EXAMINATION OF THE COLLISION FORCE METHOD FOR
ANALYZING THE RESPONSES OF SIMPLE
CONTAINMENT/DEFLECTION STRUCTURES TO IMPACT BY
ONE ENGINE ROTOR BLADE FRAGMENT

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An approximate collision analysis, termed the collision-force method, has been developed for studying impact-interaction of an engine rotor blade fragment with an initially circular containment ring. This collision analysis utilizes basic mass, material property, geometry, and pre-impact velocity information for the fragment, together with any one of three postulated patterns of blade deformation behavior termed (1) the elastic straight blade model, (2) the elastic-plastic straight shortening blade model, and (3) the elastic-plastic curling blade model. Associated with each type of behavioral model are appropriate but approximate force-deformation relations which enable one to estimate the collision-induced forces. The collision-induced forces are used to predict the resulting motions of both the blade fragment and the containment ring. Containment ring transient responses are predicted by a finite element computer code which accommodates the large deformation, elastic-plastic planar deformation behavior of simple structures such as beams and/or rings.

The effects of varying the values of certain parameters in each blade-behavior model have been studied. Comparisons of predictions with experimental data indicate that of the three postulated blade-behavior models, the elastic-plastic curling blade model appears to be the most plausible and satisfactory for predicting the impact-induced motions of a ductile engine rotor blade and a containment ring against which the blade impacts. For this theoretical blade-behavior model, the important parameters include the (assumed) perfectly-plastic yield strength of the blade, the final curl radius of the blade, and the coefficient of sliding friction between the blade fragment and the containment ring.

By employing plausible parameter values in the curling blade model, very good agreement is observed between predictions and experimental ring and blade response measurements.
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SECTION 1
INTRODUCTION

1.1 Background

In spite of intensive conscientious effort through the use of improved materials, design, fabrication, and inspection, there continue to be a not-insignificant number of failures (Ref. 1 - 6) of rotor blades and/or disks of turbines or compressors of aircraft turbojet engines. The resulting fragments might injure personnel occupying the aircraft or might cause damage to fuel lines and tanks, control systems, and/or other vital components, with the consequent possibility of a serious crash and loss of life. It is necessary, therefore, that means be devised for protecting (a) on-board personnel and (b) vital components from such fragments.

Two distinct avenues for providing this protection are evident. First, the structure surrounding the "failure-prone" rotor region could be designed to contain (that is, prevent the escape of) the rotor-burst fragments completely. Second, the structure surrounding this rotor could be designed so as to prevent fragment penetration in, and to deflect fragments away from, certain critical regions or directions but to permit fragment escape readily in other "harmless" regions or directions. One or both of these schemes could, in principle, be employed in a given design. In any event, this desired protection is sought for the least weight and/or cost penalty. If only one of these two schemes were to be adopted, one might expect that the second would be most cost and/or weight effective. However, the present (1) knowledge of the fragment-structure interaction phenomena and (2) analysis/design tools are inadequate to permit making a definitive comparative assessment at this time, although much progress has been made in both of these areas in the past few years.
As pointed out in Ref. 1, NASA has been sponsoring a research program which is designed to meet the objective of providing the necessary protection to aircraft without imposing large weight penalties. Starting about 1964, the Naval Air Propulsion Test Center (NAPTC) under NASA sponsorship has constructed and employed a spin-chamber test facility wherein rotors of various sizes can be operated at high rpm, failed, and very importantly, the interactions of the resulting fragments with various types of containment and/or deflection structures can be studied with high-speed photography, in addition to post-mortem studies of the containment/deflection structure and the fragments. Many such tests involving single fragments or many complex fragments impinging upon containment structures of various types and materials have been conducted (Refs. 2-6) and have substantially increased the body of knowledge of the attendant phenomena. For the past several years NASA has sponsored a research effort at the MIT Aeroelastic and Structures Research Laboratory (ASRL) to develop methods for predicting theoretically the interaction behavior between fragments and containment/deflection structures, as well as the transient deformations and responses of containment/deflection structures -- the principal objective being to devise reliable prediction/design procedures and containment/deflection techniques. Important cross-fertilization has occurred between the NAPTC experimental and the MIT-ASRL theoretical studies, with special supportive-diagnostic experiments and detailed measurements being designed jointly by NASA, NAPTC, and MIT personnel and conducted at the NAPTC. Subsequent analysis and theoretical-experimental correlation work has been increasing both the understanding of the phenomena involved and the ability to predict these interaction/structural-response phenomena quantitatively.

Because of the multiple complexities involved in the very general case wherein the failure of one blade leads to impact against the engine casing, rebound, interaction with other blades
and subsequent cascading rotor-failures and multiple-impact interactions of the various fragments with the casing and with each other, it is necessary to focus attention initially upon a much simpler situation in order to develop an adequate understanding of these collision-interaction processes. Accordingly, rather than considering the general three-dimensional large deformations of actual engine casings under multiple rotor-fragment attack, the simpler problem of planar structural response of containment structures has been scrutinized. That is, the containment structure is regarded simply as a structural ring lying in a plane; the ring may undergo large deformations but these deformations are confined essentially to that plane. For such a case, numerical finite-difference (Refs. 7 and 8) and finite element (Ref. 9) methods of analysis to predict the transient large-deformation responses of such structures to known impulsive and/or transient external loading have been developed at the MIT-ASRL and have been verified by evaluative comparison with high-quality experimental data to provide reliable predictions.

In the present context, therefore, the crucial information which needs to be determined (if the structural response of a containment ring is to be predicted reliably) concerns the magnitude, distribution, and time history of the loading which the ring experiences because of fragment impact and interaction with the ring. Two means for supplying this information have been considered:

(1) The TEJ concept (Refs. 10-12) which utilizes measured experimental ring position-time data during the ring-fragment interaction process in order to deduce the external forces experienced by the ring. This concept has been pursued. An important merit of this approach is that it can be applied with equal facility to ring problems involving simple single fragments such as one
blade, or to cases involving a complex multi-bladed-disk fragment. The central idea here is that if the TEJ type analysis were applied to typical cases of, for example, (a) single-blade impact, (b) disk-segment impact, and/or (c) multi-bladed disk fragment impact, one could determine the distribution and time history of the forces applied to the containment ring for each case. Such forces could then be applied tentatively in computer code response-prediction-and-screening studies for similar types of ring-fragment interaction problems involving various other materials, where guidance in the proper application of these forces or their modification could be furnished by dimensional-analysis considerations and selected spot-check experiments. It remains, however, to be demonstrated whether adequate rules can be devised to "extrapolate" this forcing function information to represent similar types of fragment attack (with perhaps different fragment material properties) against containment vessels composed of material different from that used in the aforementioned experiments.*

On the other hand, this approach suffers from the fact that experimental data must be available; the forcing function is not determined from basic material property, geometry, and initial impact information.

* It is to circumvent this tenuous extrapolation problem and to eliminate the necessity for making detailed transient response measurements now required in the TEJ concept that effort has been devoted to developing alternate methods of analysis (see the next approach in item 2).
The second approach, however, utilizes basic material property, geometry, and initial impact information in an approximate analysis. If the problem involves only a single fragment, this method can be carried out and implemented without undue difficulty, but can become very complicated and time-consuming if complex fragments and/or multiple fragments must be taken into account.

Approach 1 is explained in detail in Refs. 10-12. The present report deals with approach 2 and confines attention to problems involving only a single simple fragment; problems involving more complicated fragments are left for future consideration.

Various levels of sophistication may be employed in approach 2. One could, for example, employ finite-difference methods wherein both the containment ring and fragment are represented by a suitably fine three-dimensional spatial mesh and the conservation equations are solved in time for simple configurations by digital computer codes such as HEMP (Ref. 13), STRIDE (Ref. 14), and/or HELP (Ref. 15), which take into account elastic, plastic, strain hardening, and strain-rate behavior of the material. Such computations, while vital for certain types of problems, are very lengthy and expensive, and are not well suited for the type of engineering analysis/design purposes needed in the present problem; for complicated or multiple fragments, such calculations would be prohibitively complicated, lengthy, and expensive. A simpler, less complicated, engineering-analysis attack within this general framework is needed.

Two categories of such an engineering analysis may be identified and are termed: (a) the collision-imparted velocity
method (CIVM) and (b) the collision-force method (CFM). The essence of each method follows:

(a) **Collision-Imparted Velocity Method (CIVM)**

In this approach the local deformations of the fragment or of the ring at the collision interface do not enter explicitly, but the containment ring can deform in an elastic-plastic fashion by membrane and bending action as a result of having imparted to it a collision-induced velocity at the contact region via (a) perfectly-elastic, (b) perfectly-inelastic, or (c) intermediate behavior. Since the collision analysis provides only collision imparted velocity information for the ring and the fragment (not the collision-induced interaction forces themselves), this procedure is called the collision-imparted velocity method.

(b) **Collision-Force Method (CFM)**

In this method the primary information predicted in the collision analysis consists of the collision-induced interaction forces themselves; the associated and subsequent ring and fragment responses are also predicted.

The CIVM approach is explored in Ref. 9 for cases in which a single nondeformable rotor blade is the attacking fragment; impacts against complete free rings and restrained partial rings such as depicted in Fig. 1 are studied. Another simplification employed is that the impacting surfaces are perfectly smooth. Some comparisons of predictions against experimental data for free containment-ring examples are presented. Although favorable experimental-theoretical agreement is found, some evident deficiencies are noted, and suggestions for analysis improvements have been offered.
The principal object of the present study is to explore the utilization and merits of scheme (b): the collision-force method (CFM).

1.2 Purpose and Scope of the Present Study

The main purpose of the present study is to examine the feasibility and merits of employing the collision-force method (CFM) for predicting collision-induced forces and the resulting structural responses of an attacking fragment and a free complete containment ring. This relatively simple fragment/ring problem has been chosen for study in order to minimize the attendant complexities in this initial study of the CFM. Should the CFM prove to be suitably convenient and accurate, consideration could be given later to extending this method of analysis to treat more complex types of fragments (see Fig. 2) and/or containment/deflection structures.

In Section 2, the principal ingredients of the collision-force method are described as well as its use in conjunction with an available computer code for predicting the large-deflection elastic-plastic transient responses of simple structures such as beams and rings. An initial example discussed in Section 3 consists of a simply-supported beam impacted by a ball and is used to compare the present analysis and computer program with independent available predictions for small-deflection beam responses for cases involving (a) perfectly-elastic impact or (b) perfectly-inelastic impact.

The CFM analysis is then applied in Section 4 to the more complex ring/blade problem wherein large-deflection elastic and/or elastic-plastic ring transient structural responses are taken into account; also included are: (1) various types of postulated blade structural behavior and (2) the influence of several modeling parameters appearing in the analysis. Some comparisons of predictions with experimental observations are also given.
Section 5 contains a summary of the present study, an assessment of the merits and difficulties found with the CFM, and some suggestions for further research.
SECTION 2

OUTLINE OF THE ANALYSIS

2.1 Introduction

For present purposes, attention is restricted to analyzing the transient response of a free complete containment ring subjected to impact by a single engine-rotor blade, as depicted schematically in Figs. 1a and 3a. The transient structural responses of this ring-fragment system are assumed to lie in one plane; hence, this is termed a "2-d problem" since only two coordinates, Y, Z are needed to locate any given material point of this system.

Using this ring-fragment problem as an illustrative example, this section is devoted to a description of the general procedure used to calculate the ring/fragment transient motion in accordance with the process called the collision-force method, as indicated by the information flow schematic of Fig. 4. The key difference of this procedure from that of the collision-imparted velocity method (CIVM) of Ref. 9 is that from the collision-interaction calculation stage of this procedure one obtains collision-induced forces acting on the fragment and on the ring, whereas in the CIVM scheme one obtains only collision-imparted velocities for the fragment and the ring, not the collision-induced forces themselves; for convenient comparison, an information flow diagram for the CIVM procedure is shown in Fig. 5. Therefore, the key aspect of the present study centers on devising simple approximate methods for predicting these collision-induced forces. It is evident that more sophisticated and rigorous methods can be devised than discussed herein, but emphasis here is placed upon seeking simple prediction methods which hopefully contain most of the important behavioral ingredients.
Briefly, the analysis procedure indicated in Fig. 4 consists of the following principal steps:

1. **Motion and Position of Bodies**
   The motions of both bodies are predicted and the (tentative) region of space occupied by each body at a given instant in time is determined.

2. **Collision Inspection**
   Next, an inspection is performed to determine whether a collision has occurred during the small increment \( \Delta t \) in time from the last instant at which the body locations were known to the present instant in time at which body location data are sought.

3. **Collision-Interaction Force Prediction**
   If a collision has not occurred during this \( \Delta t \), one follows the motion of each body under zero external forces for another \( \Delta t \), etc. However, if a collision has occurred, one proceeds to carry out an appropriate collision-interaction calculation from which one obtains an estimate of the collision-interaction forces which now play the role of externally-applied forces for the next \( \Delta t \) time step of motion-and-position prediction.

   Accordingly, the general analysis procedure as well as various considerations and simplifying assumptions invoked in order to predict the collision-induced forces are discussed in Subsection 2.2.

2.2 Outline of the Collision-Force Method

2.2.1 Prediction of Containment Ring Motion and Position
   Let it be assumed that the containment ring depicted in Figs. 1a and/or 3a is about to be impacted by a single rotor blade. In the CPM procedure, the equations of motion for the
(deforming) ring and for the fragment are solved at successive instants in time \( t \) spaced \( \Delta t \) apart. As indicated in Fig. 4, the motion and position of each body are followed and compared at each instant in time to determine whether or not a collision has occurred. If no collision has occurred, the motion of each body continues under the influence of "no external forces". If inspection shows that a collision has occurred, one proceeds to carry out an appropriate collision-interaction calculation from which one obtains an estimate of the collision-induced forces acting on the fragment, with equal and opposite forces acting on the ring. These collision-induced forces act as externally-applied forces acting on each body; the resulting motion during the next small time increment \( \Delta t \) is then calculated. The resulting blade and ring position are then compared in order to see whether or not a collision has occurred during that \( \Delta t \) time step -- and the process continues as long as the analyst wishes.

In the present studies, the motion and structural response of the ring are predicted by the finite-element method of analysis described in Ref. 9; large deflection transient Kirchhoff-type responses including elastic, plastic, strain-hardening, and strain-rate sensitive behavior are included. In this method the ring is represented by an assemblage of discrete (or finite) elements compatibly joined at the nodal stations (see Fig. 3a). The behavior of each finite element is characterized by a knowledge of the four generalized displacements \( q \) at each of its nodal stations, referred to the \( \eta, \zeta \) local coordinates (see Fig. 3b); the displacement behavior within each finite element is represented by a cubic polynomial for the normal displacement \( w \) and a cubic polynomial for the circumferential displacement \( v \), anchored to the four generalized nodal displacements at each node (see Ref. 9 for further details). For present purposes, it

\[ \text{[Transverse shear deformation is excluded.] \quad 11} \]
suffices to note that the resulting equations of motion for the ring referred to the global Y,Z coordinates (referred to which the generalized nodal displacements are denoted by \( q^* \)) are:

\[
[M]\{\ddot{q}^*\} + \{P\} + [H]\{\dot{q}^*\} = \{F^*\} \tag{2.1}
\]

where \( \{\ddot{q}^*\} \) are the global generalized accelerations
\( \{q^*\} \) are the global generalized displacements
\( [M] \) represents the mass matrix for the complete structure
\( \{P\} \) represents the usual internal elastic forces and some plastic force contributions
\( [H]\{q^*\} \) represents generalized loads arising from both large displacements and plastic strains (if present)
\( \{F^*\} \) denotes the prescribed externally-applied generalized forces acting on the structure

Equation 2.1 represents the equations of motion for the "improved formulation" of Ref. 9.

These equations of motion are solved by applying an appropriate finite-difference operator to represent \( \ddot{q}^* \) at any time instant \( t_m \). For example, if the (explicit) central-difference method is used, one may write

\[
\dot{q}^*_m = \frac{q^*_{m+1} - 2q^*_m + q^*_m}{(\Delta t)^2} + O(\Delta t)^2 \quad (m > 0) \tag{2.2a}
\]

Also, the velocity at time \( t_m \) may be expressed as

\[
\dot{q}^*_m = \frac{q^*_{m+1} - q^*_{m-1}}{2(\Delta t)} + O(\Delta t)^2 \quad (m > 0) \tag{2.2b}
\]
Thus, if desired, one may obtain $q_{m+1}^*$ from Eq. 2.2a or 2.2b, respectively, as

$$Q_{tm+1}^* = -Q_{tm}^* + 2Q_{tm}^* + \frac{\ddot{q}_m^*}{2}(\Delta t)^2 \quad (m > 0) \quad (2.3a)$$

or

$$Q_{tm+1}^* = Q_{tm}^* + 2(\Delta t)\ddot{q}_m^* \quad (m > 0) \quad (2.3b)$$

At any instant $t_m$ in time, Eq. 2.1, may be written as

$$[\mathbf{M}]\{\ddot{q}_m^*\} + \{P\}_m + [\mathbf{H}]_m\{\varepsilon\}_m = \{F\}_m \quad (2.4)$$

where, in general, only the mass matrix $[\mathbf{M}]$ does not change with time. Hence, it is apparent that if all quantities in Eq. 2.4 are known at time $t_m$ as is assumed to be true, one may solve Eq. 2.4 for $\{\ddot{q}_m^*\}$ and then use Eq. 2.3a to determine $\{q_m^*\}_{m+1}$, since it is assumed that the solution has already been found at previous instants in time. Then from a knowledge of the generalized displacements $\{q_m^*\}_{m+1}$ at time $t_m$, one may employ appropriate strain-displacement relations and stress-strain relations (including plasticity and strain-rate effects) in order to determine all of the non-prescribed quantities in Eq. 2.1 written for the time instant $t_{m+1}$:

$$[\mathbf{M}]\{\ddot{q}_{m+1}^*\} + \{P\}_{m+1} + [\mathbf{H}]_{m+1}\{\varepsilon\}_{m+1} = \{F\}_{m+1} \quad (2.5)$$

This equation is then solved for $\{\ddot{q}_{m+1}^*\}$, and the solution procedure can be carried out cyclically as already described. For present purposes, this general description is considered to be adequate; one may consult Section 3 of Ref. 9 for a very detailed discussion of the solution and evaluation process.
Before concluding this brief discussion of the solution process for predicting transient structural response, it is useful to sketch the solution-starting procedure. Assuming that initially (at $t=0$), the structure is in a known condition such as $\{q^*\}_0 = \{0\}$ and $\{q^*\}_0 = \{a\}$, one can readily obtain $\{q^*\}_1$ from

$$\{q^*\}_1 = \{q^*\}_0 + \{q^*\}_0 \Delta t + \{q^*\}_0 \frac{(\Delta t)^2}{2} + O(\Delta t)^3 \quad (2.6)$$

since $\{F^*\}_0$ is prescribed and all other quantities are known. The solution can then be carried out for time instants $t_2, t_3, \ldots$, as described earlier.

In principle, the analysis procedure just described is applicable to both the containment ring and the attacking fragment (the rotor blade). This procedure is, in fact, applied for predicting the motion and structural response of the containment ring, but a simpler conceptual/mathematical model is used to predict approximately the motion and behavior of the blade, as described in Subsection 2.2.2 and in Section 4. For both of these participants, the externally-applied forces (represented as $\{F^*\}$ in Eqs. 2.1, 2.4, and 2.5) are the equal and opposite forces experienced by each as a result of collision. A key aspect of this prediction procedure, therefore, involves an estimation of these collision-induced forces. First, however, a comparison of the space occupied by each body must be made at each instant in time to determine whether or not a collision has occurred; this collision-inspection step is discussed in Subsection 2.2.3.

2.2.2 Prediction of Fragment Motion and Position

In the present analysis each of the various modelings used to describe the fragment has the property that at any given instant in time the geometry of the fragment is known (see Sections 3 and 4) such that one need only know the CG location and a
parameter characterizing the fragment's angular orientation $\theta$ in its plane in order to define the space occupancy of the fragment. Thus, for the present planar motion, the pertinent equations of motion are:

**Translational**

$M_f \ddot{Y}_{CG} = F_Y$  \hspace{1cm} (2.7a)

$M_f \ddot{Z}_{CG} = F_Z$

**Rotational**

$I_f \ddot{\theta} = T_{CG}$  \hspace{1cm} (2.8)

where $M_f$ = mass of fragment

$I_f$ = known (instantaneous) moment of inertia of the fragment about its CG

$F_Y, F_Z$ = total forces acting on the fragment in the y- or the z-direction, respectively.

$T_{CG}$ = torque about the CG from the externally applied forces.

For appropriate initial conditions, Eqs. 2.7a through 2.8 may be solved by the same procedure already described for solving Eq. 2.1.

### 2.2.3 Collision Inspection

In order to simplify the logic whereby one determines whether a given fragment has collided with a structure such as a curved ring or beam, it is convenient to approximate the geometry of the ring (or beam) by straight line segments\* between the

*Straight-line segments are used only for collision-inspection analysis purposes; correct curved ring (beam) geometry is used in the structural transient response model.
nodal stations of the finite-element model representing the structure; this approximate geometry approaches the correct geometry as one increases the number of finite elements used. Since the structural transient response calculation provides the locations in space of each nodal station of the structural model, the space occupied by a ring of given thickness is known. From motion predictions for the fragment, its space occupancy at this same instant in time can be determined. If neither body has intruded into the other's space at this time instant, \( t_{m+1} \), no collision has occurred. If, however, both bodies are found to "occupy" a small common region of space at this instant but had no such occupancy at an instant \( \Delta t \) earlier (at \( t_m = t_{m+1} - \Delta t \)), a collision is considered to have occurred between times \( t_m \) and \( t_{m+1} \). As indicated schematically in Fig. 6, the "unimpeded distance \( \alpha \) of fictitious penetration" of the fragment into the containment structure is determined from the pertinent geometry. Then, depending upon the type of approximate interaction calculation that one wishes to employ, one computes the "resulting interaction forces". Some of the considerations and simplifying assumptions which might be employed in making approximate predictions of these interaction forces are discussed in Subsection 2.2.4.

### 2.2.4 Considerations in Choosing Collision-Interaction Force Models

For present purposes, collision-interaction force predictions which are much simpler and more approximate than those of Refs. 13 through 15 are sought. Thus, it is assumed that the "immediate collision-interaction process" is highly localized near the contact region such that the impact induced forces may be taken into account in the ring transient response calculation by applying these forces only to the impacted finite element. Also various types of fragment and/or ring material behavior at the impact-interaction region are to be taken into account such as: (a) perfectly elastic, (b) perfectly plastic, and perhaps (c) intermediate type of behavior; as discussed in Section 3, the Hertz
and the Meyer laws (Ref. 16) for impact may be employed to approximate these effects. Indentation rate effects may also be taken into account.

The effects of bending, stretching, large deformations, elastic, plastic, and strain-rate behavior are taken into account in the analysis of containment/ring response itself by the finite-element method employed. In principle, one could also employ the finite-element method to analyze the bending/stretching behavior of the rotor blade which is subjected to forces arising from impact against the containment ring; in this way one could predict the blade shortening and/or curling behavior which is observed to occur experimentally. However, in the present study, a simpler more approximate behavioral model for the blade was chosen for investigation; the former approach is a matter for further study. The selected simpler approach is sometimes termed the "assumed mode method" (Refs. 17 through 19, for example). In this method, certain types and/or shapes of deformation patterns are postulated to occur, and associated with each is a plausible approximate relation for the load-carrying ability of the structure as a function of some pertinent geometric parameter of its deformation. In this context several types of blade behavior (straight and wholly elastic, straight and elastic-plastic, and curling elastic-plastic) are postulated, discussed, and their use explored in Section 4.

Another collision-interaction aspect which should be mentioned is that the equal-and-opposite forces experienced by the blade and by the containment ring during impact/interaction, in general, will not be perpendicular to the impacted surface of the ring. For analysis purposes, it is convenient to employ the coefficient-of-friction idea whereby the force tangential to the surface in opposition to relative motion is \( \mu \) times the normal force, where \( \mu \) is an appropriate coefficient of sliding friction for the blade and ring materials involved.
2.3 Summary of Analysis Steps

A concise summary of the analysis steps involved in the present procedure for predicting the responses of simple structures to fragment impact may be useful.

If the attacking fragment is a sphere or a circular disk of known size, the location of its center of gravity becomes a useful parameter for determining its space occupancy at any given instant in time, for assessing whether it has collided with another body at a known location. On the other hand if the attacking fragment is a rotor blade which (a) remains straight and a fixed length, (b) remains straight but shortens, or (c) becomes bent, other locator quantities besides the CG location may be helpful in the collision inspection process. However, the following terse analysis-step description is kept simple and parallels the information flow chart of Fig 4:

(1) The nodal positions of the target, and the position of the fragment are known.

(2) A check is made to determine whether or not the fragment has come in contact with the target.

(3) If contact has been made, the interaction distance, \( \alpha \) (the distance the fragment has "penetrated into" the ring), is calculated.

(4) Using some selected force vs. \( \alpha \) relation, the forces acting on the fragment are calculated.

(5) The forces in opposition to those acting on the fragment are distributed to the target nodes neighboring the point of impact.

(6) The forces on the fragment are used, along with the equations of motion of the fragment's center of gravity, to calculate the motion of the fragment.
The forces on the target nodes are used as input for a finite-element program, which then calculates the target node displacements.

This entire procedure is carried out for each succeeding interval of time, from before initial impact until after the collision has ceased.

The computer program which embodies this analysis procedure consists of one section for input-output, one section for the finite-element scheme which carries out step (7) of the CFM procedure, and a section called COLIS, which carries out the bulk of the CFM procedure. This subroutine differs depending on the particular problem being solved.

This general process as applied to analyzing the response of a beam subjected to ball impact is discussed in Section 3, while Section 4 is devoted to describing several versions and consequences of this approach for the case of a simple containment ring subjected to impact by a single engine-rotor blade.
SECTION 3

BEAM RESPONSE TO BALL IMPACT

3.1 Introduction

In order to assess whether or not the analysis procedure described in Section 2 and embodied in a computer program during the present investigation is functioning properly and providing accurate predictions, a simple impact problem for which independent predictions are available is analyzed first; once the present procedure has been validated, one may extend its application to more complex problems with some degree of confidence. Accordingly, the simple example problem chosen consists of transverse normal impact of a steel sphere at the midspan of a simply-supported steel beam of rectangular cross section as depicted in Fig. 7. Of principal interest are the collision-interaction force time history and the midspan deflection-time history. Independent predictions are reported in Ref. 16 for several types of assumptions concerning the collision-interaction process itself; however, only the predicted contact force histories are reported in Ref. 16.

First in Subsection 3.2 a brief review is given of some commonly-discussed types of behavior (Ch. 4 of Ref. 16) at the collision interface region. Then the present predictions (herein termed the CFM predictions, for convenience) are compared with those of Ref. 16 for two of these postulated types of collision behavior.

3.2 Review of Some Types of Idealized Collision-Force Behavior

For discussion purposes consider the impact of a solid sphere against a flat target. Two of the commonly-considered types of collision behavior will be reviewed: (1) perfectly-elastic behavior and (2) elastic-plastic behavior.
3.2.1 The Hertz Law for Perfectly-Elastic Impact

Shown in Fig. 8 is a schematic time sequence of sphere-target configurations depicting perfectly elastic impact, including prior to, during, at the end of, and after sphere-target collision. The total local deformation of the "fragment" and the target at any instant of time at the center of the impact region is denoted by \( \alpha \); note also that \( \alpha \) represents the "depth of penetration" of the undeformed sphere into the "virgin target". When collision ceases, this idealization indicates that both bodies have returned to their original undeformed state, and no longer exert an equal and opposite contact force on each other. As indicated schematically in Fig. 8, this contact force \( F \) may be expressed as a function of \( \alpha \); qualitative pictures of \( \alpha \) vs. time and \( F \) vs. time are also shown in Fig. 8.

This force-deformation relation for idealized static interaction of isotropic perfectly-elastic bodies whose surfaces can be approximated by paraboloids in the vicinity of the contact point is known as Hertz's Law of Contact and may be expressed as (Ch. 4 of Ref. 16):

\[
F = K \alpha^{3/2}
\]

(3.1)

where \( \alpha \) is the total relative indentation of the two bodies and \( K \) depends upon certain geometric and material properties of the bodies. This type of relation is applicable to a wide variety of smooth-surface configurations, but does not apply for discontinuous surfaces.

In the case of impact of a ball of radius \( R \) upon a beam wherein the ball and beam consist of different material of elastic modulii \( E_1 \) and \( E_2 \), and Poisson's ratios \( \nu_1 \) and \( \nu_2 \):
If both bodies consist of the same material, Eq. 3.2a becomes:

\[ K = \frac{4}{3} \sqrt{R} \sqrt{\left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)} \]  (3.2a)

Combining Eqs. 3.1 and 3.2b, the Hertz force-deformation relation becomes:

\[ F = \frac{2E \sqrt{R}}{3(1 - \nu^2)} \alpha^{3/2} \]  (3.3)

Thus, one may employ Eq. 3.3 to calculate the mutual contact force for any given value of the total relative indentation \( \alpha \). In applications a timewise step-by-step solution procedure in small time steps is used whereby one follows the motion of the bodies, makes a trial calculation of \( \alpha \), and then calculates \( F \) from Eq. 3.3; this \( F \) is then applied to each body in solving for the next increment of motion of each body during a selected small time interval \( \Delta t \), and the process proceeds cyclically.

Should the fragment have a hemicylindrical impact end of radius \( R \) and length \( L \), a similar but more complex Hertz Law force-deformation relation can be found, as given by Eqs. 4.23 through 4.26 of Ref. 16.

3.2.2 Elastic-Plastic Collision Behavior

For simple impact involving both elastic and inelastic behavior, force-deformation relations often have been patterned after corresponding laws which have been developed for static conditions. One of the simplest and most useful of these
relations is the Meyer law (see Art. 4.3 of Ref. 16) in which the "contact force" $F$ is expressed empirically for the force-increase phase of the loading by

$$F = N \alpha^n$$  \hspace{1cm} (3.4)

Plastic behavior is assumed to dominate during this phase of the load-deformation behavior. For perfectly-plastic behavior, $n = 1$ and $N = 2\pi R p_o$ where $p_o$ (in the terminology of Ref. 16) is the flow stress and $R$ is the radius of the sphere being pressed into the "target" surface; in general, $N$ depends upon the fragment geometry and the flow stress of the weaker of the two materials.

During unloading, $F$ is given by the empirical relation

$$F = F_m \left[ \frac{\alpha - \alpha_r}{\alpha_m - \alpha_r} \right]^q \text{ for } \alpha_r \leq \alpha \leq \alpha_m$$  \hspace{1cm} (3.5)

where subscript $m$ refers to conditions at the maximum relative indentation, and $\alpha_r$ is the permanent crater depth of the target after separation. If the unloading is wholly elastic (Hertz type behavior), $q = 3/2$.

Force-indentation estimates for a steel ball bearing into a cold-rolled mild steel plate are given in Fig. 53 of Ref. 16 according to Eqs. 3.4 and 3.5 for values of $q = 3/2$ and $q = 1$; the latter represents simply a linear unloading of the force as contrasted to the Hertz law of elastic unloading with a changing contact area which results in a nonlinear force-indentation relation during unloading. From these results, $\alpha_r = 0.85 \alpha_m$ is a reasonable approximation for these materials.

A schematic of this type of elastic-plastic collision behavior is given in Fig. 9, where qualitative depictions of
α vs. time and F vs. time are shown.

These empirical static relations are useful as a starting point. For dynamic conditions, the flow of the material during the impact process, strain hardening, and strain rate effects clearly will alter these relations. Accordingly, if one wishes to retain this framework, both n and N will depend, among other things, upon the indentation rate \( \dot{\alpha} \). However, many impact experiments and measurements have been carried out to determine these empirical "constants" for various "average impact conditions" so that convenient pseudo-static predictions may be carried out; an extensive discussion of this work may be found in Ref. 20.

Various other estimates of force-indentation behavior for elastic and plastic loading followed by elastic unloading may be found in Ch. 4 of Ref. 16.

3.3 Predictions and Comparisons

Figure 7 illustrates the example problem under discussion in this section. A 0.5-inch diameter steel sphere at a velocity of 1800 in./sec. impacts perpendicular to the span of a 1/2 x 1/2 x 30-inch simply-supported steel beam at its midspan location.

In applying the present CFM scheme, the half-beam has been modeled by 10 finite elements; advantage of symmetry was taken to minimize computations. For structural response purposes, the material properties of the steel beam were taken to be \( E = 30 \times 10^6 \) psi and \( \rho = 7.34 \times 10^{-4} (\text{lb-sec}^2)/\text{in}^4 \). Shown in Fig. 10 are CFM predictions, for \( \Delta t = 0.5 \) microsecond, of midspan beam deflection response and the contact force history only for Hertz-type elastic impact (Eq. 3.1) but for beam structural behavior assumed to be (a) elastic (EL) or (b) elastic, perfectly-plastic (EL-PP) with perfectly-plastic yield stress of 40,000 psi; no strain rate effects are taken into account. From Fig. 10 it is seen that the type of beam structural response assumed, EL or EL-PP, has very little effect on the predicted time history of
the contact force. Sphere-beam contact is seen to last about 18 microseconds for this case, which is very short compared with the quarter period of the dominant mode of transient structural response of this beam. After about 16 microseconds, the predicted beam deflection for the EL-PP case diverges from and becomes larger than that for which wholly elastic (EL) beam behavior is assumed.

Shown in Fig. 11 are CFM contact force and structural response predictions for which the impact process itself was assumed to be inelastic during loading according to Eq. 3.4 with \( n = 1 \) and \( N = 2\pi R p_0 = 2\pi(1/4) (370,000) = 5.81 \times 10^5 \text{lb/lin.} \), and during unloading to behave according to Eq. 3.5 as either (1) Hertz-type elastic with \( q = 3/2 \) or (2) linear unloading with \( q = 1 \). For these predictions a time increment \( \Delta t \) of 0.5 microsecond was used, and 10 finite elements were used to model the half-beam. For both calculations, it is assumed that the beam structural response behavior is EL-PP. It is seen that these two different types of unloading behavior have very little influence upon the predicted histories of either the contact force or the midspan deflection. Further, in view of the results in Fig. 10 for EL vs. EL-PP modeling of beam structural behavior, it is expected that the use of EL rather than EL-PP beam behavior in the present case would lead to only small changes in the contact force history shown in Fig. 11.

Since the sphere-beam impact problem defined in Fig. 7 involves small-displacement linear-elastic structural response of the beam, this problem has been analyzed (Ref. 16) by carrying out a timewise numerical step-by-step approximate solution of the Duhamel superposition integral with the well-known normal mode method in conjunction with the following force-indentation laws:

\[ n = 1 \text{ and } N = 2\pi R p_0 = 2\pi(1/4) (370,000) = 5.81 \times 10^5 \text{lb/lin.} \]

These numerical values represent those employed in the corresponding predictions in Ref. 16.
(1) Hertz law elastic impact (Eq. 3.1) and (2) the inelastic loading case given by Eq. 3.4 together with Eq. 3.5 for the unloading behavior. Shown in Fig. 12 are predicted contact force histories from numerical solutions for these two types of behavior, taking 65 excited normal modes into account; for each case, predictions are shown for solution time-step sizes of 0.1 and 2.735 microseconds. The smaller the time-step size, the less approximate is the solution. Only contact force history predictions for this problem are reported in Ref. 16; beam transient deflection predictions are not reported.

In addition to the predictions given in Fig. 12, additional predictions are given in Ref. 16 where second-order corrections are applied during the approximate solution procedure. For a 0.1 microsecond time-step size, those corrected predictions differ only slightly from those shown in Fig. 12 for $\Delta t = 0.1$ microsecond. Therefore, for each type of impact behavior, an average of the predictions for $\Delta t = 0.1$ microsecond from (a) the Fig. 12 results and (b) those cited "corrected results" is chosen as a reasonable basis for evaluating the present CFM predictions. Accordingly, the present CFM predictions given in Figs. 10 and 11 are compared in Fig. 13 with the cited "average results" from Ref. 16. In each case, reasonably good agreement is observed.

In view of the Fig. 12 results for the effects of different time step sizes in the calculation, it is believed that the $\Delta t$ size effect is mainly responsible for the discrepancy in Fig. 13 between the present CFM predictions ($\Delta t = 0.5$ microseconds) and those of Ref. 16 ($\Delta t = 0.1$ microsecond).

Having thus established reasonable confidence in the CFM prediction scheme for a simple problem, its application to a more complex situation is discussed next in Section 4.
SECTION 4

FREE CONTAINMENT RING RESPONSE TO ENGINE ROTOR BLADE IMPACT

4.1 Introduction

The problem to be discussed in this subsection involves the prediction of the transient motions and structural responses of a free initially-circular 2024-T3 aluminum containment ring impacted by a single T58 turbine rotor blade of known geometric and mechanical properties and with known pre-impact translational and rotational velocities. It is desired to predict the motion and deformation history of the ring, the blade-ring interaction force history, the blade-position history, and the plastic deformation history of the blade. The specific problem to be analyzed has been studied experimentally at the Naval Air Propulsion Test Center, Philadelphia, Pa. as NAPTC Test 91 (Ref. 21). The ring and blade data for this problem (Refs. 11 and 21) are given in Table 1, and the situation is shown schematically in Fig. 3. Photographic measurements of containment ring motion and of blade motion are available for comparison with predictions.

As pointed out in Subsection 2.2.3, the behavior of the fragment (the engine rotor blade) is to be approximated in a highly simplified manner by invoking various "postulated assumed modes of behavior", rather than by the more rigorous procedure of finite-element modeling and transient structural response analysis together with the application of momentum and energy conservation relations. It will be appreciated that these various postulated assumed modes of behavior when used in the present CFM analysis procedure greatly simplify and expedite the analysis, but, in general, will exhibit aspects inconsistent with physical reality. The effect of varying certain parameter values in each type of assumed blade behavior is studied. With parameter values lying within plausible bounds, the predicted responses for each
type of assumed blade behavior are compared with each other and with experimental data.

Three types of assumed blade behavior are examined and are termed:

(1) Straight Elastic Blade Model
(2) Straight Elastic-Plastic Shortening Blade Model
(3) Elastic-Plastic Curling Blade Model

These models are discussed, respectively, in Subsections 4.2, 4.3, and 4.4.

In Subsection 4.5, the steps in the prediction process are explained concisely. Finally, Subsection 4.6 is devoted to (1) presenting the ring-blade response predictions obtained for each type of assumed blade behavior and (2) comparing predictions with experimental data.

First, however, it is useful to describe the idealized model chosen to represent the actual blade before any impacts have occurred since this pre-impact model applies\(^*\) to all three of the aforementioned types of assumed mode blade-fragment behavior. The actual blade portion which attacks the containment ring has the following known properties (Fig. 14a and Table 1): (a) weight \(W\), (b) center of gravity (CG) location, (c) mass moment of inertia \(I_{CG}\) about the CG, and (d) a known distance \(L_1\) from its CG to the tip of the blade. The blade is twisted and has variable camber and chord along its length; from its tip end, its cross-sectional area is nearly constant over about one-half of its length and this increases near its root or butt end. In the present idealized model (Fig. 14b), the blade fragment is assumed to have a uniform cross-sectional area \(A\) (representative of its near-tip region) throughout its length; its associated weight per unit length is denoted by \(w_1\) (lb/in). At the butt end a concentrated weight \(W_2\) (lb) is included in the idealized model. Actual blade

\(^*\) An exception to this is noted in Subsection 4.4.4.
fragment characteristics which the idealized pre-impact model preserves are: (1) the total weight \( W \), (2) mass moment of inertia about the CG, and (3) the tip-to-CG distance \( L_1 \). These three conditions are satisfied, respectively, as follows by suitably choosing \( w_1, w_2, \) and \( L_2 \) (or \( L_0 \)):

\[
W = w_1 (L_1 + L_2) + W_2 \tag{4.1}
\]

\[
I_{CG} = \left[ I_{CG_1} + \frac{W_1}{g} L_1 \left( \frac{L_1^2}{2} \right) \right] + \left[ I_{CG_2} + \frac{W_2}{g} L_2 \left( \frac{L_2^2}{2} \right) \right] + \frac{W_2}{g} (L_2)^2 \tag{4.2}
\]

\[
w_1 L_1 \left( \frac{L_1}{2} \right) = w_1 L_2 \left( \frac{L_2}{2} \right) + W_2 L_2 \tag{4.3}
\]

where

\[
I_{CG_1} = \frac{1}{12} \frac{W_1}{g} L_1^2
\]

\[
I_{CG_2} = \frac{1}{12} \frac{W_2}{g} L_2^2
\]

\( g = \) gravitational acceleration \tag{4.2a}

4.2 Straight Elastic Blade Model

For this simplified model, it is postulated that the rotor blade remains straight (i.e., it cannot bend) but may shorten elastically in a quasi-static fashion by an amount \( \alpha \) appropriately computed during the blade-ring collision-response calculation (see Figs. 15a and 15b). This shortening is assumed to produce a uniform uniaxial strain given by \( \alpha/L_0 \) where \( L_0 \) is the initial pre-impact blade length from butt to tip of the idealized model of the blade shown in Fig. 14b. It is further postulated that the axial stress \( \sigma \) induced at the blade tip is given by \( \sigma = E(\alpha/L_0) \) where \( E \) is the elastic modulus of the blade material. In turn,
if \( A \) is the cross-sectional area of the blade at the tip, a quasi-steady axial compressive force given by

\[
F_A = EA \frac{\alpha}{L_0}
\]  

is assumed to act on the blade; an equal and opposite force acts on the ring at the contact station.

As indicated schematically in Figs. 15a and 15b, the ring is idealized as consisting of straight-line segments only for the purpose of estimating the fictitious penetration distance \( \alpha \) and the ring/blade impact point location \( I \) in terms of the distances \( s_{1I} \) and \( s_{2I} \) between ring nodes 1 and 2, for example, as shown in Fig. 15c. The use of the "correct curved ring geometry" for computing the quantities \( \alpha, s_{1I}, \) and \( s_{2I} \) would be more accurate but very much more complex; for present "initial exploration purposes", therefore, the simple straight-segment ring geometry is used to find \( \alpha, s_{1I}, \) and \( s_{2I} \). However, after these quantities have been calculated, one returns to a curved-ring model to estimate the surfact tangent angle \( \theta_{1} \) of the ring at the impact point as indicated in Fig. 15c, where straight-line segments between ring nodes 0, 1, 2, and 3 are illustrated; this is done since one is interested in defining the direction for force components normal and tangent to the curved ring at the impact point. Since the global coordinates \((Y_1, Z_1)\) of each ring node are known at any selected time instant, one can readily determine the angles \( \theta_{01}, \theta_{12}, \) and \( \theta_{23} \), for example. The surface tangent angles of the curved ring at nodes 1 and 2 are approximated, respectively, by

\[
\theta_1 = \frac{\theta_{01} + \theta_{12}}{2} \tag{4.5a}
\]

\[
\theta_2 = \frac{\theta_{12} + \theta_{13}}{2} \tag{4.5b}
\]
at the impact point, the surface tangent angle $\theta_I$ is approximated by linear interpolation as:

$$\theta_I = \theta_1 + \frac{s_{I1}}{S}(\theta_2 - \theta_1)$$  \hspace{1cm} (4.5c)

where $s$ is the distance from node 1 to node 2. This suffices for one to define the directions of force components normal and tangential to the surface at impact point I. Similar expressions apply if impact occurs between two other nodes.

The force $F_A$ given by Eq. 4.4 may be decomposed into components normal, $F_{AN}$, and tangential, $F_{AT}$, to the surface of the curved ring at the impact point:

$$F_{AN} = F_A \sin \bar{\theta}$$  \hspace{1cm} (4.4a)

$$F_{AT} = F_A \cos \bar{\theta}$$  \hspace{1cm} (4.4b)

where $\bar{\theta}$ is the angle between the blade axis and the tangent to the curved ring at the impact point (see Fig. 15d). However, for the case of frictionless blade/ring contact when treating the ring as an impenetrable body (as done herein), the only non-zero collision-interaction force present must be normal to the surface of the ring at the contact point. In this case $F_{AT}$ must be zero; this would be the case for the present model only at $\bar{\theta} = 90$ degrees.

To circumvent this difficulty and since the impact forces will generally be the most severe at $\bar{\theta} = 90$ degrees, the further simplifying approximation is made that the normal force $F_N$ is given by:

$$F_N = E A \frac{\alpha}{L_o}$$  \hspace{1cm} (4.5)

regardless of the value of $\bar{\theta}$. Using Eq. 4.5 for the estimated
normal force, it is next postulated that a tangential-direction frictional force $F_{Tf}$ opposing the relative tangential motion occurs when friction is present and may be approximated by

$$F_{Tf} = \mu F_N$$

(4.6)

where $\mu$ is an appropriate coefficient of sliding friction. The forces $F_N$ and $F_{Tf}$ acting, respectively, normal and tangential upon the surface of the ring have equal and opposite counterparts which act upon the blade at the contact point as depicted schematically in Fig. 15d.

In each step of the timewise step-by-step solution process, the motion of the curved containment ring and the motion of the blade are predicted as occurring independently under the action of known (zero or nonzero) forces. At the end of a given time step, the space occupancy of the fragment and of the idealized straight-segment ring model are compared. As depicted in Fig. 15b at time $t_{m+1}$, the blade appears to have "penetrated into" the ring structure by an amount $\alpha$. For the impact process itself, this model assumes that the ring is impenetrable (rigid) and, hence, quasi-instantaneous blade shortening by an amount $\alpha$ must occur and this, in turn, "leads approximately" to the strain, stress, and force conditions already postulated.

With the "externally applied loads" given by Eqs. 4.5 and 4.6 now acting appropriately on each body as indicated in Fig. 15d, the motion prediction for each body for the next $\Delta t$ time step is carried out, and the computation proceeds cyclically.

In the predictions reported in Subsection 4.6 for this model, two further simplifying assumptions which are invoked are that for the blade-motion calculation at any instant in time (a) the blade retains its original length $L_0$ and (b) its CG location
relative to its two ends remains unchanged.

4.3 Straight Elastic-Plastic Shortening Blade Model

This blade behavioral model is similar to that for the straight elastic blade model in that the blade is assumed to undergo no bending deformation. However, the blade can deform axially; further, the blade is assumed to consist of elastic, perfectly-plastic material so that permanent shortening can occur. The main attendant features of this model, which contrast with those of the previous model are: (1) the axial force is limited by the plastic yield stress and (2) permanent blade shortening with an associated reduced mass moment of inertia and altered force lever arm between the impacted tip and the blade's center of gravity (CG) occur.

Shown in Fig. 16a is a pre-impact ring/blade condition at time $t_m$, while predicted tentative blade and ring locations are depicted in Fig. 16b at time $t_{m+1} = t_m + \Delta t$; during this $\Delta t$ time interval, blade/ring collision has occurred. Since the tentative location of the blade has been determined by solving the equations of motion for translation of and rotation about the CG of the blade, the CG location and the angular orientation of the blade are regarded as known; at the same time, however, the containment ring is regarded as being impenetrable. Therefore, the total blade length is now $L_{m+1}$ (which is less than the length $L_m$ at time $t_m$). If blade/ring contact persists, $L_{m+1} = L_m - \alpha$. However, should contact between the blade and the ring be lost immediately after $t_{m+1}$, the increment of irreversible shortening of the blade is estimated as follows. First, however, an estimate of the axial force acting on the blade (consistent with the simplifying approximations invoked earlier) is needed; this situation is indicated schematically in Fig. 16c. Accordingly, the axial force acting on the blade at time $t_{m+1}$ is approximated by
\[(F_A^*)_{m+1} = (F_N)_{m+1} [\sin \bar{\theta} - \mu \cos \bar{\theta}] \quad (4.7)\]

where

\[(F_N)_{m+1} = AE \frac{\alpha_{m+1}}{L_m} \quad (4.8)\]

Then if \((F_A^*)_{m+1}/A\) is less than the yield stress \(\sigma_y\), no permanent deformation (shortening) occurs. However, if \((F_A^*)_{m+1}/A \geq \sigma_y\), the axial force is assumed to be equal to:

\[(F_A^*)_{m+1} = A \sigma_y \quad (4.9)\]

At the yield stress condition, the associated value of \(\alpha\) is called \((\alpha_{cr})_{m+1}\) and is given by the condition:

\[\sigma_y = \frac{(F_A^*)_{m+1}}{A} = \frac{(\alpha_{cr})_{m+1}}{L_m} E \quad (4.10)\]

Using Eq. 4.7 in Eq. 4.10 and solving for \((\alpha_{cr})_{m+1}\), one obtains

\[(\alpha_{cr})_{m+1} = \frac{\sigma_y L_m}{E [\sin \bar{\theta} - \mu \cos \bar{\theta}]} \quad (4.11)\]

If the blade/ring contact is lost, the incremental permanent shortening \((\Delta L)_{m+1}\) of the blade is given by:

\[(\Delta L)_{m+1} = -(\alpha - \alpha_{cr})_{m+1} = -\left[\alpha_{m+1} - \frac{L_m \sigma_y}{E (\sin \bar{\theta} - \mu \cos \bar{\theta})}\right] \quad (4.12)\]

where a negative value of \(\Delta L\) means shortening; this is shown schematically in Fig. 16c. Accordingly, the residual blade length if blade/ring contact were now lost would be
\[ L_{m+1} = L_m + \Delta L \]  

(4.13)

Since

\[ L_{m+1}(w_i)_m = L_o w_i \]  

(4.14)

one can readily calculate \((w_i)_{m+1}\) and thereby establish the distance from the known blade CG location to the blade tip of the now-deformed blade. Such calculations are carried out for each of the time steps during which blade/ring contact is predicted to occur.

In this model as in the previous model, the simplifying approximation is made that \((F_N)_{m+1} = (F_A)_{m+1}\) where \((F_A)_{m+1}\) is given by either Eq. 4.7 or 4.9, as applicable. Further, the Eq. 4.6 estimate for the surface-tangential friction force opposing blade-ring relative motion is employed. Also, the surface tangent angle \(\theta_i\) at the impact point I is given by Eq. 4.5c (see Fig. 15d).

4.4 Elastic-Plastic Curling Blade Model

4.4.1 Motivation and Features

In fragment containment experiments conducted at the NAPTC (Refs. 2-6 and 21) wherein the attacking fragment consisted either of a single engine rotor blade or of bladed-disk segments, it has been observed that often the blades undergo a curling type of blade deformation. Thus, in the present study which is concerned with exploring the consequences of employing various fragment assumed-deformation modes, an idealized blade behavioral model which includes the most prominently-observed blade curling deformation features has been given some exploratory study. The general features and postulated behavior of this model are described in the remainder of this subsection. Supportive details are given
in Subsections 4.4.2 through 4.4.4.

The present idealized model assumes that the blade has non-uniform geometric properties along its length and that it is comprised of elastic, perfectly-plastic, strain-rate-insensitive material. While the cross-sectional area of the idealized blade is constant along its length, the minimum bending area moment of inertia and its fully-plastic moment-carrying ability are permitted to vary along the length of the blade as explained later at pertinent points in the text.

The principal behavioral features of this assumed-mode elastic-plastic curling blade model may be described most clearly and conveniently, perhaps, by the following enumerated "sequence of events" (see Figs. 17a through 17e):

1. At initial impact the blade is straight and undergoes elastic-type impact loading given by Eqs. 4.7 and 4.8. As in the two previous models, the "impact forces" experienced by the ring are approximated by Eqs. 4.5 and 4.6 for the normal and the tangential component, respectively.

2. When the axial load $F_{A}$ on the blade as given by Eq. 4.7 reaches a prescribed critical value $F_{cr}$, blade curling is assumed to commence; the associated interaction distance is termed $\alpha_{cr}$ and is given by

$$\alpha_{cr} = \frac{F_{cr} L}{E A [\sin \psi + \mu \cos \psi]}$$

(4.15)

3. Blade curling is assumed to proceed by instantly forming a 90-degree circular sector having the

Strain-rate-sensitive behavior could be explored in future work.
following radius for the first time increment after initiation of curling:

\[ r_i = \frac{\alpha}{\frac{\pi}{2} - \frac{1 - \cos \psi}{\sin \psi}} \] (4.16)

(the rationale for this value is discussed in Subsection 4.4.2).

4. Taking the direction of blade curling as that depicted in Figs. 17b-17e, the curl radius \( r \) is thereafter assumed to increase (according to rules to be explained later) until a prescribed "final" value \( r_f \) is reached. This curling is defined to occur only over a 90-degree sector \( \beta \).

5. Thereafter, further blade circular-arc curling is assumed to occur with the final curl radius \( r_f \), with \( \beta \) increasing from its initial 90-degree value. The junction of the circular-arc curled portion and the straight blade portion at any given time instant is termed \( Q \). If at point \( Q \) the applied axial load and the moment stay within the boundary of the interaction curve for the fully-plastic limit moment and the fully-plastic limit axial force, no increment of blade curling is permitted. However, if that boundary is reached or "exceeded" further curling occurs, as described later.

Having outlined the postulated behavior of the blade, Subsection 4.4.2 is now devoted to describing this behavioral sequence in terms of quantitatively descriptive rules.

+ Similar but somewhat more complex logic must be employed if curling occurs in the opposite direction.
4.4.2 Blade Deformation Rules

It is convenient to describe the postulated deformation behavior for this idealized model as a sequence of five phases, the geometries for which are depicted in Figs. 17a through 17e, while the force pictures associated with most of these phases are shown in Figs. 18a through 18c.

Phase 1: Straight Blade

Before initial impact and during the phase 1 type of impact, the blade is straight and experiences only elastic impact loading. Shown in Fig. 17a for this type of impact at a given instant in time are the tentative "uncorrected location" of the blade which "penetrates" into the impenetrable ring and the corrected blade configuration wherein the blade is shortened so as not to "penetrate" the ring, while preserving the blade's CG location and angular orientation \( \theta_f \). The normal and the tangential force experienced by the curved ring at the impact point with surface-tangent angle \( \theta_I \) (Eq. 5.5c) are approximated by Eqs. 4.5 and 4.6 (which are repeated here for convenience):

\[
F_N = EA \frac{\alpha}{L_0} \tag{4.5}
\]

\[
F_{Tf} = \mu F_N \tag{4.6}
\]

As in the previous models, the axial force acting on the straight elastic blade is approximated by:

\[
F_A^* = F_N [\sin \psi + \mu \cos \psi] \tag{4.17}
\]

An exploded schematic for the forces acting on the "two independent bodies" immediately following the conceptual determination of the corrected configuration of Fig. 17a is shown in Fig. 18a.
Phase 2: Curl Initiation Instant

It is assumed that the phase 1 elastic straight-blade behavior persists until the collision-induced axial force \( F_A \) approximated by Eq. 4.17 reaches a critical value \( F_{cr} \). Accordingly, using \( F_{cr} \), Eq. 4.17, and Eq. 4.5, one obtains

\[
F_{cr} = EA \frac{\alpha}{L_o} [\sin \psi + \mu \cos \psi]
\]

(4.18)

Hence,

\[
\alpha_{cr} = \frac{F_{cr} L_o}{EA [\sin \psi + \mu \cos \psi]}
\]

(4.19)

where \( \alpha_{cr} \) is the value of \( \alpha \) needed to make \( F_A = F_{cr} \). At the first time instant at which, from tentative predictions of blade and ring positions, it is computed that \( \alpha > \alpha_{cr} \), it is postulated that the blade length portion \( \alpha \) instantly curls into a 90-degree circular sector with the (initial) radius \( r_i \). As indicated in Fig. 17b, the following two conditions of approximation are used to convert the tentative configuration to the corrected configuration for the initial curl condition:

1. The total blade length is preserved:

\[
L_o = L' + \alpha = L'' + \frac{\pi}{2} r_i
\]

(4.20)

where \( L_o \) is the total blade length before initial impact and the other quantities are defined on Fig. 17b.

2. The perpendicular distance from the butt end of the blade to the impacted ring surface is preserved:
\[ \frac{L'}{\sin \gamma} = \frac{L''}{\sin \gamma} + r_i (1 - \cos \gamma) \]  

(4.21)

Solving Eqs. 4.20 and 4.21 simultaneously,

\[ r_i = \frac{\alpha}{\frac{\pi}{2} - \frac{1 - \cos \gamma}{\sin \gamma}} \]  

(4.22)

Of the various possibilities for choosing the critical axial load \( F_{cr} \), the elastic buckling load for a uniform column of length \( L_a \) (see Fig. 17b), average bending stiffness EI, and cantilevered at the CG has been selected for convenience:

\[ F_{cr} = k_1 \frac{\pi^2 EI}{L_a^2} \]  

(4.23)

where \( k_1 = 0.25 \) for frictionless cases \((\mu = 0)\) corresponding to a cantilevered-free condition, and \( k_1 = 2.05 \) when \( \mu \neq 0 \); the latter corresponds to a cantilevered-pinned condition.

A schematic of the collision-induced forces acting immediately upon the corrected initially-curled configuration of Fig. 17b is shown in Fig. 18b.

Phase 3: Curling Blade with Increasing Curl Radius, \( r \)

For phase 3 it is postulated that after initial blade curling has occurred, further blade curling in the form of a circular-arc quadrant with an increasing radius \( r \) occurs if at point \( Q \) (the junction between the curled and the straight-blade portion) the moment \( M \) and the axial force \( N = F_A \) there resulting from only\(^\dag\) the collision-induced forces \( F_N \) and \( \mu F_N \) reach or exceed the

\(^\dag\)Strictly speaking, the evaluation of \( M \) and \( N \) at any point along the blade should include not only \( F_N \) and \( \mu F_N \) but also the inertia forces of the blade from the tip to the point of evaluation (such as point \( Q \)). In view of the many approximations made in this assumed mode procedure and because of the attendant simplified computations, these inertia forces have been neglected in evaluating \( M \) and \( N \) at point \( Q \). Accordingly, these force resultants may tend to be somewhat overestimated in the present procedure.
boundary of the interaction curve (Fig. 19) for the fully-plastic moment $M_o$ and the fully-plastic axial force $N_o$. Shown in Fig. 19 is the well-known yield boundary (Ref. 22):

$$\frac{|M|}{M_o} + \frac{N^2}{N_o^2} = 1$$  \hspace{1cm} (4.24)

Referring to Fig. 18c, the $M$ and $N$ at point $Q$ are given by

$$M_Q = F_N r \left[ \sin \psi - \mu (1 - \cos \psi) \right]$$  \hspace{1cm} (4.25)

$$N_Q = (F_N^*)_Q = -F_N \left[ \sin \psi + \mu \cos \psi \right]$$  \hspace{1cm} (4.26)

where a positive $M_Q$ causes compression on the fibers nearest to the center of curvature of the curled portion, and a positive (negative) value of $N_Q$ means tension (compression). Note that

$$\frac{M_Q}{N_Q} = \frac{-r \left[ \sin \psi - \mu (1 - \cos \psi) \right]}{\left[ \sin \psi + \mu \cos \psi \right]}$$  \hspace{1cm} (4.27)

is defined by $r$, $\psi$, and $\mu$. Where the "current condition" as represented (along ray OK, for example) on Fig. 19 depends upon the value of $F_N$. If that condition falls between points 0 and B, that condition lies within the yield boundary; in this case no additional curling of the blade is assumed to take place at that time instant. However, if the value of $F_N$ causes the condition-location point to lie either at B or between B and K, a further increment of blade curling is assumed to occur by an amount to be described shortly; also, under these conditions $F_N$ is defined to be limited by the value $F_{N_b}$ required to place the condition location on the yield boundary (that is, at point B itself). It should be recalled that the geometric properties of the blade vary along its length; also for any location of point $Q$, $M_o(Q)$ is known ($N_o = A \sigma_y$).
Now, the simple approximate procedure by which the tentative configuration (curl radius $r$) of Fig. 17c is converted to the corrected configuration with curl radius $r + \Delta r$ to remove the penetration distance $\alpha$ and achieve compatible blade contact with the ring in Fig. 17c is described next. As a simple approximation consider the bending-induced elastic deflection of a uniform curved beam of radius $r$, cantilevered at point $Q$, and subjected to the loads $F_N$ and $\mu F_N$ shown in Fig. 18c. The deflection $\delta$ in the direction of the load $F_N$ is given from the unit virtual load method by

$$
\delta = \int_0^\psi \frac{Mm}{EI} d\psi = \int_0^\psi \frac{Mm}{EI} r d\psi
$$

(4.28)

where $M$ is the moment distribution from loads $F_N$ and $\mu F_N$ from the point of application of $F_N$ to point $Q$, and $m$ is the moment distribution along the same region caused by the application of a unit virtual load at the location and in the direction of $\alpha$ (see Fig. 17c). Applying the pertinent information to Eq. 4.28 and evaluating, one obtains

$$
\delta = \frac{F_N}{EI} \frac{r^3}{6} \mathcal{K}(\mu, \psi)
$$

(4.29)

where

$$
\mathcal{K}(\mu, \psi) = \left(\frac{1}{4} \psi - \frac{1}{4} \sin 2\psi\right) + \mu (-1 + \cos \psi + \frac{1}{2} \sin^2 \psi)
$$

(4.29a)

When the applied loading is $F_{N_b}$, let the corresponding deflection be called $\alpha_b$:

$$
\alpha_b = \frac{F_{N_b}}{EI} \frac{r^3}{6} \mathcal{K}(\mu, \psi)
$$

(4.30)
Thus for \( a > a_b \), \((a-a_b)\) represents the amount of the to-be-removed penetration which will be used to increase the curl radius by an amount \( \Delta r \). Accordingly, referring to Fig. 17c, the conditions for configuration correction are:

1. The total blade length is preserved:

\[
L_o' = L' + \frac{\pi}{2} r = L'' + \frac{\pi}{2} (r+\Delta r)
\]  (4.31)

2. The perpendicular distance from the butt end of the blade to the impacted ring surface is preserved:

\[
L'_s \sin \psi + r(1-\cos \psi) - (\alpha - a_b) = L'' \sin \psi + (r+\Delta r)(1-\cos \psi)
\]  (4.32)

Solving Eqs. 4.31 and 4.32 simultaneously,

\[
\Delta r = \frac{\alpha - a_b}{\frac{\pi}{2} \sin \psi + \cos \psi - 1}
\]  (4.33)

If \( a < a_b \), \( \delta \) of Eq. 4.29 is equal to \( \alpha \); the associated \( F_N \) is obtained by solving Eq. 4.29.

This type of blade curling (that is, with an increasing curl radius) is postulated to continue until the curl radius \( r \) reaches a prescribed final value called \( r_f \).

Phase 4: Blade Curling with the Fixed Curl Radius, \( r_f \)

After the radius of the curled blade quadrant has reached the prescribed final radius \( r_f \), it is postulated that the curl radius remains fixed at \( r_f \); further blade curling will then increase the included angle \( \beta \) of the curled portion from its initial 90-degree value. Figure 17d depicts this phase-4 behavior.
In determining the collision-induced loading, the penetration distance $a$ is used and conditions at point Q of Fig. 17d are monitored as in phase 3. That is, if $a > a_b$ where $a_b$ is given by Eq. 4.30 with $r = r_f$, $F_N = F_{N_b}$; the following conditions (see Fig 17d) are used to determine the curl angle increment $\Delta\beta$:

1. The total blade length is preserved:

$$L_o = L' + \beta r_f = L'' + (\beta + \Delta\beta) r_f$$  \hspace{1cm} (4.34)

2. The perpendicular distance from the butt end of the blade to the impacted ring surface is preserved:

$$L' \sin \gamma + r_f (1 - \cos \gamma) = \alpha - a_b$$

$$= L'' \sin \gamma + r_f (1 - \cos \gamma)$$  \hspace{1cm} (4.35)

Solving Eqs. 4.34 and 4.35, one obtains

$$\Delta\beta = \frac{\alpha - a_b}{r_f \sin \gamma}$$  \hspace{1cm} (4.36)

If $a < a_b$, $\delta$ of Eq. 4.29 is equal to $\alpha$; the associated $F_N$ is obtained by solving Eq. 4.29 with $r$ replaced by $r_f$.

Phase 5: Impact of Curled Blade with No Further Curling

When the ring-blade impact point more and more closely approaches point Q (or when $\Psi$ becomes sufficiently small, Fig. 17e), the phase-4 model becomes less and less plausible; that model would predict the impact force to approach infinity as the contact point approaches point Q, but a finite impact force would occur. Therefore, when the moment arm $r_f \sin \Psi$ for $F_N$ about point Q becomes small, a rather arbitrary limit to the value which $F_N$ can reach is imposed as $F_N \sqrt{1 + \mu^2} = \sigma_{yc} A_{contact}$ where the "contact
area" $A_{\text{contact}}$ is taken for convenience to be $k_2 A$ (A being the cross-sectional area of the blade), $\sigma_{yc}$ is the compressive yield stress of the blade, and $k_2$ is a number which may be assigned such as unity, for example. When this limit is reached, the use of the phase-4 model is terminated and the following simplified model is then employed. A more accurate and rational model could be devised but such is not believed to be warranted since by this stage the impact conditions have been "de-energized" to the extent that only relatively gentle impacts with attendant small forces occur.

For this phase-5 behavior, the normal component of the impact force is estimated from Eq. 3.3, the Hertz force-deformation relation for a ball of radius $r_f$ impacting against a surface:

$$F_N = \frac{2E r_f}{3(1-\nu^2)} \alpha^{3/2}$$

This is a gross simplifying approximation but is adopted for convenience.

Alternatively, if desired, one could employ the empirical Meyer law as given by Eqs. 3.4 and 3.5 for estimating the normal component of the impact force for the phase-5 type of impact.

Throughout the 5 phases of blade-ring impact, the space occupancy and the motion of the blade at each time instant are determined (1) by predicting the translational motion of and the angular motion about the center of gravity of the blade and (2) by employing information on the "instantaneously known pre-corrected geometry" of the deformed blade. It is useful, therefore, to express the blade CG location and the $I_{CG}$ in terms of the characterizing geometric parameters of the deformed blade during each phase of its behavior. This is discussed next in Subsection 4.4.3.
4.4.3 CG Location and the $I_{CG}$ for the Deformed Blade in Terms of Characterizing Geometric Quantities

It should be recalled that the pre-impact idealized blade model has the following properties compared with the actual pre-impact blade fragment:

(1) the same total weight $W$,
(2) the same mass moment of inertia $I_{CG}$ about its CG, and
(3) the same distance $L_1$ from the blade tip to its CG.

Thus, initially, the idealized model correctly represents the rigid-body dynamic properties of the actual fragment and also includes the proper $L_1$ length which is pertinent to predicting the time and location of initial impact.

During the straight-blade phase 1 behavior, $I_{CG}$ and the blade tip-to-CG distance do not change sensibly. However, after curling commences, these quantities can change significantly and may be determined readily from the deformed-blade geometry by noting that during all deformation phases $w_1$, $w_2$, and the total blade length $L_0$ are assumed to remain unchanged. It is convenient, therefore, to deal with weight $W_2$, the straight blade section and its weight $W_s$, and the curved sector and its weight $W_c$. All curled blade conditions are accommodated by considering the configuration shown in Fig. 20a. As indicated on Fig. 20b, the CG may be located with respect to the $W_2$ location and the straight-blade portion by the two distances $\Delta_P$ and $\Delta_N$, which may be verified readily to be given by:

$$\Delta_P = \frac{W_s \left( \frac{L}{2} \right) + W_c \left( L + \bar{y}_c \right)}{W}$$  \hspace{1cm} (4.38)$$

$$\Delta_N = \frac{W_c \bar{x}_c}{W}$$  \hspace{1cm} (4.39)
where \( l = L_0 - r_\beta \)  \( \quad \) (4.40a)

\[ W_S = w_1 l \]  \( \quad \) (4.40b)

\[ W_c = w_1 r_\beta \]  \( \quad \) (4.40c)

and where \( l \) is defined in Fig. 20a, and \( \overline{x}_c \) and \( \overline{y}_c \) as identified in Fig. 20a are given by:

\[ \overline{x}_c = r \left( \frac{\beta - \sin \beta}{\beta} - \frac{h^2}{12r} \right) \]  \( \quad \) (4.41a)

\[ \overline{y}_c = \left( r + \frac{h^2}{12r} \right) \left( 1 - \cos \beta \right) \]  \( \quad \) (4.41b)

From Fig. 20b, it is seen that the mass moment of inertia \( I_{CG} \) for the curled blade about its CG may be expressed by:

\[ I_{CG} = \frac{W_s}{g} \left[ \Delta_N^2 + \Delta_P^2 \right] + I_s + \frac{w_s}{g} \left[ \Delta_N^2 + \left( \frac{l}{2} - \Delta_P \right)^2 \right] 
+ I_c + \frac{w_c}{g} \left[ \left( \overline{x}_c - \overline{A}_N \right)^2 + \left( l + \overline{y}_c - \Delta_P \right)^2 \right] \]  \( \quad \) (4.42)

where \( I_s \) and \( I_c \) are the mass moments of inertia about its CG of the straight portion and the curved portion, respectively:

\[ I_s = \frac{1}{4} \frac{W_s}{g} l^2 \]  \( \quad \) (4.43a)

\[ I_c = \frac{W_c}{g} \left[ r^2 - \left( \overline{x}_c - r \right)^2 - \overline{y}_c^2 \right] \]  \( \quad \) (4.43b)

Having found \( \Delta_N \) and \( \Delta_P \) which locates the curled blade with respect to \( W_2 \) and the straight portion of the blade, the global space occupancy of the curled blade may be readily determined since the global coordinate CG location \( (Y_{CG}, Z_{CG}) \) and the global orientation \( \theta_f \) (see Fig. 16b) of the straight portion of the
blade are known. This space occupancy information can be used at each time step of the calculation to determine whether or not this space occupancy conflicts with that of the containment ring.

4.4.4. Blade Limit Load Approximations

In discussing the deformation rules for the idealized curling blade model, the chosen criterion for further blade curling involves reaching the interaction curve for the fully-plastic limit moment $M_o$ and the fully-plastic limit axial load $N_o$ as indicated in Fig. 19. It will be recalled that a constant cross-sectional area $A$ is postulated for the idealized model and hence $N_o = \sigma_y A$ is a constant; this closely approximates the actual conditions over about one half or more of the blade's length, including the tip region. However, because the blade's camber and chord change along its length, the minimum (principal) bending area moment of inertia $I_p$ varies along the length of the blade; the associated fully-plastic bending moment $M_o$ also varies with the location along the blade. For the idealized blade model, the "built-in twist" of the actual blade is ignored. Accordingly, the idealized blade is regarded as untwisted, but at any spanwise station the idealized model is assigned a fully-plastic moment-carrying ability which approximates that of the actual blade referred to bending about the centroidal axis of the actual section with the least area moment of inertia ($I_p$).

Since typical experimental results show that blade curling often occurs over less than one half of the blade's length, modeling of the fully-plastic moment ($M_o$) resistance of the blade for about one-half or two-thirds of the span was selected as a reasonable approximation (see Fig. 21); an inaccurate representation of $M_o$ for the inner half of the span is of no practical consequence since this information will seldom if ever enter in the fragment/ring response predictions. Accordingly, the blade's pre-impact geometries at the tip, 1.094 in. from the tip, and at the initial
CG station (2.188 in. from the tip), as shown schematically in Fig. 21, were approximated as consisting of simple shapes indicated schematically in Fig. 21 at each of these three stations. For each section, the section area CG was determined and the fully-plastic moment \( M_o \) about the \( I_p \) axis was computed. The \( M_o \) values for these three stations were then used in an interpolation function to express \( M_o \) as the following function of spanwise location \( \eta/L_o \) (see Fig. 14b) (rather than the less convenient \( x/L_1 \) of Fig. 21):

\[
M_o = (M_o)_{tip} \left[ +3.627 - 2.692 \frac{\eta}{L_o} + 0.065 \left( \frac{\eta}{L_o} \right)^2 \right] \tag{4.45a}
\]

where \( (M_o)_{tip} \) represents the value at the tip. In computing \( M_o \) at each station, it was assumed that the geometry of the cross-section when \( M_o \) is reached would be unchanged from the initial geometry; this "unchanging-geometry approximation" is reasonable for the elastic range and perhaps for the early stages of plastic bending, but clearly degrades as more and more severe bending ensues (the camber of the blade where severe plastic bending occurs nearly disappears). Time and effort limitations, however, have not permitted a more thorough analysis of this matter nor have laboratory measurements been made of the \( M_o \) characteristics of this blade. Therefore, Eq. 4.45a for \( M_o \) has been employed as providing hopefully a reasonable functional dependence of \( M_o \) on \( \eta/L_o \), with the magnitude \( (M_o)_{tip} \) varied in a parametric sense to assess the fragment/ring response sensitivity to such variations. Further work would be required to obtain an accurate evaluation of the magnitude and distribution of \( M_o \) along the actual blade, including both its static and strain-rate dependence, if desired.

Finally, only for the purpose of estimating a more consistent and realistic value of \( N_o \) \( (N_o = A \sigma_y) \) as a function of location along the blade, the following blade cross-sectional area distribution was used (see Fig. 21):

\[
A = (A)_{tip} \left[ +2.164 - 0.617 \frac{\eta}{L_o} - 0.547 \left( \frac{\eta}{L_o} \right)^2 \right] \tag{4.45b}
\]
4.5 Review of Solution Process

Having described the salient features of the present simplified models for predicting free containment ring responses to engine rotor blade fragment impact, it is useful to review briefly the main steps in this solution process, which are indicated schematically in Fig. 4.

4.5.1 Collision Inspection

For the purpose of describing the essential aspects of the process of checking to determine whether or not a fragment/ring collision has occurred and where that impact takes place, consider the straight-blade fragment example shown in Fig. 22. Let the origin of the global Cartesian coordinates $Y,Z$ be located at the center of the pre-impacted free ring. At a given instant in time, the global coordinates $(Y_i,Z_i)$ of every node point of the ring are known; also known are the CG coordinates $(Y_{CG},Z_{CG})$, the angular orientation $\theta_f$ of the fragment, and the distances from the CG to the tips T1 and T2 of the fragment. The inner surface of the idealized straight-segment ring at nodes $N_1$ and $N_2$ can be readily computed from a knowledge of the nodal coordinates and the thickness $h$ of the ring as:

$$
\begin{align*}
Y_{SN1} &= Y_{N1} + \frac{h}{2} \sin \theta_{12} \\
Z_{SN1} &= Z_{N1} - \frac{h}{2} \cos \theta_{12}
\end{align*}
$$

One can then write the equations for the straight lines connecting (a) point $SN_1$ with $SN_2$ and (b) the fragment's CG with point T1; the solution for the intersection of these two lines (or their extensions) will identify the impact point "I".

In the collision inspection process, one computes the angle $\theta$ between the $+Y$ axis and the radius vector from the $Y,Z$ origin to each of the points $SN_1$, $SN_2$, $T1$, and $I$. For point I to be an impact point on the inner surface of the ring between $SN_1$ and $SN_2$, one must have $\theta_{SN1} \leq \theta_{T1} \leq \theta_{SN2}$. If $|R_{T1}| < |R_I|$, no impact has
occurred; however, if \( \| \mathbf{R}_{I1} \| \geq \| \mathbf{R}_{I} \| \), an impact has occurred at point I.

Inspections of this type can be carried if and as needed for the T2 tip.

4.5.2 Penetration Distance, Interaction Forces, and Blade Deformation

If a collision has occurred at point I (see Fig. 22), the "penetration distance" \( \alpha \) may be computed from:

\[
\alpha = \left[ (\gamma_i - \gamma_{i1})^2 + (Z_i - Z_{i1})^2 \right]^{1/2}
\]

(4.47)

Since \( \alpha \) is known\(^{+}\), one may apply the force-penetration rules associated with whichever of the previously-described models is pertinent in order to determine the interaction forces \( F_N \) and \( \mu F_N \). If \( \alpha \) is sufficiently large, (a) blade shortening \( \Delta L \) and (b) blade curling \( \Delta r \) or \( \Delta \beta \) may occur according to these already-described rules.

4.5.3 Collision Forces Acting on the Ring

Figure 23 indicates schematically the forces \( F_N \) and \( \mu F_N \) acting upon the ring segment located at point I between ring finite element nodes N1 and N2; at point I the surface-tangent inclination angle is \( \theta_I \). In the present analysis, these forces are treated as being externally applied only to the ring finite element connecting nodes N1 and N2. Since the generalized degrees of freedom for this finite element are located at these two nodes, generalized external forces acting only at those nodes must be defined in such a way as to represent the given forces \( F_N \) and \( \mu F_N \) acting at point I. By the standard variational procedure for the assumed displacement finite-element method (Ref. 9),

\(^{+}\)Similar calculations enable one to determine \( \alpha \) readily for the elastic-plastic curling blade model.
one may compute the equivalent nodal generalized forces. However, an approximate computationally simpler procedure has been employed in the present study. This consists of apportioning $F_N$ and $\mu F_N$ (in their known directions) to local tangential and normal components at nodes $N1$ and $N2$ as depicted in Fig. 23b such that the four forces $F_{N1}$, $F_{T1}$, $F_{N2}$, and $F_{T2}$ represent the same resultant force as do $F_N$ and $\mu F_N$. This force apportioning between nodes $N1$ and $N2$ is inversely proportional to the fractional distance $(s_I)$ of point I from the respective nodes. The resulting forces are given by (see Fig. 23):

$$F_{T1} = F_N \left[ 1 - \frac{s_I}{s} \right] \left[ \sin(\theta_1 + \theta_{N1}) - \mu \cos(\theta_1 + \theta_{N1}) \right]$$

$$F_{N1} = F_N \left[ 1 - \frac{s_I}{s} \right] \left[ \cos(\theta_1 + \theta_{N1}) + \mu \sin(\theta_1 + \theta_{N1}) \right]$$

$$F_{T2} = F_N \left[ \frac{s_I}{s} \right] \left[ \sin(\theta_1 + \theta_{N2}) - \mu \cos(\theta_1 + \theta_{N2}) \right]$$

$$F_{N2} = F_N \left[ \frac{s_I}{s} \right] \left[ \cos(\theta_1 + \theta_{N2}) + \mu \sin(\theta_1 + \theta_{N2}) \right]$$

4.5.4 Motions of the Ring and the Fragment

With the forces given by Eqs. 4.48a through 4.49b being the only externally-applied collision-induced forces acting on the ring, the equations of motion for the ring and their solution to determine the locations of all nodal stations at the next instant of time as described in Subsection 2.2.1 may be carried out.

Similarly, the equations of motion for the fragment (Eqs. 2.7a, 2.7b, and 2.8) with forces $F_N$ and $\mu F_N$ of known magnitude, direction, and point of application, may be solved by the procedure of Subsection 2.2.1 to obtain the CG location and the fragment orientation $\theta_f$ at the next instant in time. Thereafter, this process is carried out cyclically for as long as desired.
4.6 Predictions and Comparisons

The application of the three idealized fragment models described in Subsections 4.2, 4.3, and 4.4 is discussed in Subsections 4.6.1, 4.6.2, and 4.6.3, respectively. Predictions are compared with experiment in Subsection 4.6.4. The responses of the free 2024-T3 aluminum containment ring of NAPTC Test 91 subjected to impact by a single T58 rotor blade fragment are examined. The pre-impact geometric and material properties of the ring and of the fragment are given in Table 1. For the ring, it was assumed that the material properties may be approximated reasonably as being elastic, perfectly-plastic strain-rate sensitive\(^+\) (EL-PP-SR) with \(E = 10^7\) psi, \(\sigma_0 = 50,000\) psi, \(D = 6500\) sec\(^{-1}\), and \(p = 4\). It is assumed that the rotor blade fragment is either elastic (EL) or elastic, perfectly-plastic (EL-PP) with \(E = 30 \times 10^6\) psi and \(\sigma_0 = 120,000\) psi. Finally, it is useful to note the following pre-impact blade fragment properties:

- Velocity of CG = 7884 in/sec
- Angular Velocity = 1638.3 rad/sec
- Tip Velocity = 11,467 in/sec
- Weight = 0.084 lb.
- \(I_{CG} = 2.163 \times 10^{-4}\) in-lb-sec\(^2\)
- Tip-to-CG Distance = 2.188 in
- Translational Kinetic Energy = 6756 in-lb
- Rotational Kinetic Energy = 290.3 in-lb

In carrying out these calculations, it was assumed, for convenience, that "blade fragment release" occurred when the blade was aligned with the +Z axis where \(\theta_f = 90\) degrees -- this is termed the time \(t = 0\) condition.

For all calculations discussed in this subsection, the ring was modeled in ring quadrants Q1, Q2, Q3, and Q4 (see Fig. 3) by 10, 6, 6, and 6 equal-length finite elements, respectively. Accordingly, initial impact occurs at \(t = 554\) microseconds after "blade

\(^+\)The strain-rate sensitivity parameters \(D\) and \(p\) serve to raise the quasi-instantaneous yield stress \(\sigma_y\) at strain rate \(\dot{\varepsilon}\) according to \(\sigma_y = \sigma_0 \left[ 1 + \left| \frac{\dot{\varepsilon}}{D} \right| \right]^{1/p}\) where \(\sigma_0\) is the static uniaxial yield stress.
release". Also, a time increment $\Delta t$ of one microsecond was used in all of the predictions discussed in this subsection (this value also was used in corresponding calculations in Ref. 9).

### 4.6.1 Straight Elastic Blade Model

For this straight elastic blade model (termed SEB, for convenience), ring-fragment response predictions were carried out for frictionless impact ($\mu = 0$) and for $\mu = .15$; each of these calculations spanned a duration of 646 microseconds after initial impact. For discussion it is convenient to use the abbreviation TAI for "time after initial impact". During the 646 microsecond TAI time spans of these two calculations, several distinct periods of blade-ring contact and force interaction were noted, with smaller peak $F_N$ forces occurring with each successive period. Summarized for $\mu = 0$ and $\mu = .15$ are the following interesting quantities: time, impact duration, angular orientation and velocity of the blade, peak $F_N$, and the $\theta$-region of the ring which experienced impact:

<table>
<thead>
<tr>
<th>Impact Period</th>
<th>Time (usec)</th>
<th>Impact Duration (usec)</th>
<th>Blade Data</th>
<th>Peak $F_N$ (lbs)</th>
<th>Impacted $\theta$-Region of Ring (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>554</td>
<td>0</td>
<td>141.9, 1638.3</td>
<td>134.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>637</td>
<td>83</td>
<td>154.3, 3440.9</td>
<td>140.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>724</td>
<td>170</td>
<td>171.5, 3440.9</td>
<td>148.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>771</td>
<td>217</td>
<td>181.4, 3980.6</td>
<td>152.6</td>
<td></td>
</tr>
</tbody>
</table>

No further impacts occurred for TAI $< 646\, \mu$sec.

*The effects of various values of $\mu$ are discussed more extensively in connection with predictions for the other two blade fragment models.*
### Straight Elastic Blade Results for μ = .15

<table>
<thead>
<tr>
<th>Impact Period</th>
<th>Time (μsec) TAI</th>
<th>Impact Duration (μsec)</th>
<th>θₚ</th>
<th>ωₚ</th>
<th>Peak Fₚ N</th>
<th>Impacted θ-Region of Ring (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>554 0</td>
<td>86</td>
<td>141.9</td>
<td>1638.3</td>
<td>22,132</td>
<td>134.6</td>
</tr>
<tr>
<td></td>
<td>640 86</td>
<td></td>
<td>150.3</td>
<td>1864.7</td>
<td></td>
<td>139.8</td>
</tr>
<tr>
<td>2</td>
<td>723 169</td>
<td>70</td>
<td>159.1</td>
<td>1864.7</td>
<td>9,827</td>
<td>144.5</td>
</tr>
<tr>
<td></td>
<td>793 239</td>
<td></td>
<td>165.7</td>
<td>1383.6</td>
<td></td>
<td>148.3</td>
</tr>
<tr>
<td>3</td>
<td>911 357</td>
<td>80</td>
<td>174.9</td>
<td>1383.6</td>
<td>6,094</td>
<td>154.3</td>
</tr>
<tr>
<td></td>
<td>991 437</td>
<td></td>
<td>179.8</td>
<td>746.6</td>
<td></td>
<td>157.7</td>
</tr>
<tr>
<td>4</td>
<td>1177 623</td>
<td>*</td>
<td>187.1</td>
<td>746.6</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

*Contact was continuing when the calculation was terminated.

For the 646 microsecond time span after initial impact, it is seen that (a) only two periods of impact are observed for μ = 0 but (b) four periods of impact occurred for μ = .15. Also note that friction has a pronounced effect upon blade fragment motion as indicated by the θₚ and the ωₚ behavior; consequently, the θ-region of the ring which is impacted during each impact period is also affected.

From Fig. 24 it is seen that the time histories of Fₚ N for μ = 0 and μ = .15 are very similar during the first contact period but thereafter differ significantly. However, the blade motions are very different as is seen from Fig. 25 where the time histories of θₚ and ωₚ are given. Finally, Fig. 26 shows the pre-impact quadrant-one (Q1) ring profile compared with predictions at TAI = 646 microseconds for the μ = 0 and the μ = .15 calculation; here one sees that at this instant in time the effects of the
prolonged impact and of the larger θ-region of impact on the ring are manifest by the more severe deformation and the greater "θ-extent" of that deformation for the $\mu = .15$ case as compared with the frictionless ($\mu = 0$) case. The most striking and expected difference between these two calculations, however, is that the fragment motion is greatly affected by the value of the friction coefficient (or force).

Finally, it is interesting to compare certain pre-impact blade quantities with corresponding values at the end of these predictions ($t = 1200 \mu\text{sec}$ or TAI = 646 μsec):

<table>
<thead>
<tr>
<th></th>
<th>Immediately Before Impact</th>
<th>At TAI = 646 microseconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>CG Velocity (in/sec)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_y$</td>
<td>- 7884</td>
<td>- 3501</td>
</tr>
<tr>
<td>$V_z$</td>
<td>0</td>
<td>- 3597</td>
</tr>
<tr>
<td>$\omega_f$ (rad/sec)</td>
<td>1638.3</td>
<td>3980.6</td>
</tr>
<tr>
<td>$\theta_f$ (deg)</td>
<td>141.9</td>
<td>279.3</td>
</tr>
<tr>
<td>Kinetic Energy (in-lb)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translational</td>
<td>6756</td>
<td>2739</td>
</tr>
<tr>
<td>Rotational</td>
<td>290.3</td>
<td>1714.0</td>
</tr>
</tbody>
</table>

By TAI = 646 microseconds, both the CG translational velocity and the angular velocity of the blade for the $\mu = .15$ case have been drastically reduced compared with the frictionless case ($\mu=0$). Accordingly, the remaining "attack kinetic energy" of the fragment from translation and rotation are much smaller for $\mu = .15$ as compared with $\mu = 0$; the attendant "energy transference" from the fragment to the ring is significantly greater for the former compared with the latter case.

4.6.2 Straight Elastic-Plastic Shortening Blade Model

Recall that the elastic-plastic shortening blade model (termed EL-PP-SB, for convenience) has two features or rules which
result in different predictions from those of the straight elastic-blade (SEB) model: (1) blade shortening occurs and (2) the maximum collision force is limited by the yield stress $\sigma_o$ (or $\sigma_y$) and the given cross-sectional area of the blade. In order to compare with the SEB model results, calculations for the present EL-PP-SB model were carried out for $\mu = 0$ and 0.15; in addition, to show more fully the influence of the coefficient of friction $\mu$ on the predictions, further calculations were done for $\mu = 0.10$ and 0.20.

It was observed that although the predicted distribution and amount of the containment ring deflections were very nearly the same for $0.10 \leq \mu \leq 0.20$, the blade motion was very different. The latter is illustrated clearly in Fig. 27 where $\omega_f$ is shown as a function of TAI for $\mu = 0, 0.10, 0.15$, and 20.

To illustrate most sensitively the effects of the present EL-PP-SB model versus the SEB model discussed in Subsections 4.6.1, the predictions of $\omega_f$ and $\theta_f$ versus TAI are shown in Fig. 28a and 28b, respectively, for these two blade models for $\mu = 0$ and $\mu = 0.15$. These results indicate that blade shortening together with elastic-plastic force limiting affect the predicted blade motion significantly.

The time histories of the collision-induced forces $F_N$ for the EL-PP-SB model for $\mu = 0$ and $\mu = 0.15$ are shown in Fig. 29. It is seen that the impact durations for the several contact phases observed are much longer than for the SEB results of Fig. 24, but the peak loads are much smaller; also, more and longer periods of contact are observed for the $\mu = 0.15$ compared with the $\mu = 0$ case. Thus, it is expected that for the present EL-PP-SB case, the ring deformation in the vicinity of the impact region will be less severe than in the former elastic-blade case. This expectation is borne out as is seen from Figs. 30a and 30b which compare the predicted first-quadrant deformed-ring profiles...
for the present EL-PP-SB model with those for the straight elastic-blade model (SEB) at TAI = 346 and 646 microseconds, respectively, for μ = .15.

Finally, it is useful to compare certain pre-impact blade quantities with values of corresponding quantities at the end of the predictions (t = 1200 μsec or TAI = 646 μsec). This is done in the following tabulation for μ = 0 and μ = .15 for both the SEB model and the EL-PP-SB model:

<table>
<thead>
<tr>
<th></th>
<th>Immediately Before Impact</th>
<th>Values at TAI = 646 Microseconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>μ = 0</td>
<td>μ = .15</td>
</tr>
<tr>
<td></td>
<td>μ = 0</td>
<td>μ = .15</td>
</tr>
<tr>
<td>CG Velocity (in/sec)</td>
<td>-7884</td>
<td>-3501</td>
</tr>
<tr>
<td>V_y</td>
<td>-3543</td>
<td>4592</td>
</tr>
<tr>
<td>V_z</td>
<td>-2704</td>
<td>-2424</td>
</tr>
<tr>
<td>ω_τ (rad/sec)</td>
<td>1638.3</td>
<td>3980.6</td>
</tr>
<tr>
<td>θ_τ (deg)</td>
<td>4222.6</td>
<td>3082.4</td>
</tr>
<tr>
<td>Translational Kinetic Energy (in-lb)</td>
<td>6756</td>
<td>2739</td>
</tr>
<tr>
<td>Rotational</td>
<td>827</td>
<td>2931</td>
</tr>
<tr>
<td>Idealized Blade</td>
<td>290.3</td>
<td>1714</td>
</tr>
<tr>
<td>Length Change, ΔL (in)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.65</td>
<td>.73</td>
</tr>
</tbody>
</table>

From this comparison, it is seen that for the frictionless impact-interaction case with both the SEB and the EL-PP-SB model, the blade motion (θ_τ, ω_τ) and kinetic energy data are quite similar at TAI = 646 microseconds. For the μ = .15 condition, the larger collision-induced forces associated with the SEB model as compared with the EL-PP-SB model affect the motion and the kinetic energy condition of the blade fragment more drastically for the former than for the latter model. Also, the effects of friction produce more severe changes in blade fragment motion for the SEB than for the EL-PP-SB calculation, as one sees by comparing the prediction for μ = 0 with those for μ = .15.
4.6.3 Elastic-Plastic Curling Blade Model

One of the motivations for postulating this type of blade-deformation model was the observation that blade impact against a containment ring often results in a curled-deformed blade. Figure 31 shows both an untested T58 engine turbine rotor blade and a blade fragment which has undergone impact against a 6061-T6 aluminum containment ring* at the NAPTC (Ref. 23). The similarity between this curled blade and the postulated curled-blade model of Subsection 4.4 is evident.

In the present elastic-plastic curling blade model (termed EL-PP-CB, for convenience), there are two principal parameters which may take on a range of values for a given blade. These are (1) the perfectly-plastic yield stress \(\sigma_y\) (or \(\sigma_o\)) which in turn affects the limit quantities \(M_o\) and \(N_o\) and (2) the final curl radius \(r_f\). Accordingly, a set of calculations was carried out wherein each of these parameters was assigned selected values. As noted in Subsection 4.4.4, there is some uncertainty in determining appropriate limit moments \(M_o\) for the variable cross-section curled twisted blade as a function of the station along the blade even if \(\sigma_o\) were known accurately. Also it should be recalled that strain-rate effects are neglected for the blade; such effects would tend to increase the yield stress \(\sigma_y\) and hence also both \(M_o\) and \(N_o\). For these reasons and for parametric-sensitivity reasons, these blade-ring response calculations have utilized "effective values" of \(\sigma_y\) (and/or \(M_o\) and \(N_o\)) varying by factors of 1, 2, and 3 (i.e., \(\sigma_y = \sigma_y/\sigma_{o, ref.} = 1, 2, 3\)) from an arbitrary but useful reference value of 80,000 psi for \((\sigma_o)_{ref.}\). Final curl radius \(r_f\) values such that \(R_f = (r_f/h_{ref.}) = 4, 6, \) and 8 were used, where \(h_{ref.} = .05\) in. is taken as the maximum thickness of the blade at the tip; the respective \(r_f\) values are 0.2, 0.3, and 0.4 in.

*The dimensions of this containment ring were: thickness .175 in., axial length 1.00 in., and inner-surface radius 7.25 in.; rotor rpm at blade separation was 15,250.
By modeling the rotor blade fragment and employing the pre-impact conditions of NAPTC Test 91 (Table 1), most of the aforementioned matrix of conditions for $\overline{\sigma}_y$ and $F_f$ were used in fragment-ring impact response calculations. Also, for present purposes, predictions are discussed only for an "effective friction coefficient" $\mu = .15$; the effects of varying $\mu$ are discussed in Subsection 4.6.4.

One may gain some appreciation of the effects of the parameter variations studied by examining Fig. 32 which shows the deformed configuration of the blade at $\text{TAI} = 646$ microseconds, except for the two cases noted. For one of these cases, the entire length of the blade was "consumed" by blade curling by $\text{TAI} = 450 \mu\text{sec};$ beyond this time the calculations are not meaningful. For the other case, it was apparent that complete curling consumption would occur even earlier; this case, therefore, was not analyzed. For the cases studied one sees that for fixed $\sigma_y$ or fixed $\overline{\sigma}_y$, a larger fraction of the blade's length becomes curled as larger and larger values are assumed for the final curl radius $r_f$. Similarly, for a given value of $r_f$, the curled blade length fraction increases as the given yield stress $\sigma_y$ is reduced. Both of these observations are consistent with the physical conditions of this model.

Next, it is useful to examine the predictions for $F_N$, $\omega_f$, $\theta_f$, and the ring deformations to appreciate the effects of the present parameter variations on each of these quantities.

The predicted impact-induced normal force $F_N$ experienced by the ring for the EL-PP-CB model is shown as a function of $\text{TAI}$ in Fig. 33a for: $\overline{\sigma}_y = 3$; $F_f = 4, 6,$ and 8; and $\mu = .15$. Similar $F_N$ versus $\text{TAI}$ predictions are shown in Figs. 33b and 33c for $\overline{\sigma}_y$ values of 2 and 1, respectively. Although the $F_N$ data of Fig. 32c are limited, Figs. 33a and 33b indicate that $F_N$ has a higher value but a shorter duration for the smaller values of $r_f$. Also,
for any given $r_f$, as $\sigma_y$ is increased $F_H$ has a higher value; this is consistent with the ingredients included in the postulated EL-PP-CB model.

From the $F_N$ predictions shown in Figs. 33a-33c, one expects to observe differences in the deformation of the containment ring at any given instant of time for these various parameter combinations. Selected for convenient comparison are the predicted first-quadrant deformed ring profiles predicted at TAIL = 646 microseconds for $r_f = 4, 6$, and $8$ and $\sigma_y = 2$ (which correspond in this case to $r_f = 0.2, 0.3$, and $0.4$ in. and $\sigma_y = 160,000$ psi) as shown in Fig. 34. It is seen, as expected, that the severity of the deformation of the containment ring is reduced as one increases the assumed value for the final curl radius $r_f$. Similar ring deformation trends are observed for the other fixed ratios of $\sigma_y$.

Another quantity of interest in the present set of EL-PP-CB model predictions is the time history of the fragment's angular velocity $\omega_f$ — this is a more parameter-sensitive quantity than is the angular orientation $\theta_f$. Accordingly, shown in Fig. 35a is $\omega_f$ versus TAIL for: $\sigma_y = 3; r_f = 4, 6$, and $8$; and $\eta = 0.15$. Similar comparisons of $\omega_f$ versus TAIL are shown in Figs. 35b and 35c for $\sigma_y/(\sigma_o)_r$ values of 2 and 1, respectively. For all three values of $\sigma_y$ or $\sigma_y$, it is seen that at late times $\omega_f$ approaches larger values as $r_f$ increases; also, $\omega_f$ tends to increase as one assumes a smaller value of $\sigma_y$.

Among the factors which influence the predicted time histories of $\omega_f$ (and/or $\theta_f$) are the values of the collision-induced forces, the moment arms of these forces about the CG of the "instantaneously deformed" fragment, and the $I_{CG}$ of the deforming fragment. For convenience, the time histories of the $I_{CG}$ are shown in Figs. 36a, 36b, and 36c to correspond with the conditions of Figs. 35a, 35b, and 35c, respectively. In nearly all cases, one sees a strong correlation between the time histories of $\omega_f$ and of $I_{CG}$.  

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4.6.4 Theoretical-Experimental Comparisons

Experimental data are available from NAPTC Test 91 (see Table 1) for comparing predictions with experimental results. In NAPTC Test 91 a single T58 turbine rotor blade was impacted against a 2024-T4 aluminum containment ring. Framing camera pictures spaced about 30 microseconds apart recorded the motions of the blade and of the containment ring. A background grid and markings on the blade and the ring enable one to make quantitative measurements of the positions and motions of these two objects. Because the illumination for these pictures was not triggered early enough, the "first readable picture" shows that the blade is already in contact with the containment ring. Therefore, there is some uncertainty in identifying the instant of initial impact, that is, the instant for $TAII = 0$. However, there are certain pieces of information which enable one to make a reasonable estimate of the time interval between initial blade-ring impact and the first readable picture. This estimate is described next.

From a knowledge of the ring's inside diameter and the blade tip-to-ring clearance before the blade separated from the hub, as well as from other background grid information (see Table 3), one can establish a "length scale calibration" for the pictures. Since the blade was painted black with three white stripes running chordwise, one can determine, from the photographs, the spanwise distance from the edge of a given white stripe (1) to the blade tip and (2) to the inner surface of the ring before blade release. From the "first readable picture" the distance from the cited white-stripe edge to the inner surface of the ring was measured; it was evident that a portion of the blade's near-tip region had already "disappeared", probably by curling. From the measurements it was estimated that about .33 to .51 inches of the blade's length near the tip had disappeared from view; this estimate is the result of several measurements by various
individuals. Since the relative velocity of the blade to the ring in a direction normal to the ring's surface at impact is $4993 \text{ in/sec}$, the elapsed time interval between the instant of initial impact and the first readable picture is estimated to be about $(.33 \text{ to } .51)/(4993) = 66 \text{ to } 102 \text{ microseconds}$. 

A second means of estimating this time interval is described now. From the sequence of pictures spaced about 30 microseconds apart, measurements were made of the displacements of a point on the ring at the point of initial impact from one picture frame to the next, measured from its pre-impact location. This "displaced-distance" information was plotted as a function of time for several photographic frames using the "first readable picture" as the zero-time reference. Similar displaced-distance data were plotted from EL-PP-SB predictions ($\sigma_y = 160,000 \text{ psi}$ and $\mu = .15$) and EL-PP-CB predictions ($\sigma_y = 160,000 \text{ psi}$, $r_f = 0.3 \text{ inch}$, and $\mu = .15$). A mean fit of these predictions with measurements over the first few frames indicated that a time-axis shift of about 35 microseconds would be needed to achieve consistent comparisons between predictions and experiments.

In view of these indicators and the associated uncertainties in employing each, a convenient near-average value of 56 microseconds for the sought time interval was selected for application in correcting the origin for the time scale of the experimental results. Hence, this corrected time origin is used for the experimental data in the following comparisons between predictions and experimental measurements.

Before discussing the theoretical-experimental comparisons, it is worth noting the following facts. In all of the theoretical predictions discussed here, the containment ring was treated as having EL-PP-SR behavior with $E = 10^7 \text{ psi}$, $\sigma_o = 50,000 \text{ psi}$, $D = 6,500 \text{ sec}^{-1}$, and $p = 4$. However, the blade material assumed for the various assumed-mode blade models is:
For this blade material, it was assumed that $E = 30 \times 10^6$ psi and $\sigma_0 = 120,000$ psi. Although strain-rate properties for the blade material are not known to the authors and are not taken into account in a direct explicit fashion one can take such effects into account in a very rough way by choosing an effective yield stress for the EL-PP blade material so as to represent a plausible fixed strain-rate-dependent yield stress throughout the transient response prediction calculation. If the strain-rate mechanical behavior of the blade SEL 15 material were similar to that for mild steel ($D=40$ sec$^{-1}$, $p=5$), a $\sigma_y/\sigma_0 = 1.33$ or 2 would correspond to "effective strain rates" of 0.2 or 40 sec$^{-1}$, respectively. For $\sigma_0=120,000$ psi, these respective $\sigma_y$ values would be 160,000 and 240,000 psi, respectively. For impact-response situations of the present type, effective blade strain rates during the most important early few hundred microseconds of impact-response would likely approximate or exceed 40 sec$^{-1}$; hence, these two $\sigma_y$ values especially the latter, would appear to be very plausible. However, the authors have no information on such applicability. In the absence of better information, these effective $\sigma_0$ (or $\sigma_y$) values will be assumed to be reasonable (better information, if available, would be welcomed and used).

Further, because of the nature of the comparisons discussed in Subsection 4.6.2 between SEB and EL-PP-SB predictions, the SEB predictions are omitted from the present comparisons.

Compared in Fig. 37 for frictionless impact ($u=0$) and $\sigma_0=\sigma_y/\sigma_0 = 160,000$ psi are the following EL-PP-SB and EL-PP-CB ($r_f=.3$ in) predictions as a function of time after initial impact (TAII): (a) first-quadrant deformed ring configurations for TAIL=326 and
626 microseconds in Fig. 37a and 37b, respectively, and (b) the blade fragment orientation angle change \( \Delta \theta_f = \theta_f - (\theta_f)_I \) as a function of TAIL, where \((\theta_f)_I\) represents \( \theta_f \) at the instant of initial impact; for this case \((\theta_f)_I = 141.9 \) deg. To illustrate the effects of friction, the corresponding information for \( \mu = 0.15 \) is shown in Figs. 38a, 38b, and 38c. It should be noted that the coordinates for the deformed ring profiles are inertial coordinates centered at the pre-impact center of the ring.

Examining each of Figs. 37a, 37b, 38a, and 38b, one finds that the shortening-blade model, EL-PP-SB, results in more pronounced deformations than those predicted by the curling blade model EL-PP-CB.

Comparing Figs. 37a and 37b with Figs. 38a and 38b, respectively, it is seen that the predicted deformed ring profiles for frictionless impact-interaction (\( \mu = 0 \)) differ very little from those for \( \mu = 0.15 \). However, Figs. 37c and 38c indicate that \( \mu \) has a significant effect on the angular motion of the blade fragment as shown by the angular orientation change history of the blade; the EL-PP-SB predictions of \( \Delta \theta_f \) versus TAIL are somewhat more affected by the \( \mu \) value employed than are those of the EL-PP-CB model.

Concerning the Fig. 37 and 38 results, one can argue on physical grounds that the results for \( \mu \neq 0 \) are more plausible than those for the frictionless case \( \mu = 0 \). Further, from Figs. 38a and 38b, it appears that the deformed ring profiles predicted by the curling-blade model are in much better agreement with experiment than are those predicted by the shortening blade model. However, it should be noted that for these \( \mu = 0.15 \) calculations, the effective PP-stress of the blade \((\sigma_y)_f\) has been taken as 160,000 psi which, as noted earlier, corresponds roughly to an effective strain rate of 0.2 sec\(^{-1}\). Again, based on much transient response prediction experience, a larger "effective strain rate" would be expected under the conditions of NAPTC Test 91. Therefore, the use of a larger value for \((\sigma_y)_f\) such as 240,000 psi,
for example, which corresponds roughly to an effective strain rate of about 40 sec\(^{-1}\) would be a more plausible choice. Also, for the curling-blade model, the final radius \(r_f\) used in these figures is 0.3 inch; this value closely corresponds to the observed value for the blade fragment shown in Fig. 31. To date, rational, independent means for predicting or selecting the \(r_f\) value for use in the curling-blade model have not been developed; for present purposes, therefore, this "experimentally observed" value for \(r_f\) has been used throughout the present comparisons of predictions with experimental ring and blade fragment response. Finally, the deformed blade more closely resembles the curling blade configuration than it does the shortening blade configuration.

In view of these considerations, experimental deformed ring profiles are compared in Fig. 39 with EL-PP-CB predictions which employ \(\mu = .15, (\sigma_y)_f = 240,000\) psi, and \(r_f = 0.3\) inch. It is seen that this "more plausible" set of conditions leads to EL-PP-CB predictions which are in very good agreement with experimental data.

Not only does one desire to make accurate predictions of containment ring response to fragment impact but also one seeks to obtain accurate predictions of blade fragment motion so that reasonable estimates, for example, of subsequent collisions* of the blade fragment with one or more of the blades which remain attached to the spinning rotor could be predicted reliably. Hence, accurate predictions of \(\Delta \theta_f\) versus TAI\(\bar{I}\) are desired. As noted in connection with Figs. 37 and 38, the value of \(\mu\) employed strongly affects the predicted time history of \(\Delta \theta_f\). Accordingly, shown in Fig. 40 are EL-PP-CB predictions of \(\Delta \theta_f\) versus TAI\(\bar{I}\) for \(\mu = .15, .20, .25, .30, .40, .50\); also used in these cases are \(r_f = 0.3\) inch and \((\sigma_y)_f = 160,000\) psi. For these conditions, \(\mu\) is seen to have a stronger influence on \(\Delta \theta_f\) as TAI\(\bar{I}\) increases. Calculations have shown that the value of \((\sigma_y)_f\) used for any fixed

* No predictions have been attempted for this more complicated problem.
value of \( \mu \) has relatively little effect upon the time history of \( \Delta \theta_f \); hence, the Fig. 40 predictions would not be changed much if 
\[
(s_y)_f = 240,000 \text{ psi}
\]
were used. Thus, it appears that an effective value of \( \mu \) of about 0.25 to 0.30 would provide fairly good agreement between measurements and curling-blade-model predictions. However, the plausibility of an "effective \( \mu \)" of 0.25 to 0.30 has not yet been discussed and/or established. Another \( \mu \) consideration not taken into account in the present calculations (although modifications could be made to do so readily) is the likelihood that the coefficient of friction from sliding and/or gouging could change with time as a function of the relative velocity of sliding and/or the changed fragment-ring configuration at the "impact point".

Among the useful literature found during a literature search to find pertinent coefficient of sliding friction data are Refs. 24 and 25. For the present problem a "steel blade" is rubbing against a 2024-T4 aluminum ring with an initial relative tangential velocity of 9780 in/sec.

Reference 24 reports the following values for \( \mu \) as a function of the sliding velocity for steel-steel contact:

<table>
<thead>
<tr>
<th>Velocity (in/sec)</th>
<th>.0001</th>
<th>.001</th>
<th>.01</th>
<th>.1</th>
<th>1</th>
<th>10</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>.53</td>
<td>.48</td>
<td>.39</td>
<td>.31</td>
<td>.23</td>
<td>.19</td>
<td>.18</td>
</tr>
</tbody>
</table>

The upper range of this information extends only to sliding velocities which are two orders of magnitude less than those of present interest. Also steel-steel contact rather than aluminum-steel contact is involved.

Reference 25 presents experimental coefficient of sliding friction data for various material combinations including an aluminum prod pressing against a chrome-plated steel disk as a function of the sliding velocity from 0 to 2000 ft/sec. In the sliding

*Increasing \((s_y)_f\) tends to lower the \(\Delta \theta_f\) curve somewhat.*
velocity range of present interest, these μ data appear to scatter from about 0.4 to 0.6. The data indicate that μ tends to diminish with increasing sliding velocity.

Therefore, the curling blade calculation carried out for $r_f = 0.3$ in., $(σ_y)_f = 160,000$ psi, and for μ = 0.40 and 0.50 might appear to be pertinent. For the latter value of μ, the blade "bounced back" and reversed its $ω_f$ rotation direction; however, further blade-ring impacts occurred. For the μ = 0.40 calculation, the blade behavior was similar to that shown in earlier calculations for smaller values of μ. As seen from Fig. 40, the predicted $Δθ_f$ histories for μ = 0.40 and 0.50 are not in as good agreement with experiment as are those for μ = 0.20 to 0.30.

Finally, a curling blade calculation was carried out for $r_f = 0.3$ in., $(σ_y)_f = 240,000$ psi, and μ = 0.40. The resulting $Δθ_f$ time history falls only slightly below the μ = 0.40 result of Fig. 40. Also, the corresponding predicted deformed ring profiles are compared in Figs. 39a and 39b with those for μ = 0.15. It is seen that the use of this higher μ value leads to more pronounced ring deformations than does the use of μ = 0.15. One may note incidentally, that the use of μ = 0.20 to 0.30 would provide (1) predicted deformed ring profiles lying between the predictions shown in Figs. 39a and 39b, giving quite good agreement with experiment and (2) predicted $Δθ_f$ results also in good agreement with experiment.
4.7 Comments

Various matters that are common to Subsections 4.2 through 4.6 or that were inconvenient to discuss in those earlier subsections of Section 4 are considered now.

The three blade fragment analysis models by design differ greatly in their load-deflection characteristics. The SEB model, being entirely elastic, has no upper limit of \( F_N \). In general, since there is no upper bound to this force, there will be rather high forces present, but the collision will last for a relatively short period of time. The force history, then, for impact of the SEB model against a ring, will be characterized by parabolic Hertzian force profiles, each of short duration and high peak force, with peak forces decreasing with each impact. Both of the other models, EL-PP-SB and EL-PP-CB, have an upper limit on \( F_N \). In the case of the shortening blade model, the limit is the cross-sectional area times the compressive yield stress of the blade. For the curling blade model, the limit is determined by consideration of the interaction curve of Fig. 19, and is of the same order of magnitude, but unlike the previous model, varies with point of impact on the blade. For the straight blade, the impact must always occur at the blade tip, while for the curling blade the impact may occur at various points along the curled portion of the blade. Because of the limit on \( F_N \) for these models, the force histories will show low forces, with no sudden large increases in force. Because of the lower interaction force present, the blade and ring will tend to remain in contact longer before they bounce apart. For this reason, the force history will show impacts of long duration and small magnitude.

As a matter of curiosity, the amount of impulse applied to the ring as predicted by the SEB, EL-PP-SB, and EL-PP-CB models to a given instant in time has been computed by integrating \( F_N \, dt \) over the impact duration; for present rough comparative purposes, the part arising from \( (\mu F_N) \, dt \) has been neglected.

More than one impact will occur, and since the kinetic energy of the blade decreases with time, the peak force of successive impacts will decrease.
Tabulated below are these results to three instants in time after initial impact (TAII):

<table>
<thead>
<tr>
<th>TAI (μsec)</th>
<th>Impulse (lb-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>SEB (μ=0.15)</td>
<td>1.27</td>
</tr>
<tr>
<td>EL-PP-SB</td>
<td>0.66</td>
</tr>
<tr>
<td>(μ=0.15, σy=160,000 psi)</td>
<td></td>
</tr>
<tr>
<td>EL-PP-CB</td>
<td>0.18</td>
</tr>
<tr>
<td>(μ=0.15, rₚ=0.3 in, σy=160,000 psi)</td>
<td></td>
</tr>
</tbody>
</table>

It is seen that the applied impulse from the SEB calculation is larger than for the other models and is applied over a much shorter span of time. Hence, a much more severe ring response is expected for the SEB prediction than for the other two predictions as can be seen by comparing the ring deformation results of Figs. 26, 30, and 34. The expected relative severity of the ring deformation is evident from these figures.

The information on hand at present concerning the friction forces existing at the ring-fragment interface is quite scanty. Some numbers for the coefficient of sliding friction for steel-steel contact at high sliding speed and high contact pressure have been found, and seem to indicate a value of about .15 for μ. On the other hand, more recent work on melt lubrication has shown that for contact between aluminum and chrome-plated steel at high velocities, the value for μ might be as high as .40 to
Therefore, the values employed in the present calculations are ball park figures with widely separated upper and lower bounds. In order to compare theory with experiment with more confidence, it would be greatly desired to find reliable numbers for the appropriate friction coefficient (for the contact materials and sliding-velocity range of interest), within much narrower bounds of uncertainty than are available to date.

As things stand now, the SEB and EL-PP-SB models seem to predict blade motion quite well for values of \( \mu \) near .15. However, for the EL-PP-CB model, this value for \( \mu \) results in experimentally unobserved large angular orientations of the blade. However, if \( \mu \) were of the order of .30 to .40, then this model would do a better job of predicting blade position and motion. On the other hand, if these same higher friction values were used in the other two models, the results would deteriorate due to "bounce-back" of the blade. On the one hand, this might seem bad, because the previously obtained "good results" would become "poor results". On the other hand, just these types of poor results might be expected from these two "more-simple" models, which also may be assumed to be "poorer" models for the example problem studied. In other words, if the same coefficient of friction of about .40 were used in all three models, great improvements would be noted in both ring deflection and blade motion predictions as one proceeds from models SEB to EL-PP-SB to EL-PP-CB. At the same time, however, it should be remarked that the severity of the impact condition studied in that example problem is such as to make the EL-PP-CB model physically the most appropriate of these three models.

Of perhaps incidental interest is the predicted and the observed (apparent) shortening of the rotor blade fragment. This shortening is found to be relatively insensitive to the value of \( \mu \). Thus, shown below, for convenience, is the blade shortening.
It is seen that there is reasonably good agreement between the experimental value of $\Delta L$ and the most plausible of these three predictions (EL-PP-CB for $\sigma_y = 240,000$ psi).

Various matters associated with the curling blade model need further study. Better, more rational criteria for curl initiation need to be devised. Also, criteria need to be developed to indicate the proper direction of curling. For the latter question, one may resort to a simplified rigid-body analysis of the blade to estimate the direction, value, and location of the maximum bending moment experienced by the idealized blade of Fig. 14b subjected to loads $F_N$ and $\mu F_N$ as depicted in Fig. 18a. If one assumes that the direction of curling will be in accord with the direction of the maximum bending moment $M$ along the blade, one can thus estimate for any given blade/ring orientation angle $\psi$ (see Fig. 18b) the value of $\mu$ which would cause $M$ to change sign; this value of $\mu$ may be termed $\mu_{cr}$. One may readily calculate the bending moment arising from the applied tip forces and

<table>
<thead>
<tr>
<th>$\Delta L$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL-PP-SB ($\mu=0.15, \sigma_y = 160,000$ psi)</td>
</tr>
<tr>
<td>EL-PP-CB ($\mu=0.15, r_f=0.3$ in, $\sigma_y = 160,000$ psi)</td>
</tr>
<tr>
<td>EL-PP-CB ($\mu=0.15, r_f=0.3$ in, $\sigma_y = 240,000$ psi)</td>
</tr>
<tr>
<td>Experiment</td>
</tr>
</tbody>
</table>

$\Delta L$, at TAI=626 $\mu$sec:
the rigid-body inertia forces of the blade to be

$$M(x) = + F_L [-0.335 - 0.369x + 0.092x^2 + 0.067x^3]$$

(4.50)

where $+F_L = F_N \cos \psi - \mu F_N \sin \psi$ represents the tip force component perpendicular to the blade’s axial length, with $+F_N$ as shown in Fig. 18b, and $x$ is measured from the CG toward the impact-loaded tip. A positive $M$ causes compression on the clockwise face of the blade shown in Fig. 18b. From Eq. 4.50, one finds that the maximum bending moment occurs at $x=0.97$ in. or $1.218$ in. from the impacted tip, regardless of the value of $\mu$. The sign of $M$ depends upon the sign of $F_L$; $M$ changes sign when $F_L = 0$ which means that $\mu_{cr} = \left| \cot \psi \right|$. Hence, $\mu_{cr}$ is as follows for various $\psi$ values:

<table>
<thead>
<tr>
<th>$\psi$ (deg)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
<th>100</th>
<th>110</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{cr}$</td>
<td>0.577</td>
<td>0.364</td>
<td>0.176</td>
<td>0.0875</td>
<td>0</td>
<td>0.0875</td>
<td>0.176</td>
<td>0.364</td>
<td>0.577</td>
</tr>
</tbody>
</table>

For the present blade/ring example, $\psi = 82.17$ deg. which indicates that $\mu_{cr} = 0.138$. For $\mu = 0$, $M$ is negative at $x=0.97$ in.; thus, for $\mu < 0.138$ one might expect the blade of Fig. 18b to curl in the counterclockwise direction indicated in Figs. 18b and 18c, but to curl in a clockwise direction for $\mu > 0.138$. It would appear from this argument, if valid, that clockwise curling should have been used for all of the EL-PP-CB cases for which $\mu > 0.138$ was used. However, the consequences of employing that type of blade curling and the associated logic remain a topic for future study since time has not permitted this matter to be investigated.

Aside from predicting containment ring response and blade fragment motion, the present CPM analysis provides an estimate of the collision-induced forces experienced mutually by the ring and the fragment; provided that one has chosen the proper blade behavior model for the impact conditions at hand, these predicted
collision-induced forces are probably of the correct magnitude and duration, at least in a ball park sense. Since in the TEJ analysis of Ref. 12 any reasonable additional collision-induced force estimate available leads to an improvement in the TEJ predictions of the collision-induced forces, the present CFM predictions will be of assistance in that role.

All computing was performed on an IBM 370 Model 155 digital computer located at P.IIT. The amount of core storage needed to run a typical CFM program was 70,000 words. Most of the CPU time was spent solving the finite element ring response problem, for which there were 112 degrees of freedom in typical cases analyzed. The time spent in doing the CFM procedure was actually a very small part of the total CPU time. This is because the CFM process requires no large matrix manipulations, as does the finite element ring response prediction process. For an example of running time, the CPU time for a typical run of the curling blade model with $r_f=0.3$ in. and $\sigma_y=160,000$ psi was 8.725 minutes for 661 time increments of calculation; this is equivalent to 0.013 minutes per time increment.

Fundamental improvements to the blade/ring CFM analysis could be made in various ways. One such improvement would be to represent and analyze the blade deformation and motion response in a direct rigorous dynamic fashion rather than treating the blade as quasi-rigid as done for convenience and efficiency in the present initial exploratory investigation. Representing the blade by appropriate finite elements, for example, would be a convenient way of effecting this improvement; the pattern of blade deformation would not need to be assumed but would develop quite naturally and automatically in accordance with the applied forces, the inertial forces, and the geometric and mechanical properties of the blade/ring system.

Finally, although this initial study of the CFM scheme was restricted deliberately to the relatively simple problem of a
single blade fragment attacking an initially circular containment ring in order to examine the feasibility and practicality of this approach, more complicated situations are of interest. For example, one may be interested in analyzing the containment ring response induced by (1) impact from a single blade from a fully-bladed rotor with subsequent blade-blade and blade-ring impacts, (2) blade impact, blade breakup and subsequent multiple fragment attack, or (3) a bladed-disk fragment. Based upon the present studies, it is believed that the CFM approach could be extended feasibly with some effort and computing cost to analyze case (3). While in principle cases (1) and (2) could also be treated by the CFM approach, it is apparent that much greater complexity, computing cost, and development time would be involved.
5.1 Summary

The present study is concerned with investigating an approximate collision analysis termed the collision force method (CFM) for studying the impact-interaction of an engine rotor fragment or fragments with fragment containment or deflection structures; for the present initial study of the CFM approach, attention is restricted to analyzing the relatively simple problem of impact-interaction of a single rotor blade fragment with an initially-circular containment ring. The large-deflection elastic-plastic transient response of the containment ring is analyzed by a rigorous and validated finite element analysis method. However, the rotor blade fragment is analyzed via an assumed-mode type of analysis whereby the blade is regarded as quasi-rigid and its rigid-body motion behavior is analyzed by solving the appropriate equations of motion; in this simplified assumed-mode blade model, the blade is permitted to undergo "instantaneous" small changes of configuration at a given instant---this configuration is then employed in the next small time step of the timewise step-by-step solution procedure. Thus, the blade fragment is not treated in a rigorous dynamic fashion; accordingly, this part of the analysis may perhaps be termed "quasi-dynamic".

An objective of the present analysis is to develop an approximate method for predicting the transient responses of both a containment ring and a single rotor blade fragment which impacts and interacts with the ring. This analysis is intended to employ basic material property information for both the ring and the fragment and to utilize geometric, mass, mass moment of inertia, and pre-impact translational and rotational-velocity information for the fragment --- thereby avoiding the necessity of intro-
ducirig detailed experimentally measured transient ring/fragment response information (for the particular ring/fragment case under analysis) into the analysis itself in order "to achieve accurate predictions". This independent theoretical approach also is intended to provide estimates of the collision-interaction forces mutually experienced by the ring and fragment in order to supply the Kalman filter loop of the TEJ program of Ref. 12 with an additional collision-interaction force estimate which will result in improving the TEJ predictions of the interaction forces.

For the present blade/ring collision-interaction analysis, the following three assumed-mode blade fragment models having certain prescribed patterns of deformation have been studied: (1) the straight-elastic blade (SEB) model, (2) the straight elastic-plastic shortening blade (EL-PP-SB) model, and (3) the elastic-plastic curling blade (EL-PP-CB) model. For each model appropriate but approximate "force-deformation relations" are employed from which, having a knowledge of the blade's necessary deformation so as to be compatible with the containment ring at the end of each small time step increment \( \Delta t \) in time during the step-by-step calculation procedure, one can estimate the collision-interaction forces mutually being experienced at that instant by the containment ring and the fragment. As the nomenclature for each assumed-mode blade model implies, the blade is assumed to experience only elastic behavior in the SEB model, but may undergo elastic-plastic action in the other two models; the rules of behavior postulated for these three models are explained in Subsections 4.2-4.4. These collision-interaction forces are then employed in appropriate equations of motion which are solved in order to predict the locations of the blade and the ring at the end of the next \( \Delta t \) increment (see Fig. 4).

The application of each of these three models to predict the responses of a containment ring and a single engine rotor blade fragment to blade impact against the ring is discussed in Sub-
sections 4.6.1-4.6.3, including some studies of the effects of varying (a) certain geometric and/or material property parameters of the blade and (b) the coefficient of sliding friction between the rotor-blade fragment and the containment ring. Predictions are then compared in Subsection 4.6.4 with experimental ring/blade response results measured in NAPTC Test 91.

It should be emphasized that the present containment ring studies have been restricted to cases involving only a single ductile type (fracture free) rotor-blade fragment. No provision or attempt has been made to follow or predict the blade's response to the condition of fracture, possible multiple fragmentation, and subsequent motion and impact of these multiple fragments with the containment ring and/or each other. A more rigorous and complex analysis would be required to handle such cases.

Before carrying out the ring/blade response calculations, the collision force method of analysis was applied to a problem for which a reliable independent solution (Ref. 16) exists, in order to check both on the present analysis procedure and on the proper operation of the computer program which embodies the present analysis (see Section 7). This problem involves the lateral impact of a steel sphere at the midspan of a simply-supported steel beam. The present predictions were in acceptable agreement with those of Ref. 16.

Based upon all of these studies, certain conclusions have been reached; these are discussed in Subsection 5.2. Also, some suggestions for further research are offered in Subsection 5.3.

5.2 Conclusions

Although a number of matters remain to be studied in depth, the present collision force method of analysis which utilizes (1) a validated finite-element analysis method for predicting the large-deflection elastic-plastic responses of containment rings subjected to given externally-applied forces together with (2)
one of several postulated quasi-dynamical assumed-mode blade behavior models, permits one to make reasonable qualitative and quantitative predictions of the motion and deformation response of a containment ring subjected to impact by a deformable ductile type engine rotor blade fragment, given plausible mechanical property information, initial impact condition data, and friction coefficient values. Unfortunately, experimental data on transient strains of the fragment-impacted containment rings tested at the NAPTC against which the present predictions on containment ring transient strains could be compared and hence evaluated more critically do not exist.

To achieve reasonable ring and blade fragment response predictions by the present CFM approach, one must select the blade behavior model (SEB, EL-PP-SB, or EL-PP-CB) which is appropriate for the impact conditions under consideration. For example, for NAPTC Test 91 which involved impact of a T58 turbine rotor blade against an aluminum containment ring wherein the initial impact velocity normal to the ring was 4993 in./sec, the EL-PP-CB model is a better selection than the other two models since the severity of impact results in a curl-deformed blade.

Based upon the series of blade/ring deformation and motion response predictions with the EL-PP-SB and the EL-PP-CB model and upon comparisons of these predictions with NAPTC Test 91 data, it is found that of the variables \((U_y)_f\) and \(\mu\), the ring response is affected more by the value of the fragment yield stress \((\sigma_y)_f\) than by the coefficient of sliding friction \(\mu\). However, the motion of the idealized blade is affected more by \(\mu\) than by \((\sigma_y)_f\).

Further data search and research will be needed in order to establish sound definitive estimates of the coefficient of sliding friction and its proper functional dependence (function of sliding velocity, contact materials, etc.) for impact interaction problems involving typical types and materials of engine rotor fragments and containment rings or vessels.

*For more complex types of fragments, one would need to devise and employ one or more types of assumed-mode models with pertinent geometric properties and rules of behavior.*
The present quasi-dynamic CFM analysis appears to be a useful interim analysis tool which can provide useful blade/ring response estimates with only a modest amount of computing. Its extension to treat multiple blade fragments and/or blade-disk fragments would considerably escalate the bookkeeping complexity even within the present highly simplified assumed-mode framework.

In addition to the main general conclusions just discussed, certain observations pertaining to the specific types and ingredients of the assumed-mode blade fragment behavior models examined in this investigation may be useful. For comparatively gentle fragment impact (small relative fragment/ring impact velocity normal to the surface of the ring), the SEB analysis model would be appropriate. For higher normal-impact velocities the EL-PP-SB model would be more realistic; for still higher velocities, the EL-PP-CB model would be expected to represent the physical situation better and to provide more realistic predictions. The inappropriate use of a given such model can lead to unrealistic predictions as demonstrated for the single set of severe impact conditions used in Subsections 4.6.1, 4.6.2, and 4.6.3 for examining, respectively, the SEB, the EL-PP-SB, and the EL-PP-CB model. Unfortunately, time has not permitted the development, in this study, of criteria by which one can select the more appropriate of the three blade-fragment-behavior models devised in this investigation. Also, additional models perhaps may be useful.

For the very simple SEB and EL-PP-SB models, the parameters subject to variation are few. However, the more complex EL-PP-CB model involves many quantities whose values may vary; further, the initiation of each of the various deformation phases of this model is signaled by tentative rules which could be revised and founded perhaps upon more rational bases. The present interim rules, however, together with plausible values for the various quantities defining the EL-PP-CB model appear to provide ring and blade motion information which are in quite good agreement with
5.3 Suggestions for Future Work

With some additional cost in computing, a less simplified but still approximate CFSM analysis of the present blade/ring problem could be carried out, for example, by representing the variable cross-section twisted blade by curved-plate or flat plate finite elements and by including the elastic-plastic, strain hardening, strain-rate sensitive mechanical behavior of the blade in a direct fashion in a transient response analysis. This procedure would enable one to circumvent the invoking of the present prescribed simple patterns of blade deformation and instead allow the particular nature of the blade's deformation to come forth as a natural by-product or consequence of the analysis; also, this would circumvent the need to estimate $M_o$ and $N_o$ by the current perhaps oversimplified scheme. Whether the gain in generality and accuracy would be worth the attendant computing/analysis cost increase can not be assessed at present.

Within the present idealized framework for the curling blade model, several matters need further study. Rational criteria need to be devised to indicate both the initiation of curling and the direction of curling. Should curling occur in the direction opposite that assumed in the present calculations, the pertinent geometric, collision inspection, and force-deflection rules would need to be worked out and implemented in a computer program.

With regard to the three types of assumed-mode blade fragment models postulated in the present study, it would be helpful to develop convenient criteria by which a user could select the model which is most appropriate for his set of engine rotor blade fragment, containment ring, and impact conditions.

Further CFSM studies should be carried out to compare predictions with experimental measurements not only for containment ring motion and deformation response and blade-fragment motion but also
for transient strains of the impacted containment ring. The latter information not only is a more sensitive indicator of the appropriateness and adequacy of the prediction scheme but also is more meaningful in indicating when ring fracture conditions are being approached.

Some improvement in the modeling accuracy in the present CFM blade/ring analysis could be achieved by using the correct curved ring geometry in the collision-inspection part of the procedure rather than approximating the ring by straight segments for the particular part of the process. It is clear, however, that this improvement will lead to a greater amount of computing. Hence, the value/cost ratio for this improvement is uncertain, but the authors are dubious of its worth particularly if the ring is already modeled with enough finite elements to permit making accurate and/or converged transient response predictions.

The desirability of extending the present simplified type of CFM analysis to treat other types of engine rotor fragments such as multiple blade fragments or bladed-disk fragments should be weighed against other analysis and/or experimental alternatives. For these more complicated cases, the number of variables which must be accounted for increases rapidly whether one is concerned with theoretical analysis, experimental studies, or both. The versatility and productivity of a validated theoretical analysis could in many cases make its use more attractive than experiment for parametric studies. However, the actual fragment/containment and/or deflection problems involve various complexities that theoretical analysis can not feasibly duplicate in the reasonably near future at an acceptable cost. Therefore experimental studies will remain an indispensable part of fragment containment/deflection design and verification work. However, this essential but very expensive experimental work can be minimized by the judicious development and application of theoretical analysis.
REFERENCES


TABLE 1

DATA CHARACTERIZING NAPTC RING TEST 91

**Ring Data**

<table>
<thead>
<tr>
<th>Outside Diameter (in)</th>
<th>17.619</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Thickness (in)</td>
<td>0.152</td>
</tr>
<tr>
<td>Axial Length (in)</td>
<td>1.506</td>
</tr>
<tr>
<td>Material</td>
<td>2024-T4</td>
</tr>
<tr>
<td>Elastic Modulus E (psi)</td>
<td>$10^7$</td>
</tr>
<tr>
<td>PP Static Yield Stress $\sigma_0$ (psi)</td>
<td>50,000</td>
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**Fragment Data**

<table>
<thead>
<tr>
<th>Type</th>
<th>T-58 Single Blade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>SEL-15</td>
</tr>
<tr>
<td>Outer Radius (in)</td>
<td>7.0</td>
</tr>
<tr>
<td>Fragment Centroid from Center of Rotation (in)</td>
<td>4.812</td>
</tr>
<tr>
<td>Fragment Tip Clearance from Ring (in)</td>
<td>1.658</td>
</tr>
<tr>
<td>Fragment Length (in)</td>
<td>3.5</td>
</tr>
<tr>
<td>Fragment Length from CG to Tip (in)</td>
<td>2.188</td>
</tr>
<tr>
<td>Fragment Weight (lbs)</td>
<td>0.084</td>
</tr>
<tr>
<td>Fragment Moment of Inertia about its CG (in lb sec$^2$)</td>
<td>$2.163 \times 10^{-4}$</td>
</tr>
<tr>
<td>Failure Speed (RPM)</td>
<td>15,644.4</td>
</tr>
<tr>
<td>Fragment Tip Velocity (ips)</td>
<td>11,467.</td>
</tr>
<tr>
<td>Fragment Centroidal Velocity (ips)</td>
<td>7,884.</td>
</tr>
<tr>
<td>Fragment Initial Angular Velocity (rad/sec)</td>
<td>1,638.3</td>
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<tr>
<td>Fragment Translation KE (in lb)</td>
<td>6,756.</td>
</tr>
<tr>
<td>Fragment Rotational KE (in lb)</td>
<td>290.3</td>
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### TABLE 2

**BLADE CROSS SECTION DATA FOR THE IDEALIZED ELASTIC-PLASTIC CURLING BLADE MODEL**

<table>
<thead>
<tr>
<th>Stations Location</th>
<th>Coordinates(in.)</th>
<th>h(in)</th>
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<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td><strong>Station ( \frac{x}{L_1} = 0 ):</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_{ac} = .302 \text{ in} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y_{ac} = .180 \text{ in} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_p = 1 \text{ deg} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_x = 4.34 \times 10^{-4} \text{ in}^4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_o = 5.13 \times 10^{-3} \text{ in}^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_y = 8.03 \times 10^{-2} \text{ in}^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>.135</td>
<td>.200</td>
</tr>
<tr>
<td>C</td>
<td>.330</td>
<td>.260</td>
</tr>
<tr>
<td>D</td>
<td>.510</td>
<td>.175</td>
</tr>
<tr>
<td>E</td>
<td>.720</td>
<td>0</td>
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<tr>
<td><strong>Station ( \frac{x}{L_1} = 0.5 ):</strong></td>
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<tr>
<td>( x_{ac} = .330 \text{ in} )</td>
<td></td>
<td></td>
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<tr>
<td>( y_{ac} = .165 \text{ in} )</td>
<td></td>
<td></td>
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<tr>
<td>( \theta_p = 2 \text{ deg} )</td>
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<tr>
<td>( I_x = 2.68 \times 10^{-4} \text{ in}^4 )</td>
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<tr>
<td>( M_o = 3.44 \times 10^{-3} \text{ in}^3 )</td>
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<td>( \sigma_y = 5.78 \times 10^{-2} \text{ in}^2 )</td>
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<td>C</td>
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<tr>
<td>D</td>
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<tr>
<td>E</td>
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**TABLE 2 CONTINUED**

BLADE CROSS SECTION DATA FOR THE IDEALIZED ELASTIC-PLASTIC CURLING BLADE MODEL*

<table>
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<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>Station (\frac{x}{L_1}=1.0):</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(x_{ac} = .432)</td>
<td>(\theta = 1) deg</td>
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<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>.185</td>
<td>.170</td>
</tr>
<tr>
<td>C</td>
<td>.440</td>
<td>.170</td>
</tr>
<tr>
<td>D</td>
<td>.710</td>
<td>.100</td>
</tr>
<tr>
<td>E</td>
<td>1.0</td>
<td>0</td>
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*See Fig. 21 for pertinent nomenclature.
FIG. 1 SCHEMATICS OF THE FRAGMENT CONTAINMENT AND FRAGMENT DEFLECTION PROBLEMS
FIG. 2 SCHEMATICS OF SOME TYPES OF FAILED-ROTOR FRAGMENTS

(a) Single-Blade Fragment

(b) Multi-Bladed Disk Fragment

(c) Multiple-Blade Fragments
FIG. 3 SCHEMATICS OF A CONTAINMENT RING SUBJECTED TO SINGLE-FRAGMENT IMPACT AND OF A CURVED-BEAM ELEMENT
FIG. 4 INFORMATION FLOW SCHEMATIC FOR PREDICTING RING AND FRAGMENT MOTIONS IN THE COLLISION FORCE METHOD
FIG. 5 INFORMATION FLOW SCHEMATIC FOR PREDICTING RING AND FRAGMENT MOTIONS IN THE COLLISION-IMPARTED VELOCITY METHOD (REF. 9)
FIG. 6 SCHEMATICS OF IDEALIZED PRE-IMPACT AND COLLISION CONFIGURATIONS
0.5 IN. DIA. STEEL SPHERE
1800 IN/SEC
STEEL BEAM
15 IN  15 IN
30 IN
(a) Problem Schematic

(b) Finite Element Modeling for Analysis

FIG. 7 GEOMETRY AND MODELING SCHEMATICS FOR STEEL SPHERE IMPACT UPON A 1/2 x 1/2 x 30 INCH SIMPLY-SUPPORTED STEEL BEAM
Collision Starts
\( t = t_0 \)

During Interaction
\( t_1 > t > t_0 \)

Collision Ceases
\( t = t_1 \)
\( t > t_1 \)

Impact Force
\( \alpha \)

\( t_0 \) \hspace{1cm} \( t_1 \) \hspace{1cm} \( t \)

This reversible elastic local indentation \( \alpha \) is related to the impact force \( F \) by the Hertz Law: \( F = k\alpha^{3/2} \) where \( k \) is a constant found from the elastic properties of the colliding materials.

\( \alpha \) = Elastic Local Indentation,
Impact Force \( F \) is Given by the Hertz Law: \( F = K\alpha^{3/2} \)

FIG. 8 SCHEMATICS OF AN IMPACT SEQUENCE FOR PERFECTLY PLASTIC BEHAVIOR IN THE COLLISION-INTERFACE REGION FOR SPHERE IMPACT AGAINST A SURFACE
\[ \alpha = \text{Current Total Indentation} \]
\[ \alpha_m = \text{Maximum Indentation} \]
\[ \alpha_r = \text{Permanent Indentation of Target} \]

Impact Force \( F \) is Given Empirically by

During Loading: \( F = N \alpha^n \) for \( 0 \leq \alpha \leq \alpha_m \)

During Unloading: \( F = F_m \left( \frac{\alpha - \alpha_r}{\alpha_m - \alpha_r} \right)^q \) for \( \alpha_r \leq \alpha \leq \alpha_m \)

**FIG. 9** SCHEMATIC OF AN IMPACT SEQUENCE FOR ELASTIC-PLASTIC BEHAVIOR IN THE COLLISION-INTERFACE REGION FOR SPHERE IMPACT AGAINST A SURFACE
IMPACT BEHAVIOR IS HERTZ-TYPE ELASTIC:

\[ F = K \alpha^{3/2} \]

ELASTIC (EL) BEAM RESPONSE

ELASTIC, PERFECTLY-PLASTIC (EL-PP) BEAM RESPONSE

\[ E_{\text{Beam}} = 30 \times 10^6 \text{ PSI} \]

\[ (\sigma_0)_{\text{Beam}} = 40,000 \text{ PSI} \]

FIG. 10 CFM PREDICTIONS FOR THE SIMPLY-SUPPORTED BEAM SUBJECTED TO STEEL SPHERE IMPACT, ASSUMING HERTZ-TYPE WHOLLY ELASTIC IMPACT
BOTH CASES INVOLVE:

(1) EL-PP BEAM RESPONSE
(2) INELASTIC IMPACT LOADING, EQ. 3.4 BY $F = Na^n$

(n=1; N=581,500 LB-IN)

IMPACT UNLOADING BY EQ. 3.5:

- LINEAR ($q=1$)
- HERTZ-ELASTIC ($q=\frac{3}{2}$)

FIG. 11 CPM PREDICTIONS FOR THE SIMPLY-SUPPORTED STEEL BEAM SUBJECTED TO STEEL SPHERE IMPACT, ASSUMING INELASTIC IMPACT LOADING WITH EITHER HERTZ OR LINEAR UNLOADING
NOTE:
1. Small Deflection Linear-Elastic Beam Behavior
2. Initial Impact Velocity = 1800 in/sec.

FIG. 12 COMPUTED FORCE HISTORIES AT THE CONTACT POINT FROM FIG. 57 OF REF. 16 FOR THE CENTRAL TRANSVERSE IMPACT OF 0.5 IN. DIAMETER STEEL SPHERE ON A 0.5 X 0.5 X 30 IN. SIMPLY-SUPPORTED STEEL BEAM
FIG. 13 COMPARISON OF CFM PREDICTIONS WITH COMPUTED CONTACT FORCE HISTORIES FROM FIG. 57 OF REF. 16 FOR THE CENTRAL TRANSVERSE IMPACT OF 0.5-IN. DIAMETER STEEL SPHERE ON A 0.5 x 0.5 x 30-IN. SIMPLY-SUPPORTED STEEL BEAM
PRINCIPAL KNOWN QUANTITIES:

\( W \) = BLADE WEIGHT

\( I_{CG} \) = MASS MOMENT INERTIA

\( L_1 \) = DISTANCE FROM BLADE TIP TO CG

\( \text{CG, } I_{CG} \text{ is given} \)

(a) Schematic of the Actual Blade Equipment

![Schematic of the Actual Blade Equipment](image)

(b) Schematics of the Idealized Blade Model before Deformation

![Schematics of the Idealized Blade Model before Deformation](image)

### Preserved Knowns

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
<th>Modeling Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>( w_1 ) (lb/in)</td>
<td>1. ( w_1(L_1+L_2) + W_2 = W )</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>( W_2 ) (lb)</td>
<td>2. ( w_1L_1 \left( \frac{L_1}{2} \right) = w_1L_2 \left( \frac{L_2}{2} \right) + W_2L_2 )</td>
</tr>
<tr>
<td>( I_{CG} )</td>
<td>( L_2 ) (in)</td>
<td>3. ( I_{CG} = [I_{CG_1} + w_1L_1 \left( \frac{L_1}{2} \right)^2] ) + [I_{CG_2} + w_1L_2 \left( \frac{L_2}{2} \right)^2 ] + ( W_2(L_2)^2 )</td>
</tr>
</tbody>
</table>

FIG. 14 SCHEMATICS FOR THE ACTUAL AND THE IDEALIZED PRE-IMPACT BLADE MODEL

103
(a) Pre-Impact Condition (at time $t_m$)

(b) Predicted Locations at Time $t_{m+1} = t_m + \Delta t$

FIG. 15 SCHEMATICS AND NOMENCLATURE FOR THE STRAIGHT ELASTIC BLADE MODEL
(c) Schematics for Idealized Straight-Segment and Curved-Ring Models

FIG. 15 CONTINUED
NOTE: \( \ell = L_a - \alpha \)

(d) Exploded Schematic of Configuration and Interaction Forces at Time \( t_{m+1} \)

FIG. 15 CONCLUDED

106
(a) Pre-Impact Condition (at time $t_m$)

(b) Predicted Tentative Locations at Time $t_{m+1} = t_m + \Delta t$

FIG. 16 SCHEMATICS FOR THE STRAIGHT ELASTIC-PLASTIC BLADE MODEL

107
Axial Force at Blade Tip is $(F^*_A)_{m+1}$:

$$(F^*_A)_{m+1} = (F_N)_{m+1} [\sin \theta - \mu \cos \theta]$$

(c) Exploded Schematic of External Forces Acting at Time $t_{m+1}$

FIG. 16 CONTINUED
(d) Schematic of Blade Shortening at and Following Time $t_{m+1}$

$\Delta L_{m+1} = \text{Increment of Permanent Shortening}$

$L_m - \alpha_{m+1}$

**FIG. 16 CONCLUDED**

109
(a) Phase 1: Straight Blade at a Given Instant

TENTATIVE CONFIGURATION

CORRECTED CONFIGURATION

FIG. 17 SEQUENCE OF TENTATIVE AND CORRECTED BLADE DEFORMATION GEOMETRIES AT FIXED TIME INSTANTS DURING THE VARIOUS DEFORMATION PHASES OF THE ELASTIC-PLASTIC CURLING BLADE MODEL
CONDITIONS:
1. $L_0 = L' + \alpha = L'' + \frac{\pi r_1}{2}$ = Total Blade Length
2. $H + L_0 \sin \psi$ is Preserved; Hence,
   \[ L' \sin \psi = L'' \sin \psi + r_1 (1 - \cos \psi) \]

TENTATIVE CONFIGURATION

CORRECTED CONFIGURATION

(b) Phase 2: Curl Initiation Instant ($F_A^* = F_{cr}$)

FIG. 17 CONTINUED
CONDITIONS:
1. \( L = L' + \frac{\pi r}{2} = L'' + \frac{\pi}{2} (r + \Delta r) \) = Total Blade Length
2. Distance \( P_3 \) is Preserved (see text for \( \alpha_b \))

\[
L' \sin \psi + r(1 - \cos \psi) - (\alpha - \alpha_b) \\
= L'' \sin \psi + (r + \Delta r)(1 - \cos \psi)
\]

TENTATIVE CONFIGURATION       CORRECTED CONFIGURATION

(c) Phase 3: Curling Blade with Increasing Curl Radius, \( r \)

FIG. 17 CONTINUED
1. $L = L' + \beta r_f = L'' + (\beta + \Delta \beta) r_f$ = Total Blade Length

2. Distance $P_h$ is Preserved (see text):
   
   $L' \sin \psi + r_f(1-\cos \psi) - (\alpha - \alpha_b)$
   
   $= L'' \sin \psi + r_f(1-\cos \psi)$

(d) Phase 4: Curling Blade with a Fixed Final Curl Radius, $r_f$

FIG. 17 CONTINUED
(a) Phase 1: Impact-Induced Forces Acting on Each "Independent Body" Following the Corrected Conditions of Fig. 16a

FIG. 18 SCHEMATICS FOR THE COLLISION-INDUCED FORCES FOR VARIOUS BEHAVIORAL PHASES OF THE ELASTIC-PLASTIC CURLING BLADE MODEL
(b) Phase 2: Impact-Induced Forces Acting on Each "Independent Body" Following the Corrected Conditions of Fig. 16b.

FIG. 18 CONTINUED

116
(c) Phase 3: Impact-Induced Forces Acting on Each Body (Applicable Also to Phase 4 by Setting \( r = r_f \))

FIG. 18 CONTINUED

117
FIG. 19 INTERACTION BOUNDARY FOR PERFECTLY-PLASTIC MOMENTS AND AXIAL FORCES

\[ \frac{|M|}{M_o} + \frac{N^2}{N_o^2} = 1 \]

\[ \{ |M| = M_o \]
\[ |N| = N_o \]
(a) Schematic and Nomenclature for a Curled Blade

(b) CG Location Quantities and Associated Nomenclature

FIG. 20 SCHEMATICS OF QUANTITIES ASSOCIATED WITH DETERMINING $I_{CG}$ AND LOCATING THE CG FOR THE CURLED BLADE
FIG. 21  SCHEMATICS OF ACTUAL BLADE AND IDEALIZED BLADE CROSS SECTIONS USED FOR ESTIMATING THE FULLY-PLASTIC BENDING MOMENT CAPABILITY $M_o$
IDEALIZED RING SEGMENT IS STRAIGHT ONLY FOR COLLISION INSPECTION AND COLLISION-FORCE ANALYSIS

\[ \alpha = \text{"PENETRATION DISTANCE"} \]
\[ = |\vec{R}_{T1} - \vec{R}_I| \text{ FOR } |\vec{R}_{T1}| > |\vec{R}_I| \]

T1, T2: BLADE TIPS

FIG. 22 BLADE-RING COLLISION INSPECTION SCHEMATIC
FIG. 23 SCHEMATICS AND NOMENCLATURE FOR THE DISTRIBUTION OF THE COLLISION-INTERACTION FORCES TO THE IMPACTED CURVED RING ELEMENT
FIG. 24 PREDICTED TIME HISTORIES OF THE COLLISION-INDUCED NORMAL FORCE ON THE RING ACCORDING TO THE STRAIGHT ELASTIC BLADE MODEL FOR BLADE-RING IMPACT
FIG. 25
PREDICTED TIME HISTORIES OF THE ANGULAR VELOCITY AND ANGULAR ORIENTATION OF THE FRAGMENT ACCORDING TO THE STRAIGHT ELASTIC BLADE MODEL FOR BLADE-RING IMPACT, FOR TWO VALUES OF FRICTION COEFFICIENT.

Angular Orrientation of Fragments, \( \theta \) (Deg.)

Time after Initial Impact, TAI (Microseconds)

SEB Model

\( \mu = 0 \)

\( \mu = 0.15 \)

Angular Velocity of Fragments, \( \omega \) (Rad./Sec.)

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FIG. 26 DEFORMATION PATTERNS FOR RING QUADRANT 1 AT 646 MICROSECONDS AFTER INITIAL BLADE-RING IMPACT, PREDICTED USING THE STRAIGHT ELASTIC BLADE MODEL
FIG. 27 TIME HISTORIES OF ANGULAR VELOCITY OF THE FRAGMENT FOR VARIOUS VALUES OF FRICTION COEFFICIENT, PREDICTED USING THE ELASTIC, PERFECTLY-PLASTIC SHORTENING BLADE MODEL
FIG. 28 COMPARISON OF TIME HISTORIES OF FRAGMENT ANGULAR VELOCITY AND ORIENTATION FOR THE STRAIGHT ELASTIC BLADE MODEL AND THE ELASTIC, PERFECTLY-PLASTIC SHORTENING BLADE MODEL, FOR TWO VALUES OF FRICTION COEFFICIENT.
FIG. 28 CONCLUDED
FIG. 29 PREDICTED TIME HISTORIES OF THE COLLISION-INDUCED NORMAL FORCE ON THE RING ACCORDING TO THE ELASTIC, PERFECTLY-PLASTIC SHORTENING BLADE MODEL FOR BLADE-RING IMPACT.
RING AND BLADE POSITIONS IMMEDIATELY BEFORE IMPACT

EL-PP-SB MODEL

SEB MODEL

(a) Deformed Ring Shape and Blade Location at TAIL = 346 µsec.

FIG. 30 DEFORMATION PATTERNS FOR RING QUADRANT 1 AT TWO DIFFERENT TIMES AFTER INITIAL BLADE-RING IMPACT, COMPARING PREDICTIONS OF THE STRAIGHT ELASTIC BLADE MODEL AND THE ELASTIC, PERFECTLY PLASTIC SHORTENING BLADE MODEL, FOR A COEFFICIENT OF FRICTION OF .15.
RING AND BLADE POSITIONS
IMMEDIATELY BEFORE IMPACT

EL-PP-SB MODEL
SEB MODEL

(b) Deformed Ring Shape and Blade Location at TAI = 646 μsec.

FIG. 30 CONCLUDED
FIG. 31. PHOTOGRAPHS OF UNTESTED AND IMPACT/CONTAINMENT TESTED T58 ENGINE TURBINE ROTOR BLADES
FIG. 32 "FINAL" DEFORMED BLADE CONFIGURATIONS AT TAI1=646 MICROSECONDS FOR THE PARAMETRIC CALCULATIONS OF BLADE/RING IMPACT, OBTAINED BY USING THE EL-PP-CB MODEL.
FIG. 33 IMPACT-INDUCED NORMAL FORCE HISTORIES PREDICTED VIA THE ELASTIC, PERFECTLY-PLASTIC CURLING BLADE MODEL FOR BLADE/RING IMPACT

(a) Blade Yield Stress $\sigma_y = 240,000$ psi ($\bar{\sigma}_y = 3$)
(c) Blade Yield Stress $\sigma_y = 80,000$ psi ($\bar{\sigma}_y = 1$)

FIG. 33   CONCLUDED
FIG. 34 COMPARISON OF RING QUADRANT 1 PROFILES PREDICTED BY THE EL-PP-CB MODEL AT TAJII=646 MICROSECONDS FOR BLADE YIELD STRESS $\sigma_y=160,000$ PSI($\bar{\sigma}_y=2$)
FIG. 35 BLADE $\omega_\xi$ HISTORIES PREDICTED BY THE EL-PP-CB MODEL FOR FIXED BLADE $\bar{\sigma}_y$ VALUES AND VARIOUS $\bar{r}_\xi$ VALUES DURING BLADE/RING IMPACT INTERACTION.
(b) Blade Yield Stress $\sigma_y = 160,000$ psi ($\bar{\sigma}_y = 2$)

FIG. 35 CONTINUED
* Blade is fully curled

(c) Blade Yield Stress $\sigma_y = 80,000$ psi ($\overline{\sigma}_y = 1$)

FIG. 35 CONCLUDED
FIG. 36 BLADE YIELD STRESS VALUES, AND VARIOUS FY VALUES DURING BLADE/RING IMPACT INTERACTION.

(a) Blade yield stress σy = 240,000 psi (FY = 3)
(b) Blade Yield Stress $\sigma_Y = 160,000$ psi ($\bar{\sigma}_Y = 2$)

FIG. 36 CONTINUED
(c) Blade Yield Stress $\sigma_Y = 80,000$ psi ($\bar{\sigma}_Y = 1$)

FIG. 36 CONCLUDED
FIG. 37. COMPARISON OF EL-PP-SB and EL-PP-CB MODEL PREDICTIONS FOR FRICTIONLESS BLADE/RING IMPACT WITH EXPERIMENTAL RING DEFORMATION AND BLADE FRAGMENT ANGULAR ORIENTATION CHANGE DATA.
PRE-IMPACT PROFILE
EXPERIMENT
○ EL-PP-SB MODEL
△ EL-PP-CB MODEL
($r_f = 0.3$ IN.)

FIG. 37 CONTINUED

(b) First Quadrant of the Deformed Ring at TAIL = 626 μsec

FIG. 37 CONTINUED
FIG. 38. COMPARISON OF EL-PP-SB AND EL-PP-CB MODEL PREDICTIONS FOR $\mu = 0.15$ BLADE/RING IMPACT WITH EXPERIMENTAL RING DEFORMATION AND BLADE FRAGMENT ANGULAR ORIENTATION CHANGE DATA.
PRE-IMPACT PROFILE

EXPERIMENT

○ EL-PP-SB MODEL
△ EL-PP-CB MODEL

($r_f = 0.3$ IN)

\[ \mu = 0.15 \]
\[ (\sigma_y)_f = 160,000 \text{ PSI} \]

(b) First Quadrant of the Deformed Ring at $T_{AI} = 626 \mu$sec.

FIG. 38. CONTINUED
\[ \mu = 0.15 \]
\[ (\sigma_y)_f = 160,000 \text{ PSI} \]

(C) Blade Fragment Angular Orientation Change $\Delta \theta_f$

FIG. 38. CONCLUDED
FIG. 39 COMPARISON OF EL-PP-CB PREDICTIONS FOR $r_f = 0.3$ INCH AND $(\sigma_y)_f = 240,000$ PSI WITH EXPERIMENTAL FIRST-QUADRANT DEFORMED RING PROFILE AT TWO TAII INSTANTS

(a) TAII = 326 $\mu$sec
PRE-IMPACT PROFILE

EXPERIMENT

\( \Delta \) EL-PP-CB MODEL \((\mu = 0.15)\)

\( \circ \) EL-PP-CB MODEL \((\mu = 0.40)\)

\((\sigma_y)_f = 240,000 \text{ PSI}\)

\(r_f = 0.3 \text{ IN}\)

(b) \(\text{TAlI} = 626 \text{ \mu sec}\)

FIG. 39. CONCLUDED
FIG. 40. COMPARISON OF EL-PP-CB MODEL PREDICTIONS FOR VARIOUS VALUES OF THE FRICTION COEFFICIENT $\mu$ WITH EXPERIMENTAL BLADE FRAGMENT ANGULAR ORIENTATION CHANGE DATA.