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ASSEMBLY LANGUAGE
INTERVAL ARITHMETIC SOFTWARE

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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IBM SYSTEM/360
ASSEMBLY LANGUAGE
INTERVAL ARITHMETIC SOFTWARE

E. J. Phillips
Computing and Software, Inc.
Data Systems Division

April 1972

Goddard Space Flight Center
Greenbelt, Maryland
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ACKNOWLEDGMENT

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INTRODUCTION

This document describes some computer software designed to perform interval arithmetic. An interval is defined as the set of all real numbers between two given numbers including or excluding one or both endpoints. Interval arithmetic consists of the various elementary arithmetic operations defined on the set of all intervals, such as interval addition, subtraction, union, etc. The theoretical aspects of interval arithmetic are described in reference (2).

One of the main applications of interval arithmetic is in the area of error analysis of computer calculations. For example, it has been used successfully to compute bounds on sounding errors in the solution of linear algebraic systems\(^{(2)}\), error bounds in numerical solutions of ordinary differential equations\(^{(1)}\)\(^{(3)}\) as well as integral equations and boundary value problems. The described software should enable users to implement algorithms of the type described in these references efficiently on the IBM 360 system.

The subroutines in this package were supplied to Goddard Space Flight Center by Dr. Ronald Tost, Institute für Numerische Datenverarbeitung under an exchange agreement with Dr. C. E. Velez, GSFC Code 552. Although the subroutines were coded in IBM 360 assembly language, they were designed so that calls may be made from FORTRAN programs. The sample problem in Section VII and all of the examples in Sections I to VI are given in FORTRAN to illustrate how this package may be used in FORTRAN programs. The subroutines were tested under varying conditions by Computing and Software, Inc., personnel.

The following pages explain in detail the arithmetic operations performed by this package.
INTERVAL ARITHMETIC PACKAGE

I. Language

Assembler Language Code for IBM/360.

II. Purpose

This package is designed to perform arithmetic operations using floating point (64 bits) interval arithmetic.

III. Method

The interval members are defined as COMPLEX * 16, by the user. The basic arithmetic operations are performed along with some basic set theory applications. The arithmetic functions are: addition, subtraction, multiplication, division, exponentiation. The absolute value of an interval and a negative interval can be created. Real and integer numbers can be converted into interval numbers. The intersection and union between two sets of interval numbers can be found. An illegal interval can be tested for. There are tests to determine if zero is contained in (∈) an interval or if one interval is contained in another. The sum of two sets of interval numbers can be found by means of a scalar product. The above operations are accomplished by the following functions:

DLI, DRE, $IT$, $DT$, $ADD$, $SUB$, $MUL$, $DIV$, $POT$, $MIN$, $ABS$, $SCH$, $VER$, IZER, ICOR, $ENT$, $SUM$, $INP$.

IV. Calling Sequence

The interval numbers and functions are defined as COMPLEX * 16.

In the following set: A is an INTERVAL (AL, AR)
B is an INTERVAL (BL, BR)
C is an INTERVAL (CL, CR)
D: REAL * 8
N: INTEGER * 4

Functions:

1. \( D = DLI(A) \) \( D: = AL \) Left Corner of A
   
   Ex. \( A = (2.5, 4.0) \)
   
   \( D = DLI(A) = 2.5 \)
2. \( D = DRE(A) \quad D = AR \) Right Corner of \( A \)
   Ex. \( A = (2.5, 4.0) \)
   \( D = DRE(A) = 4.0 \)

3. \( A = IT(N) \quad A = (N, N) \) Produce an interval from an integer
   Ex. \( N = 5 \)
   \( A = IT(N) = (5.0, 5.0) \)

4. \( A = DT(D) \quad A = (D, D) \) Produce an interval from a real number
   Ex. \( D = 2.52 \)
   \( A = DT(D) = (2.52, 2.52) \)
   \( A = DT(0.5DO) = (0.5D0, 0.5D0) \)

5. \( A = ADD(B,C) \quad A = B + C \) Addition
   Ex. \( B = (3.0, 4.0) \)
   \( C = (2.0, 2.0) \)
   \( A = ADD(B,C) = (3.0 + 2.0, 4.0 + 2.0) = (5.0, 6.0) \)
   \( B = (-1.5, -0.5) \)
   \( C = (2.5, 4.5) \)
   \( A = ADD(B,C) = (-1.5 + 2.5, -0.5 + 4.5) = (1.0, 4.0) \)

6. \( A = SUB(B,C) \quad A = B - C \) Subtraction
   Ex. \( B = (3.0, 4.0) \)
   \( C = (2.0, 2.0) \)
   \( A = SUB(B,C) = (3.0 - 2.0, 4.0 - 2.0) = (1.0, 2.0) \)
   \( B = (3.0, 4.0) \)
   \( C = (-1.5, -2.5) \)
   \( A = SUB(B,C) = (3.0 - (-1.5), 4.0-(-2.5)) = (4.5, 6.5) \)

7. \( A = MUL(B,C) \quad A = B * C \) Multiplication
   Ex. \( B = (3.0, 4.0) \)
   \( C = (2.0, 2.5) \)
   \( A = MUL(B,C) = (\text{MIN}(6.0, 7.5, 8.0, 10.0), \text{MAX}(6.0, 7.5, 8.0, 10.0)) = (6.0, 10.0) \)

8. \( A = DIV(B,C) \quad A = B/C \) Division
   Ex. \( B = (3.0, 4.0) \) if \( 0 \neq (B,C) \)
   \( C = (1.0, 2.0) \)
   \( A = DIV(B,C) = (3.0, 4.0) * (0.5, 1.0) \)
   \( = (\text{MIN}(1.5, 3.0, 2.0, 4.0), \text{MAX}(1.5, 3.0, 2.0, 4.0)) = (1.5, 4.0) \)
9. $A = \$DOT (B, N) \quad A: = B**N \quad \text{Exponentiation}$

Ex. $B = (3.0, 4.0)$
$N = 2$
$A = \$DOT (B, N) = (9.0, 16.0)$

10. $A = \$MIN (B) \quad A: = -B \quad \text{Negative Interval}$

$B = (3.0, 4.0)$
$A = \$MIN(B) = (-4.0, -3.0)$

11. $A = \$ABS (B) \quad A: = B \text{ if } BL > 0 \quad A: = -B \text{ if } BR < 0 \text{ else}\quad A: = (0, \text{MAX}(BL, BR))$

Examples:

(1) $B = (4.0, 4.25)$
$A = \$ABS(B) = (4.0, 4.25)$

(2) $B = (-2.0, -1.0)$
$A = \$ABS(B) = (1.0, 2.0)$

(3) $B = (-1.5, 4.0)$
$A = \$ABS(B) = (0, \text{MAX}(-1.5, 4.0))$
$= (0, 4.0)$

12. $A = \$SCH (B, C) \quad A: = (\text{MAX}((CL, BL), \text{MIN}(CR, BR))) \quad \text{Intersection}$

Ex. $B = (4.0, 5.0)$
$\$SCH can produce illegal intervals. This can be
$C = (-1.5, 2.0)$
$\text{proved with ICOR.}$
$A = \$SCH (B, C) = (\text{MAX}(4.0, -1.5), \text{MIN}(5.0, 2.0))$
$= (4.0, 2.0) \quad (-\text{illegal interval})$

$B = (1.0, 2.0)$
$C = (-1.5, 5.0)$
$A = \$SCH (B, C) = (1.0, 2.0)$

13. $A = \$VER (B, C) \quad A: = (\text{MIN}(CL, BL), \text{MAX}(CR, BR)) \quad \text{Union}$

$B = (1.5, 2.5)$
$C = (1.0, 5.0)$
$A = \$VER (B, C) = (\text{MIN}(1.0, 1.5), \text{MAX}(5.0, 2.5))$
$= (1.0, 5.0)$

14. IF (IZER (A)) Test to see if zero (0) is contained in A

If IZER(A) < 0 GO TO
If IZER(A) = 0 GO TO
If IZER(A) > 0 GO TO
Test to see:

\[< \] if \( AR < 0 \) (if \( AR < 0 \), \( 0 \notin A \))
\[< \] if \( AL > 0 \) (if \( AL > 0 \), \( 0 \notin A \))
\[< \] else \( 0 \in A \) (zero contained in \( A \)) null interval

15. IF (ICOR(A)) \[^{<}\] , \[^{<}\] , \[^{<}\] Interval Test
   IF ICOR(A) < 0 GO TO \[^{<}\]
   IF ICOR(A) = 0 GO TO \[^{<}\]
   IF ICOR(A) > 0 GO TO \[^{<}\]

Test to see:

\[^{<}\] if \( AL > AR \) (illegal interval)
\[^{<}\] if \( AL = AR \)
\[^{<}\] if \( AL < AR \)

16. $\text{ENT} \ - \ Test \ on \ being \ contained \ in$
   CALL $\text{ENTC}(A, B, \&1, \&2, \&3, \&4)$
\[^{1}\] AL > BL and AR < BR (A is contained in B)
\[^{2}\] AL = BL and AR = BR (A and B are equal)
\[^{3}\] AL < BL and AR > BR (B is contained in A)
\[^{4}\] A and B are not fully contained in one another or are not contained in each other at all.

17. $\text{SUM} \ - \ Build \ a \ sum \ with \ a \ scalar \ product.$
   CALL $\text{SUM}(A, I, J, K)$ where \( I = J, K \)

$\text{SUM}$ store \( A, I, J, K \) and sets up a loop made to perform addition.

Along with the call to $\text{SUM}$ a multiplication statement has to follow immediately.

Ex:

\[
\begin{align*}
\text{CALL} & \quad \text{SUM}(A, I, J, K) \\
D & = \quad \text{MUL}(B, C)
\end{align*}
\]

\[
\begin{align*}
A & = (0.0, 0.0) \\
B(1) & = (1.0, 2.0) C(1) = (2.0, 3.0) \\
B(2) & = (3.0, 4.0) C(2) = (4.0, 5.0) \\
B(3) & = (5.0, 6.0) C(3) = (6.0, 7.0)
\end{align*}
\]

Example:

\[
\begin{align*}
\text{CALL} & \quad \text{SUM}(A, I, 1, 3) \\
D & = \quad \text{MUL}(B(I), C(I))
\end{align*}
\]

\[
\begin{align*}
J & = 1, K = 3 \\
\text{CALL} & \quad \text{SUM}(A, I, J, K) \\
D & = \quad \text{MUL}(B(I), C(I))
\end{align*}
\]

Correspondent

\[
\begin{align*}
\text{AKKU} & = A \\
\text{DO} & \quad I = 1, 3
\end{align*}
\]

\[
\begin{align*}
D & = (0.0, 0.0) + (2.0, 6.0) \\
& = (2.0, 6.0) + (12.0, 20.0) \\
& = (14.0, 26.0) + (30.0, 42.0) \\
& = (44.0, 68.0)
\end{align*}
\]
1. \[ \text{AKKU} = \text{AKKU} + \text{$MUL (B(I), C(I))} \]
\[ \text{D} = \text{AKKU} \]

D, AKKU, and A are intervals; B, C are interval arrays. The sequence of $SUM and $MUL cannot contain interval functions.

18. \$INP A = \$INP ('\langle INPUT\rangle') conversion routine

$\langle INPUT\rangle$ is an interval number, that is produced from an alphanumeric field of input values surrounded by quotation marks. The routine creates an interval from the set of numbers in the alphanumeric field by choosing the smallest number for the left endpoint and the largest number for the right endpoint. The interval number is greater than or equal to 1 (\(\geq 1\)) in absolute value. The real or integer numbers are separated by commas but can contain no blanks. The beginning and end of the interval number must contain at least one blank. The left side of the output is the minimum of the real or integer numbers. The right side of the output contains the maximum of the real or integer numbers.

Example:

- \[ A = \text{\$INP ('1.3, 1.56') } \rightarrow \text{ A = (1.3, 1.56)} \]
- \[ A = \text{\$INP ('-2., 4.') } \rightarrow \text{ A = (-2., 4.)} \]
- \[ A = \text{\$INP ('1') } \rightarrow \text{ No answer a blank doesn't preceed "1" on left side} \]
- \[ A = \text{\$INP ('1E45, 23, 2.5') } \rightarrow \text{ A = (2.5, 1.E45)} \]

The arithmetic functions can produce exponent overflow and divide check interrupts. Certain interval functions can produce illegal intervals and only two of the subroutines in this package can determine if the output is illegal, \text{IZER} and \text{ICOR}.

In all the arithmetic interval functions the input is tested to determine if it is illegal. No error messages are generated just a return to the calling program.

DEFINITION OF \$\langle INPUT\rangle$:

\[ \langle INPUT \rangle := ^* \langle\text{INTER}\rangle ^* \]
\[ \langle\text{INTER}\rangle := \langle\text{REAL}\rangle \]
\[ := \langle\text{INTER}\rangle, \langle\text{REAL}\rangle \]
\[ \langle\text{REAL}\rangle := \langle\text{FLOAT}\rangle \]
\[ := \langle\text{FLOAT}\rangle \langle\text{EX}\rangle \langle\text{INTEG}\rangle \]
\[
\langle \text{FLOAT} \rangle := \langle \text{INTEG} \rangle
\]
\[
:= \langle \text{INTEG} \rangle \cdot \langle \text{ZIFF} \rangle
\]
\[
\langle \text{INTEG} \rangle := \langle \text{SIGN} \rangle \langle \text{ZIFF} \rangle
\]
\[
\langle \text{ZIFF} \rangle := \langle \text{DIGIT} \rangle
\]
\[
:= \langle \text{ZIFF} \rangle \langle \text{DIGIT} \rangle
\]
\[
\langle \text{DIGIT} \rangle := 1 2 3 4 5 6 7 8 9 \emptyset < >
\]
\[
\langle \text{SIGN} \rangle := + - < >
\]
\[
\langle \text{EX} \rangle := E
\]
\[
:= D
\]

- At least one blank.

<> The empty string.

V. MATHEMATICS USED IN PACKAGE

If * is one of the symbols +, -, \cdot, \div, the arithmetic operations of intervals are defined by:

\[
[a, b] \cdot [c, d] = \{x \cdot y \mid a \leq x \leq b, c \leq y \leq d\}
\]

\[
[a, b] + [c, d] = [a + c, b + d]
\]

\[
[a, b] - [c, d] = [a - d, b - c]
\]

\[
[a, b] \cdot [c, d] = [\min (ac, ad, bc, bd), \max (ac, ad, bc, bd)]
\]

and if 0 \notin [c, d] then

\[
[a, b] / [c, d] = [a, b] \cdot \frac{1}{d}, \frac{1}{c}
\]

Normalized floating point interval arithmetic is used in this package. Under IBM O/S 360 a floating point number is represented as

<table>
<thead>
<tr>
<th>bit</th>
<th>0</th>
<th>1</th>
<th>-</th>
<th>7</th>
<th>8</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign</td>
<td>characteristic</td>
<td>fraction (binary)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exponent</td>
<td>mantissa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where bit = 0 sign of number (0-positive, 1 negative)
$1 - 7$ = the exponent indicating the power of 16 by which the fraction is multiplied. A zero in bit 1 indicates negative exponent, 1 indicates positive exponent. It is coded in excess 64-notation (decimal). That is, subtract decimal equivalent of characteristic by 64 to obtain the actual characteristic. e.g. \[ \text{0} \bigg| \text{1000000} \bigg] = 64 \\
64 - 64 = 0 = 16 \\

$8 - 63$ = fraction or mantissa. The binary points of the fraction is just before bit position 8. That is to say that a 1 in bit position 8 represents $2^{-1}$, a 1 in bit position 9 represents $2^{-2}$, etc.

The magnitudes of a floating point number is $16^{-65}$ $(5.4 \times 10^{-79})$ \\
$(1 - 16^{-14}) \cdot 16^{63} (7.2 \times 10^{75})$

Example:

1 1000001 01010100 00000000 00000000 00000000 etc.

<table>
<thead>
<tr>
<th>Sign</th>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>65</td>
</tr>
<tr>
<td>65 - 64 = 1</td>
<td></td>
</tr>
</tbody>
</table>

$- 1 \times 16^1 \times (2^{-2} + 2^{-4} + 2^{-6}) = -2^4 (2^{-2} + 2^{-4} + 2^{-6})$ \\
$= -(2^2 + 2^0 + 2^{-2})$ \\
$= -(4 + 1 + 0.25) = -5.25$

Normalized floating point numbers have a non-zero high order hexadecimal fraction digit. One or more high order hexadecimal fraction digits which are zero makes the number unnormalized. Normalization consists of shifting the fraction left until the high order hexadecimal digit is non-zero and reducing the characteristic by the number of hexadecimal digits shifted. For multiplication and division the operands are normalized before the operations are performed. For the remaining operands (+, -) the result is normalized.

A = 436D1000 00000000 = normalized number 
B = 4306D1000 00000000 = unnormalized 
= 426D1000 000000 after normalization.
VI. RESTRICTIONS

1. Only basic arithmetic operations and basic set theory applications are performed. No trigonometric operations are performed.

2. Uses long (64 bits) floating point interval arithmetic.

3. The arithmetic performed can produce illegal intervals.

4. The arithmetic performed can cause arithmetic overflow and divide check interrupts.
APPENDIX

LEVEL 20.1 (AUG 71)  OS/360 FORTRAN H

DATE 72.108/09.51.35

COMPILER OPTIONS - NAME= MAIN,OPT=01,LINECNT=58,SIZE=000K,
SOURCE, EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,TD,XREF

ISN 0002 IMPLICIT COMPLEX*16(A-H,S),REAL*8(D-Z),INTEGER(I-N)
ISN 0003 DIMENSION C(20),D(20),E(20)
ISN 0004 WRITE(6,101)
ISN 0005 I=1
ISN 0006 P=0.5DO
ISN 0007 A= (2.OOO,S.000)
ISN 0008 B= (1.000,B.000)
ISN 0009 N=3
ISN 0010 V=DLI(A)
ISN 0011 S=DLI(B)
ISN 0012 T=DLI(B)
ISN 0013 U=DLI(B)
ISN 0014 WRITE(6,203) V,S,T,U,N,I
ISN 0015 WRITE(6,222)
ISN 0016 D(1)=ITS(N)
ISN 0017 WRITE(6,108)
ISN 0018 R=DLI(D(1))
ISN 0019 S=DLI(D(1))
ISN 0020 WRITE(6,2002) I,R,S
ISN 0021 I=I+1
ISN 0022 D(12)=SDT$P(I)
ISN 0023 WRITE(6,109)
ISN 0024 R=DLI(D(11))
ISN 0025 S=DLI(D(11))
ISN 0026 WRITE(6,2002) I,R,S
ISN 0027 I=I+1
ISN 0028 R=DLI(D(11))
ISN 0029 WRITE(6,102)
ISN 0030 R=DLI(D(1))
ISN 0031 S=DLI(D(1))
ISN 0032 WRITE(6,2002) I,R,S
ISN 0033 I=I+1
ISN 0034 D(4)=SUB(A,B)
ISN 0035 WRITE(6,103)
ISN 0036 R=DLI(D(11))
ISN 0037 S=DLI(D(11))
ISN 0038 WRITE(6,2002) I,R,S
ISN 0039 I=I+1
ISN 0040 D(5)=MLU(A,B)
ISN 0041 WRITE(6,104)
ISN 0042 R=DLI(D(1))
ISN 0043 S=DLI(D(1))
ISN 0044 WRITE(6,2002) I,R,S
ISN 0045 I=I+1
ISN 0046 D(6)=SDIV(A,B)
ISN 0047 WRITE(6,105)
ISN 0048 R=DLI(D(1))
ISN 0049 S=DLI(D(1))
ISN 0050 WRITE(6,2002) I,R,S
ISN 0051 WRITE(6,223)
ISN 0052 I=1
ISN 0053 C(1)=SPOT(A,N)
ISN 0054 WRITE(6,110)
ISN 0055 R=DLI(C(1))
ISN 0056  S=DREICII)
ISN 0057  WRITE(6,2002) I,R,S
ISN 0058  I=I+1
ISN 0059  C(2)=SPOT(B,N)
ISN 0060  WRITE(6,111)
ISN 0061  R=DLDI(C(1))
ISN 0062  S=DREICII)
ISN 0063  WRITE(6,2002) I,R,S
ISN 0064  I=I+1
ISN 0065  C(3)=MIN(A)
ISN 0066  WRITE(6,112)
ISN 0067  R=DLDI(C(1))
ISN 0068  S=DREICII)
ISN 0069  WRITE(6,2002) I,R,S
ISN 0070  I=I+1
ISN 0071  C(4)=MIN(B)
ISN 0072  WRITE(6,113)
ISN 0073  R=DLDI(C(1))
ISN 0074  S=DREICII)
ISN 0075  WRITE(6,2002) I,R,S
ISN 0076  I=I+1
ISN 0077  C(5)=ABS(A)
ISN 0078  WRITE(6,114)
ISN 0079  R=DLDI(C(1))
ISN 0080  S=DREICII)
ISN 0081  WRITE(6,2002) I,R,S
ISN 0082  I=I+1
ISN 0083  C(6)=ABS(B)
ISN 0084  WRITE(6,115)
ISN 0085  R=DLDI(C(1))
ISN 0086  S=DREICII)
ISN 0087  WRITE(6,2002) I,R,S
ISN 0088  I=I+1
ISN 0089  C(7)=SCH(A,B)
ISN 0090  WRITE(6,107)
ISN 0091  R=DLDI(C(1))
ISN 0092  S=DREICII)
ISN 0093  WRITE(6,2002) I,R,S
ISN 0094  IF(ICOR(C(7))) 10,20,20
ISN 0095  10 IF(R.GT.S) WRITE(6,190) R,S
ISN 0096  I=I+1
ISN 0097  20 C(8)=VER(A,B)
ISN 0098  WRITE(6,106)
ISN 0099  R=DLDI(C(1))
ISN 0100  S=DREICII)
ISN 0101  WRITE(A,2002) I,R,S
ISN 0102  IF(I zer(I)) 28,30,29
ISN 0103  28 IF(S.LT.0) GO TO 35
ISN 0104  29 IF(R.GT.0) GO TO 35
ISN 0105  30 WRITE(A,210)
C TEST ON BEING CONTAINED IN
ISN 0106  35 WRITE(A,217)
ISN 0107  CALL SENT(A,0,636,637,638,639)
ISN 0108  36 WRITE(A,211)
ISN 0109  37 WRITE(A,212)
ISN 0114   GO TO 40
ISN 0115   WRITE(6,213)
ISN 0116   GO TO 40
ISN 0117   WRITE(6,214)
ISN 0118   A=(0.000,0.000)
ISN 0119   WRITE(6,101)
ISN 0120   T=DLIR(R)
ISN 0121   (=NREI(R)
ISN 0122   WRITE(6,203) V,S,T,U,N,P
ISN 0123   WRITE(6,217)
ISN 0124   CALL SERT(A,B,.641,.642,.643,.644)
ISN 0125   41 WRITE(6,211)
ISN 0126   GO TO 44
ISN 0127   42 WRITE(6,212)
ISN 0128   GO TO 45
ISN 0129   43 WRITE(6,213)
ISN 0130   GO TO 45
ISN 0131   44 WRITE(6,214)
ISN 0132   45 CONTINUE
ISN 0133   CALL $SUM(B,I,1,5)
ISN 0134   F=$MUL(C(I),D(I))
ISN 0135   R=DLIFF)
ISN 0136   S=DREI(F)
ISN 0137   WRITE(6,218)
ISN 0138   WRITE(6,215),R,S.
 ISN 0139   C CONVERSION ROUTINE
ISN 0140   C(I)=SINP( ' 1.3,1.56 ')
ISN 0141   C(2)=SINP( ' -2.4 ' )
ISN 0142   C(3)=SINP( ' 1 ' )
ISN 0143   C(4)=SINP( ' 1E7,23,2.5 ')
ISN 0144   C(5)=SOFS(1)
ISN 0145   WRITE(6,216)
ISN 0146   DO 9 I=1,6
ISN 0147   Continue(C(I))
ISN 0148   S=DRE(C(I))
ISN 0149   9 WRITE(6,200) I,R,S.
ISN 0150   2002 FORMAT(IX,15, 'LEFT= ',D24.16, ' RIGHT= ',D24.16)
ISN 0151   101 FORMAT(' INTERVAL ARITHMETIC PACKAGE')
ISN 0152   102 FORMAT('OA+B')
ISN 0153   103 FORMAT('OA-A')
ISN 0154   104 FORMAT('OA=R')
ISN 0155   105 FORMAT('OA+B')
ISN 0156   106 FORMAT('OA UNION B')
ISN 0157   107 FORMAT('OA INTERSECT B')
ISN 0158   108 FORMAT('OSITE(N)')
ISN 0159   109 FORMAT('OSITE(P)')
ISN 0160   110 FORMAT('OA<>N')
ISN 0161   111 FORMAT('OP<NP')
ISN 0162   112 FORMAT('OSMIN(A)')
ISN 0163   113 FORMAT('OSMIN(R)')
ISN 0164   114 FORMAT('OSABS(A)')
ISN 0165   115 FORMAT('OSABS(R)')
ISN 0166   190 FORMAT(' THIS IS AN ILLEGAL INTERVAL ('G24.16, '1',G24.16, ')')
ISN 0167   203 FORMAT(I4,'= ',D24.16,'+','D24.16/1X,'B= ',D24.16,'+','D24.16/1X,'N= ',15/1X,'P= ',D24.16)
FORMAT(' ZERO IS CONTAINED IN THE FOLLOWING INTERVAL (',G24.16,')

FORMAT(' A IS CONTAINED IN B')

FORMAT(' A AND B ARE EQUAL')

FORMAT(' B IS CONTAINED IN A')

FORMAT(' A AND B ARE NOT FULLY CONTAINED IN EACH OTHER')

FORMAT(' LEFT= ',D24.16,2X,'RIGHT=',D24.16)

FORMAT(' O$INP CONVERSION ROUTINE')

FORMAT(' O$ENT-TEST ON BEING CONTAINED IN')

FORMAT(' O$UM PRODUCED WITH SCALAR PRODUCT')

FORMAT(' OVALUES FOR ARRAY D(I)')

FORMAT(' OVALUES FOR ARRAY C(I)')

STOP

END
INTERVAL ARITHMETIC PACKAGE

A = 0.200000000000000000 01, 0.800000000000000000 01
B = 0.0

N = 3
P = 0.500000000000000000 00

SENT-TEST ON BEING CONTAINED IN
A AND B ARE NOT FULLY CONTAINED IN EACH OTHER

SUM PRODUCED WITH SCALAR PRODUCT
LEFT = -0.685000000000000000 02 RIGHT = 0.873000000000000000 03

$INP CONVERSION ROUTINE

1LEFT = 0.130000000000000000 01 RIGHT = 0.156000000000000000 01
2LEFT = -0.200000000000000000 01 RIGHT = 0.400000000000000000 01
3LEFT = -0.200000000000000000 01 RIGHT = 0.400000000000000000 01
4LEFT = 0.250000000000000000 00 RIGHT = 0.100000000000000000 46
5LEFT = 0.550000000000000000 00 RIGHT = 0.550000000000000000 01
6LEFT = 0.900000000000000000 00 RIGHT = 0.900000000000000000 01
INTERVAL ARITHMETIC PACKAGE

A = 0.20000000000000000 01, 0.50000000000000000 01
B = 0.10000000000000000 01, 0.80000000000000000 01
N = 3
P = 0.50000000000000000 00

VALUES FOR ARRAY D(I)

$SEL$(N)
1LEFT = 0.30000000000000000 01 RIGHT = 0.30000000000000000 01
$SEL$(P)
2LEFT = 0.50000000000000000 00 RIGHT = 0.50000000000000000 00
A+B
3LEFT = 0.30000000000000000 01 RIGHT = 0.13000000000000000 02
A-R
4LEFT = -0.60000000000000000 01 RIGHT = 0.40000000000000000 01
A*B
5LEFT = 0.20000000000000000 01 RIGHT = 0.40000000000000000 02
A/B
6LEFT = 0.25000000000000000 00 RIGHT = 0.50000000000000000 01

VALUES FOR ARRAY C(I)

A**N
1LEFT = 0.80000000000000000 00 RIGHT = 0.12500000000000000 03
A**N
2LEFT = 0.10000000000000000 01 RIGHT = 0.51200000000000000 03
$MIN$(A)
3LEFT = -0.50000000000000000 00 RIGHT = -0.20000000000000000 01
$MIN$(B)
4LEFT = -0.80000000000000000 00 RIGHT = -0.10000000000000000 01
$ARS$(A)
5LEFT = 0.20000000000000000 01 RIGHT = 0.50000000000000000 01
$ARS$(B)
6LEFT = 0.10000000000000000 00 RIGHT = 0.80000000000000000 01
A INTERSECT B
7LEFT = 0.20000000000000000 01 RIGHT = 0.50000000000000000 01
A UNION B
8LEFT = 0.10000000000000000 01 RIGHT = 0.80000000000000000 01

$SENT-TEST ON BEING CONTAINED IN
A IS CONTAINED IN B
References

