EXTRAPOLATION OF SONIC BOOM PRESSURE SIGNATURES BY THE WAVEFORM PARAMETER METHOD

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The waveform parameter method of sonic boom extrapolation is derived and shown to be equivalent to the F-function method. A computer program based on the waveform parameter method is presented and discussed, with a sample case demonstrating program input and output.
SYMBOLS

\( a \)  
\( a_o \)  
\( A \)  
\( A_h \)  
\( c_n \)  
\( c_o \)  
\( C_1 \)  
\( C_2 \)  
\( F(\xi) \)  
\( \Delta F_i \)  
\( m_i \)  
\( m_i^0 \)  
\( M \)  
\( \dot{N} \)  
\( p \)  
\( p_o \)  
\( \Delta p_i \)  
\( \Delta p_i^0 \)  
\( r \)  
\( r_o \)  
\( S \)  
\( t \)

- sound speed (includes sound speed perturbation due to wave passage)
- ambient sound speed
- ray tube area as cut by the wavefront
- ray tube area as cut by a horizontal plane
- speed that a wave propagates normal to itself, given by \( a_o + \vec{V}_o \cdot \vec{N} \)
- defined as \( c_n / \cos \theta \)
- atmospheric quantity defined by equation (34)
- atmospheric quantity defined by equation (35)
- an \( F \) function
- discontinuity of \( F(\xi) \) corresponding to \( \Delta p_i \)
- slope of waveform segment \( i \)
- initial value of \( m_i \)
- Mach number
- wavefront unit normal
- pressure perturbation due to wave passage
- ambient pressure
- pressure rise across shock at the juncture of waveform segments \( i \) and \( i-1 \)
- initial value of \( \Delta p_i \)
- for a conical wave, the distance from the flight path of the projectile that is generating the wave; for a spherical wave, the distance from the point where the wave originated
- initial value of \( r \)
- for a plane wave, the distance of propagation
- time
\( \Delta t \) propagation time required for the wave to traverse the distance between points on the ray path

\( T \) time associated with a waveform point; also, a time parameter defined by equation (39)

\( T \epsilon_1 \) defined by equation (25)

\( T \epsilon_2 \) defined by equation (26)

\( u \) fluid particle speed due to wave passage (positive in the direction of wave propagation)

\( \vec{V}_o \) wind velocity

\( z \) distance above ground

\( z_1 \) initial altitude, where the pressure signature is known

\( z_2 \) altitude where the pressure signature is desired

\( \epsilon_1 \) defined by sketch (c) and equation (20)

\( \epsilon_2 \) defined by sketch (c) and equation (20) (if in eq. (20) \( \epsilon_1 \) is replaced by \( \epsilon_2 \) and \( \Delta F_i \) is replaced by \( \Delta F_{i+1} \))

\( \gamma \) ratio of specific heats

\( \lambda_i \) time duration of waveform segment \( i \)

\( \lambda_i^0 \) initial value of \( \lambda_i \)

\( \rho_o \) ambient density

\( \tau \) age variable

\( \theta \) angle between \( \vec{N} \) and horizontal plane

\( \xi \) phase, a variable used to identify or “tag” points on a waveform

Subscript Notation for Derivatives

\( (\cdot)_z = \partial(\cdot)/\partial z \)

\( (\cdot)_\xi = \partial(\cdot)/\partial \xi \)

\( (\cdot)_{\xi z} = \partial^2(\cdot)/\partial z \partial \xi \)
EXTRAPOLATION OF SONIC BOOM PRESSURE SIGNATURES

BY THE WAVEFORM PARAMETER METHOD

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SUMMARY

The waveform parameter method of sonic boom extrapolation is derived and shown to be equivalent to the $F$-function method. A computer program based upon the waveform parameter method is presented and discussed, with a sample case demonstrating program input and output.

INTRODUCTION

Sonic boom pressure waves created by high-flying supersonic aircraft propagate over extremely large distances through an atmosphere with temperature, pressure, and wind gradients before reaching the ground. The atmospheric gradients have a strong effect on both the wave amplitude and the nonlinear waveform distortion. Two sonic boom extrapolation methods that account for these atmospheric effects are the $F$-function method (ref. 1) and the waveform parameter method (ref. 2). These methods are based on the same fundamental concepts; both rely on a result from geometric acoustics for the wave amplitude, and both utilize isentropic wave theory to account for nonlinear waveform distortion. They differ, however, in the manner in which they account for nonlinear effects. The $F$-function method uses an age variable, calculated from the atmospheric properties, to distort the pressure signature. Shock waves are then located from the distorted signature by means of an area-balancing criterion. No shock-finding criterion is required in the waveform parameter method. Instead, a set of waveform parameters is defined that completely describes the waveform, and equations are obtained for the time rates of change of these parameters. These equations, which can be integrated over small time increments, are derived from the same basic equations used in the $F$-function method. No additional assumptions are required, and therefore the two methods are mathematically equivalent. The waveform parameter method appears to provide a more suitable approach for automatic computation because the necessity of using the area-balancing technique for locating shocks is eliminated. Also, in the special cases of plane, conical, and spherical pressure waves propagating in a uniform medium, closed-form expressions for the waveform parameters as functions of propagation distance can be obtained.
ANALYSIS AND DISCUSSION OF RESULTS

Both the F-function method and the waveform parameter method depend, for waveform amplitude, on a result derived from geometric acoustics—conservation of the Blokhintsev energy invariant along ray tubes (refs. 3 and 4). This conservation principle may be stated as

\[ p \frac{c_n^2 A}{\sqrt{\rho_0 a_0^3}} = F(\xi) \]  (1)

The quantities \( c_n, A, \rho_0, \) and \( a_0 \) are functions only of \( z \) (altitude) and therefore vary along the ray tube. The acoustic pressure \( p \) is a function of both \( z \) and phase \( \xi \). If the sonic boom pressure wave were truly an acoustic wave, the above expression would be sufficient for calculation of the sonic boom waveform at any point along the ray tube. However, because of the extremely large propagation distances involved, and because near the aircraft the wave is not extremely weak, the actual nonlinearity of pressure wave propagation causes a distortion of the waveform that must be taken into account. Both the F-function method and the waveform parameter method account for this nonlinear distortion by assuming that the speed of propagation of any point on the waveform is given by isentropic wave theory. The propagation speed of a point on a pressure wave is equal to the value of \( u + a \) for the point. If the wave is assumed isentropic and of small amplitude, the propagation speed is given by

\[ u + a = a_o \left( 1 + \frac{\gamma + 1}{2\gamma} \frac{p}{p_o} \right) \]  (2)

or, in terms of the function \( F(\xi) \)

\[ u + a = a_o + \frac{\gamma + 1}{2} \sqrt{\frac{a_o}{\rho_o A c_n^2}} F(\xi) \]  (3)

Let the pressure signature at some point along the ray tube be described by the pressure \( p \) versus time \( T \) plot that would be produced by a microphone located at that point. Also, let \( T = 0 \) correspond to a zero disturbance point on the waveform as shown by the example signature of sketch (a). Then the amount of nonlinear waveform distortion that occurs in a propagation time increment \( dt \) is given by

\[ dT = \frac{a_o - (u + a)}{c_n} dt \]  (4)

If the altitude \( z \) of the wave is measured from the \( T = 0 \) point on the waveform, then

\[ \frac{dz}{dt} = -a_o \sin \theta \]  (5)
and, using equation (3), the total nonlinear distortion that occurs during propagation from the initial altitude $z_1$ to the altitude of interest $z_2$ is given by

$$
\Delta T(\xi) = \frac{\gamma + 1}{2} F(\xi) \int_{z_1}^{z_2} \frac{dz}{\sqrt{\rho_o a_o c_n^4 A \sin^2 \theta}}
$$

(6)

The phase $\xi$ is as yet undefined quantitatively. If $\xi$ for a point on the waveform is taken to be the value of $T$ corresponding to the same point on the initial signature—that is, the signature at $z_1$—then the required expression for $T(\xi)$ is

$$
T = \xi - \tau F(\xi)
$$

(7)

where the “age variable” $\tau$ is calculated from

$$
\tau = -\frac{\gamma + 1}{2} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\rho_o a_o c_n^4 A \sin^2 \theta}}
$$

(8)

It is pointed out by Hayes (ref. 1) that in a horizontally stratified, steady atmosphere $c_n = c_o \cos \theta$ where $c_o$ is constant along ray paths. It is possible to define the ray tube area as the area cut by a horizontal plane, rather than that cut by the wavefront. The relationship between the two areas can easily be shown to be

$$
A_h = \frac{c_n}{a_o} \frac{A}{\sin \theta}
$$

(9)

The expression for the age variable in terms of $A_h$ is then

$$
\tau = -\frac{\gamma + 1}{2c_o^{3/2}} \int_{z_1}^{z_2} \frac{dz}{\left(\rho_o a_o^2 A_h \sin^3 \theta \cos^3 \theta \right)^{1/4}}
$$

(10)
In cases where sonic cutoff occurs, or nearly occurs, the use of $A_h$ instead of $A$ presents some difficulty due to $\sin \theta$ going to zero.

The above expression for $\tau$ is identical to that obtained in reference 1 (eq. (46)), except for the exponent of $c_o$, which in reference 1 is 1 instead of $3/2$. This difference in exponent is due to a factor of $c_o^{-1/2}$ in the definition of $F(\xi)$ used in reference 1, where $F(\xi)$ is denoted $V_F(\xi)$. Since $c_o$ is constant along ray tubes, including $c_o$ in the definition of $F(\xi)$ is permissible.

The above equations are the fundamental equations of the $F$-function method. The $F$ function $F(\xi)$ is first determined using a near-field signature and equation (1). The age variable corresponding to altitude $z_2$ is then calculated from equation (8) or equation (10). Then, using equations (1) and (7), one can plot $p$ versus $T$ at altitude $z_2$. The result will normally look as shown in sketch (b). It is seen that the signature (indicated by the solid line) is multivalued in $p$. The multivalued regions are eliminated by locating shock waves (indicated by dashed lines) according to the area-balancing rule. The area-balancing rule is a consequence of the fact that the propagation speed of a weak shock is equal to the average of the propagation speeds $(u + a)$ just ahead of and just behind the shock.

In the waveform parameter approach, the waveform is approximated by an arbitrary number of linear segments. For each segment the waveform parameters $m_i$, $\Delta p_i$, and $\lambda_i$ are defined as follows: $m_i$ is the slope $\partial p / \partial T$ of segment $i$, which may be positive or negative; $\Delta p_i$ is the pressure rise across the shock at the juncture of segments $i$ and $i-1$. Often there will be no shock at the juncture, in which case $\Delta p_i$ is zero. Finally, $\lambda_i$ is the time duration $\Delta T$ of segment $i$. A completely general waveform can be described using these waveform parameters.

Consider first the waveform parameter $m_i$, which is defined by

$$m_i = \frac{P_{x_i}}{T_{x_i}}$$

(11)
Then

\[
\frac{dm_i}{dt} = -\frac{p_{\xi i} T_{\xi z i} dz}{T_{\xi i}} + \frac{p_{\xi z i} dz}{T_{\xi i}} \tag{12}
\]

The first term on the right-hand side of equation (12) is zero for a linear, nonplane wave, whereas the second term is zero for a nonlinear, plane wave propagating in a uniform, stationary medium. This means that the rate of change of the waveform parameter \( m_i \) can be obtained by superposition of the rate of change assuming the wave propagates as a linear, nonplane wave and the rate of change assuming the wave propagates as a nonlinear, plane wave. This concept of superposition of waveform parameter rates of change was first introduced, without proof, in reference 2. Equation (12) can be written as

\[
\frac{dm_i}{dt} = -\frac{m_i^2 T_{\xi z i} dz}{p_{\xi i}} + m_i \frac{\partial p_{\xi i}}{\partial t} \tag{13}
\]

Then, from

\[
p_{\xi i} = \frac{\rho_o c_o^3}{c_n^2 A} F'_i(\xi) \tag{14}
\]

\[
T_{\xi z i} = -F'_i(\xi) \tau_z \tag{15}
\]

\[
\tau_z = -\frac{\gamma + 1}{2} \sqrt{\frac{1}{\rho_o c_o^2 c_n^4 A \sin^2 \theta}} \tag{16}
\]

and equation (5), the time rate of change of the waveform parameter \( m_i \) is given by

\[
\frac{dm_i}{dt} = \frac{\gamma + 1}{2\gamma} \frac{a_o}{p_o c_n} m_i^2 + \frac{1}{2} \left( \frac{3}{a_o} \frac{da_o}{dt} \frac{1}{\rho_o} \frac{d\rho_o}{dt} - \frac{2}{c_n} \frac{dc_n}{dt} - \frac{1}{A} \frac{dA}{dt} \right) m_i \tag{17}
\]

where the fractional time rates of change of the ambient properties \( a_o, \rho_o, \) and \( c_n \) and of the ray tube area \( A \) are understood to be the rates of change as seen by an observer moving down the ray tube with the wave.
The waveform parameter $\Delta p_i$ can be expressed in terms of the jump in the $F$ function across the shock as

$$\Delta p_i = \sqrt{\frac{\rho_0 a_0^3}{c_n^2 A}} \Delta F_i$$

Then

$$\frac{d\Delta p_i}{dt} = \frac{\Delta p_i}{\Delta F_i} \frac{d\Delta F_i}{dt} + \Delta p_i \frac{c_n^2 A}{\rho_0 a_0^3 A} \frac{d}{dt} \sqrt{\frac{\rho_0 a_0^3}{c_n^2 A}}$$

The first term on the right-hand side of this equation is zero for a linear, nonplane wave, whereas the second term is zero for a nonlinear, plane wave propagating in a uniform, stationary medium. Therefore, the concept of superposition of waveform parameter rates of change is also valid for the parameter $\Delta p_i$. The fractional time rate of change of $\Delta F_i$ can be determined as follows. Sketch (c) shows a typical signature at some point along a ray tube. The dashed lines are "equalizing lines" representing the shock waves at some time increment $dt$ later. Using equation (3), the relationship between $\epsilon_1$ and $dt$ is

$$2\epsilon_1 c_n = \frac{\gamma + 1}{2} \sqrt{\frac{a_o}{\rho_0 c_n^2 A}} \Delta F_i dt$$

Since from the area-balancing rule

$$\frac{d\Delta F_i}{dt} = \frac{\epsilon_1}{dt} \left( m_i + m_{i-1} \right) \sqrt{\frac{c_n^2 A}{\rho_0 a_0^3}}$$
we have

\[ \frac{1}{\Delta F_i} \frac{d\Delta F_i}{dt} = \frac{\gamma + 1}{4\gamma} \frac{a_o}{p_o c_n} (m_i + m_i-1) \]  

(22)

and therefore the time rate of change of the waveform parameter \( \Delta p_i \) is

\[ \frac{d\Delta p_i}{dt} = \frac{\gamma + 1}{4\gamma} \frac{a_o}{p_o c_n} (m_i + m_i-1) \Delta p_i + \frac{1}{2} \left( \frac{3}{a_o} \frac{da_o}{dt} + \frac{1}{\rho_o} \frac{d\rho_o}{dt} c_n \frac{d c_n}{dt} - \frac{1}{A} \frac{dA}{dt} \right) \Delta p_i \]  

(23)

The waveform parameter \( \lambda_i \) is defined as

\[ \lambda_i = T_i+1 - T_i \]  

(24)

where \( T_i \) and \( T_{i+1} \) are the times corresponding to the shocks at each end of the waveform segment, as shown in sketch (c). Let

\[ T_{e_1} = T_i + e_1 \]  

(25)

\[ T_{e_2} = T_{i+1} - e_2 \]  

(26)

Since in time \( dt \) both \( e_1 \) and \( e_2 \) go to zero (by definition of \( e_1 \) and \( e_2 \)), then

\[ \frac{d\lambda_i}{dt} = \frac{d}{dt} \left( T_{e_2} - T_{e_1} \right) - \frac{e_1 + e_2}{dt} \]  

(27)

Let \( F_{e_1} \) and \( F_{e_2} \) be the values of the \( F \) function \( F(\xi) \) at \( T_{e_1} \) and \( T_{e_2} \), respectively. Then, from equation (7)

\[ \frac{d}{dt} \left( T_{e_2} - T_{e_1} \right) = -\left( F_{e_2} - F_{e_1} \right) \frac{d\tau}{dt} \]  

(28)

\[ = -m_i \lambda_i \sqrt{\frac{c_n^2 A}{\rho_o a_o^3}} \frac{d\tau}{dt} \]  

(29)
Thus, from equations (5), (16), (18), and (20) the time rate of change of the waveform parameter $\lambda_i$ is

$$\frac{d\lambda_i}{dt} = -\gamma + \frac{1}{\gamma} \frac{a_o}{\rho_o c_n} (\Delta p_i + \Delta p_{i+1}) - \frac{1}{2\gamma} \frac{a_o}{\rho_o c_n} m_i \lambda_i$$

(30)

We now have three ordinary, first-order, coupled differential equations for the waveform parameters $m_i$, $\Delta p_i$, and $\lambda_i$, which completely describe the deformation of the waveform. These equations can be written more concisely as

$$\frac{dm_i}{dt} = C_1 m_i^2 + C_2 m_i$$

(31)

$$\frac{d\Delta p_i}{dt} = \frac{1}{2} C_1 \Delta p_i (m_i + m_{i-1}) + C_2 \Delta p_i$$

(32)

$$\frac{d\lambda_i}{dt} = -\frac{1}{2} C_1 (\Delta p_i + \Delta p_{i+1}) - C_1 m_i \lambda_i$$

(33)

where

$$C_1 = \gamma + \frac{1}{\gamma} \frac{a_o}{\rho_o c_n}$$

(34)

$$C_2 = \frac{1}{2} \left( \frac{3}{a_o} \frac{da_o}{dt} + \frac{1}{\rho_o} \frac{d\rho_o}{dt} - 2 \frac{d\sigma_n}{dt} - \frac{1}{A} \frac{dA}{dt} \right)$$

(35)

The above equations differ from corresponding equations of reference 2 because the waveform parameters used here refer to a pressure versus time signature, while those of reference 2 refer to a pressure versus distance signature (the distance being measured normal to the wavefront).

In the general case of a wave with arbitrary wavefront shape propagating in a nonuniform atmosphere with winds the quantities $C_1$ and $C_2$ vary along the ray path. However, if these quantities are assumed to be constant over small time increments, the above waveform deformation equations can be integrated to obtain the following solutions.

$$m_i = m_i^0 e^{C_2 \Delta t}$$

(36)

$$\Delta p_i = \Delta p_i^0 e^{C_2 \Delta t}$$

(37)
\[
\lambda_i = (1 - C_1 m_i^0 T) \left[ \lambda_{i-1}^0 - \frac{\Delta p_{i-1}^0}{m_i^0 - m_{i-1}^0} \left( \sqrt{\frac{1 - C_1 m_{i-1}^0 T}{1 - C_1 m_i^0 T}} - 1 \right) \right.
\]

\[
- \frac{\Delta p_{i+1}^0}{m_i^0 - m_{i+1}^0} \left( \sqrt{\frac{1 - C_1 m_{i+1}^0 T}{1 - C_1 m_i^0 T}} - 1 \right) \right] \tag{38}
\]

where

\[
T = \frac{C_2 \Delta t}{C_2} - 1 \tag{39}
\]

The above solutions for \( m_i \), \( \Delta p_i \), and \( \lambda_i \) are used to determine the values of the waveform parameters at some point on the ray path, given \( m_i^0 \), \( \Delta p_i^0 \), and \( \lambda_i^0 \), which are the values of the parameters at the preceding point. The time increment \( \Delta t \) is the propagation time required for the wave to traverse the distance between points on the ray path. To use the above solutions for the waveform parameters one must first calculate the values of \( C_1 \) and \( C_2 \) at many points along the ray path of interest. The ray path can be calculated by the method outlined in reference 2. When the ray path is known, the fractional rates of change of the ambient properties \( a_o \), \( \rho_o \), and \( c_n \) can be determined at many points along the ray path. The fractional rate of change of ray tube area is also needed at each point along the ray path. Ray tube area may be calculated by theoretical methods, such as that of reference 1, or can be determined directly by independently calculating each of the four rays that bound the ray tube. Once the values of \( C_1 \) and \( C_2 \) are determined, the above solutions for the waveform parameters can be used to calculate the waveform at any point of interest along the ray path if the waveform near the aircraft is known.

The above expression for \( \lambda_i \) (eq. (38)) cannot be used when \( m_i^0 = m_{i-1}^0 \) or when \( m_i^0 = m_{i+1}^0 \). In the case when \( m_i^0 = m_{i+1}^0 \), or, in general, when

\[
\left| \frac{C_1 (m_i^0 - m_{i+1}^0) T}{1 - C_1 m_i^0 T} \right| < 0.001 \tag{40}
\]

we can use the approximation

\[
\frac{\Delta p_i^0}{m_i^0 - m_{i-1}^0} \left( \sqrt{\frac{1 - C_1 m_{i-1}^0 T}{1 - C_1 m_i^0 T}} - 1 \right) \approx \frac{\Delta p_i^0}{2} \frac{C_1 T}{1 - C_1 m_i^0 T} \tag{41}
\]

A similar expression can be obtained when \( m_i^0 = m_{i+1}^0 \).
As the wave propagates down the ray path, shocks often will coalesce and compression regions will steepen to form new shocks. When this occurs one or more of the \( \lambda_i \) will go to zero. When one of the \( \lambda_i \) does go to zero somewhere between two points on the ray path, the expression for \( \lambda_i \) given by equation (38) can be used to determine the value of \( \Delta t \) at which it goes to zero. The waveform parameters should then be incremented using this value of \( \Delta t \) to determine the waveform at the point where the waveform segment goes to zero. The waveform parameters must then be redefined so that the new waveform (which has one less segment and possibly one more shock) is represented correctly.

Extrapolations of plane, conical, and spherical pressure waves propagating in a uniform medium can be done easily by hand if the waveform is of a very simple shape. Solutions of the waveform deformation equations for plane, conical, and spherical waves are presented in appendix A.

A sonic boom extrapolation program, based on the waveform parameter method described above, is presented and discussed in appendix B. The program includes the effects of aircraft accelerations, aircraft flight path angle, and atmospheric temperature, pressure, and wind gradients. The program has been checked using the well-established Hayes program (ref. 1) and has been found to give nearly identical results for all types of flight conditions.

CONCLUDING REMARKS

The waveform parameter method for extrapolation of weak pressure waves has been shown to be valid, within the same limitations as the \( F \)-function theory. The waveform parameter method appears to provide a more suitable approach for automatic computation because the necessity of using the area-balancing technique for locating shocks is eliminated. The waveform parameter method also makes possible closed-form solutions for plane, conical, and spherical waves propagating in uniform media.

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SOLUTIONS OF THE WAVEFORM DEFORMATION EQUATIONS
FOR PLANE, CONICAL, AND SPHERICAL WAVES

Plane, conical, and spherical pressure waves propagating in a uniform, stationary gas can be treated very simply by the waveform parameter method. The waveform deformation equations (31), (32), and (33) can be solved in closed form in these special cases. The solutions are found to be the following.

\[
\frac{m_i}{m_i^0} = \frac{1}{Q_1 \left( 1 - \frac{\gamma + 1}{2\gamma} \frac{m_i^0}{a_i^0} Q_2 \right)}
\]

\[
\Delta p_i = \Delta p_i^0 \sqrt{\frac{m_i}{m_i^0}} \sqrt{\frac{m_i - 1}{m_i^0}}
\]

\[
\lambda_i = \frac{1}{Q_1} \frac{m_i^0}{m_i^0 - m_i^0} \left[ \lambda_i^0 - \frac{\Delta p_i^0}{m_i^0 - m_i^0} \left( \sqrt{\frac{m_i}{m_i^0}} \sqrt{\frac{m_i^0}{m_i - 1}} - 1 \right) \right.
\]

\[
- \frac{\Delta p_i^0}{m_i^0 - m_i^0} \left( \sqrt{\frac{m_i}{m_i^0}} \sqrt{\frac{m_i^0}{m_i^0}} - 1 \right)
\]

where the quantities \( Q_1 \) and \( Q_2 \) are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane</td>
<td>1</td>
<td>( S )</td>
</tr>
<tr>
<td>Conical</td>
<td>( \sqrt{\frac{r}{r_o}} )</td>
<td>( \frac{2Mr_o}{(M^2 - 1)^\frac{\gamma}{2}} \left( \sqrt{\frac{r}{r_o}} - 1 \right) )</td>
</tr>
<tr>
<td>Spherical</td>
<td>( \frac{r}{r_o} )</td>
<td>( r_o \ln \frac{r}{r_o} )</td>
</tr>
</tbody>
</table>
For a plane wave, $S$ is the distance of propagation. For a conical wave, $r$ is the distance from the flight trajectory of the projectile that is generating the wave, and $M$ is the Mach number of the projectile. For a spherical wave, $r$ is the distance from the point where the wave originated. In both cases, $r_o$ is the value of $r$ corresponding to the initial pressure signature.

If the value of $\lambda_i$ computed from the above equation is negative, the equation for $\lambda_i$ must be used to determine the value of $r$ or $S$ at which $\lambda_i$ becomes zero. The initial signature should then be redefined to be the signature at this value of $r$ or $S$.

It should be remembered that the above solutions for the waveform parameters assume a weak, isentropic wave. If $p/p_o < 0.1$ over the entire signature, this assumption is justifiable. If the wave is conical or spherical and the initial signature is near the point where the wave originated, the waveform amplitude will attenuate rapidly due to a large fractional rate of change of ray tube area. In these cases, the maximum value of $p/p_o$ can be somewhat larger than 0.1 and the above expressions for the waveform parameters will still be applicable.
APPENDIX B

A COMPUTER PROGRAM FOR SONIC BOOM EXTRAPOLATIONS

The sonic boom extrapolation program presented here is based on the waveform parameter method described in this report. This computer program has essentially the same capability as the extrapolation program of reference 1. Furthermore, both programs have the same limitations, namely, those of geometric acoustics and isentropic wave theory. The advantage of the waveform parameter approach is that it provides a more appropriate base for automatic computation since no shock-finding technique is needed. As a result, the program presented here is considerably shorter and simpler than the program of reference 1. Other advantages of the program presented here are the following. No $F$-function calculation is required, thus simplifying extrapolations of experimental signatures. No problems arise in extrapolating near field signatures corresponding to multivalued $F$ functions. The atmospheric pressure variation with altitude is computed within the program, and therefore does not have to be input. Aircraft accelerations are more conveniently specified. Sonic boom "footprints" are more easily determined since both aircraft location and ground-ray intersections are specified in terms of geographical longitude and latitude. The sonic cutoff angle is computed, thus eliminating timely trial-and-error determinations of this angle. Signatures at more than one altitude can be output on each run. Also, signatures can be output in terms of $\Delta p$ (psf), $\Delta p/\rho$, $x$ (ft), or $t$ (msec). Finally, the effect of varying aircraft length can be easily determined, without requiring recalculation of the aircraft $F$ function for each aircraft length. Because of these advantages the extrapolation program presented here is considerably more convenient to use than the program presented in reference 1.

In this extrapolation program ray path calculations are performed by the method outlined in reference 2. Ray tube areas are determined directly by independently calculating each of the four rays that bound the ray tube.

Input to the program consists primarily of aircraft flight conditions, atmospheric properties, and near-field signature data. Flight conditions include Mach number, altitude, flight path angle, and aircraft accelerations. The three components of aircraft acceleration are expressed in terms of $\dot{M}$, $\dot{\gamma}$, and $\dot{\psi}$, which are the time rates of change of Mach number, flight path angle, and heading, respectively. Atmospheric properties are input in the form of temperature and wind velocity profiles. The atmospheric pressure variation with altitude is computed in the program from the input temperature profile, using the perfect gas law and the hydrostatic equation. Winds in both the northerly and easterly directions are input, allowing wind shear. However, vertical winds and atmospheric turbulence are not accounted for in this program. The near-field signature data consist of the signature itself and its corresponding location relative to the aircraft. The signature is input in the form of $p/p_o$ (more commonly written as $\Delta p/p$) versus $x$, where $x$ is a spatial coordinate measured parallel to the aircraft velocity vector. The signature that is input should be consistent with the flight conditions specified. The flight conditions that affect the near-field signature are Mach number and lift coefficient. Therefore, the lift coefficient should first be estimated from the aircraft weight, flight altitude, Mach number, and aircraft accelerations. The near-field signature corresponding to the Mach number and required lift coefficient can then be determined either experimentally by a wind tunnel test or by theoretical means. Normally, if near-field signatures are determined in the wind tunnel, signatures are obtained at several lift coefficients, or angles of
attack, and in several azimuthal planes of the model. A near-field signature corresponding to a specific lift coefficient and location relative to the model can then be estimated by interpolation.

Program output consists of a listing of the input data and of output signatures at the desired altitudes. The location (latitude and longitude) where the output signature would be detected is also output so that sonic boom “footprints” can be determined. In the event that sonic cutoff occurs, the sonic cutoff altitude is output. Also, the minimum $\phi$ (azimuthal angle) for which sonic cutoff occurs is output. In the event that a ray tube area goes to zero, due to large aircraft accelerations, the program will output the altitude at which the ray tube area goes to zero and the location of the ground-ray intersection. The computation will then be terminated. Because the basic theory is one of geometric acoustics, the theory breaks down in regions where ray tube areas go to zero. All that can be said regarding the wave amplitude is that a “superboom” is likely to occur if the ray tube area goes to zero near the ground. Although the program cannot be used to predict the amplitude of a superboom, it can be used to predict where superbooms are likely to occur.
Program Input Variables

**TITLE**  
title card. 80 columns available.

**MACH**  
flight Mach number

**FLTALT**  
flight altitude above ground in feet.

**MDOT**  
time rate of change of MACH (/sec)

**PSIDOT**  
time rate of change of HEAD (deg/sec)

**GAMDOT**  
time rate of change of FPA (deg/sec)

**FPA**  
aircraft flight path angle (deg)

**LONGP**  
aircraft longitude (deg). West longitudes are taken to be positive. East longitudes are negative.

**LATP**  
aircraft latitude (deg). North latitudes are positive. South latitudes are negative.

**HEAD**  
aircraft heading in degrees from true north.

**POG**  
atmospheric pressure on the ground in psf. Input POG = 1 if output signatures in terms of $\Delta p/p$ are desired. Input POG = 0 for uniform atmosphere extrapolations, or for any case where constant atmospheric pressure is desired.

**NTEMP**  
number of atmospheric temperature values input. NTEMP $\geq$ 2

**NWINDE**  
number of east wind values input. NWINDE $\geq$ 2

**NWINDN**  
number of north wind values input. NWINDN $\geq$ 2

**ZT(I)**  
altitudes (in thousands of feet) at which atmospheric temperatures are given. Program logic requires ZT(NTEMP) $> \text{FLTALT}$.

**TO(I)**  
atmospheric temperatures in degrees Fahrenheit. The program assumes a linear variation of temperature between points.

**ZWE(I)**  
altitudes (in thousands of feet) at which east wind values are given. Program logic requires ZWE(NWINDE) $> \text{FLTALT}$.

**VOE(I)**  
wind speeds in east direction (blowing east) in ft/sec. The program assumes a linear variation of wind speed between points.

**ZWN(I)**  
altitudes (in thousands of feet) at which north wind values are given. Program logic requires ZWN(NWINDN) $> \text{FLTALT}$.

**VON(I)**  
wind speeds in north direction (blowing north) in ft/sec. The program assumes a linear variation of wind speed between points.
NALT  number of altitudes at which output signatures are desired.

ALT(I)  the altitudes (feet) at which signatures are desired, in descending order.

OCODE  output code. OCODE = 1 for output signatures of pressure versus time (msec). OCODE = 2 for output signatures of pressure versus distance (feet), as measured along a line parallel to the horizontal component of the aircraft velocity (relative to the ground).

REFL  sonic boom reflection factor. Signatures on the ground are multiplied by REFL.

ROVERL  radial distance (nondimensionalized by the aircraft reference length) from the flight path corresponding to the input pressure signature.

PHI  the lateral angle in degrees corresponding to the input pressure signature and aircraft bank angle. PHI is measured from the vertical plane passing through the aircraft velocity vector. Positive values of PHI correspond to rays that start out to the left of vertical, as seen from behind the aircraft.

NX  number of values input for X(I) and DPP(I)

X(I)  The near field pressure signature is input as a $\Delta p/p$ versus $x$ signature, where $x$ is a spatial coordinate measured parallel to the aircraft velocity vector. X(I) is a table of $x$ values for the near field signature. X(I) can be input in any unit of length.

DPP(I)  the value of $\Delta p/p$ corresponding to X(I). The program assumes a linear variation of $\Delta p/p$ between points.
AL  aircraft reference length in feet.

ML  model reference length in the same unit of length as used for X(I).
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<tr>
<th>SPACE SHUTTLE ROOSTER REENTRY</th>
<th>STANDARD ATMOSPHERE</th>
<th>EASTERLY WINDS</th>
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# PROGRAM OUTPUT – SAMPLE CASE

## SPACE SHUTTLE BOOSTER REENTRY

- FLIGHT ALTITUDE = 50400. FEET
- MACH NUMBER = 1.200
- HEADING = 356.5 DEGREES
- FLIGHT PATH ANGLE = -12.75 DEGREES
- M-DOT = -0.0197 /SEC
- PSI-DOT = -0.359 DEGREES/SEC
- GAMMA-DOT = 1.013 DEGREES/SEC
- AIRCRAFT LONGITUDE = 119.880 DEGREES
- AIRCRAFT LATITUDE = 27.600 DEGREES
- AIRCRAFT LENGTH = 256.0 FEET

## ATOMIC CONDITIONS

POG = 2116.20 PSF

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WAVEFORM AT ALTITUDE = 30000.0 FEET
LONGITUDE = 119.922 DEGREES
LATITUDE = 27.671 DEGREES

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GROUND-RAY INTERSECTION
LONGITUDE = 120.051 DEGREES
LATITUDE = 27.6860 DEGREES
DISTANCE FROM GROUND TRACK = 9.31 MILES

WAVEFORM AT THE GROUND REFLECTION FACTOR = 1.90

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PROGRAM LISTING

1 FORMAT(2OA4)
2 FORMAT(10F7.0)
3 FORMAT(315)
4 FORMAT(///5X,24HSONIC CUTOFF ALTITUDE = ,F7.0,6H FEET, 1/5X,17HPHI FOR CUTOFF = ,F5.0,9H DEGREES)
5 FORMAT(5X,20A4,/)  
7 FORMAT(///5X,6HPOG = ,F7.2,5H PSF)
8 FORMAT(///9X,2HZT,11X,2HT0,11X,3H2WE,11X,3HV0E,11X,3H2WN,11X,3HV0N/ 18X,4HFEET,9X,5HDEG F,9X,4HFEET,9X,6HFT/SEC,9X,4HFEET,9X,6HFT/SEC/)
9 FORMAT(F14.1,F11.1,F12.1,F16.1,F12.1)
10 FORMAT(25X,F15.1,F12.1,F16.1,F12.1)
11 FORMAT(25X,F15.1,F12.1)
12 FORMAT(52X,F16.1,F12.1)
13 FORMAT(F14.1,F11.1,27X,F16.1,F12.1)
14 FORMAT(F14.1,F11.1)
15 FORMAT(F14.1,F11.1,F15.1,F12.1)
16 FORMAT(///5X,16HINITIAL WAVEFORM,13X,6HR/L = ,F5.2,13X,6HPHI = 1,F7.2,8H DEGREES,///16X,3HX/L,17X,4HDP/P, ///(F20.3,F21.5))
19 FORMAT(///H1H)
20 FORMAT(///5X, 23HWAVEFORM AT ALTITUDE = ,F8.1,6H FEET,///9X,12HLONGITUDE = ,F8.3,9H DEGREES,///9X,11HLATITUDE = ,F7.3,9H DEGREES,///)
21 FORMAT(13X,7HT, 15X,6HP, PSF, ///(F20.1,F20.3))
22 FORMAT(13X,7HX, FEET, 14X,6HP, PSF, ///(F20.2,F20.3))
23 FORMAT(13X,7HT, 15X,4HDP/P, ///(F20.1,F20.6))
24 FORMAT(13X,7HX, FEET, 16X,4HDP/P, ///(F20.2,F20.6))
27 FORMAT(8X,34HRAY TUBE AREA GOES TO ZERO AT Z = ,F8.1,6H FEET)
28 FORMAT(///5X,28HCONSTANT PRESSURE ATMOSPHERE)
29 FORMAT(///5X,23HGROUND-RAY INTERSECTION,///9X,12HLONGITUDE = ,F8.3 1,9H DEGREES,///9X,11HLATITUDE = ,F7.3,9H DEGREES,///9X,29HDISANCE F 2RM GROUND TRACK = ,F6.2,7H MILES,///5X,22HWAVEFORM AT THE GROUND 3D,10X,20HREJECTION FACTOR = ,F4.2,///)

INTEGER OCODE
REAL MACH,N(3,4),K1,M(100),LAMDA(100),ML,MAG,MDOT,LATP,LONGP
DIMENSION TITLE(20),R(3,4),ZT(100),TO(100),ZWE(100),VOE(100), 1 ZWN(100),VON(100),ALT(50),X(100),DP(100),F1(100),F2(100), 2 P(100),DP(100),XX(100),PP(100),V0(3),A01(4),V01(3,4)
DOUBLE PRECISION MMACH,FFPA,HHEAD,MU
WRITE(6,19)

40 READ(5,1) TITLE
40 READ(5,2) MACH, FLTALT
40 READ(5,2) MDOT, PSIDOT, GAMDOT, FPA
40 READ(5,2) LATP, HHEAD, HEAD
40 READ(5,2) POG
40 READ(5,3) NTEMP, NWINDE, NWINDN
40 READ(5,2) (ZT(I), I=1, NTEMP)
40 READ(5,2) (TO(I), I=1, NTEMP)
40 READ(5,2) (ZWE(I), I=1, NWINDE)
40 READ(5,2) (VOE(I), I=1, NWINDE)
40 READ(5,2) (ZWN(I), I=1, NWINDN)
40 READ(5,2) NALT
40 READ(5,2) (ALT(I), I=1, NALT)

22
READ(5,3) OCODE
READ(5,2) REFL
READ(5,2) ROVERL, PHI
READ(5,3) NX
READ(5,2) (X(I), I=1,NX)
READ(5,2) (DPP(I), I=1,NX)
READ(5,2) AL, ML
DELTAT=.1
PHISAV=PHI
RO=ROVERL*AL
G=32.26-0.17*COS(LATP/57.296)**2
IF(POG .GT. 0.) GO TO 45
G=0.
POG=1.
45 ICUT=0
DPHI=-10.*
  IF(PHI .LT. 0.) DPHI=10.*
XOFF = X(1)
DO 50 I=1,NX
  X(I)=(X(I)-XOFF)/ML
DO 60 I=1,NTEMP
  ZT(I)=1000.0*ZT(I)
DO 70 I=1,NWINDE
  ZWE(I)=1000.0*ZWE(I)
DO 80 I=1,NWINDN
  ZWN(I)=1000.0*ZWN(I)
WRITE(6,5) TITLE
WRITE(6,6) FLTA, MACH, HEAD, FPA, MDOT, PSIDOT, GAMDOT, LONP, LATP, AL
IF(G .GT. 0.) GO TO 85
WRITE(6,28) GO TO 90
85 WRITE(6,7) POG
90 WRITE(6,8)
I=1
100 IF(I .GT. NTEMP) GO TO 110
   IF(I .GT. NWINDE) GO TO 140
   IF(I .GT. NWINDN) GO TO 160
   WRITE(6,9) ZT(I), TO(I), ZWE(I), VOE(I), ZWN(I), VON(I)
   GO TO 170
110 IF(I .GT. NWINDE) GO TO 130
   IF(I .GT. NWINDN) GO TO 120
   WRITE(6,10) ZWE(I), VOE(I), ZWN(I), VON(I)
   GO TO 170
120 WRITE(6,11) ZWE(I), VOE(I)
   GO TO 170
130 IF(I .GT. NWINDN) GO TO 180
   WRITE(6,12) ZWN(I), VON(I)
   GO TO 170
140 IF(I .GT. NWINDN) GO TO 150
   WRITE(6,13) ZT(I), TO(I), ZWN(I), VON(I)
   GO TO 170
150 WRITE(6,14) ZT(I), TO(I)
   GO TO 170
160 WRITE(6,15) ZT(I), TO(I), ZWE(I), VOE(I)
170 I=I+1
   GO TO 100
180 IF(I .LT. 10) WRITE(6,19)
   WRITE(6,16) ROVERL, PHI, (X(I), DPP(I), I=1,NX)
   WRITE(6,19)
200 IF(ICUT .EQ. 0) GO TO 230
   IF(PHISAV .LT. 0.) GO TO 210
   IF(PHI .GT. 0.) GO TO 230
   DPHI=1.
   GO TO 220
210 IF(PHI .LT. 0.) GO TO 230
   DPHI=-1.
220 PHI=0.
230 MMACH=MACH
   FFPA=FPA/57.296
   HHEAD=HEAD/57.296
   PHI=PHI/57.296
   R(1,1)=0.
   R(2,1)=0.
   R(3,1)=FLTALT
   IZERO=0
   IFORK=1
   II=1
   IRAY=2
   H=FLTALT
240 DO 250 J=2,NTEMP
   IF(ZT(J) .LT. H) GO TO 250
   A0=49.1*SQRT(TO(J-1)+(H-ZT(J-1))/(ZT(J)-ZT(J-1)))*(TO(J)-TO(J-1))
   +459.67
   GO TO 260
250 CONTINUE
260 DO 270 J=2,NWINDE
   IF(ZWE(J) .LT. H) GO TO 270
   VO(1)=VOE(J-1)+(H-ZWE(J-1))/(ZWE(J)-ZWE(J-1))*(VOE(J)-VOE(J-1))
   GO TO 280
270 CONTINUE
280 DO 290 J=2,NWINDN
   IF(ZWN(J) .LT. H) GO TO 290
   VO(2)=VON(J-1)+(H-ZWN(J-1))/(ZWN(J)-ZWN(J-1))*(VON(J)-VON(J-1))
   GO TO 300
290 CONTINUE
300 IF(IRAY .GT. 1 .OR. ICUT .EQ. 1) GO TO 340
   PO=P0G
   DO 330 J=2,NTEMP
      ZZ=ZT(J)
      IF(H .LE. ZZ) ZZ=H
      IF(TO(J-1) .EQ. TO(J)) GO TO 310
      TOZ=(TO(J)-TO(J-1))/(ZT(J)-ZT(J-1))
      PO=PO*(1.+TOZ/(TO(J-1)+459.67)*459.67)*(-1./53.353/TOZ)
   GO TO 320
310 PO=PO*EXP((ZT(J-1)-22)/53.353/(TO(J-1)+459.67))
320 IF(ZZ .EQ. H) GO TO 340
330 CONTINUE
340 GO TO (350,460,580), IFORK
350 VOXH=VO(1)
   VOYH=VO(2)
   VX=MACH*A0*COS(FPA/57.296)*SIN(HEAD/57.296)
   VY=MACH*A0*COS(FPA/57.296)*COS(HEAD/57.296)
   IRAY=0
360 IRAY=IRAY+1
   GO TO (410,390,380,370,430), IRAY
370 R(1,4)=R(1,3)
   R(2,4)=R(2,3)
   R(3,4)=R(3,3)
   GO TO 400

24
380 DT=0.17453*RO/AO/SORT(MACH**2-1.0)
MMACH=MMACH+MDOT*DT
FFPA=FFPA+GAMMDT/57.296*DT
HHEAD=HHEAD+PSIDOT/57.296*DT
PHI=PHI-0.017453
R(1,3)=R(1,1)+(VX+VO(1))*DT
R(2,3)=R(2,1)+(VY+VO(2))*DT
R(3,3)=R(3,1)+MACH*AO*SIN(FFPA/57.296)*DT
GO TO 410
390 R(1,2)=R(1,1)
R(2,2)=R(2,1)
R(3,2)=R(3,1)
400 PHI=PHI+0.017453
GO TO 420
410 MU=DARSIN(1.0/MMACH)
420 N(1,IRAY)=DCOS(FFPA)*DSIN(HHEAD)*DSIN(MU)-DCOS(HHEAD)*DCOS(MU)
1*SIN(PHI)+DSIN(HHEAD)*DCOS(MU)*DCOS(MU)
N(2,IRAY)=DCOS(FFPA)*DSIN(HHEAD)*DSIN(MU)+DSIN(HHEAD)*DCOS(MU)
1*SIN(PHI)+DSIN(HHEAD)*DCOS(MU)*DCOS(MU)
N(3,IRAY)=DSIN(FFPA)*DSIN(MU)-DCOS(FFPA)*DCOS(MU)*COS(PHI)
IF(IRAY .EQ. 4) PHI=(PHI-.017453)*57.296
GO TO 360
430 DO 440 IRAY=1,4
IF(IRAY .EQ. 1) DELT=RO/AO/DCOS(MU)
IF(IRAY .EQ. 3) DELT=DELT-DT
R(1,IRAY)=R(1,IRAY)+(AO*N(1,IRAY)+VO(1))*DELT
R(2,IRAY)=R(2,IRAY)+(AO*N(2,IRAY)+VO(2))*DELT
440 R(3,IRAY)=R(3,IRAY)+AO*N(3,IRAY)*DELT
A1=((-R(2,2)*R(2,1))+(R(3,3)-R(3,2))-(R(3,4)-R(3,1))*R(2,3)
1-R(2,2)))*N(1,1)+((R(3,4)-R(3,1))*R(1,3)-R(1,2))-(R(1,4)-R(1,1))*
2*(R(3,3)-R(3,2)))*N(2,1)+((R(1,4)-R(1,1))*R(2,3)-R(2,2))
3-(R(2,4)-R(2,1)))*N(1,3)-R(1,2))*N(3,1)
IFORK=2
IRAY=0
450 IRAY=IRAY+1
H=R(3,IRAY)
GO TO 240
460 AO1(IRAY)=AO
VO1(1,IRAY)=VO(1)
VO1(2,IRAY)=VO(2)
IF(IRAY .LT. 4) GO TO 450
IF(I*ICUT .EQ. 1) GO TO 530
PO1=PO
CN1=AO1(1)+VO1(1,1)*N(1,1)+VO1(2,1)*N(2,1)
DO 470 I=1,NX
P(I)=DPP(I)*PO
470 X(I)=X(I)/MACH*AL/CN1
K=1
J=2
NN=NX-1
M(1)=0.
DO 520 I=1,NX
IF(K .EQ. 2) GO TO 510
IF(I .EQ. NX) GO TO 500
IF(X(I) .EQ. X(I+1)) GO TO 480
DP(J)=0.
LAMDA(J)=X(I+1)-X(I)
M(J)=(P(I+1)-P(I))/LAMDA(J)
J=J+1
GO TO 520
25
K=2
DP(J) =P(I+1)-P(I)
IF(I+2 GT. NX) GO TO 490
LAMDA(J) =X(I+2)-X(I+1)
M(J) =(P(I+2)-P(I+1))/LAMDA(J)
NN=NN-1
J=J+1
GO TO 520
490 M(J) =0.
NN=NN-1
GO TO 520
500 DP(J) =0.
M(J) =0.
510 K=1
520 CONTINUE
530 KRITE=0
IF(DELTAT EQ. 2) GO TO 535
DELATAT =.002*SORT(R(1,1)**2+R(2,1)**2+(FLTALT-R(3,1))**2)
IF(DELTAT GT. 2) DELTAT = 2
530 IF(R(3,1)+AO1(1)*N(3,1)*DELTAT-ALT(II)) 540,540,550
540 DELT =R(3,1)-ALT(II)/AO1(1)/(-N(3,1))
KRITE=1
GO TO 560
550 DELT =DELTAT
560 IRAY =0
570 IRAY =IRAY+1
IF(IRAY EQ. 5) GO TO 610
R(1,IRAY) =R(1,IRAY)+(AO1(IRAY)*N(1,IRAY)+VO1(1,IRAY))*DELT
R(2,IRAY) =R(2,IRAY)+(AO1(IRAY)*N(2,IRAY)+VO1(2,IRAY))*DELT
R(3,IRAY) =R(3,IRAY)+AO1(IRAY)*N(3,IRAY)*DELT
H=R(3,IRAY)
IFORK=3
GO TO 240
580 DZ =-AO1(IRAY)*N(3,IRAY)*DELT
AOZ=(AO1(IRAY)-AO)/DZ
VOXZ=(VO1(1,IRAY)-VO(1))/DZ
VOYV=(VO1(2,IRAY)-VO(2))/DZ
AO1(IRAY) =AO
VO1(1,IRAY) =VO(1)
VO1(2,IRAY) =VO(2)
CC =-(N(1,IRAY)*VOXZ+N(2,IRAY)*VOYZ+AOZ)
DN1 =N(1,IRAY)*N(3,IRAY)*CC*DELT
DN2 =N(2,IRAY)*N(3,IRAY)*CC*DELT
DN3 =N(1,IRAY)**2+N(2,IRAY)**2)*CC*DELT
MAG=SQRT((N(1,IRAY)+DN1)**2+N(2,IRAY)+DN2)**2+(N(3,IRAY)+DN3)**2)
N(1,IRAY) =(N(1,IRAY)+DN1)/MAG
N(2,IRAY) =(N(2,IRAY)+DN2)/MAG
N(3,IRAY) =(N(3,IRAY)+DN3)/MAG
IF(N(3,IRAY) LT. -.01745) GO TO 570
IF(ICUT EQ. 0) GO TO 600
IF(ABS(DPHI) GT. 2.) GO TO 590
WRITE(6,4) HCUT,PHI
GO TO 1090
590 PHI=PHI+DPHI
GO TO 200
600 HCUT=R(3,1)
ICUT=1
IPHI=PHI
PHI=IPHI
GO TO 590

26
IF(icut .eq. 1 .or. izero .eq. 1) GO TO 921
CN =a01(1)*v01(1,1)+v01(2,1)*n(2,1)
A =((r(2,4)-r(2,1))*(r(3,3)-r(3,2))* (r(3,4)-r(3,1))* (r(2,3))
1-r(2,1))*n(1,1)+(r(3,4)-r(3,1))* (r(1,3)-r(1,2)))-(r(1,4)-r(1,1)))*
2*(r(3,3)-r(3,2))*n(2,1)+((r(1,4)-r(1,1))*(r(2,3)-r(2,2))
3-(r(2,4)-r(2,1))*(r(1,3)-r(1,2))*n(3,1)
IF(A .gt. 0.) GO TO 620
Hzero=r(3,1)
izero=1
GO TO 921
620 c1=1.714*(a01(1)+aos)/(p01+p01)/(cn+cn1)
c2=((a01(1)-aos)/(a01(1)+aos)+2.8*dz*g/(a01(1)+aos)**2-2.*cn-cn1)
1/(cn+cn1)-(a-a1)/(a+a1))/delt
p01=p0
cn1=cn
a1=a
630 delt=delt
tp=(exp(c2*delt)-1.0)/c2
n1=nn+1
n2=nn+2
k=n2
640 j1=1
650 DO 660 J=1,N2
IF(M(J) .EQ. M(K) .AND. J1 .EQ. 2 .AND. J .NE. K) M(J)=.9999*M(J)
F1(J)=1.0-C1*M(J)*TP
IF(F1(J) .LE. 0. .AND. J .NE. K) GO TO 670
660 CONTINUE
GO TO 680
670 k=j
j1=2
TP=1.0/C1/M(J)
GO TO 650
680 tk=c1*tp
IF(J1 .EQ. 1) GO TO 690
IF(DP(K) .EQ. 0. .AND. DP(K+1) .EQ. 0.) GO TO 690
IF(DP(K) .EQ. 0.) GO TO 760
IF(DP(K+1) .EQ. 0.) GO TO 780
GO TO 800
690 KF=1
J1=1
695 J=J+1
IF(J .GT. N1) GO TO 840
IF(J .EQ. K) GO TO 695
700 IF(ABS(TK*M(J)-M(J-1))/F1(J) .GT. .001) GO TO 710
IF(ABS(TK*M(J)-M(J+1))/F1(J) .GT. .001) GO TO 705
F2(J)=LAMDA(J)-(DP(J)+DP(J+1))*TK/F1(J)/2.
GO TO (730,820), KF
705 F2(J)=LAMDA(J)-DP(J)*TK/F1(J)/2-DP(J+1)/(M(J)-M(J+1))*(SQR(F1(J+1)/F1(J))-1)
GO TO (730,820), KF
710 IF(ABS(TK*M(J)-M(J+1))/F1(J) .GT. .001) GO TO 720
F2(J)=LAMDA(J)-DP(J)/(M(J)-M(J-1))*(SQR(F1(J-1)/F1(J))-1.)-DP(J+1)
1)*TK/F1(J)/2.
GO TO (730,820), KF
720 F2(J)=LAMDA(J)-DP(J)/(M(J)-M(J-1))*(SQR(F1(J-1)/F1(J))-1.)-DP(J+1)
1)/(M(J)-M(J+1))*(SQR(F1(J+1)/F1(J))-1.)
GO TO (730,820), KF
730 IF(F2(J)) 740, 735, 695
IF(J1 .EQ. 2) LAMDA(J)=1.0001*LAMDA(J)
IF(J1 .EQ. 2) GO TO 700
K=J
GO TO 695

K=J
IF(DP(K) .GT. 0.) GO TO 770
IF(ABS(TK*(M(K)-M(K+1))/F1(K)) .GT. .001) GO TO 760
TP=2.*LAMDA(K)/C1/(DP(K+1)+2.*M(K)*LAMDA(K))
GO TO 640

TP=(1.-(1.+LAMDA(K)/DP(K+1)*M(K)-M(K+1))**2)/(C1*(M(K+1)-M(K))*
1*(1.+LAMDA(K)/DP(K+1)*(M(K)-M(K+1))**2))
GO TO 640

IF(DP(K+1) .GT. 0.) GO TO 790
IF(ABS(TK*(M(K)-M(K-1))/F1(K)) .GT. .001) GO TO 780
TP=2.*LAMDA(K)/C1/(DP(K)+2.*M(K)*LAMDA(K))
GO TO 640

TP=(1.-(1.+LAMDA(K)/DP(K)*(M(K)-M(K-1))**2)/(C1*(M(K)-M(K-1))*(1.+
1+LAMDA(K)/DP(K)*(M(K)-M(K-1))**2))
GO TO 640

IF(ABS(TK*(M(K)-M(K-1))/F1(K)) .GT. .001) GO TO 800
IF(ABS(TK*(M(K)-M(K+1))/F1(K)) .GT. .001) GO TO 800
TP=2.*LAMDA(K)/C1/(DP(K)+DP(K+1)+2.*M(K)*LAMDA(K))
GO TO 640

TP1=0.
TP2=TP
KF=2
IC=0

IF(IC .GT. 20) GO TO 640
TP=(TP1+TP2)/2.
TK=C1*TP
F1(J)=1.0-C1*M(J)*TP
F1(J-1)=1.0-C1*M(J-1)*TP
F1(J+1)=1.0-C1*M(J+1)*TP
GO TO 700

IF(F2(K) .LE. 0.) GO TO 830
TP1=TP
GO TO 810

820 IF(F2(K) .GT. 0.) GO TO 830
TP2=TP
GO TO 810

840 DELT=ALOG(TP*C2+1.0)/C2
E2=EXP(C2*DELT)
IF(K .EQ. N2) GO TO 850
DPSUM=DP(K)+DP(K+1)
IF(K .EQ. 2) GO TO 870

850 K11=K-1
DO 860 J=2,K11
M(J)=M(J)/F1(J)*E2
DP(J)=DP(J)*E2/SQRT(FI(J)*FI(J-1))
860 LAMDA(J)=F1(J)*F2(J)
IF(K .EQ. N2) GO TO 920

870 K22=K+1
IF(DPSUM .EQ. 0.) GO TO 880
DP(K)=E2*(DP(K)/SQRT(F1(K)*F1(K-1))+DP(K+1)/SQRT(F1(K)*F1(K+1)))
GO TO 890

880 IF(DP(K)=M(K)) LAMDA(K)=E2
890 M(K)=M(K+1)/F1(K+1)*E2
IF(K .EQ. N1) GO TO 910
DO 900 J=K22,N1
M(J)=M(J+1)/F1(J+1)*E2
DP(J)=DP(J+1)*E2/SQRT(F1(J+1)*F1(J))
900 DP(K)=M(K+1)/LAMDA(K)*E2
IF(K .EQ. N1) GO TO 910
DO 900 J=K22,N1
M(J)=M(J+1)/F1(J+1)*E2
DP(J)=DP(J+1)*E2/SQRT(F1(J+1)*F1(J))
910 M(K)=M(K+1)/F1(K+1)*E2
LAMDA(J-1) = F1(J) * F2(J)
NN = NN - 1
DELT = DELTS - DELT
GO TO 630
DP(N2) = DP(N2) * E2 / SQRT(F1(N2) * F1(N1))
IF(KRITE .EQ. 0) GO TO 530
IF(ICUT .EQ. 0) GO TO 922
II = II + 1
IF(II .LE. NALT) GO TO 530
DFI = 1.
IF(PHI .LT. 0.) DFI = -1.
GO TO 590
IF(IZER .EQ. 0) GO TO 926
II = II + 1
IF(II .LE. NALT) GO TO 530
IF(ALT(NALT) .NE. 0.) GO TO 924
II = II - 1
GO TO 990
NALT = NALT + 1
AL(NALT) = 0.
GO TO 530
XX(1) = 0.
PP(1) = 0.
J = 2
DO 940 L = 2, N1
IF(DP(L) .GT. 0.) GO TO 930
XX(J) = XX(J-1) + LAMDA(L)
PP(J) = PP(J-1) + M(L) * LAMDA(L)
J = J + 1
GO TO 940
XX(J) = XX(J-1)
PP(J) = PP(J-1) + DP(L)
XX(J+1) = XX(J) + LAMDA(L)
PP(J+1) = PP(J) + M(L) * LAMDA(L)
J = J + 2
940 CONTINUE
IF(DP(N2) .EQ. 0.) GO TO 950
XX(J) = XX(J-1)
PP(J) = PP(J-1) + DP(N2)
J = J + 1
950 J = J - 1
IF(ALT(J) .NE. 0.) GO TO 970
DO 960 L = 1, J
PP(L) = PP(L) * REFL
970 IF(POG .NE. 1.) GO TO 990
DO 980 L = 1, J
PP(L) = PP(L) / PO
980 IF(IZER .EQ. 0) GO TO 990
RLAT = R(2,1) / (20890070. + ALT(J)) * 57.296 + LATP
RLONG = -R(1,1) / (20890070. + ALT(J)) * 57.296 / COS(RLAT/57.296) + LONP
IF(ALT(J) .NE. 0.) GO TO 1000
DGT = (R(2,1) * SIN(HEAD/57.296) - R(1,1) * COS(HEAD/57.296)) / 5280.
WRITE(6, 29) RLONG, RLAT, DGT, REFL
IF(IZER) 1010, 1010, 1080
1000 WRITE(6, 20) ALT(J), RLAT, RLONG
1010 IF(OCODE .EQ. 2) GO TO 1040
DO 1020 L = 1, J
1020 XX(L) = XX(L) + 1000.
IF(POG .EQ. 1.) GO TO 1030
WRITE(6, 21) (XX(L), PP(L), L = 1, J)
GO TO 1070
1030 WRITE(6,23) (XX(L),PP(L),L=1,J)  
GO TO 1070
1040 C=((VX+VOXH)*(N(1,1)+(VY+VOYH)*N(2,1))/SORT((VX+VOXH)**2+1(VY+VOYH)**2))  
DO 1050 L=1,J
1050 XX(L)=XX(L)*CN/C  
IF(POG .EQ. 1.) GO TO 1060
WRITE(6,22) (XX(L),PP(L),L=1,J)  
GO TO 1070
1060 WRITE(6,24) (XX(L),PP(L),L=1,J)  
1070 II=II+1  
KRITE=0  
IF(II-NALT) 530, 530, 1090
1080 WRITE(6,27) HZERO  
1090 WRITE(6,19)  
GO TO 40  
END
REFERENCES


