IBM SYSTEM/360
ASSEMBLY LANGUAGE
INTERVAL ARITHMETIC SOFTWARE

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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IBM SYSTEM/360
ASSEMBLY LANGUAGE
INTERVAL ARITHMETIC SOFTWARE

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Computing and Software, Inc.
Data Systems Division

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Goddard Space Flight Center
Greenbelt, Maryland
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>v</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>INTERVAL ARITHMETIC PACKAGE</td>
<td>2</td>
</tr>
<tr>
<td>I Language</td>
<td>2</td>
</tr>
<tr>
<td>II Purpose</td>
<td>2</td>
</tr>
<tr>
<td>III Method</td>
<td>2</td>
</tr>
<tr>
<td>IV Calling Sequence</td>
<td>2</td>
</tr>
<tr>
<td>V Mathematics Used in Package</td>
<td>7</td>
</tr>
<tr>
<td>VI Restrictions</td>
<td>9</td>
</tr>
<tr>
<td>VII Appendix (Sample problem and output)</td>
<td>10</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>16</td>
</tr>
</tbody>
</table>
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INTRODUCTION

This document describes some computer software designed to perform interval arithmetic. An interval is defined as the set of all real numbers between two given numbers including or excluding one or both endpoints. Interval arithmetic consists of the various elementary arithmetic operations defined on the set of all intervals, such as interval addition, subtraction, union, etc. The theoretical aspects of interval arithmetic are described in reference (2).

One of the main applications of interval arithmetic is in the area of error analysis of computer calculations. For example, it has been used successfully to compute bounds on sounding errors in the solution of linear algebraic systems(2), error bounds in numerical solutions of ordinary differential equations(1)(3) as well as integral equations and boundary value problems. The described software should enable users to implement algorithms of the type described in these references efficiently on the IBM 360 system.

The subroutines in this package were supplied to Goddard Space Flight Center by Dr. Ronald Tost, Institute für Numerische Datenverarbeitung under an exchange agreement with Dr. C. E. Velez, GSFC Code 552. Although the subroutines were coded in IBM 360 assembly language, they were designed so that calls may be made from FORTRAN programs. The sample problem in Section VII and all of the examples in Sections I to VI are given in FORTRAN to illustrate how this package may be used in FORTRAN programs. The subroutines were tested under varying conditions by Computing and Software, Inc., personnel.

The following pages explain in detail the arithmetic operations performed by this package.
INTERVAL ARITHMETIC PACKAGE

I. Language

Assembler Language Code for IBM/360.

II. Purpose

This package is designed to perform arithmetic operations using floating point (64 bits) interval arithmetic.

III. Method

The interval members are defined as COMPLEX * 16, by the user. The basic arithmetic operations are performed along with some basic set theory applications. The arithmetic functions are: addition, subtraction, multiplication, division, exponentiation. The absolute value of an interval and a negative interval can be created. Real and integer numbers can be converted into interval numbers. The intersection and union between two sets of interval numbers can be found. An illegal interval can be tested for. There are tests to determine if zero is contained in (ε) an interval or if one interval is contained in another. The sum of two sets of interval numbers can be found by means of a scalar product. The above operations are accomplished by the following functions:

DLI, DRE, $IT$ $16$, $DT$, $ADD$, $SUB$, $MUL$, $DIV$, $POT$, $MIN$, $ABS$, $SCH$, $VER$, IZER, ICOR, $ENT$, $SUM$, $INP$.

IV. Calling Sequence

The interval numbers and functions are defined as COMPLEX * 16.

In the following set:  
A is an INTERVAL (AL, AR)  
B is an INTERVAL (BL, BR)  
C is an INTERVAL (CL, CR)  
D: REAL * 8  
N: INTEGER * 4

Functions:

1. $D = DLI(A)$  
   $D: = AL$  
   Left Corner of A

   Ex. $A = (2.5, 4.0)$  
   $D = DLI(A) = 2.5$
2. $D = DRE(A)$  $D = AR$  Right Corner of A  
   Ex. $A = (2.5, 4.0)$  
   $D = DRE(A) = 4.0$

3. $A = \text{IT}(N)$  $A = (N, N)$  Produce an interval from an integer  
   Ex. $N = 5$  
   $A = \text{IT}(N) = (5.0, 5.0)$

4. $A = \text{DT}(D)$  $A = (D, D)$  Produce an interval from a real number  
   Ex. $D = 2.52$  
   $A = \text{DT}(D) = (2.52, 2.52)$  
   $A = \text{DT}(0.5DO) = (0.5D0, 0.5D0)$

5. $A = \text{ADD}(B, C)$  $A = B + C$  Addition  
   Ex. $B = (3.0, 4.0)$  
   $C = (2.0, 2.0)$  
   $A = \text{ADD}(B, C) = (3.0 + 2.0, 4.0 + 2.0) = (5.0, 6.0)$  
   $B = (-1.5, -0.5)$  
   $C = (2.5, 4.5)$  
   $A = \text{ADD}(B, C) = (-1.5 + 2.5, -0.5 + 4.5) = (1.0, 4.0)$

6. $A = \text{SUB}(B, C)$  $A = B - C$  Subtraction  
   Ex. $B = (3.0, 4.0)$  
   $C = (2.0, 2.0)$  
   $A = \text{SUB}(B, C) = (3.0 - 2.0, 4.0 - 2.0) = (1.0, 2.0)$  
   $B = (3.0, 4.0)$  
   $C = (-1.5, -2.5)$  
   $A = \text{SUB}(B, C) = (3.0 - (-1.5), 4.0 - (-2.5)) = (4.5, 6.5)$

7. $A = \text{MUL}(B, C)$  $A = B \times C$  Multiplication  
   Ex. $B = (3.0, 4.0)$  
   $C = (2.0, 2.5)$  
   $A = \text{MUL}(B, C) = (\text{MIN}(6.0, 7.5, 8.0, 10.0), \text{MAX}(6.0, 7.5, 8.0, 10.0)) = (6.0, 10.0)$

8. $A = \text{DIV}(B, C)$  $A = B/C$  Division  
   Ex. $B = (3.0, 4.0)$  if $0 \neq (B, C)$  
   $C = (1.0, 2.0)$  
   $A = \text{DIV}(B, C) = (3.0, 4.0) \times (0.5, 1.0) = (\text{MIN}(1.5, 3.0, 2.0, 4.0), \text{MAX}(1.5, 3.0, 2.0, 4.0)) = (1.5, 4.0)$
9. $A = $DOT (B, N) \quad A = B**N \quad \text{Exponentiation}$

Ex. $B = (3.0, 4.0)$

$N = 2$

$A = $DOT (B, N) = (9.0, 16.0)$

10. $A = $MIN (B) \quad A = -B \quad \text{Negative Interval}$

$B = (3.0, 4.0)$

$A = $MIN(B) = (-4.0, -3.0)$

11. $A = $ABS (B) \quad A = B \quad \text{if } BL > 0 \quad \text{Absolute Value}$

$A = -B \quad \text{if } BR < 0 \quad \text{else}$

$A = (0, \text{MAX}(BL, BR))$

Examples:

1. $B = (4.0, 4.25)$
   $A = $ABS (B) = (4.0, 4.25)$

2. $B = (-2.0, -1.0)$
   $A = $ABS (B) = (1.0, 2.0)$

3. $B = (-1.5, 4.0)$
   $A = $ABS (B) = (0, \text{MAX}(-1.5, 4.0))$
   $= (0, 4.0)$

12. $A = $SCH (B, C) \quad A = (\text{MAX}(CL, BL), \text{MIN}(CR, BR)) \quad \text{Intersection}$

Ex. $B = (4.0, 5.0)$ \quad $S$SCH can produce illegal intervals. This can be

$C = (-1.5, 2.0)$ \quad \text{proved with ICOR}.$

$A = $SCH (B, C) = (\text{MAX}(4.0, -1.5), \text{MIN}(5.0, 2.0))$

$= (4.0, 2.0)$ (- illegal interval)

$B = (1.0, 2.0)$

$C = (-1.5, 5.0)$

$A = $SCH (B, C) = (1.0, 2.0)$

13. $A = $VER (B, C) \quad A = (\text{MIN}(CL, BL), \text{MAX}(CR, BR)) \quad \text{Union}$

$B = (1.5, 2.5)$

$C = (1.0, 5.0)$

$A = $VER (B, C) = (\text{MIN}(1.0, 1.5), \text{MAX}(5.0, 2.5))$

$= (1.0, 5.0)$

14. \text{IF (IZER (A))} \quad \wedge, \wedge, \wedge \quad \text{Test to see if zero (0) is contained in A}$

If IZER(A) < 0 \text{ GO TO}^\wedge$

If IZER(A) = 0 \text{ GO TO}^\wedge$

If IZER(A) > 0 \text{ GO TO}^\wedge$
Test to see:

\[ \text{if } AR < 0 \ (\text{if } AR < 0, \ 0 \notin A) \]
\[ \text{if } AL > 0 \ (\text{if } AL > 0, \ 0 \notin A) \]
\[ \text{else } 0 \in A \ (\text{zero contained in } A) \quad \text{null interval} \]

15. IF (ICOR(A)) \^, \^^, \~~

\[ \text{Interval Test} \]
\[ \text{IF ICOR(A) < 0 GO TO} \^ \]
\[ \text{IF ICOR(A) = 0 GO TO} \^^ \]
\[ \text{IF ICOR(A) > 0 GO TO} \~~ \]

Test to see:

\[ \text{if } AL > AR \quad \text{(illegal interval)} \]
\[ \text{if } AL = AR \]
\[ \text{if } AL < AR \]

16. $ENT - \text{Test on being contained in}$

\[ \text{CALL } $ENTC \ A, B, \ &1, \ &2, \ &3, \ &4 \]
\[ \^1: \ AL > BL \text{ and } AR < BR \ (A \text{ is contained in } B) \]
\[ \^2: \ AL = BL \text{ and } AR = BR \ (A \text{ and } B \text{ are equal}) \]
\[ \^3: \ AL < BL \text{ and } AR > BR \ (B \text{ is contained in } A) \]
\[ \^4: \ A \text{ and } B \text{ are not fully contained in one another or are not contained in each other at all.} \]

17. $SUM - \text{Build a sum with a scalar product.}$

\[ \text{CALL } $SUM(A, I, J, K) \] \text{where } I = J, K \]

\[ \text{$SUM$ store } A, I, J, K \text{ and sets up a loop made to perform addition.} \]

\[ \text{Along with the call to $SUM$ a multiplication statement has to follow immediately.} \]

Ex:

\[ \text{CALL } $SUM \ (A, I, J, K) \]
\[ D = $MUL \ (B, C) \]
\[ A = (0.0, 0.0) \]
\[ B(1) = (1.0, 2.0) \quad C(1) = (2.0, 3.0) \]
\[ B(2) = (3.0, 4.0) \quad C(2) = (4.0, 5.0) \]
\[ B(3) = (5.0, 6.0) \quad C(3) = (6.0, 7.0) \]

Example:

\[ \text{J = 1, K = 3} \]

\[ \text{CALL } $SUM \ (A, I, 1, 3) \]
\[ D = $MUL \ (B(I), C(I)) \]

\[ \text{D} = (0.0, 0.0) + (2.0, 6.0) \]
\[ = (2.0, 6.0) + (12.0, 20.0) \]
\[ = (14.0, 26.0) + (30.0, 42.0) \]
\[ = (44.0, 68.0) \]
1. \[ AKKU = AKKU + \$MUL (B(I), C(I)) \]
   \[ D = AKKU \]

\( D, AKKU, \) and \( A \) are intervals; \( B, C \) are interval arrays. The sequence of \$SUM and \$MUL cannot contain interval functions.

18. \$INP \( A = \$INP (\{INPUT\}) \) conversion routine

   \( \{INPUT\} \) is an interval number, that is produced from an alphanumeric field of input values surrounded by quotation marks. The routine creates an interval from the set of numbers in the alphanumeric field by choosing the smallest number for the left endpoint and the largest number for the right endpoint. The interval number is greater than or equal to 1 (\( \geq 1 \)) in absolute value. The real or integer numbers are separated by commas but can contain no blanks. The beginning and end of the interval number must contain at least one blank. The left side of the output is the minimum of the real or integer numbers. The right side of the output contains the maximum of the real or integer numbers.

   Example:

   \[ A = \$INP ('1, 1.3, 1.56') \rightarrow A = (1.3, 1.56) \]
   \[ A = \$INP ('-2., 4.') \rightarrow A = (-2., 4.) \]
   \[ A = \$INP ('1') \rightarrow \text{No answer a blank doesn't preceed "1" on left side} \]
   \[ A = \$INP ('1E45, 23, 2.5') \rightarrow A = (2.5, 1E45) \]

   The arithmetic functions can produce exponent overflow and divide check interrupts. Certain interval functions can produce illegal intervals and only two of the subroutines in this package can determine if the output is illegal, IZER and ICOR.

   In all the arithmetic interval functions the input is tested to determine if it is illegal. No error messages are generated just a return to the calling program.

DEFINITION OF \( \{INPUT\} \):

\( \{INPUT\} = " \{INTER\} " \)

\( \{INTER\} = \{REAL\} \)
   \[ = \langle \{INTER\}, \{REAL\} \) \]

\( \{REAL\} = \{FLOAT\} \)
   \[ = \langle \{FLOAT\}, \{EX\}, \{INTEG\} \) \]

6
V. MATHEMATICS USED IN PACKAGE

If * is one of the symbols +, -, *, /, the arithmetic operations of intervals are defined by:

\[[a, b] * [c, d] = \{x * y | a \leq x \leq b, c \leq y \leq d\}\]

\[[a, b] + [c, d] = [a + c, b + d]\]

\[[a, b] - [c, d] = [a - d, b - c]\]

\[[a, b] * [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]\]

and if 0 ∉ [c, d] then

\[[a, b] / [c, d] = [a, b] \cdot \left[\frac{1}{d}, \frac{1}{c}\right]\]

Normalized floating point interval arithmetic is used in this package. Under IBM O/S 360 a floating point number is represented as

<table>
<thead>
<tr>
<th>bit</th>
<th>0</th>
<th>1</th>
<th>-</th>
<th>7</th>
<th>8</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sign</td>
<td>characteristic</td>
<td>fraction (binary)</td>
<td>mantissa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where bit 0 = 0 sign of number (0-positive, 1 negative)
1 - 7 = the exponent indicating the power of 16 by which the fraction is multiplied. A zero in bit 1 indicates negative exponent, 1 indicates positive exponent. It is coded in excess 64-notation (decimal). That is, subtract decimal equivalent of characteristic by 64 to obtain the actual characteristic.

\[
\text{e.g. } 0 \bar{1} \overline{0} 000000 = 64 \\
64 - 64 = 0 = 16
\]

8 - 63 = fraction or mantissa. The binary points of the fraction is just before bit position 8. That is to say that a 1 in bit position 8 represents \(2^{-1}\), a 1 in bit position 9 represents \(2^{-2}\), etc.

The magnitudes of a floating point number is \(16^{-65} \times (5.4 \times 10^{-79})\)

\((1 - 16^{-14}) \cdot 16^{63} (7.2 \times 10^{75})\)

Example:

1 1000001 01010100 00000000 00000000 00000000 etc.

<table>
<thead>
<tr>
<th>Sign</th>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>65</td>
</tr>
</tbody>
</table>

\[
65 - 64 = 1
\]

\[-1 \times 16^1 \times (2^{-2} + 2^{-4} + 2^{-6}) = -2^4 (2^{-2} + 2^{-4} + 2^{-6})
\]

\[- = (2^2 + 2^0 + 2^{-2})
\]

\[- = (4 + 1 + 0.25) = -5.25
\]

Normalized floating point numbers have a non-zero high order hexadecimal fraction digit. One or more high order hexadecimal fraction digits which are zero makes the number unnormalized. Normalization consists of shifting the fraction left until the high order hexadecimal digit is non-zero and reducing the characteristic by the number of hexadecimal digits shifted. For multiplication and division the operands are normalized before the operations are performed. For the remaining operands (+, -) the result is normalized.

A = 436D1000 00000000 = normalized number
B = 4306D1000 00000000 = unnormalized

= 426D1000 000000 after normalization.
VI. RESTRICTIONS

1. Only basic arithmetic operations and basic set theory applications are performed. No trigonometric operations are performed.

2. Uses long (64 bits) floating point interval arithmetic.

3. The arithmetic performed can produce illegal intervals.

4. The arithmetic performed can cause arithmetic overflow and divide check interrupts.
**APPENDIX**

**LEVEL 20.1 (AUG 71) OS/360 FORTRAN H**

**COMPILER OPTIONS**
- NAME= MAIN,OPT=01,LINECNT=58,SIZE=0000K,
- SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,1D,XREF

---

**ISN 0002**
+ IMPLIED COMPLEX*16(A-H,S),REAL*8(O-Z),INTEGER(I-N),

**ISN 0003**
+ DIMENSION C(20),D(20),E(20)

**ISN 0004**
+ WRITE(6,101)

**ISN 0005**
+ I=1

**ISN 0006**
+ P=0.5D0

**ISN 0007**
+ A=(2.0D0,5.0D0)

**ISN 0008**
+ B=(1.0D0,B.0D0)

**ISN 0009**
+ N=3

**ISN 0010**
+ V=DLI(A)

**ISN 0011**
+ S=DRE(A)

**ISN 0012**
+ T=DLI(B)

**ISN 0013**
+ U=DRE(B)

**ISN 0014**
+ WRITE(6,203) V,S,T,U,N,P

**ISN 0015**
+ WRITE(6,222)

**ISN 0016**
+ D(I)=SIT(N)

**ISN 0017**
+ WRITE(6,108)

**ISN 0018**
+ R=DLI(D(I))

**ISN 0019**
+ S=DRE(D(I))

**ISN 0020**
+ WRITE(6,202) I,R,S

**ISN 0021**
+ I=I+1

**ISN 0022**
+ D(l)=SIT(P)

**ISN 0023**
+ WRITE(6,109)

**ISN 0024**
+ R=DLI(D(I))

**ISN 0025**
+ S=DRE(D(I))

**ISN 0026**
+ WRITE(6,202) I,R,S

**ISN 0027**
+ I=I+1

**ISN 0028**
+ D(3)=SAD(A,B)

**ISN 0029**
+ WRITE(6,102)

**ISN 0030**
+ R=DLI(D(I))

**ISN 0031**
+ S=DRE(D(I))

**ISN 0032**
+ WRITE(6,202) I,R,S

**ISN 0033**
+ I=I+1

**ISN 0034**
+ D(4)=SUB(A,B)

**ISN 0035**
+ WRITE(6,103)

**ISN 0036**
+ R=DLI(D(I))

**ISN 0037**
+ S=DRE(D(I))

**ISN 0038**
+ WRITE(6,202) I,R,S

**ISN 0039**
+ I=I+1

**ISN 0040**
+ D(5)=SMUL(A,B)

**ISN 0041**
+ WRITE(6,104)

**ISN 0042**
+ R=DLI(D(I))

**ISN 0043**
+ S=DRE(D(I))

**ISN 0044**
+ WRITE(6,202) I,R,S

**ISN 0045**
+ I=I+1

**ISN 0046**
+ D(6)=SDIV(A,B)

**ISN 0047**
+ WRITE(6,105)

**ISN 0048**
+ R=DLI(D(I))

**ISN 0049**
+ S=DRE(D(I))

**ISN 0050**
+ WRITE(6,202) I,R,S

**ISN 0051**
+ WRITE(6,223)

**ISN 0052**
+ I=1

**ISN 0053**
+ C(I)=SPRT(A,N)

**ISN 0054**
+ WRITE(6,110)

**ISN 0055**
+ R=DLI(C(I))
ISN 0056  S=DREICII)
ISN 0057  WRITE(6,2002) I,R,S
ISN 0058  I=I+1
ISN 0059  C(2)=$PVOT(B,N)
ISN 0060  WRITE(6,111)
ISN 0061  R=DLIC(C(I))
ISN 0062  S=DREIC(C(I))
ISN 0063  WRITE(6,2002) I,R,S
ISN 0064  I=I+1
ISN 0065  C(3)=$MIN(A)
ISN 0066  WRITE(6,112)
ISN 0067  R=DLIC(C(I))
ISN 0068  S=DREIC(C(I))
ISN 0069  WRITE(6,2002) I,R,S
ISN 0070  I=I+1
ISN 0071  C(4)=$MIN(B)
ISN 0072  WRITE(6,113)
ISN 0073  R=DLIC(C(I))
ISN 0074  S=DREIC(C(I))
ISN 0075  WRITE(6,2002) I,R,S
ISN 0076  I=I+1
ISN 0077  C(5)=$ABS(A)
ISN 0078  WRITE(6,114)
ISN 0079  R=DLIC(C(I))
ISN 0080  S=DREIC(C(I))
ISN 0081  WRITE(6,2002) I,R,S
ISN 0082  I=I+1
ISN 0083  C(6)=$ABS(B)
ISN 0084  WRITE(6,115)
ISN 0085  R=DLIC(C(I))
ISN 0086  S=DREIC(C(I))
ISN 0087  WRITE(6,2002) I,R,S
ISN 0088  I=I+1
ISN 0089  C(7)=$SCH(A,B)
ISN 0090  WRITE(6,107)
ISN 0091  R=DLIC(C(I))
ISN 0092  S=DREIC(C(I))
ISN 0093  WRITE(6,2002) I,R,S
ISN 0094  IFICOR(C(7)) 10,20,20
ISN 0095  10 IF(R GT S) WRITE(6,190) R,S
ISN 0096  I=I+1
ISN 0097  20 C(8)=$VER(A,B)
ISN 0098  WRITE(6,106)
ISN 0099  R=DLIC(C(I))
ISN 0100  S=DREIC(C(I))
ISN 0101  WRITE(A,2002) I,R,S
ISN 0102  IFIZE(C(I)) 28,30,29
ISN 0103  28 IF S LT O GO TO 35
ISN 0104  29 IF(R GT O) GO TO 35
ISN 0105  30 WRITE(A,210)
ISN 0106  C TEST ON BEING.Contained_In
ISN 0107  WRITE(A,217)
ISN 0108  CALL SENT(A,0,636,637,638,639)
ISN 0110  36 WRITE(A,211)
ISN 0111  GO TO 40
ISN 0112  37 WRITE(A,212)
ISN 0114  GO TO 40
ISN 0115  38  WRITE(6,213)
ISN 0116  GO TO 40
ISN 0117  39  WRITE(6,214)
ISN 0118  40  A=(0.000,0.000)
ISN 0119  WRITE(6,101)
ISN 0120  B=DL1[R]
ISN 0121  (=DLRE(R)
ISN 0122  WRITE(6,203) V,S,T,U,N,P
ISN 0123  WRITE(6,217)
ISN 0124  CALL SENT(A,B,641,642,643,644)
ISN 0125  41  WRITE(6,211)
ISN 0126  GO TO 44
ISN 0127  42  WRITE(6,212)
ISN 0128  GO TO 45
ISN 0129  43  WRITE(6,213)
ISN 0130  GO TO 45
ISN 0131  44  WRITE(6,214)
ISN 0132  45  CONTINUE
ISN 0133  CALL SUM(B,I,1,5)
ISN 0134  F=SUM(C(I),D(I))
ISN 0135  R=DL1IF()
ISN 0136  S=DLRI(F)
ISN 0137  WRITE(A,218)
ISN 0138  WRITE(6,215)_R,S
ISN 0139  10  CONVERSION ROUTINE
ISN 0140  C(1)=SINP('1.3,1,56')
ISN 0141  C(2)=SINP('1,4')
ISN 0142  C(3)=SINP('1')
ISN 0143  C(4)=SINP('1.45,2,5')
ISN 0144  C(5)=SINP('5,500')
ISN 0145  WRITE(6,216)
ISN 0146  DO 9 I=1,6
ISN 0147  R=DLI(C(I))
ISN 0148  S=DLR(C(I))
ISN 0149  WRITE(6,202) I,R,S
ISN 0150  2002 FORMAT(X,15,'LEFT= ',D24.16,' RIGHT= ',D24.16)
ISN 0151  101  FORMAT(*INTERVAL ARITHMETIC PACKAGE*)
ISN 0152  102  FORMAT(*OA+B*)
ISN 0153  103  FORMAT(*OA-B*)
ISN 0154  104  FORMAT(*OA*B*)
ISN 0155  105  FORMAT(*OA/B*]
ISN 0156  106  FORMAT(*OA UNION B*]
ISN 0157  107  FORMAT(*OA INTERSECT B*)
ISN 0158  108  FORMAT(*OS15(N*)
ISN 0159  109  FORMAT(*OSOT(0*)
ISN 0160  110  FORMAT(*OA**N*)
ISN 0161  111  FORMAT(*0B**N*)
ISN 0162  112  FORMAT(*0S15(N*)
ISN 0163  113  FORMAT(*0S15(N*)
ISN 0164  114  FORMAT(*0S15(N*)
ISN 0165  115  FORMAT(*0S15(N*)
ISN 0166  190  FORMAT(*' THIS IS AN ILLEGAL INTERVAL ('D24.16,'',''D24.16,')*')
ISN 0167  203  FORMAT(X,'A= 'D24.16','B= 'D24.16,'X,'N= '15/X,'P= 'D24.16)
FORMAT(' ZERO IS CONTAINED IN THE FOLLOWING INTERVAL (',G24.16,')')

FORMAT(' A IS CONTAINED IN B')

FORMAT(' A AND B ARE EQUAL')

FORMAT(' B IS CONTAINED IN A')

FORMAT(' A AND B ARE NOT FULLY CONTAINED IN EACH OTHER')

FORMAT(' LEFT= ',D24.16,'X','RIGHT= ',D24.16)

FORMAT(' O$INP CONVERSION ROUTINE')

FORMAT(' O$ENT-TEST ON BEING CONTAINED IN')

FORMAT(' O$UM PRODUCED WITH SCALAR PRODUCT')

FORMAT(' OVALUES FOR ARRAY D(I)')

FORMAT(' OVALUES FOR ARRAY C(I)')

STOP

END
INTERVAL ARITHMETIC PACKAGE

\[
\begin{align*}
A &= 0.20000000000000000 D 01, \\
B &= 0.00000000000000000 D 01, \\
N &= 3, \\
P &= 0.50000000000000000 D 00
\end{align*}
\]

SENT-TEST ON BEING CONTAINED IN
A AND B ARE NOT FULLY CONTAINED IN EACH OTHER

SUM PRODUCED WITH SCALAR PRODUCT

\[
\begin{align*}
\text{LEFT} &= -0.68500000000000000 D 02, \\
\text{RIGHT} &= 0.87300000000000000 D 03
\end{align*}
\]

$\text{INP CONVERSION ROUTINE}$

\[
\begin{align*}
1\text{LEFT} &= 0.13000000000000000 D 01, \\
2\text{LEFT} &= -0.20000000000000000 D 01, \\
3\text{LEFT} &= -0.20000000000000000 D 01, \\
4\text{LEFT} &= 0.25000000000000000 D 01, \\
5\text{LEFT} &= 0.55000000000000000 D 01, \\
6\text{LEFT} &= 0.90000000000000000 D 01
\end{align*}
\]
INTERVAL ARITHMETIC PACKAGE

A = 0.20000000000000000 01, 0.50000000000000000 01
B = 0.10000000000000000 01, 0.80000000000000000 01
N = 3
P = 0.50000000000000000 00

VALUES FOR ARRAY D(1)

$\text{SIN}(N)
\begin{align*}
1\text{LEFT} &= 0.30000000000000000 01 & 1\text{RIGHT} &= 0.30000000000000000 01 \\
2\text{LEFT} &= 0.50000000000000000 00 & 2\text{RIGHT} &= 0.50000000000000000 00
\end{align*}

A+B
\begin{align*}
3\text{LEFT} &= 0.30000000000000000 01 & 3\text{RIGHT} &= 0.13000000000000000 02 \\
4\text{LEFT} &= -0.60000000000000000 01 & 4\text{RIGHT} &= 0.40000000000000000 01
\end{align*}

A-B
\begin{align*}
5\text{LEFT} &= 0.20000000000000000 01 & 5\text{RIGHT} &= 0.40000000000000000 02 \\
6\text{LEFT} &= 0.25000000000000000 00 & 6\text{RIGHT} &= 0.50000000000000000 01
\end{align*}

VALUES FOR ARRAY C(1)

A**N
\begin{align*}
1\text{LEFT} &= 0.80000000000000000 01 & 1\text{RIGHT} &= 0.12500000000000000 03 \\
2\text{LEFT} &= 0.10000000000000000 01 & 2\text{RIGHT} &= 0.51200000000000000 03
\end{align*}

$\text{MIN}(A)$
\begin{align*}
3\text{LEFT} &= -0.50000000000000000 01 & 3\text{RIGHT} &= -0.20000000000000000 01 \\
4\text{LEFT} &= -0.80000000000000000 01 & 4\text{RIGHT} &= -0.10000000000000000 01
\end{align*}

$\text{ABS}(B)$
\begin{align*}
5\text{LEFT} &= 0.20000000000000000 01 & 5\text{RIGHT} &= 0.50000000000000000 01 \\
6\text{LEFT} &= 0.10000000000000000 01 & 6\text{RIGHT} &= 0.80000000000000000 01
\end{align*}

A INTERSECT B
\begin{align*}
6\text{LEFT} &= 0.20000000000000000 01 & 6\text{RIGHT} &= 0.50000000000000000 01
\end{align*}

A UNION B
\begin{align*}
3\text{LEFT} &= 0.20000000000000000 01 & 3\text{RIGHT} &= 0.50000000000000000 01 \\
8\text{LEFT} &= 0.10000000000000000 01 & 8\text{RIGHT} &= 0.80000000000000000 01
\end{align*}

$\text{SENT-TEST ON BEING CONTAINED IN}$

A IS CONTAINED IN B

15
References

