Final Report

STUDY ON MULTI-SATELLITE,
MULTI-MEASUREMENT OF THE STRUCTURE
OF THE EARTH'S BOW SHOCK

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Space Sciences Department
TRW Systems Group
One Space Park
Redondo Beach, California 90278
RESUME

NASA Study Contract NASW-2186 was conducted to verify and extend a pulsation model of the earth's bow shock. The model proposed a nonuniform shock having both perpendicular (abrupt, monotonic) and oblique (oscillatory, multigradient) properties simultaneously, depending on local orientation of the shock surface to the interplanetary field $B_{SW}$ in parallel planes defined by $B_{SW}$ and solar wind velocity $V_{SW}$. The study proceeded through the use of multiple, concurrent, satellite observations of the shock and solar wind conditions.

Twenty-six potentially useful intervals of concurrent Explorer 33 and 35 data acquisition were examined, of which six were selected for closer study. In addition, two years of OGO-5 and HEOS-1 magnetometer data were examined for possible conjunctions of these spacecraft having applicable data. Several intervals of unusual interest were selected for closer study.

No counterexamples were found to the proposition that field orientation determines shock structure. One case of clear nonuniformity and several of field-dependent structure were documented. A computational aid, called pulsation index $I_p$ was developed. The background and results of the study are enumerated and recommendations for further work are made. Results certifiable at the time this is written are summarized in the report as answers to questions with which the study was initiated. Detailed results appear in Appendices prepared as scientific papers. Two are soon to be published; the third is under submission.
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I. INTRODUCTION

The original premise which the study was initiated to investigate has proven correct and opened a whole new approach to the macroscopic plasma structure of the earth's shock and magnetosheath. Application of the results of the study should effect a substantial improvement in our experimental description of the earth's shock as a collisionless plasma phenomenon.

No difficulty was encountered in acquiring adequate data appropriate to the investigation. Indeed, more data were accumulated in a relatively short time than could possibly be treated within the scope of the initial study. This situation arose unexpectedly when a chance meeting with P. C. Hedgecock of Imperial College opened the excellent HEOS-1 magnetometer records to this investigation, enlarging significantly the pool of accessible data. While the study still relied heavily on data from Explorers 33 and 35 and OGO 5, addition of HEOS-1 measurements improved enormously the prospects for correlated simultaneous observations of specific phenomena. Problems of representation and documentation occurred, but some solutions to the documentation have been developed that helps and that will facilitate, even accelerate, future work considerably.

The following sections review the background of the study, summarize the results, and discuss the direction which further activity should take.

II. BACKGROUND

Purpose. This program was initiated to test a model postulating field-dependent, nonuniform structure of the earth's bow shock and to develop and extend the model, if verified. The means employed by the program were to be
empirical, using existing data from simultaneous multiple spacecraft observations.

B-X Geometry. An important facet of the investigation was to be the proposed reliance on analysis of observations in a B-X frame. Local shock structure was assumed to be determined by the local orientation of the interplanetary field $B_{SW}$ to a cross section of the shock in a plane containing $B_{SW}$ and the solar ecliptic $X$ axis, actually an approximation for the plane formed by $B_{SW}$ and solar wind velocity $V_{SW}$. According to the model, the shock should exhibit an abrupt profile at any point where $B_{SW}$ is approximately tangent to the nominal shock contour in a B-X plane through the point and a pulsation, or wave-train profile at any point where $B_{SW}$ is approximately perpendicular to the nominal B-X contour through the point. The curvature of the bow shock in relation to a given $B_{SW}$ and to the relatively fixed direction of $V_{SW}$ should, it was postulated, produce a generally nonuniform shock, with jump structure in some portion and pulsation structure in the remainder. Continuous change in $B_{SW}$ should cause a continuous change in the distribution pattern of nonuniform structure around the shock (Greenstadt et al., 1970).

Oblique and Perpendicular Shocks. The nonuniform-bow-shock model was developed from empirical considerations alone. Data on magnetic fields in and near the shock led heuristically to the notions synopsized in the preceding paragraph. There does exist, however, a growing body of plasma theoretical literature on collisionless shocks that leads naturally to more or less the same conceptualization of the earth's bow shock, although the nonuniform representation does not seem to have been explicitly formulated with regard to the earth's shock. Theoretical and laboratory experimental treatments of high $\beta$, high
Mach number, collisionless plasma shocks, which is the category in which the earth's shock fits most of the time, are still being developed and refined. In fact, some of the available results most applicable to shocks in the solar wind was in preparation or being reported concurrently with analysis and publication of the dual satellite work (Greenstadt et al., 1970) that led to the present study.

In non space-oriented research, a distinction between perpendicular and oblique shocks is universally recognized, where the terms refer to the orientation of a uniform field $\mathbf{B}$ with respect to the direction of propagation of a plane, infinite shock moving through the plasma. Whistler wave precursors are among the phenomena commonly associated with oblique shocks (Drummond and Kolb, 1968); shock broadening, ion reflection, and upstream "foot" formation are associated with perpendicular shocks of high Mach number $M_A > M_A^*$ where $M_A^* \approx 3$ is the critical Mach number (Paul, 1969), and may be associated with oblique shocks as well. Unfortunately, the high $\beta$, high $M_A$, oblique shock is the least researched type. Judging by what has been learned, however (Paul, 1969; Robson, 1969), and extrapolating from low $\beta$ or low $M_A^*$ results, a distinction between perpendicular and oblique structures should not be unexpected in the bow shock. Nonuniform curved shocks have been made in the laboratory (Robson, 1969).

Shock theoretical formulations are, almost entirely two-dimensional, with plane shocks and homogeneous conditions assumed (essentially in a B-V plane). Nonuniformity of earth's curved bow shock is a natural conclusion for a hyperbolic surface of revolution in the solar wind for all $\beta$ and $M_A$, if the distinction between perpendicular and oblique shocks is maintained by the plasma.
Objective. This study opened with a list of specific questions to which answers were to be sought. Foremost, of course, was the question of whether the nonuniform shock model postulated from an earlier investigation of some spatially and temporally limited data was generally valid. Additional questions of detail depending on affirmation of the model were posed. When the study was proposed, an extensive and promising list of paired observations in which either Explorer 33 or 35 was expected to be in or near the shock while the other satellite was in the solar wind had been prepared. The nature of the data for those observations was unknown. The initial objective was to answer as many questions as the prepared list of paired observations, or any others that could be formed, would allow.

III. RESULTS OF STUDY

Specific Questions. The questions approached by the study, paraphrased for brevity, are listed below, each followed by a short statement describing its status as this is written. The original form of the questions appeared in TRW's Proposal No. 17873.000, Section III, August 1970.

1. Do contrasting structures coexist?

Definitely. A case of contrasting profiles on opposite sides of the shock was found for low $\beta$ and moderate $M_A$, the contrast evidently caused by differing field orientations. A report on this result has been accepted for publication by JGR and will appear in the issue of April 1972. It is included with this report as Appendix 1.

2. What is the rate of broadening of the pulsation shock following field reorientation?

No answer yet.
3. Does the outer envelope of the pulsation shock expand?

No answer yet.

4. How far inward toward the magnetopause do pulsations extend?

This question has been partially answered by inspection of magnetosheath passes recorded by the HEOS 1 magnetometer. A collection of "portraits" of the sheath showing a variety of structures will be the subject of a paper in preparation at the end of this study. Figure 1 displays the magnetosheath passes to be discussed in the paper. The most notable examples show magnetosheath fields of extreme irregularity, under very quiet conditions (QQ), with large-amplitude fluctuations occurring at many distances between the magnetopause and the solar wind. Space-time separation ambiguity prevents a definitive interpretation, but shock-like pulsations and field gradients certainly appear to extend all the way to the magnetopause at times. The paper, by the present investigator and P. C. Hedgecock, a HEOS-1 experimenter, will include upstream observations by Explorers 33 and 35 to aid in interpreting the selected examples.

5. Does the shock ever lose its sudden, large gradients entirely?

No documented example of this behavior has been found, but ill-defined boundaries which might correspond to such a picture have been observed.

6. Can a general outline, or "map," of the nonuniform shock be established?

No illustration better than the one originally proposed can yet be drawn.

7. How does the interplanetary (solar wind) field magnitude $B_{SW}$ affect shock structure?

Evidently not much, except insofar as it may contribute to $\beta$ and $M_A$. This is one of the subjects to be treated in the HEOS report.
8, 9. What is the influence on shock structure of solar wind parameters other than $B_{SW}$ orientation, and how are various nonclassical features of the shock interrelated?

These are fundamental questions of potentially wide scope which define the essence of collisionless plasma shock investigation. An attack on the general problem has begun with study of a set of observations of the moving shock, at low Mach number, seen by OGO 5, HEOS 1, and Explorer 33 on 12 February 1969. Initial results of the study, dealing only with some new estimates of high shock velocity, have been summarized in a report which has been accepted by J. Geophys. Res., for the March 72 issue. It is attached as Appendix 2. A detailed examination of the events of 12 February had been initiated before the end of the study period, also in cooperation with P. C. Hedgecock and C. T. Russell.

General Remarks. Several questions not on the original list, but which arose during the study, have also been answered in the analysis of HEOS 1's magnetosheath passes. For example, the structure of the shock seems to be unrelated to the "southwardness" of $B_{SW}$ or to the existing level of storminess or disturbance.

The quality and content of the coordinated Explorer 33, 35 data sections selected for study were unknown before this study began. It was established early not only that the data were of satisfactory quality, but that they supported the model the study was attempting to confirm. However, although finding cases of simultaneous observations seeming to corroborate a field-dependent shock structure was easier than expected, the ease of documenting these cases was overestimated. Few cases exhibit $B$-$X$ configurations readily visualized or long constant. Transformation of most subjectively evaluated cases into persuasive, objectively documented examples would have required a prodigious volume of transformations of data to $B$-$X$ representations.
If the early, graphic method were employed, the level of effort would not only have been prohibitive, but publication of the bulk of diagrams would be out of the question. Also, most of the investigator's effort would have been purely mechanical, with little time available for further analysis. Attention was therefore turned to creation of an automatic, or semiautomatic, means of evaluating the B-X arrangement in a given case, and a "pulsation index" $I_p$ was devised to measure the likelihood that the shock will be in one form or the other, i.e., abrupt or oscillatory, for a given position on the nominal shock and in a given interplanetary field orientation. The index has proved valuable in early application and is described fully in a paper being submitted for publication as this report is completed. A draft of the paper, included as Appendix 3, gives the definition of the index, a brief account of how it is computed, and three illustrations of its application. The illustrations demonstrate the validity of the relationships postulated as necessary for the wavetrain shock.

IV. CONCLUSION

Several specific questions have been answered. Most importantly, the nonuniform shock has been found to be a reality, and the dependence of local structure on field orientation in B-X geometry has been confirmed in a few cases. The groundwork has been laid, through invention of a wavetrain index, for thorough documentation of the postulated model. Two reports (Appendices 1, 2) have been accepted for publication, a third (Appendix 3) is being submitted, and two more are in various stages of completion. A number of interesting cases of correlated measurements have been discovered which could benefit from application of the newly formulated index.
V. RECOMMENDATIONS FOR FUTURE WORK

Further multisatellite study of the bow shock will entail three categories of interest: Documentation, further investigation of unanswered questions, and structural analysis.

Many examples of nonclassical shock behavior are available in existing data to provide convincing evidence of field-dependent structure once the new index is applied. Analysis of such examples and illustration of the results constitute an important documentary activity which should be pursued in the immediate future.

Unanswered Questions. Questions, or parts of questions, originally posed but as yet unanswered should be addressed. Application of index $I_p$ to intervals selected for additional documentation on field-dependent structure and non-uniformity may provide, as a byproduct, answers to some or all of the remaining questions. If not, a substantial quantity of data as yet unexamined may furnish the desired results. The last pair of original questions (see 8,9 in the Results section, above) define a large subject, as already explained, which may be designated "structural analysis," and which is treated separately in the following paragraphs.

Structural Analysis. Demonstration of field-dependent nonuniformity of the bow shock and development of the index opens up a field of investigation for which satellite observations in the collisionless plasma surrounding the earth are eminently suitable. Effort can now be directed toward analyzing the macrostructure of the shock in a systematic way. The solar wind provides a preshocked plasma at every field direction and over a wide range of $\beta$, 
mostly at high Mach number, but occasionally at $M_A < 3$ (Formisano et al., 1971). Shock observations below and above the critical Mach number, around $M_A^* = 3$, can be separated and categorized as perpendicular or oblique by use of the index. It should also be possible to isolate cases of various $B_{SW}$ orientations over a range of $\beta$, so that comprehensive sets of $\beta, M_A, \theta$ combinations can be studied, where $\theta$ denotes the field angle relative to the local shock normal. It should be possible to isolate factors responsible for the various shock "forms" noted, for example, by Heppner et al. (1967) and Fredricks et al. (1970), provided only that a suitable set of correlated spacecraft observations is available. A catalogue of empirical shock profiles, classified according to the vital plasma parameters contributing to them is now a distinct possibility and will be regarded as an important objective to be pursued. Wherever possible, data from the OGO-5 plasma wave detector will be included in selected examples to aid in determining the plasma processes responsible for limiting the observed gradients.

Future analysis of shock structures along the lines described above will involve extensive use of index $I_p$, which will in turn require development of improved capability in computing the index. In practice, a single shock observation point and a single interplanetary field value are seldom of interest. A continuous set of observations over some time interval, as illustrated by the examples of Appendix 3, usually provides the data to be analyzed. For this reason, a rapid, automatic computation of $I_p$ is desirable. Computation of $I_p$ has been programmed in this study for a Hewlett-Packard desk calculator, which is suitable for handling short data intervals. Since the data must be inserted in the machine through the keyboard, large quantities of data, which should be entered directly from data tape, cannot yet be
processed. An important task is to acquire the capability to handle masses of data.

Aside from this mechanical difficulty, one shortcoming limits the really wide application of $I_p$ as it is presently designed: It cannot be used to estimate the likelihood of disturbed conditions far outside the shock (upstream waves), or deep in the magnetosheath (large amplitude turbulence). In effect, the index shows whether a point of observation on a nominal shock of nonuniform profile is located in the region of wavetrain behavior, in the region of step behavior, or exactly in between. In order to use the index for points away from the shock, it is necessary, first, to have some independent knowledge of shock location (a third satellite, perhaps) and, second, to have a means of estimating the point on the shock which might be responsible for conditions at an off-shock point of observation. At present, application of the index is confined to predicting the character of the shock at times within a few minutes of its direct observation, as was done for the first two examples of Appendix 3. If suitable coincidences of spacecraft measurements can be found, it may be possible to overcome this limitation and attack the question of what the pattern of downstream turbulence might be in the entire magnetosheath, for a given nonuniform pattern on the nominal shock surface. Determination of the magnetosheath structure is one of the objectives, admittedly a difficult one, of further analysis using the index.

Clearly, an index of some kind will be of major importance in future work, and continued attention should be given to ways of improving the present one and of accelerating its computation for large quantities of data. One potential refinement would be the development of a computer program to accept
solar wind plasma velocity and direction as well as magnetic field as input, in order to operate in a true \( B-V_{SW} \) frame. The feasibility of such a refinement should be assessed.

VI. NEW TECHNOLOGY

We believe that pulsation index \( I_p \), developed under this study as a computational aid in assessing local shock obliquity, may constitute new technology. Attention is therefore called to the index here under this specific heading. The index is described in Appendix 3 and will be the subject of a separate technology report to follow.

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REFERENCES


APPENDIX 1

AN OBSERVATION OF NONUNIFORM STRUCTURE
OF THE EARTH'S BOW SHOCK CORRELATED
WITH INTERPLANETARY FIELD ORIENTATION

E. W. Greenstadt
Space Sciences Department

12 July 1971

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Space Sciences Department
TRW Systems Group
One Space Park
Redondo Beach, California 90278
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ABSTRACT

Explorer 33 and 35 magnetometers, the first on the western, the second on the eastern flank of the earth's bow shock, observed the boundary concurrently between 0130 and 0430 UT, 30 October 1968. Contrasting shock structures were recorded: Explorer 35 saw a quiet, abrupt shock; while Explorer 33 saw an irregular, noisy boundary with much upstream wave activity. The interplanetary field was roughly in the average archimedean spiral angle, and was therefore approximately tangent to the shock at Explorer 35 and normal to the shock at Explorer 33. Gross motions and variable tilting of the aberrated shock probably contributed to the peculiar sequence of shock crossings at the two spacecraft. The observations support a model of the shock in which perpendicular and oblique collisionless structures coexist, forming a nonuniform magnetosheath outer boundary.
INTRODUCTION

The local structure of a magnetized, collisionless, plasma shock depends on plasma thermal to field energy ratio $\beta$, Alfvén Mach number $M_A$, and the angle $\alpha$ between the field and the direction of shock propagation (Kennel and Sagdeev, 1967; Paul, 1969; Robson, 1969; Friedman et al., 1970; Coroniti, 1970). To a magnetometer passing through such a shock, its large-scale, or macroscopic structure may consist of a monotonic rise in field, with or without an upstream "foot," a double shock, a step rise with a precursor pulse, a series of upstream or downstream waves of various amplitudes and damping distances, or composites of these features depending on the prevailing combination of parameters $\beta$, $M_A$, and $\alpha$. One outstanding and consistent characteristic of plasma shocks is their broadening, generally with the establishment of an upstream pulse or wavetrain, when $\alpha$ is not close to 90°, i.e., when the shock is an oblique shock (Iskol'dskii et al., 1965; Patrick and Pugh, 1969). A shock is called perpendicular if $\alpha = 90^\circ$, meaning the field in the unshocked plasma is perpendicular to the direction of propagation of the shock, hence tangent to the shock front; otherwise, the shock is oblique. This paper reports the first observation of coexisting perpendicular and oblique structure in the earth's bow shock.

A plane shock propagating into a homogeneous plasma containing a uniform field will be either perpendicular or oblique everywhere and should display a uniform structure. A curved shock, in contrast, can be locally perpendicular, or nearly so, at one section of its surface, but oblique everywhere else and can have a nonuniform structure of varying thickness. The existence of such nonuniformity has been demonstrated in the laboratory (Robson, 1969). Since the interplanetary field in the solar wind is, as a rule, reasonably uniform over the dimensions of the earth's magnetosphere-magnetosheath system, the
curved, hyperbolic bow shock enclosing the system should be expected, in general, to span both perpendicular and oblique local conditions and therefore display a nonuniform structure.

The frame in which shock structure is examined in plasma theoretical and laboratory investigations is usually two-dimensional, consisting of the plane containing the plasma flow velocity and its embedded magnetic field vector. The earth's bow shock is, of course, unavoidably three-dimensional, and the solar wind parameters are continuously changing, but the familiar frame of examination can be constructed by considering cross sections of the shock in planes formed by instantaneous values of interplanetary field \( \mathbf{B} \) and the solar wind velocity \( \mathbf{V}_{\text{SW}} \). \( \mathbf{V}_{\text{SW}} \) approximately parallels the solar ecliptic X axis, so the appropriate frame in which to examine bow shock structure to first approximation is a plane containing \( \mathbf{B} \) and X. The instantaneous B-X plane, or planes, through a point, or points, of observation will intersect the bow shock to form a B-X shock contour, or contours. It is in the frame of these contours that nonuniform structure should be apparent, if it exists. An empirical model of the shock incorporating just such nonuniformity in this frame has been suggested from examination of spacecraft data.

In a study of the earth's shock with two relatively closely-spaced satellites, Greenstadt et al. (1970) observed a thick, pulsation boundary alternating with a simple step shock, and inferred that the structure of the shock was locally dependent on direction of the interplanetary field \( \mathbf{B} \), being abrupt when \( \mathbf{B} \) was tangent to the nominal shock contour on an intersecting B-X plane, and of pulsation character when \( \mathbf{B} \) was at large angle to the B-X contour. A model of the shock as a whole was then proposed according to which an interplanetary field uniform over the dimensions of the magnetosheath would
necessarily imply orthogonal field-to-boundary orientations on opposite sides of the sheath, hence a nonuniform shock with coexisting step and wavetrain structures. Numerous examples of diverse shock behavior reported in the literature were cited by Greenstadt et al. to underscore the ubiquity of the pulsation, or wavetrain, type of structure encountered by single satellites. Wolfe and Intriligator (1968) also suggested, on the basis of their review of satellite shock observations, the possible importance of curvature-dependent local field orientation on shock and magnetosheath structure. In this paper, the experimental analysis of the macrostructure of the outer magnetosheath boundary is carried a step further and the model of Greenstadt et al. tested by using dual shock observations by Explorers 33 and 35 at wide spacing.

The only data heretofore reported which furnished a comparison of structures on opposite sides of a curved planetary shock supported the model of Greenstadt et al. (1970) but were not simultaneous and were obtained near Venus and Mars rather than at the earth's shock (Greenstadt, 1970). The data described in the present report were recorded simultaneously by identical magnetometers on opposite sides of the earth's shock. As the sequel will explain, contrasting structures were observed whose characteristics were correlated with the local relative orientation of the ambient interplanetary field in the way prescribed by the curved-shock model.

In the following sections the locations of the two Explorer spacecraft are given, the observations described, and explanations for the appearance of the observed sequence of boundary crossings offered. The possible influence of \( \beta \), Mach number, and field direction on the structure is assessed, and the \( B \times X \) configurations at the two spacecraft are presented, showing the respective correlations of tangent and normal fields with abrupt and wavetrain boundaries.
The data used in this study were obtained by the NASA Ames Research Center magnetometers of Explorers 33 and 35 around 0300 UT, 30 October 1968. The instruments have been described in the literature (Mihalov et al., 1968).

LOCATIONS OF THE SATELLITES

The positions of Explorers 33 and 35 during the joint observations of this report are depicted in Figure 1. At the top are the projections of the satellite loci on the Y-Z plane through the earth, i.e., the plane \( X = 0 \), looking from the sun in solar ecliptic coordinates. Explorer 33 was on the west, or midnight-to-noon, side of the earth; Explorer 35 was on the east, or noon-to-midnight side.

The lower half of the figure shows the spacecraft locations rotated onto a common \( X-\rho \) plane, assuming cylindrical symmetry, where \( \rho = (Y^2 + Z^2)^{1/2} \). The east-west distinction has been preserved, with west above, east below the \( X \)-axis, and the solar wind blowing from the left. Nominal shock contours, without solar wind aberration, are shown passing through the locations of both satellites at the boundary crossings described in this report. Both vehicles were on the flanks of the boundary behind the dawn-dusk meridian.

OBSERVATIONS

The magnetic field data recorded during the subject interval are shown in Figure 2, with Explorer 33's measurements in the upper box, Explorer 35's in the lower box. Magnitudes of \( B \) have been plotted adjacent to each other in the center to facilitate comparison. Angles \( \lambda \) and \( \phi \) represent latitude and
longitude of the field vector in solar ecliptic coordinates. The plotted values are 82-second averages of samples obtained roughly every six seconds. To emphasize the contrast in character between the data at the two spacecraft, the highest and lowest values of $B$ in each 82-second averaging unit have been plotted surrounding the averages.

The shock, or magnetosheath envelope, had been seen to expand past the two satellites some four hours before the beginning of the illustrated interval. A contraction of the shock returned both spacecraft to the solar wind by 0310, as depicted in Figure 2.

The transition from magnetosheath to solar wind, which took place at the two satellites between 0200 and 0310, provided two markedly different magnetic signatures. In general, field magnitudes and directions were more agitated at Explorer 33 than at Explorer 35. This had been true as well for the hours preceding the interval in the figure. The most obvious difference in fluctuation content appeared in the magnitudes, as shown in the figure. Certain events, for example the dips in $\lambda$ and $\phi$ between 0300 and 0340, were interplanetary, showing similar, but not identical, profiles at both spacecraft, with a corotation delay of a few minutes from Explorer 35, on the eastern, to Explorer 33, on the western, side. The events designated as shock crossings, best identified as the reductions in average field magnitude from magnetosheath to interplanetary levels, were clearly dissimilar at the two satellites, both on the average and in detail.

Gross changes in field direction accompanied the approach of the boundary to Explorer 33, as the field rotated from its "draped" direction in the third
quadrant, paralleling the magnetopause (Dryer and Faye-Petersen, 1966; Spreiter et al., 1966; Fairfield, 1967), underwent a series of large oscillations, and finally settled, by 0400, into an interplanetary orientation in the fourth quadrant, much the same as that measured at the same time by Explorer 35. Average field magnitudes were 25 percent higher just behind the shock at Explorer 33. The steps from sheath to interplanetary magnitude, or the reverse, were abrupt and quiet at Explorer 35, but irregular and noisy at Explorer 33. The noise limits, i.e., the differences between high and low values, were never more than 1\textmu G at Explorer 35, whether behind, inside, or in front of the boundary; the noise limits were never less than 1\textmu G at Explorer 33, within half an hour of the apparent boundary, and were as much as 5 to 6\textmu G at several points. Explorer 33 detected continuous upstream wave activity in the solar wind, Explorer 35 found none.

Shock crossings, or encounters, are easily defined at Explorer 35: There was an emergence into the solar wind at 0237, a reentry into the sheath at 0250, and a final exit into the solar wind at 0302. Crossings at Explorer 33 are not so simply delineated: The average field apparently dropped from sheath to interplanetary level at 0214, then rose immediately to sheath level and dropped down again at 0225, forming a shape that could have been either a recrossing or a large amplitude pulse in B. The average field then climbed gradually almost to what had been sheath level at the earlier "boundary"; then, at 0308-9, the mean field descended quickly to solar wind level where it remained, modulated, as noted, by upstream waves.
BOUNDARY MOTION AND SHOCK STRUCTURE

The dissimilar signatures in the observations elaborated in the preceding section are related to the structural character of the shock. They occur, however, in association with certain other features of the measurements that may be connected with shock motion and which have a bearing on interpretation of the data. Shock motion and structure will therefore be discussed together.

The relatively low average field strength at Explorer 33 between 0225 and 0250 suggests that Explorer 33 was in the solar wind in that interval. Explorer 35 was clearly behind the shock, however, between 0225 and 0237, and considerably farther away from the earth than Explorer 33. According to the geometrical arrangement of Figure 1, then, Explorer 33 could not have been observing solar wind, but should have been seeing magnetosheath fields well inside the shock. Does a highly variable magnetosheath field of average magnitude comparable to that of the interplanetary field make sense? Why were more pulsations, high field excursions, or higher field magnitude characteristic of the downstream side of the shock's envelope not recorded at Explorer 33?

Four explanations are possible: First, it may be imagined that striking inhomogeneities of the scale of the magnetosheath diameter persisted in the solar wind for up to twelve minutes (0225-0237), allowing two entirely different, and inconsistent, shock segments to coexist. This seems unlikely enough to be dismissed as an explanation, but, if true, would establish a temporary, although not intrinsic, inhomogeneity of the shock.
A second explanation would be that the "shock" boundary on Explorer 33's side consisted of an extensive region of fluctuating field on the order of $18 \, R_e$ thick. This region would have been composed of a combination of precursor and downstream wavetrains none of which would have been associated, strictly speaking, with unaffected solar wind. The latter would have existed only beyond the outer envelope of the entire wavetrain region. A conspicuous field gradient earthward from Explorer 33 might have separated the precursor from the post-shock region, and the generally lowered average field from 0225 to 0310, closer to solar wind level than before 0225, would presumably indicate Explorer 33's residence in the precursor regime. This is a purely "structural" explanation.

As a third possibility, consider the last, definite appearance of solar wind at both satellites, which occurs at Explorer 35 at 0302 and seven minutes later at Explorer 33 at 0309 (the end of the last large field gradient): If we take $18 \, R_e$ as the normal distance through which the shock contracted (Figure 1), and say the last high maximum at Explorer 33 was in fact the shock, then the shock would have been moving inward at some 270 Km/sec. The Explorer magnetometers sample the field approximately every six seconds, with the signals prefiltered to avoid aliasing. Rapid oscillations of the sort frequently seen in the shock and sheath (Greenstadt et al., 1970) could have been doppler shifted completely out of the band of sensitivity in a system moving at such high speed past the satellite. The real nature of the wavetrain boundary at Explorer 33 may therefore not have been resolved.

Finally, the prospect remains that the direction of the solar wind bulk velocity during the group of boundary crossings reported here was so far from
the antisolar direction that the entire magnetospheric system, including the
sheath and shock, was tilted at a great enough angle to expose Explorer 33 to
the solar wind while Explorer 35 remained in the sheath between 0214 and 0237,
and again between 0250 and 0302. Sudden restoration of the bulk flow direction
toward a more usual angle would then have allowed Explorer 33 to have its
final encounter with the shock at 0308, after Explorer 35 was already in the
solar wind. Brief rocking, or gross in-and-out movements, of the shock could
have produced the pulse at Explorer 33 at 0220; gross movement would have
cau sed the last pair of crossings at Explorer 35.

There is supplementary data supporting this last explanation, at least
in principal. Although neither Explorer plasma experiment was delivering data
at the time of the observations reported here, inquiries revealed that Vela 4B
recorded a 45-minute section of data on 30 October, between 0200 and 0245, in
the location shown in Figure 3 in Y-Z and X-p coordinates. The Vela plasma data,
graciously furnished the author by Dr. Michael Montgomery of Los Alamos, were
unambiguously characteristic of the magnetosheath.

The solid curve in the right-hand sketch of Figure 3 represents an
unaberrated shock contour through Explorer 33's position, the dashed curve an
aberrated and expanded one with an axis of symmetry 10° from the sun-earth
line. Since Vela, like Explorer 33, was on the morning side, it seems likely
that an unaberrated shock inside of Explorer 33 at 0230 would also have
crossed Vela, at least briefly, exposing it, too, to the solar wind. The mea-
surement of magnetosheath field at Explorer 35 would then have been impos-
sible. An aberrated shock, on the other hand, could have accounted for the
observed combination of regimes at all three spacecraft: The aberrated con-
tour would have placed Explorer 33 in the solar wind, while leaving Explorer
35 and Vela 4B in the sheath. The variable signal at Explorer 33 from 0225-0300 would then have been caused by precursor waves, which could also have affected the solar wind much as in the second explanation. The specific configuration of the tilted shock in Figure 3 is illustrative only, but serves to indicate that a plausible combination of subsolar point, standoff distance, and aberration angle could explain the measurements in the locations occupied by Vela and the two Explorers. An evening-side "bulge" in the shock (Behannon, 1968) would reduce the amount of aberration needed to put Explorer 35 in the sheath.

No basis exists for a definitive statement on which explanation, or what combination of explanations, actually produced the peculiar combination of field levels in Figure 2 and the sequence in which they occurred. The fourth possibility probably dominated, but it would not exclude the others. Both rocking and gross shock expansion and contraction undoubtedly took place. The shock "front" could have moved so rapidly past Explorer 33, from either cause, that small scale details of structure would have been rendered instrumentally unresolvable, and the exact relationship of any particular segment of the signal to the principal field gradients would have been obscured. None of the possibilities does away with the nonuniformity apparent in the boundary structure, while the third, the instrumental one, suggests that the noisy structure may have been even more irregular than is evident in the data.

PLASMA PARAMETERS

Consideration of \( \beta, M_A \), and field orientation is necessary in defining the context in which the measurements were obtained. Systematic east-west differences in one or more of these parameters should account for the distinct shock structures.
Energy ratio $\beta$. There was no access in the interval reported here to concurrent plasma data in the solar wind, as already mentioned, but the Vela 4B observations in the magnetosheath are helpful in evaluating $\beta$.

The solar wind near the earth is, on the average, a naturally-occurring collisionless, moderate-to-high-$\beta$ plasma, i.e., $\beta \approx 1$ (Coroniti, 1970; Ness et al., 1969; Ness et al., 1971), the quiet-time proton temperature is about $4 \times 10^6$K, and the proton density is 5 to 8 cm$^{-3}$ (Hundhausen, 1968; Ness et al., 1971). The interplanetary field measured just after the magnetosheath encounter of this report was 10-15\gamma (Figure 2), so $\beta$ was probably two to three, and the magnetic energy density, $B^2/8\pi$, four to nine, times the usual quiet value. The magnetosheath proton temperature measured at 0200-0245 by Vela 4B was 1.5 x $10^6$K, while the density varied from 2 to 8 cm$^{-3}$, with a group of measurements at 4 cm$^{-3}$. The observed sheath temperatures should have been in the range 8-35 times the corresponding solar wind temperatures and the densities 2.8 times the solar wind densities (Argo et al., 1967; Dryer, 1971). Hence the solar wind proton temperatures can be placed in the range 4 to 20 x $10^6$K and the density in the range .7 to 2.9 cm$^{-3}$, or one to five and one-tenth to six-tenths times the corresponding quiet values of these quantities. Combined with the observed field magnitudes, these values would put $\beta$ between extremes of .013 and .6. An intermediate density of 1.4 cm$^{-3}$, a post-to-pre-shock temperature ratio of 20, and an interplanetary field of 11\gamma would give $\beta = .09$. It thus appears that $\beta$ was below its usual value, perhaps considerably so, which would tend, if anything, to have made the shock structure simpler and more laminar than it would be for high $\beta$. There is no reason to believe $\beta$ was radically different on Explorer 35's side of the shock, but it must be conceded that no systematic
study of the change in plasma parameters across differing bow shock structures has been carried out.

**Mach number.** A range for the Alfvén Mach number can also be evaluated on the basis of qualitative arguments and Vela's plasma data. We may infer that the overall prevailing Mach number was lower than usual for the early part of the observation interval because B was high and expansion of the shock past Explorer 35's relatively distant location would have required the subsolar distance of the shock to be large. At the same time, the bulk proton velocity $V_{SW}$ measured in the sheath by Vela was abnormally high ($\approx 500 \text{ Km/sec}$), and had been measured at over 700 Km/sec outside the shock around 1700 on the 29th. A high wind speed implies at least an average to perhaps a large solar wind pressure, which would make similar expansion of the magnetopause unlikely. Hence a large standoff distance between shock and magnetopause should have prevailed.

A numerical estimate bears out this reasoning: The Alfvén Mach number $M_A$ is proportional to $V_{SW} \sqrt{n}/B$, where $n$ and $B$ are the solar wind's density and field magnitude. The average value for this quantity is 196, in mixed units, if we take 400 Km/sec, 6 cm$^{-3}$, and 5Y as quiet values of $V_{SW}$, $n$, and $B$. If at the time of measurement $V_{SW} \approx 700 \text{ Km/sec}$ (Vela's measurement of $V$ in the sheath was 500 Km/sec, or about .7 of this), and $n$ and $B$ had the ranges quoted in the preceding discussion of $\beta$, then $V_{SW} \sqrt{n}/B$ would have been between 39 and 120, or 20 to 61 percent of its average quiet value. Since $M_A$ averages around 11 (Ness et al., 1971), this would put $M_A$ between 2.2 and 6.7 during the observation interval. Values of 1.4 cm$^{-3}$ and 11Y for $n$ and $B$ give an intermediate $M_A = 4.3$. It appears that the solar wind Mach number was somewhat lower than usual, although not extremely so, and very probably equal to or greater than the critical Mach number $M_A \lesssim 3$. 
If the angle of the velocity to the local shock normal is taken into account, the effective, normal Mach number is lowered and there is the marginal prospect that $M_A$ was below the critical value on the flanks of the shock or perhaps even below it at Explorer 35 but above it at Explorer 33. Thus there could conceivably have been some contribution to the distinction in shock structure by slightly differing Mach numbers at the two observations points. This seems an improbable eventuality for two reasons. First, east-west separation by critical, normal Mach number would require a fairly delicate coincidence, the two normal angles having differed by no more than about 10° even for the skewed shock proposed early in the data interval (Figure 3). Second, the Mach number could not have remained steady near its critical value throughout the observations. The interval studied, although between real storms, was a relatively disturbed one, as attested by the shock's net inward motion and its apparent multiple crossings by Explorer 35. Both of these phenomena must have been occasioned by variations in the solar wind and local normal Mach number as well as by shifts in solar wind direction. Of course, the instantaneous Mach number cannot be estimated without a complete record in the solar wind, free of the shock altogether, but we may argue that the shock contracted at 0300 because of a rise in $M_A$ toward its average level, well above 3. This seems quite probable, since Vela's $n$ was rising in the sheath between 0200 and 0245. Because of the increase in $n$, the distinction between the two shock signatures would not likely have been formed by a critical east-west difference in $M_A$ after about 0240. But the fields were consistently different throughout the observations, so it seems doubtful that the Mach number played a crucial role in distinguishing the field behavior at the two Explorers at any time during the interval.
In sum, the data do not support an influence on shock nonuniformity by small differences in Mach number centered around a critical value.

Interplanetary field orientation. A uniform field in the solar wind defines a nonuniform relationship of local field to local shock, and a differential in angle between \( \mathbf{B} \) and \( \mathbf{V}_{SW} \) on the two sides of the shock is a plausible source of the inhomogeneity of the boundary. If the direction of \( \mathbf{V}_{SW} \) is taken along the X-axis, then the Y-Z intercepts of the planes containing \( \mathbf{B} \) and \( \mathbf{V}_{SW} \) (the B-X planes) through the observation points at 0241 (during the penultimate, 9-minute solar wind observation at Explorer 35 in Figure 2) were as shown at the top in Figure 4. The \( \mathbf{B} \) vector (SEC rectangular) at that time was \( (9.4, -6.6, 3.74)_{Y} \). It was therefore pointing toward the sun, 18° above the ecliptic, not far (15°) from the 45° archimedean angle. The contours formed on the B-X planes by nominal shocks through the satellite locations are shown at the bottom in Figure 4, looking in the direction of the large arrow in the upper sketch. The contours were obtained by the method of Greenstadt et al. (1970, Part II, Appendix). We see that the interplanetary field was almost tangent to the contour at Explorer 35, where the abrupt shocks were seen, and about normal to the contour at Explorer 33, where the noisy boundary was seen. In terms of the relationship between the interplanetary field and the component of solar wind velocity normal to the shock, local conditions at Explorer 35 corresponded to a nearly perpendicular (\( \alpha \approx 90° \)), those at Explorer 33 to a parallel (\( \alpha \approx 0° \)), collisionless shock in conventional plasma phraseology, so the structures should have differed regardless of \( \beta \) or \( M \) (Robson, 1969; Friedman et al., 1970). No significant change in the relationships of Figure 4 occurs if an aberrated shock is used with \( \mathbf{V}_{SW} \) at small angle to X.
The data of this report provide an example in which the contracting outer boundary of the earth's magnetosheath crossed Explorers 33 and 35 more or less simultaneously, with the first on the predawn, and the second on the postdusk, flank. The loci of the two spacecraft with respect to an unaberrated, or nominally aberrated, magnetosheath system did not favor simultaneous shock observations, and the precise order in which apparent shock crossings occurred at the two vehicles would have been impossible unless either the magnetosheath system had been tilted at a large aberration angle, some of the "crossings" at Explorer 33 were not crossings at all, or the noisy boundary at Explorer 33 contained considerably more structure than the magnetometer revealed. A combination of the above factors is suggested, with heavy weight on the first. Regardless of the above, the character of the outer sheath and boundary was markedly different at the two satellites, implying that the magnetosheath had a nonuniform envelope with the character of an abrupt shock locally at Explorer 35 and a wavetrain of large amplitude oscillations locally at Explorer 33. The noisy, wavetrain boundary corresponded, in B-X coordinates, to an interplanetary field locally normal to the shock, and the abrupt, quiet boundary to an interplanetary field locally tangent to the shock. Energy ratio $\beta$ was less than average, probably on the order of 0.1; the Mach number was lower than average, probably between 2 and 7. In the context of theoretical and laboratory studies of collisionless plasma shocks (see, especially, Robson, 1969), of the earlier dual satellite study of Greenstadt et al. (1970), and of the evidence of nonuniform boundaries in the Venus and Mars data (Greenstadt, 1970), the example described here substantiates the nonuniformity of the earth's shock postulated by Greenstadt et al. and supports the source of the nonuniformity in the local relative orientation of shock to interplanetary field.
DISCUSSION

Multiple satellite studies are at last beginning to develop a firm basis for linking macroscopic plasma shock theory to measurements in the extraterrestrial interaction region, which is supposed to be an ideal place for experimental investigation of collisionless plasma phenomena. The present case is, of course, still far from ideal. The nominal shock frame during the episode reported here was not static, but complicated the measurements by its inward, outward, and rocking motions. At the same time, the noisy, or wavetrain, boundary seen on Explorer 33's side did not present nearly as irregular a profile as has been recorded in other observations of the "pulsation shock" (Greenstadt et al., 1970). Shock motion could have affected the profile so that an extensive wavetrain, representing a thick region composed of some combination of upstream and downstream waves, was moved too rapidly past Explorer 33 to leave a clear signature. High bow shock speeds above 50 Km/sec have been reported elsewhere (Greenstadt et al., 1971). It does seem, from the visible evidence, that in this case the boundary was fairly well defined at both satellites despite the obvious structural dissimilarities. It is interesting to note that the weaker and more irregular profile occurred at Explorer 33, closer to the nose, where the shock should presumably have been stronger, and, moreover, that severe tilting of the magnetosheath system would have transformed Explorer 33 even closer to, and Explorer 35 even farther from, the subsolar point, making the contrast between the shock strengths all the more significant from a theoretical viewpoint. In particular, the maximal field enhancement in the "parallel" case was evidently reduced substantially to only around 70 percent of the enhancement in the perpendicular case, even though the former was closer to the nose.
If the combination of shock movement and rocking proposed in this paper to explain the bilateral sequence of shock observations is correct, it appears that in this example most of the shock structure at Explorer 33 was manifested as a wavetrain extending upstream from the major field gradient. Such a profile is expected for collisionless, oblique shocks (Robson, 1969; Friedman et al., 1970). The dawn shock structure was apparently dominated by low frequency dispersion whereas the dusk shock structure was evidently determined by either high enough dissipation or high-frequency wave dispersion (Fredricks et al., 1970), or both, to produce a monotonic transition. The difference in local orientation of $\mathbf{B}$ produced the distinct structures.

It is hoped that additional examples will be uncovered in which a third spacecraft will have been upstream, full observations by plasma probes as well as magnetometers will have been recorded, and a relatively static situation will have prevailed.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Figure 1. Locations of Explorers 33 and 35 during observation interval 0130 UT 29 October to 0430 UT 30 October 1968. Above, projection on SEC Y-Z plane; below, rotation on common X-\(\rho\) plane, where \(\rho = (Y^2 + Z^2)^{1/2}\). Unaberrated shocks of nominal shape are shown.

Figure 2. Magnetic field data in spherical solar ecliptic coordinates during bilateral shock observations by Explorers 33 and 35. Curves represent, from the top, field latitude, field longitude, 82-second high, average, and low magnitudes at Explorer 33 (upper box); 82-second high, average, and low magnitudes, field latitude, and field longitude at Explorer 35 (lower box).

Figure 3. Position of Vela 4B in relation to Explorers and illustrative shock contours.

Figure 4. Above, edge-on views of B-X planes through Explorer 33 and 35 positions; below, configurations of shock contours on B-X planes through Explorer positions, looking down in direction of large arrow of upper figure.
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APPENDIX 2

LARGE SCALE COHERENCE AND HIGH VELOCITIES
OF THE EARTH'S BOW SHOCK
ON 12 FEBRUARY 1969

E. W. Greenstadt
TRW Systems Group
Redondo Beach, California 90278

P. C. Hedgecock
Imperial College of Science and Technology
London S.W. 7, England

C. T. Russell
University of California at Los Angeles
Los Angeles, California 90024

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Space Sciences Department
TRW Systems Group
One Space Park
Redondo Beach, California 90278
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OF THE EARTH'S BOW SHOCK
ON 12 FEBRUARY 1969

ABSTRACT

The earth's bow shock exhibited a clean, laminar profile, at low Mach number, as it crossed and recrossed OGO 5, HEOS 1, and Explorer 33 on 12 February 1969. The approximate 120 $R_e$ distance between HEOS and Explorer during one set of crossings indicated the abrupt character of the laminar shock "front" and the absence of magnetosheath turbulence both in the dayside hemisphere above the ecliptic and in the flank of the shock 75 $R_e$ behind the earth, below the ecliptic. The abruptness of the shock and the coplanarity of the loci of OGO and HEOS with the local shock normal permit the most reliable estimates yet obtained of shock velocities along the normal. These mean velocities ranged from 11 to at least 100 Km/sec over distances of 2-7 earth radii.
INTRODUCTION

The earth's collisionless bow shock, even when appearing as an isolated clean jump both in magnetic field amplitude and in all measurable plasma parameters, is nevertheless a movable entity, located at different places during consecutive passes of the satellite through a region or even appearing at different locations during a single pass in response to changes in the solar wind characteristics. It is natural that the speed with which the shock might move from one position to another should be a subject of investigation. Unfortunately observational limitations in the past have led experimenters to use either single satellite observations of multiple crossings, together with assumptions about regular oscillations of shock position (Holzer et al., 1966; Heppner et al., 1967; Kaufmann, 1967), or multiple satellite observations at different locations with assumptions of shock coherence and symmetry (Greenstadt et al., 1968). Despite their limitations, previous estimates of shock velocity have reinforced each other, all yielding values below 28 Km/sec and averaging around 10 Km/sec.

Changes in the apparent local position of the "bow shock" may be caused either by changes in the solar wind momentum or Mach number altering the scale of the magnetopause/bow shock system as a whole, or by positional instabilities associated with some wave generating mechanism occurring even with constant solar wind input conditions. The latter might take forms ranging from "roving surface corregations" to "regionally coherent radial pulsations" depending on the wavelengths involved. It is change of scale with which we deal here. The estimates we report are based on elapsed time measurements between shock transits past two satellites, one located a few earth radii northwestward of the
other approximately in the plane of local shock normal. This almost ideal geometrical relationship avoids the necessity for assumptions of large scale coherent and rotationally symmetric shock motion. Nevertheless, simultaneous observations with a third satellite located 120 $R_e$ back along the flank of the shock indicate that in one case the extreme outward displacement of the shock could be represented to a first approximation by a simple change in scale size of a hyperbolic shock surface. Four shock velocity estimates result. One is comparable to those previously obtained; the other three are significantly higher.

This report uses the field magnitude data from the magnetometers of HEOS 1 (Imperial College), OGO 5 (UCLA), and Explorer 33 (NASA/Ames Research Center); the data interval is 0000 to 1700, February 12, 1969. The HEOS plasma data for this interval have been displayed and extensively treated in a paper by Formisano et al. (1971), where they are interpreted as observations of the bow shock at low varying Mach number, generally between 1.2 and 3, and at unusually great distances from earth. Several of the multiple encounters with the shock recorded by HEOS on 12 February were also observed by one or both of the other spacecraft listed above. It is these on which we report.

GEOMETRY

The locations and trajectories of the three satellites during the data interval are shown in Figure 1. Coordinates are solar ecliptic rectangular; distances are in earth radii, $R_e$ ($1 R_e = 6380$ Km). At the top are projections of trajectory segments on the Y-Z plane. The dashed radial line in the second quadrant has been inserted to demonstrate the relatively constant ratio $Z/Y$ of
the HEOS-1 and OGO-5 coordinates throughout the interval; i.e., that both satellites were essentially coplanar in the geocentric solar ecliptic cylindrical \( X-p \) system in which \( p = (Y^2 + Z^2)^{1/2} \). Explorer 33 was in the third quadrant, well below the ecliptic (the \( Z = 0 \) plane), approximately 125 \( R_e \) from earth.

The trajectory segments on a common \( X-p \) plane containing the shock normal (in an unaberrated system) are illustrated in the lower half of Figure 1. Nominal hyperbolic shock outlines are shown passing through the points of the Explorer 33 and HEOS crossings for the low Mach numbers which prevailed early on 12 February (solid curve) and, for perspective, through more typical positions where Explorers 33 and 35 and OGO 5 had placed the shock on the 5th, 6th, and 11th, respectively (interrupted curve). We emphasize again that the HEOS and OGO trajectories were almost coplanar on the 12th, requiring negligible rotation around \( X \) to form the illustrated path segments, but that Explorer 33 coordinates had to be rotated some 90° around the \( X \)-axis to appear on the same plane.

The fortuitous positions of HEOS 1 and OGO 5 offered the first opportunity to make dual satellite velocity measurements without appeal to assumptions of east-west shock symmetry or even to symmetry over a moderate rotation angle, as was necessary in obtaining the previous elapsed-time estimates of Greenstadt et al. (1968). If the concept of the shock front moving as a locally coherent surface along its own normal is a valid representation of the shock transient response to changes in solar wind parameters, then a more favorable arrangement for direct measurement of velocities along the normal can hardly be anticipated, at least at present.
DATA

Field magnitude data recorded by the magnetometers of the three spacecraft from 0000 to 1700 UT, 12 February 1969, are shown in Figure 2. The sharp, laminar nature of all field jumps at all three magnetometers as the shock crossed and recrossed the satellites is evident. Examination of the boundary crossings displayed on scales affording a view of the individual measurements shows the jumps to differ little at high time resolution from the way they appear here in the averaged data. Detailed investigation of the shock crossings will be the subject of a separate report.

As the day progressed, the value of interplanetary field $B_{SW}$ steadily diminished, increasing the magnetosonic Mach number. The shock appropriately followed HEOS and OGO earthward, alternately expanding and contracting, and finally remaining inside the position of OGO until the two satellites entered the magnetosheath again hours later. Outbound Explorer 33, already having observed the shock around 0100-0200 at considerably greater distance than it would ordinarily be expected to, continued into the solar wind, and recorded no more shock crossings. Sets of sequential observations of the shock, in which it crossed one vehicle, then another, are numbered 1 to 4 in the figure. Primes indicate the second member of each set of matched shock crossings between OGO and HEOS. These were the sets used for estimating velocities.

COHERENCE

The coherent shock concept is evidently suitable for the data of the 12th. At the time of the shock's first expansion, between crossings 1 and 2 seen by all three spacecraft, Explorer 33 was some $86 R_e$ behind HEOS in X-distance, some
120 $R_e$ behind HEOS in a line running obliquely along the flank of the shock curve, and, as already pointed out, a full quadrant away around the sun-earth line. Comparison of Figures 1 and 2, then, suggests the applicability of the hyperbolic laminar shock profile over the very large distance and angular separation of Explorer from the other two spacecraft. The representation of the shock locally as a unified "front," with an identifiable and measurable normal velocity during the alternating appearances at HEOS and OGO throughout the 12th, seems acceptable in view of the shock's evidently uniform behavior on the much larger scale, including Explorer 33, during the first hours of the day. Moreover, even taking the worst case view that each shock displacement might have occurred as a step discontinuity propagating westward along the shock with the local solar corotation velocity, the propagation time would be no more than 17 percent of the delay between HEOS and OGO in all but the fourth case. The assumption that the local movement was adequately represented by an inward- or outward-moving plane front is therefore a justifiable one.

**SHOCK VELOCITIES**

Figure 3 illustrates the positions of HEOS, OGO, and the shock at the times numbered in Figure 2. The shock segments are parallel to those given in Figure 3b of the paper of Formisano et al. (1971), but have been extended by eye a few radii to cover the positions of OGO 5. The numbers designate paired sets of observations at OGO and HEOS, as in Figure 2.

Velocities were computed by dividing the normal distance between like-numbered shock positions by the difference between primed and unprimed times. Normal distances were approximated by graphic measurement. Two slightly different values were measured for each numbered pair, the difference depending on
whether the normal distance was measured outward from OGO or inward from HEOS. Table 1 is a compilation of times, distances, and computed velocities derived from the paired observations. Positive signs denote motion outward, negative signs denote motion inward. Times are given to the nearest minute except in Case 4 where the HEOS sampling period was a substantial fraction of the total delay. There, the times relate to the two measurements nearest to the observed step in B and are used to derive upper and lower limits on the transit velocity. Normal distances were estimated to the nearest .1 R\textsubscript{e} leading to individual shock speeds correct to ± 5 percent. This is separate from the uncertainty introduced by the unknown propagation times of the governing solar wind conditions between OGO to HEOS discussed below.

The above calculations are formally correct only if the movement of the shock was perfectly uniform from one spacecraft to another, i.e., if altered plasma conditions in the solar wind were felt instantaneously over the region of the shock observed by the two spacecraft. The actual situation would have departed from this ideal if a significant correction were necessary to account for the finite time taken by solar wind features to sweep along the shock. The correction would arise because the shock could already have moved locally past the first satellite before it even began moving toward the second, located downstream. The measured intersatellite time delays would then not represent the true delays in motion of the shock along its own normal.

Time corrections can be calculated by assuming features moving with co-rotation or solar wind velocities over solar azimuthal or radial projections of the distance between the satellites. These "sweep" delays over at most 8 R\textsubscript{e}, at 400 Km/sec, could not have exceeded about 17 percent of the measured delays.
in the first two cases, 4 percent in the third, and 50 percent in the last.
There is reason to believe, however, that the appropriate corrections are much smaller. In the first three cases, the interplanetary field was almost tangent to the shock locally and almost coplanar with the two spacecraft and the local bow shock normal. Solar wind features paralleling the field would then also have paralleled the local shock, affecting it uniformly, and little or no sweep time correction would be necessary. In the fourth case, the interplanetary field was still nearly tangent to the shock locally, but no longer coplanar with the spacecraft and the local normal. However, around the time of the last crossing pair, slight fluctuations in the field were recorded by Explorer 35 that were repeated about six minutes later at HEOS 1. Explorer 35 was on the ecliptic some 10 $R_e$ sunward and 39 $R_e$ westward of HEOS. Pure radial motion of solar wind structure at the measured solar wind velocities above 375 Km/sec could only have accounted for three minutes or less of the 6 min delay from Explorer 35 to HEOS 1, while the west-to-east order of observation seemed to rule out east-to-west corotation. The most reasonable inference consistent with all the data available is that the solar wind feature responsible for the fourth bow shock contraction swept past the magnetosphere from west to east. It would then have contacted the shock near HEOS and OGO almost uniformly necessitating little, certainly much less than 50 percent, correction to the measured time delay and the dependent bow shock velocities of Table 1. Thus, the available data do not support significant corrections for sweep times, and it may be concluded that the time measurements contributing to the velocities in the table are in error by no more than, say, 10 to 20 percent. We believe the velocities are indications of real shock motion along the normal.
The estimated velocities represent motion of the shock as it adjusted itself from one scale to another either in a steady solar wind following a sudden change in solar wind parameters or in a variable solar wind of gradually changing composition. In the latter case, the shock might have expanded or contracted in synchronism with the rate of change of solar wind composition, and the velocity would be a direct measure of the gradient of some plasma parameter passing the earth at solar wind speed. Only detailed examination of plasma measurements could determine whether this circumstance actually took place during the observations.

No attempt has been made to use the Explorer 33 crossings for obtaining additional velocity estimates because the locus of Explorer was not rotationally coplanar with the others and because extrapolation of the shock contours to the extreme position of Explorer would have been too imprecise to provide a reliable "normal distance" over which to assess the shock's movement. The order of shock observations among the spacecraft when Explorer 33 is included was not straightforward and will not be treated here.

DISCUSSION

The data exhibited above yield two important new results.

1. The earth's bow shock, despite having a variable, even elusive character (Greenstadt et al., 1970a; Wolfe and Intriligator, 1970), can appear to be a coherent surface, or "front," over an enormous span. Although evidence of the shock's presence has been detected as far as 75 to 200 \( R_e \) behind the earth (Behannon, 1968; Bavassano et al., 1971), the boundary has not previously been recorded as a single identifiable step in B extending at once from the dayside hemisphere to a distance 40 \( R_e \) further than the moon. The present data establish such a boundary for a situation of low \( \beta \) and low prevailing Mach
number, when clear identification of the very weak shock in the distant flank might have been least expected (H. Howe, private discussion). One factor which may have contributed to the local appearance of the shock as a clear step in field level was the orientation of the interplanetary field vector $B_{SW}$ (Greenstadt et al., 1970b), which was primarily tangent to the shock in a plane containing $B$ and the solar wind velocity at all three spacecraft, for all four cases. An analysis of the geometry will be reported with details of the shock structure in another paper. The virtual lack of turbulence behind this shock, as indicated by the field signature, is consistent with the expected form of a low $\beta$ (< .05), low $M$ (< 3), perpendicular, collisionless, plasma shock.

2. Average apparent bow shock speeds can exceed 50 Km/sec, and may even reach 200 Km/sec. The velocities of Table 1 are, in three out of four cases, higher than any previously reported. The velocity in Case 4 may even have been as much as an order of magnitude higher than any found in earlier estimates. We believe that these velocities are due to gross changes in the scale of the shock due to the changing Mach number observed by Formisano et al (1971). These are mean velocities measured over considerable distances between spacecraft approximately along the local shock normal. Both lower and higher instantaneous speeds probably occurred as the bow shock readjusted its location to meet the changed solar wind conditions.

It is noteworthy that for the first and last of the four February 12 shock observations listed in Table 2 of the Formisano et al paper, which are the same as the first and last cases in Table 1 of this report, the shock velocities found here have the correct directions needed to bring the computed
and measured post-shock fields of Formisano et al. into closer agreement. Formisano et al. had suggested that rapid shock motion could have been responsible for the few discrepancies in their table. The shock speeds needed to reconcile their discrepancies are within the range of our average speeds, so it is reasonable to suppose that the instantaneous velocities required to match their measured to their computed fields might have occurred at the times of observation.

High speeds, if common, can have a profound influence on any deduction of boundary structure and thickness from single satellite measurements where a shock velocity must be assumed in order to interpret the data. The utmost caution is required because velocities of the magnitudes reported here can, through doppler shifting, make details of boundary structure invisible to instruments of limited frequency response. For example, a shock showing structure over a distance of a few Km - i.e., a few electron inertial lengths -- moving at a velocity of 100 Km/sec would appear as if it had a step function field signature when observing with a magnetometer of 3 Hz bandwidth. Distinctions - or similarities - of profile between one shock observation and another could then be entirely illusory.
ACKNOWLEDGMENTS

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<table>
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<th>Number Pair</th>
<th>Time</th>
<th>Δt (sec)</th>
<th>Normal Distance Outward (R_e) at OGO</th>
<th>Normal Distance Outward (R_e) at HEOS</th>
<th>Normal Velocities (Km/sec) at OGO</th>
<th>Normal Velocities (Km/sec) at HEOS</th>
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<tr>
<td>1</td>
<td>0049 0101</td>
<td>720</td>
<td>6.2</td>
<td>7.0</td>
<td>55</td>
<td>62</td>
</tr>
<tr>
<td>2</td>
<td>0215 0203</td>
<td>-720</td>
<td>5.5</td>
<td>6.6</td>
<td>-49</td>
<td>-59</td>
</tr>
<tr>
<td>3</td>
<td>0223 0311</td>
<td>2880</td>
<td>5.0</td>
<td>6.1</td>
<td>11.1</td>
<td>13.5</td>
</tr>
<tr>
<td>4</td>
<td>1628:30 1626:37 to 1627:30</td>
<td>-60 to -117</td>
<td>1.9</td>
<td>2.1</td>
<td>-104 to -202</td>
<td>-115 to -224</td>
</tr>
</tbody>
</table>
Figure 1. Position of HEOS 1, OGO 5, and Explorer 33 during the observation interval. Above, projection on solar ecliptic Y-Z plane, with dashed radial line showing steady proximity of HEOS and OGO to common X-\(\rho\) plane (\(\rho = \sqrt{Y^2+Z^2}\)). Below, trajectories on common X-\(\rho\) plane, with shock, as observed early on 12 February 1969 (solid curve) and more usual, high Mach no. shock, as observed on earlier days (broken curve).
Figure 2. Magnetic field magnitudes measured by Explorer 33 (82-second averages), HEOS 1 (individual sample every 48 seconds), and OGO 5 (1-minute averages). Numbers denote multisatellite observations of shock motion. In paired crossings between OGO and HEOS, used for elapsed time estimates of shock velocities, the later member of each pair is primed. Note the lack of magnetosheath field amplitude fluctuations even in the HEOS-1 individual samples.
Figure 3. Locations of shock segments at times of observed crossings at OGO 5 and HEOS 1. Order of observations as in Figure 2.
APPENDIX 3

A BINARY INDEX
FOR ASSESSING LOCAL BOW SHOCK OBLIQUITY

E. W. Greenstadt
Space Sciences Department

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Short Title: Shock Pulsation Index

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Space Sciences Department
TRW Systems Group
One Space Park
Redondo Beach, California 90278
A BINARY INDEX FOR ASSESSING LOCAL BOW SHOCK OBLIQUITY

ABSTRACT

The earth's collisionless plasma bow shock has, in general, a non-uniform structure whose magnetic profile is simultaneously that of a monotonic or laminar perpendicular shock and of a multigradient, oblique shock, depending on local orientation of the interplanetary field to the nominal shock surface. A "pulsation index" \( I_p \) has been devised from empirical results to provide a simple, convenient means of assessing the probable local character of the shock's structure: \( I_p = 0 \) or \( 1 \) according to whether local field geometry favors perpendicular or oblique structure, respectively, at a chosen point of observation on the nominal shock surface. Computation of the index begins with an average hyperbolic shock, changes its scale appropriately to place the observation point on it, finds the orientation of the interplanetary field \( \mathbf{B}_{SW} \) to it at the observation point (from upstream measurements) in a plane containing \( \mathbf{B}_{SW} \) and the solar ecliptic X axis, and assigns a value 1 or 0 to \( I_p \), depending on whether protons reflected from the shock with velocity \( p \) times the solar wind speed along \( \mathbf{B}_{SW} \) will or will not remain in the solar wind ahead of the shock. Examples to which the index has been applied demonstrate the utility of the index, the consistency of appearance of oblique structure when the index predicts it, and an empirical preference for \( p \approx 1.6 \).
INTRODUCTION

The existence of a distinct class of bow shock structures characterized by large amplitude magnetic pulsations, irregular proton energy spectra, and a minimal thickness of 1 earth radius \( (R_e) \) was established by analysis of data obtained during a fortuitous conjunction of two spacecraft in 1966 (Greenstadt et al., 1970a). An implication was drawn from the data that the bow shock as a whole should normally be nonuniform (Greenstadt et al., 1970c), and it was recently demonstrated that nonuniform structure of the earth's shock does indeed occur and is consistent with what should be expected for a curved shock simultaneously encompassing both perpendicular and oblique geometry with respect to a relatively uniform solar wind field (Greenstadt, 1972), just as in certain laboratory experiments (Robson, 1969).

To be sure, the macroscopic structures of the oblique profiles of the earth's shock so far obtained have not duplicated exactly those found in the laboratory or described by theory (Paul, 1969; Robson, 1969; Rossow, 1967), but recognition of, and emphasis on, the obliquity, nonuniformity, and mutability of the bow shock have set the stage for systematic exploitation of spacecraft measurements to study collisionless shock structure and for a merger of extraterrestrial, laboratory, and theoretical approaches to understanding shock processes. A key factor in systematization of bow shock structural descriptions is the determination whether any given shock observation by satellite has occurred under perpendicular or oblique conditions, a matter that can be settled completely in a given case only by reference to upstream magnetic field and plasma measurements made by a separate spacecraft. Unfortunately, simultaneous upstream measurements usually record
continual change in direction of the solar wind and its field, which means that proper interpretation of shock observations requires sample-by-sample reappraisal of the geometrical situations at the local point of observation in the nominal shock surface. Moreover, estimation of local conditions involves interpretation of the shock's geometry in an appropriate B-V coordinate plane formed by interplanetary field $B_{SW}$ and solar wind velocity $V_{SW}$ (Greenstadt et al., 1970b,c; Greenstadt, 1970, 1972).

The short-term variability of $B_{SW}$ and the importance of examining shock processes in B-V geometry are unhappy complications. Few cases exhibit B-V configurations readily visualized or long constant, and transformation of most subjectively evaluated cases into persuasive, objectively documented examples requires a prodigious effort. If the early, graphic method were to be employed (Greenstadt et al., 1970b, appendix), the level of effort in most examples would be prohibitive, and publication of the bulk of the diagrams produced would be out of the question. Attention has therefore been turned to creation of an automatic, or semiautomatic, means of evaluating the B-V arrangement at any instant, and a "pulsation index" $I_p$ has been devised to measure the likelihood that the shock will be in one form or the other, i.e., abrupt or oscillatory, for a given position on the nominal shock and a given interplanetary field orientation.

In the report which follows, the empirical results on which the definition of $I_p$ is based are briefly summarized, the index is described, its construction detailed, and its successful application to three examples of satellite shock traversals is illustrated.
Local Broadening of the Bow Shock. Construction of index $I_p$ is predicated on the following empirical results: Upstream waves, i.e., deca-second magnetic oscillations in the solar wind ahead of the bow shock, are evidently the result of a reflection phenomenon (probably back-streaming of solar wind protons) which travels along the interplanetary field lines at velocity $U = pV_{SW}$ ($p \approx 1.6$) with respect to a frame in which the earth is at rest (Fairfield, 1969; Greenstadt et al., 1970b). These waves appear to be correlated one-to-one with, and are an intimate part of, a broadening, or thickening, of the shock in which the shock macrostructure is recorded magnetically as a sequence of large amplitude fluctuations, or pulsations (Greenstadt et al., 1970a) rather than a single, abrupt step in ambient field $B$. The waves, and hence the shock pulsations, can only appear outside or at a given point if they, or the reflected protons believed responsible for them, are able to progress into the solar wind a finite distance along interplanetary field $B_{SW}$ before it itself is swept by the solar wind downstream through the shock and into the magnetosheath. Thus the factors governing the waves and broadened shock structure are $p$, $B_{SW}$, and solar wind velocity $V_{SW}$, and evaluation of the conditions of local obliquity may concentrate on the plane in which these factors operate, the $B_{SW}-V_{SW}$ plane.

For purposes of this report, the view has been adopted that plasma streaming effects on shock orientation are of second order so that the solar ecliptic $X$-axis may be substituted for the direction of $V_{SW}$, approximating the $B-V$ plane by the $B-X$ plane. The direction of $V_{SW}$ is then along the $-X$ axis, but reference to the $B-X$ plane always means a plane parallel to the SEC $X$-axis, containing $B_{SW}$ and passing through the point of observation $P$, not necessarily a plane containing the line of symmetry of the
shock. Coordinates in this plane will be designated $X$, $n$, and it will also be referred to as the $X$-$n$ plane, as described further below. The $B$-$X$, or $X$-$n$, plane intersects the shock, forming a contour in the plane similar to the familiar shock outline in the ecliptic or noon-midnight meridian plane. The $X$-$n$ planes do, of course, occasionally intersect the $X$ axis.

A Binary Index of Oblique Shock Broadening. The local orientation of $B_{SW}$ to the $B$-$X$ shock contour at a point of observation $P$ will, according to the process outlined above, determine the structure of the shock to be either a step or a wavetrain. The purpose of the index is to quantify only the distinction, so it is defined simply as 1 or 0, depending on whether wavetrains should be present or absent according to the model. A marginal case, expected to occur physically in rare instances or mathematically as a result of round-off error, is given a value $1/2$ for completeness.

It is important to emphasize at the outset that the nature and behavior of the index $I_p$ as defined is determined by two fundamental considerations:

1. $I_p$ is not an indicator of plasma-theoretical, geometrical obliquity in any general sense; it represents an attempt to codify the conditions for local bow shock obliquity evidenced in satellite measurements, and is defined only in an empirical, space-physical context. Naturally, it is hoped that the relationship of $I_p$ to general geometrical parameters governing shock structure will ultimately emerge through its use.

2. $I_p$ is not a measure, but an index of local shock obliquity; its purpose is to separate monotonic from pulsation conditions by a simple quantification of the empirically-determined, field-orientation effect, and not to provide a continuous gauge of any angle of orientation.
The value assigned to \( I_p \) for a given, local \( B-X \) configuration is determined as follows: An incipient wave will not appear outside of the shock, nor, it is assumed, will shock pulsations appear if the wave is immediately blown behind the shock, hence behind the local tangent to the shock. The tangent to contour \( n = C(X) \) in the \( X-n \) plane is given by

\[
\eta = \frac{dn}{dX} (X - X_M) + n_M,
\]

where \((X_M, n_M)\) are the coordinates of \( P \) in the \( X-n \) frame. We proceed, from \( P \), \( U/V_{SW} = p \) units along \( B_{SW} \) and 1 unit along \(-X\) to test point \( P_T \):

\[
P_T = (X_T, n_T) = (X_M, n_M) \pm p \, B_{SW} - i
\]

\[
= (X_M \pm p \frac{B_X}{B} - 1, n_M \pm p \frac{B_n}{B}),
\]

where \( \hat{B}_{SW} \) denotes the unit vector \( \left( \frac{B_X}{B}, \frac{B_n}{B} \right) \) in \( X-n \) coordinates, \( B \equiv |B_{SW}| \), \( i \equiv (1, 0) \), and the \( \pm \) sign indicates that it is necessary to begin by moving outward along \( B_{SW} \) regardless of the sense of \( B_{SW} \), which may point into the magnetosheath. Figure 1 illustrates these relationships.

If \( P_T \) is behind the line tangent to \( C(X) \) at \( P \), we set \( I_p = 0 \) (perpendicular step shock); if \( P_T \) is sunward of the tangent, we set \( I_p = 1 \) (oblique pulsation shock); if \( P_T \) is on the tangent line, we set \( I_p = 1/2 \) (borderline).

The formal development of the above criteria and the geometry underlying it follow.
CONSTRUCTION OF THE PULSATION INDEX

Pulsation index $I_p$ is derived by the following routine, after selection of a shock observation point of interest and a simultaneous, or appropriately timed, observation of the interplanetary magnetic field $B_{SW}$, preferably well outside the magnetosheath:

1. A nominal shock surface through the shock observation point is found by using the scaling law $R(\alpha) = MR'(\alpha)$ (Formisano et al., 1971), where $R(\alpha)$ is the radial vector, in solar ecliptic coordinates, from the origin to the shock at angle $\alpha$; $\alpha$ is the angle between the SEC $X$-axis and $R$ or $R'$. $M = \frac{R(\alpha)}{R(o)}$ is constant for a given observation; $R(o)$ denotes the radial vector to a symmetric version of Fairfield's average shock, the "reference shock" (Fairfield, 1971), at angle $\alpha$.

2. The appropriate $B$-$X$ plane is found that contains $B_{SW}$, passes through the shock observation point, and parallels the $X$-axis.

3. An inequality is examined that tests whether a wave generation phenomenon which starts at the shock observation point and travels upstream along $B_{SW}$ at velocity $U = p V_{SW}$ ($V_s$ is the solar wind velocity, $p$ is a constant $> 1$), while $B_{SW}$ flows downstream at speed $V_{SW}$, will remain outside the contour formed by the intersection of the nominal shock with the $B$-$X$ plane; i.e., the inequality tests whether vector sum $\pm p \frac{B_{SW}}{|B_{SW}|} + \frac{V_{SW}}{|V_{SW}|}$ leads to a point sunward of the tangent line to the $B$-$X$ countour through the point of observation. The $\pm$ sign is inserted to emphasize that the first vector must point outward from the shock regardless of the sense of $B_{SW}$.
4. A value of $I_p$ is assigned, either 1, 0, or 1/2, depending on whether the above vector sum is upwind from, downwind from, or exactly on, the tangent line, as described in the preceding section. Values 1 and 0 should correspond to pulsation and abrupt shocks, respectively, if the index is designed properly. Value 1/2 is used as an indicator of borderline conditions.

Reference Shock. The reference shock used for scaling to obtain a nominal shock through arbitrary position $P$ is based on the best fit conics of Fairfield (1971). Fairfield's conic sections, which he derived by several methods, were all asymmetric because of the influence of the solar wind aberration angle. Since no aberration is included in the first-order approach described here, a rotationally symmetric version of one of Fairfield's hyperbolas has been chosen. Its equation is

$$\frac{(X-75.25)^2}{60.75^2} - \frac{\bar{\rho}^2}{34.95^2} = 1,$$

where the constants define a hyperbola passing through the points $(0, \pm 25.55, 0)$, $(10, \pm 13.7, 0)$, $(14.5, 0, 0)$, and $\bar{\rho} = (Y + Z)^{1/2}$. The $Y$-values for $X = 0$ and 10 are mean values taken from east (dusk) and west (dawn) sides of the curve of Fairfield (1971, Figure 1). The compromise reference shock is compared with the two branches of Fairfield's shock in Figure 2. Discrepancies become appreciable only at negative $X$ where the empirical shock is least well determined and should have negligible effect on $I_p$ anyway. The reference shock is virtually identical to the experimental curve of Egidi et al. (1970) for positive $X$. 
Scaling. It is assumed that the nominal shock passing through measurement point $R_M$ is given by $R(a) = M R(a)$ where $a$ is the angle from the $X$-axis to $R(a)$, $R(a)$ is the vector from the origin to the average hyperboloid given by Equation (1), and $M$ is a scalar constant. Figure 3 illustrates the scaling geometry in the $X-\rho$ plane that passes through $R_M$. Quantities $\rho_M$, $\rho$, $\bar{\rho}_M$, $\bar{\rho}$, to be used later, are defined in the figure. $R_M = (X_M, Y_M, Z_M)$ is the known point of measurement, so $\cos \alpha_M = X_M/R_M$.

It will be convenient to substitute letters for the numbers in Equation (1), which becomes

$$\frac{(X-X_0)^2}{a^2} - \frac{\rho^2}{b^2} = 1,$$  \hspace{1cm} (1a)

and can be rewritten

$$\rho^2 = \frac{b^2}{a^2} \left[ \frac{(X-X_0)^2}{a^2} - a^2 \right],$$  \hspace{1cm} (1b)

where $a = 60.75$, $b = 34.95$, $X_0 = 75.25$. The point $R_M(\alpha_M)$, abbreviated $R_M$, on the reference shock corresponding to observation point $R_M(\alpha_M)$ at angle $\alpha_M$ is given by

$$R_M = (X_M^2 + \rho_M^2)^{1/2}$$

$$= \left\{ R_M^2 \cos^2 \alpha_M + \frac{b^2}{a^2} \left[ \frac{(X_M-X_0)^2}{a^2} - a^2 \right] \right\}^{1/2}$$

$$= \left\{ R_M^2 \cos^2 \alpha_M + \frac{b^2}{a^2} \left[ \frac{(R_M \cos \alpha_M - X_0)^2}{a^2} - a^2 \right] \right\}^{1/2}$$

(see Figure 2), which yields the relation
The constant $M$ needed for the scaling law is then

$$M \equiv \frac{R(\alpha)}{R(\alpha)} = \frac{R_M(\alpha_M)}{R_M(\alpha_M)} ,$$

with $R_M(\alpha_M)$ given by Equation (2).

The **B-X Plane**. Figure 4 shows two semicircular cross sections of the observed and reference shocks as they would appear on a plane $X = X_M$, looking toward the earth. The figure defines several quantities with reference to the plane through the observation point, paralleling $X$, and containing $BYZ = (0, B_Y, B_Z)$, seen edge-on. This is the B-X plane. $P_M$ is the fixed distance from the $X$-axis to the B-X plane; $\eta$ is the coordinate orthogonal to $X$ in the B-X plane; the observation point on the B-X plane is at $(X_M, \eta_M)$.

The **X-\eta Contour of the Shock**. We seek the expression for the contour $\eta = C(X)$ formed by the observed shock on the B-X plane.

From Figure 2,

$$P_M^2 = \rho_M^2 - \eta_M^2 = \rho_M^2 - (\rho_M \cdot \hat{B}_{YZ})^2 ,$$

where $\hat{B}_{YZ} = B_{YZ}/|B_{YZ}|$, $\rho_M = (0, Y_M, Z_M)$.

For cross sections in planes $X = \text{const. } \neq X_M$, the above relationships and the arrangement of the diagram are preserved, but subscript $M$ is deleted from quantities $X, \eta, \rho$, while $P_M$ remains unchanged, i.e., if we
slide the diagram forward or backward along the X-axis, we obtain for arbi-
trary \( X, \eta \)

\[
\eta^2 = \rho^2 - P_M^2 , \tag{4}
\]
as long as \( X \) is less than its value at the subsolar point of the contour.

From Figure 3,

\[
X/X = \rho/\rho = R/R = M ;
\]
hence, using (1b),

\[
\rho^2 = M^2 \rho^2 = M^2 \frac{b^2}{a^2} \left[ (X-X_0)^2 - a^2 \right] = M^2 \frac{b^2}{a^2} \left[ \left( \frac{X}{M} - X_0 \right)^2 - a^2 \right].
\]

Returning to Equation (4),

\[
\eta^2 = M^2 \frac{b^2}{a^2} \left[ \left( \frac{X}{M} - X_0 \right)^2 - a^2 \right] - P_M^2 \\
\eta^2 = \frac{b^2}{a^2} \left[ (X-MX_0)^2 - M^2 a^2 \right] - P_M^2 . \tag{5}
\]

This is the equation for the B-X cross section of the observed shock through \( R_M \). The tangent to the cross section \( \eta = C(X) \), through \( R_M \), which will be needed below, is

\[
\eta = \frac{dn}{dX} \left| \left( \eta_M \right) \right| (X-X_M) + \eta_M \\
\eta = \frac{b^2}{a^2} \frac{X_M - MX_0}{\eta_M} (X-X_M) + \eta_M . \tag{6}
\]
Representation of \( n = C(X) \). We adopt the convention that \( n_M \geq 0 \), so \( n_M = |v_M \cdot BYZ| \). Thus the positive sense of the \( n \)-axis is determined by \( n_M \), which is equivalent to saying that in drawing the \( B-X \) shock contour, \( X \) is always positive to the left and the observation point \((X_M, n_M)\) is always above the \( X \)-axis. This necessitates care in obtaining the representation of \( B = (B_X, B_Y) \) that gives the correct orientation of \( B \) with respect to the contour. The proper \( B \) results if \( B_\eta = [\text{sign}(\rho_M \cdot BYZ)] |BYZ| \).

**Pulsation Index \( I_p \).** We are now in a position to compute Index \( I_p \). We begin by proceeding along \( B_{SW} \) to the first test point

\[
P_{T1} = P_M + p B_{SW} = (X_M, n_M) + p (B_X, B_\eta)
\]

\[
= (X_M + p B_X, n_M + p B_\eta).
\]

The \( X \)-coordinate of tangent line (6) at \( n = n_M + p B_\eta \) is \( X = X_M + p B_\eta \cdot \left( \frac{a}{b} \right)^2 \frac{n_M}{X_M - MX_0} \). If this value of \( X \) is sunward of the \( X \)-coordinate of \( P_{T1} \), i.e., if

\[
X_M + p B_\eta \cdot \left( \frac{a}{b} \right)^2 \frac{n_M}{X_M - MX_0} < X_M + p B_X,
\]

then \( B_{SW} \) must have pointed inward from the shock, and we must proceed instead to

\[
P_{T2} = P_M - p B_{SW}
\]

\[
= (X_M - p B_X, n_M - p B_\eta)
\]

If, on the other hand,
\[ X_M + \frac{B_n}{B_{SW}} \left( \frac{a}{b} \right)^2 \frac{n_M}{X_{M-MX_0}} > X_M + \frac{B_X}{B_{SW}}, \]

then we subtract unit vector \( \hat{\mathbf{i}} \) from \( P_{T1} \) to reach test point

\[
P_{T1} = P_M + p \frac{\mathbf{B}_{SW}}{B_{SW}} - \hat{\mathbf{i}}
\]

\[
= \left( X_M + p \frac{B_X}{B_{SW}} - 1, n_M + p \frac{B_n}{B_{SW}} \right).
\]

If the \( X \) coordinate of \( P_{T1} \) is sunward of the tangent, i.e., if

\[ X_M + p \frac{B}{B_{SW}} \left( \frac{a}{b} \right)^2 \frac{M}{X_{M-MX_0}} < X_M + p \frac{B_X}{B_{SW}} - 1, \]

then waves can be expected in front of the shock at \( \mathbf{M}_s \), and \( I_p = 1 \). If the last inequality is reversed, no waves are expected and \( I_p = 0 \). In the case of equality, we set \( I_p = 1/2 \).

The definitions of \( I_p \) are the same with respect to point \( P_{T2} \) except for some minus signs:

If \( X_M - p \frac{B_n}{B_{SW}} \left( \frac{a}{b} \right)^2 \frac{n_M}{X_{M-MX_0}} < X_M - p \frac{B_X}{B_{SW}} - 1, I_w = 1; \)

if \( X_M - p \frac{B_n}{B_{SW}} \left( \frac{a}{b} \right)^2 \frac{n_M}{X_{M-MX_0}} > X_M - p \frac{B_X}{B_{SW}} - 1, I_w = 0; \)

equality again defines \( I_p = 1/2 \).

The above expressions can be simplified by cancelling like terms on opposite sides of each inequality. A summary decision matrix for \( I_p \) is as follows:
Determination of $I_p$

<table>
<thead>
<tr>
<th>$B_n \left( \frac{a}{b} \right)^2 \frac{nM}{X_M-MX_0}$</th>
<th>$&gt; B_X$</th>
<th>$&lt; B_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; B_X - \frac{B_{SW}}{p}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$&lt; B_X + \frac{B_{SW}}{p}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$&lt; B_X - \frac{B_{SW}}{p}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$&gt; B_X + \frac{B_{SW}}{p}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$= B_X - \frac{B_{SW}}{p}$</td>
<td>$1/2$</td>
<td></td>
</tr>
<tr>
<td>$= B_X + \frac{B_{SW}}{p}$</td>
<td>$1/2$</td>
<td></td>
</tr>
</tbody>
</table>

Thus, for example, if $B_n \left( \frac{a}{b} \right)^2 \frac{nM}{X_M-MX_0}$ is greater than $B_X$ and less than $B_X - \frac{B_{SW}}{p}$, $I_p = 1$ (third row, first column).

There is one remaining condition, namely, $B_n \left( \frac{a}{b} \right)^2 \frac{nM}{X_M-MX_0} = B_X$, which occurs when $B_{SW}$ coincides with the tangent at $P$. In this case, $I_p = 0$. 
APPLICATION

Three examples of shock observations by spacecraft magnetometers illustrate the successful use of index $I_p$ to characterize the temporary, local orientation of interplanetary field to the nominal bow shock.

A Broad, Oblique Shock Crossing. Figure 5 shows an application of the index to a broad, multigradient shock structure observed by OGO 5 outbound on 18 December 1968. The bottom graph is a plot of the one-minute average of ambient field magnitude $B$ at OGO; just above it is a plot of the rms deviation of $B$ from the average each minute; at the top is a plot of $I_p$ evaluated at the position of OGO, using $B_{SW}$ measured simultaneously by Explorer 33 upstream in the solar wind ahead of the shock and $p = U/V_{SW} = 1.6$, the best value found by Greenstadt et al. (1970b). OGO 5 was at $R_M \sim (9,3,12) R_E$; Explorer 33 was at $R \sim (30,-21,0) R_E$.

In passing from the magnetosheath at 1700 to the solar wind at 1745, the OGO magnetometer (UCLA) observed a noisy, highly variable field with a gradual net decline from some 25$\gamma$ to around 7$\gamma$ and a region of upstream waves outside the high noise boundary. Examination of the OGO data at high resolution showed that at no time did OGO witness a classical jump shock during its approach to the solar wind, although large gradient pulses were recorded. Thus the data of Figure 1 constitute an example of a true, broadened, multigradient shock in so far as a single satellite can establish such an entity.

The index demonstrates that throughout the time of the noisy field observations the orientation of $B_{SW}$ to the nominal shock was oblique according to the criteria by which $I_p$ is computed. In contrast, $B_{SW}$ was largely perpendicular to the local shock normal at other times. Indeed, the figure
suggests that shock pulsations were convected to OGO as soon as the field provided a favorable orientation for their development. Thus obliquity and non-classical structure were well correlated, as the model and theory demand.

Multiple Perpendicular & Oblique Crossings During a Single Pass. A second example shows that this correlation was no chance coincidence. Figure 6 exhibits a series of multiple crossings with a changing shock profile recorded by OGO 5 outbound on 3 December 1968. Although pulsation or wavetrain shocks are common in OGO shock data, the example of the figure is somewhat unusual in offering a clear contrast between the two shock forms in close time sequence during the same pass. The example also provides some fortuitous relationships which serve to corroborate the validity of the wavetrain model as well as the utility of the index. OGO 5 was at $R_M \approx (9, 8, 13.5) R_E$; Explorer 33 at $R \approx (58.5, 30, -42.5) R_E$.

The six quantities plotted in Figure 6 are, from bottom to top:

1. One-minute averages of field magnitude $B$ at OGO 5.
2. Corresponding one-minute rms deviations of field magnitude, hence a representation of rapid change in $B$.
3. The index $I_p$ computed from field components measured in the solar wind by Explorer 33, applied to the position of OGO 5 when in or near the shock, with $p = 1.6$.
4,5,6. Latitude $\lambda_{B_{SW}}$, longitude $\phi_{B_{SW}}$ (SEC), and magnitude $B_{SW}$ of the interplanetary field at Explorer 33, all represented by 82-second averages.

Explorer 33 was located some 50 $R_E$ upstream and eastward from OGO, so a delay could be expected in the appearance of a given $B_{SW}$ at OGO.
Index \( I_p \) was computed only for times when the shock was within a few minutes of crossing OGO, with allowances made for reasonable delay between the satellites. This is the reason for the gaps in the \( I_p \) graph.

There were two pairs of abrupt crossings accompanied by moderate rms deviations \( \Delta B \) at OGO in the intervals 0400-0415 and 0525-0550, followed by a sequence of large amplitude fluctuations with higher \( \Delta B \), from 0612 to 0720. The large amplitude oscillations in \( B \) from 0612 to 0720 were clearly unrelated to any corresponding fluctuation in \( B_{SW} \), which was rather quiet throughout the depicted interval.

The two direction angles of \( B_{SW} \) underwent a sudden change at around 0510 (probably a tangential discontinuity), between the two pairs of abrupt crossings. Thus two entirely different orientations of \( B_{SW} \), both producing \( I_p = 0 \), were evidently unfavorable to breakup of the shock, which exhibited a classical jump profile before and after the direction change. Half an hour after the direction change in \( B_{SW} \), and right between the two last abrupt crossings at OGO, the latitude \( \lambda_{B_{SW}} \) began to drift slowly toward its original value (the value before 0510), while the longitude \( \phi_{B_{SW}} \) remained more or less at its new level. There was a gap in the Explorer data from 0600 to 0614. When the data resumed, \( \lambda_{B_{SW}} \) was where it had been before 0510, giving a third orientation to \( B_{SW} \), and a value of 1 to \( I_p \). Meanwhile, the large pulsations in \( B \) had appeared at OGO. These are taken to represent the wavetrain profile of the shock. Shortly after 0700, \( \lambda_{B_{SW}} \) drifted northward again, \( I_p \) returned to 0, and the pulsations subsided.

Figure 6 sustains three conclusions. First, a special orientation of \( B_{SW} \) produced the wavetrain shock profile at the position of OGO; second,
the special orientation was the same as the one determined in earlier studies, which is what $I_p$ purports to measure; third, index $I_p$ is a valid and useful substitute for the tedious geometrical constructions heretofore employed in testing the local orientation of $B_{SW}$ to the cross section of the scaled, nominal shock in the appropriate B-X plane. Note that the southward turn of $B_{SW}$ between 0600 and 0614 was not important to the shock's structure, since the field also turned southward just before 0400, when the first abrupt crossing occurred.

The example of Figure 6 provides a good case for illustrating the way $I_p$ substitutes for, and improves on, visual examination of B-X configurations or computation of field angles alone. Figure 7 shows seven views of shock cross sections in X-n coordinates for selected times during the data interval of Figure 6. Just below the cross sections, the trace of B at OGO is reproduced with the approximate times at which the X-n diagrams should apply indicated by arrows under the time scale. The diagrams depend on measurements of $B_{SW}$ at Explorer 33, to which the time under each of them refers, so a 10-minute shift has been allowed in assigning the position of the arrows, as a rough estimate of the average delay from Explorer to OGO, which may have been anywhere from 5 to 20 minutes. At the bottom of the figure is a plot of the angle $\theta$ between $B_{SW}$ at Explorer and the local shock normal at OGO, in three dimensions, without regard to B-X geometry. In the absence of the B-X criterion, $\theta$ would be the most likely quantity to be chosen for assessing local shock obliquity. The graph of $\theta$ was supplied by Dr. C. T. Russell of the University of California, Los Angeles. In Figure 7, it has been shifted 10 minutes to the right to approximate the proper correspondence between $\theta$ and shock conditions at OGO.
The first three X-\eta configurations (top, a,b,c) give $I_p = 0$, before the multigradient shock crossing, the next three, d,e,f, give $I_p = 1$, during the pulsation observations, and the last one, g, gives $I_p = 0$, after the pulsations subsided. The clear switches of $B_{SW}$ from perpendicular to oblique orientations between 0556 and 0620 and back to perpendicularity by 0720 are readily apparent in the diagrams. The distinction between conditions at 0408 (a) and 0631 (e), however, is of special interest.

Refering to diagrams a and e, $B_{SW}$ appears to have had an oblique orientation with respect to the shock in both cases, differing only slightly in the one case from the other. The relative positions of the satellite (OGO 5), however, are significantly different, as are the corresponding shock observations. The dashed and wavy lines in the two diagrams define the regions of upstream waves according to the $p = 1.6$ criterion. The nonuniform shock model asserts that the portion of the shock downstream from, and abutting, the upstream wave region should have pulsation structure, the remainder should have a monotonic profile. The spacecraft clearly was outside the pulsation region at 0408, inside it at 0631. $I_p$ made the correct distinction, hardly obvious by visual inspection of $B_{SW}$ alone. Moreover, $I_p$ can be computed for hundreds of measurements in the time necessary to produce a single X-\eta diagram.

The value of $I_p$ is emphasized further on examination of the graph of $\theta$. The $B_{SW}$-shock normal angle was certainly closer, but not equal, to 90° when the perpendicular shocks were observed than when the pulsation structure was recorded, but the distinction of some 30° in $\theta$ between, say 0408 and 0631, would not by itself be very helpful in assessing expected shock conditions.
It is not yet known whether $\theta$ can be disregarded entirely in favor of $B-X$ criteria alone. This distinction deserves additional study. What Figure 7 does demonstrate is the clear utility and easy interpretability of $I_p$ compared to two other ways of obtaining assessments or predictions of local shock conditions.

Possible Response of the Shock to a Transient Change in Local Field Orientation. A third example demonstrates the use of $I_p$ in defining the local plasma conditions at the shock and suggests the sensitivity of $I_p$ to $p$ as presently composed.

Two recent papers by Formisano et al. (1971) and Greenstadt et al. (1972) described a sequence of laminar, low mach number shock crossings seen for many hours on 12 February 1969. It was noted in the second paper that the sequence of shock crossings observed on 12 February may have owed their step-like character to the orientation of the interplanetary magnetic field, which was roughly tangent to the nominal shock during all the depicted crossings. This seemed especially significant because the observations were on the morning side of the shock, where tangency could occur only because the field maintained, for many hours, either a steady longitude perpendicular to the average stream angle or a high inclination to the ecliptic or both. The statistical likelihood of this orientation is small (Ness et al., 1971).

The situation changed a little over an hour after the end of the interval discussed in the second paper, as shown in Figure 8. Although the shock, contracting inward was not seen again by HEOS 1 after 1630, it did re-cross OGO 5 at 1741 and 1752, at about the same time as the interplanetary field made a radical change in direction. The quantities plotted in
Figure 8 are, from bottom to top, B at OGO, and $B_{SW}$, $\phi_{BSW}$, and $\lambda_{BSW}$ at HEOS. The positions of the satellites at 1800 were: OGO 5, $R_M = (18,-2,7)$ $R_e$; HEOS 1, $R = (-74,-82,-64)$ $R_e$.

Detailed examination of the crossing interval shows that the re-orientation of $B_{SW}$ took place close to the second crossing, when the solar wind returned to OGO 5, and that while the magnetosheath entrance at 1741 was abrupt, the exit at 1752 was accompanied by considerable field fluctuation. Figure 9 presents the data of the interval from 1737 to 1757 at high time resolution. The quantities plotted are, from bottom to top: 1) Field magnitude at OGO 5, as recorded at the 1-kilobit sampling rate; 2) three versions of $I_p$ at OGO 5 as derived from $B_{SW}$ at HEOS 1 for $p = 1.4, 1.6, \text{ and } 1.8$; 3) field latitude angle $\lambda_{BSW}$ at HEOS; $B_{SW}$ at HEOS. Latitude $\lambda_{BSW}$, second from the top, shows the timing of the direction change in $B_{SW}$ at HEOS, which was accompanied by a dip in $B_{SW}$. $I_p$, which depends, of course, on $\phi_{BSW}$ as well as on $\lambda_{BSW}$, held a steady value of 0 before the change, switched to 1 near the end of the change, and returned to 0 afterward where it remained beyond the end of the figure. The proper corotation delay was about 150 sec from OGO to HEOS at 1750, so the event responsible for the large, sudden irregularities in field at OGO should have reached HEOS about 2-1/2 minutes later. This would rule out the big, conspicuous jumps in $\lambda_{BSW}$ and the dips in $B_{SW}$, which occurred too soon at HEOS, if the pattern formed by the HEOS data is taken at face value.

It is tempting, but possibly coincidental, to associate the two extreme, narrow minima in B at OGO with the two values of 1 achieved by $I_p$ for $p = 1.6$, which have both the corotation delay and the correct separation,
The estimated delay may not have been precise, and the relatively slow sampling rate of the HEOS-1 magnetometer may have prevented that instrument from recording all the important magnitudes and directions of \( B_{SW} \), however.

What the figure shows is, first, that \( I_p \) is a valuable tool for giving a simple assessment of the local geometric relations between \( B_{SW} \) and the shock, and, second, the remarkable stability and apparent significance of the value \( p = 1.6 \). If not for \( I_p \), the breaks in \( B \) at OGO might have been credited entirely to a change in mach number or \( \beta \) brought about by the dip in \( B_{SW} \); it would have been difficult to prove otherwise. The value \( \frac{U}{V_{SW}} = p = 1.6 \), which is simply adopted from the study of Greenstadt et al. (1970a, b, c) dealing with data from different satellites in a different place and a different year, seems to represent a physical phenomenon characteristic of the pulsation shock. The best candidate for this phenomenon is a stream of reflected protons at energy about 2.5 times the energy of the solar wind protons, as predicted by Sonnerup (1969). The graph of \( I_p \) shows that when \( p = 1.4 \) was used to compute \( I_p \), only a single value of 1 was obtained; 1.6 gave two 1's; 1.8 gave five 1's. Thus, if the interpretation is correct that the structure at OGO was caused by transient obliquity of field orientation to the shock in a \( B-X \) system, the index seems to provide a very sensitive indicator of the geometric factor with \( U \approx 1.6 V_{SW} \). A substantially lower or higher value of \( U/V_{SW} \) results in an under- or overestimate of the geometric factor. Such sensitivity, if substantiated by further test, could be used as a means of measuring the energy of backstreaming protons when direct measurement of them is unavailable. One of the results of Sonnerup's calculations was that the energy of reflected protons should be dependent on position along the curved shock. This might be verifiable
with the use of $I_p$ and test values of $p$ for observations at many locations.

**DISCUSSION**

Demonstration of field-dependent nonuniformity of the bow shock and development of a first-order index opens up a field of investigation for which satellite observations in the collisionless plasma surrounding the earth are eminently suitable. Approximation of the B-V by the B-X plane is a practical step that compensates the compound difficulty often encountered in obtaining not just correlated, upstream satellite measurements during selected intervals of interest, but measurements by more than one instrument. Both fields and plasma data would be necessary to operate in a true B-V system. The straightforward assessment of local obliquity provided by $I_p$ can aid spacecraft investigation of shock structure given only concurrent data on the magnetic field recorded by any spacecraft outside the sheath.

Index $I_p$ should be widely applicable to general studies of planetary shocks in the solar wind, including those of Venus, Mars, and Jupiter. Its greatest utility, however, should be in both statistical and case studies of the structure of earth's bow shock, where reliable evolution of the obliquity factor will allow isolation and examination of the influence of other plasma parameters. Laboratory studies of oblique shocks have not yet been extended to angles less than about 45° between field and shock normal (Robson, 1969), whereas the bow shock is typically closer to the parallel case (0°) locally over much of its "surface," most of the time.

Effort can now be directed toward analyzing the macrostructure of the shock in a systematic way, with special emphasis on parallel, or nearly
parallel geometry. The solar wind provides a preshocked plasma at every field direction and over a wide range of $\beta$, mostly at high mach number, but occasionally at $M_A < 3$ (Formisano et al., 1971). Shock observations below and above the critical mach number, around $M_A = 3$ (Paul, 1969), can be separated and categorized as perpendicular or oblique by use of the index. It should also be possible to isolate cases of various $B_{SW}$ orientations over a range of $\beta$, so that comprehensive sets of $\beta$, $M_A$, $\theta$ combinations can be studied, where $\theta$ denotes the field angle relative to the local shock normal. It should be possible to isolate factors responsible for the various shock "forms" noted, for example, by Heppner et al. (1967) and Fredricks et al. (1970), provided only that a suitable set of correlated spacecraft observations is available. A catalogue of empirical shock profiles, classified according to the vital plasma parameters contributing to them is now a distinct possibility and should be regarded as an important objective to be pursued.

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FIGURE CAPTIONS

Figure 1. Relationships between $I_p$ values and local shock conditions are shown for observation points $P$ at three representative positions on the nominal bow shock. Waves generated by a shock-originated phenomenon, presumably reflected solar wind protons, arrive at test points $P_T$ by traveling $p$ units along field lines $B_{SW}$ during the time the lines are carried downstream 1 unit with the solar wind. The first $P_T$ is inside the upstream wave, pulsation shock region ($I_p = 1$), the second is on the tangent line dividing the upstream-wave region from unaffected solar wind ($I_p = 1/2$), and the third is behind the nominal shock ($I_p = 0$).

Figure 2. The reference shock approximates the asymmetric average shock of Fairfield (1971).

Figure 3. Relationships among quantities defined by observed shock $R(\alpha)$ and reference shock $\overline{R}(\alpha)$ are shown in a cross section through the $X$ axis.

Figure 4. Relationships among quantities defining the $B-X$, or $X-\eta$ plane are seen on plane $X = X_M$.

Figure 5. Application of index $I_p$ demonstrates that a condition of local magnetic obliquity prevailed during a nonclassical shock crossing by OGO 5.

Figure 6. Use of $I_p$ demonstrates changing perpendicular and oblique local field geometry corresponding to changing abrupt and pulsation shock structures observed during a single outbound pass by OGO 5.
Figure 7. Index $I_p$, as computed for the shock crossings of 3 December 68 (Figure 6), represents succinctly the conditions displayed in selected B-X or $X_\eta$ configurations a through g, differentiates correctly between two similar configurations, a and e, and avoids the difficulty of assigning a proper interpretation to quantities of continuous scale like $\theta$, at the bottom.

Figure 8. A sudden rotation of the interplanetary field was seen at HEOS 1 at time of brief double shock crossing by OGO 5 at 1750, 12 February 69.

Figure 9. An apparent correspondence between a brief condition of local obliquity at OGO 5, when $I_p = 1$, and an interval of large field fluctuations in the shock occurred during the rotation of the interplanetary field at around 1750. Slight differences in $I_p$ are shown for $p = 1.4, 1.6, 1.8$. 
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