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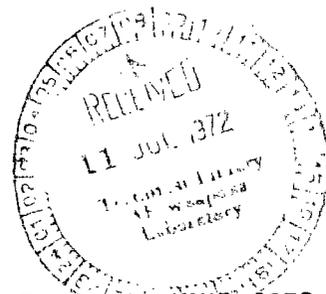


INFLUENCE OF POROUS-WALL THERMAL EFFECTIVENESS ON TURBULENT-BOUNDARY-LAYER HEAT TRANSFER

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SUMMARY

In view of the interest in employing porous wall materials for which the wall thermal effectiveness η' is less than unity, the influence of η' on the convective heat transfer coefficient is examined. A Couette-flow model of the turbulent boundary layer shows that the familiar expression for the Stanton number with mass transfer at the boundary is modified by a correction factor which accounts for the wall thermal effectiveness and the effectiveness of the film layer in protecting the surface from the hot gas stream. The correction term is found to be of considerable importance for low values of blowing rate and η' .

INTRODUCTION

An analytical model of the turbulent boundary layer over a transpired surface which accounts for the effects of wall thermal effectiveness is proposed.

Most theoretical analyses of transpiration cooling are based on the assumption that the temperature of the coolant leaving the porous wall T_{c2} is equal to the outside wall temperature T_{w2} (refs. 1 and 2). Likewise, extensive experimental studies of the transpired boundary layer, some of which are summarized in reference 3, have verified the theoretical predictions of the influence of coolant blowing on wall heat flux under conditions with T_{c2} equal to T_{w2} . However, in transpiration cooling applications employing thin porous walls (e. g. , turbine blade cooling) or for many so-called pseudo-transpiration cooled walls with discrete hole injection, it is known that T_{c2} can be considerably less than T_{w2} . This effect is frequently characterized in terms of the thermal effectiveness of the porous wall defined as

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$$\eta' = \frac{T_{c2} - T_{ci}}{T_{w2} - T_{ci}} \quad (1)$$

(All symbols are defined in the symbol list.) The analysis presented herein examines a simplified Couette-flow model of the blown boundary layer in order to derive an expression showing the influence of the wall effectiveness η' on the convective heat-transfer coefficient. This influence of η' on the heat-transfer coefficient was discussed in reference 4.

ANALYSIS

Experimental studies of the transpired boundary layer (ref. 5) have verified the applicability of the common theoretical expression

$$\frac{St}{St_0} = \frac{\ln(1+B)}{B} \quad (2)$$

where $St/St_0 = h/h_0$ reflects the reduction in the convective heat-transfer coefficient h due to coolant blowing. The blowing parameter B is defined as

$$B = \frac{F}{St} \quad (3)$$

where

$$F = \frac{(\rho V)_c}{(\rho V)_g} = \frac{G_c}{G_g}$$

An alternate form of equation (2) giving (St/St_0) explicitly is

$$\frac{St}{St_0} = \frac{\frac{F}{St_0}}{e^{F/St_0} - 1} \quad (4)$$

Equation (4) or (2) has been derived and experimentally verified under conditions with T_{c2} equal to T_{w2} . Colladay (ref. 4) has suggested that this commonly accepted

expression represents a limiting form as $\eta' \rightarrow 1.0$. A more general expression might be expected to assume the form

$$\frac{St}{St_0} = \left[\frac{\frac{F}{St_0}}{e^{\frac{F}{St_0}} - 1} \right] f(\eta') \quad (5)$$

where $f(\eta') \rightarrow 1.0$ as $\eta' \rightarrow 1.0$. A simple expression for $f(\eta')$ is derived in the next section.

Boundary Layer Model

Consider a transpiration cooled wall with free-stream, wall, and coolant temperatures as illustrated in figure 1. The analysis presented here follows that outlined in

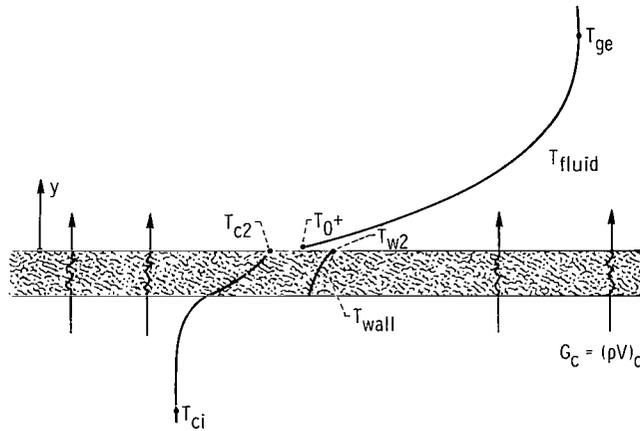


Figure 1. - Porous wall model.

references 1 and 2. The Couette-flow model provides a simplified, but reasonable, analysis for the turbulent boundary layer by neglecting all derivatives in the x direction. With this simplification, the energy and continuity equations can be written as

$$\rho V C_p \frac{dT}{dy} - \frac{d}{dy} \left(k_e \frac{dT}{dy} \right) = 0 \quad (6)$$

$$\frac{d}{dy} (\rho V) = 0 \quad (7)$$

The effective thermal conductivity in the boundary layer k_e includes both molecular and turbulent effects ($k_e = k + k_{\text{turb}}$). These are frequently characterized in terms of molecular and eddy diffusivities, $k_e = (\alpha + \epsilon_H)\rho C_p$. The form of equation (6) also implies that C_p is a constant throughout the boundary layer. The effects of viscous dissipation are approximated by using an effective gas temperature T_{ge} to characterize the free-stream flow. For low velocity flow, $T_{ge} = T_g =$ free-stream total temperature. When dissipation is important, T_{ge} is equal to recovery temperature.

Integration of equation (7) shows that $\rho V = \text{constant} = G_c =$ mass flux of coolant at $y = 0$. Thus, equation (6) becomes

$$G_c C_p \frac{dT}{dy} - \frac{d}{dy} \left(k_e \frac{dT}{dy} \right) = 0 \quad (8)$$

Some caution must be exercised in specifying the boundary conditions for the temperature of the fluid in the boundary layer. At the outer edge of the thermal boundary layer, it is apparent that

$$y = \delta \text{ and } T = T_{ge} \quad (9a)$$

At the wall, there is some tendency to use the boundary condition $T = T_{c,2}$ at $y = 0$. However, this can be done only when the exit coolant temperature and the outer wall temperature are equal (i. e. , $\eta' = 1$). Such a condition exists in the limit when the surface provides a truly uniform coolant injection at $y = 0$, or for a surface with finite pore openings when the blowing rate is decreased and the thickness is increased. When the convection effectiveness is less than one, a transition layer of infinitesimal thickness is required in the one-dimensional Couette model to permit an adjustment of the exit cooling air to a uniform distribution of mass flux G_c and the fluid temperature at $y = 0^+$. Within the transition layer, a temperature (or velocity) profile is undefined, and the wall boundary condition must be specified at $y = 0^+$ where $V_y = G_c/\rho$, $V_x = 0$, and $T = T_{0+}$.

The boundary-layer model is illustrated schematically in an enlarged view in figure 2. The characteristics of the transition layer are primarily influenced by the surface geometry; that is, distribution and size of pore openings.

The presence of this transition layer has not been of importance in many past studies of transpiration cooling because most theoretical treatments have assumed that the boundary-layer temperature approaches T_{c2} continuously as y tends to zero. Practically all experiments have employed porous walls sufficiently thick to approximate this

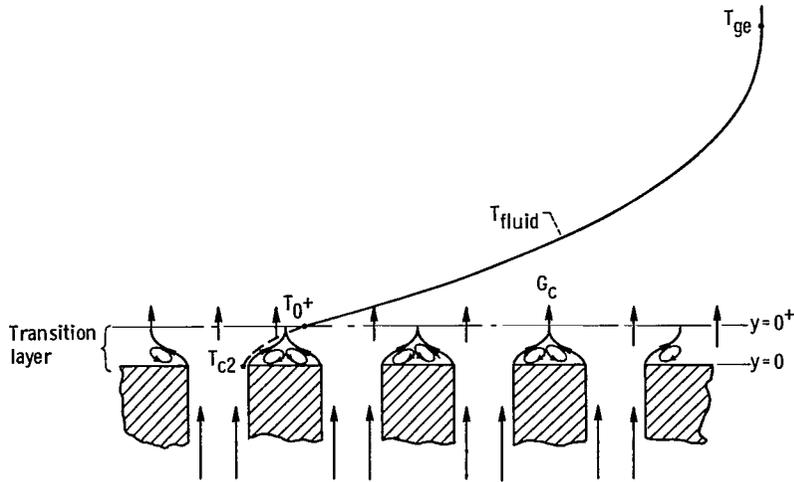


Figure 2. - Boundary-layer model.

condition. However, with the use of very thin porous walls as are required for transpiration cooled turbine blades, the influence of the porous wall thermal effectiveness can no longer be ignored. It has been demonstrated experimentally that the coolant can leave the wall at a temperature well below the average outside wall temperature (i. e. , $T_{c2} < T_{w2}$ or $\eta' < 1.0$).

In establishing the condition T_{0+} it is possible to write an energy balance for the transition layer. A change in temperature from T_{c2} to a uniform T_{0+} results from the mixing of the ejected cooling air (at T_{c2}) with entrained coolant which scrubs the surface at $y = 0$ between the pore openings.

The energy transfer to the coolant is then

$$q_{\text{layer}} = G_c C_p (T_{0+} - T_{c2})$$

It is assumed that this energy transfer will be proportional to the outside wall-to-coolant temperature difference ($T_{w2} - T_{c2}$) and the solid surface area at $y = 0$. Thus, the energy balance for a unit surface area becomes

$$q_{\text{layer}} = G_c C_p (T_{0+} - T_{c2}) = X_m \left(1 - \frac{A_c}{A} \right) (T_{w2} - T_{c2}) \quad (10)$$

where A_c/A is the fraction of frontal area consisting of coolant pores and X_m is a coefficient of proportionality characterizing the transition layer. From equation (10),

using the definition of η' to eliminate T_{c2} ,

$$T_{0+} = T_{ci} + \left[\eta' + \frac{X_1}{G_c C_p} (1 - \eta') \right] (T_{w2} - T_{ci}) \quad (11)$$

where

$$X_1 = X_m \left(1 - \frac{A_c}{A} \right)$$

and η' is given by equation (1).

Equation (11) shows that as $X_1 \rightarrow 1.0$ (i.e., uniform injection at $y = 0$) $T_{0+} \rightarrow T_{c2}$. Likewise, as $\eta' \rightarrow 1.0$ then $T_{0+} \rightarrow T_{c2} \rightarrow T_{w2}$. Thus, the present model is consistent with previous studies. It should be noted that in the study of transpired surfaces where η' is known to be less than unity, the existence of some such transition layer cannot be ignored. Otherwise there is a physical inconsistency in prescribing uniform injection at $y = 0$ (i.e., infinitesimal pores) while considering $\eta' < 1.0$.

The boundary condition for $y \rightarrow 0$ can now be prescribed as

$$y = 0^+ \quad T = T_{0+} \quad (9b)$$

where T_{0+} is given by equation (11). An additional condition required is obtained by writing an overall energy balance for the coolant. Thus

$$G_c C_p (T_{0+} - T_{ci}) = k_e \left. \frac{dT}{dy} \right|_{0^+} = q_{0+} \quad (12)$$

where q_{0+} represents the heat flux transferred from the free stream at $y = 0^+$.

Equation (8) can now be integrated once to give

$$G_c C_p T - \left(k_e \frac{dT}{dy} \right) = C$$

Applying equations (9b) and (12) at $y = 0^+$ shows that

$$C = G_c C_p T_{ci}$$

So

$$G_c C_p T - \left(k_e \frac{dT}{dy} \right) = G_c C_p T_{ci} \quad (13)$$

Equation (13) can now be integrated, applying the limits indicated in equations (9a) and (9b)

$$G_c \int_{0^+}^{\delta} C_p \frac{dy}{k_e} = \int_{T_{0^+}}^{T_{ge}} \frac{dT}{T - T_{ci}}$$

$$G_c \int_{0^+}^{\delta} C_p \frac{dy}{k_e} = \ln \frac{T_{ge} - T_{ci}}{T_{0^+} - T_{ci}} \quad (14)$$

Equation (14) can be transformed to the familiar form by following the method of reference 1. Defining a general mass transfer conductance g and driving force B related by

$$G_c = g \cdot B \quad (15)$$

equation (14) becomes

$$\frac{g}{g_o} = \frac{\ln(1 + B)}{B} \quad (16)$$

where

$$g_o = \lim_{B \rightarrow 0} g$$

Reference 2 provides a physical interpretation of the quantities g and B by noting that the conductance of the boundary layer g can be defined by the expression

$$q_{0^+} = \left(k_e \frac{dT}{dy} \right)_{0^+} \equiv g C_p (T_{ge} - T_{0^+}) \quad (17)$$

The heat flux transferred from the free stream at $y = 0^+$ can also be expressed in terms of a convective heat-transfer coefficient defined as

$$q_{0^+} = \left(k_e \frac{dT}{dy} \right)_{0^+} \equiv h'(T_{ge} - T_{0^+}) \quad (18)$$

It should be noted that T_{0^+} in equation (18) is properly identified as the fluid temperature at $y = 0^+$, since h' is defined to reflect the conductance of the fluid boundary layer. In most applications, the heat-transfer coefficient is conventionally defined in terms of a gas-to-wall temperature difference. However, for the transpiration cooled surface with $\eta' < 1.0$ (i. e. , $T_{w2} \neq T_{c2}$ for the one dimensional model) the appropriate fluid boundary condition T_{0^+} must be employed in the definition of h' .

The heat-transfer coefficient h' and the mass-transfer conductance g as defined by equations (17) and (18) are seen to be related as

$$g = \frac{h'}{C_p} \quad (19)$$

By introducing the definition of Stanton number

$$St' = \frac{h'}{G_g C_p} \quad (20)$$

we see that

$$\frac{g}{G_g} = St' \quad (21)$$

Equations (16) and (21) then combined to give

$$\frac{g}{g_0} = \frac{h'}{h_0} = \frac{St'}{St_0} = \frac{\ln(1+B)}{B} \quad (22)$$

where from equations (15) and (21)

$$B = \frac{F}{St'} \quad (23)$$

In terms of F/St_o , equation (22) becomes

$$\frac{St'}{St_o} = \frac{h'}{h_o} = \frac{\frac{F}{St_o}}{e^{F/St_o} - 1} \quad (24)$$

Although equation (22) or (24) gives a familiar expression for St'/St_o , it should be noted that since h' is defined in terms of the coolant temperature at $y = 0^+$ rather than the customary wall temperature, St' does not permit a direct evaluation of the wall heat flux q_w or the wall surface temperature T_{w2} . For the special case when $\eta' = 1.0$, then $T_{0+} = T_{c2} = T_{w2}$, and equation (22) or (24) is applicable for computing q_w , which is equal to q_{0+} . However, for the more general case when $\eta' < 1.0$, q_w is not equal to q_{0+} because of the energy absorbed in the transition layer. The relation between the heat flux leaving the free stream (q_{0+}) and the net heat flux to the wall (q_w) is developed in the next section.

Relation Between Transition Layer and Wall Heat Fluxes

The net heat flux to the transpired wall ($y < 0$) is given by

$$q_w = q_{0+} - q_{layer} \quad (25)$$

with q_{layer} and q_{0+} given by equations (10) and (18), respectively. In many applications it is convenient to express the wall heat flux in terms of a gas-to-wall temperature difference by the expression

$$q_w = h(T_{ge} - T_{w2}) \quad (26)$$

The relation between the heat-transfer coefficients h and h' is obtained by combining equations (1), (10), (18), (25), and (26) to yield the following expression

$$\frac{h}{h'} = \frac{T_{ge} - T_{0+}}{T_{ge} - T_{w2}} - \frac{X_1(1 - \eta')}{h'} \left(\frac{T_{w2} - T_{ci}}{T_{ge} - T_{w2}} \right) \quad (27)$$

To simplify further, the following expression for the first temperature difference ratio in equation (27) can be obtained by combining equations (12) and (18) and introducing the

Stanton number ratio given by equation (24)

$$\frac{T_{ge} - T_{0+}}{T_{ge} - T_{w2}} = \frac{1}{\phi} \left(\frac{e^{F/St_o} - 1}{e^{F/St_o}} \right) \quad (28)$$

where

$$\phi = \frac{T_{ge} - T_{w2}}{T_{ge} - T_{ci}} \quad (29)$$

Also, the second temperature ratio in equation (27) can be expressed as

$$\frac{T_{w2} - T_{ci}}{T_{ge} - T_{w2}} = \frac{1 - \phi}{\phi} \quad (30)$$

Substituting equations (28) and (30) into equation (27) and using equation (24) to eliminate h' inside the brackets yield

$$\frac{h}{h'} = \frac{e^{F/St_o} - 1}{\phi} \left[\frac{1}{e^{F/St_o}} - \frac{X_1}{G_c C_p} (1 - \eta')(1 - \phi) \right] \quad (31)$$

Equating the heat flux expressions given by equations (12) and (18) and introducing the Stanton number ratio from equation (24) give

$$T_{0+} = T_{ci} + (T_{ge} - T_{ci}) \frac{1}{e^{F/St_o}} \quad (32)$$

Substituting equation (11) into (32) yields,

$$1 - \phi = \frac{T_{w2} - T_{ci}}{T_{ge} - T_{ci}} = \frac{1}{e^{F/St_o} \left[\eta' + \frac{X_1}{G_c C_p} (1 - \eta') \right]} \quad (33)$$

Finally, substituting equation (33) into (31) to eliminate φ gives the desired relation between h and h'

$$\frac{h}{h'} = \frac{\left(e^{F/St_o} - 1\right)\eta'}{e^{F/St_o} \left[\eta' + \frac{X_1}{G_c C_p} (1 - \eta')\right] - 1} \quad (34)$$

To obtain the general expression for St/St_o to be used in conjunction with equation (26), equation (24) is introduced to give

$$\frac{h}{h_o} = \frac{St}{St_o} = \left(\frac{\frac{F}{St_o}}{e^{F/St_o} - 1}\right) f \quad (35)$$

where

$$f = \frac{\left(e^{F/St_o} - 1\right)}{e^{F/St_o} - \frac{1}{\eta'} + e^{F/St_o} \frac{X_1}{G_c C_p} \left(\frac{1}{\eta'} - 1\right)} \quad (36)$$

The term f can be interpreted as a correction factor for wall thermal effectiveness η' . When $\eta' = 1.0$, the correction term f equals 1.0, and (St/St_o) is in agreement with previous analytical and experimental studies (eq. (4)). However, for porous walls having an η' value less than unity, the corrected Stanton number used in predicting q_w is dependent on the thermal effectiveness and the blanketing effect of the film layer $X_1/G_c C_p$.

It is noted from equation (36) that an explicit evaluation of the correction f requires a knowledge of the empirical parameter $(X_1/G_c C_p)$ introduced in the definition of the transition layer. The physical significance of this empirical parameter can be clarified by examining the variation of q_w and T_{w2} with η' . An energy balance for the coolant shows that q_w can be expressed as

$$q_w = G_c C_p (T_{c2} - T_{ci}) = G_c C_p \eta' (T_{w2} - T_{ci}) \quad (37)$$

It is noted from this expression that the condition of an adiabatic transpired surface can only occur when $\eta' = 0$. The corresponding outer wall temperature for an adiabatic

transpired surface T_{aw} can be obtained from equation (33) with $\eta' = 0$:

$$\frac{T_{aw} - T_{ci}}{T_{ge} - T_{ci}} = \frac{1}{e^{\frac{F}{St_o} \left(\frac{X_1}{G_c C_p} \right)}} \quad (38)$$

For the adiabatic condition, $q_w = 0$ implies that $T_{c2} = T_{ci}$. Thus, equation (38) can be written as

$$\frac{T_{ge} - T_{aw}}{T_{ge} - T_{c2}} = 1 - \frac{1}{e^{\frac{F}{St_o} \left(\frac{X_1}{G_c C_p} \right)}} \quad (39)$$

Note that the temperature difference ratio in equation (39) is, by definition, the film effectiveness commonly used to describe localized film cooling.

$$\eta_{film} = \frac{T_{ge} - T_{aw}}{T_{ge} - T_{c2}} \quad (40)$$

Introducing the expression for film effectiveness into equation (39) results in

$$\frac{X_1}{G_c C_p} = \frac{1}{e^{\frac{F}{St_o} (1 - \eta_{film})}} \quad (41)$$

The empirical parameter $(X_1/G_c C_p)$ is related to a film effectiveness for the transpired surface.

The condition of an adiabatic transpired surface ($q_w = 0$, $\eta' = 0$) could be approached by making the porous wall successively thinner without changing the character of the external surface of the wall. Thus, the parameters X_1 and η_{film} , which reflect the blanketing effect of the coolant, can be expected to be independent of η' . Therefore, although equation (41) was established for the condition $\eta' = 0$, it is applicable for all values of η' .

Equation (41) can be employed to reduce equations (33) and (36) to these final forms:

$$\frac{T_{w2} - T_{ci}}{T_{ge} - T_{ci}} = \left[e^{\frac{F}{St_o}} \eta' + \frac{(1 - \eta')}{(1 - \eta_{film})} \right]^{-1} \quad (42)$$

$$q_w = h(T_{ge} - T_{w2}) \quad (26)$$

$$\frac{St}{St_o} = \frac{h}{h_o} = \frac{\frac{F}{St_o}}{\left(e^{\frac{F}{St_o}} - 1 \right)} f \quad (5)$$

$$f = \frac{e^{\frac{F}{St_o}} - 1}{e^{\frac{F}{St_o}} - \frac{1}{\eta'} + \frac{\left(\frac{1}{\eta'} - 1 \right)}{(1 - \eta_{film})}} \quad (43)$$

It is often desirable to express the heat flux to the wall and the surface temperature in terms of the total temperature rise of the cooling air due to convection from the wall; that is, an overall thermal effectiveness η'_T defined as

$$\eta'_T = \frac{T_{0+} - T_{ci}}{T_{w2} - T_{ci}}$$

As the cooling air approaches a porous wall with an upstream supply temperature T_{ci} , passes through the wall, and is injected into the boundary layer at T_{0+} , three distinct heat-transfer regions are defined. The heat picked up by the cooling air approaching the wall is discussed in reference 6. The internal porous wall-to-coolant heat transfer has been studied in detail and is usually lumped with the upstream region to give the thermal effectiveness η' . The heat transferred from the wall to the cooling air in the transition layer, q_{layer} , can now be included to yield η'_T .

Expressing q_{0+} in equation (25) in terms of η'_T

$$q_{0+} = G_c C_p \eta'_T (T_{w2} - T_{ci}) \quad (44)$$

yields the following expression for the overall thermal effectiveness

$$\eta_T' = \eta' + \frac{1 - \eta'}{e^{F/St_0} (1 - \eta_{\text{film}})} \quad (45)$$

Equations (42) and (43), when written in terms of η_T' , become

$$1 - \phi = \frac{T_{w2} - T_{ci}}{T_{ge} - T_{ci}} = \frac{1}{e^{F/St_0} \eta_T'} \quad (42a)$$

$$f = \frac{\eta' (e^{F/St_0} - 1)}{\eta_T' (e^{F/St_0} - \frac{1}{\eta_T'})} \quad (43a)$$

DISCUSSION

Equation (42) shows that the wall temperature T_{w2} for the transpired surface is a function of η' and η_{film} as well as the coolant blowing parameter F/St_0 . To illustrate the variation of T_{w2} with F/St_0 , equation (42) is shown in figure 3 with η' varied parametrically over a range of η_{film} values. Note from the figure that a critical value in F/St_0 of

$$\frac{F}{St_0} = \ln \left(\frac{1}{1 - \eta_{\text{film}}} \right)$$

occurs where T_{w2} is independent of η' . Whether the wall temperature increases or decreases with increasing η' depends on whether F/St_0 is respectively greater than or less than its critical value.

It should be noted that for a given transpiration cooled wall, η' and η_{film} are not independent of F/St_0 so there are combinations of these parameters represented by the curves in figures 3 that correspond to impossible states. For example, as F/St_0 tends to zero, η' and η_{film} must approach 1 and 0, respectively. The dependence of η_{film} on F/St_0 will be included in equation (42) later in the discussion.

When $\eta_{\text{film}} = 1.0$, then $T_{aw} = T_{c2}$, which represents the maximum in coolant blanketing effectiveness. Furthermore, when $\eta_{\text{film}} = 0$ and $\eta' = 0$, the wall temperature reaches its maximum value of $T_{aw} = T_{ge}$.

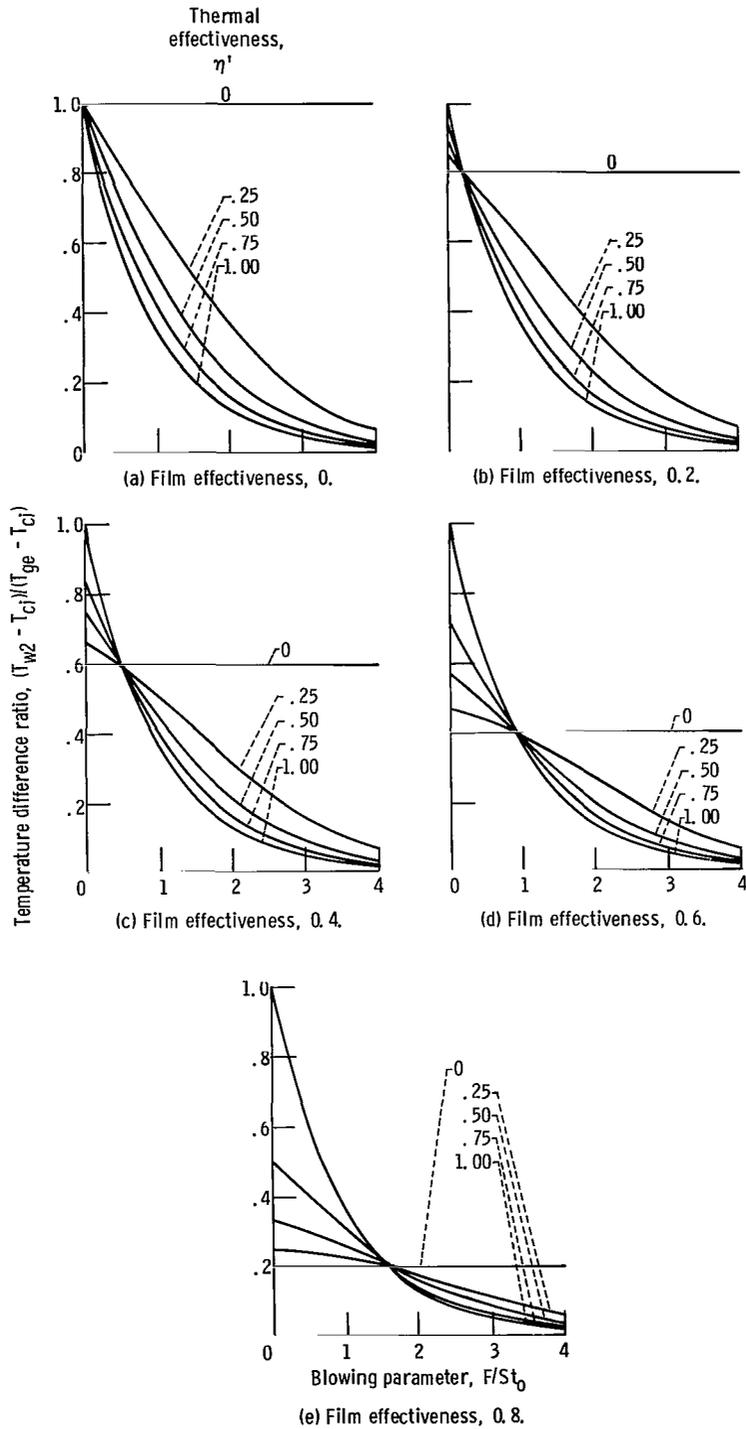


Figure 3. - Wall temperature dependence on thermal effectiveness.

The variation of \overline{St}/St_0 with η' is shown by equations (5) and (43). It can be seen that, when $\eta' = 1.0$, then $f = 1.0$ and

$$\left. \frac{\overline{St}}{St_0} \right|_{\eta'=1} = \frac{\frac{F}{St_0}}{e^{\frac{F}{St_0}} - 1}$$

which is in agreement with previous studies. Likewise, when $\eta' = 0$ (the adiabatic case), then $f = 0$ and

$$\left. \frac{\overline{St}}{St_0} \right|_{\eta'=0} = 0$$

When $\eta_{\text{film}} = 1.0$, then $f = 0$, but this condition can only exist when $F/St_0 \rightarrow \infty$. At the other extreme, when $\eta_{\text{film}} = 0$, then $f = 1.0$. However, this condition implies that $T_{\text{aw}} = T_{\text{ge}}$. Since it is reasonable to expect some blanketing effect of the coolant (i. e. , $T_{\text{aw}} < T_{\text{ge}}$) for any finite rate of coolant flow, η_{film} falls in the range $0 < \eta_{\text{film}} < 1.0$ for realistic surface and coolant blowing conditions. Thus f lies in the range $0 < f < 1.0$ such that \overline{St}/St_0 , as predicted by equation (5), is less than the value commonly expected; that is,

$$\left. \frac{\overline{St}}{St_0} \right|_{\eta', \eta_{\text{film}}} < \frac{\frac{F}{St_0}}{e^{\frac{F}{St_0}} - 1}$$

Figure 4 illustrates the variation of f for arbitrary values of η' and η_{film} other than the limiting cases just discussed.

From the foregoing discussion, it is seen that, for the assumed boundary-layer model, equations (42), (5), and (43) provide new expressions for evaluating $T_{\text{w}2}$ and \overline{St}/St_0 for the general case of transpiration cooling, accounting for the influence of the thermal effectiveness η' as well as the coolant blowing parameter F/St_0 . The results of recent experimental (ref. 7) and analytical (ref. 6) studies provide realistic values of η'_T and η' , respectively, for some porous wall configurations. However, in view of the absence of any experimental evidence regarding typical values of η_{film} for a transpired surface, it is necessary to draw on the background of localized injection film cooling to estimate η_{film} . Numerous gas film cooling experiments have demonstrated the conven-

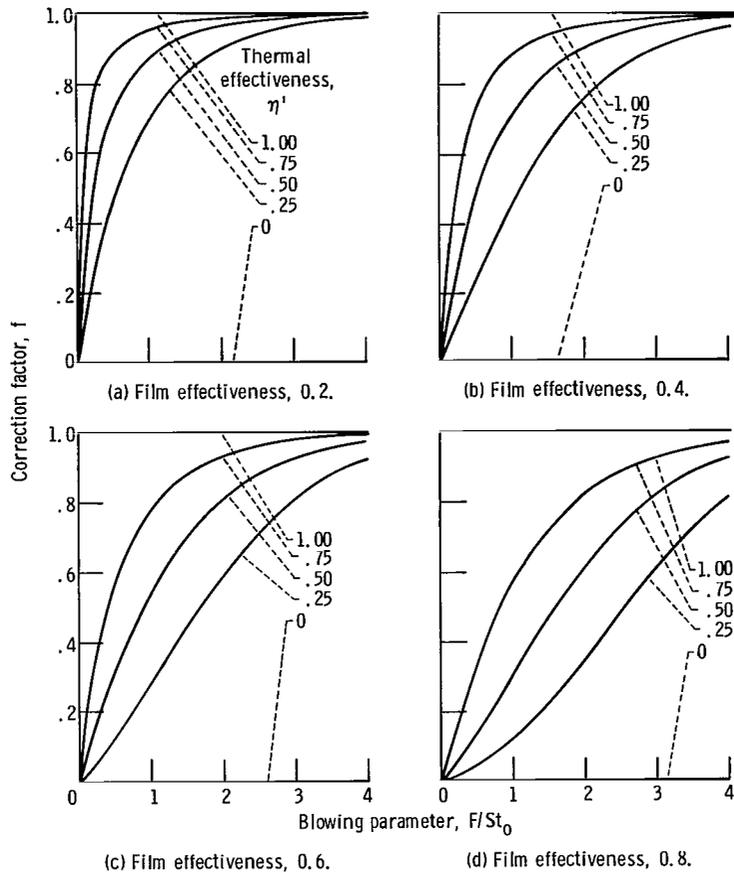


Figure 4. - Correction to $(St/St_0)_{\eta'=1}$ for wall thermal effectiveness (eq. 43).

ience of the film effectiveness parameter η_{film} in characterizing the blanketing effect of the coolant and in evaluating the heat flux to a cooled wall. It has been demonstrated for film cooling with slots or discrete holes that T_{aw} , as characterized by η_{film} , can be employed in calculating the wall heat flux by using the following expression.

$$q_w = h_o(T_{\text{aw}} - T_{w2}) \quad (46)$$

where h_o is the convective coefficient without gas film cooling.

It is assumed herein that this expression is equally valid for the transpiration cooled surface based on the results of reference 8 which show equation (46) to be valid at low blowing rates. The utility of this assumption is illustrated by considering the special case of transpiration cooling for which $\eta' = 1.0$. Under this condition q_w can also be written as

$$(q_w)_{\eta'=1.0} = h'(T_{ge} - T_{w2}) \quad (47)$$

where

$$h' = h_o \frac{\frac{F}{St_o}}{e^{\frac{F}{St_o}} - 1} \quad (24)$$

By combining equations (46), (47), and (24) we obtain

$$T_{aw} = T_{w2} + \frac{\frac{F}{St_o}}{e^{\frac{F}{St_o}} - 1} (T_{ge} - T_{w2}) \quad (48)$$

Substituting equation (48) into the definition of η_{film} , equation (40) gives for the transpiration cooled surface ($T_{w2} = T_{c2}$ for $\eta' = 1.0$)

$$(\eta_{film})_{\eta'=1.0} = 1 - \frac{\frac{F}{St_o}}{e^{\frac{F}{St_o}} - 1} \quad (49)$$

Although equation (49) was established for the special case of $\eta' = 1.0$, it was assumed previously that η_{film} is not a function of η' . On the basis of this assumption and the assumed validity of equation (46) for a transpired surface, the film effectiveness given by equation (49) is applicable for the general case. Thus,

$$\eta_{film} = 1 - \frac{\frac{F}{St_o}}{e^{\frac{F}{St_o}} - 1} \quad (49a)$$

It can be seen that the film effectiveness for the transpired surface (and also the parameter X_1) is a function of F/St_o with $\eta_{film} \rightarrow 0$ as $F/St_o \rightarrow 0$ and $\eta_{film} \rightarrow 1.0$ as $F/St_o \rightarrow \infty$.

By combining equation (49a) with equations (5) and (43) the following expressions are obtained:

$$\frac{St}{St_0} = \frac{\frac{F}{St_0}}{e^{\frac{F}{St_0}} - 1} f_1 \quad (50)$$

$$f_1 = \left[\frac{\left(e^{\frac{F}{St_0}} - \frac{1}{\eta'} \right)}{\left(e^{\frac{F}{St_0}} - 1 \right)} + \frac{\left(\frac{1}{\eta'} - 1 \right)}{\frac{F}{St_0}} \right]^{-1} \quad (51)$$

Thus, on the basis of the assumptions for evaluating η_{film} for the transpired surface, the correction term f_1 is given explicitly as a function of η' and F/St_0 . In view of the conventional expression for St/St_0 at $\eta' = 1.0$, equation (50) shows that f_1 can be interpreted as

$$f_1 = \frac{\left(\frac{St}{St_0} \right)_{\eta'}}{\left(\frac{St}{St_0} \right)_{\eta'=1.0}} \quad (52)$$

The convective coefficient h determined from the foregoing is used in conjunction with equation (26) in evaluating q_w .

The significance of the correction term f_1 is illustrated in figure 5, which presents f_1 as a function of F/St_0 for selected values of η' . As can be seen, f_1 exhibits approximately a linear variation with respect to F/St_0 over the range of blowing parameter of interest. For values of $\eta' < 1.0$, f_1 is less than 1.0, indicating a correction of as much as 30 percent to the conventional expression for $(St/St_0)_{\eta'=1.0}$.

It is interesting to note, as shown by equations (50) and (51) and figure 5, that St/St_0 for the transpired surface decreases as the wall effectiveness η' decreases. Thus, h , and consequently q_w , decrease with decreasing η' .

Recall from the discussion of figure 3 that if

$$\frac{F}{St_0} > \ln \left(\frac{1}{1 - \eta_{\text{film}}} \right)$$

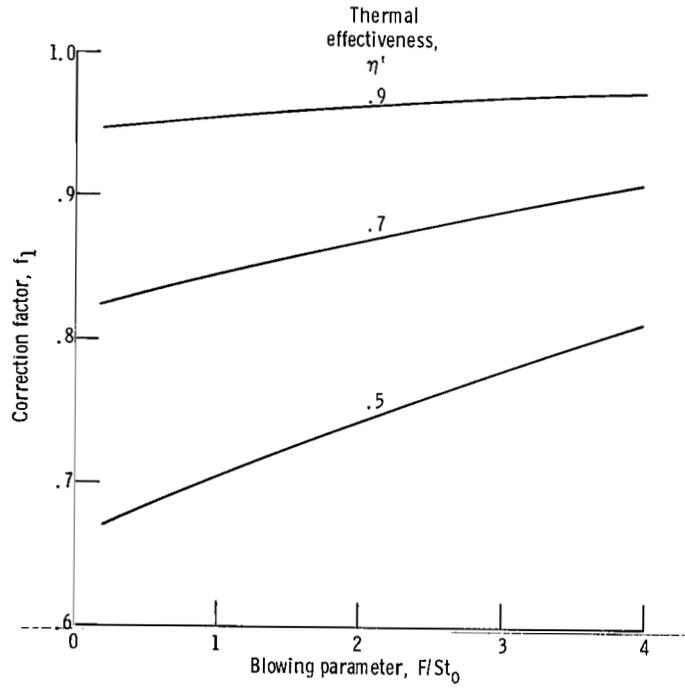


Figure 5. - Correction to $(S/USt_0)_{\eta'=1.0}$ for wall effectiveness.

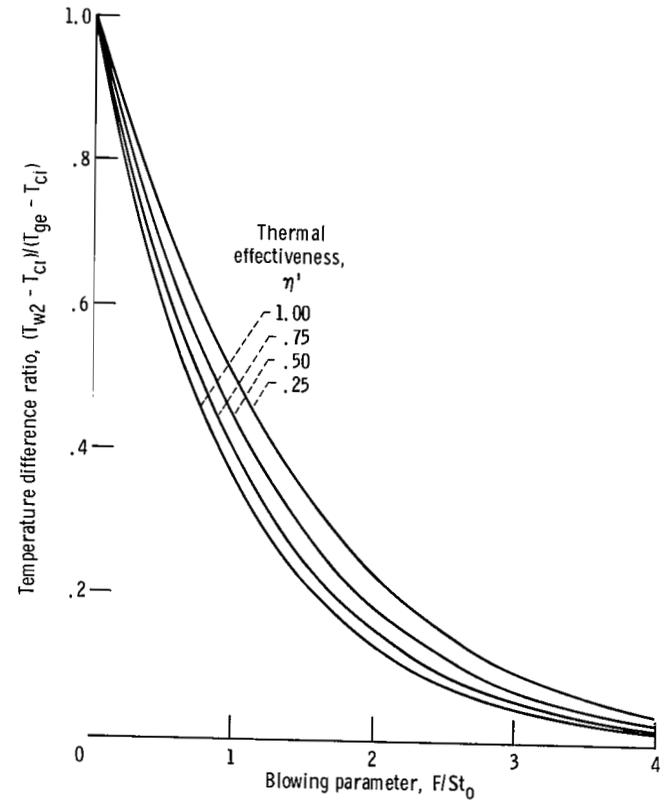


Figure 6. - Wall temperature dependence on thermal effectiveness (eq. (54)).

or, alternatively, if

$$\eta_{\text{film}} < \left(1 - \frac{1}{e^{F/St_o}}\right) \quad (53)$$

then T_{w2} decreases with increasing η' . With η_{film} given by equation (49a) the inequality in equation (53) is satisfied and equation (42) becomes

$$\frac{T_{w2} - T_{ci}}{T_{ge} - T_{ci}} = \left[\eta' \left(e^{F/St_o} - \frac{e^{F/St_o} - 1}{\frac{F}{St_o}} \right) + \frac{e^{F/St_o} - 1}{\frac{F}{St_o}} \right]^{-1} \quad (54)$$

Equation (54) is illustrated graphically in figure 6 for various values of η' . As it may be noted from the figure, the effect of η' (i. e., $\eta' < 1.0$) is to increase the transpired surface temperature T_{w2} when η' decreases from unity. For example, at a value of blowing parameter F/St_o of 1.5, a gas temperature T_{ge} of 2200 K (3500^o F), and a coolant supply temperature T_{ci} of 811 K (1000^o F), the difference in T_{w2} between $\eta' = 1.0$ and $\eta' = 0.5$ is 100 K (180^o F).

Equations (50), (51), and (54), provide the desired expressions applicable for the general case of transpiration cooling. These expressions correspond to the surface averaged results in the transpiration section of reference 4.

CONCLUSIONS

A simplified Couette-flow model of the turbulent boundary layer shows that for the general case of transpiration cooling

$$\frac{St}{St_o} = \frac{\frac{F}{St_o}}{e^{F/St_o} - 1} f_1$$

$$f_1 = \left[\frac{\left(e^{F/St_o} - \frac{1}{\eta'} \right)}{\left(e^{F/St_o} - 1 \right)} + \frac{\left(\frac{1}{\eta'} - 1 \right)}{\frac{F}{St_o}} \right]^{-1}$$

when the convection coefficient is defined in terms of a gas-to-wall temperature difference. The subscript o refers to zero blowing conditions and St is the Stanton number, F is the coolant-to-gas mass flux ratio, and η' is the convection effectiveness. The foregoing expressions demonstrate an influence of porous wall convection effectiveness on St/St_o . The correction term f_1 is found to be of considerable importance for low values of F/St_o .

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National Aeronautics and Space Administration,
Cleveland, Ohio, April 12, 1972,
764-74.

APPENDIX - SYMBOLS

A	wall surface area
A_c	surface area of pore openings
B	general mass transfer driving force, F/St'
C_p	specific heat
F	coolant-to-gas ρV ratio
f	Stanton number ratio correction factor
f_1	Stanton number ratio correction factor incorporating $\eta_{film} = \eta_{film} (F/St_o)$
G	mass velocity, ρV
g	general mass-transfer conductance
g_o	limiting value of g
h	heat-transfer coefficient based on wall temperature
h'	heat-transfer coefficient based on T_{0+}
k	thermal conductivity
0^+	outer edge of transition layer
q	heat flux rate
St	Stanton number, $h/(G_g C_p)$
St	Stanton number, $h'/(G_g C_p)$
T	temperature
V	velocity
X_m	coefficient of proportionality
X_1	coefficient of proportionality, $X_m(1 - A_c/A)$
y	spacial coordinate
α	molecular diffusivity
δ	thermal boundary-layer thickness
ϵ_H	eddy diffusivity
η'	thermal effectiveness, $(T_{c2} - T_{ci})/(T_{w2} - T_{ci})$
η'_T	total thermal effectiveness, $(T_{0+} - T_{ci})/(T_{w2} - T_{ci})$
η_{film}	film effectiveness, $(T_{ge} - T_{aw})/(T_{ge} - T_{c2})$

ρ density
 φ temperature difference ratio, $(T_{ge} - T_{w2}) / (T_{ge} - T_{ci})$

Subscripts:

aw adiabatic wall
c coolant
ci coolant supply
c2 coolant exit from pores
e effective
g gas
ge effective gas
layer transition layer, $0 < y < 0^+$
o no-blowing condition
turb turbulent
w wall
w2 outer wall surface
x spacial coordinate
y spacial coordinate
 0^+ location at $y = 0^+$

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