the focus of an earthquake, and sound radiated into the atmosphere by the seismic waves.

is called a \( P \) wave. The second to arrive is a transverse or shear wave traveling more slowly and designated as an \( S \) wave. The third wave, which travels entirely on the surface, is the Rayleigh wave. These three are accompanied by many others, for example, by \( P \) and \( S \) waves reflected from the boundary between the mantle and core 2900 km below the surface. From measurements of arrival times of the waves received at several seismological stations, the epicenter and focal depth of an earthquake can be determined very accurately.

**Montana Earthquake**

The great earthquake in Montana at 0637.27 UT on August 18, 1959, produced seismic waves strong enough in the Washington, D. C. area to cause easily measured infrasonic waves in the atmosphere. The epicenter in Montana was 2860 km away from the Washington infrasonic station on a great-circle bearing 66° west of north.

The sound radiated by the \( P \) wave arrival was obscured by wind noise and microbaroms and could not be distinguished with certainty. The arrival of the \( S \) waves (shear waves) produced measurable infrasound, from which it was deduced that the shear waves came from 44° west of north with a period of oscillation of 11 sec and a horizontal trace velocity of 6.0 km/sec. The
arrival was from a direction 22° north of the great-circle bearing. The earth's displacement in Washington, deduced from a peak-to-peak sound pressure of 0.8 dyn/cm², was 0.34 mm.

The very strong Rayleigh waves had periods initially of about 15 sec, which shortened to about 8 sec after a minute or so. This change came about because the group velocity of the 8-sec waves was less than for the 15-sec waves, and so it took longer for the 8-sec waves to travel across the country to Washington. The earth's displacement, calculated from the large peak-to-peak sound pressure of 5 dyn/cm², was 3.0 mm. The average trace velocity of the Rayleigh waves was 3.8 km/sec. They came mainly from the great-circle direction of the epicenter, with the later waves coming from a slightly more northerly direction.

We can only conjecture about the reason for the arrival of the seismic waves from directions mostly north of the great-circle bearing. Refraction on passing from the Appalachian Mountains onto the Piedmont Plateau might have been the cause.

**Acoustical Radiation Zones**

An interesting feature of the sound radiated into the atmosphere by Rayleigh waves is that the sound pressure at any point on the surface is due to the integrated effect of an extensive area of the traveling waves. This is in contrast to a seismometer, which measures the earth's displacement only at the spot where the instrument is located.

We present the results of an analysis showing how much of the traveling wave motion is effective in producing sound pressure at a point just above the surface. For a long train of sinusoidal surface waves of wavelength $\lambda_o$, the waves in a circular area of radius $4 \lambda_o$ contribute at least 70 percent to the amplitude of the sound pressure at the center of the circle. We call this circular area the "radiation zone" for the sound pressure produced by the surface waves. If the Rayleigh waves have a period $T = 25$ sec, then $\lambda_o = 88$ km, and so $R = 4 \lambda_o \approx 350$ km; this is the radius of the radiation zone for such long waves. Fuller details have been given by Cook (1965).

**Ionospheric Motions**

At the time of the Alaskan earthquake on March 28, 1964, Rayleigh waves of considerable amplitude passed across the United States. The sound waves which they produced at infrasonic frequencies were propagated upward and
caused substantial motions of the ionosphere.

We examine first the equation for propagation of a sound wave into the less dense regions of the upper atmosphere. We recall that the sound wave travels almost vertically upward in a direction $\arcsin (c/c_0) \approx 6^o$ from the vertical. The main features of the propagation can therefore be seen from a consideration of plane waves of sound traveling vertically upward parallel to the $z$-axis in an approximately isothermal atmosphere. The differential equation for the particle velocity $v$ in the waves is

$$\frac{\partial^2 v}{\partial z^2} - \frac{\gamma g}{c^2} \frac{\partial v}{\partial z} = \frac{1}{c^2} \frac{\partial^2 v}{\partial t^2}$$

(2)

In this $c^2/\gamma g = H = the scale height of the atmosphere \approx 8.4 \text{ km}$ (in the lower atmosphere), for $\gamma = 1.40$, and $g = 9.8 \text{ m/sec}^2$, $c = 340 \text{ m/sec}$. The density $\rho$ of the atmosphere as a function of altitude $z$ is given by $\rho = \rho_0 \exp(-z/H)$. The particle velocity $v$, obtained from equation (2) for sinusoidal waves $(\omega = 2\pi/T)$ is

$$v = |v_0| \exp(z/2H) \cos \left[\omega t - z \sqrt{k^2 - (1/2H)^2}\right],$$

(3)

where $|v_0|$ = amplitude of vibration at ground level $(z=0)$. From this we see that for vertically traveling sound waves in the atmosphere, the amplitude of vibration varies inversely as the square root of the atmospheric density.

Let us see how this applies to the Rayleigh waves from the Alaskan earthquake. The infrasonic stations at Boulder and Washington measured sound pressures of about 20 dyn/cm$^2$, the period $T$ of the waves being of the order of 25 sec. These very substantial sound pressures correspond to vertical surface motions $|v_0| \approx 0.5 \text{ cm/sec}$. A wave that starts out with an amplitude of 0.5 cm/sec will increase to an amplitude of about $10^4 \text{ cm/sec}$ at an altitude of 160 kilometers.

A little before it reaches this altitude the sound wave becomes a discontinuous shock wave. From the analysis that follows we can estimate the altitude $Z$ at which this occurs. In a real gas the pressure-crests in a sound wave continually gain on the troughs, since the crests have the excess velocity $|v|$ computed above. The atmospheric waves start out sinusoidal, and the time $T'$ at which the crests overtake the troughs and discontinuity begins is given by

$$\frac{\lambda}{2\pi} = \int_0^{T'} |v| \ dt = \int_0^{T'} |v_0| \exp(ct/2H) \ dt = 2(H|v_0|/c) \left[\exp(cT'/2H)-1\right].$$

(4)
The foregoing is based on Lord Rayleigh's (1945) analysis. Continuing, we find that the altitude \( z \) is approximately

\[
Z = cT' = 2H \log \left[ 1 + \frac{c\lambda}{4\pi |v_0|H} \right] \approx 138 \text{ km.} \tag{5}
\]

From the foregoing analysis we estimate that the oscillatory vertical motions of the atmosphere at higher altitudes, caused by the Rayleigh waves, are of the order of hundreds of meters per second.

We consider next the results of observations on ionospheric motions near Boulder, Colorado. These observations were made by Mr. Donald M. Baker of ESSA's Boulder Laboratories. He sent radio waves almost vertically upward and observed the waves reflected from the ionosphere back down to a receiving station on the ground. At the time the Rayleigh waves passed through the Boulder area, Doppler shifts of more than 3 Hz occurred for the radio waves at 4 MHz. Such shifts correspond to a vertical motion of the ionosphere, at the 4 MHz reflection height of about 240 kilometers, of more than 200 meters per second. Similar Doppler shifts occurred in the 10-MHz radio wave propagated from the standard-frequency station WWV in Hawaii and received in Boulder, but it is difficult to estimate the geographical area of the ionosphere from which reflections might have occurred.

The Doppler shifts started about 9 min after the Rayleigh waves arrived at Boulder. The sound waves travel vertically upward with an average velocity \( c \) of about \( 1/3 \text{ km/sec} \) up to an altitude of 140 km. At higher altitudes the velocity \( c \) is about 700 m/sec. The computed transit time to the ionosphere at an altitude of 240 km is therefore less than 10 min, which is confirmed by the observed transit time of 9 min.

We conclude from the foregoing that the Rayleigh waves traveling across the continental United States from the Alaskan earthquake produced sound waves which, on propagation upward through the atmosphere, caused substantial motions of the ionosphere.

The absorption and dissipation into heat energy of the sound waves takes place in the ionosphere. This can be seen from the results of the measurements. These show that the intensity on entering the ionosphere is at least of the same order of magnitude as the computed intensity, the latter being based on no absorption in the lower atmosphere. We can estimate the total energy dissipated. The intensity of the sound waves traveling upward was about 10 erg/(cm\(^2\) sec) for about 300 sec. Therefore the total sound energy carried up into the ionosphere, and there dissipated as heat, was roughly
6 x 10^{20} \text{ ergs for the area of North America} \text{ (about 20,000,000 km}^2\text{). This surprisingly large energy is to be compared with the total estimated seismic energy released by the Alaskan earthquake, which was about 10^{24} \text{ ergs.}

4.4 Microbaroms

Natural sounds in the atmosphere having dominant periods of oscillation between 4 and 7 sec seem to be particularly widespread and are commonly called microbaroms. They are characterized by their persistence at a given geographic location for many hours, by a rather constant period of oscillation, and by an amplitude which seldom exceeds 3 or 4 \text{ dyn/cm}^2 \text{ in the area of Washington, D. C. Microbaroms of 4-sec periods were observed by Gutenberg and Benioff (1941) at Pasadena in southern California. They found the microbaroms to be sound waves traveling approximately parallel to the earth's surface and coming from a direction southwest of Pasadena. Saxer (1945, 1953-54) and Dessauer et al (1951) observed microbaroms at Fribourg in Switzerland. They found the waves traveling parallel to the earth's surface to come from a direction northwest of Fribourg, with periods \approx 5 \text{ sec, and sound pressures } \approx 0.5 - 1 \text{ dyn/cm}^2. \text{ Furthermore, the sound pressures seemed to be correlated with the heights of water waves in the north Atlantic Ocean due to storms and with the strength of microseisms in the earth observed at Strasbourg (about 200 km north of Fribourg). Observations on microbaroms in the Washington area will be discussed later.}

What causes microbaroms? Similarities between them and microseisms on the earth's surface suggest that one causes the other. But it can be easily shown that, on the one hand, the sound waves radiated into the atmosphere by microseisms are much weaker than microbaroms. On the other hand, sound pressure of the latter on the earth's surface is not strong enough to produce observable microseisms (Cook and Young, 1962).

It had long been conjectured (e.g., Daniels, 1962) that the gravity waves created by storms on the surfaces of the seas radiate sound into the atmosphere. In the analysis that follows we shall introduce quantitative expressions showing that long trains of such waves cannot radiate sound power, because their phase velocities are less than the phase velocity $c$ of sound in the atmosphere. On the other hand sound power radiation can occur (a) when the waves come to an abrupt stop as, for example, on the beach, and (b) when waves traveling in different directions have an interference pattern which causes periodic oscillations in the potential energy of the atmosphere over the waves.
Radiation by an infinite train of surface waves on water

We consider first a sinusoidal gravity wave on water, of amplitude $A$ and period $T$, the wave fronts being straight lines parallel to the $z$-axis. (See Figure 9). We suppose the depth to be substantially greater than

$$\frac{1}{k_o} = \lambda_o / 2\pi \quad (\lambda_o = \text{wavelength}),$$

and so the deep-water wave speed is $c_o = gT/2\pi = g/w$, where $g$ = acceleration of gravity. Therefore the surface displacement can be represented by

$$y_o = A \exp i(\omega t - k_o x) \quad (6)$$

for all $x$.

In the atmosphere above the water a distribution of sound pressure and particle motions is caused by the surface wave. The velocity potential for this distribution must satisfy the sound wave equation and the boundary condition at the water surface. The final result is that the sound pressure in the atmosphere above the water is

$$p = \frac{pc \omega A}{\sqrt{\beta^2 - 1}} \exp \left[-\sqrt{k_o^2 - k^2} y + i(\omega t - k_0 x)\right] \quad (7)$$
where \( \rho = \) density of the atmosphere, \( c = \) speed of sound, \( \beta = c/c_o \), and \( k = \omega/c \).

Note that the sound pressure at the water surface is in phase with the displacement. Hence no net work is done and no sound power is radiated into the atmosphere.

For water waves of period \( T = 2\pi = 6.28 \) sec, \( \omega = 1.0/\text{sec}, \) \( c_o = 9.80 \text{ m/sec}, \) and \( \lambda_o = 61 \) meters. Suppose the displacement amplitude \( A = 100 \) cm. Then the sound pressure at the water surface is \( |p| \approx 120 \text{ dyn/cm}^2 \). But at a height of \( 100 \) m above the surface the sound pressure is reduced by a factor of \( e^{-10} = 1/22000 \) to less than \( 10^{-2} \text{ dyn/cm} \).

Radiation by a semi-infinite wave train

We consider next a sinusoidal gravity wave on water coming from \(-\infty\) and stopping abruptly at \( x = 0 \), the wavefronts again being straight lines parallel to the \( z\)-axis (see Figure 9). The surface displacement \( y_o \) is the same as in Eq. (6) for \(-\infty < x < 0\), and \( y_o = 0 \) for \( x > 0 \). We imagine the line \( x = 0 \) in the \( xz\)-plane to be the beach, and the region \( x > 0 \) to be the landward side.

The velocity potential \( \psi \) for the total wave field at \( y = 0 \) is readily found by superposition of the hemicylindrical waves generated by each line element, parallel to the \( z\)-axis and of width \( du \), of the surface waves.

\[
\psi = -\frac{\rho A e^{i\omega t}}{2} \int_{-\infty}^{0} \left[ J_{0}(kx - ku) - i Y_{0}(kx - ku) \right] e^{-i k_x u} du
\]

for \( x > 0 \).

A good approximation to this integral is found as follows. We first replace the Bessel functions \( J_0 \) and \( Y_0 \) by the first terms of their asymptotic expansions. This leads to a Fresnel integral expression for \( \psi \), whose asymptotic form for large \( x \) yields

\[
\psi \sim -\frac{\rho A}{\sqrt{2\pi} (\beta-1)} \frac{(-i e^{\pi i/4} i(\omega t - kx))}{\sqrt{kx}}
\]

From this we find the sound pressure on the landward side \( (x > 0) \) to be

\[
|p| = \frac{\rho c A}{\sqrt{(\beta-1)T}} \sqrt{\frac{\lambda}{x}},
\]

where \( \lambda = \) wavelength of the atmospheric sound. As before we suppose the water waves to have a period \( T = 2\pi \) sec and \( A = 100 \) cm. Then \( \beta = 35 \) and \( \lambda = 2130 \) m. At a distance \( x = 185 \) km from the beach, the sound pressure will be \( 2.1 \text{ dyn/cm}^2 \).

Suppose the wave starts abruptly at the beach \( x = 0 \) and travels towards \( x = -\infty \). We find the sound pressure on the landward side to be almost the same as before; for such a wave the factor \( \beta - 1 \) in Eq. (10) above is replaced
by \( \beta + 1 \). Therefore a standing wave caused by reflection of an incoming wave by a beach will also give rise to a radiated sound field.

Since an infinite wave train radiates no sound power, whereas there is radiation by a semi-infinite train stopping abruptly at \( x = 0 \), we can imagine that the radiated power is due to a line source on the beach. The assumption is not strictly correct, but we can use it to estimate the power by means of Eq. (10). For the water waves of the period \( T \) and amplitude \( A \) considered above, we find the radiated sound power to be about 30 kW per kilometer of beach.

**Radiation by a standing wave**

A theory, analogous to the Longuet-Higgins analysis for the generation of microseisms, explains the generation of microbaroms by standing water waves associated with marine storms. The theory is based on the vertical oscillations of the center of gravity of the atmosphere immediately above the standing waves, which might be near a beach as well as out at sea. The frequency of oscillation of the atmosphere's gravitational potential energy is twice that of the ocean waves. The varying potential energy has a radiated sound field associated with it, with sound waves at twice the ocean wave frequency. Full details of the analysis have been given by Posmentier (1967) and by Brekhovskikh (1968).

**Comparison with observed microbaroms**

The foregoing (for waves on a beach) analysis is for the idealized case of straight-line wavefronts of infinite length, the waves being perpendicularly incident on a straight-line beach. But natural beaches are not very straight; the waves arriving at one point might not be coherent with those arriving at a point on the beach a few kilometers away; the strength of the wave can be affected by reflection from the upper atmosphere; etc. The mathematically derived sound field of Eq. (10) can therefore be expected to yield only order-of-magnitude estimates for sound pressures at a large distance from a beach.

Microbaroms have been observed in the Washington area with the infrasonic system described in Section 3. The Atlantic Ocean beach is 185 km east-south-east of the infrasonic station. Generally speaking, the microbarom sound waves come from the east and travel parallel to the earth's surface. They appear at almost all times of the year, and occasionally have sound pressures as great as 6 dyn/cm\(^2\). For a typical recording made on March 11, 1961, \( T \approx 5.5 \) sec and \( p \approx 1 \) dyn/cm\(^2\). On the basis of the analysis given above, microbaroms
at Washington could be caused by waves on the Atlantic Ocean beaches ranging in amplitude from about 20 cm to 100 cm.

Fribourg (Switzerland) is about 650 km from the Atlantic Ocean beach on the west coast of France. Microbaroms observed at Fribourg often had daily average sound pressures $\approx 0.4 \text{ dyn/cm}^2$. From the above analysis the expected pressure at Fribourg due to ocean waves of $A = 100 \text{ cm}$ would be $1.0 \text{ dyn/cm}^2$. But the sound waves were reported as arriving from a northwest direction. The Atlantic Ocean is about 1500 km away in this direction, beyond north Ireland and Scotland. It seems that the standing-wave hypothesis can account for the microbaroms observed at Fribourg by Saxer and Dessauer.

4.5 Subsonic Oscillations

The passage of a jet stream in the atmosphere over the eastern (Atlantic) seaboard of the United States is occasionally accompanied by large oscillations in barometric pressure at infrasonic frequencies. The jet stream is a thin layer of fairly high speed wind. The location of the layer in the atmosphere is in the neighborhood of the tropopause, at an altitude of about 10 km. The thickness of the stream is about three kilometers. The wind speed along the axis is at least 30 m/sec, and sometimes as great as 80 m/sec. The wind blows towards a direction between northeast and southeast, and the width of the stream (in a direction transverse to the direction of flow) is generally at least 100 km.

An important characteristic is that the periods of oscillation are usually greater than the resonant period $T_R \approx 300 \text{ sec}$ of the atmosphere (see Section 2.4). Since, as we recall, the phase velocities for plane waves of such long periods are substantially less than the high-frequency sound velocity, the waves may be called subsonic oscillations.

The results of observations made at our station in Washington show that almost all sound waves coming from subsonic oscillations of the jet stream have wavefront surfaces of constant phase which are almost plane. The sound pressure has the following features when the jet stream is blowing. (1) The direction of propagation of lines of constant-phase sound pressure across the Washington area is very close to the direction of the jet stream over Washington. (2) The horizontal phase velocity $c_o = 30$ to 100 m/sec is about the same as the speed of the jet. (3) The sound pressures are mainly in the range $50 - 400 \text{ dyn/cm}^2$. (4) Periods of oscillation $T = 300$ to 1000 sec.
Figure 10. Observations on jet stream oscillations.

A brief summary of a few observations made in Washington is given in Figure 10. The data show the correlations between the features of the sound pressure, and the characteristics of the jet stream causing the sound pressure. The waves observed at the Washington station have been studied in detail by Mary W. Hodge and her associates (1968). Further data on the Washington waves have been summarized by A. J. Bedard, Jr. (1966). Sound pressures caused by the jet stream have been also observed elsewhere; a comprehensive report on waves in the Boston, Massachusetts, area has been prepared by Elizabeth A. Flauraud and her associates (1954).

The sound pressure probably has its origin in flow instability of the jet stream. The mechanism of the instability is not known. We can conjecture
that it arises from a combination of viscous shear between the jet stream and the surrounding atmosphere, and unstable temperature gradients. We assume that the jet stream oscillations force the atmosphere into oscillation. The well-known equations of motion for sound waves in a wind-free atmosphere (see Lamb, 1945) can be used to determine the relationship between the sound pressure measured at the ground and the assumed oscillatory displacement of the jet. The basic idea is that the atmosphere between the oscillating jet stream and the surface of the ground is filled with downward-traveling plane waves, and reflected upward-traveling waves, with both waves having a forward component of phase velocity the same as the speed of the jet stream. Figure 11 is a schematic drawing for the mathematical analysis that has been carried out in detail by Cook (1968), under the following physical assumptions. (1) The atmosphere is isothermal and wind-free. (2) The waves are sinusoidal in time, and all quantities vary like \( \exp(\text{i}\omega t) \). (3) All motions are in the \( x-z \) plane, and so the particle velocity, with components \( u, v, w \), has its \( y \)-component \( v = 0 \). (4) The traces of the straight lines of constant-phase sound pressure on the \( x-y \) plane have a phase velocity \( c_o = \omega/k_o \), and so all quantities vary as \( \exp(-\text{i}k_o x) \); the waves are advancing in the (+x) direction. The equations of motion finally yield the following expression for the amplitude of the vertical component of particle displacement at the altitude \( z = 10 \text{ km} \) of the jet stream:
\[ \Delta z = 2H(k_o^2 - k^2) |P_o|/\beta Y B k^2 \]  
\hspace{1cm} (11)

where \( H = \) scale height of the atmosphere = \( c^2/\gamma g \) = 8.1 km for an isothermal atmosphere with \( c = 333 \text{ m/sec} \). Also \( c = \omega/k, \gamma = 1.40, \) and \( \beta \) is a real number for a typical subsonic oscillation. As an example, consider a wave with \( T = 500 \text{ sec}, c_o = 33 \text{ m/sec} (\approx c/10), \) and \( |P_o| = 200 \text{ dyn/cm}^2 \). We find \( \Delta z = 60 \text{ m}, \) which must be the jet stream's vertical oscillatory displacement at an altitude of 10 km necessary to produce the measured sound pressure at the ground.

\[ \beta z/H = \frac{\pi}{2}, \frac{3\pi}{2} \]

Figure 12. Horizontal phase velocities at various oscillation periods. Solid lines = theoretical velocities. 0 = observed velocities.

Standing-Wave Hypothesis

The mechanism of the oscillation is not known, and there is no obvious limitation on the vertical component \( \beta/H \) of the wave-number vectors. We examine the conjecture that the vertical component \( w \) of the particle velocity (which is zero at the ground) has a maximum at the jet stream's nominal altitude of 10 km (see Figure 11). The analysis shows that this occurs approximately when

\[ \beta z/H = \pi/2, 3\pi/2, 5\pi/2, \ldots \]  
\hspace{1cm} (12)

Use of these values for \( \beta \) leads to a series of curves showing how the horizontal trace velocity \( c_o \) varies with the period of oscillation \( T \). Two of the curves are shown in Figure 12. The curves all start with \( c_o = 0 \) at \( T_y = 337 \text{ sec} \), this being the Väisälä period of stability oscillations for the isothermal atmosphere \( (c = 333 \text{ m/sec}) \) under consideration. Also plotted are some
observed values of horizontal phase velocities corresponding to well-defined periods of oscillation for the jet stream over Washington during January 1964. The data do not seem to confirm the hypothesis of Eq. (12). But gravitational forces evidently play a substantial role in the generation mechanism, since most of the observed oscillations occur at periods greater than the Väisälä T.'

**Propagation to the Ionosphere**

The displacement and velocity amplitudes in the jet-stream oscillation can be expected to serve as sources for radiation of subsonic wave power upward into the ionosphere. The equations of motion show that the amplitude of the vertical component $w$ of the particle velocity increases exponentially with altitude.

$$|w| = w_j \exp\left[\frac{(z-10)}{2H}\right],$$

where $w_j = (2\pi/T)\Delta z$ = the amplitude at $z = 10$ km. For the example given above, $T = 500$ sec, etc., we find that at an altitude of $z = 100$ km, $|w| \approx 100$ m/sec and $\Delta z \approx 9$ km. This estimate for $|w|$ is greater than the phase velocity of the wave ($\approx 30$ m/sec). It appears that subsonic waves traveling upward will probably undergo substantial waveform changes, e.g., taking on a shock-wave configuration, well before reaching the ionosphere.

**4.6 Shock Waves from Satellite Entry**

The entry of a meteorite, artificial satellite, or other solid object into the upper atmosphere at supersonic speeds will generally produce an acoustical shock wave. Sometimes the shock wave strength is great enough so that it can be measured at an infrasonic station. For example, the entry of the Cosmos 213 rocket body into the atmosphere on April 19, 1968, was accompanied by a Mach cone whose shock wave was observed and measured at ESSA's infrasonic station in Boulder, Colorado. The paper-chart recordings of the shock wave's sound pressure at ground level resembled a single sine wave — a pseudo N-waveform — with a peak-to-peak sound pressure of 1.2 dyn/cm$^2$ and a duration of about 2.5 sec. The rocket body passed overhead through the ionosphere near Boulder at an elevation of 112 km, on a path almost parallel to the earth's surface.

The measurement system used at Boulder was the same as that described earlier (see Section 3.1). The band-pass filter was N6 S6 (see Figure 4). But an N-waveform of duration 2.5 sec has a Fourier transform with a substantial spectral density at higher frequencies, outside the N6 S6 "window." In short, the paper-chart recording was that of the N-waveform sound pressure.
appreciably modified by the filter.

If an N-waveform is recorded and measured accurately, it can be used to deduce some information about the object producing the wave. For example, one can obtain the altitude of an artificial satellite during atmospheric entry, even if its large Mach number \( M \approx 25 \) is not accurately known. This is because the shock strength and duration in the so-called far field, at a miss-distance \( h \) (= altitude) moderately large relative to the greatest linear dimension \( t \) of the object, is determined essentially by the following factors: (1) The geometrical size and shape of the object. (2) The ambient atmospheric pressure \( B \) at the altitude of the object. (3) The Mach number \( M \) of its supersonic speed. (4) The miss-distance \( h \). The pressure jump at the head of the N-waveform is given by

\[
\Delta p = B (M^2 - 1)^{1/8} \times (t/h)^{3/4} \times (K_s D/t) ,
\]

where \( D \) = an equivalent maximum diameter for the object, and \( K_s \) = its aero-dynamic shape factor for supersonic speeds. The pressure jump \( \Delta p \) observed will be increased, because of the approximately exponential increase in ambient pressure, by a factor of about \( e^{h/2H} \) when the shock wave propagates down to the ground. It will be increased also by a factor of 2 because of reflection at the ground surface.

REFERENCES


