INVESTIGATION OF CHARACTERISTICS OF FEED SYSTEM INSTABILITIES

Prepared by
R. D. Vaneg
J. F. Fuller
R. A. Zehnle

Prepared for
National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

MARTIN MARIETTA CORPORATION
P. O. Box 179
Denver, Colorado 80201
FOREWORD

This final report is submitted in accordance with the requirements of the Statement of Work for Contract NAS8-26266, and documents the work accomplished during the contract period 1 July 1970 through 1 June 1972. This study was performed for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration, and was administered technically by Mr. Raymond Spink of the Science and Engineering Directorate, Astronautics Laboratory.
In the investigation of structure-propulsion system coupled longitudinal oscillations (POGO), the relationship between the structural and feed system natural frequencies is of major importance. The structural frequencies can be adequately defined by existing analytical techniques. The feed system frequencies are usually very dependent upon the compressibility (compliance) of cavitation bubbles that exist to some extent in all operating turbopumps. The lack of an accurate analytical prediction method for determining cavitation compliance has delayed the completion of POGO stability analyses until after turbopumps have been built and tested.

This document includes: a complete review of cavitation mechanisms; development of a turbopump cavitation compliance model; an accumulation and analysis of all available cavitation compliance test data; and a correlation of empirical-analytical results. The analytical model is based on the analysis of flow relative to a set of cascaded blades, having any described shape, and assumes phase changes occur under conditions of isentropic equilibrium. The model is restricted to incipient blade cavitation and does not include the effects of blade tip clearance or back flow.

Analytical cavitation compliance predictions for the J-2 LOX, F-1 LOX, H-1 LOX and LR87 oxidizer turbopump inducers do not compare favorably with test data. The model predicts much less cavitation than is derived from the test data. This implies that mechanisms other than blade cavitation contribute significantly to the total amount of turbopump cavitation. A current related technology contract (NAS8-27731) is extending the empirical evaluation of test data presented in this document.
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1. Introduction
1. Introduction

1.1 Purpose - Longitudinal oscillation instabilities (POGO) due to closed loop coupling between structural modes and propulsion feed system modes have been encountered on most liquid propellant launch vehicles (Reference 1). Experimental evaluation of feed system dynamics in these vehicles has shown that turbopump cavitation is usually the major source of feed system compliance which, along with the effective fluid mass, determines the feed system resonant frequencies. Compliance (C) is defined as the rate of change of fluid mass (W) with respect to pressure (P) for a constant volume; i.e.,

\[ C = \frac{\partial W}{\partial P} \quad \text{lb m in.}^2 \text{ or in.}^2 \quad (1.1) \]

Cavitation bubble compliance (\(C_b\)) is given by the rate of change of the mass of propellant stored in the turbopump (\(W_p\)) with respect to inlet pressure (\(P_s\)). Changes in \(W_p\) can be related to changes in cavitation vapor volume (\(V_v\)) by

\[ C_b = \frac{\partial W}{\partial P_s} \approx \rho \frac{\partial V_v}{\partial P_s} \quad (1.2) \]

where \(\rho\) is the liquid density and the mass of vapor is small relative to the mass of liquid.* Some previous analytical and semi-empirical attempts (Section 2.3) have been made, usually using average geometry parameters and flow conditions through the turbopump, to predict the amount of cavitation. Confidence in these methods has never been sufficient to eliminate the requirement to perform dynamic response tests on new turbopump components.

* In some of the literature \(C_b\) is defined as \(\frac{\partial V_v}{\partial P_s}\) which can be related to the values presented in this report by multiplying by the appropriate propellant density.
configurations. The lack of available test hardware during design phases is one of the major reasons that POGO suppression has been worked as a post flight effort on all past launch vehicles. The technology effort documented in this report is aimed at producing increased confidence in pre-test POGO stability analysis on future launch vehicle programs like the Space Shuttle Vehicle. This study is closely related to three other current Space Shuttle Technology programs: contract NAS8-26250, "Research on Cavitating Pump Instabilities", Hydronautics Incorporated; contract NAS8-25919, "Analysis of Propellant Feedline Dynamics", South West Research Institute; and contract NAS8-27731, "Empirical Evaluation of Pump Inlet Compliance", Aerospace Corporation.

1.2 Objectives - The intent of this investigation is to establish the relationships between turbopump inlet compliance and the pump parameters and fluid properties that control or define the compliance mechanism. The correlation is to be established with an analytical and/or semi-empirical model which is verified with existing test data. It is desired that the correlations be formulated and presented such that the frequency response characteristics of a cryogenic feed system can be evaluated for a given vehicle configuration. It is desired that the deviation in feed system resonant frequency between analytical and empirical results not exceed ±10%.

1.3 Scope - This study deals primarily with the determination of turbopump cavitation compliance, the largest element of uncertainty in dynamic modeling of feed systems. Feed system models, incorporating cavitation compliance, are well understood and range from very simple models (Paragraph 4.1.3) to
fairly complicated models (References 2 and 3). A general feed system computer model was provided to NASA under contract NAS8-23511. Also, detailed feed system modeling is currently being performed under contract NAS8-25919. For these reasons, no complex feed system models are presented in this report.

1.3.1 Since cavitation compliance is required for determination of feed system frequency response characteristics for use in POGO stability analysis, only linear response characteristics are of interest. Also, only the normal flight operating range of a turbopump is considered. These two conditions permit the analysis of turbopump cavitation to be restricted to the region of incipient cavitation, and does not consider the region of gross cavitation where turbopump operating performance is significantly reduced. The analytical model developed deals with thermal vapor cavitation between the turbopump blades. It does not include gaseous cavitation, or cavitation resulting from back flow or blade tip clearance flow, although these effects are discussed.

1.4 General Approach - The general approach taken to meet the study objectives within the scope specified was to:

a. Review all literature relative to turbopump cavitation;
b. Develop an analytical model to predict cavitation compliance;
c. Analyze all available cavitation compliance test data;
d. Perform an evaluation of the test data;
e. Correlate the analytical and test results.

The model development portion of this study is a continuation of a portion of a general POGO technology contract conducted for the Air Force Rocket Propulsion Lab (References 4 and 5).
The analytical cavitation model is a fluid dynamic/thermodynamic model which employs the compressible flow equations in finite difference form and solves them iteratively. Isentropic conditions of thermodynamic equilibrium between the vapor (cavitation) and liquid phases are assumed. The solution yields the amount of vapor at many grid points in a turbopump stream sheet of revolution (an annulus between two blades). A combination of several streamsheets at different blade radii gives the total turbopump cavitation, which when related to a change in inlet pressure, results in cavitation compliance. This approach accounts for varying conditions of fluid flow and blade geometry throughout the turbopump.
2. Review of Cavitation Mechanism
2. REVIEW OF CAVITATION MECHANISM

2.1 Turbopump Operation - Turbopumps used in present rocket propulsion systems are generally of the mixed flow design. The pump fluid while in the impeller has, in addition to angular velocities, both axial and radial velocity components as opposed to the predominantly radial velocities associated with centrifugal water pumps. In most cases, the turbopump will have an inducer section upstream of the main impeller that improves fluid angle of attack and increases the pressure at the inlet to the impeller. This allows further reduction in NPSH before blade stall and a loss in head rise (pressure increase from inlet to discharge) occurs. In some configurations, the impeller and inducer are of one piece construction, the inducer blades transitioning into impeller blades with additional impeller blades starting at some distance into the turbopump. The pressure rise through the inducer makes it probable that the majority of the turbopump cavitation occurs in the inducer. This is because the average pressure at the impeller inlet is generally too high for any local blade surface pressure to be reduced to the vapor pressure.

2.1.1 Shrouded blades of proper design, operating at the design point and free of vapor represent an analytical idealization in that channel type flow exists between the blades. Most turbopumps, however, are not designed with completely shrouded blades, and the channel flow idealization cannot be realized. In this situation, the flow picture is complicated by a tip clearance flow between the moving blades and the stationary shroud. The tip clearance flow, which is from the pressure side of a blade to the suction side, often induces a vortex
near the outer edge of the blade on the suction side. If the blades are improperly designed or are being operated off the design conditions, a liquid-filled separation cavity may be attached to the leading edge. This is more likely when the blade leading edges are quite sharp. Although the impeller will still pump fluid with an associated pressure rise, it is accomplished at a reduced efficiency.

2.1.2 The complexity of the flow situation increases with the appearance of vapor phases. If the velocity gradients are not large and the inducer blades have large radii of curvature associated with the leading edge region, the vapor phases, which consist of dissolved gas coming out of solution and/or a change in phase of the pump fluid, will appear as a region of bubbles moving with the fluid. On the other hand, if a liquid-filled separation cavity exists before the appearance of vapor, the cavity may fill with vapor when it evolves and expand with further reduction in suction pressure. Such cavities that originate near the leading edge of the blade may reattach on the downstream side of the blade or, if severely low pressures exist, extend downstream into the discharge portion of the pump. Although separation cavities may not exist before the appearance of vapor, the density change and disturbances associated with the evolution of vapor may induce separation of the boundary layer and create a cavity. In both types of two-phase flow, the variations of density add considerable complexity to the analysis problem.

2.2 Sources of Cavitation - Cavities may exist in the turbopump liquid propellant due to the presence of either vapor bubbles produced by liquid boiling, hereafter referred to as thermal cavitation; or contaminant gas bubbles, hereafter referred to as gaseous cavitation.
2.2.1 Thermal Cavitation - Thermal cavitation results when the ambient pressure drops below the saturation pressure, or the fluid temperature rises above the saturation temperature. Pressure changes may be associated with quasi-steady fluid motion, transient fluid motion, or acoustic excitation. Temperature changes can result from heat transfer across the boundaries of the system, fluid motion, and phase changes of the fluid (latent heat of vaporization and condensation). For an isentropic process, a change in pressure produces a phase change, which results in a change in fluid temperature, which in turn tends to impede the phase change. With sufficient time, thermodynamic equilibrium is reached and there exists a given amount of vapor for a given pressure. For bubble growth and decay under conditions of non-thermodynamic equilibrium, the mathematics defining the rate of change of bubble size are given in Appendix A.

2.2.2 Gaseous Cavitation - Gaseous cavitation can occur from the following sources:
   a. Dissolved gases coming out of solution;
   b. Undissolved gases mixed with the propellant;
   c. Chemical reaction (corrosion) between the propellant and the turbopump.

Substantial concentrations of both dissolved and undissolved foreign gases, such as atmospheric air or blanket gases used to hold the fluid under pressure before entrance to the pump, may also be present in propellants which have been stored for either long periods of time or under very low gravity conditions. Changes in the ratio of dissolved to undissolved gas can result from changes in fluid velocity, acoustic excitation, or heat
transfer. Undissolved gas may also exist as a result of a Gas Bubbling POGO Suppression Device (References 6 and 7). The problem of corrosion is not normally encountered in current propulsion systems. Even if storage and utilization of propellants are controlled so that an insignificant amount of gaseous cavitation occurs, small amounts of dissolved gases may be instrumental in the initial formation of a thermal cavitation vapor bubble (Paragraph 2.3.2).

2.3 Previous Cavitation Analyses - Analysis of turbopump cavitation compliance was initiated with a review of existing knowledge on cavitation. Discussion of this review and the conclusions derived follow.

2.3.1 Turbopump vs Other Types of Cavitation - Considerable investigation has been performed in the field of cavitation. Areas that have received the most attention have been pump cavitation, hydrofoil and hydrodynamic propeller cavitation, cavitation on such underwater vehicles as submarines and torpedos, and cavitation induced by sound waves. The majority of investigations associated with pumps have been experimental or semiempirical with the objective of preventing cavitation damage by determination of incipient cavitation conditions (References 8 and 9). Investigations of hydrofoils, propellers, and underwater vehicles have been aimed at predicting lift and drag coefficients and reducing noise associated with cavitation bubble collapse. Ultrasonic cavitation research has been directed primarily toward assessing sound energy and frequency requirements to induce cavitation, and examining the attenuation and distortion of sound waves caused by the cavitation bubbles.
2.3.2 Nucleation - If gas-filled voids exist in a fluid (References 10 and 11), changes in the concentration of dissolved gases and changes in phase within the fluid will take place at the boundary of the voids as well as at the fluid surface. The presence of voids in a fluid is suggested by the cohesive strength of water. Predictions of cohesive strength based on breaking the van der Waals intermolecular bonds exceed experimental observations by several orders of magnitude. The source of this large discrepancy is attributed to the presence of contaminants in the water that form voids of nuclei on the order of $10^{-5}$ to $10^{-2}$ cm in diameter. Since it is known that large amounts of air can be dissolved in water, it is hypothesized that the voids of nuclei are filled with contaminant gases such as air or mixtures of gas and fluid vapor. This hypothesis only partially explains the observed fracture strength of water. A nucleus containing contaminant gas and vapor will be in static equilibrium in the fluid if

\[ P_v + P_G - P_\infty = 2\sigma/R \]  

(2.1)

where

- $P_v$ = vapor pressure
- $P_G$ = sum of partial pressures of contaminant gases
- $P_\infty$ = ambient pressure
- $\sigma$ = surface tension constant
- $R$ = radius of nucleus

If the nucleus contains only vapor and its radius is given by Equation (2.1), it is in a condition of unstable equilibrium and will either grow or collapse upon being disturbed.

2.3.2.1 Gas-filled nuclei on the order of $10^{-5}$ to $10^{-2}$ cm in diameter present in an undersaturated or saturated solution of
water (saturation in this context refers to the concentration of dissolved gas) dissolve in a few minutes or seconds depending on the radius of the nuclei and the dissolved gas concentration. The dissolving process is assisted by the surface tension force \(2\sigma/R\). A nucleus of this size, if present in a supersaturated solution of water, grows by diffusion of gas into the nucleus and floats to the surface of the water where it escapes. The rate of rise of a nucleus of \(10^{-5}\) cm in diameter is very slow and may take several hundred hours to escape from the liquid. Water that has been allowed to set a long time still fails to demonstrate the expected cohesive strength. A mechanism or mechanisms, therefore, that prevents the diffusion of gas out of the nuclei and/or prevents nuclei from rising to the surface of the fluid must be acting. Two hypotheses advanced are:

a. Surface films composed of algae or other contaminants form around the nuclei and act as barriers to the diffusion process;

b. The nuclei are held in surface cracks of the fluid container or on dust particles suspended in the fluid.

The first hypothesis has been demonstrated by Bernd (Reference 12) in experiments where the cohesive strength of various fluids has been manipulated by control of the algae content. Rosenberg (Reference 13) has shown analytically and experimentally that the walls of the fluid containers can have cracks in which nuclei can be attached in a stable condition. In the same work, it was shown that dust particles or colloidal matter in the fluid can have cracks upon which the nuclei can be stabilized. An aspect of Rosenberg's investigation that will require further attention is the observed difference in susceptibility of various liquids to contamination by dust.
2.3.2.2 The nuclei are the focal points that govern changes in dissolved gas concentration and changes in phase (Reference 14). Regarding changes in phases, nucleation action occurs not only for the growth of vapor bubbles, but also for their collapse. The presence of contaminant dust particles in the bubble and on its surface serve as nuclei upon which additional condensation can take place.

2.3.2.3 The preceding discussion shows that cavitation in a turbopump will depend on the number of nuclei present in the fluid, their size, and conditions that will affect their dynamic behavior. It is important, therefore, that methods be developed for assessing the effects of fluid characteristics on nucleation.

2.3.3 **Diffusion** - A decrease in the concentration of dissolved gas in the pump fluid will result from diffusion of dissolved gas into nuclei present in the fluid. If the diffusion process continues, the nuclei will grow into gas bubbles and the phenomenon of gaseous cavitation will be observed. The diffusion process in a pump can be driven by changes in either pressure or temperature. Once the temperature and pressure conditions have been altered to a condition that favors diffusion of gas out of the cavitation bubble, the bubble will decrease in size. Although conditions return to those existing at inception of bubble growth, the contracting bubble may reach an equilibrium volume substantially greater than that of the original nucleus. This phenomenon is demonstrated by tests of the cohesive strength of water. Water that has been subjected to gaseous cavitation has a fracture strength considerably less than it had before cavitation. This results from the presence of nuclei after cavitation that are larger than those that existed before cavitation. Plesset and Epstein (Reference 15),
ignoring the effects of the motion of the bubble boundary on the concentration gradient in the liquid, have derived the equation governing the diffusion of gas into and out of a static bubble. Their results, if applied directly to turbopumps, show that the characteristic time for bubble growth is too large to cause cavitation. The boundary conditions for the derivation, however, differ considerably from those that will exist in a turbopump. The effects of bubble boundary motion and turbulence on the gaseous concentration gradient serve to accelerate the bubble growth. Future work, therefore, should incorporate these effects into a more accurate description of the diffusion process.

2.3.4 Acoustic Cavitation - As previously stated, changes in the ratio of dissolved to undissolved gas in a fluid can occur due to acoustic excitation. The generation of sound in a fluid results in an oscillatory pressure throughout the fluid. The pressure disturbances in turn result in an oscillation of the boundary of nuclei in the fluid. With a fluid saturated or supersaturated with dissolved gas, the nuclei may grow into gas bubbles given the proper amplitude and frequency of acoustic excitation. The growth is achieved through the rectified diffusion of gas into the bubble. During the low-pressure or expansion phase of bubble oscillation, conditions result in diffusion of gas into the bubble. During compression of the bubble, gas diffuses out; however, due to the large time surface area associated with the bubble expansion, a net inflow of gas occurs. Because of the extreme noises associated with rocket engines, this type of cavitation should not be ignored when analyzing rocket turbopumps.
2.3.5 Thermodynamics - The cavitation problem requires consideration of many thermodynamic effects. Literature on the subject of thermal cavitation may be divided into two areas. The first of these deals with the effect of heat transfer on cavitation bubble growth. The nonsteady heat diffusion problem with moving boundaries has been solved by Plesset and Zwick (Reference 16). The same authors have combined the results of the heat diffusion problem with the equations of motion for bubble growth to obtain a solution for the case of constant ambient pressure (Reference 17). Skinner and Bankoff (Reference 18) as well as Forster and Zuber (Reference 19) have taken slightly different approaches and obtained similar results. The bubble growth problem with variable ambient pressure and inclusion of terms containing $P_v$ is still unsolved.

2.3.5.1 Other investigators such as Stepanoff (References 20 and 21) and Jakobsen (Reference 22), rather than examine thermal cavitation on a microscopic basis, choose to derive semiempirical macroscopic descriptions. The results of this approach suffer from inability to correlate a particular value of pump head dropoff with volume of vapor present over a wide range of operating conditions. Furthermore, the results cannot justifiably be used to predict the vapor volume, because a number of assumptions and empirical factors do not accurately describe the vapor formation process.

2.3.5.2 Future work on thermal cavitation should take two directions. The first, referred to as the equilibrium approach, should examine the thermal cavitation phenomenon assuming thermal equilibrium phase changes. For pumps that have gradual changes in pressure through the system and low fluid velocities, the
equilibrium approach may give entirely satisfactory results; and, thereby, negate the need for a microscopic examination. The second method should be a microscopic examination of cavitation in which large pressure gradients and velocities produce metastable changes of phase. For cryogenic fluids, the effects of heat transfer across the boundaries of the system should be included in both methods of analysis. The thermodynamic properties of the fluid should be examined closely to determine whether certain characteristics simplify or complicate the solution of the thermal cavitation equations.

2.3.6 Fluid Mechanics - Like thermodynamics, there are many branches of fluid mechanics drawn on for examination of turbopump cavitation. The flow of liquid through a turbopump to the inception of cavitation can be examined with the incompressible flow equations. In the pump, cavitation may appear in a number of forms. If the leading edges of the impeller blades are very sharp, a separation cavity may be attached to the suction side of the blade. Depending on the fluid velocity, pressure, and boundary layer conditions, the cavity may close on the downstream suction side of the blade or may extend through the pump in a condition known as supercavitation. Examinations of separation cavities as related to turbopump cavitation have been made by Stripling and Acosta (References 23 and 24). This method predicts the geometry of the cavity up to the point of maximum height based on fluid momentum considerations; however, the reattachment or cavity closure conditions remain arbitrary. The work of Wade (Reference 25) applies to the conditions of cavities closing on the blade. The most recent and most elegant application of the Stripling and Acosta method is that developed by Davis, Coons and Schoer (Reference 26). This application
couples the Stripling-Acosta model to a two dimensional impeller flow field including boundary layer displacement effects. The primary objective of this model was to predict blade loading under cavitating conditions. Comparing its results with test data showed that it accomplished this purpose very well. Its use for predicting cavitation compliance is much more questionable due to a greater sensitivity to the assumed closure conditions. After documentation of the Stripling and Acosta work, different investigators have claimed that various pieces of experimental data (mostly photographic data, e.g., Reference 32) support either the separation cavity theory or the thermal equilibrium mixed flow concept. Our own review of this data suggests that it is inconclusive for the most part in offering substantial supporting evidence for either theory.

2.3.6.1 Unfortunately, the assumptions for the mathematical models, such as a blade leading edge radius of curvature equal to zero are seldom if ever physically realizable. If the blade is very thick and has a large leading edge radius of curvature, the flow may remain attached, and cavitation will appear as a mixture of vapor bubbles and liquid that demonstrates compressible flow characteristics. In this type of cavitation, the growth and collapse of vapor bubbles require fluid-mechanics examination. Plesset (References 17, 27 and 28) has examined the growth phenomena under conditions of constant ambient pressure and bubble vapor density. Both Gilmore (Reference 29) and Hunter (Reference 30) have taken into account compressibility and shock effects encountered during bubble collapse.

2.3.6.2 The effect of gas bubbles on the sonic velocity in a fluid represents another area that has received attention in
cavitation investigations. Figure 2.1 shows the variation in sonic velocity as a function of gas content for a mixture of water and air. It is seen that the sonic velocity can drop to a very low value that may result in sonic choking in the turbopump. Ghahremani (Reference 31) uses the work of Jakobsen (Reference 22) and assumes that fully choked flow exists at head breakdown (no pressure rise through the turbopump). Somewhat arbitrary assumptions, requiring empirical correlation, are then made to relate the fully choked conditions to normal operating conditions of unchoked or partially choked flow. The Ghahremani approach is unique in that the theory includes the effects of blade tip clearance backflow which, according to his results, produce more cavitation than occurs on the blade suction surface.

2.3.6.3 Attention in future fluid-mechanic investigations of cavitation should be given to determination of the conditions necessary for and which influence the geometry of separation cavities; and development of the flow equations for a vapor-liquid mixture in a turbopump. Incorporated into these equations should be the effects of bubble growth and collapse on the surrounding fluid. Also, the approach related to sonic choking at head breakdown should be refined.

2.4 A General Cavitation Analysis - From the preceding review of existing investigations of cavitation, it is possible to construct a plan for solution to the general cavitation compliance problem. This plan, which itemizes the various areas of investigation and integrates these areas into a completely general analysis, is outlined in the following paragraphs. The analytical investigations conducted during this program were restricted to fluid-mechanics with thermodynamic cavitation, which occurs on an equilibrium basis.
2.4.1 **Nucleation**

a. Number of nuclei;

b. Size distribution of nuclei;

c. Conditions in fluid or vapor that affect the dynamic behavior of nuclei.

2.4.2 **Acoustic Cavitation**

a. Identification of sound sources;

b. Assessment of power and frequency characteristics of each source;

c. Wave transmission in the fluid mechanic system.

2.4.3 **Diffusion**

a. Identification of dissolved gas species and assessment of their relative concentrations;

b. Evaluation of diffusion rates into nuclei or out of gas bubbles under both laminar and turbulent flow conditions;

c. Determine effects of contaminants on diffusion rates.

2.4.4 **Thermodynamics**

a. Examination of thermal cavitation on a microscopic or dynamic basis;

b. Examination of thermal cavitation on a thermal equilibrium basis;

c. Thermodynamic properties of fluids;

d. Heat transfer across system boundaries.

2.4.5 **Fluid Mechanics**

a. Hydrodynamics of turbomachinery without cavitation;

b. Hydrodynamics of turbomachinery with cavitation, but without separation cavities;

c. Bubble hydrodynamics;

d. Hydrodynamics of turbomachinery with separation cavities attached to blades;

e. Compressibility effects in fluid including shock phenomena;

f. Effects of tip clearance flow and backflow.
Figure 2.1 Variation of Isothermal Velocity of Sound in Water Containing Air Bubbles
3. Turbopump Cavitation Model
3. TURBOPUMP CAVITATION MODEL

3.1 Model Requirements - As outlined in Section 2.4, a complete analysis of turbopump cavitation compliance will require a complex model of the turbopump based upon the physical equations describing the fluid mechanic and thermodynamic phenomena occurring in the pump. The purpose of this program was to develop such a model; but, on a very fundamental basis, in order to evaluate the validity of such a model and identify the associated programming and numerical analysis problems. If the feasibility and engineering usefulness of a basic program could be demonstrated the more complex effects of fluid viscosity, gas diffusion, and non-equilibrium thermodynamics could be added to the program with considerably more confidence of success.

3.1.1 A prime objective of the cavitation compliance model development was to derive mathematical descriptions that could be related directly to the physical situation in a turbopump. Semi-empirical approaches were discarded because of their inability to account for all the different design considerations. This is particularly true considering the lack of any empirical data which relates changes in cavitation compliance to changes in specific turbopump geometry parameters. The required mathematical descriptions, which are consistent with the objectives and scope of this study, are:

a. Basic turbopump flow equations into which two-phase flow phenomena can be incorporated and which could later be expanded to include more complex flow situations;

b. A thermal cavitation model which is independent of time and conditions of nucleation and which can be
combined with the flow equations to give a description of turbopump cavitation compliance;

c. A finite difference iteration algorithm which allows solution of the flow and cavitation equations for any given blade geometry and flow conditions.

3.2 Model Assumptions - The assumptions made in the development of the turbopump model pertain to the cavitation process, the fluid-mechanics, and the turbopump configuration. The assumptions were required in order to obtain a solution for cavitation compliance within the scope of this study. The first two assumptions related to the cavitation process determine the basic approach of the analytical effort.

3.2.1 Channel Flow - The fundamental assumption of the turbopump model is that channel flow exists approximately between the pump inducer and impeller blades. This assumption can be used to separate the three-dimensional flow problems into two-dimensional problems. The first problem is that of defining the flow streamlines in the meridional plane. This can be accomplished by the method described in Appendix A, or by a more approximate method wherein the streamlines and associated streamtube width (b in Figure 3.2) is related to the inducer or impeller hub and shroud geometry by a suitable function. With a meridional plane description of the streamlines, one can proceed with the development of the blade to blade flow equations along a surface generated by rotating a meridional plane streamline about the impeller axis. This development is presented below in Section 3.3.

3.2.2 Thermal and Velocity Equilibrium - The liquid-vapor phase change is assumed to occur under isentropic conditions of
thermodynamic equilibrium with both phases in velocity equilibrium. An equation that deals with nonequilibrium changes of phase (vapor bubble growth) for a single bubble is presented in Appendix B. This derivation includes the effects of heat transfer at the bubble wall, varying ambient pressure, and variable density of the vapor within the bubble. Unfortunately, a complete solution to the resulting integro-differential equation was not obtained. Solutions were found in the literature for simplified versions of the equation; however, the solutions sacrificed the inertial effects to gain a description of the thermodynamic effects or vice versa. The assumed condition of equilibrium applies to the bubble growth and decay both as the fluid passes through the turbopump encountering different local pressures, and as the local pressures change as a result of changes in the turbopump inlet pressure. Some test results (Reference 4) at very low static inlet pressure (6 to 10 psi) and large pressure oscillation amplitudes (10 to 20 psi peak to peak) indicate that the cavitation process is not in equilibrium, and that the amount of compliance is a function of the frequency of the pressure oscillations. The extrapolation of this data to small amplitudes and flight pressures is not possible. Since this model is more concerned with cavitation compliance (rate of change of cavitation with respect to pressure) than with the amount of cavitation, the equilibrium assumption should be more valid because the period of pressure oscillation is greater than the average bubble life (Paragraph 3.2.5).

3.2.3 Tip Clearance and Backflow - An additional restriction which is implied by the channel flow assumption is that there is no tip clearance flow or backflow within the pump.
The model only computes cavitation as a result of channel flow between cascad ed blades and does not consider cavitation which may occur from either blade tip clearance flow or from backflow into the suction line (backflow is produced by tip clearance flow). These other sources may have a significant effect when considering unshrouded blades. Based on analysis, Ghahremani (Reference 31) theorizes that tip flow cavitation is much larger than blade cavitation. The only tests concerned with tip clearance flow evaluated the effect on performance for a gas medium (Reference 33) instead of a liquid, and for a variation in the axial clearance of the impeller tip (Reference 34) instead of a radial clearance of the inducer tip. A test program to investigate the effect of tip clearance flow and backflow would be very beneficial to the understanding of the complete turbopump cavitation process.

3.2.4 Incipient Cavitation - The model is developed for conditions of incipient cavitation only. This restriction is necessary because in deep cavitation local fluid velocities may approach the local speed of sound, whereupon the finite difference solution scheme becomes invalid. Since turbopumps do not usually operate in the region of deep cavitation, this is not considered to be a severe restriction.

3.2.5 Steady Flow - Since cavitation compliance is related to an oscillatory change in inlet pressure, unsteady flow conditions are implied. However, since the period of oscillation is typically 50 times greater than the time required for a fluid element to pass through the cavitation region, it is valid to assume quasi-steady flow; i.e., cavitation compliance can be obtained from a steady state solution at different turbopump inlet pressures.
3.2.6 **Inviscid Flow** - A further assumption which must be used is that the flow is inviscid. A solution of the complete viscous flow equations would compound the overall computational problems and is not warranted until the usefulness of the basic inviscid approach is demonstrated.

3.2.7 **Separation Cavities** - The computer model of turbo-pump cavitation was developed on the basis that no separation cavities were present in the blade system. This restriction is actually necessary only when the vapor phase is present in the pump. Solutions can be obtained with separation cavities for incompressible or non-cavitating flow; however, diverging solutions appear whenever two phase flow is encountered.

3.2.8 **Identical Blades** - The final assumption of the model development requires that all blades within the pump inducer or impeller are identical. This restriction was necessary in order to simplify the computer programming and to meet core limitations on the computer. In most pumps, the inducer section where most cavitation occurs is made up of identical blades. In the impeller section, however, partial blades are quite often placed between the main blades. The capability of treating non-identical blade systems can be added to the program, but would require an overlay technique and, consequently, some re-programming.

3.3 **Equation Development** - The development of the non-separated, thermal equilibrium cavitation flow equations for a blade-to-blade analysis are discussed below along with the pump blade coordinate transformations used to simplify the numerical solutions. Figure A.1 shows the coordinate system used for
derivation of the flow equations. The coordinate system is rotating about the Z axis with angular velocity \( \omega \). The velocities shown, therefore, are relative to the pump blades. The equation development derives two basic equations: a fluid flow equation, and an energy equation. The flow equation is derived from potential theory utilizing a continuity equation and the condition of irrotational flow. The energy equation is derived by relating the fluid energy to the inlet energy and the work done by the turbopump. Changes in thermal energy are obtained from the assumed condition of thermal equilibrium. The final form of the two equations is written in terms of the stream function, \( \psi \), and the density \( \rho \).

3.3.1 Flow Equation - From a solution to the flow problem in the meridional plane (Appendix A), a streamline and its associated streamtube can be defined as shown graphically in Figure 3.1. Rotation of the streamtube about the impeller axis results in a streamtube of revolution while a stream surface is generated by the meridional streamline. With reference to Figure 3.2, which shows a segment of the streamtube, the flow continuity equation is derived as follows. Using the segment of the streamtube as a control volume, the conservation of mass is expressed by

\[
\sum \left( \dot{W}_{\text{in}} - \dot{W}_{\text{out}} \right) = \frac{d}{dt} (W) = 0
\]

where \( \dot{W} \) is the flowrate. \( \frac{d}{dt} (W) \) is the time rate of change of the weight of fluid in the control volume, which is zero for steady flow. In the M direction,
In the $\theta$ direction,

$$\dot{W}_{in} - \dot{W}_{out} = \rho V_\theta b \frac{dr}{\sin \alpha} - \left[ \rho V_\theta + \frac{\partial}{\partial \theta} \left( \rho V_\theta \right) d\theta \right] b \frac{dr}{\sin \alpha}$$

$$= - \frac{\partial}{\partial \theta} \left( \frac{\rho V_\theta b}{\sin \alpha} \right) d\theta \ dr$$

Using the above relationship, Equation (3.1) becomes:

$$\rho V_M b \ dr \ d\theta + \frac{\partial}{\partial r} \left( \rho V_M b \right) dr \ d\theta + \frac{\partial}{\partial \theta} \left( \rho V_M b \right) dr^2 d\theta$$

$$+ \frac{\partial}{\partial \theta} \left( \frac{\rho V_\theta b}{\sin \alpha} \right) dr \ d\theta = 0$$

Dividing through by $r \ dr \ b \ \frac{r}{\sin \alpha}$, and taking the limit as $dr$ and $d\theta$ approach zero, Equation (3.4) becomes

$$\frac{\partial}{\partial r} \left( \rho V_M b \right) + \frac{\partial}{\partial \theta} \left( \frac{\rho V_\theta b}{\sin \alpha} \right) = 0$$

A stream function $\psi$ that satisfies Equation (3.5) is then defined by

$$\frac{\partial \psi}{\partial \theta} = \rho V_M b \ r$$

$$= - \frac{\rho V_\theta b}{\sin \alpha}$$

With the further assumptions that the fluid is inviscid and that its absolute motion is irrotational, another equation for
fluid motion can be derived. For absolute irrotational flow the circulation, \( \Gamma \), around the fluid segment (Figure 3.3) must be zero.

If \( \Gamma = 0 \), then \( d\Gamma = 0 \), or

\[
d\Gamma = 0 = \left[ \frac{\partial}{\partial r} \left( r\omega + V_\theta \right) r \theta \right] dr - \frac{\partial}{\partial \theta} \left( \frac{V_M}{\sin \alpha} \right) d\theta
\]

(3.8)

Differentiating:

\[
2r\omega + V_\theta + r \frac{\partial V_\theta}{\partial r} - \frac{\partial V_M}{\partial \theta} \frac{1}{\sin \alpha} = 0
\]

(3.9)

At this point, a transformation of the pump blade coordinates facilitates programming of the problem for computer solution. The transformation will depend on the blade shape. However, the objective of the transformation is to straighten the blade such that the leading edge becomes the maximum of the blade angular coordinates and the trailing edge the minimum. For an inducer a transformation such as \( dE = dZ \) and \( dF = d\theta \) may be the most appropriate. For an impeller having logarithmic spiral blades, the following transformation is most convenient.

\[
dE = \frac{1}{\sin \alpha} \frac{dr}{r}
\]

(3.10)

\[
dF = d\theta
\]

(3.11)

Carrying the equation development through, using this last transformation, Equation (3.9) becomes

\[
2r\omega + V_\theta + \frac{1}{\sin \alpha} \left[ \frac{\partial}{\partial E} \left( V_\theta \right) - \frac{\partial}{\partial F} \left( \frac{V_M}{E} \right) \right] = 0
\]

(3.12)
Combining Equations (3.6), (3.7) and (3.12) results in

\[ 2r_w \sin \alpha - \frac{\sin \alpha}{\rho_{br}} \frac{\partial \psi}{\partial E} + \frac{1}{\rho_{br}} \frac{\partial \rho}{\partial E} \frac{\partial \psi}{\partial E} + \frac{\sin \alpha}{b_{br}} \frac{\partial \psi}{\partial E} \]

\[ + \frac{1}{b_{br}} \frac{\partial b}{\partial E} \frac{\partial \psi}{\partial E} - \frac{1}{\rho_{br}} \frac{\partial^2 \psi}{\partial E^2} - \frac{1}{\rho_{br}} \frac{\partial^2 \psi}{\partial F^2} \]

\[ + \frac{1}{\rho_{br}} \frac{\partial \psi}{\partial F} \frac{\partial F}{\partial F} = 0 \] \hspace{1cm} (3.13)

which in turn reduces to

\[ 2r_w b_{br} \sin \alpha + \frac{\partial \psi}{\partial E} \left[ \frac{\partial}{\partial E} (\mathcal{L}_n \rho) \right] + \frac{\partial \psi}{\partial E} \left[ \frac{\partial}{\partial E} (\mathcal{L}_n b) \right] - \frac{\partial^2 \psi}{\partial E^2} - \frac{\partial^2 \psi}{\partial F^2} \]

\[ + \frac{\partial \psi}{\partial F} \left[ \frac{\partial}{\partial F} (\mathcal{L}_n \rho) \right] = 0 \] \hspace{1cm} (3.14)

From the meridional plane solution a relationship between \( \alpha \), \( b \), and \( r \) can be defined. Equation (3.10) can then be integrated to give \( E \) as a function of \( r \) with the condition that \( E = 0 \) at \( r = r_t \). Also, from Equation (3.11) \( F = \theta \). The relationship between the stream surface on which \( E \) and \( F \) lie and the \( r, \theta, Z \) coordinate system is shown for the general case in Figure 3.4. In the \( E,F \) plane, the pump blades are as shown in Figure 3.5. With a relationship between fluid density, stream function, and known inlet conditions, Equation (3.14) can be solved numerically in the \( E,F \) plane and the results transformed back to the \( r, \theta, Z \) physical plane.

3.3.2 Energy Equation - The completion of the solution to Equation (3.14) depends on a relationship between the fluid density, \( \rho \), the streamfunction, \( \psi \), and known pump inlet conditions. The energy equation for a steady flow fluid system such as a turbopump is given by
\[ h + \frac{V^2}{2g} = h_{o_u} + W \]  \hspace{1cm} (3.15)

where

\( h \) = static enthalpy/\( lb \)
\( V' \) = absolute fluid velocity
\( h_{o_u} \) = total enthalpy/\( lb \) at the pump inlet
\( W \) = work done on the fluid per pound of fluid/unit time

Equation (3.15) expressed in terms of components of the absolute velocity is

\[ h + \frac{1}{2g} \left[ \left( \omega r + V \theta \right)^2 + V^2_M \right] = h_{o_u} + W \]  \hspace{1cm} (3.16)

But the rate of work addition between a station, \( u \), upstream (where all flow properties are known) and the station being considered is equal to the rate of change in moment of angular momentum between the stations or

\[ W = \frac{\omega}{g} \left[ r \left( r \omega + V \theta \right) - r_u \left( r_u \omega + V \theta_u \right) \right] \]  \hspace{1cm} (3.17)

The quantity \( r_u \left( r_u \omega + V \theta_u \right) \) is commonly referred to as the pump prewhirl, which is either specified for the problem or is obtained from a viscous flow solution to the upstream flow problem. If it is assumed that the flow in the impeller undergoes isentropic changes of state and that for cavitation conditions the vapor and liquid phases are in thermal and velocity equilibrium, a relationship between pressure and average fluid density, \( \rho \), can be obtained from a state diagram for the working fluid. Referring to Figure 3.6, which represents a temperature-entropy diagram for a typical pump fluid,
the isentropic compression process might be represented by the vertical line CAB. Assuming the inlet properties of the fluid correspond to Point A, the fluid experiences a decreasing pressure as it enters the pump and ultimately reaches conditions corresponding to Point B in the vicinity of the blade leading edge. Cavitation is fully developed at this point. Downstream of the blade leading edge region, the work input to the pump goes into compressing the fluid that exists from the pump having properties corresponding to Point C. Using oxygen as the pump fluid and assuming that the flow process in the pump is isentropic and that velocity and thermal equilibrium exist throughout, the variation of density with pressure is shown in Figure 3.7. The data of the figure is based on a saturation temperature corresponding to 15 psia. The data from which the curve was derived were taken from Reference 35. Similar relationships can be obtained for different saturation temperatures as well as different fluids. Combining Equations (3.16), (3.17), and relating $h$ to $P/\rho$ yields

$$h \left( \frac{P}{\rho} \right) - \frac{(\omega r)^2}{2g} + \frac{1}{2g} \left( V^2 + V_M^2 \right) = \text{CONST} \quad (3.18)$$

where CONST = inlet energy conditions. Upon application of Equation (3.6), (3.7), (3.10) and (3.11), Equation (3.18) is transformed to its final form

$$h \left( \frac{P}{\rho} \right) + \frac{1}{2g} \left( \frac{1}{\rho Br} \right) \left[ \left( \frac{\partial \psi}{\partial F} \right)^2 + \left( \frac{\partial \psi}{\partial E} \right)^2 \right] - \frac{(r \omega)^2}{2g} = \text{CONST} \quad (3.19)$$

3.4 Solution Technique - In order to define the turbopump cavitation flow field, Equations (3.14) and (3.19) must be solved throughout the field between two blades of the pump and for a number of streamtubes selected from the meridional plane.
The results are then integrated throughout the pump to obtain the total cavitation compliance. The solution of Equations (3.14) and (3.19) is accomplished in a finite difference form on the CDC 6000 series computer. The problem is initiated by transforming the pump blades from the physical plane (Figure 3.4) to the E, F plane (Figure 3.5) through Equations (3.10) and (3.11). Next, a gridwork is established between the blades as shown in Figure 3.5. Equations (3.14) and (3.19) are written in finite difference form at each grid point and a relaxation method of solution employed. Solution of the problem is accomplished by specifying the upstream and downstream boundary conditions, assuming values of $ at each grid point within the boundaries, and checked to see if Equations (3.14) and (3.19) are satisfied at each point. If it is not, the left hand side of Equation (3.14) will be equal to a residual $R$. Then, values of $ at each grid point are systematically adjusted until the residuals are reduced to an acceptable level. Once this condition is reached, a solution is achieved. This initial solution may not correspond to the correct angular velocity on the upstream boundary. A scheme is included in the program for adjusting the upstream boundary and reapplying the relaxation solution until the correct value of angular velocity is obtained. A complete discussion of the relaxation method of solving systems of partial differential equations is given in Reference 36. The equations and solution technique described above have been developed into a computer program known as the turbopump cavitation flow program. User instructions for the computer model are given in Appendix D, and program listings are given in Appendix E. The analysis of the computer results required to obtain the cavitation compliance of the total turbopump is given in Section 5.1.
3.5 Model Applications - In addition to cavitation compliance, the turbopump model is capable of generating other information which is of interest in the analysis of turbopump response and the design of turbopump blades. Turbopump discharge dynamic pressure gain can be determined as a function of inlet pressure (pump gain, $\frac{\partial P_d}{\partial P_s}$), exit flow (pump resistance, $\frac{\partial P_d}{\partial \omega_d}$), and blade speed (speed gain, $\frac{\partial P_d}{\partial N}$). These parameters are also important in POGO stability analysis. Unlike cavitation compliance, there are currently methods available for estimating these parameters; however, the use of a cavitating turbopump model may result in a significant improvement. This model can also be used for design analysis of turbopump blades. This could include blade pressure loading, and the influence of blade shape on cavitation, separation, etc.
Figure 3.1 Meridional Plane of Pump Impeller

Figure 3.2 Segment Streamtube of Revolution for Continuity Equation
Figure 3.3 Segment Streamtube of Revolution for Irrotational Flow

Figure 3.4 E,F Surface Relative to r, Z and \( \theta \) Coordinates
Figure 3.5 Pump blades in E,F Plane

Figure 3.6 Pump Fluid Property Diagram
Figure 3.7  LOX Density for Isentropic Phase Change

Mixture Density \( \rho \) ~ lb/in^3

- P vap = 15 psi

Pressure ~ psi

LOX Density for Isentropic Phase Change
4. Empirical Cavitation Data
4. EMPIRICAL CAVITATION DATA

4.1 Test Data Analysis

4.1.1 Objectives - The objectives of the test data analysis are:

a. Determine the true cavitation compliance from all available test data on as many different turbopump configurations as possible;

b. Considering turbopump and propellant parameters which influence cavitation, attempt to present all the test data in a nondimensional correlated form;

c. Provide test results of specific turbopump configurations for verification of the analytical model.

Completion of the first objective will provide all currently available turbopump cavitation data in a single document. An empirical evaluation of the data, in terms of nondimensional parameters, is a parallel approach to the purely analytical turbopump cavitation model. The pump configurations selected for verification of the analytical model should meet the following requirements:

a. Accurate determination of cavitation compliance from test data;

b. Controlled and known test conditions;

c. The turbopump should be typical of those of interest in POGO analysis;
The turbopump should be consistent with the assumptions of the analytical model.

Considering these requirements, the J-2 LOX and F-1 LOX turbopumps were originally selected for model verification. In addition, the J-1 LOX and LR87 oxidizer pumps were selected for less detailed study. These selections were based on both compatibility with the analytical model and confidence in existing test data, as discussed in the following sections.

4.1.2 Data Sources - Table 4.1 shows all the different turbopump configurations for which cavitation data is known to exist. In all of these cases, cavitation data was derived from tests whose objectives were to determine the natural frequencies of the propulsion feed system for use in POGO analysis. Although turbopump cavitation usually has an important influence on feed system frequency, testing and data reduction was only concerned with determining an equivalent cavitation compliance. For determination of feed system frequency it was not required to separate the true cavitation compliance from other sources of mechanical compliance in the vicinity of the turbopump.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Stage</th>
<th>Engine</th>
<th>Oxidizer</th>
<th>Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>S-1B</td>
<td>H-1</td>
<td>LOX</td>
<td>RP-1</td>
</tr>
<tr>
<td>Saturn</td>
<td>S-1C</td>
<td>F-1</td>
<td>LOX</td>
<td>RP-1</td>
</tr>
<tr>
<td>Saturn</td>
<td>S-11/8-LVB</td>
<td>J-2</td>
<td>LOX</td>
<td>LH₂</td>
</tr>
<tr>
<td>Titan</td>
<td>1</td>
<td>LR87</td>
<td>N₂O₄</td>
<td>Aerozine 50*</td>
</tr>
<tr>
<td>Titan</td>
<td>11</td>
<td>LR91</td>
<td>N₂O₄</td>
<td>Aerozine 50*</td>
</tr>
<tr>
<td>Thor</td>
<td>1</td>
<td>MB-3</td>
<td>LOX</td>
<td>RP-1</td>
</tr>
</tbody>
</table>

* 50% Hydrazine and 50% UDMH
Test data related to this study comes from one of the following sources:

a. System tests with flowing propellant and an operating turbopump;
b. Feed system tests with non-flow propellant in the absence of an operating turbopump;
c. Feed line component tests on segments whose compliance cannot be accurately calculated;
d. Flight data.

Almost all of the available ground system test data is pulsed; i.e., the system response is measured relative to some known forcing function. The only test results which include the effects of turbopump cavitation are the flow system tests and the flight data. These test results required separation of the cavitation effects from other compliance effects. Some flow system tests have flight feed systems while others have facility feed systems; and some have hot firing engines while others are "bobtailed" (turbopump is driven in normal mode of operation but propellants are not mixed and burned in the main thrust chamber).

4.1.3 Determination of Cavitation Compliance From Natural Frequency - Cavitation compliance cannot be measured directly during a turbopump test and must be determined through use of analysis. A typical procedure for this determination is as follows:
a. Run a flow systems test;
b. Assume an analytical model of the test configuration;
c. Calculate, estimate, or determine from separate tests all model inputs except cavitation compliance;
d. Determine value of cavitation compliance for which model best fits test data.

Since cavitation compliance has its strongest influence on feed system natural frequency (as opposed to gain, damping, etc.) the above procedure is normally reduced to a correlation between test and analytical natural frequencies. These frequencies are a function of the inertance and compliance of the total system. For those unfamiliar with these hydraulic terms an analogy with a spring mass system is given in Figure 4.1. Inertance can be accurately calculated from the geometry of the feed line. The system compliance includes the distributed compressibility of the fluid and the radial flexibility of the suction line, axial flexibility due to a feed line area change, local flexibility of a line joint or bellows, and the compressibility of the cavitation vapor bubbles in the turbopump. Only the combined effect of all the system compliances can be determined from a dynamic systems test; thus, the line and fluid compliance must be known before cavitation compliance can be accurately determined. The distributed fluid and line compliance can be calculated fairly accurately. Local flexibilities can be determined from analysis, component tests, and/or system tests without the turbopump operating. Suction line bellows are local flexibilities which often represent a significant portion of the feed system compliance but cannot be determined accurately due to insufficient test data and a lack of analytical methods. A current technology contract (NAS8-25919)
should result in improved analytical methods for determination of all suction line elements. The exact equations relating cavitation compliance to natural frequency are a function of the analytical model used. The more representative the model, the more accurate the derived cavitation compliance. Two computer programs which were developed for Titan and Saturn V POGO analysis were modified for general test analysis in this study. These programs consist of: 1) a modal analysis program (Reference 37) for determination of natural frequencies for any distribution of line inertance and compliance; and 2) a transfer function program (unpublished) which utilizes the modal data to generate the transfer function of suction pressure per excitation as a function of excitation frequency. For a lightly damped system with negligible feed line and fluid compliance the cavitation bubble compliance, $C_b$, can be approximated by

$$C_b = \frac{1}{I \omega_1^2}$$

(4.1)

where $I$ is the feed system inertance and $\omega_1$ is the first natural frequency of the feed system. For a uniformly distributed suction line and fluid compliance, which yields an open-closed organ pipe frequency ($\omega_o$), and a lumped duct compliance near the pump inlet ($C_d$), the cavitation bubble compliance can be approximated by

$$C_b = \frac{1}{I \omega_1^2} \left[ \frac{(\omega_1/\omega_o)^2 - 1}{(\omega_1/2 \omega_o)^2 - 1} \right] - C_d$$

(4.2)
4.1.4 Determination of Natural Frequency From Test Data -
A fairly standard procedure for determining feed system
natural frequency from test data is as follows:

a. Configure a test set up which resembles the
    flight feed system as closely as possible;

b. Excite the feed system dynamic response with a
    measured forcing function;

c. Generate frequency domain perturbation transfer
    function (amplitude ratio and phase) of response
    per excitation from the measured results;

d. Natural frequency occurs near frequency of
    maximum amplitude ratio and phase shift of 90°.

For most of the test results analyzed herein, the excitation
has been some type of near sinusoidal wave pulsing of suction
flowrate. In this case the pulsor frequency has been changed
in steps (or in very slow ramp) and the resulting pressure
oscillations recorded at each frequency increment. This,
together with the measured excitation, yields a perturbative
transfer function of turbopump inlet pressure per excitation. Another type of excitation which has been successfully used
(Reference 38) is the random noise associated with the engine
combustion process. In this case auto- and/or cross-spectral
analysis is required to determine the frequency domain response
of the system. In most feed system configurations the first
natural frequency is very near the frequency at which the
amplitude ratio is maximum simultaneous with a phase shift of
43

90° for an appropriate feed system transfer function. In POGO analysis the most important transfer function is turbopump inlet pressure oscillation per turbopump acceleration perturbation \((\partial P_s/\partial g_p)\). Any transfer function which has the same natural frequency as \(\partial P_s/\partial g_p\) is an appropriate transfer function. Pulser acceleration and flow acceleration are legitimate excitations whereas transfer functions with respect to pulser pressure can yield significant errors in natural frequency. This effect is shown in the analysis of the J-2 test results. If additional information besides natural frequency (e.g. static gain and damping) are desired from the test results, it is required that a best fit between an analytical and test transfer function be obtained.

4.1.4.1 Test Frequency Correction - In reviewing test data two possible situations were recognized which could cause small errors in the test results. First is the determination of the system natural frequency from a transfer function of suction pressure per pulser pressure \((\partial P_s/\partial P_p)\) in lieu of suction pressure per pulser flow \((\partial P_s/\partial W_p)\). Second is the presence of facility lines which do not exist in the flight configuration. Both of these conditions existed on the S-II LOX line tests and each represents an error in frequency determination of about 5% in opposite directions. The effect of using the \(\partial P_s/\partial P_p\) transfer function is shown in Appendix C for a simplified system and results in a frequency which is 5% too high. The S-II LOX line test set up had a facility line running from the sump to the facility tank. This line was isolated from the suction line by a large accumulator at the sump; however, the residual effect of the facility line yields a system
natural frequency which is 5% too low. A model of the test and flight configuration is shown in Figure 4.2. A comparison of the $\frac{\partial P_s}{\partial P}$ transfer function for the test set up with the correct $\frac{\partial P_s}{\partial g}$ transfer function for the flight configuration is shown in Figure 4.3. Neither of the two possible discrepancies are known to exist in any other test data.

4.1.5 **Test Results** - No attempt has been made to duplicate previous analysis of pertinent test results. However, in many cases the analysis had to be extended in order to separate cavitation compliance from other sources of compliance. Also, in several instances independent tests on the same configuration produced conflicting results. In these cases, if a review of both results could not favor one over the other, the discrepancy was carried through the analysis and yields a tolerance on the results. Cavitation compliance is a function of many propellant and turbopump parameters; however, for a given configuration at a fixed operating point the only significant parameter which undergoes a planned variation during tests is turbopump inlet static pressure ($P_s$). Thus, the cavitation results presented in the following paragraphs are given either as a function of $P_s$ or as a non-dimensional form of $P_s$ defined by

$$K = \frac{P_s - P_v}{\sqrt[1/2]{\rho V_r}}$$

(4.3)

where

- $K$ = cavitation index
- $P_v$ = propellant vapor pressure
- $\rho$ = propellant mass density
- $V_r$ = inducer relative tip velocity
This is a convenient parameter since it has been found that for most turbopump configurations cavitation compliance is a linear function of a constant raised to a power which is proportional to \( K \), i.e., a straight line on semi-log graph paper. Whenever the scatter in the test data allows, this functional relationship will be observed in the presentation of the following test data. The data necessary for calculation of cavitation index is presented in Table 4.2.

4.1.5.1 F-1 LOX Cavitation Compliance - The F-1 LOX turbopump was tested with a S-IC outboard feed line and an outboard Arrowhead PVC (Pressure-Volume Compensator) duct. The feed line properties were taken from Reference 39. A LOX compressibility of \( 0.135 \times 10^6 \text{ psi} \) (\( T = -296\degree\text{F} \)) was calculated from velocity of sound data given in Reference 40. The compliance of the main line area change (above the prevalve) was investigated and found to be negligible. The PVC compliance was calculated from component test by Arrowhead, MSFC and Boeing; and from non-flow system dynamic tests by MSFC and Boeing. These data were then used in a model in which cavitation compliance was varied to match Bobtail (Reference 41) and Single Engine (Reference 42) tests results.

4.1.5.1.1 F-1 LOX PVC Component Test Results - The total PVC compliance, \( C_{\text{PVC}} \), includes the combined effect of the fluid compressibility and radial expansion (\( C_f \)) and axial flexibility of the upper and lower annulus area (\( C_x \)).
\[
C_{PVC} = \frac{\Delta V}{\Delta P} = \frac{d(pV)}{dp} = C_x + C_f
\]  
(4.4)

where

\[
C_x = \rho A \frac{\Delta X}{\Delta P} = \rho \frac{A^2}{K}
\]  
(4.5)

and

\[
C_f = \rho \int \frac{d\Delta}{dP} + \kappa \frac{d\rho}{dP}
\]  
(4.6)

\(C_f\) was calculated to be .011 in\(^2\) and is included as part of the distributed compliance of the food line model. The results of the component test are summarized in Table 4.1, and discussed below. Figures 4.4 and 4.5 contain unpublished results, supplied by MSFC, of PVC component tests run by Aerrowhead. From Figure 4.4 total PVC compliance can be calculated from \(dV/dP\); however, this result is considerably large compared with all other data. This can only be explained by the presence of air in the system. Two observations can still be made from these results. 1) \(dV/dP\) appears to be nonlinear with respect to pressure, and 2) \(dV/dP\) is approximately the same whether or not the PVC ends are restrained (this agrees with an analysis of the areas and spring rates). A more reliable estimate of PVC compliance can be made using Equation (4.5) and the results given in Figure 4.5. This result is independent of the amount of trapped air in the system.

Component tests run at MSFC (Reference 43) gave a value of \(dX/dP\) of .0056 (2 x .0028 for both the upper and lower annulus) for an Aerrowhead inboard PVC, and .007 for Flexonics outboard PVC. The results were very linear over the pressure range of 40 to 130 psig. The corresponding values of \(C_x\) are .060 and .075 in\(^2\) respectively. An equivalent PVC spring rate of \(K = 34250 \text{ lb/in}\) was determined from Boeing component tests on a Aerrowhead outboard PVC (Reference 39), and this was also
fairly linear over the pressure range of 50 to 125 psia. This yields (Equation 4.5) a Cx value of .081 in2. An analysis of the flexibility of supporting structure used in the Boeing water tests (Reference 39) showed that for the PVC installed in the line Cx could increase to .096 in2; however, it could never decrease below .081 (a completely rigid mounting of the upper spool and lower flange). Component tests (ΔX/ΔP) show that the outboard Aerrowhead and Flexonics PVC's have approximately the same compliance while inboard Aerrowhead PVC has about 25% less compliance.

4.1.5.1.2 F-1 LOX PVC Systems Test Results - Analytical feed system natural frequencies as a function of the sum of the cavitation bubble compliance (Cb) and the PVC annulus compliance (Cx) are given in Figure 4.6. Since these two compliances are located fairly close together, the frequencies are practically independent of the distribution between Cb and Cx. The second resonance of a pressure/pulser flow transfer function is shown to be a function of the location of the pulser line. Figure 4.7 gives MSFC dynamic test data with the turbopump isolated from the feedline. Some non-flow tests were run with a Flexonics outboard PVC and some with an Aerrowhead PVC; however, since the component tests indicate they have essentially the same compliance no attempt was made to differentiate between the two. A similar method of obtaining Cx from the Boeing Water Tests (Reference 39) yielded a Cx of .125 in2 at 80 psia and .086 at 140 psia. This compliance is high enough to suspect that there may be some air trapped in the system, which is always a possibility in non-flow tests.
4.1.5.1.3 F-1 LOX Cavitation Compliance - Figure 4.8 shows MSFC dynamic test results (References 41 and 42) with the turbopump running. Also shown is the sum of the cavitation and PVC annulus compliance, $C_b$ and $C_x$, required to make the analytical results match the test results. The resulting cavitation compliance is shown in Figure 4.9 for two different $C_x$ functions. One is the maximum $C_x$ variation as derived from a best fit of the MSFC non-flow tests (Figure 4.7). This variation is much more than can be justified by any component tests, and also results in a cavitation compliance which shows less variation with pressure than is expected. The other $C_x$ function used is a constant value of 0.081 in$^2$, as derived from some of the component tests. Since some of the component tests indicate some variation in $C_x$ with pressure, a constant value is probably conservative. The true cavitation compliance should lie between the limits shown in Figure 4.9.

4.1.5.2 F-1 Fuel Cavitation Compliance - The Rocketdyne evaluation of fuel pump inlet compliance (Reference 44) from the F-1 Bobtail Test results (Reference 41) are presented in Figure 4.10. Also shown are cavitation data derived from feed system frequencies obtained from the F-1 Bobtail tests and the S-IC Single Engine Test (Reference 42). The results of this analysis yield higher values of pump inlet compliance than were obtained by Rocketdyne. The variation in test frequency data presented in Figure 4.11 accounts for the lower values of pump inlet compliance analytically derived from the S-IC Single Engine Tests. The only differences between the Bobtail and Single Engine Test are that the Bobtail Test configuration had an outboard PVC and used discharge pulsing, whereas the Single Engine
Test configuration had an inboard PVC and used suction pulsing. Discharge pulsing should yield the same results as suction pulsing; however, obtaining good reliable feed system frequencies by discharge pulsing usually presents severe data reduction problems. The differences in results are larger than the anticipated differences due to changing PVC ducts. It was concluded that the S-IC Engine Test data was more reliable because it was derived from suction line pulsing, it is more recent data, and it showed less scatter. Since compliance derived from the S-IC Engine Test is less than that derived from the F-1 Bobtail Tests it should contain less PVC duct compliance. Since no PVC duct compliance was available it is assumed that turbopump inlet compliance derived from the F-1 Engine Tests is equal to cavitation compliance.

4.1.5.3 J-2 LOX Cavitation Compliance - Dynamic test data exists for four different J-2 LOX feed systems. They are the S-II inboard test facility feed system, the S-II outboard test facility feed system, the S-IVB test facility feed system, and the Rocketdyne turbopump test facility feed system. These tests were run for a range of turbopump inlet pressures and three different PU (propellant utilization) settings. As expected, the feed system natural frequencies (and thus turbopump cavitation) varied greatly with inlet pressure; however, the effect of PU setting was within the scatter of the data. Thus, for a given inlet pressure, reduced test results from all the different J-2 LOX feed systems should yield the same cavitation compliance. Two independent detailed analyses of test results have been performed. Brown Engineering analyzed the results of the S-II and S-IVB feed system tests (Reference 45) and Rocketdyne analyzed their test facility results.
(References 46 and 47). Both analyses yield approximately the same predominant resonances when related to common feed systems. This is shown in Figure 4.12 for the J-2 and S-IVB flight feed systems (which is somewhat different than the test facility feed systems). Brown Engineering assumed a single compliance turbopump model whereas Rocketdyne derived a turbopump flow impedance transfer function, G(S), from their test data which implies a dual compliance turbopump model. For the same test results (i.e., the same predominant feed system frequency) the two different turbopump models will yield different values of cavitation compliance.

4.1.5.3.1 J-2 LOX Duct Compliance - Also of importance is the amount of fluid and duct compliance present in the suction line. The Brown Engineering analysis derived a lumped compliance from dynamic tests which had the suction line isolated from the pump. Rocketdyne, on the other hand, coupled their test derived turbopump impedance function to flight suction line models and added suction line compliance until the analytical frequencies agreed with flight observed resonances. These results are compared in Table 4.5 and in both cases represent equivalent lumped values at, or near, the turbopump inlet. Both of these approaches are valid and should yield approximately the same results, whereas, the differences shown represent a very significant portion of the total feed system compliance.
4.1.5.3.2 J-2 LOX Turbopump Model - The single compliance model (see Figure 4.13a) used in the Brown Engineering analysis has a flow impedance given by

\[
G(S)_1 = \frac{P}{W_o} = \frac{R}{K_p} \left( \frac{\frac{L}{R} S + 1}{\left( \frac{CL}{K_p} S^2 + \frac{CR}{K_p} S + 1 \right)} \right) \tag{4.7}
\]

In this case the resistance, \( R \), and inductance, \( L \), were calculated; and the pump gain, \( K_p \), and the cavitation compliance, \( C \), were derived from test data. This impedance function and Brown Engineering duct compliance shown in Table 4.4 yield cavitation compliances shown in Figure 4.14.

The Rocketdyne analysis fit an impedance function to test data. The form required to give good correlation is given by

\[
G(S)_2 = \frac{P}{W_o} = \frac{K \left( \frac{S}{\omega_2} \right)^2 + \left( \frac{2 \omega_2}{\omega_2} \right) S + 1}{\left( \frac{S}{\omega_1} + 1 \right) \left[ \frac{S}{\omega_3} + \left( \frac{2 \omega_3}{\omega_3} \right) S + 1 \right]} \tag{4.8}
\]

One possible physical representation which gives this type of response is given in Figure 4.13b. In terms of the physical parameters, the impedance functions become

\[
G(S)_2 = \frac{P}{W_o} = \frac{K \left[ \left( \frac{LC_2 R_2}{R} \right) S^2 + \left( \frac{L + R_1 R_2 R_2}{R} \right) S + 1 \right]}{\left( \frac{LC_2 R_2}{K_p} \right) S^3 + \frac{C_1}{K_p} \left( L + R_1 R_2 C_2 \right) S^2 + \left( \frac{C_1 R}{K_p} + C_2 R_2 \right) S + 1} \tag{4.9}
\]
Equating coefficients of Equations (4.8) and (4.9) will yield the physical model parameters in terms of the frequency and damping parameters. Table 4.5 gives these results for the latest frequency and damping data given in Reference 47. In this model, $C_1$ is the main inlet cavitation compliance. It is the most important turbopump parameter in determining the predominate feed system frequency and is independent of pump gain ($K_p$). To illustrate the effect of the other turbopump parameters consider the case where the impedance beyond the cavitation compliance is very high. Then both of the above turbopump models approach an impedance function given by

$$G(S)_3 = \frac{1}{CS}$$

where $C$ is the cavitation compliance. Using Rocketdyne feed system frequencies (Figure 4.12) and duct compliances (Table 4.4) cavitation compliances for both $G(S)_2$ and $G(S)_3$ are shown in Figure 4.15. Although $G(S)_3$ is not a good pump model, the effect of different turbopump models is illustrated.

4.1.5.3.3 J-2 LOX Test Cavitation Compliance - The difference in the Brown Engineering and the Rocketdyne derived cavitation compliance can be attributed to different suction duct compliances and different pump impedance functions. There is considerable test data which indicates that a double compliance model, $G(S)_2$, is a better representation of the turbopump than a single compliance model, $G(S)_1$. From this point of view the Rocketdyne data should be more accurate; however, the Brown Engineering analysis is a more conventional approach which has been used on several other turbopump configurations.
For purposes of this study, both of these results (as shown in Figure 4.16) are considered to be equally valid. It is thus assumed that the true inlet cavitation compliance can be anywhere between these limits.

4.1.5.3.4 **S-II Flight Data** - AS-509 S-II Stage flight data (Reference 48) were reviewed and the observed LOX feed system oscillations compared with test results. Two things were evident from the flight data. First, the observed S-II outboard feed system contourgram frequency was higher than anticipated; and, second, there was an observed frequency change at engine mixture ratio (EMR) shift. Prior to EMR shift, the observed outboard frequency was approximately 33% higher than the predicted value which was based on S-II "bobtail" and J-2 test results for inlet NPSH in the vicinity of 60 ft. Furthermore, if the 65 to 75 Hz inboard suction pressure oscillation, observed during and after accumulator fill, represents a response of the second inboard line ("short stack") mode, this result is approximately 28% higher than would be analytically predicted. The following theories have been advanced by different Saturn V POGO analysts as to the reason for these apparent frequency discrepancies:

   a. The observed oscillation is not a natural frequency of the feed system but a 1/3 sub-harmonic of a 90 Hz turbopump self-induced oscillation;
b. the section line structure is such that the
pressure rise in the horizontal section does
not affect the feed system frequency on the
flight configuration.
c. For some unknown reason less turbopump cavitation
exists in the flight vehicle than in the ground
test.

The true cause remains unresolved; however, if it is due
to reduced cavitation compliance ($C_b$), the flight results for
both the S-11 inboard and outboard feed systems imply that $C_b$
is at least 70% lower than predicted from test data. This
possible discrepancy must be considered when utilizing the
test results. In the later portion of S-11 burn, the engine
mixture ratio (EMR) changed from 5.5 to 4.8. This results in
a predicted decrease in the flow coefficient ($\phi$) of 10%. A
reduced flow coefficient results in the turbopump inlet flow
entering the inducer blades with a larger angle of attack.
This should produce increased cavitation compliance and result
in a lower feed system frequency. The observed flight results
were just the opposite. When the EMR changed from 5.5 to 4.8
the S-11 outboard 105 contourgram frequency appeared to increase
from 25 to 31 Hz. The reason for this contradiction is unknown.

4.1.5.4 J-2 Fuel Cavitation Compliance - Turbopump cavitation
compliance for the J-2 axial flow fuel pump is shown in Figure
4.17. This data was derived from a Rocketdyne single compli-
ance math model (Reference 46). Since these tests were run at
a Rocketdyne test facility it is assumed that rigid suction lines were used, and thus the results do not contain any pump inlet duct compliance. An analytical model of the S-II outboard feed system was used to determine J-2 fuel pump inlet compliance from a frequency data point (9.5 Hz) supplied by MSFC for Saturn POGO analysis. Although present available data on fuel bellows compliance is incomplete, a value of 2.004 in$^2$ was assumed after a review of the J-2 oxidizer configuration and inlet duct compliance test values. The resulting pump inlet compliance derived from this datum falls within the scatter of pump compliance data derived by Rocketdyne as shown in Figure 4.17.

4.1.5.5 H-1 LOX and Fuel Cavitation Compliance - Brown Engineering and Rocketdyne derived values of pump inlet compliance (Reference 49) based on S-IB Bobtail Tests (Reference 50) are shown in Figure 4.18. The mathematical models differed only in the suction line representation while turbopump and discharge line representation were comparable. The characteristic resonant frequencies derived from the test data by Brown Engineering and Rocketdyne also differed since the method employed by each in the interpretation of the test data varied. Both suction line and discharge line pulsing data were available. The use of spectral analysis of the test results by Brown Engineering has shown that discharge line pulsing did not give the correct characteristic frequencies (Reference 51). Lack of data points for the fuel pump inlet compliance in the Brown Engineering analysis is due to the inability of their data reduction procedure to always determine values of feed
system frequencies. Since Rocketdyne did not indicate this to be a problem, their procedure was considered more reliable and will be used exclusively in this analysis. The Bobtail test configuration does not have any suction line bellows located at the pump inlet. Since there are small line bellows located at several points in the fuel and oxidizer lines, it is assumed that their compliance is accounted for in the distributed compliance of the suction lines. Thus, for the H-1 feed systems, the derived pump inlet compliance shown in Figure 4.18 is assumed to result exclusively from turbopump cavitation.

4.1.5.6 MB-3 LOX and Fuel Cavitation Compliance - MB-3 cavitation compliance data was obtained from an Aerospace Corporation evaluation of feed system frequencies on the THOR vehicle (Reference 52). These data are shown in Figure 4.19 as a function of cavitation index (K). They are presented in this report for reference only, as MB-3 turbopump geometry and operating parameters were not available to permit evaluation of the data.

4.1.5.7 LR87 and LR91 Oxidizer and Fuel Cavitation Compliance - Cavitation compliance of the Titan Stage I and II turbopumps is shown in Figure 4.20. This data was determined by combined Martin Marietta Corporation and Aerospace Corporation analysis of pulsed and/or non-pulsed hot firing engine tests, bobtailed turbopump tests, suction line non-flow tests, and flight data. The final results have evolved over several evaluations (particularly in the case of the LR87 data) and no concise documentation exists. The Martin results, presented here, agree closely (except for a density scale factor) with the
Aerospace results given in Reference 31. The LR87 oxidizer feed system contains the largest amount of non-cavitation compliance. In this case the suction line distributed compliance was calculated and compared with non-flow test results (unpublished results of Martin Marietta Corporation tests). These results indicated that the line bellows located near the pump inlet contain very little compliance. This is assumed to be true for the other Titan lines which use similar line bellows.

4.1.6 General Test Data Assessment - The preceding cavitation compliance test data generally have large uncertainties associated with the results. In most cases this can be related to the fact that the objective of these tests was to determine feed system frequency, not cavitation compliance. In several cases the results even show large dispersions in feed system frequency for a given test series and unexplained disagreement between results of different tests of the same feed system. Assuming the feed system frequency is accurately known the following error sources exist for determining cavitation compliance.

a. Unknown feed line compliance;

b. Frequency insensitive to cavitation compliance, conversely cavitation compliance is very sensitive to frequency dispersions;

c. Unknown turbopump model (test data will not fit a physical model).

In addition to dispersions in the results there are unknowns associated with parameters which affect the amount of cavitation which occurs. Some of these unknown factors are:
4. Attest distribution at the turbopump inlet;
5. Propellant back flow into the suction line;
6. Propellant pressure at the inlet;
7. Amount of absorbed gas in the propellant;
8. Inlet propellant temperature at the turbopump inlet;

The accumulated effect of all the unknowns must be carefully considered when judging the merits of either an empirical or analytical prediction technique.

4.2 Empirical Data Evaluation

4.2.1 Influential Parameters - in order to evaluate all the different turbopump cavitation compliance test data one has to postulate as to what are the important variables, and attempt to group these into non-dimensional parameters which will yield the same cavitation compliance for all existing turbopump configurations. If this could be accomplished with some degree of success, it would provide a method for predicting the amount of cavitation compliance that will occur on a new turbopump design. The turbopump operating and geometry parameters, and propellant variables which could affect the amount of turbopump cavitation are:

\[ P \] = pump inlet static pressure
\[ \dot{m} \] = propellant flow rate
\[ H_i \] = inducer head rise
\[ N \] = inducer speed
\[ \alpha_i \] = inducer angle of attack
4.2.2 Non-Dimensional Parameters - Ideally there exists a non-dimensional combination of parameters which uniquely describes a cavitation parameter as a function of operating and configuration parameters for all conditions and configurations. The most generally used non-dimensional cavitation parameter is cavitation index, K (Equation 4.3), which combines $P_s$, $P_v$, and $V_r$. The data required to compute cavitation index is given in Table 4.2 for several turbopump configurations. All of the available cavitation compliance data is shown in Figure 4.21 vs cavitation index. When two sources of equally valid
data exist, an average of the two values was assumed for comparison with other data. With a set of data exhibiting the same trend and lies within an order of magnitude band, the variation is much too large to eliminate the need for obtaining cavitation test data on a new turbopump configuration. Turbopump size can be easily accounted for by non-dimensionalizing cavitation compliance ($C_b$) with respect to size variables.

$C_b/\eta_1 D_1^2$ vs $\xi$ is shown in Figure 4.22. This in general narrows the band of data except for the LR-91 oxidizer data. The inducer leading edge angle of attack, assuming no propellant pressure swirl at the pump inlet, can be calculated by

$$\alpha_i = \beta_i - \tan^{-1} \phi$$ (4.4)

where $\beta_i$ is the inducer blade angle and $\phi$ is the flow coefficient.

Comparing the values of $\alpha_i$ given in Table 4.2 with the cavitation data given in Figure 4.21 and 4.22 shows no particular correlation. This may be partly due to the fact that all of the LR series turbopumps have cambered inducers which operate at near zero leading edge angle of attack. All the other turbopumps have flat inducer blades which require a leading edge angle of attack to generate a pressure rise. Thus, $\alpha_i$ is not a good universal cavitation measurement parameter.

A turbopump performance parameter, pump specific speed (SS), defined in non-dimensional form as

$$SS = 8136 \left[ 1 - \frac{D_i^2}{D_h^2} \right]^{1/2} \phi^{1/3} \left[ \frac{\xi}{\sigma} \right]^{3/4}$$
was considered as a means for correlating cavitation compliance. Figure 4.23 shows non-dimensional cavitation compliance (related to inducer inlet area $D_i^2 - D_h^2$) as a function of $1/SS$. Comparison of Figures 4.22 and 4.23 shows that $1/SS$ is not significantly better than $K$. Thor(MB-3) data is not shown because the necessary geometry parameters were not known. Information on other influential parameters given in the preceding paragraph was not obtained for enough turbopump configurations to permit non-dimensional evaluation of their affect. However, it is doubtful if the existing spread in data shown in Figures 4.22 and 4.23 can be significantly reduced. Simple non-dimensional parameters cannot account for such important affects as blade shape, flow separation, and propellant phase change. The non-dimensional data presented here could be used to predict an order of magnitude cavitation compliance on a new turbopump configuration.

4.2.3 Effect of Inlet Pressure - It is of interest to note what the functional relationship is between inlet pressure ($P_s$) and cavitation compliance ($C_b$) as observed from the test data. Assuming that

$$C_b = \text{Constant} / P_s^n$$

then

$$n = - \frac{P_s}{C_b} \frac{\partial C_b}{\partial P_s} = - \left( K + \frac{P_s v}{q} \right) \frac{\partial C_b}{\partial K}$$
where  \[ K = \frac{(P_s - P_v)}{q} \]

and  \[ q = \rho v_r \frac{\partial}{2g} \]

Measuring average values of \( K, C_b, \) and \( \frac{\partial C_b}{\partial K} \) from the test results (Figure 4.21), values of "n" were calculated (see Table 4.6). The results show that for the Aerojet turbopumps, "n" fall in the range of 2 to 4. For several other turbopumps, "n" falls in the range of .5 to 1. Due to scatter in the test data, there is a fairly large tolerance associated with \( \frac{\partial C_b}{\partial K} \); however, if the trend is correct, these results imply that cavitation compliance, in different turbopumps, is proportional to different powers of \( P_s \). This indicates that an analytical derivation of cavitation compliance, in terms of average flow field parameters, cannot yield good agreement with test results for all configurations. The work of F. Ghahremanzadeh (Reference 31) indicates that blade cavitation is inversely proportional to \( P_s^2 \), while backflow cavitation is inversely proportional to \( P_s^3 \). For the Aerojet turbopumps, this formulation should show good slope agreement (which it does) with test results; however, poor slope agreement could result for some of the other pumps. Current studies are being performed under Contract NAS8-27731 to evaluate this approach with respect to additional turbopump configurations (Reference 56).
Table 4.2  
Turbopump Data

<table>
<thead>
<tr>
<th>Vehicle Stage</th>
<th>TITAN</th>
<th>SATURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAGE I</td>
<td>OXID</td>
<td>F1</td>
</tr>
<tr>
<td></td>
<td>FUEL</td>
<td>OXID</td>
</tr>
<tr>
<td>Engine Identification</td>
<td>LR-87</td>
<td>LR-91</td>
</tr>
<tr>
<td>Mixture Ratio</td>
<td>1.917</td>
<td>1.67</td>
</tr>
<tr>
<td>Propellant</td>
<td>$N_2O_4$</td>
<td>$N_2O_4$</td>
</tr>
<tr>
<td>Density ($\rho$) lb/in$^3$</td>
<td>0.0525</td>
<td>0.033</td>
</tr>
<tr>
<td>Suction Line Area in$^2$</td>
<td>37.2</td>
<td>27.2</td>
</tr>
</tbody>
</table>

Pump Dimensional Parameters

- Diameter-Inducer Eye ($D_i$) in: 7.10 6.64 5.10 3.80 15.75 15.71 6.75 7.80 7.60 6.12
- Diameter-Inducer Hub ($D_h$) in: 2.15 2.24 1.73 0.84 3.51 6.61 1.375 2.93 2.0 2.00
- Area Inducer Inlet in$^2$: 36.0 30.7 18.0 10.8 185.0 159.6 34.3 41.0 42.3 26.3
- Design Clearance ($\ell$) in: 0.045 0.045 0.045 0.045
- Number of Inducer Blades (n): 3 3 3 3 3 4 3 4 4 4
- Diameter of Impellertip in: 9.42 10.75 8.75 4.93 19.50 23.42 10.20 Axial 11.0 13.75
- Blade Angle ($\beta$) deg. at tip: 5.7 5.7 5.7 5.7 9.0 8.6 9.9 7.0 11.3 10.4

Pump Performance Parameters

- Pump Speed (N) RPM: 8350 9175 8366 23576 5550 5550 8050 25800 6750 6750
- Inducer Tip Velocity ($V_{i}$) ft/sec: 258 266 186 390 381 380 237 876 224 180
- Propellant Velocity (U) ft/sec: 23.0 22.4 17.2 27.3 41.4 30.6 22.9 62.3 24.7 26.6
- Relative Velocity$^2$ ($V_{r}^2$) ft$^2$/sec$^2$: 67093. 71258. 34892. 152845. 146875. 145336. 56693. 771257. 50787. 32943.
- Flow Rate (W) lb/sec: 522.55 272.57 195.05 116.9 3765. 1697. 386. 78. 514.4 243.
- Flow Coefficient ($\phi = U/V_i$): 0.089 0.084 0.093 0.070 0.109 0.081 0.097 0.071 0.110 0.1475
- Inducer Tip Angle of Attack ($\alpha_i$) deg.0.6: 0.9 0.4 1.7 3.6 4.0 5.1 3.7 6.5 3.1
Table 4.3
PVC Annulus Compliance \( (C_x \approx \text{in}^2) \)

<table>
<thead>
<tr>
<th>Test Conductor</th>
<th>Source</th>
<th>Press=80psia</th>
<th>Press=140psia</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Aerrowhead Inboard PVC)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aerrowhead</td>
<td>( \Delta X/\Delta P )</td>
<td>.073</td>
<td>.046</td>
</tr>
<tr>
<td>MSFC</td>
<td>( \Delta X/\Delta P )</td>
<td>.060</td>
<td>.060</td>
</tr>
<tr>
<td>MSFC</td>
<td>1</td>
<td>.062</td>
<td>.060</td>
</tr>
</tbody>
</table>

| (Aerrowhead Outboard PVC) |              |              |               |
| Aerrowhead              | \( \Delta X/\Delta P \) | .085         | .060          |
| Boeing                  | \( \Delta X/\Delta P \) | .081         | .081          |
| Boeing                  | 2            | .096         | .096          |
| Boeing                  | 1 3          | .125         | .086          |
| MSFC                    | 1            | .068         | .025          |

| (Flexonics Outboard PVC) |              |              |               |
| MSFC                    | \( \Delta X/\Delta P \) | .075         | .075          |

1 Match to no flow dynamic test results.
2 \( \Delta X/\Delta P \) results plus mounting flexibility.
3 Possibility of air in the system.
Table 4.4

J-2 LOX Suction Duct and Fluid Compliance

<table>
<thead>
<tr>
<th>Feed System</th>
<th>Compliance (in²)</th>
<th>Brown Eng* (Reference 45)</th>
<th>Rocketdyne** (Reference 47)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-II Inboard</td>
<td>.0112</td>
<td>.0031</td>
<td></td>
</tr>
<tr>
<td>S-II Outboard</td>
<td>.0077</td>
<td>.0055</td>
<td></td>
</tr>
<tr>
<td>S-IVB</td>
<td>.0055</td>
<td>.0015</td>
<td></td>
</tr>
</tbody>
</table>

* Located approximately 10" above turbopump inlet

** Located at turbopump inlet

Table 4.5

J-2 LOX Physical Model Parameters For $G(S)_2$

<table>
<thead>
<tr>
<th>NPSH</th>
<th>$P_s$</th>
<th>$C_1$</th>
<th>$C_2 Kp$</th>
<th>$R_1 / Kp$</th>
<th>$R_2 / Kp$</th>
<th>$L / Kp$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>33.7</td>
<td>.0189</td>
<td>.0158</td>
<td>.418</td>
<td>3.47</td>
<td>.016</td>
</tr>
<tr>
<td>45</td>
<td>36.2</td>
<td>.0175</td>
<td>.0173</td>
<td>.371</td>
<td>3.18</td>
<td>.014</td>
</tr>
<tr>
<td>50</td>
<td>38.1</td>
<td>.0142</td>
<td>.0163</td>
<td>.376</td>
<td>3.29</td>
<td>.014</td>
</tr>
<tr>
<td>55</td>
<td>41.1</td>
<td>.0134</td>
<td>.0140</td>
<td>.346</td>
<td>3.22</td>
<td>.014</td>
</tr>
<tr>
<td>60</td>
<td>43.6</td>
<td>.0123</td>
<td>.0119</td>
<td>.318</td>
<td>3.15</td>
<td>.015</td>
</tr>
<tr>
<td>65</td>
<td>46.0</td>
<td>.0102</td>
<td>.0112</td>
<td>.304</td>
<td>3.23</td>
<td>.014</td>
</tr>
<tr>
<td>70</td>
<td>48.4</td>
<td>.00813</td>
<td>.0105</td>
<td>.327</td>
<td>3.22</td>
<td>.014</td>
</tr>
</tbody>
</table>
Table 4.6  
Suction Pressure Power

<table>
<thead>
<tr>
<th>Turbopump Configuration</th>
<th>Average Test Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_b$</td>
</tr>
<tr>
<td>LR87 Fuel</td>
<td>.019</td>
</tr>
<tr>
<td>LR87 Ox</td>
<td>.014</td>
</tr>
<tr>
<td>LR91 Ox</td>
<td>.027</td>
</tr>
<tr>
<td>F-1 Fuel</td>
<td>.088</td>
</tr>
<tr>
<td>F-1 Ox</td>
<td>.095</td>
</tr>
<tr>
<td>J-2 Fuel</td>
<td>.019</td>
</tr>
<tr>
<td>J-2 Ox</td>
<td>.011</td>
</tr>
<tr>
<td>H-1 Fuel</td>
<td>.012</td>
</tr>
<tr>
<td>H-1 Ox</td>
<td>.014</td>
</tr>
<tr>
<td>MB-3 Fuel</td>
<td>.0023</td>
</tr>
<tr>
<td>MB-3 Ox</td>
<td>.027</td>
</tr>
</tbody>
</table>
\[ (m S^2 + dS + k) x = F \]  \hspace{1cm} (2)

\[ W = \rho A \dot{x} \]
\[ mg_c = \rho A \dot{l} \]
\[ F = PA \]  \hspace{1cm} (1)

Substitute (1) into (2) or (3) gives:

- Inertance, \( I = \frac{m}{\rho A^2} = \frac{\ddot{x}}{\rho g_c} \)
- Resistance, \( R = \frac{d}{\rho A^2} \)
- Compliance, \( C = \frac{\rho A^2}{k} \)

where

- \( A \) = line area,
- \( \dot{x} \) = line length,
- \( g_c \) = gravitational constant,
- \( \rho \) = fluid density.

Figure 4.1 Comparison of Spring-Mass and Fluid Systems
Figure 4.2 Analytical Model of S-II LOX Test Suction Systems
Inboard PVC (Unrestrained)

Conversion: 16.39 milliliters/in$^3$

Figure 4.4 S-IC LOX PVC Volume Change With Pressure
Aerrowhead Test Data

- □ Inboard PVC
- △ Outboard PVC

Figure 4.5  S-IC LOX PVC Length Change With Pressure
Figure 4.6  S-IC LOX Feed System Frequency Variation
Figure 4.7  S-IC LOX Non-Flow Feed System Data
Figure 4.8  S-IC LOX Feed System Data
Figure 4.9  F-1 LOX Turbopump Cavitation Compliance

PVC Annulus Compliance Varied To Match MSFC Non-flow Dynamic Test Data

PVC Annulus Compliance = 0.081 (Constant)
Figure 4.10  F-1 Fuel Turbopump Cavitation Compliance
Figure 4.11  F-1 Fuel Feed System Frequency Data
Figure 4.12  J-2 LOX Feed System Resonance
Figure 4.13  J-2 LOX Analytical Turbopump Models
Figure 4.14  J-2 LOX Cavitation Compliance Derived by Brown Engineering
Figure 4.15  Effect of J-2 LOX Turbopump Model on Cavitation Compliance
Figure 4.16  J-2 LOX Cavitation Compliance
Figure 4.17  J-2 Fuel Turbopump Cavitation Compliance
Figure 4.18 H-1 Oxidizer and Fuel Turbopump Cavitation Compliance
Figure 4.19 MB-3 Cavitation Compliance
Figure 4.20 Titan Turbopump Cavitation Compliance
Figure 4.21 Oxidizer and Fuel Turbopump Cavitation Compliance
Figure 4.22 Nondimensionalized Turbopump Cavitation Compliance
Figure 4.23  Nondimensionalized Turbopump Cavitation Compliance
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5. Empirical-Analytical Correlation
5. EMPIRICAL - ANALYTICAL CORRELATION

5.1 Analytical Results - The turbopump cavitation model described in Section 3. was used to analyze the J-2 LOX, F-1 LOX, H-1 LOX, and LR87 oxidizer turbopumps. These four configurations were selected because they are typical of those of interest in the determination of cavitation compliance for POGO analysis. Also, considerable test data exists for these configurations and their geometry and operation are in reasonable agreement with the assumptions of the analytical model. Because of the large pressure rise which occurs through the inducers (for example, 100 psi through the J-2 inducer and 300 psi through the F-1 inducer) it is assumed that any mismatch between the inducer exit flow and the impeller blades is not sufficient to produce a significant amount of cavitation in the impeller. For this reason only the inducers were analyzed.

5.1.1 Blade Section Analysis - The complete analysis of an inducer requires that the computer model be run for different streamsheets corresponding to different blade cross sections at different inducer radii. These results are then interpolated and integrated to yield the total cavitation compliance for all the inducer blades from the hub to the tip. Initial analyses performed for the J-2 and F-1 inducers employed the blade sectional data as tabulated on the inducer design drawings (Figures 5.1 and 5.2, respectively). This data defines the blade geometry for a constant distance off the inducer hub, and is normally only given for a few blade sections. The turbopump cavitation compliance program was thus restricted by both the limited amount of input data and the fact that the data supplied for a given blade section was associated with a constant distance off the inducer hub, and not a fixed percent of the distance between the hub and the feedline wall. The results of these
analyses indicated significantly different behavior of the cavitation phenomenon between the hub and blade tip, and did not agree favorably with test data. Recent analyses performed for all four inducers have utilized a different form of input blade geometrical data. A computer program was written to interpolate the supplied data and calculate blade geometrical data for five blade sections located at 10%, 30%, 50%, 70%, and 90% of the blade span. Details of this computer program are given in Appendix F. This simulation improvement resulted in an order of magnitude reduction in predicted cavitation.

5.1.1.1 Figures 5.5 through 5.8 show the blade sectional data derived from the inducer design drawings (Figures 5.1 through 5.4) for the J-2 LOX, F-1 LOX, H-1 LOX, and LR87 oxidizer turbopumps. It is noted that this data has been normalized to the trailing edge. Figures 5.9 through 5.12 show the results of interpolating this data to five blade sections, each at a constant percent of blade span. As shown by these figures, \( \Delta Z/\Delta \theta \) is constant and independent of \( r \) for the J-2, F-1, and H-1 inducers. These inducers are a constant pitch helical screw design and are symmetrical about the chord. In contrast, the LR87 inducer is a twisted flat plate cambered in the vicinity of the leading edge.

5.1.2 Calculation Procedure - Because most of the inducers analyzed exhibit a linear relationship between blade coordinates \( Z \) and \( \theta \), the most suitable transformation from the inducer coordinates \( (r, \theta, Z) \) to the streamsheet coordinates \( (E, Z) \) is simply

\[
E = Z \quad (5.1)
\]

\[
F = \theta \quad (5.2)
\]
This transformation is used in lieu of the logarithmic spiral transformation, Equations (3.10), which is more suitable for the impeller portions of a turbopump. For channel flow the streamsheet width varies as a function of $Z$ and is defined by

$$b(Z) = b(Z_o)(r_t(Z) - r_h(Z))/C \quad (5.3)$$

where:
- $r_t$ = the blade tip radius
- $r_h$ = the hub radius
- $b(Z_o)$ = selected streamsheet width at $Z_o$
- $C = (r_t(Z_o) - r_h(Z_o))$

The feedline axial velocity, $U$, can have any radial distribution provided that continuity is satisfied, i.e.,

$$\dot{W} = 2 \pi \rho_{\ell} \int_{r_h}^{r_t} U r \, dr \quad (5.4)$$

where $\rho_{\ell}$ is the inlet fluid density. For a uniform inlet fluid velocity distribution the flow in a streamsheet annulus is given by

$$\dot{W}_{ss} = 2 \pi r(Z_o) b(Z_o) \rho_{\ell} U \quad (5.5)$$

For this analysis, a constant value of $b(Z_o)$ was used for all streamsheets in each inducer, causing $\dot{W}_{ss}$ to vary as a function of radius. Other local angles such as the blade angle, $\beta$, the inlet flow angle, $\phi$, and the angle of attack relative to the upstream undisturbed flow, $\alpha$, are defined by
\[ \beta = \tan^{-1} \left( \frac{1}{r} \frac{\Delta Z}{\Delta \theta} \right) \quad (5.6) \]

\[ \phi = \tan^{-1} \left( \frac{1}{r} \frac{U}{\omega} \right) \quad (5.7) \]

\[ \alpha = \beta - \phi \quad (5.8) \]

In the \((\xi, \eta)\) coordinate system the blade angle is \(\Delta Z/\Delta \theta\) and the flow angle is \(U/\omega\), both of which tend to be independent of the radius or of which streamsheet is under consideration. All of these parameters are given in Table 5.1. Each pump inducer was analyzed for two inlet flow angles, \(U/\omega\), and a range of inlet pressures, \(P_s\), for the streamsheets described above. Each calculation produced a description of the flow field in the particular streamsheet in terms of the streamfunction, \(\psi\), pressure field, \(P\), and the weight of propellant in the streamsheet, \(W_{ss}\). Figure 5.13 shows a computer output plot of the streamlines in a J-2 LOX inducer streamsheet which corresponds to a 30% blade section. For the purpose of calculating cavitation compliance the prime model output is \(W_{ss}\), which is computed by

\[ W_{ss} = \sum_i \rho_i A_i b_i \quad (5.9) \]

where

- \(\rho_i\) = density of the two phase fluid at grid point \(i\)
- \(A_i\) = area between grid points
- \(b_i\) = streamsheet width at grid point \(i\)

From Equation (1.2) the cavitation compliance in a streamsheet...
between two blades is given by

\[ C_{ss} = \frac{\Delta W_{ss}}{\Delta P_s} \]  \hspace{1cm} (5.10)

An example of \( C_{ss} \) derived from computer output is shown in Table 5.2. The total turbopump cavitation compliance for \( N \) blades is given by

\[ C_b = N \int_{r_h}^{r_t} \frac{\partial C_{ss}}{\partial r} \, dr \]  \hspace{1cm} (5.11)

where \[ \frac{\partial C_{ss}}{\partial r} = \frac{C_{ss}}{b} \]  \hspace{1cm} (5.12)

Since most of the cavitation occurs near the inlet; \( b = b(Z_o) \) which was chosen to be the same at each radius section. Equation (5.11) thus becomes

\[ C_b = \frac{N}{b(Z_o)} \int_{r_h}^{r_t} C_{ss} \, dr \]  \hspace{1cm} (5.13)

5.1.3 J-2 Results - Analytical values of \( W_{ss} \) were obtained from the computer model for inlet pressures from 32 to 50 psia, values of \( U/\omega \) (inlet flow direction) of .33 and .20, and five blade sections. The resulting streamsheet compliance is shown in Figure 5.14 for five inlet pressures and the nominal flow direction of \( U/\omega = .33 \). These results are relatively well be-
have and exhibit the expected trend with variations in inlet pressure. Also, the variation along the blade appears to be reasonable in view of the following factors:

a. For constant streamsheet inlet thickness the tip streamsheet has a larger fluid flow (Equation 5.5);
b. The blades are thinner at the tip which results in a sharper leading edge;
c. The blades are thinner at the tip which also results in less venturi effect between the blades;
d. The angle of attack at the tip is lower than at the hub (Equations 5.6, 5.7, and 5.8).

The first two factors would tend to produce higher streamsheet compliance at the tip than at the hub while the last two factors have the opposite effect. The graphical integration of these results (Figure 5.14), along with similar results for an inlet flow direction of \( U/\omega = .20 \), according to Equation (5.13) give the following values of cavitation compliance.

<table>
<thead>
<tr>
<th>( U/\omega ) (in/rad)</th>
<th>0.33</th>
<th>0.20</th>
<th>Inlet Press (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavitation Compliance ( \sim ) in²</td>
<td>0.0044</td>
<td>0.0063</td>
<td>33.</td>
</tr>
<tr>
<td></td>
<td>0.0033</td>
<td>0.0053</td>
<td>36.</td>
</tr>
<tr>
<td></td>
<td>0.0023</td>
<td>0.0039</td>
<td>40.</td>
</tr>
<tr>
<td></td>
<td>0.0012</td>
<td>0.0023</td>
<td>44.</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0014</td>
<td>48.</td>
</tr>
</tbody>
</table>

5.1.3.1 The sensitivity to inlet flow direction requires consideration of the factors involved. The actual analytical inlet flow direction is determined from the slope of the computed streamlines upstream of the blades. The upstream boundary conditions are then automatically adjusted until the computed slope matches the desired slope calculated with respect to the
undisturbed flow. Computation of the potential flow solution at different distances into the suction line (Figure 5.18) showed that propagation of blade disturbances extend approximately one inch upstream. This was found to be true for all the inducers analyzed. The pressure field upstream of this point remains essentially constant (+1 psi). An additional error source, not included in the model, is fluid prerotation produced by fluid viscosity (Figure 5.19). As stated previously, all flow was assumed to be inviscid. An upper bound of viscous produced prerotation of 67% of the turbopump speed ($\omega_r = 2/3\omega$) was assumed for analytical evaluation. This yielded a new inlet flow direction of $U/(\omega + \omega_r)$, or 60% of the nominal $U/\omega$ computed without considering prerotation. This effect is not intended to be representative of actual prerotation values, but is only used to demonstrate the influence on cavitation results.

5.1.4 F-I Results - Analytical values of streamsheet fluid weight, $W_{ss}$, were obtained from the computer model for inlet pressures from 60 to 140 psia, $U/\omega$ values of .86 and .52, and five blade sections. For the nominal value of inlet flow direction ($U/\omega = .86$), computed without consideration for viscous induced prerotation, the model predicted no cavitation at any of the blade sections for the range of inlet pressures considered. That is, the minimum pressure predicted by the potential flow solution was always greater than the LOX vapor pressure. Reducing the flow direction to 60% of nominal to account for neglected prerotation resulted in small amounts of blade cavitation for inlet pressures below 100 psia. As shown in Figure 5.15 cavitation was observed at the 30%, 50%, and 70% blade sections. Reasons for variations at different blade sections are the same as discussed in Paragraph 5.1.3. Initial predictions, based only on the two blade sections
defined on the drawing (Figure 5.2), assumed that the amount of cavitation increased toward the blade tip. These results indicate that was a bad assumption. Local conditions can produce cavitation at a mid section, while for the same inlet conditions none occurs at either the hub or the blade tip. Applying Equation (5.13) to the results in Figure 5.15, and to the computed results for \( U/\omega = .86 \), gives the following values of cavitation compliance.

<table>
<thead>
<tr>
<th>( U/\omega ) (in/rad)</th>
<th>.86</th>
<th>.52</th>
<th>Inlet Press. (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>.0020</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>.0009</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>.0005</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>.0002</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>105</td>
</tr>
</tbody>
</table>

5.1.5 H-1 Results - Analytical values of streamsheet fluid weight, \( W_{ss} \), were computed for inlet pressures from 40 to 90 psia, \( U/\omega \) values of .42 and .25, and five blade sections. The results showed that essentially no cavitation was predicted for the nominal flow direction of \( U/\omega = .42 \), and very little cavitation for the flow direction reduced to account for possible viscous pre-rotation effects. Figure 5.16 shows the minimum pressure obtained in the H-1 LOX inducer which occurs at a grid point near the blade leading edge of the 30% section. For non-cavitating conditions the pressure increment between the inlet static pressure and the minimum pressure grid point is essentially constant for fixed flow conditions. This is reasonable since compressibility effects should be minimal if there is no cavitation vapor present. Figure 5.16 shows that
cavitation starts at an inlet pressure of 46 psia for nominal 
$U/\omega$ and 59 psia for $U/\omega$ reduced to 60% of nominal. However, 
even for pressures below these values, so few grid points 
reach vapor pressure that no significant cavitation is pro-
duced. At blade sections other than the 30% sections no cavi-
tation was predicted.

5.1.6 LR87 Oxidizer Results — The computer model generated 
values of streamsheet fluid weight, $W_{ss}$, for inlet pressures 
from 40 to 90 psia, $U/\omega$ values of .32 (nominal) and .19, and 
five blade sections. The results showed no measurable change 
in $W_{ss}$ implying no blade cavitation. For the nominal inlet 
flow direction the potential flow solution predicts that the 
minimum pressure grid point is only 17 psi below the inlet 
pressure (Figure 5.17). This means that the inlet pressure 
would have to be reduced to 31 psia before the minimum pressure 
reaches the vapor pressure resulting in cavitation. This com-
puted pressure reduction from the inlet to the minimum pressure 
point is considerably less for the LR87 than the other inducers.

These values, along with some of the influential parameters, 
are shown in Table 5.3 for comparison. Not all of the para-
meters are in the right direction (lower angle of attack, 
thinner blade, and lower dynamic pressure); however, the com-
bination could justify the smaller pressure increment for the 
LR87. For the inlet flow direction reduced by 60% ($U/\omega = .19$) 
to simulate a worst case viscous pre-rotation, the minimum 
pressure point is 36 psi lower than the inlet pressure 
(Figure 5.17). In this case cavitation just begins for an in-
let pressure of 49 psia. For the lowest pressure case analyzed, 
40 psia, no significant cavitation had developed.
5.2 **Comparison With Test Data** - The first task performed in the comparison of the analytical and test results was to compare the predicted inducer pressure rise with available test data. Figure 5.20 shows J-2 LOX inducer head rise test data (Reference 53) using water as the test fluid. This indicates a head rise of 172 ft (75 psi for water) for the nominal operating flow rate of 2540 gal/min (U/ω = .33 in/rad). Figure 5.21 shows the corresponding pressure profiles predicted by the computer model at the 50% blade section for an inlet pressure of 100 psia (230 ft NPSH). This shows a predicted pressure rise of greater than 60 psi, and it could easily be 75 psi depending on where the measurement is taken. The model predicts slightly greater pressure rises at a hub blade section (10%) and slightly less pressure rises at a tip blade section (90%). In the actual case, radial mixing will occur and tend to give a uniform pressure rise (in the radial direction) through the inducer. Thus the 50% blade section is felt to be most representative even though the pressure measurement is assumed to be taken on the pump housing nearest to a tip blade section. The J-2 LOX inducer head rise test data was the only pressure data available for comparison with analytical predictions. This single point comparison tends to confirm the overall accuracy of the potential flow equations used to compute pressures through the inducer blades. Analytical-empirical correlation of the cavitation compliance is obviously not as good as the pressure correlation since little or no cavitation was predicted in three out of the four inducers analyzed. On the other hand, test data (Figure 4.21) indicates that a significant amount of cavitation occurs in all turbopumps. Figures 5.22 and 5.23 present a comparison between test data and the cavitation compliance predicted by the analytical model for the J-2 and F-1 LOX inducers, respectively. Since no cavitation was predicted for either the H-1 or LR87 inducers, comparative plots are not presented. Plots of test data for these inducers were presented in Figures 4.18 and 4.20, respectively.
No model assumptions have been identified which could account for the lack of correlation observed. Reasonable variations in inlet flow direction to simulate viscous prerotation tended to improve predictions but failed to yield adequate correlation. Also, since no cavitation was predicted in some inducers, the analytical results can not be simply scaled to agree with test data. The entire results of this analysis indicate that some other mechanism besides blade cavitation contributes significantly to total turbopump compliance. Other mechanisms presented (Section 2.) as having potential significance are: blade tip clearance flow, circulation flow within the turbopump, and circulation flow back into the feedline.

5.2.1 Since the effect of cavitation compliance on feed system natural frequency is of prime interest, it is important to determine how uncertainties in one propagate into uncertainties in the other. The first feed system natural frequency, \( \omega_1 \), can be defined by

\[
\omega_1 = \sqrt{\frac{1}{I \left( \frac{1}{C_b} + \frac{1}{\ell} \right)}}
\]

(5.14)

where

- \( I \) = suction line fluid inertance
- \( C_b \) = cavitation compliance
- \( C_\ell \) = equivalent total suction line and fluid compliance (except pump cavitation) related to the pump inlet.

Differentiating Equation (5.14) yields

\[
\frac{d \omega_1}{\omega_1} = - \frac{2(C_b)}{(C_\ell + C_b)} \frac{dC_b}{C_b} \tag{5.15}
\]

which shows that the percentage change in \( \omega_1 \) is at most 1/2 the percentage change in \( C_b \), and may be much less if \( C_\ell \) is large relative to \( C_b \). For the S-II/J-2 LOX feed system, \( C_\ell \) is .003 to .005 in.\(^2\) (Rocketdyne results, Table 4.4). Using
these values of $C_{\ell}$ and test values of $C_{b}$ in Equation (5.15), it may be concluded that for a maximum uncertainty of 10% in S-II feed system natural frequency cavitation compliance must be known to within 25%. A similar evaluation on the other systems of concern results in the following required accuracies in cavitation compliance for a 10% accuracy in frequency: 70% for S-IC/F-1 LOX; 25% for S-IB/H-1 LOX; and 35% for Titan/LR87 Ox.

As previously stated, an objective of the cavitation model development is that it be capable of predicting feed system frequency to within a 10% accuracy. The results predicted by the current model clearly do not meet this objective. Although additional refinements and extensions to the existing model framework could be recommended, it is felt that none of them have a high probability of resulting in adequate correlation with test data.
Table 5.1 Inducer Streamsheet Parameters

<table>
<thead>
<tr>
<th>Inducer</th>
<th>J-2 LOX</th>
<th>F-1 LOX</th>
<th>H-1 LOX</th>
<th>LR-870X</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Blades</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Flow Rate (W) lb/sec</td>
<td>386.</td>
<td>3765.</td>
<td>514.</td>
<td>522.</td>
</tr>
<tr>
<td>Pump Speed (ω) rad/sec</td>
<td>841.</td>
<td>581.</td>
<td>706.</td>
<td>874.</td>
</tr>
<tr>
<td>Inlet Velocity (U) in/sec</td>
<td>275.</td>
<td>497.</td>
<td>294.</td>
<td>276.</td>
</tr>
<tr>
<td>U/ω in</td>
<td>.33</td>
<td>.86</td>
<td>.42</td>
<td>.32</td>
</tr>
<tr>
<td>Chord ΔZ/Δθ in</td>
<td>.59</td>
<td>1.25</td>
<td>.76</td>
<td>Fig. 5.12</td>
</tr>
<tr>
<td>Inlet Thickness (b(Zₜ)) in</td>
<td>.323</td>
<td>.331</td>
<td>.317</td>
<td>.294</td>
</tr>
<tr>
<td>Tip Radius (rₜ) in</td>
<td>3.375</td>
<td>7.875</td>
<td>3.80</td>
<td>3.55</td>
</tr>
<tr>
<td>Tip Blade Angle (β) deg</td>
<td>9.0</td>
<td>9.9</td>
<td>11.3</td>
<td>5.7</td>
</tr>
<tr>
<td>Tip Flow Angle (Ø) deg</td>
<td>5.6</td>
<td>6.2</td>
<td>6.3</td>
<td>5.2</td>
</tr>
<tr>
<td>Tip Angle of Attack (α) deg</td>
<td>3.4</td>
<td>3.7</td>
<td>5.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5.2 Streamsheet Cavitation Compliance
J-2 LOX Inducer, 50% Section, U/ω = .33

<table>
<thead>
<tr>
<th>Pₛ</th>
<th>Wₛₛ</th>
<th>ΔWₛₛ</th>
<th>ΔP</th>
<th>Cₛₛ</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>.17938</td>
<td></td>
<td></td>
<td>.00004</td>
</tr>
<tr>
<td>48</td>
<td>.00015</td>
<td>4</td>
<td>.00011</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>.17923</td>
<td></td>
<td></td>
<td>.00007</td>
</tr>
<tr>
<td>44</td>
<td>.00026</td>
<td>4</td>
<td>.00011</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>.17897</td>
<td></td>
<td></td>
<td>.00014</td>
</tr>
<tr>
<td>40</td>
<td>.00042</td>
<td>4</td>
<td>.00014</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>.17855</td>
<td></td>
<td></td>
<td>.00019</td>
</tr>
<tr>
<td>36</td>
<td>.00056</td>
<td>4</td>
<td>.00019</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>.17799</td>
<td></td>
<td></td>
<td>.00019</td>
</tr>
<tr>
<td>33</td>
<td>.00038</td>
<td>2</td>
<td>.00019</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>.17761</td>
<td></td>
<td></td>
<td>.00019</td>
</tr>
</tbody>
</table>
### Table 5.3 Factors Affecting Minimum Inducer Pressure

<table>
<thead>
<tr>
<th>Inducer</th>
<th>$\alpha_{\text{tip}}$ (deg)</th>
<th>$\alpha_{\text{hub}}$ (deg)</th>
<th>Blade Thickness $^{(1)}$ (% of channel)</th>
<th>$1/2 \rho V_r^2$ (psi)</th>
<th>$P_s - P_{\text{min}}$ $^{(2)}$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-2 LOX</td>
<td>3.4</td>
<td>15.1</td>
<td>32</td>
<td>433.</td>
<td>(3)</td>
</tr>
<tr>
<td>F-1 LOX</td>
<td>3.7</td>
<td>9.4</td>
<td>18</td>
<td>1120.</td>
<td>59</td>
</tr>
<tr>
<td>H-1 LOX</td>
<td>5.0</td>
<td>14.5</td>
<td>26</td>
<td>388.</td>
<td>30</td>
</tr>
<tr>
<td>LR87 Ox</td>
<td>5.5</td>
<td>8.9</td>
<td>19</td>
<td>655.</td>
<td>17</td>
</tr>
</tbody>
</table>

1. At the 50% blade section
2. Nominal inlet flow direction
3. $P_{\text{min}} = \text{vapor pressure}$
### BLADE SURFACE COORDINATES

All R & Ø VALUES ARE GAGE DIMENSIONS
REFER TO N-D MSC-68-270 FOR ADDITIONAL INFORMATION

<table>
<thead>
<tr>
<th>Ø</th>
<th>0.2</th>
<th>0.5</th>
<th>1.2</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.025</td>
<td>0.05</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>0.25</td>
</tr>
<tr>
<td>0°</td>
<td>0.05</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>45°</td>
<td>0.075</td>
<td>0.15</td>
<td>0.3</td>
<td>0.45</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>60°</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 5.1  J-2 LOX In
SECTION B-B
TYPICAL FAIRING SYMMETRICAL
ABOUT ALL LEADING AND
TRAILING EDGES

SECTION A-A
3 BLADES EQUALLY SPACED
WITHIN 0° 15'

Reducer
F-1 LOX Inducer
Figure 5.3 H-1 LOX Inducer
1. Machine per 24003-002
2. Normal within +0.001 total (A & B surface only)
3. Parallel within 0.001 total (A & B surface only)
4. Concentric within 0.002 T.I.R.
5. Identity by electrochemical etch per R604-008
6. Chromic acid anodize per MIL-A-8625 Type I (R40103-021 Type I)
7. For in plant handling protect blades by packaging per EB 23040
8. Key slot may be rotated at random with respect to blades
9. Serialize by electrochemical etch per R604-001
10. Heat treat after rough machining to T-6 condition per R601-001
11. Penetrate inspect per MIL-I-6866 (RAIDS-116 Type III A)
12. Plan machining per R603-006

NOTE: UNLESS OTHERWISE SPECIFIED
1.075 REF HUB R
7.45

TOLERANCE ZONE
IN AREA OF 4.011
RADIUS BASIC

SE DETAILS
SCALE: 1

5.980 REF
4.885

1.00 AT INTER-
SECTION OF SURF
.842 AT INTER-
SECTION OF SURF

SECTION A-A

18. ALL DM ARE FINAL, APPLY AT 68°F ONLY

5.4 LR87 Oxidizer Inducer
Figure 5.5 J-2 LOX Inducer Blade Sections
Figure 5.6 F-1 LOX Inducer Blade Sections
Figure 5.7 H-1 LOX Inducer Blade Sections
Figure 5.8 LR87 Oxidizer Inducer Blade Sections
Figure 5.9 J-2 LOX Inducer Interpolated Blade Sections
Figure 5.10  F-1 LOX Inducer Interpolated Blade Sections
Figure 5.11 H-1 LOX Inducer Interpolated Blade Sections
Figure 5.12 LR87 Oxidizer Inducer Interpolated Blade Sections
Figure 5.13 J-2 LOX Inducer Streamlines
Blade Sections Analyzed

\[ \frac{U}{\omega} = 0.33 \text{ (nominal)} \]

Figure 5.14 J-2 LOX Inducer Streamsheet Compliance
Blade Sections Analyzed

$U/\omega = 0.52$ (60% of nominal)

Figure 5.15 F-1 LOX Inducer Streamsheet Compliance
Pump Inlet Static Pressure ($P_s$) vs. psia

Minimum Inducer Pressure ($P_{min}$) vs. psia

30% Blade Section

$P_{min} = P_s$

$\psi/\omega = 0.42$ (nominal)

$\psi/\omega = 0.25$

$P_{min} = $ Vapor Pressure

Figure 5.16 H-1 LOX Minimum Predicted Inducer Pressure
Figure 5.17 LR37 Oxidizer Minimum Predicted Inducer Pressure
Figure 5.18  Effect of Inlet Boundary Conditions
Figure 5.19 Effect of Prerotation on Inlet Flow Vector
Figure 5.20  J-2 LOX Inducer Suction Performance
FLUID = WATER
INLET PRESSURE = 100 psia
$\frac{u}{\omega} = 0.33$

50% BLADE SECTION

Figure 5.21  J-2 Inducer Pressure Profiles
Figure 5.22  J-2 Lox Analytical-Empirical Comparison
Figure 3.23  F-1 LOX Analytical-Empirical Comparison

Test Data (Ref Fig 4.9)

Analytical Data

U/ω = .52
(worst case prerotation)

Note: For U/ω = .86 (nominal), no cavitation predicted.

Pump Inlet Static Pressure ~ psia
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6. Conclusions and Recommendations
6. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are based on the analysis of available test data, the turbopump cavitation model development, the analysis of the model results, and the correlation between test data and model predictions.

a. Large uncertainties exist in most cavitation compliance values derived from test data. This is because the objective of the tests was to determine natural frequency, and cavitation compliance must usually be derived from an assumed relationship.

b. Cavitation compliance test results for all available turbopump configurations do not correlate with any simple nondimensional combination of turbopumps and fluid parameters.

c. Compliance derived from a phase change process is a function of the local flow conditions and, unlike compressibility of a gas, is not necessarily directly proportional to the vapor volume.

d. The turbopump pressure field, derived from a potential solution, will not predict a large enough blade surface cavitation region to yield agreement with test results.

e. Mechanisms other than blade cavitation contribute the major amount of total turbopump compliance.

f. Turbopump pulse tests, using accurate inlet and outlet dynamic flow meters, should be conducted for the purpose of investigating cavitation compliance. These tests should vary the following parameters
one at a time: test fluid, dissolved gas, operating conditions (pressure, speed, and flow), tip clearance, natural frequency and oscillation amplitude (effect of nonequilibrium phase changes), etc.

g. Precise analytical simulation of the cavitation process can not be obtained until a dedicated test program (item f.) is performed.
7. References
7. REFERENCES


52. ATM66(6182-03)-866-2433, 3-22, Aerospace Corporation, El Segundo, California, March 25, 1966 (Unpublished).


Appendixes
APPENDIX A

Equations of Motion in Impeller Meridional Plane

For a \((r, \theta, z)\) coordinate system (Figure A.1), the equations of relative motion for a turbopump impeller rotating with angular velocity \(\omega\) about \(z\) (Reference 53) are:

\[
\frac{dV}{dt} + \frac{V}{r} \frac{\partial V}{\partial r} + \frac{V \omega}{r} \frac{\partial V}{\partial \theta} + \frac{V}{z} \frac{\partial V}{\partial z} = \frac{(V + \omega r)^2}{r}
\]

\[
= - \frac{\partial P}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 V}{\partial \theta^2} - \frac{1}{r} \frac{\partial V}{\partial \theta} + \frac{\partial^2 V}{\partial z^2} \right] + \rho g_r \quad (A.1)
\]

\[
\frac{\partial^2 V}{\partial \theta^2} + \frac{V}{r} \frac{\partial V}{\partial \theta} + \frac{V}{r} \frac{\partial V}{\partial \theta} + \frac{V}{z} \frac{\partial V}{\partial z} + \frac{V}{r} \frac{\partial V}{\partial r} + 2 \omega V_r
\]

\[
= - \frac{1}{r} \frac{\partial P}{\partial \theta} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 V}{\partial \theta \partial \theta} + \frac{\partial^2 V}{\partial z \partial \theta} \right] + \rho g_{\theta} \quad (A.2)
\]

\[
\frac{\partial^2 V}{\partial z^2} + \frac{V}{r} \frac{\partial V}{\partial z} + \frac{V}{r} \frac{\partial V}{\partial z} + \frac{V}{z} \frac{\partial V}{\partial z}
\]

\[
= - \frac{\partial P}{\partial z} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial z} \right) + \frac{1}{r} \frac{\partial^2 V}{\partial \theta \partial z} + \frac{\partial^2 V}{\partial z^2} \right] + \rho g_z \quad (A.3)
\]

where \(\rho \cdot \frac{V^2}{r}\) is the centrifugal force. It gives the effective force in the \(r\) direction due to fluid motion in the \(\theta\) direction. The term \(\rho \cdot V_r \cdot V_{\theta}/r\) is the coriolis force. It is the effective force in the \(\theta\) direction when there is flow in both the \(r\) and \(\theta\) directions. For steady inviscid flow in the absence of gravity equations (A.1) through (A.3) can be written

\[
\frac{dV}{dt} - \left( \frac{V_r + \omega r}{r} \right)^2 = - \frac{1}{\rho} \frac{\partial P}{\partial r} \frac{\partial V}{\partial r} + \frac{\partial V}{\partial r} \frac{\partial V}{\partial \theta} + \frac{\partial V}{\partial z} \frac{\partial V}{\partial z} - \frac{(V + \omega r)^2}{r} \quad (A.4)
\]
Figure A.1 Fluid Element Coordinates
\[
\frac{dV}{dt} + \frac{V}{r} \frac{dV}{dr} + 2r \frac{dV}{dr} = -\frac{1}{\rho} \frac{\partial P}{\partial r}
\]

\[
= \frac{V}{r} \frac{\partial V}{\partial r} + \frac{V}{r} \frac{\partial V}{\partial \theta} + \frac{V}{z} \frac{\partial V}{\partial z} + \frac{V}{r} \frac{\partial V}{\partial \theta} + 2r \frac{dV}{dr}
\]  
(A.5)

\[
\frac{dV}{dt} = -\frac{1}{\rho} \frac{\partial P}{\partial z} = \frac{V}{r} \frac{\partial V}{\partial r} + \frac{V}{r} \frac{\partial V}{\partial \theta} + \frac{V}{z} \frac{\partial V}{\partial z}
\]  
(A.6)

Assuming the vane of the impeller guide the fluid or that channel flow exists approximately, a stream surface may be constructed half way between blades (Figure A.2). The stream surface \( S \) can be described by

\[
S = S(r, \theta, z)
\]  
(A.7)

Solving for \( \psi \),

\[
\psi = \psi(r, z)
\]  
(A.8)

The static pressure in a turbopump is generally a function of \( r \), \( \theta \) and \( z \):

\[
P = P(r, \theta, z).
\]  
(A.9)

On \( S \)

\[
P^* = P(r, \psi, (r,z), z)
\]  
(A.10)

since \( \psi \) on the surface is specified by Equation (A.8). The relation between the partial derivatives of static pressure in the three-dimensional field to that on the stream surface "S" is:

\[
\frac{\partial P^*}{\partial r} = \frac{\partial P}{\partial r} + \frac{\partial P}{\partial \psi} \frac{\partial \psi}{\partial r}
\]  
(A.11)

\[
\frac{\partial P^*}{\partial z} = \frac{\partial P}{\partial z} + \frac{\partial P}{\partial \psi} \frac{\partial \psi}{\partial z}
\]  
(A.12)

Substituting Equations (A.11) and (A.12) into (A.4) and (A.6),

\[
\frac{dV}{dt} - \frac{(V_0 + r \omega)^2}{r} = -\frac{1}{\rho} \left( \frac{\partial P^*}{\partial r} - r \frac{\partial \psi}{\partial r} - \frac{1}{r} \frac{\partial P}{\partial \psi} \right)
\]  
(A.13)
The circumferential pressure gradient $\frac{1}{r} \frac{\partial P}{\partial \theta}$ can be eliminated from Equations (A.13) and (A.15) by (A.14)

$$\frac{d}{dt}(r \dot{V}_r) + r \ddot{V}_r + 2 \omega V_r = - \frac{1}{\rho} \frac{\partial P}{\partial r}$$

(A.14)

$$\frac{d}{dt}(r \dot{V}_\theta) + r \ddot{V}_\theta = - \frac{1}{\rho} \left( \frac{\partial P}{\partial z} - r \frac{\partial \theta}{\partial z} \cdot \frac{1}{r} \frac{\partial P}{\partial \theta} \right)$$

(A.15)

Figure A.2 Axial View of Impeller

The circumferential pressure gradient $\frac{1}{r} \frac{\partial P}{\partial \theta}$ can be eliminated from Equations (A.13) and (A.15) by (A.14)

$$\frac{d}{dt}(r \dot{V}_r) - \left( \dot{V}_\theta + \omega r \right)^2 = - \frac{1}{\rho} \left[ \frac{\partial P}{\partial r} + r \frac{\partial P}{\partial r} \left( \frac{d}{dt} \frac{\dot{V}_r}{r} + \frac{V_r}{r} \frac{\dot{V}_\theta}{r} + 2 \omega \dot{V}_r \right) \right]$$

(A.16)

$$\frac{d}{dt}(r \dot{V}_\theta) - \frac{\dot{V}_\theta^2}{r} = - \frac{1}{\rho} \left[ \frac{\partial P}{\partial r} + r \frac{\partial P}{\partial r} \left( \frac{1}{r} \frac{\dot{V}_r}{r} \left( r \dot{V}_\theta \right) \right) \right]$$

(A.16)

where $\dot{V}_\theta = V_\theta + \omega r$ and $dr/dt = V_r$. 
\[
\frac{dV_z}{dt} = -\frac{1}{\rho} \left[ \frac{\partial \rho}{\partial z} + r \frac{\partial}{\partial z} \left( \frac{dV_z}{dt} + \frac{V_z V_r}{r} + \frac{V_r^2}{r^2} \right) \right]
\]

\[
\frac{dV_r}{dt} = -\frac{1}{\rho} \left[ \frac{\partial \rho}{\partial z} + r \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{d}{dt} \left( rV_r \right) \right) \right]
\]  

(A.17)

If the flow is restricted to a streamline on the stream sheet and the streamline is projected on the meridional plane (Figures A.3 and A.4), the tangent to the projected streamline at any point makes an angle \( \alpha \) with the impeller axis.
Projected Stream Sheet
Streamline
Shroud

Figure A.4 Meridional Plane of Impeller

Figure A.5 Meridional Streamline
The velocity components $V_r$ and $V_z$ of the stream sheet streamline lead to a velocity $V_m$ in the meridional plane where:

$$V_m^2 = V_r^2 + V_z^2 \tag{A.18}$$

$$V_r = V_m \sin \alpha \tag{A.19}$$

$$V_z = V_m \cos \alpha \tag{A.20}$$

Differentiating,

$$\frac{dV_r}{dt} = \frac{dV_m}{dt} \sin \alpha + V_m \frac{d\alpha}{dt} \cos \alpha \tag{A.21}$$

$$\frac{dV_z}{dt} = \frac{dV_m}{dt} \cos \alpha - V_m \frac{d\alpha}{dt} \sin \alpha \tag{A.22}$$

$\alpha$ is related to the radius of curvature of the projected streamline by

$$dM = r_c \, d\alpha \tag{A.23}$$

or

$$\frac{1}{r_c} = \frac{d\alpha}{dM} = \frac{d\alpha}{dM} = \frac{1}{V_m} \frac{d\alpha}{dt} \tag{A.24}$$

$$\therefore \frac{d\alpha}{dt} = \frac{V_m}{r_c} \tag{A.25}$$

Combining Equations (A.16) through (A.25),

$$\frac{dV_m}{dt} \cos \alpha - \frac{V_m^2}{r_c} \sin \alpha = - \frac{1}{\rho} \left[ \frac{\partial P}{\partial z} + r \frac{\partial \phi}{\partial z} \left[ \frac{1}{r} \frac{d(rV_m)}{dt} \right] \right] \tag{A.26}$$
\[
\frac{dV}{dt} \sin \alpha \cdot \cos \beta + \frac{V^2}{r_c} \cos \beta = \frac{V^2}{r_c} - \frac{1}{r_c} \left\{ \frac{\partial P^*}{\partial r} \cos \beta - \frac{P^*}{N} + r \rho \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{d(rV'_r)}{dt} \right) \sin \beta \right\} \quad (A.27)
\]

\[
\frac{dP^*}{dN} = \frac{\partial P^*}{\partial r} \frac{r}{N} + \frac{\partial P^*}{\partial z} \frac{z}{N} \quad \text{(A.28)}
\]

\[
\frac{r}{N} = \cos \alpha \quad \text{and} \quad \frac{z}{N} = -\sin \alpha \quad \text{(A.29)}
\]

\[
\frac{\partial P^*}{\partial N} = \frac{\partial P^*}{\partial r} \cos \beta - \frac{P^*}{\partial z} \sin \beta \quad \text{(A.30)}
\]

where \( \frac{d}{dN} \) is the derivative with respect to the normal to the streamline.

Multiplying Equation (A.26) by \( \sin \alpha \) and substituting Equation (A.30)

\[
\frac{dV}{dt} \sin \alpha \cdot \cos \beta - \frac{(V_M \sin \beta)^2}{r_c} = \]

\[- \frac{1}{r_c} \left\{ \frac{\partial P^*}{\partial r} \cos \beta - \frac{P^*}{N} + r \rho \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{d(rV'_r)}{dt} \right) \sin \beta \right\} \quad (A.31)
\]

Multiplying Equation (A.27) by \( \cos \alpha \)

\[
\frac{dV}{dt} \sin \alpha \cdot \cos \beta + \frac{(V_M \cos \beta)^2}{r_c} - \frac{V^2}{r} \cos \alpha = \]

\[- \frac{1}{r} \left\{ \frac{\partial P^*}{\partial r} \cos \beta + r \rho \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{d(rV'_r)}{dt} \right) \cos \beta \right\} \quad (A.32)
\]

Subtracting Equation (A.31) from (A.32)

\[
\frac{V^2}{r_c} \sin \alpha = \frac{V^2}{r} - \frac{1}{r} \left\{ \frac{\partial P^*}{\partial r} \cos \beta + \frac{1}{r} \left( r \rho \frac{\partial}{\partial z} \sin \alpha - r \rho \frac{\partial}{\partial r} \cos \alpha \right) \left( \frac{d(rV'_r)}{dt} \right) \right\} \quad (A.33)
\]

From Figure A.3 the velocity along the stream sheet streamline \( V \) is related to the projected velocity \( V_M \) and the velocity \( V_t \) by
\[ V_M = V \cos \beta \quad (A.34) \]

\[ V_\rho = V \sin \beta \quad \text{or} \quad V'_\phi = V \sin \beta + \omega r \quad (A.35) \]

Equation (A.33) becomes

\[ \left( \frac{V \cos \beta}{r} \right)^2 - \left( \frac{V \sin \beta + \omega r}{r} \right)^2 \cos \phi = - \frac{1}{\beta} \frac{\partial P}{\partial N} \left\{ \frac{V \cos \beta}{r} \right\} \]

\[ \left[ \left( \frac{\rho \sin \phi}{r} \sin \phi - \frac{\rho \cos \phi}{r} \cos \phi \right) \left( \frac{\partial V}{\partial r} + \frac{V \sin \beta \sin \phi}{r} + 2 \sin \phi \sin \phi \right) \right] \quad (A.36) \]

Multiplying Equation (A.4) by \( V_r = \frac{dV}{dt} \), Equation (A.5) by \( V_\theta = r \frac{d\theta}{dt} \), and Equation (A.6) by \( V_z = \frac{dz}{dt} \) yields:

\[ V \frac{dV_r}{dt} - V \left( \frac{V \rho + \omega r}{r} \right)^2 = - \frac{1}{\beta} \frac{\partial P}{\partial N} \quad (A.37) \]

\[ V \frac{dV_\theta}{dt} + V \frac{V_\theta}{r} + 2 \omega V \frac{V_\rho}{r} \theta = - \frac{1}{\beta} \frac{\partial P}{\partial N} \quad (A.38) \]

\[ V \frac{dV_z}{dt} = - \frac{1}{\beta} \frac{\partial P}{\partial N} \quad (A.39) \]

Adding the above three equations,

\[ V \frac{dV_r}{dt} - V \omega^2 r^2 \frac{r}{r} + V \frac{dV_\theta}{dt} + V \frac{dV_z}{dt} = - \frac{3}{\beta} \frac{\partial P}{\partial N} \quad (A.40) \]

\[ V^2 = \frac{V_r^2}{r} + \frac{V_\theta^2}{r} + \frac{V_z^2}{r} \quad (A.41) \]

and

\[ \frac{dV^2}{dt} = 2V_r \frac{dV_r}{dt} + 2V_\theta \frac{dV_\theta}{dt} + 2V_z \frac{dV_z}{dt} \quad (A.42) \]

\[ \therefore \frac{1}{2} \frac{dV^2}{dt} = V \omega^2 r^2 \frac{r}{r} = - \frac{3}{\beta} \frac{\partial P}{\partial N} \quad (A.43) \]
Integrating Equations (A.42) and (A.43) along a streamline between a station in the pump inlet, \( i \), and a point in the pump

\[
\frac{1}{2} \left( v^2 - v_{i1}^2 \right) - \frac{\omega^2}{2} \left( r^2 - r_{i1}^2 \right) = -3 \int_i \frac{dP}{\rho} \tag{A.44}
\]

But

\[
v'_{i1} = v_{i1}^2 + v_r^2 + v_z^2 \tag{A.45}
\]

\[
v'_{t1} = v_{t1} + \omega r \tag{A.46}
\]

\[
v'^2 = v^2 + 2v_r v_{t1} + \omega^2 r^2 \tag{A.47}
\]

\[
v'^2 = v^2 + 2\omega r v_{t1} + \omega^2 r^2 \tag{A.48}
\]

\[
v'^2 = v^2 + 2\omega r v'_{t1} - \omega^2 r^2 \tag{A.49}
\]

and along a streamline Bernoulli's equation is

\[
\frac{1}{2} \rho v^2 = \frac{1}{2} \rho v'^2 \tag{A.50}
\]

If \( \rho = \text{const} \): Equation (A.44) can be written

\[
\frac{\frac{v'^2}{2}}{2} - \frac{\omega^2 r^2}{2} + \frac{2\omega r v_{t1}}{2} - \frac{3P}{\rho} = -\frac{3P}{\rho} \tag{A.51}
\]

Taking the derivative of Equation (A.51) with respect to \( N \)

\[
v \frac{dv}{dN} = \omega \frac{dr}{dN} + \omega \frac{d}{dN} \left( r v'_{t1} \right) - \frac{3}{\rho} \frac{dP}{dN} = -\frac{3}{\rho} \frac{dP}{dN} \tag{A.52}
\]
Substituting Equation (A.52) into Equation (A.36)

\[
V \frac{dV}{dN} + \frac{r \rho}{\rho} \frac{dN}{dN} = -\frac{3}{N} \left( \frac{1}{r} \frac{U}{dN} \right) + \frac{3(V \cos \theta)^2}{r_c} - \frac{3(V \sin \theta + \omega)^2}{r} \cos \alpha
\]

\[-3 \frac{V \cos \theta}{r} \left( \rho \frac{dr}{dz} \sin \alpha - \rho \frac{dV}{dr} \cos \theta \right) \left( \frac{dV}{dN} + \frac{V \sin \theta \sin \alpha}{r} + 2 \omega \sin \alpha \right)
\]

Equation (A.53) combined with hub and shroud boundary conditions, inlet conditions, and the continuity equation in the form:

\[
\frac{\dot{V}}{N_b} = \int_{N_1}^{N_2} \int_{\theta_s}^{\theta_p} V \cdot r \cdot d\theta \cdot dN
\]

where: \( \dot{V} \) = total pump flow rate

\( N_1, N_2 \) = streamline numbers

\( \theta_p \) = \( \theta \) on pressure surface of blade

\( \theta_s \) = \( \theta \) on suction surface of adjacent blade

\( L \) = number of blades

provides a solution to the incompressible flow problem in the meridional plane. The solution involves the numerical integration of Equations (A.53) and (A.54) from streamline to streamline in the meridional plane.
A thermal cavitation bubble appears in a turbopump when the local static pressure drops below the vapor pressure of the liquid. The bubble growth begins either on a small gas nucleus lodged in the walls of the fluid container, on dust and colloidal matter suspended in the media, or on a small bubble of contaminated gas free in the fluid. Bubble growth is due to essentially three mechanisms. First, additional contaminant gas can diffuse into the bubble; second the bubble grows because of a decrease in ambient pressure; and third, growth results from a phase change occurring at the bubble wall.

The initial nucleus is composed solely of contaminant gases or a mixture of contaminant gas and liquid vapor. The effect of the initial contaminant gas and the additional contaminant gas that diffuses into the nucleus during its growth is important only during the initial stages of growth. Due to the large surface tension force, the initial growth of the bubble is slow. However, once the bubble radius has increased by an order of magnitude, the presence of the contaminant gases is relatively unimportant.

The flow of fluid surrounding a single bubble can be treated as incompressible and irrotational and, hence, can be described by a potential function, \( \psi \)

\[
- \frac{\dot{r}}{r} = \dot{r} \tag{B.1}
\]

where \( r \) is the radius from the center of the bubble to any point in the fluid and \( \dot{r} \) is the velocity of the fluid at that point. The boundary conditions \( \dot{r} = \hat{R} \) at \( r = R \), where \( R \) is the bubble radius, and \( \dot{r} = 0 \) at \( r = \infty \) establish the potential function to be

\[
\psi = \frac{R^2 \cdot \hat{R}}{r} \tag{B.2}
\]
Since the fluid is considered incompressible with gravitational effects negligible, the work done by a growing bubble appears only as a change in kinetic energy of the fluid surrounding the bubble. The increment of work done by the bubble in expanding from $R$ to $R + \Delta R$ is

$$\Delta W = \Delta P \cdot 4\pi R^2 \cdot \Delta R$$  \hspace{1cm} (B.3)

where

$$\Delta P = P_R - P_\infty$$

In terms of the rate of change of work with respect to $R$

$$\frac{dW}{dR} = (P_R - P_\infty) \cdot 4\pi R^2$$  \hspace{1cm} (B.4)

The kinetic energy of the fluid between $R$ and $r$ is

$$KE = \frac{1}{2} \rho L v^2 = \frac{1}{2} \int_{R}^{r} 4\pi \rho L r^2 \frac{dr}{r^2}$$  \hspace{1cm} (B.5)

where $\rho_L$ is the mass density of the liquid.

But from Equation (B.2)

$$\dot{r} = \frac{d}{dt} = \frac{R^2 \dot{R}}{r^2}$$  \hspace{1cm} (B.6)

or

$$\because KE = 2\pi \rho L R^4 \int_{R}^{r} \frac{dr}{r^2}$$  \hspace{1cm} (B.7)

or

$$KE = 2\pi \rho L R^4 \left( \frac{1}{R} - \frac{1}{r} \right)$$  \hspace{1cm} (B.8)

letting $r \to \infty$

$$KE = 2\pi \rho L R^3 R^2$$  \hspace{1cm} (B.9)
The rate of change of kinetic energy with respect to \( R \) is:

\[
\frac{d}{dR} (KE) = 2\pi\rho \frac{d}{dR} \left( R^3 \frac{v}{2} \right)
\]

Setting Equation (B.10) equal to Equation (B.10)

\[
(P_R - P_\infty) 4\pi R^2 = 2\pi\rho \frac{d}{dR} \left( R^3 \frac{v}{2} \right)
\]

But

\[
\frac{d}{dR} = \frac{dt}{dR} \quad \frac{d}{dt} = \frac{1}{R} \frac{d}{dR}
\]

\[
\therefore \quad \frac{P_R - P_\infty}{L} = \frac{1}{2R^2} \frac{d}{dt} \left( R^3 \frac{v}{2} \right)
\]

Equation (B.12) therefore, is the equation of motion governing bubble growth. The same results can be obtained starting with the Bernoulli equation

\[
\frac{P - P_\infty}{\rho L} = \frac{1}{2} \frac{v^2}{L} + \frac{\mathcal{V}}{L}
\]

The temperature at the bubble wall will be controlled by the evaporation process. If it is assumed that the pressure in the bubble is uniform and at the vapor pressure, \( P_v \), of the liquid corresponding to the temperature at the bubble wall, then \( P_R \) is related to \( P_v \) by:

\[
P_R = P_v - 2 \frac{\mathcal{V}}{R}
\]

where \( \mathcal{V} \) is the surface tension of the fluid

\[
\therefore \quad \frac{1}{2R^2} \frac{d}{dt} \left( R^3 \frac{v}{2} \right) = \frac{1}{\rho L} \left( P_v - P_\infty - \frac{2\mathcal{V}}{R} \right)
\]

If the boiling curve of a fluid is linear or nearly so over the region in which bubble growth takes place, the saturation pressure \( P_s \) can be related to the saturation temperature \( T_s \) by
\[ P_s = A \cdot T_s + B \]  \hspace{1cm} (B.16)

\[ P_v - P_s = A(T_R - T_o) + A(T_o - T_p) \]  \hspace{1cm} (B.17)

where \( T_o \) is the fluid temperature a great distance from the bubble and \( T_p \) is the saturation temperature corresponding to the time-independent ambient pressure.

Equation (B.15) then becomes

\[ \frac{1}{2R^2 \rho} \frac{d}{dt} \left( R^3 \right) = \frac{A}{\rho_L} \left( T_R - T_o \right) + \left( T_o - T_p \right) - \frac{2\sigma}{\rho_L R} \]  \hspace{1cm} (B.18)

If it is assumed that the temperature a great distance from the bubble, \( T_o \), remains constant during bubble growth the quantity \( T_R - T_o \) can be obtained from the solution to the problem of non-steady heat diffusion with boundary motion of Plesset and Zwick (Reference 16). The equations will not be derived here. However, the final results can be expressed by the following equation:

\[ T_R - T_o = \left( \frac{D}{t} \right)^{1/2} \int_0^t \frac{R^2(x) \cdot (\partial T/\partial r)_{r=R(x)}}{\int_0^t R^4(y) \, dy}^{1/2} \, dx \]  \hspace{1cm} (B.19)

where \( D \) is the thermal diffusivity of the fluid and the variable \( y \) is associated with a translation of the time axis. The derivative \( (\partial T/\partial r) \) is the temperature gradient at the bubble wall.

Equation (B.18) is then

\[ \frac{1}{2R^2 \rho} \frac{d}{dt} \left( R^3 \right) = \frac{A}{\rho_L} \left( T_o - T_p \right) - \frac{A}{\rho_L} \left( \frac{L}{t} \right)^{1/2} \int_0^t \frac{R^2(x) \cdot (\partial T/\partial r)_{r=R(x)}}{\int_0^t R^4(y) \, dy}^{1/2} \, dx - \frac{2\sigma}{\rho_L R} \]  \hspace{1cm} (B.20)
The temperature gradient at the bubble wall can be obtained from a mass and heat balance as follows. The heat transfer at the bubble wall per unit time is:

$$\dot{Q} = 4\pi R^2 k \left(\frac{dT}{dr}\right)_{r=R}$$  \hspace{1cm} (B.21)

where $k$ is the thermal conductivity.

This heat goes into vaporizing liquid at a rate

$$\frac{dm_L}{dt} = \frac{\dot{Q}}{L}$$  \hspace{1cm} (B.22)

where $L$ is the latent heat of vaporization.

The rate at which liquid is evaporated is equal to the rate of mass addition to the bubble:

$$\frac{dm_L}{dt} = \frac{dm_v}{dt}$$  \hspace{1cm} (B.23)

where $m_v$ = mass of vapor

but

$$m_v = \frac{4}{3} \pi R^3 \rho_v$$  \hspace{1cm} (B.24)

and

$$\frac{dm_v}{dt} = \left(\frac{4}{3}\right) \pi \frac{d}{dt} \left(R^3 \rho_v\right) = \left(\frac{4}{3}\right) \pi \left(3R^2 \dot{R} \rho_v + R^3 \ddot{\rho}_v\right)$$  \hspace{1cm} (B.25)

$$\therefore \frac{4\pi R^2 k}{L} \left(\frac{dT}{dr}\right)_{r=R} = \left(\frac{4}{3}\right) \pi \left(3R^2 \dot{R} \rho_v + R^3 \ddot{\rho}_v\right)$$  \hspace{1cm} (B.26)

or

$$\left(\frac{\frac{dT}{dr}}{r}\right)_{r=R} = \frac{L}{k} \dot{\rho}_v \dot{R} + \frac{LR}{3k} \ddot{\rho}_v$$  \hspace{1cm} (B.27)

The growth of a vapor bubble under conditions of variable ambient pressure and variable vapor density is described by a solution to Equations (B.20) and (B.27).
APPENDIX C

Simplified Test Feed System Transfer Functions:

A simplified analytical model of the S-II inboard LOX suction line with the by-pass pulser line is shown in Figure C.1. Solution of these equations gives the following transfer functions:

\[
\frac{\partial P_s}{\partial \psi_p} = \frac{-I_1}{\left(1 + 2 \zeta \frac{\omega_1}{\omega_1^2} + \frac{\omega_1^2}{\omega_1^2}\right)} \tag{C.1}
\]

\[
\frac{\partial P_s}{\partial P_p} = \frac{(I_{1P}/I_{1P})}{\left(1 + 2 \zeta \frac{\omega_1}{\omega_1^2} + \frac{\omega_1^2}{\omega_1^2}\right)} \tag{C.2}
\]

\[
\frac{\partial I_{1P}}{\partial P_p} = \frac{\partial P_s}{\partial P_p} \left(1 + \frac{\omega_1^2}{\omega_0^2}\right) \tag{C.3}
\]

where \( I = \frac{f}{Agc} \)

\( \omega_1^2 = \frac{1}{[C (I_1 + I_2)]} \)

\( \omega_0^2 = \frac{1}{CI_2} \)

\( I_{1P} = \frac{I_1 I_p}{(I_1 + I_p)} \)

\( \zeta = \frac{1}{2} \omega (I_1 + I_2) \frac{\partial \dot{W}_d}{\partial P_s} \)
\[ c_1 = \frac{1}{2} \omega_1 (l_{1p} + l_2) \frac{d\dot{w}_d}{dP_s} \]

\[ \frac{d\dot{w}_d}{dP_s} = \text{engine flow transfer function} \]

The true system transfer function of suction pressure with respect to pump acceleration is:

\[ \frac{dP_s}{d\dot{w}_p} = \frac{(l_1 + l_2) \rho A \omega c}{1 + 2 \xi \frac{S}{\omega_0} + \frac{S^2}{\omega_0^2}} \quad \text{(C.4)} \]

and, therefore, the only test transfer function that has the right dynamics (correct natural frequency) is \( \frac{dP_s}{d\dot{w}_p} \). However, since pulser flow acceleration is not easy to measure accurately, frequency can also be determined from \( \frac{dP_s}{dP_p} \) if the proper correction is applied. For the S-II inboard LOX line, the natural resonance of the \( \frac{dP_s}{d\dot{w}_p} \) transfer function will be approximately 5% too high. This is independent of where the line pressure is measured except as the pressure transducer moves up the line an anti-resonance will approach the resonance from above.

For other test configurations, the \( \frac{dP_s}{dP_p} \) transfer function will give approximately the correct natural frequency provided that the pulser line inertance (from the suction line to the pressure transducer) is large relative to the inertance in the suction line between the tank and pulser line.
Test Setup (S-II LOX I/B)

Model

\[
\begin{align*}
I_1 w_1 s^2 &= -P_1 \\
I_p wp s^2 &= P_1 - P_p \\
I_2 w_2 s^2 &= P_1 - P_s \\
w_2 - W_d &= C P_s \\
w_1 &= W_p + W_2
\end{align*}
\]

Approximate Values

\[
\begin{align*}
I_1 &= 0.00173 \text{ sec}^2/\text{in}^2 \\
I_2 &= 0.00107 \text{ sec}^2/\text{in}^2 \\
I_p &= 0.01 \text{ sec}^2/\text{in}^2
\end{align*}
\]

FIGURE C.1 Simplified Suction Line Model for Pulse Test
The turbopump cavitation flow program has been written in the Fortran IV program language for the CDC 6000 series computer. The program is made up of a main controlling program and seven subroutines. The main program controls the sequence of solution steps and the adjustment of boundary conditions to meet proper inlet and exit flow conditions.

The first subroutine employed (FIBK) reads in the input data, sets up the grid system, and establishes initial estimates of the streamfunction at each grid point. Subroutine RELAX is used next to solve the flow and energy equations throughout the field. This solution is accomplished by applying relaxation techniques to a finite difference form of the equations. The first series of relaxation solutions is done with the density throughout the field set equal to the liquid density. This solution is referred to as the incompressible or uncoupled solution. The relaxation solution is then continued with the completely coupled two phase flow equations. Following this solution, boundary conditions are checked, adjusted, and the relaxation solution repeated until the proper inlet and exit conditions are satisfied. The last operation of the program prepares and prints out the output data.

Subroutine TABL is an interpolation subroutine used by the main program and many of the subroutines. Subroutine PREWRIT prepares the output data for printing while subroutine SETUP prepares the output data for computer plotting. Subroutines WRITOUT and PLOTT respectively do the printing and plotting of the output.

A flow diagram for the program is given in Figure D.1. With the exception of the plotting capabilities, this program should be compatible with any computer which has a FORTRAN IV compiler. The plotting routines are documented in Reference 55. The routine ALTFILE is required on the CDC 6000 computer so that the TAPE9 buffer area may also be used for TAPE10 through TAPE99.

The problem solution is initiated by plotting the inducer or impeller blade sections (Figures 5.3 and 5.4) and the relationship between density and pressure for equilibrium.
phase changes (Figure 3.7). All input variables are defined in the program listing of subroutine FEER (Appendix E). Input dimensions and internal program variables are shown in Figures D.2 through D.4.

Parts of a sample output from the turbopump cavitation flow program are presented at the end of this Appendix. The first data printed is the input data. Next is a tabulation of the iterations required for solution along with the value of the maximum residual (RESIM) throughout the field for each iteration. Following the last iteration, which corresponds to the uncoupled solution, the pressure and density is printed out at each grid point in the system. The next item to be printed (volume) is the weight of propellant in the streamsheet analyzed. Finally, for the uncoupled solution the values of the streamfunction, $\psi$, theta, $\theta$, potential function, $\phi$, circumferential velocity, $V$, and meridional velocity, $U$, are printed for increments along the $E$ axis. At this point the solution is continued on a coupled basis and the pressure data printed out for each iteration. After the final iteration of the coupled solution the pressure field, the weight of fluid in the streamsheet, and the streamfunction - velocity fields are printed out.
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167

BEGIN MAIN PROGRAM
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STEP NO. ONE COMPLETE
INCREMENT E AND THEN THETA

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VTHT2 =  477.00

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Figure D.1 Turbopump Program Computation Sequence
Figure D.2 THETA, E Dimensions
Figure D.3  Grid Increment Number
Figure D.4  Grid Increment For Extrapolation to Blade Surfaces
APPENDIX E

Turbopump Program Listing
PROGRAM MAIN ( INPUT, OUTPUT, TAPE5 = INPUT, TAPE6 = OUTPUT, *
* TAPE9, FILMPL )

C
C COMPRESSIBLE - INCOMPRESSIBLE FLOW TURBO PUMP PROGRAM

C
COMMON ISIS(150), IPIB(150), W(150), X(150), Y(150), Z(150),
* JTHL(150), JTHB(150), PSI(100,100),
* RS(100), RP(100), ES1(100), RC(100),
* EC(150), RD(150), BD(150), SIAO(150), MNM, IL, JL, IIB,
* IE, B, DELTA, RESIM, OPSIP, KK, DEX, ACC, JLE, BN,
* PSIPR, RHO(100,100), G, WDOT, WW, KKL, RHOIN, POIN, HOIN,
* RCIN, VTHEN, VMOIN, PS, RR(150), BZ(150), SIA(150), RT,
* INPUF, RRL, ITIP, PDPL, JPL

C ZERO OUT ARRAYS
ITIP=0
DO 100 JJ = 1, 150
   W(JJ) = 0.
   Y(JJ) = 0.
100 CONTINUE

C DO 150 II = 1, 150
   X(II) = 0.
   Z(II) = 0.
150 CONTINUE

C DO 250 JJ = 1, 100
   DO 200 II = 1, 100
      PSI(II,JJ) = 0.
   200 CONTINUE
250 CONTINUE

C CALL PEER TO READ INPUT DATA AND SET UP GRID
C G = 386.4
300 CALL PEER
   ER = PSIPR/ 70.
   ACC = ER
C
C OBTAIN FIRST RELAXATION
C CALL RELAX
C
   JEPSI=-PSIPR/50.
   ILMI=IL-I
   VTHTA=(PSI(ILMI,1)-PSI(IL,1))/DELTA
   VTHTA=VTHTA/(RHOIN*BZ(IL)*RR(IL))
   WRITE(6,399) VTHTA
399 FORMAT(10X,* VTHTA = * F10.2)

400 IF (ABS(VTHTA-WW)*LT.15.0) GO TO 900
   JTB=JTHB(IL)
   JTL=JTHL(IL)
   JJ=0
   DO 800 JN=JTB,JTL
   JJ=JJ+1
800 CONTINUE
GO TO 300
GO TO 300
PSI(IL,JJ) = PSI(IL,JJ) + DEPSI

800 CONTINUE
CALL RELAX
ILMI = IL - 1
VTHT2 = (PSI(ILMI,1) - PSI(IL,1)) / DELTA
VTHT2 = VTHT2 / (RMOIN*BZ(IL)*RR(IL))
WRITE(6, 899) VTHT2

899 FORMAT(10X, * VTHT2 = *, F10.2)
DVTHD = (VTHT2 - VTHTA) / DEPSI
DEPSI = (WW - VTHT2) / DVTHD
VTHTA = VTHT2
GO TO 400
C WRITE OUT FIRST RELAXATION
C
900 ITRIP = 1
DO 1000 JDEL = 1, JDPL
POIN = PCIN - PDEL
WRITE(6, 989) POIN

989 FORMAT(10X, * PCIN = *F10.3)
CALL RELAX
CALL PREWRIT
CALL WROUT
1000 CONTINUE
GO TO 300
END
SUBROUTINE TAF_L ( AI, BO, CI, CD, M, J, K )
C
C. MONO-VARIANT TABLE LOOK UP ROUTINE
C. EXTRAPOLATION = LINEAR BASED ON FIRST OR LAST TWO POINTS
C. INTERPOLATION = LINEAR, QUADRATIC, OR CUBIC
C
C. SUBROUTINE ARGUMENTS
C. AI = GIVEN INDEPENDENT VARIABLE
C. BO = DESIRED DEPENDENT VARIABLE
C. CI = SET OF INDEPENDENT VARIABLES
C. CD = SET OF DEPENDENT VARIABLES
C. M = ORDER OF INTERPOLATION (1, 2, 3)
C. J = FIRST POINT IN TABLE (USUALLY 1)
C. K = LAST POINT IN TABLE
C
DIMENSION CI(1), CD(1)
C
8001 FORMAT (* UNSUCCESSFUL TABLE LOOK UP *)
C
C. IS AI INSIDE RANGE OF TABLE
C
IF ( AI .GT. CI(K) ) GO TO 100
IF ( AI .LT. CI(J) ) GO TO 200
GO TO 300
C
C. EXTRAPOLATE IF AI OUTSIDE TABLE RANGE
C
100 BO = CD(K) + ( CD(K) - CD(K-1) ) * ( AI - CI(K) ) / (CI(K) * 
C. - CI(K-1) )
GO TO 1700
200 BO = CJ(J) + ( CJ(J+1) - CJ(J) ) * ( AI - CI(J) ) / 
C. * ( CI(J+1) - CI(J) )
GO TO 1700
C
C. DOES AI = POINT IN TABLE
C
300 DD 400 IN = J, K
I = IN
IF ( ( ABS ( AI - CI(I) ) ) .LT. 0.00001 ) GO TO 500
400 CONTINUE
GO TO 600
500 BO = DD(I)
GO TO 1700
C
C. LOCATE POSITION IN TABLE
C
600 DD 700 IO = J, K
I = IO
IF ( CI(I) .GT. AI ) GO TO 800
700 CONTINUE
WRITE (6,8001)
CALL EXIT
C
800 GO TO ( 900, 1000, 1300 ), M
C
C. LINEAR INTERPOLATION
C

900 Y1 = DD(I-1)
Y2 = DD(I)
X1 = CI(I-1)
X2 = CI(I)
DEX = X2 - X1
DY1 = Y2 - Y1
BD = Y1 + (AI - X1) * DY1 / DEX
GO TO 1700

C
C. QUADRATIC INTERPOLATION
C

1000 IF ( I .EQ. K ) GO TO 1100
Y1 = DD(I-1)
Y2 = DD(I)
Y3 = DD(I+1)
X1 = CI(I-1)
X2 = CI(I)
X3 = CI(I+1)
GO TO 1200

1100 Y1 = DD(I-2)
Y2 = DD(I-1)
Y3 = DD(I)
X1 = CI(I-2)
X2 = CI(I-1)
X3 = CI(I)

1200 B1 = Y1 * (AI - X2) * (AI - X3) / (X1 - X2) * (X1 - X3)
B2 = Y2 * (AI - X1) * (AI - X3) / (X2 - X1) * (X2 - X3)
B3 = Y3 * (AI - X1) * (AI - X2) / (X3 - X1) * (X3 - X2)
BJ = B1 + B2 + B3
GO TO 1700

C
C. CUBIC INTERPOLATION
C

1300 IF ( I .EQ. K ) GO TO 1400
IF (( I - 1 ) .EQ. J ) GO TO 1500
Y1 = DD(I-2)
Y2 = DD(I-1)
Y3 = DD(I)
Y4 = DD(I+1)
X1 = CI(I-2)
X2 = CI(I-1)
X3 = CI(I)
X4 = CI(I+1)
GO TO 1600

C

1400 Y1 = DD(I-3)
Y2 = DD(I-2)
Y3 = DD(I-1)
Y4 = DD(I)
X1 = CI(I-3)
\[ X_2 = CI(I-2) \]
\[ X_3 = CI(I-1) \]
\[ X_4 = CI(I) \]

\[
CG TC 1600
\]

\[
1500 \quad Y_1 = CD(I-1) \\
Y_2 = DD(I) \\
Y_3 = DJ(I+1) \\
Y_4 = DD(I+2) \\
X_1 = CI(I-1) \\
X_2 = CI(I) \\
X_3 = CI(I+1) \\
X_4 = CI(I+2) \\
\]

\[
CG 1600 BD = Y_1 \times (AI - X_2) \times (AI - X_3) \times (AI - X_4) / ( \\
\times (X_1 - X_2) \times (X_1 - X_3) \times (X_1 - X_4) ) \\
\times + Y_2 \times (AI - X_1) \times (AI - X_3) \times (AI - X_4) / ( \\
\times (X_2 - X_1) \times (X_2 - X_3) \times (X_2 - X_4) ) \\
\times + Y_3 \times (AI - X_1) \times (AI - X_2) \times (AI - X_4) / ( \\
\times (X_3 - X_1) \times (X_3 - X_2) \times (X_3 - X_4) ) \\
\times + Y_4 \times (AI - X_1) \times (AI - X_2) \times (AI - X_3) / ( \\
\times (X_4 - X_1) \times (X_4 - X_2) \times (X_4 - X_3) )
\]

\[
1700 \quad RETURN \\
\quad END
\]
SUBROUTINE PEMX

C. INPUT AND SETUP ROUTINE
C. SETUP INCLUDES INITIALIZATION, TRANSFORMATION, GRID SET UP, AND
C. FIRST GUESS AT STREAM FUNCTION AND DENSITY VALUES
C. INPUT DEFINITION
C. N = NO. OF INPUT BLADE COORDINATES (SUCTION SURFACE)
C. L = NO. OF INPUT BLADE COORDINATES (PRESSURE SURFACE)
C. K = NO. OF INPUT BLADE COORDINATES (CORD LINE)
C. NN = NOT USED
C. LL = NOT USED
C. MM = NO. OF INPUT STREAM TUBE COORDINATES
C. B = ANGLE BETWEEN BLADES (RAD) = 6.28/BN
C. D = NO. OF F (THETA) GRID INCREMENTS FROM TRAILING TO
C. LEADING EDGE OF BLADE
C. RT = BLADE TIP RADIUS (IN)
C. ENC = LENGTH OF FLOW FIELD INFRONT AND BEHINE BLADE (IN)
C. RP(I) = R OR Z COORDINATE OF PRESSURE SURFACE (I=1,L) (IN)
C. RS(I) = R OR Z COORDINATE OF SUCTION SURFACE (I=1,N) (IN)
C. RC(I) = R OR Z COORDINATE OF CORD LINE (I=1,K) (IN)
C. THETS(I) = THETA COORDINATE OF SUCTION SURFACE (I=1,N) (DEG)
C. THETP(I) = THETA COORDINATE OF PRESSURE SURFACE (I=1,L) (DEG)
C. THETC(I) = THETA COORDINATE OF CORD LINE (I=1,K) (DEG)
C. RD(I) = RADIUS COORD. OF STREAM TUBE CENTER LINE (I=1,MNM) (IN)
C. BD(I) = WIDTH COORD. OF STREAM TUBE RADIAL DIR. (I=1,MNM) (IN)
C. SLAD(I) = SIN(A) COORD. OF STREAM TUBE CENTER LINE (I=1,MNM) (ND)
C. A = ANGLE BETWEEN IMPELLER CO. L. AND STREAM TUBE C.
C. ED(I) = AXIAL COORD. OF STREAM TUBE CENTER LINE (I=1,MNM) (IN)
C. NOTE **** ALL BLADE AND STREAM TUBE COORDINATE DATA STARTS AT
C. AT BLADE TRAILING EDGE AND GOES +VE IN UPSTREAM DIR.
C. KKL = NO. OF POINTS IN RHO-H-P TABLE
C. CONDITIONS OR INTERIOR POINTS DEPENDING ON FLAG (INPUF
C. INPUF = FLAG FOR DIFFERENT INLET ANJ EXIT BOUNDARY CONDITIONS
C. = 0 FOR STREAM LINES PARALLEL TO BLADE CORO LINE
C. = 1 EQUIVALENT TO 2 AT INLET AND 0 AT EXIT
C. = 2 INPUT STREAM FUNCTION AT ALL GRID POINTS
C. = 3 INPUT BOTTOM STREAM FUNCTION VALUE
C. = 4 EQUIVALENT TO
C. BN = NO. OF IMPELLER BLADES
C. WW = PUMP SPEED (RAD/SEC)
C. WDOT = FLOW RATE IN STREAM TUBE ANULUS (LB/SEC)
C. ROHIN = PROPELLANT DENSITY AT INLET (LB/IN**3)
C. PCIN = STATIC PRESSURE AT INLET (LB/IN**2)
C. HOIN = ENTHALPY AT INLET (FT/SEC)
C. VMIN = CIRCUMFERENTIAL FLUID VELOCITY AT INLET (IN/SEC)
C. PS = PROPELLANT VAPOR PRESSURE (LB/IN**2)
C. RCIN = RADIUS OF CENTER OF STREAM TUBE ANULUS AT INLET (IN)
C. = RJ(MNM)
C. RHT(I) = PROPEL DENSITY VALUES NEAR SATURATION (I=1,KKL) (LB/IN3
C. HTH(I) = PROPEL ENTHALPY VALUES REF. TO HOIN (I=1,KKL) (FT/SEC
C. PT(I) = PROPEL PRESSURE VALUES NEAR SATURATION (I=1,KKL) (LB/IN2
C. PSI(I,J) = STREAM FUNCTION VALUES AT GRID POINTS, BOUNDARY
C. CONDITIONS OR INTERIOR POINTS DEPENDING ON FLAG (INPUF)
C

7997 FORMAT(8A10)
7998 FORMAT(1H1,10X,8A10)
7999 FORMAT(10X,8A10)
8000 FORMAT(6IS,F10.4)
8001 FORMAT(4E10.3)
8002 FORMAT(3E10.3)
8003 FORMAT(IS5//,10E8.3)
8004 FORMAT(10F8.3)
8005 FORMAT(IS3//)
8006 FORMAT(10F8.3)
8007 FORMAT(IS3//)
8008 FORMAT(10F8.3)
8009 FORMAT(IS3//)
8010 FORMAT(IS3//)
8011 FORMAT(IS3//)
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8038 FORMAT(IS3//)
8039 FORMAT(IS3//)
8040 FORMAT(IS3//)

C

COMMON / ABC/
* RHOT(100), HT(100), PT(100)
COMMON / CBA/
* ENC
COMMON / ABD/
* NNDG, LNDG, KNDG, IMLNDG, THETMIN, THETMAX, EMIN,
* EMAX, ICNT(99), THETS(100), THETC(100), THETP(100),
* EC1(100), EP2(100), ES1(100), KNT1, KNT2
COMMON / ISIL(150), IP18(150), W(150), X(150), Y(150), Z(150),
* JTHL(150), JT1B(150), PSI(100,100),
* RMS(100), RP(100), ES1(100), RC(100),
* ED(150), RD(150), BZ(150), SIAJ(150), RMM, IL, JL, IIB,
* IIE, B, DELTA, RESIM, DPSIP, KK, DEX, ACC, JLE, BN,
* PSIPR, RHO(100,100), G, WDOT, WW, KHOIN, POIN, H1N,
* ROIN, VH1N, VM0IN, PS, RR(150), BZ(150), SIA(150), RT,
* INP18, RRL, ITRIP, PJEL, JPL

C

READ INPUT DATA
C
READ(5,997) I1, I2, I3, I4, I5, I6, I7, I8
IF (EOF,5) 50, 60
50 CALL EXIT
60 WRITE(6,998) I1, I2, I3, I4, I5, I6, I7, I8
READ(5,997) I1, I2, I3, I4, I5, I6, I7, I8
WRITE(6,999) I1, I2, I3, I4, I5, I6, I7, I8
READ (5,8000) N, L, K, NN, LL, MNM, B, J, RT, ENC, BNC
WRITE (6,8011) N, L, K, NN, LL, MNM, B, D, RI, ENC, BNC
READ (5,8004) (RP(I), I=1, L)
READ (5,8004) (RS(I), I=1, N)
READ (5,8004) (RC(I), I=1, K)
READ (5,8004) (THET(I), I=1, N)
READ (5,8004) (THETP(I), I=1, L)
READ (5,8004) (THETC(I), I=1, K)
C
READ (5,8001) (RD(I), BD(I), SIAD(I), ED(I), I=1, MNM)
READ (5,8003) KKL, INPUF, JOPL, BN, WW, WDOT, ROIN, POIN, * HOIN, VTHIN, VMOIN, PS, ROIN, PSINU, PSIND, PDEL
READ (5,8002) (RHO(I), HT(I), PT(I), I=1, KKL)
C
C. TRANSFORM FROM R,THETA PLANE TO E,THETA PLANE
C. FOR THIS CASE BOTH ARE LINEAR TRANSFORMATIONS
C
DO 100 I = 1, N
ES(I) = RS(I)
THETS(I) = THETS(I) / 57.2958
100 CONTINUE
DO 200 I = 1, L
EP2(I) = RP(I)
THETP(I) = THETP(I) / 57.2958
200 CONTINUE
DO 300 I = 1, K
EC(I) = RC(I)
THETC(I) = THETC(I) / 57.2958
300 CONTINUE
C
C. OVER RIDE BD INPUT
C
CALL TABL ( ES1(N), RRL, ED, RD, 1, 1, MNM )
DELTA = THETC(K) / D
C
WRITE OUT INPUT DATA AND VALUES CALCULATED FROM INPUT
C
WRITE (6,8009) ((RS(I), THETS(I), ES1(I)), I=1, N)
WRITE (6,8018) ((RP(I), THETP(I), EP2(I)), I=1, L)
WRITE (6,8018) ((RC(I), THETC(I), EC(I)), I=1, K)
WRITE (6,8018) ((RD(I), BD(I), SIAD(I), ED(I)), I=1, MNM)
WRITE (6,8018) KKL, INPUF, BN, WW, WDOT, ROIN, POIN, * HOIN, VTHIN, VMOIN, PS, ROIN, PSINU, PSIND
WRITE (6,8017) (RHO(I), HT(I), PT(I), I=1, KKL)
C
THESL = 0 " DELTA
JL = IFIX ( THETP(L) / DELTA ) + 1
THJL = FLOAT ( JL - 1 ) * DELTA
E = - FLOAT ( IFIX ( ENC / DELTA ) + 1 ) * DELTA
IL = IFIX ( ( EC1(K) + ENC ) / DELTA ) + IFIX ( BNC / * DELTA ) + 1

**C**

DO 500 IJK= 1, N
ESI1(IJK) = ESI1(IJK)
500 CONTINUE

**C**

IIB = IFIX ( ENC / DELTA ) + 1
IIE = IFIX ( EC1(K) / DELTA ) + IFIX ( ENC / DELTA ) + 1
RESIM = 0.
DPSIP = PSIPR / 20.
JPTE = IFIX ( B / DELTA ) + 1
JLE = IFIX(D) + 1
PEX = ( FLOAT ( IFIX ( EC1(K) / DELTA ) ) * DELTA ) + *
DELTA - EC1(K)
IILE = IIE + 1
IF ( PEX .EQ. 0. ) GO TO 600
GC TO 700

600 PEX = JELTA
700 Dex = PEX / DELTA
IF ( ( THJL .EQ. THETP(L) ) ) GO TO 800
GO TO 900

**C**

800 JL = JL - 1
THJL = THJL - DELTA
900 THETA = -DELTA
WRITE (6,8014) DELTA, THESL, E, THJL, JL, IL, IIB, IIE, JLE
WRITE (6,8019)

**C**

INCORRECT THEMA AND CALCULATE E DIMENSION BETWEEN GRID POINTS AND **C** BLADE SURFACES

DO 2100 JJ= 1, JL
THETA = THETA + DELTA
IF ( ABS ( 1. - ( THETA / THESL ) ) - .0001 ) 1300, 1300, 1000
1000 IF ( THETA .GT. THESL ) GO TO 1300

**C**

E INCREMENT NEXT TO SUCTION SURFACE = Z(JJ)

**C**

1100 CALL TABL ( THETA, ESLI, THETS, ES1, 2, 1, N )
ISIL(JJ) = IFIX ( ESLI / DELTA ) + IFIX ( ENC / DELTA ) + 1
ESLII = FLOAT ( IFIX ( ESLI / DELTA ) ) * DELTA
IF ( ESLII .EQ. ESLI ) GO TO 1200
GO TO 1200
C
1200 ISIL(JJ) = ISIL(JJ) + 1
ESLII = ESLII - DELTA
GO TO 1200
C
1300 ISIL(JJ) = ISIL(JJ) - 1
ESLII = 0.
ESLI = DELTA
C
1400 IF ( THETA .GT. B ) GO TO 1500
IPIB(JJ) = 2
GO TO 1600
C.
E INCREMENT NEXT TO PRESSURE SURFACE = X(JJ)
C
1500 CALL TABL ( THETA, EPBI, THETP, EP2, 2, 1, L )
IPIB(JJ) = IFIX ( EPBI / DELTA ) + IFIX ( ENC / DELTA ) + 2
EPBII = ( FLOAT ( IFIX ( EPBI / DELTA ) + 1 ) ) * DELTA
1600 IPB = IPIB(JJ)
ISL = ISIL(JJ)
X(JJ) = 1.
Z(JJ) = 1.
DO 2000 IN = IPB, ISL
IF ( IN .EQ. ISIL(JJ) ) GO TO 1700
GO TO 1800
C
1700 Z(JJ) = ( ESLI - ESLII ) / DELTA
GO TO 2000
1800 IF ( ( THETA .GT. B ) .AND. ( IN .EQ. IPIB(JJ) ) ) GO TO 1900
GO TO 2000
C
1900 X(JJ) = ( EPBII - EPBI ) / DELTA
2000 CONTINUE
2100 CONTINUE
WRITE (6, 6020)
C.
INCREMENT E AND I) LOOK UP STREAM TUBE DIMENSIONS, 2) CALCULATE THE
C.
DIMENSIONS BETWEEN GRID POINTS AND BLADE SURFACES, AND 3) PROVIDE
C.
FIRST GUESS OF STREAM FUNCTION AND DENSITY AT GRID POINTS
C
DO 3800 II = 1, IL
W(II) = 1.
Y(II) = 1.
E = E * DELTA
CALL TABL ( E, RRR, ED, RD, 1, 1, MNM )
RR(II) = RRR
CALL TABL ( E, BZZ, EJ, BJ, 1, 1, MNM )
BZ(II) = BZZ
C
BZ(II) = ( BJ(MNM) / ( RRR**2.) ) * ( RD(MNM)*2. )
C
CALL TABL ( E, SJO, EJ, SJAJ, 1, 1, MNM )
SIA(I) = SJOO
IF ( E .GT. 0. ) GO TO 2800
C
C REGION DOWN STREAM OF BLADES
C THETA = -DELTA
C
C EXTRAPOLATE BACK ALONG BLADE CORD LINE
C
CALL TABL ( E, (HEC, EC1, THETC, 1, 1, K )
JTHL(II) = IFIX ( B / DELTA ) + 1
BJL = FLOAT ( JTHL(II) - 1 ) * DELTA
JTHB(II) = 1
IF ( BJL .EQ. B ) GO TO 2200
GO TO 2300

2200 JTHL(II) = JTHL(II) - 1
BJL = B - JELTA
2300 IF ( ( ABS(E) ) .LT. 0.0001 ) GO TO 2400
GO TO 2500

2400 THETA = 0.
JTHB(II) = 2
2500 JTB = JTHB(II)
JTL = JTHL(II)
JJ = 0
DO 2700 JN=JTB, JTL
JJ = JJ + 1
THETA = THETA + JELTA
PSI(II,JJ)= ( THETA - THEC ) * PSIPR / B
RHO(II,JJ)= RHOIN
IF ( JN .EQ. JTHL(II) ) GO TO 2600
GO TO 2700

2600 Y(II) = ( B - BJL ) / DELTA
2700 CONTINUE
GO TO 3000

2800 IF ( E .GT. EC1(K) ) GO TO 3500
C
C REGION BEW EEN BLADES
C
CALL TABL ( E, THEIS, ES1, THETS, 2, 1, N )
C
C THETA INCREMENT NEXT TO PRESSURE SURFACE = Y(II)
C
CALL TABL ( E, THEIP, EP2, THEIP, 2, 1, L )
JTHL(II) = IFIX ( THEIF / DELTA ) + 1
THL = FLOAT ( JTHL(II) - 1 ) * DELTA
IF ( THL .EQ. THEIP ) GO TO 2900
GO TO 3000

2900 THL = THL - DELTA
JTHL(II) = JTHL(II) - 1

C
C THETA INCREMENT NEXT TO SUCTION SURFACE = W(II)
C

3000 JTHB(II) = IFIX ( THEIS / DELTA ) + 2
THB = FLOAT ( JTHB(II) - 1 ) * DELTA
THETA = THR - DELTA
DELT H = THEIP - THEIS
JTB = JTHB(II)
JTL = JTHL(II)
JJ = 0

C

DO 3400 JN= JTB, JTL
JJ = JJ + 1
THETA = THETA + DELTA
PSI(II, JJ) = ( THETA - THEIS ) * PSIPR / DELTH
RHO(II, JJ) = RHOIN
IF ( JN .EQ. JTHB(II) ) GO TO 3100
GO TO 3200

C

3100 W(II) = ( THB - THEIS ) / DELTA
3200 IF ( JN .EQ. JTHL(II) ) GO TO 3300
GO TO 3400

C

3300 Y(II) = ( THEIP - THL ) / DELTA
3400 CONTINUE
GO TO 3B00

C

REGION UPSTREAM OF BLADES
C

3500 JTHL(II) = JL

C

EXTRAPOLATE FORWARD ALONG BLADE CORD LINE
C

CALL TABL ( E, THEC, EC1, THEC, 1, 1, K )
J2 BLADE AND INLET FLOW SLOPE
JTDE = 1.7
DWDU = 3.0
DELD = DWDU - DTDE
IF ( E.LT.(EC1(K) + .5)) THEC = THEC + 1.0*DELD*(E - EC1(K))**2
IF ( E.GE.(EC1(K) + .5)) THEC = THEC + .25*DELD + DELD*(E - EC1(K) -.5)
JTHB(II) = IFIX (O) + 1
THETA = ( O - 1.0 ) * DELTA
JTB = JTHB(II)
JTL = JTHL(II)
JJ = 0

C

DO 3700 JN= JTB, JTL
JJ = JJ + 1
THETA = THETA + DELTA
PSI(II, JJ) = ( THETA - THEC ) * PSIPR / B
RHO(II, JJ) = RHOIN
IF ( JN .EQ. JTHL(II) ) GO TO 3600
GO TO 3700

C

3600 Y(II) = ( THETP(L) - THJL ) / DELTA
3700 CONTINUE
JLIM = JTL - JTB + 1
IF ( INPUF .EQ. 1 ) READ (5,015) ( PSI(I,I,J), JJ = 1, JLIM )
3800 CONTINUE
C
IF ( INPUF .GT. 1 .AND. INPUF .LT. 4 ) GO TO 4000
3900 IF ( INPUF .GE. 3 ) GO TO 4200
4000 PSIUP = PSINU
C
DO 4100 JJ= 1, JLIM
  PSIUP = PSIUP + JELPS
  PSI(I,I,JJ) = PSIUP
4100 CONTINUE
GO TO 3900
C
4200 PSION = PSIND
DO 4300 JJ= 1, JPTE
  PSION = PSION + DELPS
  PSI(I,JJ) = PSION
4300 CONTINUE
C
4400 DO 4600 I = 1, 110
  DO 4500 J = 1, 55
    RHO(I,J) = RHOIN
4500 CONTINUE
4600 CONTINUE
C
RETURN
END
SUBROUTINE RELAX

RELAXATION SUBROUTINE

8001 FORMAT ( 20H ITERATION NUMBER = , I4, 14H RESIM = , F10.4 )
I = *, E15.7, * PCAL = *, E12.5 )
8003 FORMAT ( * RHOCM = *, E12.5, * P = *, E12.5, * IN = *, I5,
* JC = *, I5 )

COMMON /PLT/
* R(100,100), RESIO(100)
COMMON ISIL(150), IPIB(150), W(150), X(150), Y(150), Z(150),
* JTHL(150), JTHB(150), PSI(100,100),
* PS(100), RIP(100), ESI(100), RC(100),
* E(150), R(150), B0(150), SIAO(150), MNM, IL, JL, IIB,
* IL, B, DELTA, RESIM, DPSIP, KK, DEX, ACC, JLE, BN,
* PSIPI, RHQ(100,100), G, WDOT, WW, KKL, RHOIN, POIN, HCIN,
* RCIN, VTHIN, VMQIN, PS, RR(150), BZ(150), SIA(150), RT,
* INPUF, RRL, ITRIP, DDEL, JDPL
COMMON /ABC/
* RHOT(100), H(100), PT(100)
DATA NUM /0/

ARITHMETIC STATEMENT FUNCTIONS FOR INTERPOLATION

AA (DEX) = ( ( DEX / 3. ) + ( 0.5 * DEX ** 2 ) + ( ( DEX ** 3 )
* / 6. ) ) * ( -1. )
BB (DEX) = ( ( 3. * DEX / 2. ) + ( 2. * DEX ** 2 ) + ( ( DEX **
* 3 ) / 2. ) )
CC (DEX) = ( 3. * DEX + ( ( 5. * DEX ** 2. ) / 2. ) + ( ( DEX
* ** 3. ) / 2. ) ) * ( -1. )
DD (DEX) = ( 1. + ( 11. * DEX / 6. ) + ( DEX ** 2. ) + ( ( DEX
* ** 3. ) / 6. )

RESI2 = 1.0000
ILM1 = IL - 1
CKK = 1.
CCG = 1.

100 DPSI = ( -11. * PSI(IL,1) + 18. * PSI(IL-1,1) - 9. * 
# PSI(IL-2,1) + 2. * PSI(IL-3,1) ) / DELTA
VTHC = - CCG * DPSI1 / ( RHQ(IL,1) * BZ(IL) * RR(IL) * 6. )
JENO = 1./(RHQIN*BZ(IL)*RR(IL))**2.
BOIN = JENO * ( ( ( DO1 DO2 ) / DELTA ) ** 2 + ( ( E1 E2
* ) / DELTA ) ** 2 )

DO 3400 JJ = 1, JL
IPB = IPIB(JJ)
ISL = ISIL(JJ)
II = 0

DO 3300 IN = IPB, ISL
II = II + 1
JC = JJ + JTHB(IN) + 1
JCIPI = JJ + JTHB(IN+1) + 1
JCIM1 = JJ - JTHB(IN-1) + 1

C IF ((IN < IIB) .OR. (IN > IIE)) GO TO 1100
200 IF (JJ .EQ. JTHB(IN)) GO TO 300
   GO TO 500

C 300 PSIJ = 0.
   RHIJ = RHO(IN,JC) * (1. + W(IN)) - RHO(IN,JCI) * W(IN)
   IF (IN .EQ. ISI(JJ)) GO TO 400
      GO TO 2100

C 400 PSI = 0.
   RHI = RHO(IN,JC) * (1. + Z(JJ)) - RHO(IN-1,JCIM1) * Z(JJ)
   BIJ = BZ(IN)
   BI = (1. - Z(JJ)) * BZ(IN) + Z(JJ) * BZ(IN+1)
   GO TO 2100

C 500 IF (IN .EQ. IPIB(JJ)) GO TO 600
   GO TO 700

C 600 PSI(IN-1,JCI) = PSI(IN,JCI1)
   BI = BZ(IN+1)
   BI = BZ(IN)
   RHO(IN-1,JCI1) = (1. + X(JJ)) * RHO(IN,JC) - X(JJ) *
      RHO(IN+1,JCI1)
   PSI = PSI(IN,JCI-1)
   RHI = RHO(IN,JCI-1)
   PSI = PSI(IN+1,JCI1)
   RHI = RHO(IN+1,JCI1)
   IF (IN > IIE) GO TO 2100

C 700 IF (JJ .EQ. JTHL(IN)) GO TO 800
   GO TO 900

C 800 PSI(IN,JCI) = PSI(IN,JCI1)
   RHO(IN,JCI) = RHO(IN,JC) * (1. + Y(IN)) - RHO(IN,JCI-1) * Y(IN)
   900 IF (IN .EQ. ISI(JJ)) GO TO 1000
      IF (IN .EQ. IPIB(JJ)) GO TO 2200
      GO TO 2000

C 1000 IF (IN > IIE) GO TO 2000
   PSI = 0.
   BI = (1. - Z(JJ)) * BZ(IN) + Z(JJ) * BZ(IN+1)
   BI = BZ(IN)
   RHI = RHO(IN,JC) * (1. + Z(JJ)) - RHO(IN-1,JCIM1) * Z(JJ)
   RHI = RHO(IN,JC) * (1. + W(IN)) - RHO(IN,JCI+1) * W(IN)
   PSI = PSI(IN,JCI-1)
   GO TO 2300

C 1100 IF (JJ .EQ. JTHB(IN)) GO TO 1200
   GO TO 1600

C 1200 JJJ = JTHL(IN) - JTHB(IN) + 1
   PSI = PSI(IN,JJL) - PSI(IN)
   RHI = RHO(IN,JJL)
   W(IN) = Y(IN)
   IF (IN < IIB) GO TO 1300
C
1300 DEX = 1.0
PSIJ = 4.0 * PSI(IN, JC+1) - 6.0 * PSI(IN, JC+2) + 4.0 *
* PSI(IN, JC+3) - PSI(IN, JC+4)
RHIJ = 4.0 * RHO(IN, JC+1) - 6.0 * RHO(IN, JC+2) + 4.0 *
* RHO(IN, JC+3) - RHO(IN, JC+4)
1400 IF ( ( IN .EQ. ISL ) .AND. ( IN .LT. IIB ) ) GO TO 1500
GO TO 2100
C
1500 PSII = 0.0
RHI = RHO(IN, JC) * ( ( 1.0 + Z(JJ) ) - RHO(IN-1, JCIM1) ) * Z(JJ)
BII = ( ( 1.0 - Z(JJ) ) ) * AZ(IN) + Z(JJ) * BZ(IN+1)
BIJ = BZ(IN)
GO TO 2200
1600 IF ( JJ .EQ. JTHL(IN) ) GO TO 1700
GO TO 2000
C
1700 PSI(IN, JC+1) = PSI(IN, 1) + PSI PR
RHO(IN, JC+1) = RHO(IN, 1)
IF ( IN .LT. IIB ) GO TO 1800
GO TO 1900
C
1800 DEX = Y(1)
PSI(IN, JC+1) = AA(DEX) * PSI(IN, JC-3) + BB(DEX) * PSI(IN, JC-2) + CC(DEX) * PSI(IN, JC-1) + DD(DEX) * PSI(IN, JC)
RHO(IN, JC+1) = AA(DEX) * RHO(IN, JC-3) + BB(DEX) * RHO(IN, JC-2) + CC(DEX) * RHO(IN, JC-1) + DD(DEX) * RHO(IN, JC)
C
1900 IF ( ( IN .EQ. IPIB(JJ) ) .AND. ( IN .GT. IIE ) ) GO TO 200
2000 PSI J = PSI(IN, JC-1)
RHI J = RHO(IN, JC-1)
2100 PSI II = PSI(IN+1, JCP1)
BII = BZ(IN+1)
BIJ = BZ(IN)
RHI = RHO(IN+1, JCIP1)
C
2200 WWW = 1.0
XXX = 1.0
YYY = 1.0
ZZZ = 1.0
IF ( JJ .EQ. JTHL(IN) ) YYY = Y(IN)
IF ( JJ .EQ. JTHB(IN) ) WWW = W(IN)
IF ( IN .EQ. IPIB(JJ) ) XXX = X(JJ)
IF ( IN .EQ. ISIL(JJ) ) ZZZ = Z(JJ)
C
D1 = ZZZ * ( PSI(IN-1, JCIM1) - PSI(IN, JC) ) / 
* ( XXX * ( XXX + ZZZ ) )
D2 = XXX * ( PSI(IN, JC) - PSI II ) / ( ZZZ * ( XXX + 
* ZZZ ) )
E1 = YYY * ( PSI J - PSI(IN, JC) ) / ( WWW * ( YYY + 
* WWW ) )
E2 = WWW * ( PSI(IN, JC) - PSI(IN, JC+1) ) / 
* ( YYY * ( YYY + WWW ) )
C
GO TO 1400

GC TO 1400

C
1300 DEX = 1.0
PSIJ = 4.0 * PSI(IN, JC+1) - 6.0 * PSI(IN, JC+2) + 4.0 *
* PSI(IN, JC+3) - PSI(IN, JC+4)
RHIJ = 4.0 * RHO(IN, JC+1) - 6.0 * RHO(IN, JC+2) + 4.0 *
* RHO(IN, JC+3) - RHO(IN, JC+4)
1400 IF ( ( IN .EQ. ISL ) .AND. ( IN .LT. IIB ) ) GO TO 1500
GO TO 2100
C
1500 PSII = 0.0
RHI = RHO(IN, JC) * ( ( 1.0 + Z(JJ) ) - RHO(IN-1, JCIM1) ) * Z(JJ)
BII = ( ( 1.0 - Z(JJ) ) ) * AZ(IN) + Z(JJ) * BZ(IN+1)
BIJ = BZ(IN)
GO TO 2200
1600 IF ( JJ .EQ. JTHL(IN) ) GO TO 1700
GO TO 2000
C
1700 PSI(IN, JC+1) = PSI(IN, 1) + PSI PR
RHO(IN, JC+1) = RHO(IN, 1)
IF ( IN .LT. IIB ) GO TO 1800
GO TO 1900
C
1800 DEX = Y(1)
PSI(IN, JC+1) = AA(DEX) * PSI(IN, JC-3) + BB(DEX) * PSI(IN, JC-2) + CC(DEX) * PSI(IN, JC-1) + DD(DEX) * PSI(IN, JC)
RHO(IN, JC+1) = AA(DEX) * RHO(IN, JC-3) + BB(DEX) * RHO(IN, JC-2) + CC(DEX) * RHO(IN, JC-1) + DD(DEX) * RHO(IN, JC)
C
1900 IF ( ( IN .EQ. IPIB(JJ) ) .AND. ( IN .GT. IIE ) ) GO TO 200
2000 PSI J = PSI(IN, JC-1)
RHI J = RHO(IN, JC-1)
2100 PSI II = PSI(IN+1, JCP1)
BII = BZ(IN+1)
BIJ = BZ(IN)
RHI = RHO(IN+1, JCIP1)
C
2200 WWW = 1.0
XXX = 1.0
YYY = 1.0
ZZZ = 1.0
IF ( JJ .EQ. JTHL(IN) ) YYY = Y(IN)
IF ( JJ .EQ. JTHB(IN) ) WWW = W(IN)
IF ( IN .EQ. IPIB(JJ) ) XXX = X(JJ)
IF ( IN .EQ. ISIL(JJ) ) ZZZ = Z(JJ)
C
D1 = ZZZ * ( PSI(IN-1, JCIM1) - PSI(IN, JC) ) / 
* ( XXX * ( XXX + ZZZ ) )
D2 = XXX * ( PSI(IN, JC) - PSI II ) / ( ZZZ * ( XXX + 
* ZZZ ) )
E1 = YYY * ( PSI J - PSI(IN, JC) ) / ( WWW * ( YYY + 
* WWW ) )
E2 = WWW * ( PSI(IN, JC) - PSI(IN, JC+1) ) / 
* ( YYY * ( YYY + WWW ) )
C
C IF((IN.EQ.ISL).AND.(ISL.EQ.ILM1).AND.(JC.EQ.3)) GO TO 2210
C GO TO 2200
C2210 BOIN = DENC*(((D1+C2)/DELTA)**2+((E1+E2)/DELTA)**2)
2220 IF ( ITRIP .EQ. 0 ) GO TO 2600
WRSQ = ( WW * RR(IN) ) ** 2
DENO = 1. / ( ROHIN * BZ(IN) * RR(IN)) ** 2.
IF(IN.LT.IIE) GO TO 2300
COIN = BOIN - ((RR(IN)*WW)**2,)
GO TO 2400
C
2300 COIN = BOIN - (( RRL * WW ) ** 2,)
2400 JOIN = COIN + ( POIN * 2. * G / ROHIN )
P = ROHIN * ( DENO * ( ( D1 + D2 ) / DELTA ) ** 2 +
* ( ( E1 + E2 ) / DELTA ) ** 2 ) - WRSQ - DOIN ) * (-1."
* ) / ( G )
WRITE(6,8002)P,IN,JJC,RHO(IN,JJC)
IF((IN.EQ.ISL).AND.(ISL.EQ.ILM1))GO TO 2510
IF(P.LT.POIN) GO TO 2510
GO TO 2500
2500 WRITE(6,8002)P,IN,JJC,RHO(IN,JJC)
C
2500 IF ( P.LT. PS ) GO TO 2700
GO TO 3300
2600 RHIJ = AMAX1 ( RHIJ, 0.0000006 )
RHO(IN,JJC+1) = AMAX1 ( RHO(IN,JJC+1), 0.0000006 )
RHIJ = AMAX1 ( RHIJ, 0.0000006 )
RHO(IN-1,JJC1) = AMAX1 ( RHO(IN-1,JJC1), 0.0000006 )
C
A1 = ( 2. * ( RR(IN) ** 2 ) * WW * BZ(IN) * RHO(IN,JJC)
* SIA(IN) ) * DELTA * DELTA
C1 = ZZZ * ( ALOG ( RHO(IN-1,JJC1) * BIJ ) - ALOG ( RHO(IN
*,JJC) * BZ(IN)) ) / ( XXX * ( XXX + ZZZ ) )
C2 = XXX * ( ALOG ( RHO(IN,JJC) * BZ(IN)) - ALOG ( RHIJ *
* PII ) ) / ( ZZZ * ( XXX + ZZZ ) )
R1 = ( PSI(IN-1,JJC1) - PSI(IN,JJC) ) / ( XXX *
* XXX + ZZZ )
R2 = ( PSI(IN,JJC) - PSIJ ) / ( ZZZ * ( ZZZ + XXX ) )
R3 = ( PSI(IN,JJC+1) - PSI(IN,JJC) ) / ( YYY * ( YYY +
* WMW ) )
R4 = ( PSI(IN,JJC) - PSIJ ) / ( WMW * ( WMW + YYY ) )
C3 = WMW * ( ALOG (RHIJ) - ALOG ( RHO(IN,JJC)) ) / ( YYY *
* YYY + WMW )
C4 = YYY * ( ALOG (RHO(IN,JJC)) - ALOG ( RHO(IN,JJC+1)) ) / *
* ( WMW * ( WMW + WMW ) )
C
R(IN,JJC) = - ( A1 + ( D1 + D2 ) * ( C1 + C2 ) * CCC - 2 * ( R1
* - R2 ) * CCC + ( R3 - R4 ) * CKK ) + ( E1 + E2 ) * ( C3 + C4 ) * CKK )
C
IF ( ITRIP .EQ. 0 ) GO TO 3300
IF(P.GT.POIN) GO TO 3300
C WRITE (6,8002) P, IN, JC, RHO(IN,JJC)
GO TO 3300
C
C
CALL FABOL (P, RHIP, PT, RHOT, 1, 1, KKL)
IF (RHIP.LT..000006) RHIP=.000006
WRITE(6,8002) P, IN, JC, RHIP
RHOO(IN, JC) = RHIP
GO TO 3300
GO TO 2710

CALL RELAX
IF (ABS((RHIP/RHOD(IN, JC)) -.10).LT.001) GO TO 2710
RHOD(IN, JC) = .5*(RHOD(IN, JC)+RHIP)
RHOCM = RHOD(IN, JC)
DENO = 1./(RHOCM*ZB(IN)*RR(IN))**2
P = RHOCM * ( DENO * ( ( (D1 + D2) / DELTA ) ** 2 + ( (E1 + E2) / DELTA ) ** 2 ) - WRSQ - DOIN ) * (-1.)
* /
GO TO 2700
2705 WRITE (6,8002) P, IN, JC, RHIP
GO TO 2600
2710 WRITE (6,8002) P, IN, JC, RHOD(IN, JC)
GO TO 2600
IF (RHIP .LE. 0.) RHIP = 0.000006

CALL RELAX
2800 RHOCM = RHIP
IF (ITRIP .EQ. 1.) RHOD(IN, JC) = RHOCM
WRITE (6,8003) RHOCM, P, IN, JC
IF (ITRIP .EQ. 1.) GO TO 3300

DENO = 1./(RHOCM*ZB(IN)*RR(IN))**2
PCAL = RHOCM * ( DENO * ( ( (D1 + D2) / DELTA ) ** 2 + ( (E1 + E2) / DELTA ) ** 2 ) - WRSQ - DOIN ) * (-1.)
* /
GO TO 2900
2900 WRITE (6,8002) P, IN, JC, RHOD(IN, JC), PCAL

3000 RHOD(IN, JC) = RHOIN - 0.05 * (RHOIN(IN, JC) - RHOCM)
IF (RHC(IN, JC) .LT. 0.00002) RHOD(IN, JC) = RHOTN
GO TO 2600

3100 IF (RHOCM .LE. 0.) RHOCM = 0.000006
3200 RHOD(IN, JC) = RHOCM
GO TO 2600
3300 CONTINUE
3400 CONTINUE
IF (ITRIP .NE. 0) GO TO 4800

3500 DO 4300 JJ = 1, JL
IPB = IPIB(JJ)
ISL = ISIL(JJ)
II = 0

DO 4200 IN = IPB, ISL
II = II + 1
WHW = t
XXX = 1.0
YYY = 1.0
ZZZ = 1.0

IF ( JJ .EQ. JTHL(IN) ) YYY = Y(IN)
IF ( JJ .EQ. JTHB(IN) ) WWW = W(IN)
IF ( IN .EQ. ISIL(JJ) ) XXX = X(JJ)
IF ( IN .EQ. ISIL(JJ) ) ZZZ = Z(JJ)

JC = JJ - JTHB(IN) + 1
JCIPI = JJ - JTHB(IN+1) + 1
JCIM1 = JJ - JTHB(IN-1) + 1
DPSI = R(IN,JC) / (( 1. / WWW ) + ( 1. / XXX ) + ( 1. / YYY )
R(IN,JC) = 0.
PSI(IN,JC) = FSI(IN,JC) + DPSI

IF ( ( IN .LT. IIA ) .OR. ( IN .GT. IIE ) ) GO TO 3600
IF ( ( JJ .EQ. JTHB(IN) ) .AND. ( IN .EQ. ISIL(JJ) ) ) GO TO 4100
IF ( JJ .EQ. JTHB(IN) ) GO TO 4000
IF ( IN .EQ. ISIL(JJ) ) GO TO 4100
GO TO 3900

3600 IF ( JTHB(IN) .EQ. JJ ) GO TO 3700
IF ( JTHL(IN) .EQ. JJ ) GO TO 3800
GO TO 3900

3700 JHL = JTHL(IN) - JTHB(IN) + 1
JB = JTHL(IN)
R(IN,JHL) = R(IN,JHL) + DPSI * ( 1. / YYY )
IF ( IN .EQ. ISIL(JJ) ) GO TO 4100
GO TO 4000

3800 JJH = JTHB(JJ)
R(IN,1) = R(IN,1) + DPSI
R(IN,JH-1) = R(IN,JH-1) + DPSI * ( 1. / WWW )
GO TO 4000

3900 R(IN,JH-1) = R(IN,JH-1) + DPSI * ( 1. / WWW)
4000 R(IN+1,JCIPI) = R(IN+1,JCIPI) + DPSI * ( 1. / ZZZ )
4100 R(IN,JH+1) = R(IN,JH+1) + DPSI * ( 1. / YYY )
R(IN-1,JCIM1) = R(IN-1,JCIM1) + DPSI * ( 1. / XXX )
4200 CONTINUE
4300 CONTINUE

ILM1 = IL - 1
DO 4500 II = 2, ILM1
RESID(II) = 0.
JJ = 0
JTB = JTHB(II)
JTL = JTHL(II)

DO 4400 JN = JTB, JTL
JJ = JJ + 1
IF ( ( ABS(R(II,JJ) ) ) .GT. RESID(II) ) RESID(II) = ABS(R(II,JJ))
4400 CONTINUE
4500 CONTINUE
C
   DO 4600 II = 2, ILM1
   IF ( RESID(II) .GT. RESIM ) RESIM = RESID(II)
4600 CONTINUE
   IF ( RESIM .GT. RESI2 ) STOP
   RESI2 = 2.0 * RESIM
   IF ( RESIM .GT. ACC ) GO TO 4700
   GO TO 4800
C
4700 NUM = NUM + 1
   WRITE (6,8001) NUM, RESIM
   RESIM = 0.
   IF ( NUM .EQ. 50 ) GO TO 4710
   GO TO 100
4710 CALL PREWRT
   CALL WRTOUT
   GO TO 100
C
4800 NUM = NUM + 1
   WRITE (6,8001) NUM, RESIM
   RESIM = 0.
   RETURN
END
SUBROUTINE PREWRT

ROUTINE TO CALCULATE DPSIDO, DPSIOW, AND PHI VALUES BEFORE GC INTO WRTOUT

6001 FORMAT (// /*+++++++ VOLUME = *, E12.5 )

COMMON ISIL(150), IPIB(150), W(150), X(150), Y(150), Z(150),
* JTHL(150), JTHB(150), PSI(100,100),
* RS(100), RP(100), ES1(100), RC(100),
* ED(150), RD(150), BD(150), SIAD(150), MNM, IL, JL, IIB,
* IE, B, DELTA, RESIM, DPSIP, KK, DEX, ACC, JLE, BN,
* PSIPR, RHO(100,100), G, WDOT, MW, KKL, RHOIN, POIN, HOIN,
* ROIN, VTHIN, VPOIN, PS, RR(150), BZ(150), SIA(150), RT,
* INPMF, RRL, ITRIP, PDEL, JOPL

COMMON / CEA /
* ENC
COMMON / NDG/
* NNJG, LNJG, KNJG, ILNJG, THETMIN, THETMAX, EMIN,
* EMAX, ICNT(99), THETS(100), THETC(100), THETP(100),
* EC1(100), EP2(100), ESS1(100), KNT1, KNT2

HDELTA = DELTA * 0.5
SUM = 0.
ILL = IL - 1
E = -(FLOAT (IFIX (ENC / DELTA )) ) * DELTA ) - DELTA

10 3000 II= 1, ILL
E = E + DELTA
JTB = JTHB(II)
JTL = JTHL(II)
JJ = 0

20 2900 JN= JTB, JTL
JJ = JJ + 1

2100 COSA = COS (ASIN (SIA(II) ))
THETA = FLOAT (JN-1) * DELTA
IF (II .LE. IIB ) GO TO 2200
IF (JN .EQ. JTB ) GO TO 2400
IF (JN .EQ. JTL ) GO TO 2600

VOLUME = DELTA / COSA * DELTA * RR(II) * BZ(II)
GO TO 2800

2200 DELT = DELTA
DELE = DELTA
IF (II .EQ. 1 ) DELE = HDELTA
IF (JN .EQ. JLT ) DELT = HDELTA + DELTA * Y(1)
IF (JN .EQ. 1 ) DELT = HDELTA
VOLUME = DELE / COSA * RR(II) * DELT * BZ(II)
GO TO 2800

2300 DELE = DELTA
IF (II .EQ. ISIL(1) ) DELE = HDELTA + DELTA * Z(1)
VOLUME = DELTA / COSA * RR(II) * HDELTA * BZ(II)
GO TO 2800

C 2400 IF ( II .GT. IIE ) GO TO 2500
EU1 = E + HDELTA
EL1 = E - HDELTA
CALL TABL ( EU1, THETU1, ESS1, THET, 2, 1, NNDG )
CALL TABL ( EL1, THETL1, ESS1, THET, 2, 1, NNDG )
C THETU2 = THETA + HDELTA
THETL2 = THETA - HDELTA
CALL TABL ( THETU2, EU2, THET, ESS1, 2, 1, NNDG )
CALL TABL ( THETL2, EL2, THET, ESS1, 2, 1, NNDG )
C EU1C = AMIN1 ( EU1, EU2 )
THE1LC = AMIN1 ( THETU1, THETU2 )
EHGT = ( EU1C - EL1 ) / COSA
TLNGTH = ((( THETU2 - THE1LC ) + ( THETU2 - THETL1 ) ) / 2.0
VOLUME = EHGT * RR(II) * BZ(II) * TLNGTH
GO TO 2800
C 2500 VOLUME = DELTA / COSA * RR(II) * HDELTA * BZ(II)
GO TO 2800
C 2600 IF ( II .GT. IIE ) GO TO 2700
EU1 = E + HDELTA
EL1 = E - HDELTA
CALL TABL ( EU1, THETU1, EP2, THETP, 2, 1, LNDG )
CALL TABL ( EL1, THETL1, EP2, THETP, 2, 1, LNDG )
C THETU2 = THETA + HDELTA
THETL2 = THETA - HDELTA
CALL TABL ( THETU2, EU2, THETP, EP2, 2, 1, LNDG )
CALL TABL ( THETL2, EL2, THETP, EP2, 2, 1, LNDG )
C EU1C = AMAX1 ( EL1, EL2 )
THE1LC = AMAX1 ( THETL1, THETL2 )
EHGT = ( EU1 - EU1C ) / COSA
TLNGTH = ((( THETU1 - THE1LC ) + ( THE1LC - THETL2 ) ) / 2.0
VOLUME = EHGT * RR(II) * TLNGTH * BZ(II)
GO TO 2800
C 2700 DELT = HDELTA + DELTA * Y(II)
VOLUME = DELTA / COSA * RR(II) * DELT * BZ(II)
C 2800 VOLUME = VOLUME * RHO(II, JJ)
SUM = SUM + VOLUME
2900 CONTINUE
3000 CONTINUE
WRITE (6, 8001) SUM
RETURN
END
SUBROUTINE WRTOUT

C ROUTINE TO PROCESS AND PRINT OUTPUT

C

C

8001 FORMAT (5H E = ,F10.4,7H I = ,I4,7H R = ,F10.4,7H B = ,F10.4,9H SIA = ,F10.4)

8002 FORMAT(1X6HPSI = F10.4,10H THETA = F10.4,8H PHI = F10.4,6H V = F10.4,6H U = F10.4)

8003 FORMAT( * IJK IS GREATER THAN 25 * )

C

COMMON ISIL(150), IP1B(150), W(150), X(150), Y(150), Z(150),
* JTHL(150), JTHB(150), PSI(100,100),
* RS(100), RP(100), ESI(100), RC(100),
* ED(150), RD(150), BD(150), SIAO(150), MNM, IL, JL, IIB,
* IIE, B, DELTA, RESIM, DPSIP, KK, DEX, ACC, JLE, BN,
* PSIPR, RHO(100,100), G, WDOT, WW, KKL, RHOIN, POIN, HOIN,
* RCIN, VTHIA, VMOIN, PS, RR(150), BZ(150), SIA(150), RT,
* INPUF, RRL, ITIP, PDEL, JDPL

COMMON /FLT/
* THTA(100)
COMMON /CRA/
* ENC
COMMON /NG/  
* NNDG, LNDG, KNDG, ILNDG, THETMIN, THETMAX, EMIN,
* EMAX, ICNT(99), THETS(100), THETC(100), THETP(100),
* EC1(100), EP2(100), ESI(100), KNT1, KNT2

C

ARITHMETIC STATEMENT FUNCTIONS FOR INTERPOLATION

C

AA (DEX) = ( ( DEX / 3. ) + ( 0.5 * DEX ** 2 ) + ( ( DEX ** 3 ) / 
* 6. ) ) * ( -1. )
BB (DEX) = ( ( 3. * DEX / 2. ) + ( 2. * DEX ** 2 ) + ( ( DEX ** 3 * 
) / 2. ) )
CC (DEX) = ( 3. * DEX + ( ( 5. * DEX ** 2 ) / 2. ) + ( ( DEX ** 
+ 3. ) / 2. ) ) * ( -1. )
OD (DEX) = ( 1. + ( 11. * DEX / 6. ) + ( DEX ** 2. ) + ( ( DEX ** 
+ 3. ) / 6. ) )

C

E = -( FLOAT ( IFIX ( ENC / DELTA ) ) ) * DELTA ) - DELTA
EMIN = E + DELTA
EC1 = EC1(KNDG)
THETCC = THETC(KNDG)
THETPP = THETP(LNDG)

C

IJKNDG = 160
SIMIN = -1.05 * PSIPR
KNT1 = 0
KNT2 = 0
LFILE = 9
ILL = IL - 1

C

DO 700 II = 1, ILL
E = E + DELTA
CALL TABL ( E, RPRNT, ED, RD, J, 1, MNM )
CALL TABL ( E, BPRNT, ED, BD, J, 1, MNM )

700 CONTINUE
CALL TABL (E, SIAPRT, EO, SIAD, 3, 1, MNN);

KKK = 0
LLL = 0
JTB = JTHB(II)
JTL = JTHL(II)
JJ = 0
MM = 0

DO 500 JN= JTB, JTL
IC = II - IPIB(JN) + 1
JJ = JJ + 1
MM = MM + 1
IF ( (II .LT. IIIB) .OR. (II .GT. IIE ) ) GO TO 300
IF ( JN .EQ. JTB ) GO TO 100
GO TO 300

100 THTA(MM) = ( (FLOT(JN-1)) - W(II) ) * DELTA
DEX = W(II)
RC(MM) = AA(DEX) * PSI(II, JJ+3) + BB(DEX) * PSI(II, JJ+2) *
* CC(DEX) * PSI(II, JJ+1) + DD(DEX) * PSI(II, JJ)
MM = MM + 1
GO TO 300

200 MM = MM + 1
THTA(MM) = ( (FLOT(JN-1)) + Y(II) ) * DELTA
DEX = Y(II)
RC(MM) = AA(DEX) * PSI(II, JJ-3) + BB(DEX) * PSI(II, JJ-2) *
* CC(DEX) * PSI(II, JJ-1) + DD(DEX) * PSI(II, JJ)
GO TO 400

300 THTA(MM) = (FLOT(JN-1)) * DELTA
RC(MM) = PSI(II, JJ)
IF ( JN .EQ. JTL ) GO TO 200
400 K = MM
500 CONTINUE

PSIP = SIMIN
JJK = 0

DO 600 IJK= 1, IJKNDG
PSIP = PSIP + DPSIP
IF ( PSIP .LT. -0.0001 .AND. E .LT. ECC1 ) GO TO 600
IF ( FSIP .GT. PSIPR .AND. E .GT. 0. ) GO TO 700

CALL TABL (PSIP, THETA, RC, THTA, 3, 1, K)
IF ( THETA .LT. 0. .OR. THETA .GT. THETPP ) GO TO 600
IF ( E .GE. ECC1 .AND. THETA .LT. THETCC ) GO TO 600
IF ( E .LE. ESSI(1) .AND. THETA .GT. THETP(1) ) GO TO 600

IF ( MOD(IJK,2) .EQ. 0 ) GO TO 600

IFILE = IFIX ( (PSIP + PSIPR) / (DPSIP * 2.0) + 9.25)
ICNT(IFILE) = ICNT(IFILE) + 1
IF ( FILE .EQ. 44 ) IFILE = 2
CALL ALTFILE (LFILE, IFILE, NOUM)
LFILE = IFILE
WRITE (IFILE) E, THETA, PSIP
600 CONTINUE
700 CONTINUE

CALL ALTFILE (LFILE, 9, NOUM)
EMAX = E
CALL PLOTT
KK = 1
RETURN
END
SUBROUTINE PLOT

SUBROUTINE TO PLOT OUTPUT DATA

COMMON /PLT/
* E(200), THETA(200), PSI(200)
COMMON /NODG/
* NNDG, LNDG, KNDG, ILNDG, THETMIN, THETMAX, EMIN,
* EMAX, ICNT(99), THETS(100), THETC(100), THETP(100),
* EC1(100), EP2(100), ES1(100), KNT1, KNT2

DATA KNTO / 0 /
IF ( KNTO .NE. 0 ) GO TO 100
CALL INIT280
KNTO = 1
100 CONTINUE

EMIN = AMIN1 ( EMIN, ES1(1), EP2(1), EC1(1) )
EMAX = AMAX1 ( EMAX, ES1(LNDG), EP2(LNDG), EC1(KNDG) )
THETPP = THETP(1)
CALL SETUP

DO 200 I = 1, NNDG
   E(I) = THETS(I) + THETPP
200 E(I+100) = EMAX - ( ES1(I) - EMIN )
DO 300 I = 1, LNDG
   THETA(I) = THETP(I) - THETPP
300 THETA(I+100) = EMAX - ( EP2(I) - EMIN )
DO 400 I = 1, KNDG
400 PSI(I) = EMAX - ( EC1(I) - EMIN )

DO 500 I = 1, 2
   CALL LINES ( E(I), THETP(I), NNDG, LNDG, KNDG )
   CALL LINES ( THETA(I), THETP(I), KNDG )
   CALL LINES ( PSI(I), THETC(I), KNDG )
   CALL LINES ( E(I), E(I), NNDG )
   CALL LINES ( THETA(I), THETA(I), LNDG )
   CALL LINE ( E(I), THETPP, EMAX, THETPP )
   CALL LINE ( EMAX, THETPP, EMAX, THETMIN )
   CALL LINE ( EMIN, THETP(LNDG), THETA(LNDG+100), THETP(LNDG) )
   CALL LINE ( EMIN, THETC(KNDG), PSI(KNDG), THETC(KNDG) )
500 CONTINUE

LFILE = 9
DO 800 I = 9, 99
   IJK = ICNT(I)
   ICNT(I) = 0
   IF ( IJK .EQ. 0 ) GO TO 800
   II = I
   IF ( I .EQ. 44 ) II = 2
   CALL ALTFILE ( LFILE, II, NDUM )
   LFILE = II
   REWIN
800 IF ( IJK .EQ. 1 ) GO TO 800
DO 600 J = 1, IJK
READ (II) E(J), THETA(J), FSI(J)
E(J) = EMAX - ( E(J) - EMIN )
600 CONTINUE
REWIND II
C
DO 700 J = 1, 2
CALL LINES ( E(1), THETA(1), IJK )
700 CONTINUE
C
800 CONTINUE
CALL ALTFILE ( LFILE, 9, NNUM )
CALL FRAME
RETURN
END
SUBROUTINE SETUP

SUBROUTINE TO SETUP GRID FOR PLOTS

COMMON /NDG/
*  NMDG, LMDG, KMDG, ILMDG, THTMIN, THTMAX, EMIN,
*  EMAX, ICNT(99), THETS(100), THEC(100), THEP(100),
*  ECL(100), EP2(100), ES1(100), KNI1, KNI2

CALL CHAROPT ( 0, 0, 1, 0, 0 )
CALL LINEOPT ( 0, 1 )
CALL ABSBEAM ( .15, .993 )
CALL ABSVECT ( .15, .214 )
CALL ABSVECT ( .930, .214 )

CALL LINEOPT ( 0, 0 )
CALL MAP ( EMIN, EMAX, THTMIN, THTMAX, .15, .930, .214, .993 )
DO 100 K = 1, 2
CALL ABSBEAM ( .5, .15 )
CALL SYMBOL ( 3HE$ )
CALL ABSBEAM ( .05, .6 )
CALL SYMBOL ( 3HF$ )
CALL ABSBEAM ( .4, .08 )
100 CALL SYMBOL ( 2THCOORDINATES IN E, F PLANES$ )
RETURN
END
APPENDIX F

Input Data Interpolation Program

For all of the turbopump inducers analyzed in this study, blade geometrical data was derived from inducer design drawings. This data is normally tabulated on the drawing for only a few blade sections at a constant distance off the inducer hub. A computer program was written to provide input data for additional blade sections located at a constant percent of blade span. Figure F.1 illustrates this procedure. Sections $R_1$ and $R_2$ represent typical blade sections for which blade geometrical data is supplied. This data is linearized to yield associated geometry for blade sections at 10%, 30%, 50%, 70%, and 90% of the blade span, shown as dashed lines in this figure.

Figures F.2 through F.6 show a typical sequence of data manipulation for the J-2 LOX inducer. Figure F.2 is a plot of the tabulated data given on the inducer design drawing (Figure 5.1). This data is normalized to the blade leading edge (Figure F.3), non-dimensionalized (Figure F.4), and linearly interpolated (Figure F.5) to yield the required input data for the five blade sections (Figure F.6). The final form of the data is then punched on computer cards for input to the turbopump cavitation compliance program.

The following pages of this appendix present a listing of the input data interpolation program. The liberal use of comment cards makes the program operation self-explanatory. Sample input/output listings are also provided.
PROGRAM BLADE2(INPUT,OUTPUT,TAPES=INPUT,TAPES=OUTPUT,FILMPL,PUNCH)BD2 2
C.

PROGRAM TO INTERPOLATE BLADE INPUT DATA FOR TWO SECTIONS BD2 4
C.

DATA STATEMENTS BD2 6
C.

DIMENSION TIT1(53), TIT2(53), X(50), Y11(50), Y12(50), Y21(50), Y22(50),
Y30M(50), Y31M(50), Y20Y45M(50), Y40M45M(50), X1(50), X2(50), BD2 10
X2(50), XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5),
XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5), XLEDG(5), BD2 12
X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), X1(51), BD2 14
1, Y11(51), Y12(51), Y21(51), Y22(51), Y20Y45M51, Y40M45M51, BD2 16
X1(51), X1(51), X1(51), X1(51), X1(51), BD2 18
C.

DEFINITION OF INPUT DATA BD2 22
C.

CARD 1 TITLE = TITLE COMMON TO ALL CASES (5A11 FORMAT) BD2 24
C.

CARD 2 TITLE2 = TITLE PEDEUAR TO EACH CASE (8A10 FORMAT) BD2 26
C.

CARD 3 PUNCH=FLAG TO CONTROL PUNCHING OF OUTPUT DATA BD2 28
C.

INTERP=FLAG TO CONTROL INTERPOLATION OF INPUT DATA BD2 30
C.

CARD 4 ALF1,ALF2,FSR,HL,HX (5F10.4 FORMAT) BD2 32
C.

ALF1 = HALF-CONE ANGLE OF HUB (DEG) BD2 34
C.

ALF2 = ANGLE OF BLADE LEADING EDGE LARGE (DEG) BD2 36
C.

FSR = FIELDLINE UPSTREAM RADIUS (IN) BD2 38
C.

HL = FORWARD HUB RADIUS (IN) BD2 40
C.

HX = HUB EQUVALENT LENGTH (IN) BD2 42
C.

CARD 5 S1,S2,XSLFC (5F10.4 FORMAT) BD2 44
C.

S1 = LOCATION OF INNERMOST BLADE SECTION (IN) BD2 46
C.

S2 = LOCATION OF OUTERMOST BLADE SECTION (IN) BD2 48
C.

XSLFC=FLAG TO CORRECT INPUT DATA IF Z MEASURED BD2 50
C.

PARALLEL TO CENTERLINE (NO) BD2 52
C.

CARD 6 BNUM,DSING,ENC,BNC,STWNOM (5F10.4 FORMAT) BD2 54
C.

BNUM = NUMBER OF BLADES BD2 56
C.

DSING = NUMBER OF GRID INCREMENTS BD2 58
C.

ENC = UPSTREAM EXTENSION OF FLOWFIELD SOLUTION BD2 60
C.

B12=DOWNSTREAM EXTENSION OF FLOWFIELD SOLUTION BD2 62
C.

STWNOM= NINAL WIDTH OF STREAMTUBE BD2 64
C.

CARD 1 X,Y11,Y12,Y21,Y22 (5F11.4 FORMAT) BD2 66
10 READ (5,P55) TIT1 BD2 68
10 READ (5,P55) TIT2 BD2 70
IF (TITLEZT1,STOP) GO TO 205 BD2 72
READ (5,280) PUNCH,INTERP BD2 74
READ IN INDUCER GEOMETRY DATA BD2 76
READ (5,260) ALFL,ALF2,FSR,HL,HX BD2 78
READ(5,260) ALFL,ALF2,FSR,HL,HX BD2 80
RAD=57.29578 BD2 82
ANGL1=ALF1/RAD BD2 84
ANGL2=ALF2/RAD BD2 86
TMLTAN=ABS(TAN(ANGL1)) BD2 88
TMLCSN=ABS(COS(ANGL1)) BD2 90
FSL=FSR-HL BD2 92
HI=HX-TMLTAN+HL BD2 94
FST=FSL+HI BD2 96
READ (5,260) S1,S2,XSLFC BD2 98
C.

READ IN INDUCER ANALYSIS DATA BD2 100
READ (5,260) BNUM,DSING,ENC,BNC,STWNOM BD2 102
DO 15 I=1,50 BD2 104
READ (5,260) X1(I),Y11(I),Y12(I),Y21(I),Y22(I) BD2 106
IF (X1(I)=0.0) GO TO 20 BD2 108
CONTINUE BD2 110
20 NPTS=I-1 BD2 112
C.

ALL INPUT COMPLETE - PRODUCE ECHO PRINTOUT BD2 114
WRITE (6,265) TIT1,TIT2 BD2 116
WRITE (6,273) S1,S2 BD2 118
C. CORRECT INPUT DATA FOR Z MEASURED PARALLEL TO CENTERLINE
   DO 25 I = 1, NPTS
   Y11(I) = Y11(I) / TM.CSN
   Y12(I) = Y12(I) / TM.CSN
   Y21(I) = Y21(I) / TM.CSN
   Y22(I) = Y22(I) / TM.CSN

25 CONTINUE
   WRITE (6, 265) TIT1, TIT2
   WRITE (6, 211)
   WRITE (6, 270) S1, S2
   WRITE (6, 260) (X(I), Y11(I), Y12(I), Y21(I), Y22(I), I = 1, NPTS)

30 CONTINUE
   TP1 = X(NPTS)
   T Y1 = Y12(NPTS)
   TP2 = X(NPTS)
   T Y2 = Y22(NPTS)
   WRITE (6, 275) TP1, TY1, TP2, TY2
   ECHO OF INPUT COMPLETE

C. LINEARIZE R/Z VERSUS ANGULAR DISPLACEMENT
   RMAX = FSL / TMLCSN
   RMIN = FST / TMLCSN
   XLSQ1(1) = X(1)
   XLSQ2(1) = X(NPTS)
   YLSQ1(1) = RMAX
   YLSQ2(1) = RMIN
   CALL LSQ (2, XLSQ, YLSQ, RAVG, RAVG, RSLOPE, PE, SRSQ)

3. LINEARIZATION COMPLETE *** R(Phi) = RAVG + RSLOPE * PHI

C. PLOT INPUT DATA NON-NORMALIZED
   CALL PLOTRI (X, Y11, Y12, Y21, Y22, NPTS, TIT1, TIT2, 1)

C. NORMALIZE ALL INPUT TO TRAILING EDGE COORDINATES
   DO 35 I = 1, NPTS
      JEND = NPTS + 1 - I
      X1(I) = X(NPTS) - X(JEND)
      Y1DU1(I) = Y12(NPTS) - Y11(JEND)
      Y2DU1(I) = Y12(NPTS) - Y12(JEND)
      Y3DU1(I) = Y22(NPTS) - Y21(JEND)
      Y4DU1(I) = Y22(NPTS) - Y22(JEND)
35 CONTINUE
   DO 40 I = 1, NPTS
      X1(I) = X1(I)
      Y11(I) = Y1DU1(I)
      Y12(I) = Y2DU1(I)
      Y21(I) = Y3DU1(I)
      Y22(I) = Y4DU1(I)
40 CONTINUE
   WRITE NORMIALIZED INPUT DATA
   WRITE (6, 265) TIT1, TIT2
   WRITE (6, 211)
   WRITE (6, 260) (X(I), Y11(I), Y12(I), Y21(I), Y22(I), I = 1, NPTS)
   C. NORMALIZATION COMPLETE

C. PLOT NORMIALIZED INPUT DATA
   CALL PLOTRI (X, Y11, Y12, Y21, Y22, NPTS, TIT1, TIT2, 2)
   IF (INTERP.L.E.0) GO TO 150

C.
C. NON-DIMENSIONALIZE ALL INPUT DATA WITH RESPECT TO CHORD LENGTH

DO 45 I=2,NPTS
   IF (Y11(I)*EQ.Y12(I)) GO TO 50
   CONTINUE
50 NPARI S1=1
   DO 55 I=2,NPTS
      IF (Y21(I)*EQ.Y22(I)) GO TO 50
   CONTINUE
   NPARI S2=1
55 CONTINUE
DO 65 I=1,NPTS
   XI(I)=X(I)/XI(NPAIRS1)
   X2(I)=X(I)/XI(NPAIRS2)
   IF (X2(I).GT.1.) X2(I)=1.0
   CONTINUE
65 CONTINUE

C. STORE LEADING EDGE AXIAL COORDINATES FOR LATER CURVE FIT
YLDOL=Y11(NPAIRS1)
YLOG2=Y21(NPAIRS2)

C. NON-DIMENSIONALIZE ALL INPUT DATA WITH RESPECT TO AXIAL CJORD

DO 70 I=1,NPTS
   Y11(I)=Y11(I)/Y11(NPAIRS1)
   Y12(I)=Y12(I)/Y12(NPAIRS1)
   Y21(I)=Y21(I)/Y21(NPAIRS2)
   Y22(I)=Y22(I)/Y22(NPAIRS2)
   IF (Y21(I).GT.1.) Y21(I)=1.
   IF (Y22(I).GT.1.) Y22(I)=1.
70 CONTINUE

C. WRITE NON-DIMENSIONALIZED INPUT DATA
WRITE (6,265) TITL1,TITL2
WRITE (6,220)
WRITE (6,255) (XL(I),Y11(I),Y12(I),X2(I),Y21(I),Y22(I),I=1,NPTS)

C. PLOT NON-DIMENSIONALIZED DATA
CALL PLOTR2 (X1,Y11,Y12,X2,Y21,Y22,NPAIRS1,NPAIRS2,TITL1,TITL2,1)

C. NON-DIMENSIONALIZATION IS NOW COMPLETE

C. BEGIN INTERPOLATION SCHEME

YT11(I)=Y11(I)
YT12(I)=Y12(I)
YT21(I)=Y21(I)
YT22(I)=Y22(I)
NSEX=51
DO 95 J=2,NSEX
   XT(J)=XT(J-1)+0.02*XTINC
95 CONTINUE

C. WRITE NON-DIMENSIONALIZED INPUT DATA
WRITE (6,265) TITL1,TITL2
WRITE (6,220)
WRITE (6,255) (XT(I),YT11(I),YT12(I),XT(I),YT21(I),YT22(I),I=1,NSEX)

C. CALL PLOTR2 (XT,YT11,YT12,XT,YT21,YT22,NSEX,NSEX,TITL1,TITL2,2)
RSTREAM(2) = RNAUT + KSLPE * XT(I) * X(NPAIRS2)
RAD(1) = S1 / RSTREAM(1)
RAD(2) = S2 / RSTREAM(2)
YLSQ(1) = Y11(I)
YLSQ(2) = Y21(I)
CALL LSQ1(2, RAD, YLSQ1, YZER1, YSLP1, PE, SRSQ)
YLSQ(1) = Y12(I)
YLSQ(2) = Y22(I)
CALL LSQ1(2, RAD, YLSQ, YZER2, YSLP2, PE, SRSQ)
DO 100 J = 1, 5
RING = J - 1
RADIUS = J + 20 * RING
Y1(J, I) = YZER1 + YSLP1 * RADIUS
Y2(J, I) = YZER2 + YSLP2 * RADIUS
100 CONTINUE
CONTINUE
WRITE (6, 265) TITI1, TITL2
WRITE (6, 235)
DO 110 I = 1, NSEX
WRITE (6, 243) (XT(I), Y1(J, I), Y2(J, I), J = 1, 5)
110 CONTINUE
CONTINUE

C. PLOT NON-DIMENSIONALIZED DATA FOR INTERPOLATED BLADE SECTIONS
C.
CALL PLOT3(XT, Y1, Y2, NSEX, TITI1, TITL2, 1)
C.
LS2 FIT TO LEADING EDGE COORDINATES
C.
YLSQ(1) = X(NPAIRS1)
YLSQ(2) = X(NPAIRS2)
RSTREAM(1) = RNAUT + RSLPE * X(NPAIRS1)
RSTREAM(2) = RNAUT + RSLPE * X(NPAIRS2)
XLSQ(1) = S1 / RSTREAM(1)
XLSQ(2) = S2 / RSTREAM(2)
CALL LSQ1(2, XLSQ, YLSQ, YLEZ, YLES, PE, SRSQ)
DO 115 J = 1, 5
XM = J - 1
XINC = 10 + 20 * XM
YLEDG(J) = YLEZ + YLES * XINC
115 CONTINUE
CONTINUE
C.
LS2 FIT TO LEADING EDGE AXIAL COORDINATES
YLSQ(1) = YLOG1
YLSQ(2) = YLOG2
CALL LSQ1(2, XLSQ, YLSQ, YLDZ, YLOS, PE, SRSQ)
DO 120 J = 1, 5
XM = J - 1
XINC = 10 + 20 * XM
YLEDG(J) = YLDZ + YLOS * XINC
120 CONTINUE
CONTINUE
C.
COMPUTE INTERPOLATED BLADE SECTIONS
DO 130 J = 1, 5
DO 135 I = 1, NSEX
XS(J, I) = XT(I) * XLEDG(J)
130 CONTINUE
CONTINUE
DO 140 J = 1, 5
DO 135 I = 1, NSEX
Y1(J, I) = Y1(J, I) * YLEDG(J)
Y2(J, I) = Y2(J, I) * YLEDG(J)
135 CONTINUE
CONTINUE
WRITE (6, 255) TITI1, TITL2
C.
WRITE (6,245)
DO 145 I=1,NSEX
WRITE (6,250) (XBS(I,J),Y1(J,I),Y2(J,I),J=1,3)
CONTINUE
* POST NORMALIZED INTERPOLATED DATA
CALL PLOT4 (X35,Y1,Y2,NSEX,TITL1,TITL2,1)
50 CONTINUE
IF (PUNCHLEJ;J) GO TO 10
NSEX=NPTS
* SET UP STREAMTUBE COORDINATES
* LINEARIZE R/Z VERS VS AXIAL DISPLACEMENT
XLSQ(1)=0,
XLSQ(2)=HX
YLSQ(1)=FSI
YLSQ(2)=FSL
CALL LSQ1 (2,XLSQ,Y_SQ,RAXZ,RAXS,PE,3RSQ)
* LINEARIZATION COMPLETE *** R(AX)=RAXZ+RAXS*AX
* SET UP AX MATRIX OF AXIAL DISPLACEMENTS
(-33 PRINT TO +66 PRINT SECTIONAL SPAN)
AX(I)=-HX/3,
AX(NSEX)=5.*HX/3,
XNP1=NSEX-1
AXDEL(T)=(AX(NSEX)-AX(I))/(XNP1)
DO 155 I=2,NSEX
AX(I)=AX(I-1)+AXDEL
55 CONTINUE
* C寅UATE RX MATRIX OF FREESTREAM RADIAL DIMENSIONS AS F(AX)
DO 160 I=1,NSEX
RX(I)=RAXZ+RAXS*AX(I)
160 CONTINUE
* PUNCH INPUT FOR CAVITATION PROGRAM *** Y1=PRESSURE,Y2=SUCRTION
BRAO=2.**SR
NDUM=1
BANG=6.26318/3NUM
PUNCH255,TITL1
PUNCH257,TITL2
KSTPRI=5
IF (INTERP.GT.0) GO TO 170
KSTPRI=2
K=1
DO 165 I=1,NPTS
Y1(K,I)=Y1(I)
Y2(K,I)=Y2(I)
XBS(K,I)=X(I)
165 CONTINUE
K=2
DO 170 I=1,NPTS
Y1(K,I)=Y1(I)
Y2(K,I)=Y2(I)
170 CONTINUE
DO 209 J=1,KSTPRI
INC=10+(K-1)*20
PUNCH255,INC
PUNCH255,NSEX,YSEX,NSEX,NDUM,NDUM,NSEX,NDUM,YSEX,BANG,3INC,BRAO,3VNC
PUNCH255,R,R,S COORDINATES
PUNCH255(Y1K,I,I=1,NSEX)
C. PUNCH SUCTION R,Z COORDINATES
   PUNCH295,(Y2(K,I),I=1,NSEX)
C. COMPUTE CHORD COORDINATES
   DO 175 I=1,NSEX
   YCC(I)=(Y1(K,I)+Y2(K,I))/2.
   CONTINUE
C. PUNCH CHORD R,Z COORDINATES
   PUNCH295,(YCC(I),I=1,NSEX)
C. PUNCH SUCTION THEETA COORDINATES
   PUNCH295,(XBS(K,I),I=1,NSEX)
C. COMPUTE PRESSURE ANUGULAR COORDINATES
   DO 180 I=1,NSEX
   TPAC(I)=XBS(K,I)+35J/3NJM
   CONTINUE
C. PUNCH PRESSURE THEETA COORDINATES
   PUNCH295,(TPAC(I),I=1,NSEX)
C. PUNCH CHORD THEETA COORDINATES
   PUNCH295,(XBS(K,I),I=1,NSEX)
C. COMPUTE RD MATRIX OF STREAMTUBE RADIUS FOR PUNCH BLADE SECTION
   XJ=K-1
   XINC=.10+.20*XJ
   DO 185 I=1,NSEX
   RDI(I)=RX(I)*XINC+H_(1+AX(I))*TMLTAN
   CONTINUE
C. COMPUTE BD(I) = STREAMTUBE WIDTH AS F(AX)
   VMID=NSEX/2
   RMID=RX(NMID)
   DO 190 I=1,NSEX
   BD(I)=SWNOM*RX(I)/RMID
   CONTINUE
   DYBETA=RDI(I)-RDI(NSEX)
   DXBETA=2.*AX
   BETA=ATAN2(DYBETA,DXBETA)
   SIAD=SIN(BETA)
   DO 195 I=1,NSEX
   PUNCH3J,RO(I),BD(I),SIAD,AX(I)
   CONTINUE
200 CONTINUE
GO TO 11
215 STOP

C. 210 FORMAT (/2X51H CORRECTED INPUT DATA FOR Z MEASURED PARALLEL TO S/L/BD2 6

1)
215 FORMAT (2X4JH NORMALIZED INPUT DATA FOR SECTIONAL DATA,/6X2MX ,2(8XBD2 6
12HY1,8X2HY2)/)
220 FORMAT (2X4JH NON-DIMENSIONALIZED INPUT DATA NORMALIZED TO TRAILING/BD2 6
1 EDGE FOR SECTIONAL INPUT,)/2(8X2HX,8X2HY1,8X2HY2)/)
225 FORMAT (5F10.4)
230 FORMAT (2X45H INCREMENTAL PERCENT CHORD DERIVED DATA POINTS,/8X2HX BD2 6
1,128X2HY1,8X2HY2)/)
235 FORMAT (2X48H BLADE SECTIONAL DATA EXTRAPOLATED FOR 5 SECTIONS,/5(3BD2 7
1X2HT,5X2HY1,6X2HY2)/)
240 FORMAT (16F8.4)
245 FORMAT (2X53H NORMALIZED INPUT DATA FOR INTERPOLATED BLADE SECTIONS/BD2 7
1,1/5(6X2HX,6X2HY1,5X2HY2)/)
250 FORMAT (5(F8.2,2F8.4))
255 FORMAT (6A10)
260 FORMAT (5F10.4)
265 FORMAT (1H1,8A1J//8A1J)/)
270 FORMAT (I4X53HNPJT DATA FOR CONTINUOUS BLADE SECTIONS AT LOCATIONS/BD2 7
SUBROUTINE LSQ1 (N,X,Y,A,B,PE,SRSQ)
LINEAR LEAST SQUARES SUBROUTINE FIT
R A ZEHNELE.. NOVEMBER 1968
EQUATION...Y = A + BX
PROGRAM VARIABLES
N NUMBER OF PAIRS OF DATA POINTS (X,Y)
X ARRAY OF INDEPENDENT VARIABLES
Y ARRAY OF DEPENDENT VARIABLES
A, B CONSTANTS OF STANDARD FIRST ORDER EQUATION
PE PROBABLE ERROR OF FIT OF DATA TO CURVE
SRSQ SUM OF THE RESIDUALS SQUARED
DIMENSION X(1), Y(1)
IF (N.LE.1) GO TO 15
SX=0.
SXSQ=0.
SY=0.
SXY=0.
SRSQ=0.
DO 5 I=1,N
5 SY=SY+Y(I)
SXY=SXY*X(I)*Y(I)
SX=SX+X(I)
SXSQ=SXSQ+X(I)**2
D=N*SXSQ-SX*SX
A=(SXSQ*SY-SX*SXY)/D
B=(N*SXY-SX*SY)/D
DO 10 I=1,N
10 SRSQ=SRSQ+(Y(I)-A-B*X(I))**2
PE=.675*SORT(SRSQ/(N-2))
RETURN
ERROR MESSAGE INDICATOR
WRITE (6,20) N
RETURN
FORMAT (1X36,'ERROR IN INPUT TO LSQ1 N INPUT AS *IS')
END LSQ1
SUBROUTINE PLOT1(X*Y11*Y12*Y21*Y22*N*TITL1*TITL2*IS)
DIMENSION X(1)*Y11(1)*Y12(1)*Y21(1)*Y22(1)
$TITL1(8)*TITL2(8)*TITL1(9)*TITL2(9)
DATA XSYM$*YSYM$18*16*$H%*2M$*
DATA XSYM$*YSYM$18*16*$H%*2M$*
CALL INIT280
DO 1 I=1,8
T1(I)=TITL1(I)
1 TL2(I)=TITL2(I)
XMIN=0.
YMIN=0.
XMAX=X(1)
YMAX=Y12(1)
DO 2 I=2,N
IF(X(I).GT.XMAX) XMAX=X(I)
IF(Y12(I).GT.YMAX) YMAX=Y12(I)
2 CONTINUE
CALL MAPG(XMIN*XMAX*YMIN*YMAX*15,15)
CALL LINEOPT(0,0,0,0)
CALL ABSEXAM(*01,98)
CALL SYMOL(TL1)
CALL SYMOL(TL2)
CALL ABSEXAM(*5,07)
CALL SYMBOL(XSYM)
CALL ABSEXAM(*10,02)
IF(IS.EQ.1) CALL SYMBOL(36HECHO PLOT OF INPT DATA SECTIONS)
IF(IS.EQ.2) CALL SYMBOL(36HECHO DATA NORMALIZED TO TRLNG EDGE)
CALL CHAROPT(0,0,1,0)
CALL ABSEXAM(*01,0)
CALL SYMBOL(XSYM)
NST=N-1
DO 45 I=1,NST
IF(Y11(I).EQ.0.) OR(Y11(I+1).EQ.0.) GO TO 44
IF(Y11(I).EQ.Y12(I)) GO TO 44
CALL LINE(X(I)*Y11(I),X(I+1)*Y11(I+1))
CALL LINE(X(I)*Y12(I),X(I+1)*Y12(I+1))
44 CONTINUE
45 CONTINUE
DO 47 I=1,NST
IF(Y21(I).EQ.0.) OR(Y21(I+1).EQ.0.) GO TO 46
IF(Y21(I).EQ.Y22(I)) GO TO 46
CALL LINE(X(I)*Y21(I),X(I+1)*Y21(I+1))
CALL LINE(X(I)*Y22(I),X(I+1)*Y22(I+1))
46 CONTINUE
47 CONTINUE
CALL FRAME
RETURN
END
SUBROUTINE PLOTR2(X1,Y11,Y12,X2,Y21,Y22,N1,N2,T1,T2,IS)
DIMENSION X1(1),Y11(1),Y12(1),X2(1),Y21(1),Y22(1),T1(8),T2(8),
T1(8),T2(8)
DATA X1(9),Y1(9),Y2(9),Y12(9),Y11(9),Y22(9)/
DATA XSYM@,YSYM@,1OOTHETA /*IOHAXIAL 2 $*/
CALL INIT280
DO 1 I=1,N
TL1(I)=T1(I)
1 TL2(I)=T2(I)
XMIN=0.
YMIN=0.
XMAX=X1(1)
YMAX=Y12(1)
DO 2 I=2,N1
IF(X1(I).GT.XMAX) XMAX=X1(I)
IF(Y12(I).GT.YMAX) YMAX=Y12(I)
2 CONTINUE
CALL MAPS (XMIN,XMAX,YMIN,YMAX,1.,9.,15.,9)
CALL LINEOPT(O,1)
CALL CHAROPT(0,0,1,0,0)
CALL ABSBEAM(.01,.98)
CALL SYMBOL(TL1)
CALL ABSBEAM(.01,.94)
CALL SYMBOL(TL2)
CALL ABSBEAM(.5,.07)
CALL SYMBOL(YSYM@)
CALL ABSBEAM(.10,.21)
IF (IS.EQ.1) CALL SYMBOL (36HNON-DIMEN DATA FOR INPUT DATA PTS $)
IF (IS.EQ.2) CALL SYMBOL (36HNON-DIMEN DATA FOR 51 DATA POINTS $)
CALL CHAROPT(0,0,1,0,0)
CALL ABSBEAM(.01,.5)
CALL SYMBOL(YSYM@)
CALL LINES(X1,Y11,N1)
CALL LINES(X1,Y12,N1)
CALL LINES(X2,Y21,N2)
CALL LINES(X2,Y22,N2)
CALL FRAME
RETURN
END
SUBROUTINE PLOT34 (X,Y1,Y2,N,TITL1,TITL2,IS)
DIMENSION X(1),Y1(5,1),Y2(5,1),XP(51),YP1(51),YP2(51),
$TITL1(8),TITL2(8),TL1(9),TL2(9)
DATA TL1(9),TL2(9)/2H$,2H$,2H$,TITL1(8),TITL2(8)/
DATA XSYM,YSYM/10H0101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010101010
SUBROUTINE PLOTR4 (X,Y1,Y2,N,TITLE1,TITLE2,IS)
DIMENSION X(5,1),Y1(5,1),Y2(5,1),XP(51),YP1(51),YP2(51)
$TITLE(8),TITL2(8),TL1(9),TL2(9)
DATA TL1(9),TL2(9) / 24, 24, 24, 24, 24, 24, 24, 24, 24
DATA XSYM,XSYM/1CHART, 1CHART, 1CHART, 1CHART, 1CHART, 1CHART, 1CHART, 1CHART, 1CHART
CALL INIT28
DO 1 I=1,N
TL1(I)=TITLE1(I)
1 XL2(I)=TITLE2(I)
XMIN=0.
YMIN=0.
XMAX=X(1,1)
YMAX=Y1(1,1)
DO 2 I=1,N
IF (X(I,1).GT.XMAX) XMAX=X(I,1)
IF (Y(I,1).GT.YMAX) YMAX=Y1(I,1)
2 CONTINUE
CALL MAPG (XMIN,XMAX,YMIN,YMAX,1,9,15,9)
CALL LINEOPT(0,1)
CALL CHAROPT(0,0,1,0,0)
CALL ABSBEAM(*.01,.98)
CALL SYMBOL(TL1)
CALL ABSBEAM(*.01,.94)
CALL SYMBOL(TL2)
CALL ABSBEAM(*.01,.94)
CALL SYMBOL(XSYM)
CALL ABSBEAM(*.10,.02)
CALL SYMBOL(36HNRMLZD DATA FOR 5 INTRPLTD SECTS *)
IF (IS.EQ.1) CALL SYMBOL(36HNRMLZD DATA FOR 5 INTRPLTD SECTS *)
CALL CHAROPT(0,0,1,0,0)
CALL ABSBEAM(*.01,.5)
CALL SYMBOL(YSYM)
DO 6 G J=1,5
DO 44 G I=1,N
XP(I,J)=X(J,I)
YP1(I,J)=Y1(J,I)
YP2(I,J)=Y2(J,I)
44 CONTINUE
JL=1
45 K=JL+1
IF (YP1(K)-YP1(JL)).GT.0.0) GO TO 48
JL=JL+1
48 CONTINUE
NP=N
IF (JL.EQ.1) GO TO 50
49 IK=I+JL-1
XP(I,J)=XP(IK)
YP1(I,J)=YP1(IK)
YP2(I,J)=YP2(IK)
NP=I
I=I+1
IF (IK.LT.N) GO TO 49
50 CONTINUE
CALL LINES(XP,YP1,NP)
CALL LINES(XP,YP2,NP)
50 CONTINUE
CALL FPAME
RETURN
END
Sample Input Data
TURBOPUMP INJECTOR CAVITATION ANALYSIS FOR NASA-26266 F/0, 17 JAN 1972

F-1 LUX, 2 INPUT BLADE SECTIONS INTERPOLATED TO 10, 30, 50, 70, 90 PRCNT SECTION

INPUT DATA FOR CONTINUOUS BLADE SECTIONS AT LOCATIONS
R1 = 0.750    R2 = 4.500

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INPUT TRAILING EDGE COORDINATES

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Table: Sample Output Data for 1 Blade Span

| Blade Span | Sample Data for Specific Sections |
|------------|----------------------------------|---|
| 1          |                                  |   |

The table contains data for specific sections of a blade span, likely related to a fluid dynamics or engineering analysis.
Figure F.1 Typical Inducer Showing Blade Sections
Figure F.2 Input Blade Sectional Data
Figure F.3 Input Data Normalized To Trailing Edge
Figure F.4  Non-Dimensionalized Input Data
Figure F.5 Non-Dimensionalized Data For 5 Blade Sections
Figure F.6 J-2 LOX Data For 5 Blade Sections Normalized To Trailing Edge