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(GEM 1 & 2)

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May 1972

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GRAVITATIONAL FIELD MODELS FOR THE EARTH
(GEM 1 & 2)

ABSTRACT

Two models of the earth's gravitational field have been computed at Goddard Space Flight Center. The first, Goddard Earth Model 1 (GEM 1), has been derived from satellite tracking data. The second, Goddard Earth Model 2 (GEM 2), has been derived from a combination of satellite tracking and surface gravimetric data. The geopotential models are represented in spherical harmonics complete to degree and order 16 for the combined solution and complete to degree and order 12 for the satellite solution. Both solutions include zonal terms to degree 21 and related satellite resonant coefficients to degree 22. The satellite data consisted primarily of optical data processed on 300 weekly orbital arcs for 25 close earth satellites. Surface gravity data were employed in the form of 5° x 5° mean free-air gravity anomalies providing about 70% world coverage. Station locations were obtained for 46 tracking sites by combining electronic, laser, and additional optical tracking data with the above satellite data. Analysis of the radial positions of these stations and a value of mean gravity on the geoid indicated a mean equatorial radius for the Earth of about 6378145 meters. Results of geopotential tests on satellite data not used in the solution show that better agreement was obtained with the GEM 1 and GEM 2 models than with the 1969 Smithsonian Standard Earth II model.

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GRAVITATIONAL FIELD MODELS FOR THE EARTH (GEM 1 & 2)

I. INTRODUCTION

The establishment of an accurate Earth model (geometric and gravimetric) is an essential requirement of any earth physics program(1). Progress in this area has been made at the Smithsonian Astrophysical Observatory using optical tracking data (mainly) and analytic techniques in the computation of orbits(2,3). Work at Goddard Space Flight Center has proceeded with the object of extending the use of this optical data and also to the inclusion of highly accurate and dense electronic satellite tracking data available since 1965 (i.e. Tranet Doppler, Goddard Range and Range-Rate, and laser). Numerical integration, providing a precise solution for modeled forces, has been used extensively at GSFC for the computation of orbits. With this approach a preliminary solution for a geopotential model has already been obtained using 17 satellites and Baker-Nunn optical data only(4). In this solution the geopotential field was complete to degree and order 8 in spherical harmonics and it included station coordinates for 13 optical tracking sites. The combined solution reported here is complete to degree and order 16 and is based on surface gravity as well as satellite optical data. The station coordinate solution has also been extended to 46 stations. In this new solution considerable improvement has been made through refinement in the modeling for drag and satellite resonant effects.

Tests of the new solution are presented which demonstrate improvement over other solutions in satellite orbit determination and better agreement with ground survey data. Consistent results have been obtained for an adjusted scale of the reference ellipsoid from analysis of both station positions and surface gravimetric data.

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II. DESCRIPTION OF SOLUTIONS

A geopotential solution (GEM 1) has been derived from tracking data (primarily optical) on 25 close earth satellites. Spherical harmonic terms in the geopotential are complete to degree and order 12 with zonals extending to degree 21 and selected satellite resonant coefficients to degree 22. Surface gravity data, consisting of 21,000 1° x 1° mean free-air gravity anomalies and used in the form of 5° x 5° mean anomalies, have been combined with the satellite data to derive a geopotential (GEM 2) complete to degree and order 16 with similar higher degree terms as in the satellite solution.

Station locations were obtained for 46 tracking sites by combining satellite electronic, laser, and additional optical data with the data used in the satellite geopotential solution described above. Tracking systems consisted of 13 Baker-Nunn cameras, 23 Minitrack Optical Tracking System cameras (MOTS), 2 lasers, 3 Goddard Range and Range-Rate (GRARR) systems, and 5 NWL Tranet Doppler systems. A value of mean gravity for the reference system was determined through a simultaneous adjustment of this quantity with the geopotential coefficients using both the satellite and surface gravity data.

Data Used in the Solutions

a. Satellite Data: Data used in the satellite geopotential solution (GEM 1) is presented in Table 1. Approximately 300 weekly orbital arcs of optical data on 23 satellites formed the basis of the solution. Also included in the solution were 21 weekly arcs of Minitrack interferometer data on TIROS 9 and Allouette 2 satellites, which were principally used to support the determination of satellite resonant coefficients. Additional information is presented in Table 1 on satellite orbit geometry and the number of orbital arcs, observations and observation residuals.

In the solution for the 46 station locations, the data in the above 300 weekly orbital arcs (120,000 optical observations) were combined with 15 weekly GEOS-I and II orbital arcs of densely covered electronic and laser data (150,000 observations) including some additional optical data. Also, some 66 one- and two-day arcs of GEOS-I and II flashing light optical data were combined with the above arcs, mainly to strengthen the solution of the MOTS stations.

b. Surface Gravity Data: The source of most of the gravimetry data was the U. S. Aeronautical Chart and Information Center which provided 19,000 one-degree by one-degree mean free-air gravity anomalies. A further set of 2000 mean gravity anomalies were obtained from a number of other sources. These data were used to form 1707 five-degree by five-degree mean gravity anomalies by a straight averaging of the one-degree by one-degree mean anomalies, and provided a total coverage of about 70% of the earth's surface. See Figure 1 for a map of surface gravity data coverage.

Starting Values for Solutions

The *a priori* values of the geopotential coefficients and station locations used in the solution were as follows:

Zonals harmonics to degree 21,	Kozai (1969)(5)
Tesseral harmonics	SAO S.E. I (1966)(2)
Satellite resonant coefficients	GSFC preliminary analysis(6)
Station coordinates	Marsh, et al. (1971)(7)

The reference ellipsoid that was adopted for the solution was the same as that used by SAO in the Standard Earth II (1969)(3); viz

Mean equatorial radius, a_e = 6378155 meters
Flattening, f = 1/298.255

Product of mass of the earth and gravitational constant,

$$GM = 3.986013 \times 10^{14}\ m^3/sec^2.$$

III. ANALYSIS TECHNIQUES

The motion of the satellite was obtained by numerical integration of the equations of motion with the following forces being modeled: the earth's gravity field, atmospheric drag, solar radiation pressure, and lunar and solar gravity. The integration was performed in a cartesian, geocentric, inertial coordinate system referenced to the true equator and equinox of the epoch. The time frame was A1 time and modeling in the reference system included luni-solar precession and nutation of the earth and polar motion, the latter being obtained from the U. S. Naval Observatory who also provided the adjustment from UT1 to A1. The origin of the coordinate system for the pole positions was the mean pole of 1900-1905. The technique of weighted least squares was applied to the satellite observation equations with the standard deviations presented in Table 2.

The surface gravity data was given in terms of 1° x 1° mean gravity anomalies (mgal) referenced to the International System. This data was used as described below in terms of the reference system defined by GM, a_e, f, and ω (rotation rate of the Earth).

Let W be the gravity potential of the Earth so that

$$W = V + \Phi \tag{1}$$

where V is the gravitational potential, and Φ the centrifugal potential, and where

$$V = \frac{GM}{r}\left[1 + \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\varphi)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right]$$

$$\Phi = \frac{1}{2}(r\,\omega\cos\varphi)^2$$

r, φ, λ spherical coordinates of radial distance, latitude, and longitude, $P_n^m(\sin\varphi)$ associated Legendre polynomial of degree n, order m and argument $\sin\varphi$, C_{nm}, S_{nm} spherical harmonic coefficients.

On a equipotential surface (i.e., mean sea level), we may write $W = W_o$ so that gravity g_0, on this surface is given by

$$g_0 = |\Delta W_o|$$

The quantity g_0 is related to the International System for gravity measurement as follows

$$g_0' = \Delta g_I + \gamma_I$$

$$g_0 = g_0' - 13.7 \tag{2}$$

where

γ_I is normal gravity on the International Ellipsoid[8]

Δg_I is gravity anomaly for the International System

-13.7 mgal is the Pottsdam correction.

The 5° x 5° mean gravity anomalies were formed from a straight average of the given 1° x 1° mean anomalies for each 5° x 5° section and were represented at the mid-point of the section in latitude and longitude. A value of g_0 was then computed from (2) using each of the 5° x 5° mean anomalies, and the resultant g_0 was taken as the observed quantity. A residual, $g_0 - g_c$, was then formed for the equation of condition and expressed in terms of linear adjustments to the starting values of the spherical harmonic coefficients. The quantity g_c was computed as

$$g_c = \left\{ \left(\frac{\partial W}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial W}{\partial \varphi}\right)^2 \right\}^{1/2} \tag{3}$$

where the partials are obtained from (1) with the use of our starting coefficients and reference system parameters. A constant term $\Delta \bar{g}_0$[9], variation of mean gravity from the reference system, was added to the equation of condition for simultaneous adjustment with the spherical harmonic coefficients. Weighted least squares normal equations were then formed for simultaneous adjustment with the satellite normal equations.

In forming the weighted least squares normal equations the variance σ^2 of g_0 (observed gravity value corresponding to the 5° x 5° mean anomalies) was obtained from

$$\sigma^2 = \frac{\sigma_0^2}{N + 1} \tag{4}$$

where

N is the number of 1° x 1° mean anomalies within a 5° x 5° section and σ_0^2, the computed variance of the 21,000 1° x 1° observed anomalies, is 33^2 ($mgal^2$).

For an unobserved 5° x 5° area, a value of zero was adopted with a variance of σ_0^2 . The weighting of the gravity data was according to the formula

$$w = \frac{\cos \phi}{\sigma^2} \tag{5}$$

where the cosine of the latitude was introduced to account for the unequal area covered by the 5° x 5° section. For a 5° x 5° mean anomaly derived from 25 1° x 1° means, σ = 6 mgal.

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IV. RESULTS

1. Solutions

The geopotential and station coordinate solutions were described in a previous section, Description of Solutions. The values of the geopotential coefficients, for the satellite solution (GEM 1) and the combined satellite/surface gravity (GEM 2) solution, are listed in Table 3. The station coordinates for the 46 tracking stations are listed in Table 4. The ellipsoid heights for the 13 Baker-Nunn stations have been lowered by 15 meters to account for an error in the computer program for the parallactic refraction correction. The solution value for $\Delta\bar{g}_0$, the variation of mean gravity from the reference system, was 3.28 mgal.

2. Zonal Coefficients

Zonal coefficients complete to degree 21 in the GEM 2 solution are obtained from the combined effects of both satellite and surface gravity data. These zonal values are compared in Table 5 with the SAO 1966 (M1) and 1969 Standard Earth II values derived by Kozai from secular and long period orbit perturbations. The table also contains the zonal values from our satellite only solution. Since weekly satellite arcs were used in our solution the satellite zonal coefficients are obtained from the short term zonal effects. The comparison in the table indicates remarkable agreement with SAO S. E. II in the zonal values from our satellite only solution and even better agreement in our combined solution.

A zonal profile of geoid height is given in Figure 2 for the GSFC satellite and combined solutions and for the SAO Standard Earth II solution. Very little difference exists except in the region of 0 to $\pm 20^{\circ}$ latitude which is given on an enlarged scale in Figure 3 where differences of a few meters may be seen.

Generally high correlation exists between consecutive odd and consecutive even zonal coefficients in satellite solutions from secular and long term orbital effects. Correlations, although very large (generally greater than 0.9) for consecutive odd or even zonals, fall off as odd or even pairs separate in degree. However, in our combination solution the coefficients do not exhibit these high correlations and have essentially become decoupled through the surface gravity data.

3. Geoid Height Maps (Undulations)

Geoid height contour maps at ten meter intervals are presented in Figure 4 for the GSFC combined and the Standard Earth II solutions. The two solutions generally compare very favorably but differences of 8 meters can be seen between the relative highs and lows on the maps. An rms of 5 meters was estimated between the GSFC combined and the S E. II solution from a more detailed analysis. A maximum difference of 15 meters exists at -70° lat. and 20° W long., where the GSFC combination geoid is shaped differently from the SAO solution.

4. Degree Variances of Gravity Anomalies

Degree variances of gravity anomalies have been obtained, for each degree n = 3 to 21, as follows

$$\sigma_{g_n}^2 = \bar{\gamma}^2 (n - 1)^2 \sum_{n=0}^{n} (\bar{C}_{nm}^2 + \bar{S}_{nm}^2) \text{ (m gal)}^2 \tag{6}$$

where $\bar{\gamma}$ is a mean value of normal gravity and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the spherical harmonic coefficients in normalized form. These values are tabulated in Table 6 for our combined and satellite only solutions and for the S.E. II solution for purposes of comparison. Although the total sums of the degree variances differ by 3 $mgal^2$ between the S.E. II and our combined solution, individual differences are as large as 8 $mgal^2$. Comparisons of the solution with observed gravity anomalies are in process but are not available at the present time.

5. Geopotential Coefficients and Standard Deviations

Tables 3a and 3b show the geopotential coefficients obtained in the GSFC satellite and combination solutions. Table 7 presents an rms of geopotential coefficient differences with the Standard Earth II for each degree for a variety of solutions. Two other solutions in addition to the GSFC solutions are presented. These are the SAO 1966 Standard Earth I field and another SAO solution, the B13.1 which is similar to the Standard Earth II. It is noted that the smallest rms total is for the GSFC combined solution and the largest total is for the Standard Earth I.

The rms differences per degree between the SAO S.E. II and the GSFC satellite solution have been used in an analysis for determining the relative weighting factor between the surface gravity normal equations and the satellite normal equations. In Figure 5 the average standard deviations of the coefficients for each degree n are plotted for the satellite and surface gravity solutions. It might be expected that the two curves would cross at some point, such as at degree ten, above which the surface gravity would provide a relatively stronger contribution to the higher degree coefficients. However, the disparity between the two curves suggests the satellite values are unrealistic. A more realistic standard deviation of the satellite coefficients can probably be obtained from the rms of coefficient differences between two solutions containing satellite data. This was done with the GSFC satellite solution and the Standard Earth II and the resulting curve is shown in Figure 5. This curve crosses the surface gravity curve at degree 10 with a value of about 5.5 x 10   The satellite standard deviation at this point is 2.0×10^{-8} so that the ratio for scaling of the satellite standard deviations is 2.7. This corresponds to a relative increase of weight of 7 for the surface gravity normal equations. In the combined solution a weight of 5 was finally used because the standard deviations for the surface gravity equations, although more realistic, may be somewhat optimistic.

It should be pointed out that the correlations among the non-zonal geopotential coefficients are generally very small. Under these conditions the relative weight of separate normal equations in estimating a parameter for a combined solution is nearly inversely proportional to the ratio of the squares of the standard deviations associated with the separate normal equations.

6. Comparison of Station Heights with Local Survey

The station heights above the geoid have been obtained for 46 stations using our combined satellite-surface gravity geopotential and station coordinate solutions. These heights were then compared with the mean sea level heights given by local survey. The comparison is presented in Figure 6. The mean sea level height, MSLH, for each of the stations was obtained from the survey sheets in the NASA Directory of Observation Station Locations published at Goddard Space Flight Center. The MSLH heights are listed in Table 8 for each of the stations along with the corresponding geoid height, N, obtained from our solution. The survey values of MSLH are generally listed with an accuracy of a meter or better. The full station coordinates obtained in our combined solution are listed in Table 4 which gives the ellipsoidal height (h), latitude, and longitude of all 46 stations.

These 46 stations are plotted in Figure 6 in terms of a letter signifying the tracking station data type, namely B - Baker-Nunn (13), M - MOTS (23), D - Doppler (5), G - GRARR (3), and L - Laser (2).

The quantity plotted in Figure 6 for each of the 46 stations is

$$\Delta H = h - N - MSLH \qquad (7)$$

as a function of latitude of the station. Since the geoid heights, N, and the station heights, h, are referenced to our ellipsoid of $a_e = 6378155$, the zero value corresponds to this value of a_e on the plot. The average ΔH is close to -10 meters, implying $a_e = 6378145$ meters is a better reference radius for this set of stations. This value of $\Delta H = -10$ meters is based upon the MOTS, laser GRARR, and doppler coordinates; the Baker-Nunn coordinates were excluded from this calculation because of the adjustment for parallactic refraction made to the station heights, as noted above in section 1.

In Figure 7 a similar set of ΔH results were computed using the Standard Earth II[3] geopotential and station coordinates for 37 sites. These results show an average value of about $a_e = 6378137$. There appears to be a systematic difference in the scale implied for the Northern than for the Southern Hemisphere, due principally to the European stations. These stations are circled in the figure and when excluded an average $a_e = 6378143$ results.

7. Adjusted Ellipsoid Scale (a_e) Based upon the Mean Value of Gravity Variation ($\Delta\bar{g}_0$)

The mean value of gravity variation $\Delta\bar{g}_0$ from the reference system was obtained as part of a general solution as previously described. The value of $\Delta\bar{g}_0 = 3.28$ mgal was obtained. Using the simple relation

$$g_e \simeq \frac{GM}{a_e^2} \tag{8}$$

the variational relationship becomes,

$$\frac{\Delta g_e}{g_e} = \frac{\Delta GM}{GM} - \frac{2\Delta a_e}{a_e} \tag{9}$$

Since the GM of the reference ellipsoid includes the atmosphere*, and the relative mass of the atmosphere to the earth is 0.85×10^{-6} and further, since

$$\frac{\Delta g_e}{g_e} = \frac{\Delta g_0}{g_e} = 3.35 \times 10^{-6}$$

where $g_e = 9.780291 \text{ m sec}^{-2}$, we can obtain

$$\Delta a_e = -\frac{a_e}{10^6} \frac{(3.35 + 0.85)}{2}$$

or -13 meters. This implies that $a_e = 6378142$ meters is a better scale for our reference ellipsoid.

*GM was obtained by the Jet Propulsion Laboratory[10] from space probes and thus includes the mass of the atmosphere, while the free-air gravity anomalies were made at the surface of the earth and reduced to sea-level.

V. TESTS OF SOLUTIONS

Two tests have been applied to the solutions that have been obtained so far. Both concern the ability of the gravity fields to represent the motion of close earth satellites.

The first test was on 22 six-hour orbital arcs of the Beacon Explorer C (BE-C) spacecraft composed of laser range data from the Goddard experimental laser. The rms of fit to the data on the 22 arcs using the Standard Earth II gravity field varied between 0.9 meters and 4.3 meters with a mean of about 3 meters. With the GSFC satellite solution these rms values dropped to between 0.6 meters and 2.0 meters with a mean of about 1.3 meters. This considerable improvement was obtained even though no laser data on BE-C was used in the solution. These rms values are shown in Figure 8.

The second test involved the long-term behavior of INTELSAT 2-F1, Cosmos 41 rocket and Cosmos 382 rocket. The variation of mean elements over periods of hundreds of days have been compared using the Standard Earth II and GSFC combination gravity fields. The results are shown in Table 9 from which it is evident that a three-fold improvement has been obtained with the GSFC combination solution. Further, it should be remembered that none of these satellites were used in the solutions for the gravity field.

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VI. SUMMARY AND CONCLUSIONS

The first comprehensive GSFC Earth Model has been obtained. This consists of a combined solution for the geopotential field using satellite and surface gravity data (GEM 2) as well as the associated satellite only solution (GEM 1). It also includes a set of 46 geocentric station positions. Tests and comparisons presented in the report yield encouraging results for these solutions, particularly in their capability for improved orbit determination as seen in the section on Test Results. Analysis is continuing with additional satellite and surface gravity data to derive a more complete geopotential model with spherical harmonics complete to degree 20. Satellite data from over 71 tracking stations are being processed for this solution.

In the present solution good agreement has been obtained between analyses of surface gravity data and station height data to provide for an adjusted scale of the equatorial radius (a_e) of a mean earth reference ellipsoid. A value of $a_e = 6378145$ is adopted. Our present solutions, data used, and results obtained in this report are summarized in Table 10 for convenient reference.

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REFERENCES

1. NASA CR 1579 (1970), "The Terrestrial Environment: Solid-Earth and Ocean Physics," Washington, D. C.

2. Lundquist, C. A. and Veis, G., (1966), "Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth," SAO Special Report 200.

3. Gaposchkin, E. M., and Lambeck, K., (1970), "1969 Smithsonian Standard Earth (II)," SAO Special Report 315.

4. Lerch, F. J., et al. (1971), "Preliminary Goddard Geopotential Using Optical Tracking Data and a Comparisonwith SAO Models," GSFC X-552-71-143.

5. Kozai, Y., (1969), "Revised Zonal Harmonics in the Geopotential," SAO Special Report 295.

6. Nickerson, K. G., et al. (1971), "Summary of GEOSTAR Operations Activities," CSC Report No. 5035-14200-02 TM, under NASA contract NAS-5-11790.

7. Marsh, J. G., Douglas, B. C. and Klosko, S. M., (1971), "A Unified Set of Tracking Station Coordinates Derived from Geodetic Satellite Tracking Data," GSFC X-553-71-370.

8. Heiskanen, W. A. and Moritz, H., (1967), Physical Geodesy, Textbook published by W. H. Freeman and Company.

9. Rapp, R. H., (1972), "A 300 n.m. Terrestrial Gravity Field," presented at the AGU Annual Meeting, Washington, D. C.

10. Sjogren, W. L., et al. (1966), "Physical Constants as Determined from Radio Tracking of the Ranger Lunar Probes," JPL Technical Report No. 32-1057

11. United States Air Force, ACIC, (1971), "1^{o} x 1^{o} Mean Free-Air Gravity Anomalies," ACIC Reference Publication No. 29.

12. "NASA Directory of Observation Station Locations," (1970), Metric Data Branch, Goddard Space Flight Center.

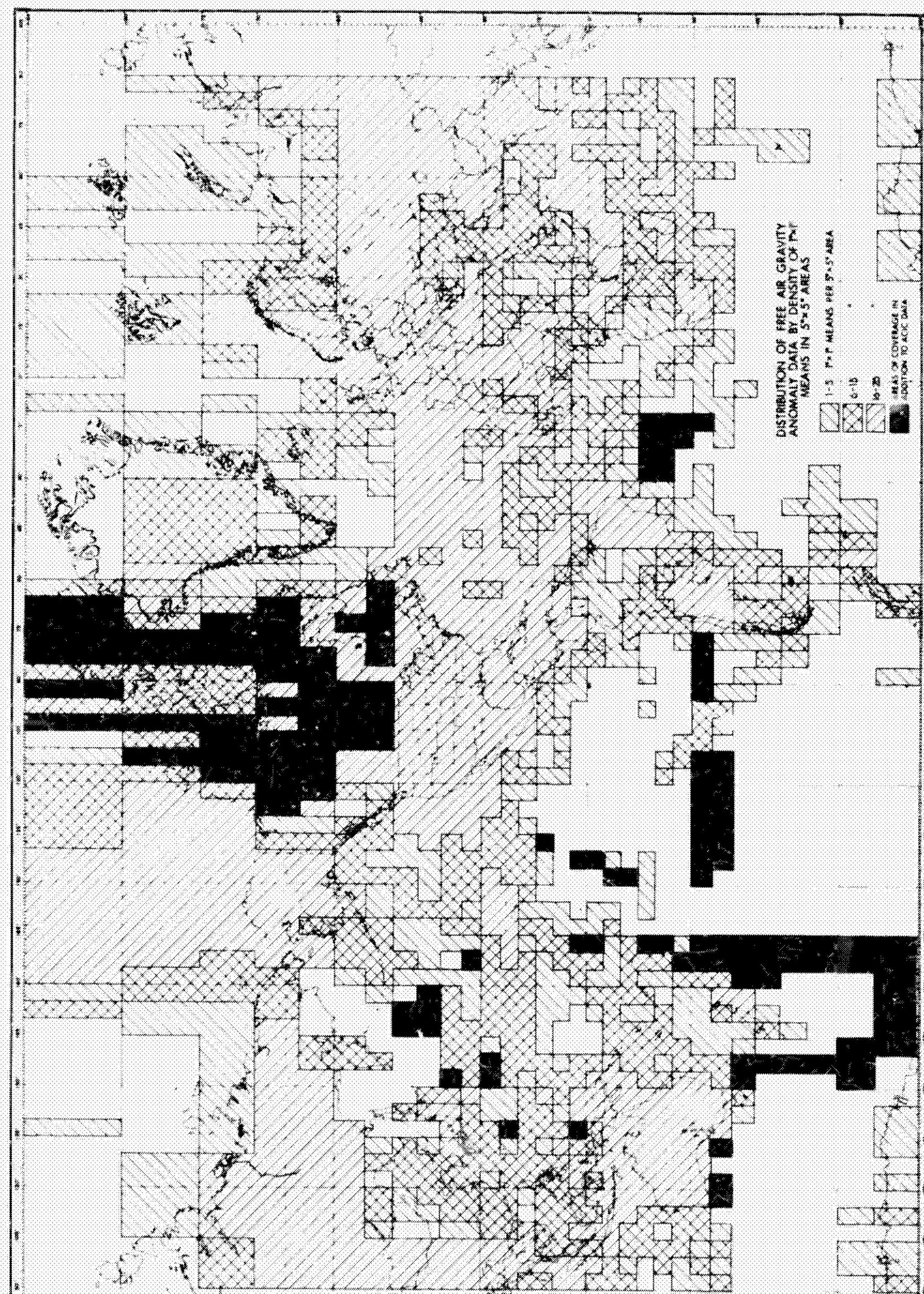

Figure 1. Surface Gravity Data 5° x 5° Mean Gravity Anomalies

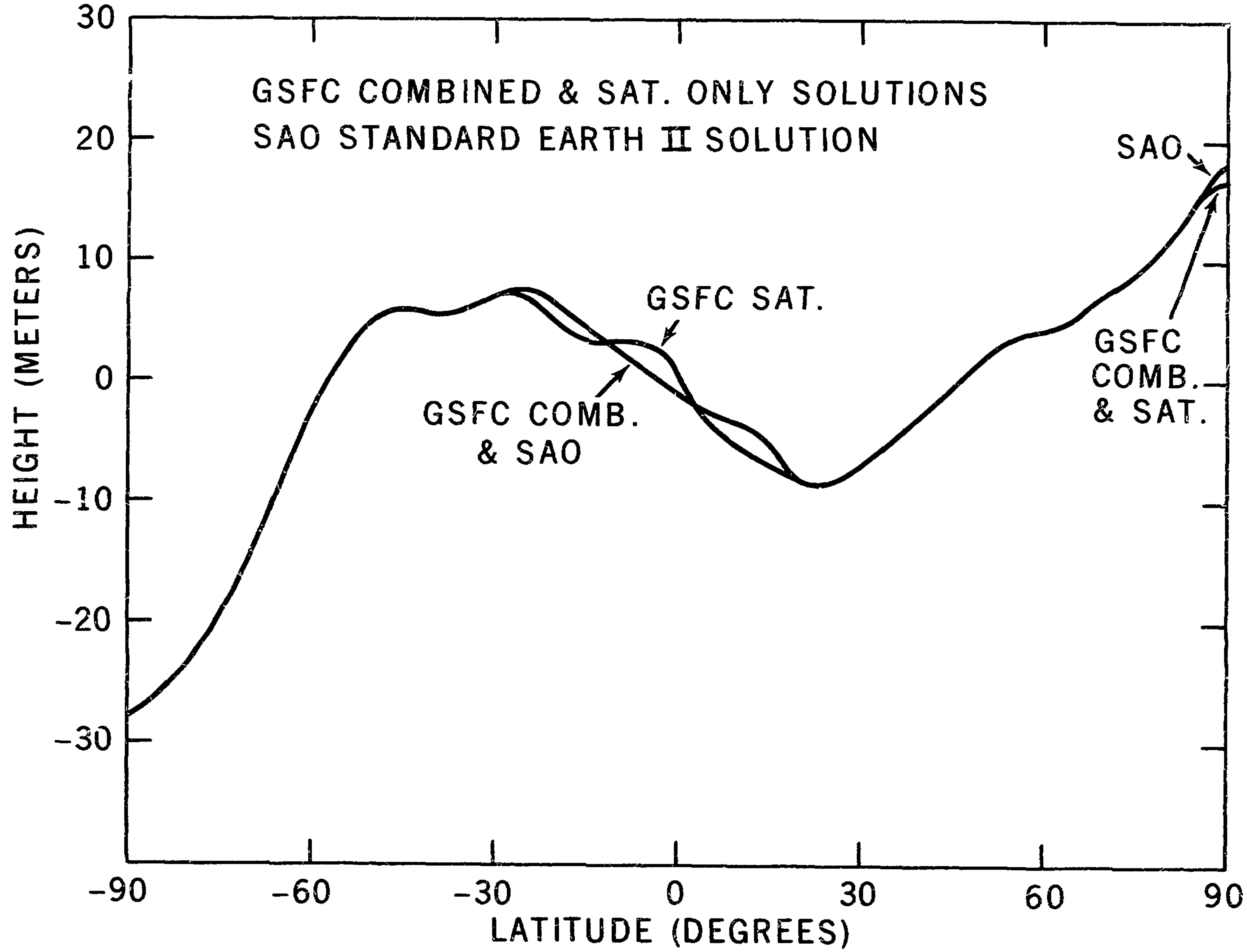

Figure 2. Geoid Height: Zonal Profile

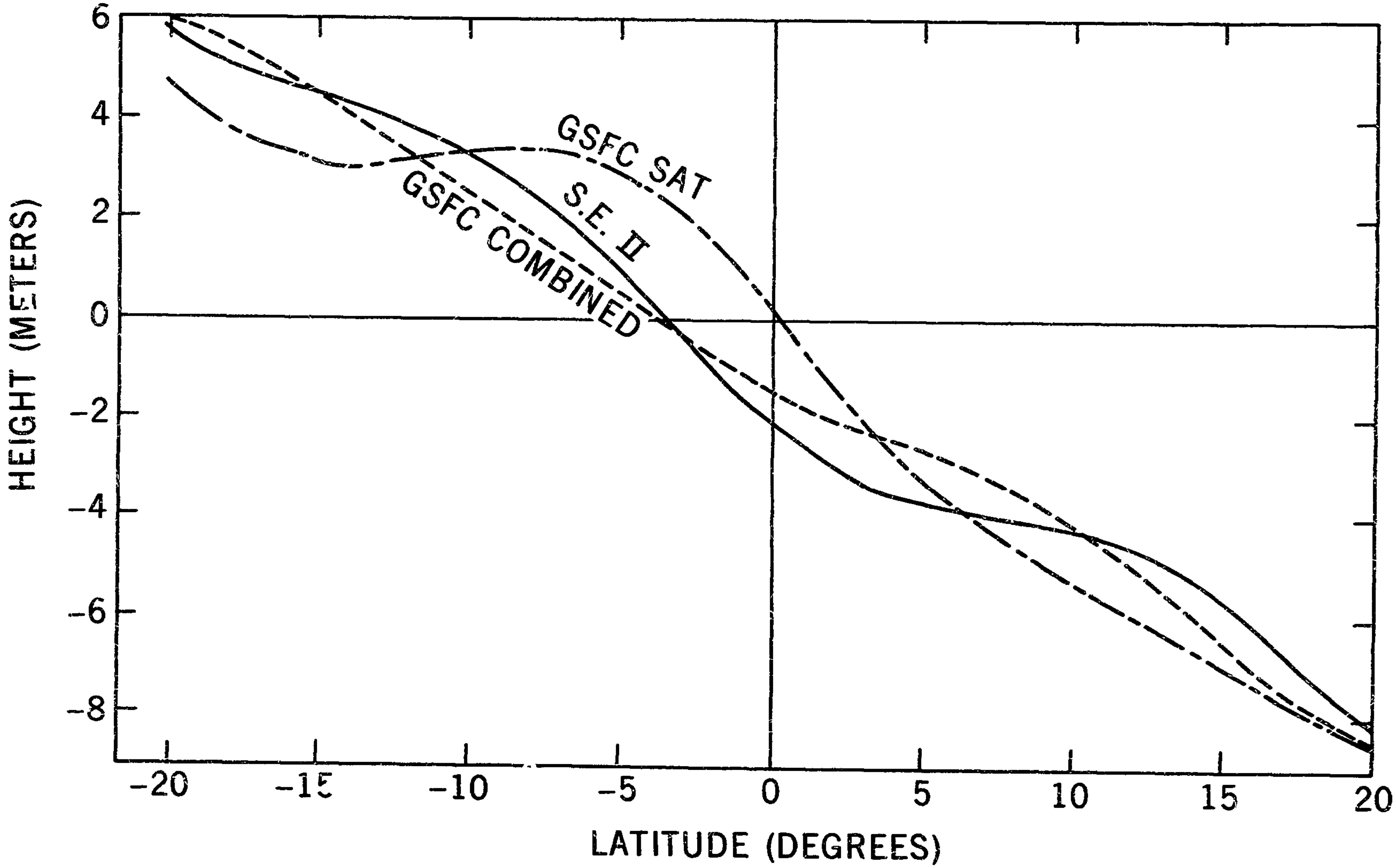

Figure 3. Geoid Height: Zonal Profile to ±20° Lat.

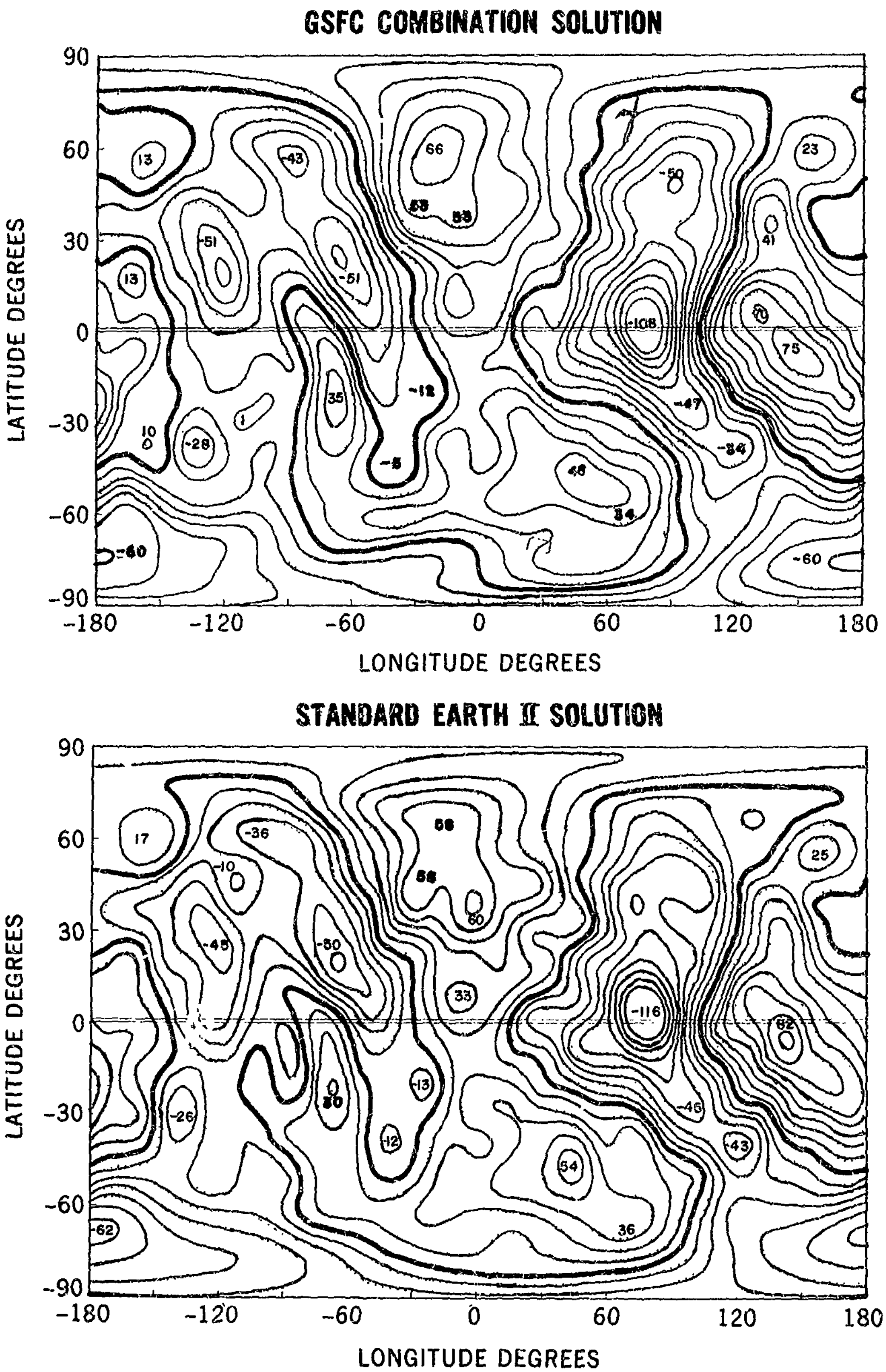

Figure 4. Geoid Heights (Meters)

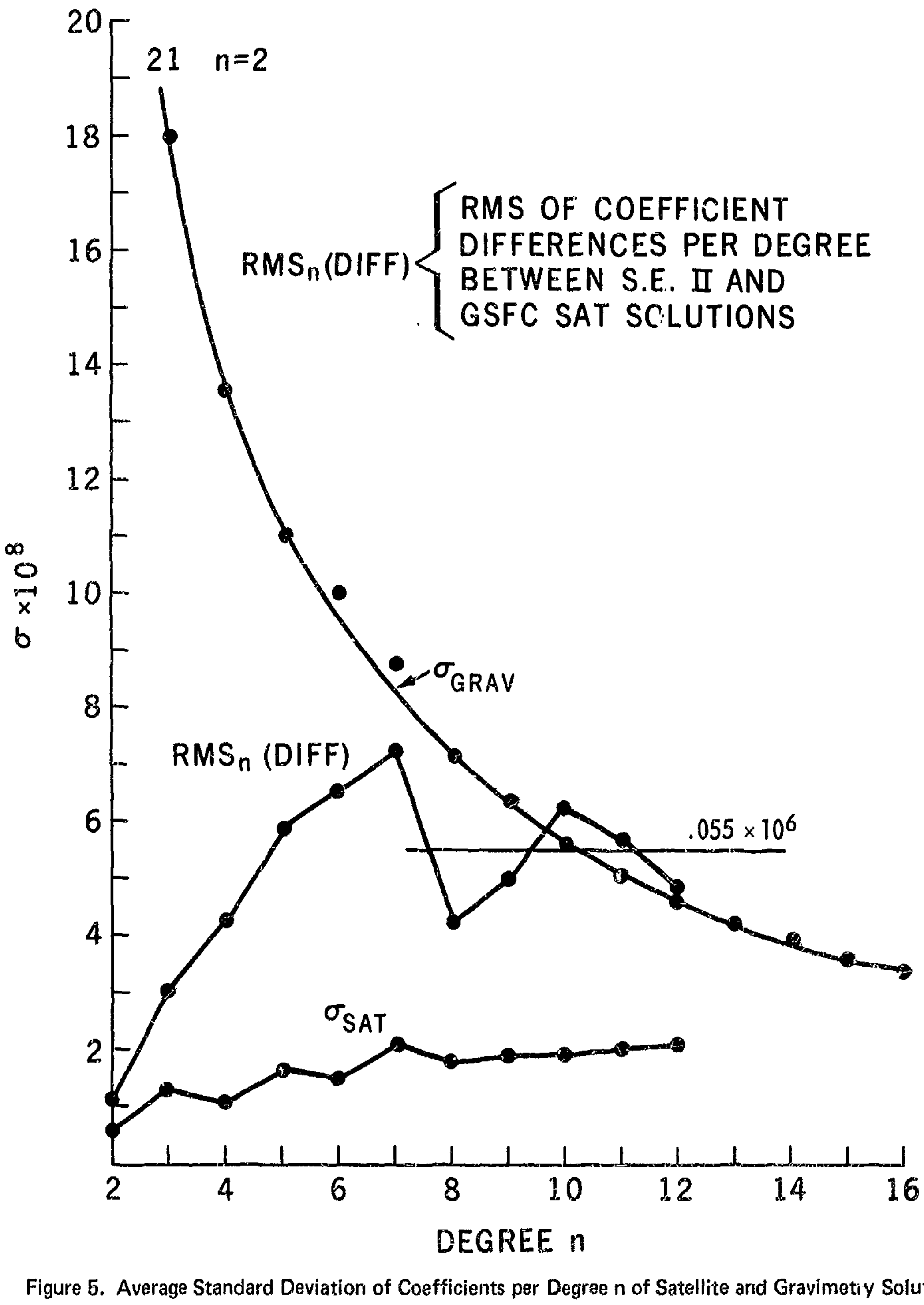

Figure 5. Average Standard Deviation of Coefficients per Degree n of Satellite and Gravimetry Solutions

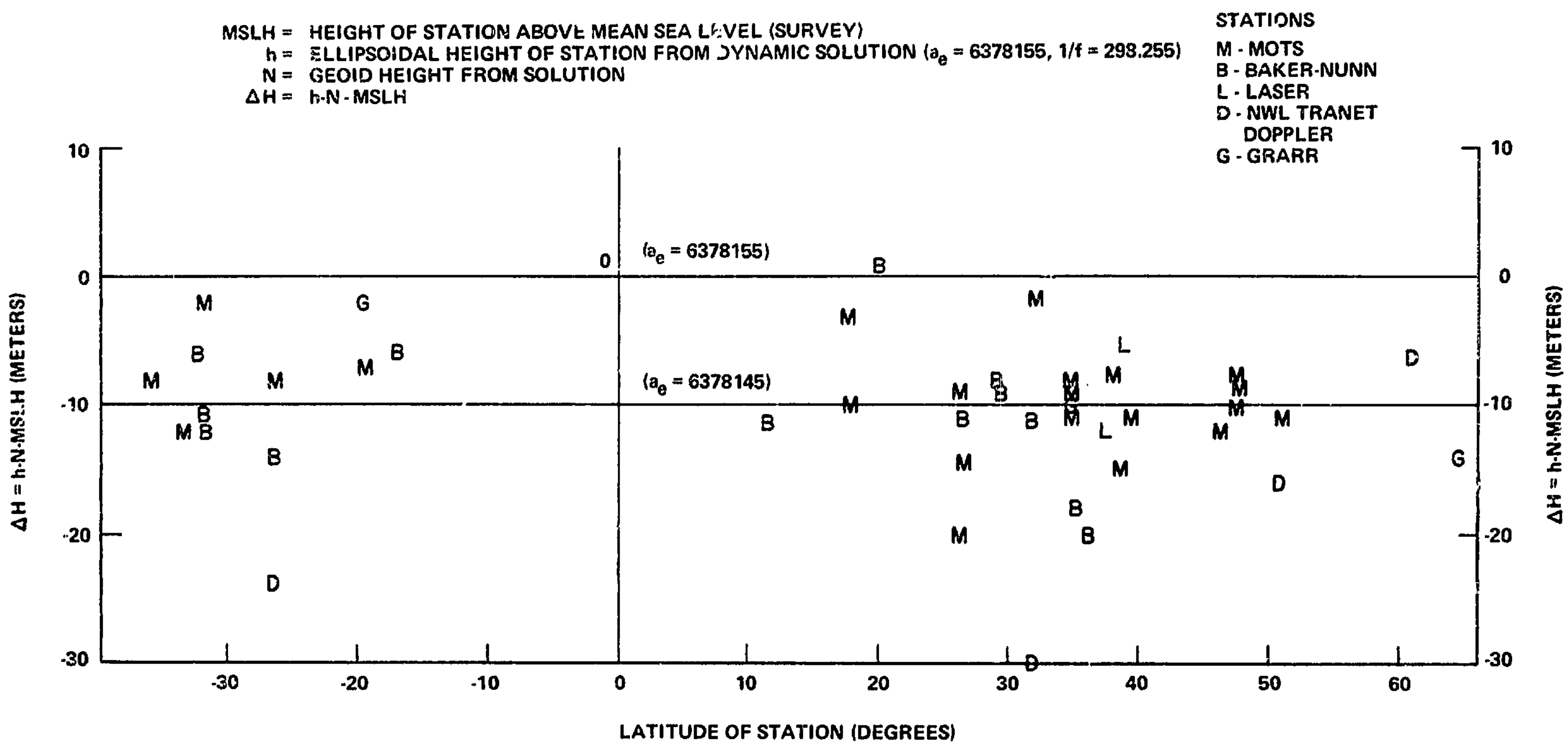

Figure 6. Station Height Above Geoid vs Survey (GSFC Solution)

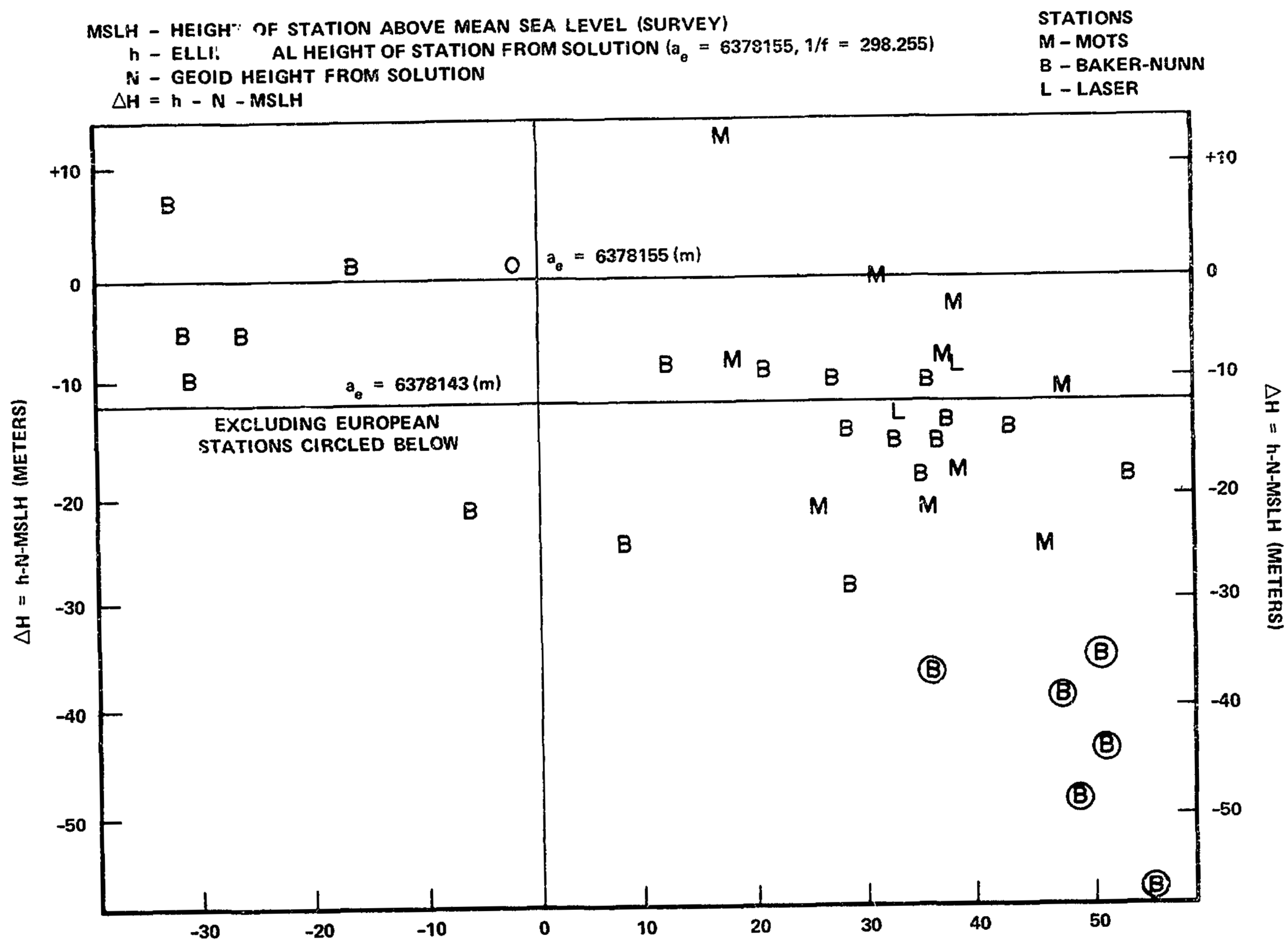

Figure 7. Station Height Above Geoid vs Survey (SAO Solution)

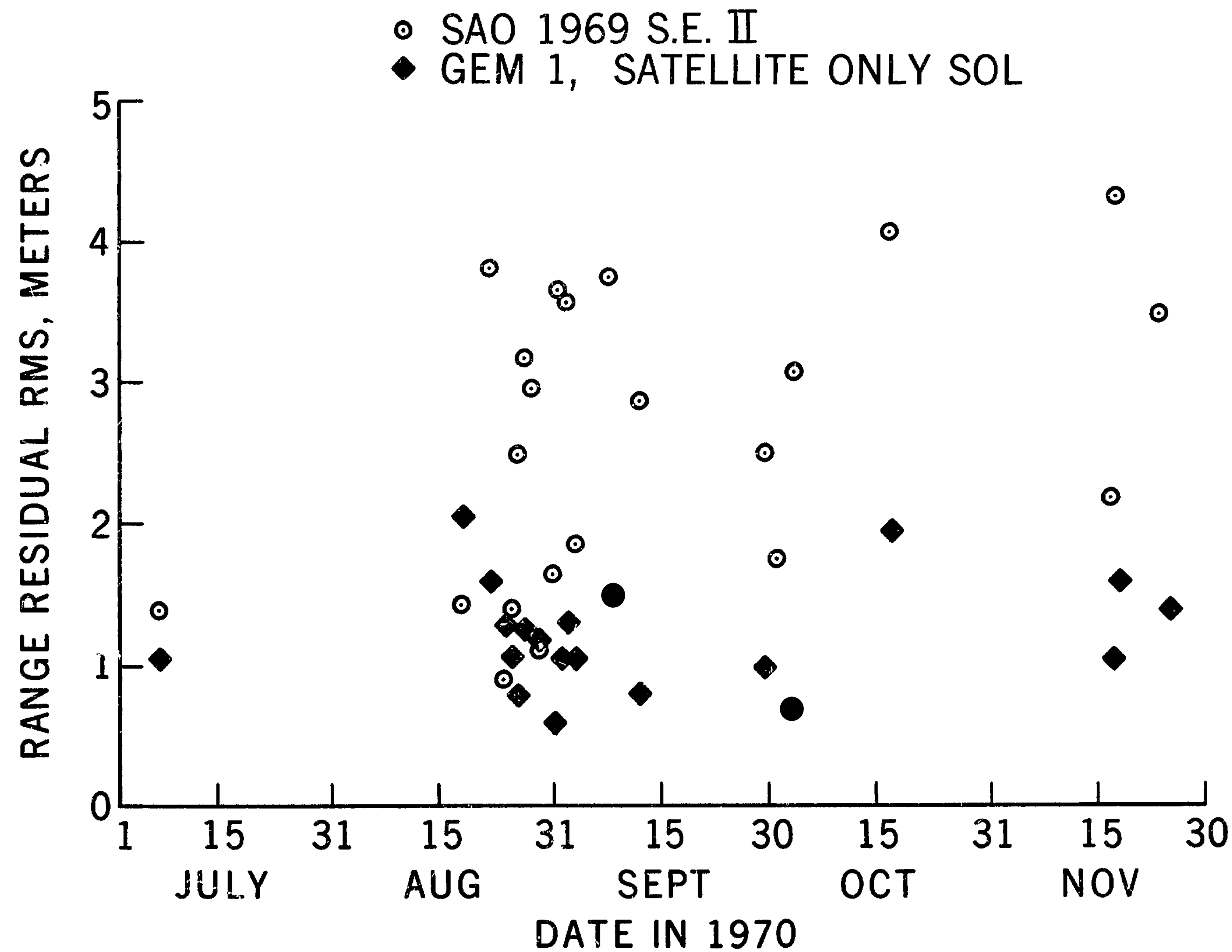

Figure 8. Gravity Field Comparison Using BE-C Laser Range Data

Table 1

Satellite Data

300 Weekly Sat. Arcs Opt. Data (Primarily SAO Baker-Nunn)

SAT. NAME	A (METERS)	E	I (DEG)	PERI HGT (KM)	PERIOD (REV/DAY)	NO ARCS	NO OBS	AVG NO OBS/ARC	AVG RMS (WEIGHTED)
TELSTAR-1	9669530.1	0.2471	44.79	951.3	9.13	16	1946	121	2.70
TIROS-9	8020761.2	0.1167	96.42	706.7	12.09	14	1525 *	108	1.16
GEOS-1	8067353.6	0.0725	59.37	1107.5	11.98	35	45555 **	1301	1.28
SECOR-5	8154869.9	0.0801	69.23	1140.1	11.79	4	290	72	2.38
OV1-2	8314700.2	0.1835	144.27	414.8	11.45	4	910	227	1.93
ALOU-2	8097474.4	0.1508	79.83	502.0	11.91	6	590 *	98	0.89
ECHO-1RB	7968879.1	0.0121	47.22	1501.0	12.20	18	2240	124	2.24
DI-D	7614681.9	0.0842	39.45	569.0	13.07	9	6386	709	1.86
BE-C	7503563.5	0.0252	41.17	941.9	13.36	22	4947	224	1.59
DI-C	7344163.4	0.0526	40.00	586.6	13.79	4	902	225	2.53
ANNA-1B	7504950.8	0.0070	50.13	1075.8	13.35	40	4153	104	1.51
GEOS-2	7710806.6	0.0308	105.79	1114.2	12.82	24	25315 **	1054	1.75
OSCAR-7	7404041.3	0.0242	89.70	847.7	13.53	4	1780	445	2.34
SBN-2	7463226.9	0.0058	89.95	1062.5	13.47	5	355	71	5.17
COURIER-1B	7473289.0	0.0174	28.34	966.5	13.44	12	3375	281	1.66
GRS	7228209.3	0.0604	49.72	421.3	14.13	5	369	73	3.06
TRANSIT-4A	7321521.7	0.0079	66.83	896.0	13.86	14	1316	94	1.92
BE-B	7354785.0	0.0143	79.70	901.8	13.74	4	469	117	1.87
OGO-2	7345633.6	0.0739	87.37	424.8	13.79	7	461	65	3.47
INJUN-1	7312542.4	0.0076	66.81	895.0	13.88	9	768	85	2.15
ACENA-R	7297251.5	0.0010	69.91	920.2	13.93	7	1005	143	2.86
MIDAS-4	9995760.5	0.0121	95.84	3504.8	8.69	20	14879	743	1.20
VANGUARD-2	8306759.8	0.1645	32.49	560.7	11.47	11	379	34	1.13
VANGUARD-2S	8339120.5	0.1648	32.37	562.2	11.46	5	615	123	2.29
VANGUARD-3S	8511504.6	0.1906	33.35	517.9	11.06	15	996	66	2.29
					TOTALS	314	121556	388	3.4"

*MINITRACK **MOTS 46,000 OBS.

Table 2

Standard Deviations Assigned to the Observations

Observation	Standard Deviation
GRARR $\dot{R}$ (range)	10 meters
R (range-rate)	3 cm/sec
Laser R	1 meter
Optical δ (declination)	2 seconds of arc
(cos δ) a (right ascension)	2 seconds of arc
Minitrack direction cosines	3 x 10^{4}
NWL Doppler $\dot{R}$	4 cm/sec
Gravity Data,	$33/(N + 1)^{1/2}$ mgal
5^0 x 5^0 mean gravity anomalies	
($N \equiv$ no. of points given in 5^0 x 5^0 section)	

Table 3

GSFC Geopotential Solutions (Normalized Coefficients X 10^6) 1

	L	M	GSFC COMB.	GSFC SAT.
C	2	0	484.1671	-484.1766
S	2	0	0.0	0.0
C	3	0	0.9548	0.9619
S	3	0	0.0	0.0
C	4	0	0.5368	0.5572
S	4	0	0.0	0.0
C	5	0	0.0726	0.0623
S	5	0	0.0	0.0
C	6	0	-0.1452	-0.1775
S	6	0	0.0	0.0
C	7	0	0.0869	0.1051
S	7	0	0.0	0.0
C	8	0	0.0397	0.0799
S	8	0	0.0	0.0
C	9	0	0.0329	0.0076
S	9	0	0.0	0.0
C	10	0	0.0648	0.0211
S	10	0	0.0	0.0
C	11	0	-0.0551	-0.0198
S	11	0	0.0	0.0
C	12	0	0.0211	0.0594
S	12	0	0.0	0.0
C	13	0	0.0432	0.0017
S	13	0	0.0	0.0
C	14	0	-0.0089	-0.0369
S	14	0	0.0	0.0
C	15	0	0.0039	0.0470
S	15	0	0.0	0.0
C	16	0	-0.0260	-0.0131
S	16	0	0.0	0.0
C	17	0	0.0072	-0.0350
S	17	0	0.0	0.0
C	18	0	0.0228	0.0180
S	18	0	0.0	0.0
C	19	0	0.0153	0.0050
S	19	0	0.0	0.0
C	20	0	-0.0006	-0.0016
S	20	0	0.0	0.0

	L	M	GSFC COMB.	GSFC SAT.
C	21	0	-0.0117	-0.0305
S	21	0	0.0	0.0
C	22	0	0.0	0.0
S	22	0	0.0	0.0
C	2	1	-0.0016	-0.0010
S	2	1	-0.0030	-0.0106
C	3	1	1.9929	2.0015
S	3	1	0.2366	0.2381
C	4	1	-0.5343	-0.5326
S	4	1	-0.4640	-0.4426
C	5	1	-0.0688	-0.0600
S	5	1	-0.0926	-0.0992
C	6	1	-0.0776	-0.0798
S	6	1	0.0103	-0.0306
C	7	1	0.2510	0.2533
S	7	1	0.1204	0.1134
C	8	1	0.0253	0.0245
S	8	1	0.0533	0.0981
C	9	1	0.1521	0.1637
S	9	1	0.0100	-0.0018
C	10	1	0.0873	0.0817
S	10	1	-0.1401	-0.1919
C	11	1	-0.0260	-0.0191
S	11	1	0.0255	0.0245
C	12	1	-0.0695	-0.0766
S	12	1	-0.0538	-0.0106
C	13	1	0.0408	0.0
S	13	1	-0.0606	0.0
C	14	1	-0.0296	-0.0150
S	14	1	0.0423	0.0053
C	15	1	0.0739	0.0
S	15	1	0.0202	0.0
C	16	1	-0.0291	0.0
S	16	1	0.0147	0.0
C	2	2	2.4230	2.4266
S	2	2	-1.3825	-1.3741
C	3	2	0.9244	0.9287
S	3	2	-0.6213	-0.6084

	L	M	GSFC COMB.	GSFC SAT.
C	4	2	0.3428	0.3461
S	4	2	0.6755	0.6777
C	5	2	0.6649	0.6403
S	5	2	-0.3065	-0.3038
C	6	2	0.0592	0.0474
S	6	2	-0.3613	-0.3478
C	7	2	0.3430	0.3283
S	7	2	0.0890	0.0922
C	8	2	0.0473	0.0444
S	8	2	0.0843	0.0688
C	9	2	0.0512	0.0021
S	9	2	-0.0002	0.0092
C	10	2	-0.0500	-0.0537
S	10	2	-0.0599	-0.0335
C	11	2	0.0098	0.0297
S	11	2	-0.1132	-0.1093
C	12	2	-0.0370	-0.0122
S	12	2	0.0443	0.0104
C	13	2	0.0211	0.0
S	13	2	-0.1134	0.0
C	14	2	-0.0313	0.0
S	14	2	0.0969	0.0
C	15	2	-0.0039	0.0
S	15	2	-0.0672	0.0
C	16	2	0.0091	0.0
S	16	2	0.0220	0.0
C	3	3	0.6876	0.7351
S	3	3	1.4571	1.3977
C	4	3	0.9830	0.9875
S	4	3	-0.2080	-0.2134
C	5	3	-0.4316	-0.4022
S	5	3	-0.1785	-0.2487
C	6	3	0.0313	0.0365
S	6	3	0.0191	-0.0060
C	7	3	0.2458	0.290[illegible]
S	7	3	-0.1728	-0.2299
C	8	3	-0.0198	-0.0232
S	8	3	-0.0460	-0.0687

	L	M	GSFC COMB.	GSFC SAT.
C	9	3	-0.1381	-0.1040
S	9	3	-0.0365	-0.1377
C	10	3	-0.0380	-0.0465
S	10	3	-0.0851	-0.1164
C	11	3	-0.0173	0.0006
S	11	3	-0.0577	-0.1283
C	12	3	0.1264	0.0872
S	12	3	0.0273	0.0810
C	13	3	-0.0215	0.0
S	13	3	0.0214	0.0
C	14	3	0.0408	0.0
S	14	3	-0.0065	0.0
C	15	3	0.0044	0.0
S	15	3	0.0402	0.0
C	16	3	0.0252	0.0
S	16	3	-0.0203	0.0
C	4	4	-0.1703	-0.1708
S	4	4	0.3116	0.3023
C	5	4	-0.2511	-0.2946
S	5	4	0.0410	0.0359
C	6	4	-0.0778	-0.0974
S	6	4	-0.4524	-0.4491
C	7	4	-0.2591	-0.2795
S	7	4	-0.1424	-0.0968
C	8	4	-0.2135	-0.2330
S	8	4	0.0308	0.0527
C	9	4	0.0313	0.0213
S	9	4	0.0106	0.0463
C	10	4	-0.0748	-0.0713
S	10	4	-0.1073	-0.1127
C	11	4	0.0053	0.0516
S	11	4	-0.0854	-0.0746
C	12	4	-0.0392	-0.0052
S	12	4	-0.0224	-0.0421
C	13	4	-0.0326	0.0
S	13	4	-0.0616	0.0
C	14	4	0.0292	0.0
S	14	4	0.0142	0.0

	L	M	GSFC COMB.	GSFC SAT.
C	15	4	-0.0006	0.0
S	15	4	-0.0146	0.0
C	16	4	0.0333	0.0
S	16	4	0.0732	0.0
C	5	5	0.2023	0.1338
S	5	5	-0.6721	-0.6662
C	6	5	-0.2600	-0.2613
S	6	5	-0.5639	-0.5310
C	7	5	0.0395	-0.0017
S	7	5	0.0526	0.0580
C	8	5	-0.0631	-0.0740
S	8	5	0.0649	0.0588
C	9	5	-0.0125	-0.0278
S	9	5	-0.0249	-0.0933
C	10	5	-0.0800	-0.1134
S	10	5	-0.0535	-0.0745
C	11	5	0.0873	0.0228
S	11	5	0.0571	0.0338
C	12	5	0.0203	0.0176
S	12	5	-0.0126	-0.0027
C	13	5	0.0343	0.0
S	13	5	0.0528	0.0
C	14	5	0.0411	0.0
S	14	5	-0.0309	0.0
C	15	5	0.0212	0.0
S	15	5	-0.0101	0.0
C	16	5	0.0126	0.0
S	16	5	0.0260	0.0
C	6	6	0.0186	0.0276
S	6	6	-0.2498	-0.2430
C	7	6	-0.3030	-0.3205
S	7	6	0.1784	0.1675
C	8	6	-0.0632	-0.0599
S	8	6	0.2871	0.3092
C	9	6	0.0339	0.0615
S	9	6	0.2071	0.2565
C	10	6	-0.0346	-0.0125
S	10	6	-0.0948	-0.0658

Table 3

GSFC Geopotential Solutions (Normalized Coefficients X 10^6) 2

	L	M	GSFC COMB.	GSFC SAT.
C	11	6	0.0045	-0.0058
S	11	6	0.0157	[illegible]
C	12	6	0.0522	0.0594
S	12	6	0.0103	0.0546
C	13	6	-0.1120	0.0
S	13	6	0.0042	0.0
C	14	6	0.0234	0.0
S	14	6	-0.0094	0.0
C	15	6	-0.0018	0.0
S	15	6	-0.0569	0.0
C	16	6	-0.0354	0.0
S	16	6	-0.0219	0.0
C	7	7	0.0406	0.0153
S	7	7	0.0194	0.0980
C	8	7	0.0640	0.0788
S	8	7	0.0805	0.0691
C	9	7	-0.0810	-0.0760
S	9	7	-0.0456	-0.0469
C	10	7	0.0394	0.0009
S	10	7	-0.0153	-0.0330
C	11	7	0.0071	0.0060
S	11	7	-0.1018	-0.1115
C	12	7	-0.0298	-0.0102
S	12	7	-0.0049	-0.0080
C	13	7	-0.0416	0.0
S	13	7	0.1067	0.0
C	14	7	0.1021	0.0
S	14	7	-0.0687	0.0
C	15	7	-0.0116	0.0
S	15	7	0.0696	0.0
C	16	7	0.0296	0.0
S	16	7	-0.0317	0.0
C	8	8	-0.1237	-0.0897
S	8	8	0.0935	0.0848
C	9	8	0.2420	0.1765
S	9	8	0.0467	-0.0214
C	10	8	0.0157	0.0286
S	10	8	-0.1318	-0.1346

	L	M	GSFC COMB.	GSFC SAT.
C	11	8	0.0210	-0.0167
S	11	8	0.0855	0.0579
C	12	8	-0.0304	-0.0269
S	12	8	0.0164	-0.0240
C	13	8	0.0396	0.0
S	13	8	-0.0074	0.0
C	14	8	-0.0041	0.0
S	14	8	-0.0518	0.0
C	15	8	-0.1339	0.0
S	15	8	0.0228	0.0
C	16	8	-0.0318	0.0
S	16	8	-0.0141	0.0
C	9	9	-0.0233	-0.0362
S	9	9	0.0935	0.0885
C	10	9	0.0909	0.1172
S	10	9	-0.0755	-0.0321
C	11	9	-0.0433	0.0329
S	11	9	0.0911	0.0484
C	12	9	-0.0105	0.0330
S	12	9	0.0151	0.0631
C	13	9	0.0239	0.1087
S	13	9	0.1181	0.0679
C	14	9	-0.0026	0.0413
S	14	9	0.0447	0.0758
C	15	9	0.0128	0.0713
S	15	9	0.0681	0.0483
C	16	9	0.0204	0.0
S	16	9	-0.0620	0.0
C	17	9	0.0	0.0
S	17	9	0.0	0.0
C	18	9	0.0	0.0
S	18	9	0.0	0.0
C	19	9	0.0	0.0
S	19	9	0.0	0.0
C	20	9	0.0	0.0
S	20	9	0.0	0.0
C	21	9	0.0	0.0
S	21	9	0.0	0.0

	L	M	GSFC COMB.	GSFC SAT.
C	22	9	0.0	0.0
S	22	9	0.0	0.0
C	10	10	0.1044	0.0574
S	10	10	-0.0370	-0.0398
C	11	10	-0.0567	-0.1206
S	11	10	-0.0219	-0.0067
C	12	10	0.0261	-0.0014
S	12	10	-0.0084	0.0309
C	13	10	-0.0120	0.0
S	13	10	0.0128	0.0
C	14	10	0.0208	0.0
S	14	10	-0.1030	0.0
C	15	10	0.0383	0.0
S	15	10	0.0124	0.0
C	16	10	-0.0456	0.0
S	16	10	-0.0013	0.0
C	11	11	0.0683	0.0539
S	11	11	-0.0319	-0.0360
C	12	11	-0.0058	-0.0028
S	12	11	0.0276	0.0402
C	13	11	-0.0680	0.0
S	13	11	-0.0084	0.0
C	14	11	0.1045	0.0
S	14	11	-0.0436	0.0
C	15	11	-0.0618	0.0
S	15	11	0.0574	0.0
C	16	11	0.0089	0.0
S	16	11	-0.0061	0.0
C	17	11	0.0	0.0
S	17	11	0.0	0.0
C	18	11	0.0	0.0
S	18	11	0.0	0.0
C	19	11	0.0	0.0
S	19	11	0.0	0.0
C	20	11	0.0	0.0
S	20	11	0.0	0.0
C	21	11	0.0	0.0
S	21	11	0.0	0.0

	L	M	GSFC COMB.	GSFC SAT.
C	22	11	0.0	0.0
S	22	11	0.0	0.0
C	12	12	-0.0183	-0.0180
S	12	12	-0.0128	-0.0114
C	13	12	-0.0290	-0.0288
S	13	12	0.1005	0.1004
C	14	12	0.0031	0.0026
S	14	12	-0.0431	-0.0401
C	15	12	-0.0389	-0.0371
S	15	12	0.0142	0.0145
C	16	12	0.0232	0.0202
S	16	12	-0.0153	-0.0119
C	17	12	0.0058	0.0108
S	17	12	-0.0053	-0.0043
C	18	12	-0.0611	-0.0606
S	18	12	-0.0264	-0.0230
C	19	12	-0.0347	-0.0310
S	19	12	-0.0380	-0.0364
C	20	12	0.0024	-0.0003
S	20	12	-0.0115	-0.0054
C	21	12	-0.0464	-0.0355
S	21	12	-0.0204	-0.0204
C	22	12	-0.0414	-0.0433
S	22	12	0.0111	0.0053
C	13	13	-0.0620	-0.0633
S	13	13	0.0723	0.0704
C	14	13	0.0225	0.0174
S	14	13	0.0524	0.0464
C	15	13	-0.0257	-0.0264
S	15	13	0.0020	0.0005
C	16	13	0.0203	0.0163
S	16	13	-0.0016	-0.0034
C	17	13	0.0072	0.0056
S	17	13	0.0116	0.0111
C	18	13	-0.0024	-0.0087
S	18	13	-0.0293	-0.0380
C	19	13	-0.0268	-0.0285
S	19	13	-0.0459	-0.0454

	L	M	GSFC COMB.	GSFC SAT.
C	20	13	0.0124	0.0021
S	20	13	0.0140	0.0015
C	21	13	-0.0275	-0.0290
S	21	13	0.0007	0.0010
C	22	13	-0.0334	-0.0413
S	22	13	0.0121	0.0050
C	14	14	-0.0459	-0.0457
S	14	14	-0.0034	-0.0005
C	15	14	0.0047	0.0053
S	15	14	-0.0203	-0.0185
C	16	14	-0.0160	-0.0208
S	16	14	-0.0385	-0.0387
C	17	14	-0.0128	-0.0137
S	17	14	-0.0007	-0.0029
C	18	14	-0.0118	-0.0084
S	18	14	-0.0049	-0.0055
C	19	14	0.0012	0.0023
S	19	14	-0.0001	0.0032
C	20	14	0.0065	0.0069
S	20	14	-0.0124	-0.0096
C	21	14	0.0112	0.0122
S	21	14	0.0042	0.0069
C	22	14	-0.0095	-0.0124
S	22	14	0.0006	0.0028
C	15	15	-0.0575	0.0
S	15	15	0.0249	0.0
C	16	15	-0.0522	0.0
S	16	15	0.0179	0.0
C	17	15	0.0	0.0
S	17	15	0.0	0.0
C	18	15	0.0	0.0
S	18	15	0.0	0.0
C	16	16	-0.0169	0.0
S	16	16	-0.0115	0.0

Table 4

Station Coordinates

	STN	LATITUDE DDMM	LATITUDE SS.SS	LONGITUDE DDDMM	LONGITUDE SS.SS	HEIGHT METERS
ANCHOR	14	6117	0.51	21010	29.69	68.7
LACRES	103	3216	44.68	25314	45.42	1149.0
LASHM2	106	5111	9.30	35858	26.32	221.5
APLMND	111	39 9	48.67	283 6	12.78	97.3
1BPOIN	1021	3825	49.77	28254	48.71	-40.1
1FTMYR	1022	2632	53.45	278 8	4.33	-31.1
1OOMFR	1024	-3123	25.19	13652	15.89	129.9
1SATAG	1028	-33 8	58.49	28919	53.60	707.2
1MOJAV	1030	3519	47.90	243 5	59.30	888.8
1JOBUR	1031	-2553	0.66	2742	26.59	1539.0
1NEWFL	1032	4744	29.84	30716	46.34	69.0
1GFORK	1034	48 1	21.27	26259	19.87	219.4
1WNKFL	1035	5126	46.48	35918	8.38	104.0
1ROSMN	1037	3512	7.13	277 7	41.75	865.4
1ORORL	1038	-3537	32.15	14857	14.80	939.5
1ROSMA	1042	3512	7.33	277 7	41.44	865.5
1TANAN	1043	-19 0	31.72	4717	59.73	1364.4
MADGAS	1123	-19 1	14.19	4718	11.75	1390.8
ROSRAN	1126	3511	45.51	277 7	26.20	828.9
ULASKR	1128	6458	19.05	21229	12.44	341.6
PRETOR	2115	-2556	48.21	2820	51.88	1579.8
1UNDAK	7034	48 1	21.41	26259	19.60	217.3
1EDINB	7036	2622	46.76	26140	7.73	21.4
1COLBA	7037	3853	36.18	26747	41.11	227.3
1BERMD	7039	3221	49.86	29520	35.37	-15.2
1PURIO	7040	1815	28.94	294 0	23.77	-5.7
1GSFCP	7043	39 1	15.32	28310	20.33	0.6
1DENVR	7045	3938	48.02	25523	38.87	1759.3
GODLAS	7050	39 1	14.53	28310	18.33	12.3
WALLAS	7052	3751	36.19	28429	23.76	-42.1
1JUM40	7072	27 1	14.54	27953	13.07	-30.6
1JUBC4	7074	27 1	14.79	27953	13.11	-34.2
1SUDBR	7075	4627	20.40	279 3	10.99	233.3
1JAMAC	7076	18 4	34.78	28311	27.12	422.4
1ORGAN	9001	3225	25.04	25326	49.06	1616.4
1OLFAN	9002	-2557	35.95	2814	52.83	1553.5
WOOMER	9003	-31 6	1.98	13647	3.63	151.0
1SPAIN	9004	3627	46.74	35347	37.13	56.9
1TOKYO	9005	3540	23.00	13932	16.65	81.0
1NATAL	9006	2921	34.71	7927	27.60	1869.6
1QUIPA	9007	-1627	56.74	28830	24.82	2476.5
1SHRAZ	9008	2938	13.87	5231	11.52	1578.2
1CURAC	9009	12 5	25.19	291 9	44.71	-28.8
1JUPTR	9010	27 1	14.15	27953	13.55	-26.8
1VILDO	9011	-3156	34.67	29453	36.93	618.[illegible]
1MAUIO	9012	2042	26.15	20344	33.98	3041.1
AUSBAK	9023	-3123	25.82	13652	43.96	128.4

Table 5

Comparison of Zonal Coefficients

Normalized Coefficients Multiplied by 10^6

DEGREE	GSFC COMB.	S.E.II	GSFC SAT.	SE I	DIFFERENCE WITH S.E.II COMB.	DIFFERENCE WITH S.E.II SAT.
2	-484.1671	-484.1660	-484.1766	-484.1735	-0.0012	-0.0106
3	0.9548	0.9593	0.9619	0.9623	-0.0045	0.0027
4	0.5368	0.5310	0.5572	0.5497	0.0058	0.0262
5	0.0726	0.0693	0.0623	0.0633	0.0033	-0.0070
6	-0.1452	-0.1392	-0.1775	-0.1792	-0.0060	-0.0383
7	0.[illegible]69	0.0935	0.1051	0.0860	-0.0066	0.0117
8	0.0397	0.0286	0.0799	0.0655	0.0111	0.0512
9	0.0329	0.0229	0.0076	0.0122	0.0100	-0.0154
10	0.0648	0.0772	0.0211	0.0118	-0.0124	-0.0561
11	-0.0551	-0.0421	-0.0198	-0.0630	-0.0129	0.0223
12	0.0211	0.0084	0.0594	0.0714	0.0127	0.0510
13	0.0432	0.0237	0.0017	0.0219	0.0196	-0.0220
14	-0.0089	0.0136	-0.0369	-0.0332	-0.0224	-0.0505
15	0.0039	0.0313	0.0470	0.0	-0.0273	0.0158
16	-0.0260	-0.0326	-0.0131	0.0	0.0066	0.0194
17	0.0072	-0.0144	-0.0350	0.0	0.0216	-0.0207
18	0.0228	0.0380	0.0180	0.0	-0.0151	-0.0200
19	0.0153	0.0346	0.0450	0.0	-0.0192	0.0104
20	-0.0006	0.0008	-0.0016	0.0	-0.0014	-0.0024
21	-0.0117	-0.0220	-0.0305	0.0	0.0103	-0.0085
22	0.0	0.0	0.0	0.0	0.0	0.0
				RMS	0.0136	0.0285

Table 6

Degree Variances for Gravity Anomalies (Mgal^2)

DEGREE	GSFC COMB.	S.E.II	GSFC SAT.
3	33.577	32.842	33.344
4	21.509	21.805	21.611
5	20.359	17.785	19.790
6	19.651	15.652	18.764
7	17.849	15.491	19.741
8	9.029	6.639	10.311
9	10.538	12.651	11.454
10	9.951	12.860	11.158
11	7.374	12.234	8.174
12	4.415	5.099	4.877
13	13.375	11.121	5.001
14	11.333	8.431	2.424
15	11.204	13.215	2.301
16	6.096	13.844	0.630
TOTAL	196.	199.	

Table 7

Comparison of Coefficients with S. E. II

Normalized Coefficients Multiplied by 10^6

RMS OF COEFFICIENT DIFFERENCES BY DEGREE

DEGREE	GSFC COMB.	GSFC SAT.	SE I	B13.1
3	0.021	0.032	0.112	0.068
4	0.039	0.043	0.064	0.030
5	0.049	0.059	0.055	0.056
6	0.062	0.065	0.072	0.044
7	0.059	0.073	0.093	0.060
8	0.031	0.042	0.065	0.061
9	0.046	0.050	0.0	0.051
10	0.052	0.063	0.0	0.058
11	0.066	0.057	0.0	0.056
12	0.044	0.048	0.0	0.050
13	0.065	0.055	0.0	0.066
14	0.049	0.023	0.0	0.060
15	0.047	0.025	0.0	0.048
16	0.034	0.018	0.0	0.017
TOTAL RMS	0.050	0.052	0.077	0.053

Table 8

Station Geoid Heights from the Combination Solution

and MSL Heights from Survey

DATA TYPE	STATION NAME	STATION NUMBER	LAT* (DEG)	LON* (DEG)	MSL (M)	GEOID (M)
D	ANCHOR	14	61.28	210.17	68.	7.
D	LACRES	103	32.27	253.23	1203.	-24.
D	LASHM2	106	51.18	358.97	190.	48.
D	APLMND	111	39.15	283.10	143.	-38.
D	PRETOR	2115	-25.93	28.33	1580.	24.
M	1BPOIN	1021	38.42	282.90	6.	-38.
M	1FTMYR	1022	26.53	278.13	5.	-27.
M	1OOMFR	1024	-31.38	136.87	133.	-1.
M	1SATAG	1028	-33.13	289.32	693.	26.
M	1MOJAV	1030	35.32	243.08	929.	-30.
M	1JOBUR	1031	-25.88	27.70	1522.	25.
M	1NEWFL	1032	47.73	307.27	69.	8.
M	1GFORK	1034	48.02	262.98	253.	-26.
M	1WNKFL	1035	51.43	359.30	67.	48.
M	1ROSMN	1037	35.20	277.12	909.	-35.
M	1ORORL	1038	-35.62	148.95	932.	16.
M	1ROSMA	1042	35.20	277.12	909.	-35.
M	1TANAN	1043	-19.00	47.28	1378.	-7.
M	1UNDAK	7034	48.02	262.98	253.	-26.
M	1EDINB	7036	26.37	261.67	60.	-19.
M	1COLBA	7037	38.88	267.78	273.	-31.
M	1BERMD	7039	32.35	295.33	31.	-45.
M	1PURIO	7040	18.25	294.00	50.	-46.
M	1GSFCP	7043	39.02	283.17	53.	-38.
M	1DENVR	7045	39.63	255.38	1790.	-20.
M	1JUM40	7072	27.02	279.88	14.	-31.
M	1SUDBR	7075	46.45	279.05	281.	-36.
M	1JAMAC	7076	18.07	283.18	446.	-21.
G	MADGAS	1123	-19.02	47.30	1399.	-7.
G	RCSRAN	1126	35.18	277.12	874.	-35.
G	ULASKR	1128	64.97	212.48	349.	7.
L	GODLAS	7050	39.02	283.17	55.	-38.
L	WALLAS	7052	37.85	284.48	9.	-39.
B	1ORGAN	9001	32.42	253.43	1651.	-24.
B	1OLFAN	9002	-25.95	28.23	1544.	24.
B	1SPAIN	9004	36.45	353.78	26.	51.
B	1TOKYO	9005	35.67	139.53	60.	39.
B	1NATAL	9006	29.35	79.45	1927.	-49.
B	1QUIPA	9007	-16.45	288.50	2452.	31.
B	1SHRAZ	9008	29.63	52.52	1596.	-9.
B	1CURAC	9009	12.08	291.15	7.	-25.
B	1JUPTR	9010	27.02	279.88	15.	-31.
B	1VILDO	9011	-31.93	294.88	598.	27.
B	1MAUIO	9012	20.70	203.73	3034.	6.
B	AUSBAK	9023	-31.38	136.87	141.	-2.
B	WOOMER	9003	-31.38	136.87	162.	-2.

*See Table 4 for more complete values.

Table 9

Long-Arc Tests on Mean Elements for Resonant Satellites

	INTELSAT* 2-F1	Cosmos 41* Rocket	Cosmos 382** Rocket
Beat Period (days)	900	900	125
Data Span (days)	1100	2000	430

Gravity Fields	RMS Mean Anomaly (degs.)	RMS Mean Anomaly (degs.)	RMS Semi-Major Axis (meters)
1. S.E. II	2.0	6.5	55.5
2. GSFC-Comb.	0.5	3.6	15.6

Resonant Coefficients

*(2,2), (3,2), (4,2) ....; (4,4), (5,4), (6,4) ....; (6,6), (7,6) .... etc.

**(9,9), (10,9), (11,9) .... etc.

Table 10

Summary of Solution, Data, and Results

A. SOLUTION

Geopotential solution in spherical harmonic series (degree n, order m).

Type Solution	Complete (n × m)	Zonals Complete	Resonant Coefficients
Satellite (GEM 1)	12 × 12	21	m = 12, 13, 14, n = m to 22 m = 9, n = 9 to 16 m = 11, n = 11, 12, 14
Satellite & Gravimetry (GEM 2)	16 × 16	21	Complete as above

46 Center of mass tracking station locations (see tracking systems below)

B. DATA

Satellite Data

Tracking Systems	25 Satellites (Data Processed)
13 Baker-Nunn Cameras	300 weekly arcs of optical data
23 MOTS Cameras	15 weekly arcs of electronic/laser data
3 GRARR (range/range-rate)	66 one/two-day arcs of GEOS-I and II flashing light data
2 LASERS	20 weekly arcs of Minitrack data
5 DOPPLER	

Gravimetry Data

19,000 1° × 1° mean gravity anomalies of ACIC data, and
2,000 1° × 1° mean gravity anomalies from other sources, used to form
1,705 5° × 5° means, providing 70% world coverage

C. RESULTS

1. Station heights indicate a_e = 6378145 meters.
2. Mean value of gravity ($\Delta \bar{g}_0$ = 3.28 mgal) indicates a_e = 6378142 (m).
3. Solution for 327 zonal and tesseral coefficients including $\Delta \bar{g}_0$ from satellite and gravimetry data.
4. Solution using optical and electronic tracking data for location of 46 tracking stations.