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RELATIONS AMONG PURE-TONE SOUND STIMULI, NEURAL ACTIVITY, AND THE LOUDNESS SENSATION

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16. Abstract

Both the physiological and psychological responses to pure-tone sound stimuli are used to derive formulas which (1) relate the loudness, loudness level, and sound-pressure level of pure tones; (2) apply continuously over most of the acoustic regime, including the loudness threshold; and (3) contain no undetermined coefficients. Some of the formulas are fundamental for calculating the loudness of any sound. Power-law formulas relating the pure-tone sound stimulus, neural activity, and loudness are derived from published data.

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RELATIONS AMONG PURE-TONE SOUND STIMULI, NEURAL ACTIVITY, AND THE LOUDNESS SENSATION

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SUMMARY

The psychoacoustic law and the fundamental formulas relating loudness, loudness level, and sound-pressure level of a 1-kilohertz-tone stimulus are reviewed in turn. These formulas are extended to include any tone at suprathreshold loudness. Next, the formulas are further generalized to include the loudness threshold. To accomplish this generalization the nature of neural activity in the auditory system is considered.

Published data indicate that, for suprathreshold loudnesses, the amplitude of the summed action potential at a given station along the neural pathway in the auditory system is a power-law function of the sound-pressure amplitude, particularly in the peripheral nervous system. This relation represents a "physioacoustic law" which when combined with the psychoacoustic law yields another power law, a "psychophysiological law," which relates the amplitude of the summed action potential to loudness. Data indicate that the exponents in the psychoacoustic and physioacoustic laws are equal. This leads to the conclusion that loudness is proportional to the amplitude of the summed action potential at suprathreshold loudnesses.

To account for the presence of appreciable neural activity at the loudness threshold, it is assumed that loudness is proportional to the amount by which the whole-nerve, action-potential amplitude at the stimulus frequency exceeds the amplitude at the sensation threshold. This, when combined with the physioacoustic law, yields a generalized psychoacoustic law originally proposed by Lochner and Burger to fit loudness judgment data extending to near threshold. The generalized law indicates that, if the origin of the loudness scale is shifted by an amount proportional to a fractional power of the mean-square sound pressure associated with the loudness threshold, then this shifted loudness is in the same proportion to the same fractional power of the mean-square pressure of the tone. Restarting with this generalized psychoacoustic law results in a new set of relations among loudness, loudness level, and sound-pressure level. These relations apply for any stimulus frequency.

INTRODUCTION

Loudness is defined as the magnitude of the auditory sensation produced by an acoustic stimulus. A quantitative scale relating the sound stimulus to the loudness sensation was established by judging the relative loudnesses of 1-kilohertz tones presented at different stimulus magnitudes (ref. 1). This scaling was extended to other tones by determining stimulus magnitudes at which the other tones and a 1-kilohertz tone are equally loud (ref. 1). Pure tones represent desirable reference stimuli because they are readily reproducible and possess a simple mathematical representation, an advantage in theory and experiment.

Usually the stimulus-sensation relations are exhibited in graphical form. However, in developing a useful theoretical description of the auditory system and loudness prediction procedures it is necessary to express the stimulus-sensation relations in mathematical form. Thus, the purpose of the present report is to provide this mathematical description by reevaluating the basis for previous partial descriptions, by incorporating observed neural phenomena in the mathematical development, and by extending the formulation to cover essentially the entire loudness regime and different source-listener configurations.

REVIEW

For many years it was commonly believed that the relation between a sound stimulus and the loudness sensation was given by Fechner's law. Fechner's law implies that loudness is proportional to the sound-intensity level, where "level" denotes that the magnitude is expressed in decibels. However, Knauss (ref. 2), using loudness data for a 1-kilohertz-tone stimulus obtained by Fletcher and Munson (ref. 1), concluded that, for loudness well above threshold (suprathreshold), loudness is, instead, proportional to a numerical power of the intensity. The amplitude of a sound stimulus is ordinarily measured by using a sound-level meter, which effectively responds to sound pressure rather than intensity. Thus, Stevens (ref. 3) recognized that, according to the measurements, loudness is more properly proportional to a numerical power of the mean-square sound pressure, which, however, is proportional to intensity for plane and spherical sound waves. Consequently, for a 1-kilohertz stimulus at suprathreshold loudness,

$$\mathscr{L}_1 = k_1\left(\overline{p_1^2}\right)^{\alpha} \tag{1}$$

where $\mathscr{L}$ is the loudness; p is the pressure perturbation; $\overline{p^2}$ is the mean-square sound pressure, expressed by

$$\overline{p^2} = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} p^2(t)dt \tag{2}$$

t is time; k and α are constants; and the subscript 1 refers to a 1-kilohertz tone. (All symbols are defined in appendix A.) Tests (refs. 4 to 7) indicate that, for a 1-kilohertz-tone stimulus, $1/4 \leq \alpha \leq 1/2$. Equation (1) is called the "psychoacoustic law."

Because the range of sound pressures, hence of loudness, is very large, it is convenient to adopt logarithmic measures of sound pressure and loudness. Thus, the free-field, sound-pressure level S is defined by

$$S = 10 \log\left(\frac{\overline{p^2}}{\overline{p_0^2}}\right) \tag{3}$$

where all logarithms are to the base 10, and $\left(\overline{p_0^2}\right)^{1/2} = 2\times10^{-5}$ newton-meter^{-2} is a widely accepted reference which is assumed to be the free-field sound pressure at the threshold of binaural hearing for a 1-kilohertz tone imposed on a listener as plane waves from the front. Because it is the sound-pressure level S, rather than the mean-square sound pressure $\overline{p^2}$, which is indicated by a sound-level meter, it is also more convenient to express loudness as a function of sound-pressure level. Thus, for a 1-kilohertz tone stimulus, it follows from equations (1) and (3) that

$$S_1 = \frac{10}{\alpha}\left\{\log \mathscr{L}_1 - \log\left[k_1\left(\overline{p_0^2}\right)^{\alpha}\right]\right\} \tag{4}$$

If the tone is imposed with a free-field, sound-pressure level $S_1 = 40$ decibels, the sound is said to have a loudness of 1 sone. This value of S_1 happens to be the approximate minimum for which the psychoacoustic law (eq. (1)) is valid. Hence, suprathreshold loudnesses are those for which $\mathscr{L}_1 \geq 1$ sone. For example, with $\alpha = 1/3$, as estimated by Stevens (ref. 4), the preceding equation becomes

$$S_1 = 30 \log \mathscr{L}_1 + 40 \tag{4a}$$

where $k_1\left(\overline{p_0^2}\right)^{1/3} = 0.0464$ so that $k_1 = 63$ newton$^{-2/3}$-meter$^{4/3}$. Except for a slight difference in the value of the coefficient of $\log \mathscr{L}_1$ due to the choice of the value of α, equation (4a) agrees with the commonly used formulas given in references 8 and 9 (wherein the coefficient of $\log \mathscr{L}_1$ is 33.3).

A listener can judge loudness ratios, but not loudness differences. Thus, a sound n times as loud as a 1-kilohertz tone at $S_1 = 40$ decibels is said to have a loudness of n sones.

The logarithmic measure of loudness, namely, the loudness level L, is defined in terms of the sound-pressure level S_1 of a 1-kilohertz tone stimulus by

$$L_1 = S_1 \tag{5}$$

Although the units of L must be decibels because S is in decibels, it is common practice to express L in phons, equal to decibels, in order to accentuate the psychological, rather than physical, nature of L. By virtue of equation (5) it follows that suprathreshold loudnesses are those for which $L_1 \geq 40$ phons.

Loudness and loudness level at suprathreshold conditions can be related by combining equations (4) and (5). Thus,

$$L_1 = \frac{10}{\alpha}\left\{\log \mathscr{L}_1 - \log\left[k_1\left(\overline{p_0^2}\right)^{\alpha}\right]\right\} \tag{6}$$

or, for example,

$$L_1 = 30 \log \mathscr{L}_1 + 40 \tag{6a}$$

if $\alpha = 1/3$.

The psychoacoustic law can be applied for stimulus frequencies other than 1 kilohertz and to other source-observer configurations by adjusting the value of the coefficient k. The range of validity of the law is quite limited at low frequencies. For a number of discrete frequencies other than 1-kilohertz, Robinson and Dadson (ref. 10) expressed the loudness level as a quadratic-polynomial function of the sound-pressure level and tabulated the polynomial coefficients. Although the relation between sound-pressure level and loudness level at all frequencies is likely to be of great practical importance in predicting the loudness of arbitrary sounds, the particular formulation chosen by Robinson and Dadson possesses no obvious physical interpretation and differs in form from a formulation which results when physical considerations are taken into account. This alternative formulation will be presented.

For a 1-kilohertz-tone stimulus the psychoacoustic law (eq. (1)) fails for sound-pressure levels less than 40 decibels. Equation (1) implies that $\overline{p_1^2}$ vanishes at the observed loudness threshold, whereas, in fact, $\overline{p_1^2} > 0$ at the loudness threshold (ref. 11).

It also follows from equation (1) that, at the loudness threshold, $S_1 = L_1 = -\infty$, whereas the proper result should be $L_1 = S_1 = 0$ because by definition $\overline{p_1^2} = \overline{p_0^2}$ at the loudness threshold. Whatever nonvanishing, sound-pressure value is associated with the loudness threshold for a 1-kilohertz tone, it is apparent that, by accepting equation (1), the predicted loudness does not vanish at the nonvanishing sound pressure for which the judged loudness vanishes. Knauss (ref. 2) synthesized, for a 1-kilohertz-tone stimulus, a formula intended to extend the psychoacoustic law to the vicinity of the loudness threshold. However, like equation (1), Knauss' formula leads to the erroneous result $\overline{p_1^2} = 0$ at the loudness threshold. Subsequent formulations intended to remedy this fault were reviewed in reference 12. The proposals specifically attributed to psychoacoustics are, for a 1-kilohertz-tone stimulus, specializations of

$$\mathscr{L}_1 \pm a_1 = k_1\left(\overline{p_1^2} \pm b_1\right)^{\alpha} \tag{7}$$

where a_1 and b_1 are constant coordinate translations, and, as before, $1/4 \leq \alpha \leq 1/2$, according to tests. Note that a_1 has the dimensions of loudness, and b_1 has the dimensions of mean-square pressure. Note also that, because p_1 is assumed to be sinusoidal, $\overline{p_1^2}$ may be replaced by the squared amplitude P_1^2. In equation (7) the desired condition $\overline{p_1^2} > 0$ at the loudness threshold is achieved by shifting the origin of either the stimulus coordinate or the sensation coordinate or both. The differences among the various forms of equation (7) for a 1-kilohertz-tone stimulus are usually within experimental error. However, a general understanding of the auditory system includes the necessity to understand threshold phenomena. Moreover, the choice of a particular psychoacoustic equation can affect the simplicity of any consequent mathematical theory of loudness. Therefore, it is desirable to have some rational argument for choosing a particular equation. In the present instance, because of the complexity of the auditory system, the choice can best be made by developing a phenomenological theory for the psychoacoustic law based on a variety of existing observations, both psychoacoustic and physiological. In this way a preferred formulation of the psychoacoustic law will be selected.

In experimental work the mean-square sound pressure at the judged loudness threshold may be denoted by $\overline{p_t^2}$, which may, or may not, equal the value assumed for $\overline{p_0^2}$. Therefore, it is common to define a sensation level $\mathscr{S}$ given by

$$\mathscr{S} = 10 \log\left(\frac{\overline{p^2}}{\overline{p_t^2}}\right)$$

For any given tone,

$$\mathcal{S} = S - C$$

where, by definition,

$$C = 10 \log\left(\frac{\overline{p_t^2}}{\overline{p_0^2}}\right)$$

is a function of the tone frequency because $\overline{p_t^2}$ is a function of the tone frequency. For a 1-kilohertz-tone stimulus C may, or may not, approach zero, whereas for high and low frequencies C is definitely large (up to 65 dB at 20 Hz) because the sound pressure at the loudness threshold is well above that for a 1-kilohertz tone. In this report it will be assumed that $C_1 = 0$, that is, $\overline{p_t^2} = \overline{p_0^2}$ for a 1-kilohertz-tone stimulus.

The loudness calculation procedure for source-observer geometries other than plane waves incident from the front is implicit in the subsequent analysis.

FORMULAS VALID FOR SUPRATHESHOLD LOUDNESSES

Extension to Other Frequencies

Consider a listener exposed to any pure-tone sound stimulus imposed from the front as plane waves. Presumably, loudness judgments made at suprathreshold loudnesses by the listener yield a psychoacoustic law similar to equation (1). For low-frequency stimuli the loudness is observed to fluctuate (ref. 13). Equation (1) does not incorporate this fluctuation. The loudness fluctuation can be introduced in the formulation by noting that the auditory system integrates the intensive attribute of a stimulus for approximately 0.2 second (ref. 14) rather than for infinite time, as implied by equation (2). Thus, the psychoacoustic law may be rewritten as

$$\mathcal{L} = k\left(\widetilde{p^2}\right)^{\alpha} \tag{8}$$

where

$$\widetilde{p^2} = \frac{1}{\tau}\int_{t-\tau}^{t} p^2(t)dt \tag{9}$$

replaces equation (2), and $\tau = 0.2$ second. The loudness computed by using equations (8) and (9) fluctuates appreciably for frequencies of the order of 5 hertz, or less. (It is noteworthy that the loudness fluctuations theoretically vanish at precisely 5 Hz according to eqs. (8) and (9). This failing can be overcome by making the reasonable assumption that the earlier integrated information is weighted decreasingly as the integration proceeds. The details of this process are beyond the scope of this report.)

Equation (8) applies not only for a 1-kilohertz tone but also for any other tone at suprathreshold loudnesses. Test results indicate that the coefficient k is frequency dependent, but, essentially, α is not. The measured sound-pressure level S, from which $\tilde{p}^2$ must be determined, should be obtained by averaging p^2 over the auditory integration time τ rather than over infinite time, which is obviously impossible. Thus, the measured sound-pressure level is more properly defined by

$$S = 10 \log\left(\frac{\tilde{p}^2}{\tilde{p}_0^2}\right) \tag{10}$$

Equations (4) to (6) are more general than previously indicated. For any other tone with the same loudness as the 1-kilohertz tone, these equations become, respectively,

$$S_1 = \frac{10}{\alpha}\left\{\log \mathscr{L} - \log\left[k_1\left(\tilde{p}_0^2\right)^{\alpha}\right]\right\} \tag{11}$$

$$L = S_1 \tag{12}$$

$$L = \frac{10}{\alpha}\left\{\log \mathscr{L} - \log\left[k_1\left(\tilde{p}_0^2\right)^{\alpha}\right]\right\} \tag{13}$$

Equations (11) and (13) are displayed graphically in figure 1 for $\alpha = 1/3$ and $1/2$. Note that the loudness and loudness level of any tone are determined from the sound-pressure level S_1 of the equally loud, 1-kilohertz-tone stimulus, not from the sound-pressure level S of the arbitrary tone.

The psychoacoustic quantities loudness $\mathscr{L}$ and loudness level L have been quantitatively defined in terms of the subjective sensation produced by a 1-kilohertz-tone stimulus imposed on the listener from the front as plane waves at a free-field, sound-pressure level of 40 decibels. The magnitude of the loudness sensation produced by tones at other frequencies can be found by equating their loudnesses to that of a 1-kilohertz tone. These measurements result in equal-loudness curves, as shown, for example, in figure 2. The exact form of the family of equal-loudness curves depends on the mode of listening, namely, monaural or binaural, and through earphones or by direct exposure to the sound source without earphones. Only the case of direct exposure and binaural listening will be considered. The form of the equal-loudness curves also depends on the extent and orientation of the source relative to the listener. The curves shown in figure 2 derived from reference 10 are for direct, binaural listening to plane, progressive waves incident from the front. This form of stimulus presentation is common and easily repeatable; hence, it is an attractive reference configuration.

Transmittance Functions

The equal-loudness curves result as a consequence of the test procedure. They represent an alternative display of transmittance curves, which are more commonly used in the physical sciences. In transforming the equal-loudness curves into transmittance curves it is worthwhile to analyze their content into an external contribution preceding the eardrum and an internal contribution succeeding the eardrum. The external contribution implicitly includes diffraction of the incident waves by the head and propagation of the waves along the external auditory meatus (ear canal). The external contribution is a function of the source-observer geometry, but not of the sound-pressure level. The internal contribution implicitly includes the propagation of the stimulus within the head, successively in mechanical, hydrodynamical, and electrochemical form, until it reaches that undetermined location in the brain at which the loudness sensation originates. The internal contribution is independent of the source-observer geometry but does depend on the sound-pressure level.

The psychoacoustic law relates an input (mean-square sound pressure) and output (loudness) of an open-loop transmission system, that is, a system in which the transmission characteristics are independent of the output. However, the transmission characteristics do adapt to the input and, hence, are variable, as indicated by the fact that the family of curves in figure 2 is not parallel. In order to evaluate the external and internal transmittance functions it is desirable to rewrite the psychoacoustic law in terms of dimensionless coefficients. Thus, let

$$\mathscr{L} = l\left(\frac{\mathscr{T}\widetilde{p^2}}{\widetilde{p_0^2}}\right)^{\alpha} \tag{14}$$

where l is a dimensionless psychoacoustic conversion factor, and, by definition,

$$\mathscr{T} = \frac{\tilde{p}_e^2}{\tilde{p}^2} \tag{15}$$

is the power transmittance of the external auditory system when p_e is defined as the pressure perturbation at the eardrum. The subscript e refers to conditions at the eardrum. For a 1-kilohertz-tone stimulus assume that $\mathscr{T}_1 = 1$, which is approximately correct (compare appendix B). In addition, with $\alpha = 1/3$, for example, it follows that $l_1 = 0.046$ because $\mathscr{L}_1 = 1$ when $S_1 = 40$ decibels. If the ratio of the loudness of any tone is written relative to that of a 1-kilohertz tone by using equation (14), the resulting expression is

$$\frac{\mathscr{L}}{\mathscr{L}_1} = \left(\frac{l}{l_1}\right)\left(\frac{\tilde{p}_e^2}{\tilde{p}_1^2}\right)^{\alpha}$$

where, of course, $\tilde{p}_1^2$ expresses the mean-square pressure of the 1-kilohertz tone at the eardrum, as well as in the free field, because $\mathscr{T}_1 = 1$. Now, define

$$\mathscr{N} = \left(\frac{l}{l_1}\right)^{1/\alpha} = \left(\frac{\tilde{p}_1^2}{\tilde{p}_e^2}\right)\left(\frac{\mathscr{L}}{\mathscr{L}_1}\right)^{1/\alpha} \tag{16}$$

where $\mathscr{N}$ represents the internal power conversion (to loudness) factor for any tone stimulus relative to that for a 1-kilohertz tone.

In other words, for any tone, $\mathscr{N}^{-1}$ measures the efficiency with which the mean-square pressure at the eardrum is converted to loudness relative to the corresponding result for a 1-kilohertz tone. Then, in the new notation

$$\frac{\mathscr{L}}{\mathscr{L}_1} = \left(\frac{\mathscr{N}\mathscr{T}\tilde{p}^2}{\tilde{p}_1^2}\right)^{\alpha} \tag{17}$$

reexpresses the psychoacoustic law in terms of dimensionless power transmittance and power conversion coefficients. From the practical standpoint it is more convenient to deal with the equivalent of equation (17) expressed as decibels. Thus, define the external, power-transmittance level T according to

$$T = 10 \log \mathscr{T} \tag{18}$$

and the internal, power-conversion level N according to

$$N = 10 \log \mathscr{N} \tag{19}$$

Then it follows from equations (17), (10), (13), (18), and (19) that

$$L - L_1 = N + T + S - S_1 \tag{20}$$

expresses the difference between the loudness level of any given tone imposed at a free-field, sound-pressure level S and that of a 1-kilohertz tone imposed at a sound-pressure level S_1. Finally, because of equation (5), the loudness level of any tone is given by

$$L = N + T + S \tag{21}$$

For a 1-kilohertz tone, $N_1 = T_1 = 0$, so that equation (21) then reduces to equation (5), as required.

Specializations of Loudness-Level Equation

Certain other specializations of equation (20) are of interest. The functions N and T are characteristics of the auditory system. Assume that their functional dependences on sound-pressure level and stimulus frequency are known. The equal-loudness ($L = L_1$) curves are given by

$$S = S_1 - N - T \tag{22}$$

Suppose two tones (as always, with one tone being a 1-kHz tone) are successively imposed on the listener at the same free-field, sound-pressure level, $S = S_1$. Then the loudness level of the tone of arbitrary frequency is obtained from equation (21). Suppose the same two tones are successively imposed at the same sound-pressure level at the eardrum, that is, $T + S = S_1 = L_1$. Then, equation (20) becomes

$$N = L - L_1 = \left(\frac{10}{\alpha}\right) \log \left(\frac{\mathscr{L}}{\mathscr{L}_1}\right) \tag{23}$$

which provides a means for evaluating N, as will be shown. The last form of equation (23) results by applying equation (13).

Loudness

By combining equations (23) and (4) and the condition $S_1 = S + T$, it can be shown that, as a function of S, the loudness of any tone is given by

$$\mathscr{L} = k_1 \left(\tilde{p}_0^2\right)^{\alpha} \text{antilog} \left[\left(\frac{\alpha}{10}\right)(N + T + S)\right] \tag{24}$$

at suprathreshold loudnesses. Recall, for example, that $k_1 \left(\tilde{p}_0^2\right)^{\alpha} = 0.0464$ if $\alpha = 1/3$.

Evaluation of Transmittance Functions

In order to apply any of the preceding equations, the functions T and N must be evaluated.

The external transmittance level T has been measured as a function of frequency (ref. 15) and has also been estimated indirectly in a manner described in appendix B. Both results are shown in figure 3 for plane waves imposed from the front. A weighted average of these results is presented in figure 4 and in table I. The external effects, particularly propagation of the sound down the ear canal, are seen to amplify the sound. For a different source-listener geometry the function T is, of course, different. Another example of the function T is presented in figure 4 and table I for a diffuse source. These results were obtained by combining data from reference 16 with those for plane waves in figure 4 (or table I).

The internal conversion level N can be evaluated from the equal-loudness curves in figure 2 if the function T is known. The equal-loudness-level curves are represented by equation (20) with $L = L_1 = \epsilon$, where ϵ is a different constant value for each curve. Adding the function T to the equal-loudness curves results in a new set of equal-loudness curves wherein the ordinate is now $S + T = S_e$. These new curves, the light dashed curves in figure 2, are represented by $L = N + S_e = \epsilon$. If the ordinate and parameter are interchanged, that is, if the parameter L is made the ordinate and S_e becomes the parameter, unnormalized, conversion-level curves are the consequence. These curves, shown in figure 5, are given by $S_e = \nu$, where ν is another constant whose value differs for each curve. Then, equation (21) becomes $N = L - \nu$. With L the ordinate, these curves are unnormalized. For a 1-kilohertz tone, $N_1 = L_1 - \nu = 0$. Therefore, $N = L - L_1$ (compare eq. (23)). By equating the sound-pressure level of any tone at the eardrum with that of a 1-kilohertz tone, it follows that $S_e = S_1 = L_1$. When presented with N as the ordinate, these curves coincide at 1 kilohertz and are normalized relative to 1 kilohertz, as shown in figure 6.

A mathematical formulation for T has not been derived, although partial analyses are already available (ref. 17). Even for computer computations it is sufficient to tabulate T for any given source-observer configuration, as in table I, because for a given source-observer geometry T is only a function of frequency. Hence, the amount of data to be tabulated is relatively small.

A graphical representation of the function N, which is a function of sound pressure as well as frequency, would be sufficient if its only use were to indicate the loudness of a pure tone and if the number of loudness values to be determined on any one occasion were small. Otherwise, the graphical procedure is inadequate. If a large number of loudness values are desired, it would be much more convenient to have an equation for N which could be evaluated by using a programmed computer. Most importantly, it has been found that the function N not only is useful for determining the loudness of pure tones, but also is fundamental in calculating the loudness of broad-band noise (ref. 18). In the latter procedure large numbers of values of N corresponding to various frequencies and sound-pressure levels S_e are required. Hence, a mathematical formulation for N is almost mandatory.

Because of the requirements just stated, a formula has been devised which continuously fits the power-conversion level function N quite well over most of the audible range and does not involve unphysical constants as does the Robinson-Dadson polynomial representation (ref. 10). Thus, let $\omega_l/2\pi$ and $\omega_u/2\pi$ designate, respectively, lower and upper cutoff frequencies and $\omega_m/2\pi$ the frequency of the maximum of the function N. Then N is given approximately by the formula

$$N = 60 \log\left\{\left(\frac{\omega}{\omega_m}\right)^2 \left[1 + \left(\frac{\omega_m}{\omega_l}\right)^2\right]\left[1 + \left(\frac{\omega_m}{\omega_u}\right)^2\right]\left[1 + \left(\frac{\omega}{\omega_l}\right)^2\right]^{-1} \left[1 + \left(\frac{\omega}{\omega_u}\right)^2\right]^{-1}\right\} \tag{25}$$

where

$$\omega_l/2\pi = \text{antilog}\ (-0.005706\ S_e + 2.1761)\ \text{Hz}$$

$$\left.\begin{aligned} \omega_u/2\pi &= 18\ 000\ \text{Hz} \\ \omega_m/2\pi &= 1000\ \text{Hz} \end{aligned}\right\}\ \text{independent of } S_e$$

Equation (25) has important theoretical implications in that it is the simplest formula found which would fit the data, would represent (aside from the logarithm) a low-pass and high-pass filter in series (as expected of the auditory system), and can be easily manipulated in a general theory of loudness (ref. 19). (Eq. (25) is a modification of the formula for listening with earphones given in ref. 19. Eq. (25) is, of course, for direct listening and frontal incidence.) Most importantly, equation (25) allows the loudness level and loudness for any given source-listener configuration to be predicted solely from knowledge of the external transmittance level T, the free-field sound-pressure level S, and the frequency of the stimulus.

Although the loudness level of any tone can be read directly from the equal-loudness curves presented in figure 2, it is important to recall that these curves apply only for plane waves incident from the front. The importance of the preceding analysis is that

(1) It separates the major effects (internal and external) of the auditory system.

(2) Only the external transmittance T need be modified to account for various source-listener geometries.

(3) The separate contributions are more amenable to theoretical analysis.

(4) The pure-tone formulas and the mathematical (rather than graphical) specification of the transmittance allow a mathematical formulation for predicting the loudness of any noise (ref. 18).

GENERALIZED FORMULAS FOR LOUDNESS

Since the psychoacoustic law (eq. (8)) is valid only for suprathreshold loudnesses ($L \geq 1$ sone), the derivation of a more general law whose validity extends to the loudness threshold becomes of interest. The best alternative formula contained in equation (7) cannot be deduced from psychoacoustic tests because the alternatives presently yield results within the experimental errors of the tests. However, the psychoacoustic law, which relates sensation to stimulus, implicitly involves physiological phenomena occurring in the auditory system between the ear and the brain. Measurements of these phenomena may be used to choose that alternative psychoacoustic law which is more compatible with the physiological data as well as the psychoacoustic data.

Electrophysiological Considerations

In consecutive order a sound is represented mechanically (middle and inner ear), electrochemically (nervous system), and psychoacoustically (brain) in the auditory system. The mechanical system consists, effectively, of a linear transducer and transmission system which filters and transports a mathematically continuous representation of the sound stimulus to the peripheral nervous system. (Note that mechanical nonlinearities have a negligible effect on overall loudness, except possibly near the threshold of pain.) The nervous system consists fundamentally of a nonlinear transducer and transmission system which carries information supplied by the mechanical system along a maze of neuronal pathways to the auditory cortex. Along each of these afferent pathways the information is coded as discontinuous signals observed essentially as impulses of electric potential of uniform amplitude, the "action" potential, moving at speeds less than 100 meter-second^{-1}. In the peripheral nervous system, measured neural activity (ref. 20) suggests that the history of the instantaneous sum of impulses, the "summed action potential," passing equivalent stations along the manifold of pathways at successive instants may determine a filtered, half-wave rectification of the waveform of the sound stimulus.

Suppose that the impulses of electric potential over all neural pathways passing any given station in the peripheral nervous system are periodically sampled and summed. (The sampling time intervals should be no greater than the reciprocal of twice the upper response frequency of the auditory system so as to detect all expected frequencies in the signal (ref. 21).) The resulting history of the summed potential magnitude at the given station is represented by the envelope of the sums. Thus, the discrete data determine a continuous signal which is probably best represented mathematically by applying the "sampling theorem" (refs. 21 and 22) in preference to other empirical formulas which might be used. The sampling theorem is preferred because it yields a formula involving frequency resolution of the envelope.

Although it has been demonstrated experimentally from period histograms that the summed action potential in individual pathways yields a half-wave rectified reproduction of a periodic stimulus (ref. 20), it is practically impossible to demonstrate that the summed action potential over all fibers will similarly reproduce a stimulus waveform, simply because simultaneous measurements of electrical activity in a large number of pathways (about 30 000 pathways in the auditory nerve) are not feasible. The half-wave-rectified reproduction of a stimulus by the summed action potential over all pathways must be assumed. However, the desired measurement can be approximated by using a gross electrode to record electrical activity in the whole nerve, that is, to record non-uniformly weighted electrical activity from all pathways at once.

Relation Between Electric Potential and Loudness: Generalized Psychoacoustic Law

The consecutive forms (mechanical, neural, and psychoacoustic) of the imposed acoustic signal are schematically represented by

$$p \rightarrow \varphi \rightarrow \mathscr{L}$$

where φ is the mathematically continuous electric potential fluctuation (summed action potential), and the arrows imply a transformation of form. In the present context the pressure fluctuation p is sufficient to describe the signal in the mechanical system, as well as the sound stimulus, itself, because the signal is effectively undistorted. The psychoacoustic law and its generalizations relate p^2 and $\mathscr{L}$. The effect of the intermediate, electric-potential fluctuation φ has apparently not been considered previously.

However, the experimental relation between φ and p should be helpful in choosing the most plausible generalization of the psychoacoustic law because φ is a necessary intermediary between p and $\mathscr{L}$.

If there were no externally imposed stimulus, it might be expected that nothing could be heard. However, in the absence of an externally imposed stimulus, impulses are spontaneously generated in the peripheral neurons by somatic activity. The spontaneously generated impulses are randomly distributed in time and, hence, may be expected to define a broad spectrum of electrical noise. It is believed that this noise influences the loudness threshold. The spontaneous activity may be represented by

$$\varphi_s \rightarrow \mathscr{L}_s$$

where, possibly, $\mathscr{L}_s = 0$.

Now, suppose a minimal detectible, external, pure-tone stimulus is introduced. Most neuronal pathways continue to display only a spontaneous response or no response at all. However, along certain pathways a modified response is observed (refs. 23 to 25). At least for stimulus frequencies less than 5 kilohertz, the temporal distribution of impulses tends to become redistributed from randomness to approximate synchronization with a given phase of the stimulus waveform. At the lowest magnitudes of response the average rate at which impulses pass a given station along each of these afferent pathways is unchanged. At higher stimulus magnitudes the average passage rate of impulses increases. The synchronization, or phase locking, with the stimulus remains. Moreover, new pathways now exhibit response to the external stimulus.

Assume, as before, that the amplitude of the summed action potential recorded from the whole nerve at a given station along the peripheral nervous system is a function of the amplitude of the stimulus and is an indicator of the loudness (refs. 14 and 26). Then, it follows from this assumption and the preceding discussion that the modification of neural activity along a solitary pathway cannot independently account for changes of loudness because the temporal redistribution of impulses corresponds to frequency modulation, not amplitude modulation. However, as the stimulus magnitude is increased, and, hence, the number of pathways and activity along each pathway increase, amplitude modulation of the whole-nerve signal results by virtue of the phase locking and summation of simultaneous impulse amplitudes at equivalent points along different pathways. It is important to recall that this increase in signal amplitude is not superposed upon the original spontaneous noise amplitude, which exists in the absence of signal, because the signal is determined in part by the transposition of this noise into signal. Therefore, as the signal amplitude increases, the overall, absolute, spontaneous-noise amplitude decreases.

At some signal amplitude the pure-tone stimulus can be detected subjectively. This threshold of loudness corresponds to consecutive forms

$$p_t \rightarrow \varphi_t \rightarrow \mathscr{L}_t$$

where φ_t is the summed action potential at the stimulus frequency, and $\mathscr{L}_t \rightarrow 0$ is the loudness of the stimulus.

When a gross electrode is positioned to produce maximum response to a given pure-tone stimulus the amplitudes of the sound pressure and summed action potential have been found to obey a power law - as in the psychoacoustic law - down to the threshold of detection of the potential fluctuations (refs. 26 and 27). These data of Derbyshire and Davis (ref. 26) and Boudreau (ref. 27) are reproduced in figures 7 and 8, respectively. If the potential as a function of the sound pressure were determined by Fechner's law, then a graph of the potential as a function of sound-pressure level would be a straight line. On the other hand, if the potential as a function of the sound pressure were determined by a power law, then a graph of the logarithm of the potential as a function of sound-pressure level would be a straight line. It is immediately evident by comparing the data plotted fully logarithmically in figure 7 with the same data plotted semilogarithmically in reference 26 - which data yield an S-shaped curve on a semilog basis - that the potential is more nearly a power-law function of the sound pressure. Specifically,

$$\Phi = \lambda P^{2\beta} \tag{26}$$

where P is the amplitude of p, Φ is the amplitude of φ at the stimulus frequency, and λ and β are constants. This might be called the "physioacoustic law" since it relates the sound stimulus to a physiological quantity, the action potential.

The measurements indicate that equation (26) applies over only part (about one-half) of the range of sound-pressure levels constituting the normal hearing range. To explain this, consider that the gross electrode can only come in close proximity to a limited number of nerve fibers. The electrode was positioned to record maximum response for a relatively high stimulus magnitude. Hence, it must have contacted a maximum fraction of those pathways which would transport the stimulus at lower stimulus magnitudes. At higher stimulus magnitudes new pathways would be excited which would not contact the electrode. The signal strength from these pathways would be attenuated at the electrode location by its distance from the source and by conductivity of the medium. This, and especially the tapering off of neural activity along the more sensitive pathways at high stimulus magnitudes, probably accounts for the observed reduction in slope of the curves of summed potential as a function of sound pressure at the highest stimulus magnitudes. At the lowest stimulus magnitudes it is again possible that most of the active pathways are remote from the electrode. This would tend to increase the slope of the potential-sound-pressure curve, an effect not observed, however. Equation (26) is obeyed down to the threshold of detection of the summed potential due to the stimulus.

Equation (8), the psychoacoustic law, which applies only at suprathreshold levels, can be rewritten in a form corresponding to equation (26), that is,

$$\mathscr{L} = \kappa P^{2\alpha} \tag{27}$$

where κ is a constant. When combined, equations (26) and (27) yield

$$\mathscr{L} = \mu\Phi^{\delta} \tag{28}$$

where

$$\mu = \kappa\left(\frac{1}{\lambda}\right)^{\delta}$$

$$\delta = \frac{\alpha}{\beta}$$

Equation (28) might be called the "psychophysiological law" for acoustic stimuli, because it relates the psychoacoustic sensation, loudness, to a physiological quantity, the action potential. The psychoacoustic law, the physioacoustic law, and the psychophysiological law are all power laws.

Stevens (ref. 4) concluded that psychoacoustic tests imply that $\alpha = 1/3$. The gross electrophysiological measurements (fig. 7) by Derbyshire and Davis (ref. 26) on the auditory nerve of cats imply that, for a 1-kilohertz tone, $\beta \approx 1/3$ if the signal is "weakly" equilibrated, that is, if individual nerve fibers do not respond within each cycle of the stimulus oscillation. Also, the gross electrophysiological measurements (fig. 8) by Boudreau (ref. 27) on the superior olivary complex of cats indicate that, for an 800-hertz tone, $\beta \approx 1/3$ if the signal is equilibrated, that is, if the potential amplitude is associated with a time greater than 2 seconds after imposition of the stimulus.

On the other hand, Warren (ref. 7) has argued that most reported studies of the psychoacoustic law were subject to known experimentally induced biases. His psychoacoustic tests, intended to eliminate the known biases, have yielded the result, $\alpha \approx 1/2$. Correspondingly, as shown in figure 7, the electrophysiological measurements by Derbyshire and Davis result in $\beta \approx 1/2$ if the weakly equilibrated signal is corrected to obtain the expectation when equilibrated (ref. 26, fig. 14(c)). Hence, there exist sets of psychoacoustic and electrophysiological data which imply that $\alpha = \beta$. The condition $\alpha = 1/3$ is probably more generally accepted. However, the condition $\alpha = \beta = 1/2$ is a newer estimate which simplifies the psychoacoustic relation, as will be shown, and is, therefore, aesthetically more acceptable from the standpoint of physics.

If it is assumed that $\alpha = \beta$, it follows that

$$\mathscr{L} = \frac{\kappa}{\lambda} \Phi \tag{29}$$

at suprathreshold levels. As the loudness threshold is approached, $\Phi \to \Phi_t$ and $\mathscr{L} \to \mathscr{L}_t \to 0$. The results at both threshold and suprathreshold levels can be incorporated in one plausible equation simply by assuming that loudness is proportional to the amount by which the whole-nerve potential amplitude at the stimulus frequency exceeds the corresponding amplitude at the threshold of sensation, that is,

$$\mathscr{L} = \frac{\kappa}{\lambda} (\Phi - \Phi_t) \tag{30}$$

Since $\Phi \to \Phi_t$ when $P \to P_t$, it follows from equation (26) that $\Phi_t = \lambda P_t^{2\beta}$. Therefore, the generalized psychoacoustic law becomes

$$\mathscr{L} = \kappa\left(P^{2\beta} - P_t^{2\beta}\right) \tag{31}$$

or

$$\mathscr{L}_1 = \kappa_1\left(P_1^{2\beta} - P_0^{2\beta}\right) \tag{32}$$

for a 1-kilohertz tone, where P_0 is the amplitude of p at threshold for a 1-kilohertz tone (compare eq. (3)). Equations (31) and (32) are in terms of pressure amplitude. In terms of mean-square pressure the equivalent equations are, respectively,

$$\mathscr{L} = k\left[\left(\widetilde{p^2}\right)^\beta - \left(\widetilde{p_t^2}\right)^\beta\right] \tag{33}$$

and

$$\mathscr{L}_1 = k_1\left[\left(\widetilde{p_1^2}\right)^\beta - \left(\widetilde{p_0^2}\right)^\beta\right] \tag{34}$$

This function, along with that represented by equation (8), is shown in figure 1 with $\widetilde{p_1^2}$ expressed in decibels. In essence, equation (34) was originally proposed by Lochner and Burger (ref. 6) to fit loudness judgment data extending to near the loudness threshold. If $\beta = 1/2$, equations (31) to (34) are especially simple. Then, for example,

$$\mathscr{L} = k\left[\left(\widetilde{p^2}\right)^{1/2} - \left(\widetilde{p_t^2}\right)^{1/2}\right] \tag{35}$$

The loudness is proportional to the amount by which the root-mean-square sound pressure exceeds its value at the threshold of the loudness sensation.

It is important to recognize that, although each separate term in equations (31) to (35), for example, $\kappa P_t^{2\beta}$ in equation (31), possesses the dimensions of loudness, the effect is only to shift the origin of the psychoacoustic coordinate. It does not imply loudness summation in the usual sense whereby loudnesses of individual spectral contributions to sensation are often summed (ref. 18).

The measured potential of the whole nerve oscillates at the stimulus frequency. However, at least in the case of Boudreau's data (fig. 5 in ref. 27), the measured waveform of the gross potential in response to a pure-tone sound stimulus deviates in magnitude from a sine wave by up to 30 percent. Nevertheless, this seemingly large defect represents a negligible effect on loudness. The deviation represents an amplitude level at least 10 decibels below, or a loudness level at least 10 phons below (compare eq. (29)), the level determined from the sine wave alone. Since the frequencies of the stimulus and the defect differ, they are incoherent. Hence, the contribution of the defect to the overall loudness level is, therefore, less than 0.5 decibel (phon), which is certainly negligible.

Loudness

For suprathreshold loudnesses and any stimulus frequency the loudness and loudness level are related by equation (13) and the loudness and sound-pressure level by equation (24). These formulas will now be generalized by a rederivation which utilizes the generalized psychoacoustic law (eq. (33)) rather than the psychoacoustic law (eq. (8)), which applies only for suprathreshold loudnesses.

From equations (33) and (10) it follows that, for any tone,

$$\mathscr{L} = k\left(\widetilde{p_t^2}\right)^{\beta}\left\{\text{antilog}\left[\left(\frac{\beta}{10}\right)(S - C)\right] - 1\right\} \tag{36}$$

where, by definition,

$$C = 10 \log\left(\frac{\widetilde{p_t^2}}{\widetilde{p_0^2}}\right) \tag{37}$$

represents the elevation of the loudness threshold as a function of the stimulus frequency. For a 1-kilohertz-tone stimulus, equation (36) reduces to

$$\mathscr{L}_1 = k_1\left(\widetilde{p_0^2}\right)^{\beta}\left[\text{antilog}\left(\frac{\beta S_1}{10}\right) - 1\right] \tag{38}$$

which generalization replaces equation (4) and reduces to equation (4) for suprathreshold loudnesses. Since by definition, $S_1 = L_1$, it follows that, for any tone,

$$\mathscr{L} = k_1\left(\widetilde{p_0^2}\right)^{\beta}\left[\text{antilog}\left(\frac{\beta L}{10}\right) - 1\right] \tag{39}$$

which, of course, reduces to equation (13) for suprathreshold loudnesses. Equations (38) and (39) are represented graphically in figure 1. Finally, for any tone,

$$L = S_1 \tag{12}$$

as before, where S_1 is the sound-pressure level of an equally loud, 1-kilohertz tone.

The transmittance function $\mathscr{T}$ and conversion function $\mathscr{N}$, as well as the respective levels T and N, may be defined as before, except that equation (33) replaces equation (8), so that

$$\mathscr{L} = \mathscr{l}\mathscr{T}^{\beta}\left[\left(\widetilde{p^2}\right)^{\beta} - \left(\widetilde{p_t^2}\right)^{\beta}\right]\left(\widetilde{p_0^2}\right)^{-\beta} \tag{40}$$

replaces equation (14). Otherwise, proceeding as in the suprathreshold case, but using the corresponding generalized formulas, leads to the conclusion that

$$\begin{aligned}\left(\frac{10}{\beta}\right)\left\{\log\left[\text{antilog}\left(\frac{\beta L}{10}\right) - 1\right] - \log\left[\text{antilog}\left(\frac{\beta L_1}{10}\right) - 1\right]\right\} &= N + T + S - S_1 \\ &+ \left(\frac{10}{\beta}\right)\left\{\log\left[1 - \text{antilog}\left(\beta\frac{C - S}{10}\right)\right] - \log\left[1 - \text{antilog}\left(\frac{-\beta S_1}{10}\right)\right]\right\}\end{aligned} \tag{41}$$

replaces equation (20), where the additional terms in equation (41) are significant only near the loudness threshold, as expected. Since $L_1 = S_1$, equation (41) reduces to

$$\left(\frac{10}{\beta}\right)\log\left[\operatorname{antilog}\left(\frac{\beta L}{10}\right) - 1\right] = N + T + S + \left(\frac{10}{\beta}\right)\log\left\{1 - \operatorname{antilog}\left[\frac{\beta}{10}(C - S)\right]\right\} \tag{42}$$

which applies for any frequency.

CONCLUDING REMARKS

The preceding serves as the basis for developing loudness evaluation procedures for any sound.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 17, 1972,
132-15.

APPENDIX A

SYMBOLS

a	loudness coordinate translation
b	mean-square pressure coordinate translation
C	pure-tone, loudness-threshold level relative to loudness threshold for 1-kHz tone
k	dimensional proportionality constant in psychoacoustic law
L	loudness level of any tone
$\mathscr{L}$	loudness of any tone
l	dimensionless psychoacoustic conversion factor
N	internal, power-conversion level
$\mathscr{N}$	internal, power-conversion (to loudness) factor
P	sound-pressure amplitude
p	sound pressure
S	sound-pressure level
$\mathscr{S}$	sensation level
T	external, power-transmittance level
$\mathscr{T}$	external power transmittance
t	time
α	constant exponent in psychoacoustic law
β	constant exponent in physioacoustic law
δ	constant in psychophysiological law; $\delta = \alpha/\beta$
ϵ	parameter for equal-loudness-level curves; $\epsilon = L$
κ	dimensional proportionality constant in psychoacoustic law based on sound-pressure amplitude
λ	dimensional proportionality constant in physioacoustic law
μ	dimensional proportionality constant in psychophysiological law
ν	parameter for loudness-conversion level curves; $\nu = S_e$
τ	auditory integration time; $\tau \approx 0.2$ sec
Φ	summed-action-potential amplitude

φ summed action potential

ω rotational frequency; $\omega = 2\pi \times$ frequency

Subscripts:

e at eardrum

l lower cutoff

m at maximum of function N

s under condition of spontaneous neural activity alone

t at loudness threshold

u upper cutoff

0 for 1-kHz tone at effective loudness threshold

1 for 1-kHz tone

Superscripts:

$-$ infinite time average

$\sim$ finite time average

APPENDIX B

EVALUATION OF T AND N

The function T shown in figure 4 is not a duplicate of that presented in figure 5 of reference 15, but rather, represents a judgment based on data in both references 10 and 15. It is commonly believed that the oscillations of equal-loudness contours obtained by direct listening, as in reference 10 and figure 2, are caused by diffraction of the plane-wave stimulus by the human head and propagation of the waves down the external auditory canal to the eardrum. Hence, the oscillations must be associated with the external auditory system. Wiener and Ross's data (ref. 15) justify this belief. To demonstrate this, the equal-loudness contours in figure 2 were treated in the following manner. It was assumed that the internal transmission function must be smooth and free of oscillations. Thus, each equal-loudness contour was visually matched by a smooth curve, concave upward, which fit the original contour wherever possible, but passed as a minimum through the point $S_1 = L_1$ (and thus provided an appropriate reference for the internal conversion level curves) and came as close as possible to intercepting the maxima of the oscillations without introducing sudden changes of curvature. These smoothed curves are the light dashed curves in figure 2. It was assumed that the difference between the original and smoothed equal-loudness curves determined the external transmittance level T. This function was compared with that determined directly by Wiener and Ross and found to be in good agreement generally, as shown in figure 3. Even the small systematic dip in Robinson and Dadson's equal-loudness contours near 400 hertz leads to a match with the Wiener and Ross data. The Wiener and Ross data must have been influenced by the probe microphone inserted in the ear canal and only extended up to 8 kilohertz. Therefore, in determining the function T shown in figure 4 the two sets of data were averaged and the result was smoothed for frequencies less than 8 kilohertz. For frequencies greater than 8 kilohertz the function T determined from the Robinson-Dadson data, alone, was smoothed.

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TABLE I. - EXTERNAL, POWER-TRANSMITTANCE-LEVEL FUNCTION

One-third octave band midfrequency, Hz	External, power-transmittance level function, T_e, dB	
	Plane wave incident frontally	Diffuse field
<160	0	0
160	.5	.5
200	1	1
250	1.5	1
315	2	1
400	2	.5
500	2	0
630	1.5	-1
800	.5	-2
1 000	0	-2.5
1 250	0	-2
1 600	1.5	0
2 000	4	3.5
2 500	7	7.5
3 150	10	11
4 000	12	12.5
5 000	10	8.5
6 300	6	2
8 000	0	-5
10 000	2	-1
12 500	9	9
16 000	5	----
20 000	0	----

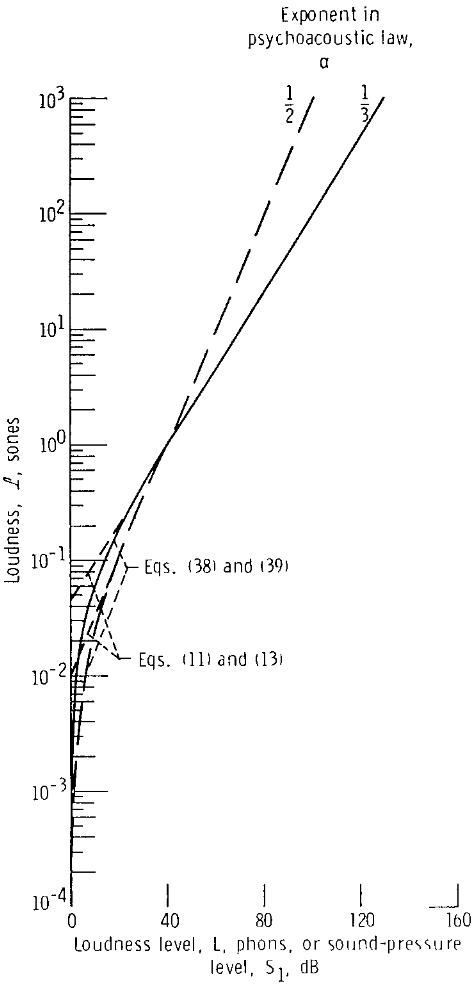

Figure 1. - Relations among loudness, loudness level, and sound-pressure level.

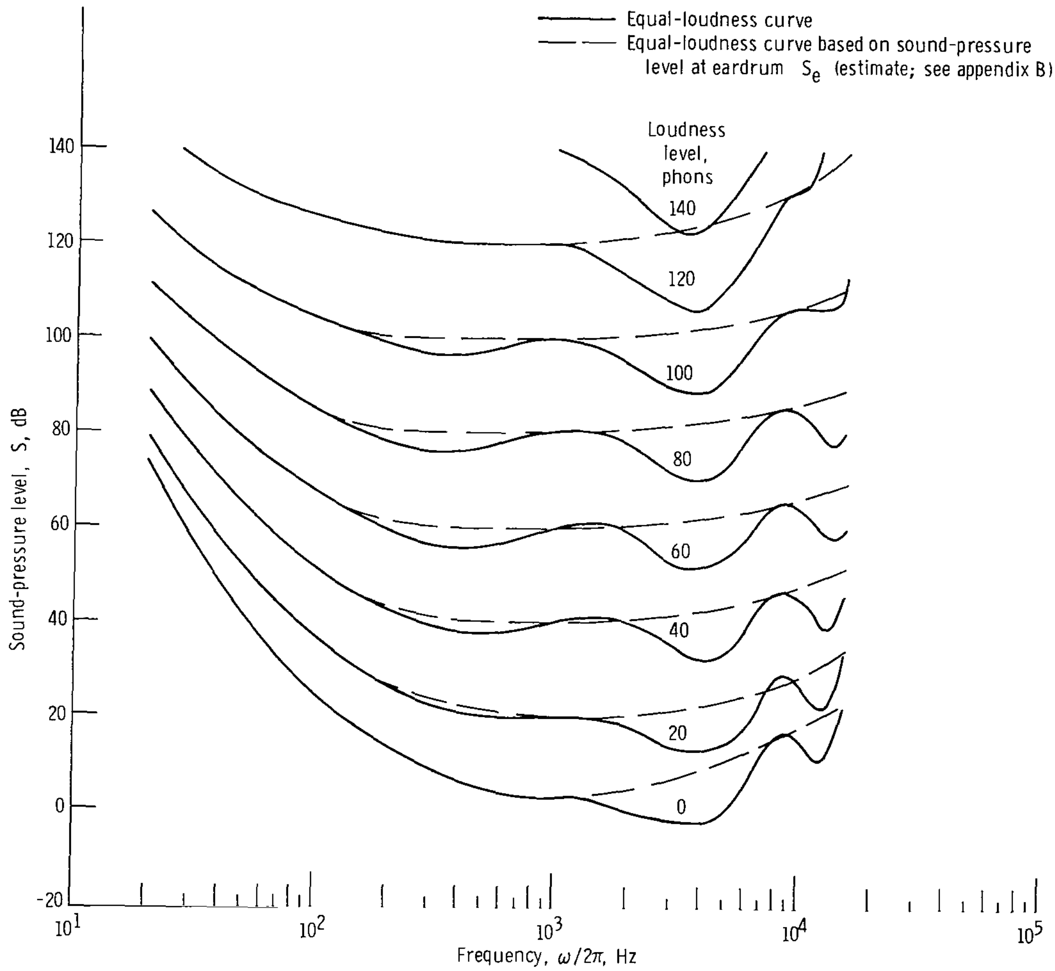

Figure 2. - Equal-loudness curves for plane waves incident from front. Data from reference 10.

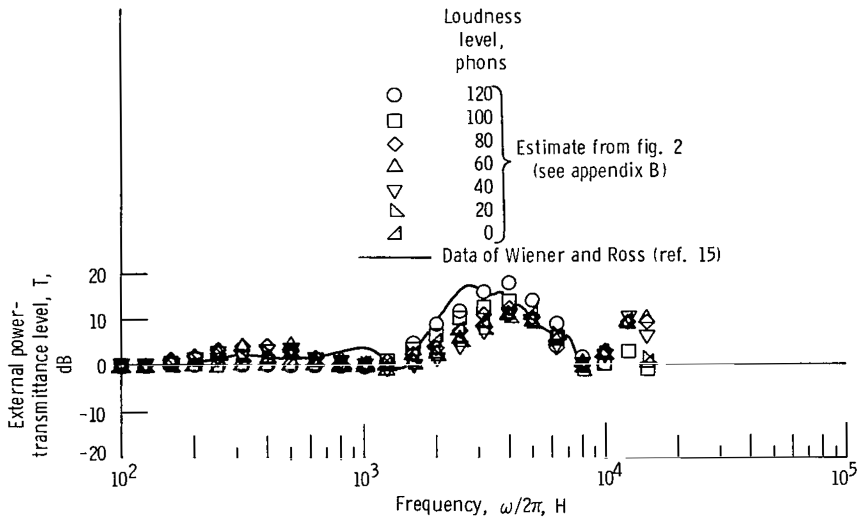

Figure 3. - Comparison of direct measurements of external power-transmittance level with estimates from figure 2.

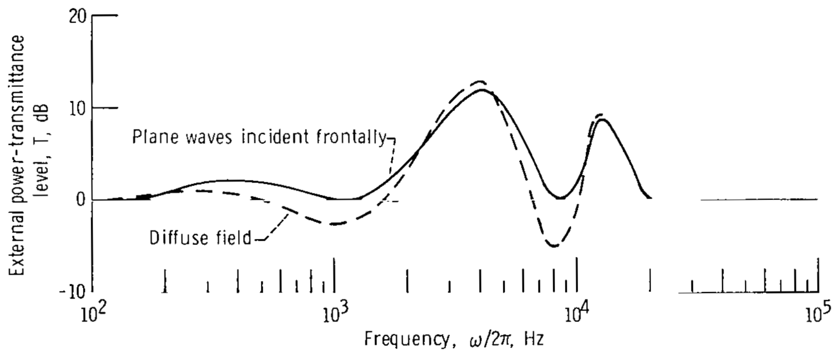

Figure 4. - Weighted average of direct measurements and estimates of external power-transmittance level. External-power transmittance level is equal to 10 log (mean square pressure at eardrum/mean square pressure in free field).

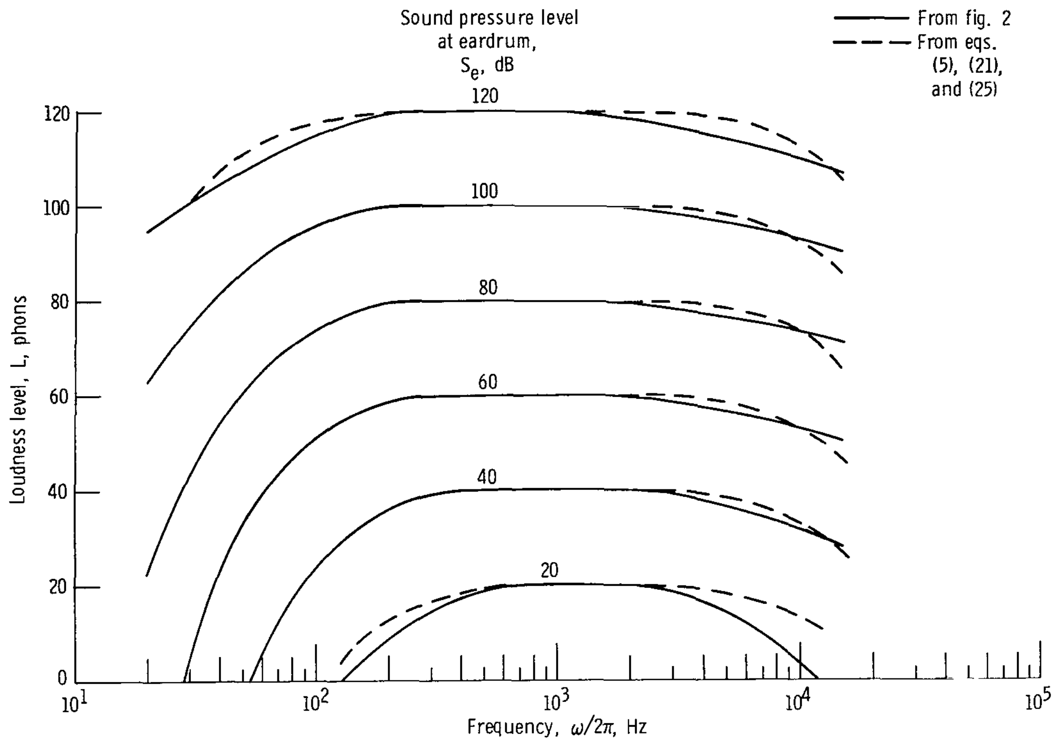

Figure 5. - Internal power-conversion level (unnormalized).

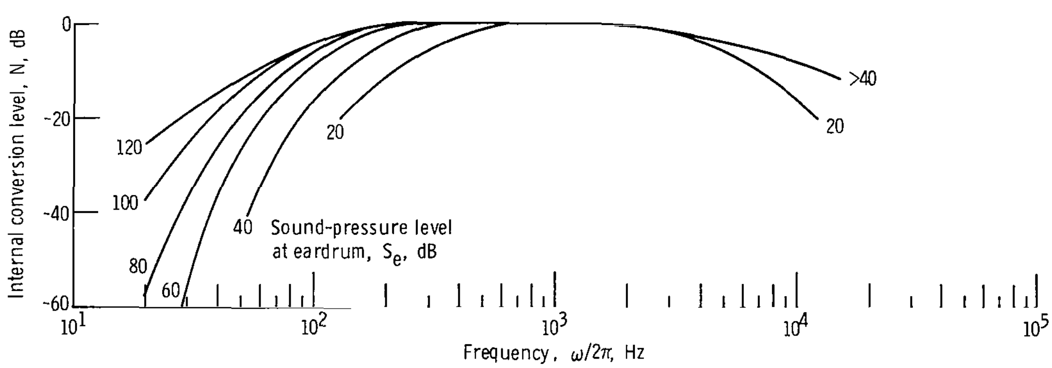

Figure 6. - Internal, power-conversion level (normalized).

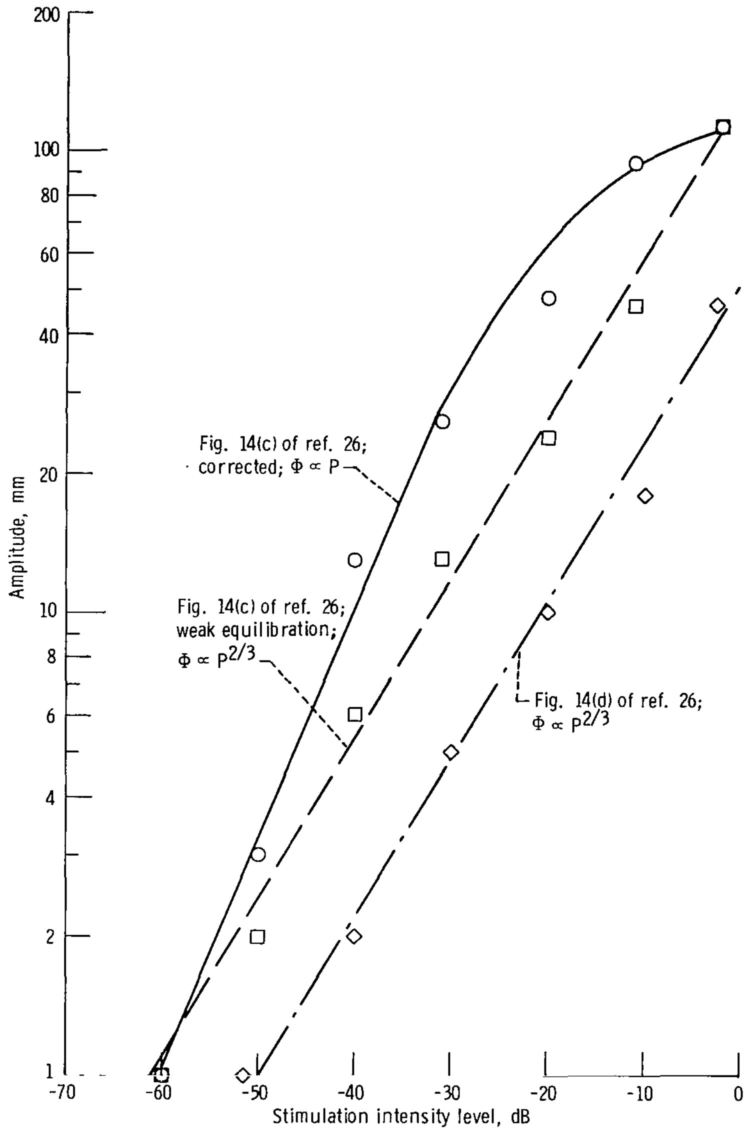

Figure 7. - Amplitude of action potential in auditory nerve of cat as function of intensity level of sound stimulus. Stimulus frequency, 1000 hertz.

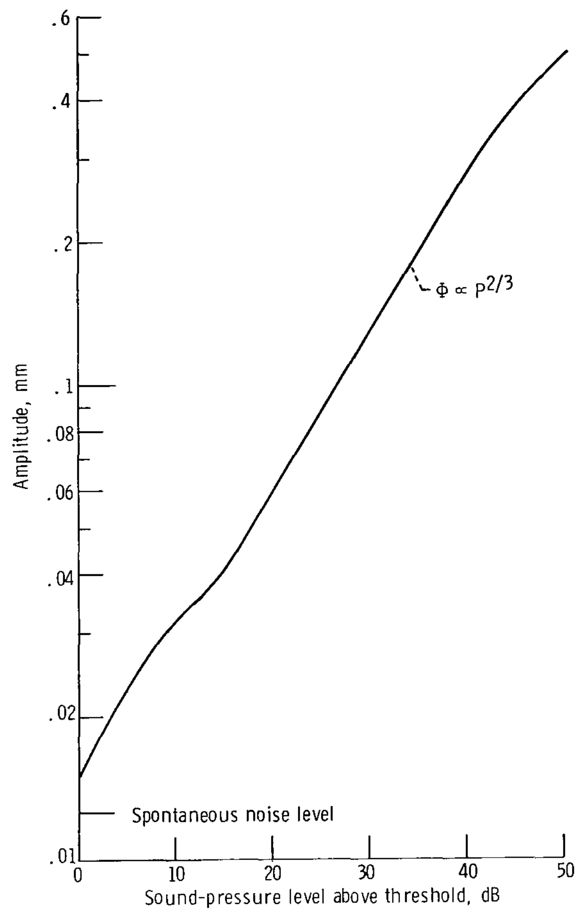

Figure 8. - Double amplitude of frequency-following response in superior-olivary complex of cat as function of sound-pressure level of 800 hertz sound stimulus. Data from figure 8 of reference 27.

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