A NUMERICAL STUDY OF ELECTROMAGNETIC SCATTERING FROM OCEAN-LIKE SURFACES

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The integral equations describing electromagnetic scattering from one-dimensional conducting surfaces is formulated and numerical results are presented. The results are compared with those obtained using approximate methods such as physical optics, geometrical optics, and perturbation theory. The integral equation solutions show that the surface radius of curvature must be greater than 2.5 wavelengths for either the physical optics or geometric optics to give satisfactory results. It has also been shown that perturbation theory agrees with the exact fields as long as the root mean square surface roughness is less than one-tenth of a wavelength.
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The scattering of electromagnetic waves from the ocean surface has been of great interest for some time. In this work the scattering from one dimensional sea-like random surfaces is examined by a variety of computational methods, with a view to establishing what practical limitations must be satisfied on such surface parameters as radius of curvature, mean squared height, etc., in order that the statistical properties of the scattered radiation may be calculated with reasonable accuracy. The results of the computations are then used to discuss the applicability of the several theoretical models for sea-surface scattering (geometrical optics, physical optics, perturbation theory and the composite model) and the prospect for direct calculation of the scattered fields from the actual sea surface.

During the past few years, theoretical and experimental work here and abroad (Refs. [1]-[7]) has led to an understanding of the mechanisms responsible for scattering and emission of microwaves by the ocean. For off-normal backscatter, the "Bragg-scatter" from capillary and short wavelength components of the ocean surface, which can be calculated by perturbation theory, has explained the angular
and polarization dependence of the microwave radar return. When combined with the known height spectrum (Ref. [8]) of the ocean surface, it explains the weak dependence of backscatter on electromagnetic wavelength and wind velocity. Near the specular direction, i.e., near normal incidence for backscatter, the scattering is controlled by the slope distribution of the large scale structure of the surface. This part of the scattering is calculated by geometrical optics, and explains the dependence of the emissivity of the surface on wind velocity.

Nevertheless, the many assumptions required in finding the scattered fields by the perturbation or geometrical optics approximations, particularly assumptions about the Gaussian character of the surface height statistics, and the applicability of the theoretical approximations to the actual sea surface, have led to considerable discussion about the validity of the various theoretical solutions (Ref. [9]). Since straightforward verification by measurement is not practical, partly because of difficulty in the measurement process itself and partly because of the difficulty in specifying exactly what the surface was when the measurement was being made, it is desirable to have a direct method for calculating the scattering from a specific realization of the ocean surface. Direct calculations will allow a realistic assessment of the validity of the various theories, without any assumptions about the statistical properties of the surface. If a statistical average of the scattered fields over an ensemble of surface representations is required, it can be obtained (albeit at
some cost) by a direct summation of the scattered fields from the individual surface representations.

The specific surfaces considered here are cylindrical perfectly conducting surfaces as shown in Fig. 1. The surface generators are parallel to the z axis, and the surface elevation is specified by \( y = H(X) \). The incident field is a plane wave whose direction of propagation lies in the \( x,y \) plane and makes an angle of \( \theta_{\text{THI}} \) with the positive \( x \) axis, while the observation direction makes an angle of \( \theta_{\text{THS}} \) with the positive \( x \) axis. Time dependence is assumed to be \( e^{j\omega t} \) and has been suppressed throughout. All distances are measured in centimeters.

Three different methods for calculating the fields from such a surface are developed here. Although the details are discussed later it is desirable to outline each technique at this time.
The first approximate method is the geometrical optics technique (G.O.). For a given surface, and given scattering and incidence angles, the program locates the specular points on the surface (points where the local incidence angle equals the local scattering angle) and evaluates the radius of curvature at each specular point. The scattered far field is then found by summing the contribution from each of the specular points, including an extra 90° phase shift for the fields scattered from concave up portions of the surface. Shadowing of one section of the surface by another section may be taken into account.

The next approximation is the physical optics (P.O.) technique. For a given surface the scattered field is computed by integrating over the approximate surface current

\[
(1) \quad \vec{J}_s = 2\hat{n} \times \vec{H}^l
\]

where \( \hat{n} \) is the outward normal to the surface and \( \vec{H}^l \) is the incident magnetic field. Shadowing is always taken into account, as this is implicit in the physical optics formulation.

The last method developed here is based on a point matching solution to the integral equation satisfied by the true surface current \( \vec{J}_s \). The scattered fields are then found by integrating over the surface currents. Test cases (e.g., the wedge problem) have shown this method to be by far the most accurate; hence it is used as a standard to which all others are compared. However, because of computer storage limitations, this program can not handle surfaces whose arclengths are greater than \( \sim 60 \) electrical
wavelengths, whereas the G.O. and P.O. programs can, in principle, handle surfaces of any length provided sufficient computer time is available.

In order to avoid edge effects, tapering of the incident field is necessary in the integral equation solutions. The same tapering has been applied in both the G.O. and P.O. solutions so that they can be directly compared to the exact fields. The tapering applied here is illustrated in Fig. 14 of Chapter IV.

In the succeeding chapters each of these methods will be described in detail. By comparing the results for a series of test surfaces, the limitations of each method are established.
CHAPTER II
THE GEOMETRICAL OPTICS METHOD

The first approach to examining the scattering from a one dimensional rough surface is the geometrical optics method. By this is meant that the scattered field is computed by finding the specular points on the surface, and associating with each such point a scattered field amplitude and phase which depend on the geometrical properties of the surface at the specular point.

A. Geometrical Optics

Conservation of energy flux along a ray path will provide us with the geometrical optics field strengths (Ref. [10]).

Consider the two dimensional ray tube shown in Fig. 2. If \( u_0 \) is the field strength at some reference point at a distance \( \rho \) from the caustic and \( u \) is the field strength at distance \( \rho + \delta \) from the caustic, then the conservation of energy in the ray tube requires

![Ray tube geometry](image)

Fig. 2.—Ray tube geometry.
(2) \[ u_0^2 \rho \, d\theta = u^2 (\rho + \xi) \, d\theta \]

so that one may write

(3) \[ u(\xi) = u_0 \sqrt{\frac{\rho}{\rho + \xi}} \, e^{-jk\xi} \]

The factor \( e^{-jk\xi} \), with \( \lambda_e \) the electrical wavelength and

(4) \[ k = \frac{2\pi}{\lambda_e} \]

accounts for the phase shift between \( \rho \) and \( \rho + \xi \). Equation (3) fails at \( \xi \) equal to \(-\rho\). This location (at the confluence of the rays) is termed a caustic. Kay and Keller (Ref. [11]) have demonstrated that at points beyond the caustic (\( \xi \) less than \(-\rho\)) Eq. (3) is still valid if a phase shift of \(+90^\circ\) is introduced.

To use geometrical optics it is necessary to find all points on the scattering body at which the law of reflection is satisfied locally for the particular set of THI and THS under consideration. Once these points are located Eq. (3) is used to calculate the scattered field. Figure 3 shows the geometry for the calculation of the scattered field from one such specular point. By the law of reflection, the local incidence and scattering angles are equal and are marked \( \text{ANG} \) in the figure. The distances marked \( r_c \) and \( \rho \) are the radius of curvature and the distance from the specular point to the optical image of the source (i.e., the caustic distance) respectively. The distance \( \rho \) is given by a cylindrical mirror formula as
Fig. 3.--Specular point geometry.

\[
(5) \quad \frac{1}{\rho} = \frac{2}{|r_c| \cos (\text{ANG})} + \frac{1}{\ell_o}.
\]

In the cases considered here the distance to the line source, \( \ell_o \), will be assumed to be infinite, hence

\[
(6) \quad \rho = \frac{|r_c| \cos (\text{ANG})}{2}.
\]

If the specular point is taken as the reference position then Eq. (3) gives \( u_s \), the scattered field at the observation position

\[
(7) \quad u_s = R u_i \sqrt{\frac{\rho}{\rho + \ell}} e^{-jk\ell}
\]

\[
= R u_i \sqrt{\rho} e^{-jk\ell/\sqrt{\rho}} \text{ for } \ell \gg \rho \text{ (far field)}
\]
where \( u_i \) is the incident field evaluated at the specular point and
\( R \) is a reflection coefficient. If the electric field is parallel
to the surface generators (T.M. case) and \( u_i \) is taken as the inci-
dent electric field, then \( u_s \) is taken as the scattered electric
field with \( R = -1 \). If the magnetic field is parallel to the surface
generators (T.E. case) and \( u_i \) is taken as the incident magnetic field,
then \( u_s \) is the scattered magnetic field and \( R = +1 \). For dielectric
scatterers the corresponding Fresnel reflection coefficients are to
be used for \( R \). This makes the geometrical optics program the easiest
to convert from perfectly conducting bodies to penetrable bodies.

Up to this point the scattering surface has been assumed to
be concave down at the specular point. If the body is concave up
at the specular point then the caustic position is above the surface
instead of below, the scattered rays pass through the caustic on
the way to the observation point if the observer is in the far field,
and thus a phase shift of +90 degrees must be introduced. The
distant scattered fields may then finally be written

\[
E^S_z(\mathbf{r}) = -E_z^i \left| r_c \right| \cos(\text{ANG}) \frac{e^{-jk\rho}}{\sqrt{\lambda}} \varepsilon
\]

for the T.M. case and

\[
H^S_z(\mathbf{r}) = H_z^i \left| r_c \right| \cos(\text{ANG}) \frac{e^{-jk\rho}}{\sqrt{\lambda}} \varepsilon
\]
for the T.E. case, where $\varepsilon$ is +1 if the surface is concave down at the specular point and +j if the surface is concave up at the specular point.

On an actual surface there may be several specular points contributing to the total scattered field, so it is important to preserve the phase relationships among them. Phase reference is taken at the origin, and an incident wave of unit amplitude is assumed, i.e.,

\begin{align}
(10) \quad E_z^i &= e^{jK \cdot \bar{R}} \quad \text{(T.M. case)} \\
(11) \quad H_z^i &= e^{jK \cdot \bar{R}} \quad \text{(T.E. case)}
\end{align}

where

\begin{align}
(12) \quad \bar{K} \cdot \bar{R} &= \frac{2\pi}{\lambda_e} (-x \cos (\Theta_I) - H(x) \sin (\Theta_I)).
\end{align}

With the aid of the geometry shown in Fig. 4, the scattered far field is found from Eqs. (8) and (9), with $\varepsilon = \varepsilon_1 + \varepsilon_2$, where

\begin{align}
(13) \quad \varepsilon_2 &= -R \cdot \hat{D}_S = -x \cos (\Theta_S) - H(x) \sin (\Theta_S),
\end{align}

and

\begin{align}
(14) \quad \hat{D}_S &= \cos(\Theta_S) \hat{x} + \sin(\Theta_S) \hat{y}.
\end{align}
is the unit vector in the scattering direction. Since $\lambda_1 >> \lambda_2$, Eq. (8) becomes, for the T.M. case

$$E_z^S(\lambda_1) = -\frac{|r_c| \cos (\text{ANG})}{2} e^{-jk\lambda_1} \frac{e}{\sqrt{\lambda_1}} \epsilon e^{jkQ(x)}$$

where

$$Q(x) = x (\cos (\text{THI}) + \cos (\text{THS})) + H(x) (\sin (\text{THI}) + \sin (\text{THS})).$$

Similarly, for the T.E. case

$$H_z^S(\lambda_1) = \sqrt{\frac{|r_c| \cos (\text{ANG})}{2}} e^{-jk\lambda_1} \frac{e}{\sqrt{\lambda_1}} \epsilon e^{jkQ(x)}$$
The total scattered field in the THS direction is the sum of the fields scattered by each of the specular points. The numerical values of the scattered fields as calculated by the programs of Appendix A, and plotted in the various figures of Chapter V are denoted by $E_Z^S$ and $H_Z^S$, and have been normalized with respect to the actual fields $E_Z(\varphi_1), H_Z(\varphi_1)$ by

\[
\begin{align*}
\begin{bmatrix}
E_Z^S \\
H_Z^S
\end{bmatrix}
= \sqrt{\frac{k}{\lambda_1}} e^{jk_1\rho} 
\begin{bmatrix}
E_Z(\varphi_1) \\
H_Z(\varphi_1)
\end{bmatrix}.
\end{align*}
\]

It is clear that Eqs. (15) and (17) fail if the radius of curvature is infinite at the specular point. This is because the source was assumed at infinity, i.e., $\lambda_0 \rightarrow \infty$. If $\lambda_0$ were to be held finite then from Eq. (5)

\[
(19) \quad \lim_{r_c \rightarrow \infty} \rho = \lambda_0
\]

and the singularity in Eqs. (15) and (17) would not occur. In addition to the singularities caused by an infinitely distant source, there are a number of other shortcomings of the G.O. approximation. Among them are: a failure to account for wedge diffraction effects (radius of curvature goes to zero), a failure to account for diffraction from shadow boundaries into shadowed regions (Ref. [12]), a failure to properly predict the scattered fields if the surface
features subtend only a few Fresnel zones (Ref. [13]), and finally a failure to predict any scattered field if no specular point exists on the body.

Implicit in the geometrical optics technique is the concept of shadowing, that is, a specular point cannot contribute to the scattered field unless it can be seen by both the source and the observer. The program developed here can account for shadowing of this type.

B. Discussion of the Geometrical Optics Program

For geometrical optics calculations the first order of business is the location of the specular points. Figure 5 shows the geometry.

![Fig. 5. --Geometry for specular point location.](image)

The surface height profile is described by \(H(X)\) and the regions under investigation lies between \(X_{\text{START}}\) and \(X_{\text{STOP}}\). \(\text{THI}\) and \(\text{THS}\) have already been defined; \(\text{THN} \) (\text{THETA of the NORMAL}) is the angle between the normal \((\hat{n})\) to the surface and the positive \(x\) axis. Clearly
(20) \[ \text{THN}(x) = \frac{\pi}{2} + \tan^{-1} \left( \frac{dH(x)}{dx} \right). \]

The law of reflection gives \((x, H(X))\) as a specular point when

(21) \[ \text{THS} - \text{THN}(x) = \text{THN}(x) - \text{THI} \]

i.e.,

(22) \[ \frac{(\text{THS} + \text{THI})}{2} = \text{THN}(x). \]

The program calculates the function

(23) \[ E(X) = \frac{(\text{THS} + \text{THI})}{2} - \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{dH(x)}{dx} \right) \right) \]

for many points in the interval \((X_{\text{STRT}}, X_{\text{STOP}})\) and when this function changes sign a specular point has been located. The collection of points so located is stored in an array \(XN(J)\). To save running time two searches are made, first a coarse grain search and then, in the neighborhood of each specular point, a finer grain pass is made.

The search must satisfy two requirements. First, it must be fine enough to locate all specular points; this requires that the surface must be sampled often enough to get an adequate description of its structure. For example if the surface were described by a Fourier series then one would expect that sampling every twentieth of the minimum mechanical wavelength would be sufficient. Secondly, the specular positions must be located to within a small fraction of an electrical wavelength so that the phase relationships among the various specular points are correctly maintained. In the light of
these considerations a first search might be made at a step size of 
(the minimum mechanical wavelength)/20. The fine grain search would 
then be made with a step size of say ($\lambda_e/20.0$) or (1st step size/2.0) 
whichever is the smallest. In the program, the coarse step size is 
called DLTAX (Delta X) and the fine step size is called DLTAX00. The 
local angle of incidence for each specular point is stored in an 
array ANG(J). This angle is used in the computation of the scat-
tered field and is shown in Fig. 5. Once a complete pass is made 
over the surface, the scattered fields are computed. It should be 
noted that whenever any one of THI, THS, H(X) is changed, the 
complete pass must be made again.

The actual program, given in Appendix A, makes the scattered 
field computation for two cases:

1) all specular points contributing,

2) scattering from concave up specular points neglected 
when calculating the scattered field.

The second case, clearly incorrect, was an attempt to see how the 
computed fields would correspond to the results of certain statisti-
cal theories which neglect the concave up specular points. In the 
program the electric field calculated from the first case is called 
ESCNS (ELECTRIC FIELDS SCATTERED WITH NO SHADOWING) and from the 
second case ESCDNS (ELECTRIC FIELD SCATTERED FROM CONCAVE DOWN 
POINTS WITH NO SHADOWING).

Geometrical optics allows shadowing to be taken into account 
without much extra effort. The three types which may occur 
(specular point not illuminated by source, specular point not visible
to observer, both) are shown in Fig. 6. Each point in the array of specular points, $XN$, is examined for inbound shadowing in the following way. A line is passed through the specular point $XN_j$, $H(XN_j)$ with slope $\tan(THI)$. The equation of the line is

$$YI(X) = \tan(THI)x + (H(XN_j) - \tan(THI)XN_j)$$

Fig. 6.--Specular point shadowing.
Then \( x \) is incremented in the proper direction until one of the following occurs. The first possibility is that at some point \( x \), \( YI(x) \) becomes greater than the maximum value that \( H(x) \) can attain for any value of \( x \) in the interval \( XSTRT, XSTOP \). This value of \( H(x) \) is called \( HMAX \) and must be supplied for each surface being considered. If the surface is a sum of sinusoids then \( HMAX \) is equal to the sum of the individual magnitudes. The second possibility is that at some point the value of \( x \) is incremented out of the interval \( (XSTRT, XSTOP) \) being considered. The third and final possibility is that at some point \( x \) the line \( YI(x) \) intersects the surface profile \( H(x) \). When the first or second case occurs the specular point is not shadowed. In the third case the specular point is inbound shadowed and for that particular \( j \), \( XN(j) \) is set equal to a number much larger than \( XSTOP \). This allows \( XN_j \) to be skipped when the contribution from each of the specular points is being computed. A very similar test is applied for outbound shadowing.

When both the inbound and the outbound shadowing tests are completed the array of specular point positions contains values which are either in the range \( XSTRT < X < XSTOP \) or \( XN_j >> XSTOP \). The scattered field is calculated as in the case where shadowing is neglected except that when \( XN_j > XSTOP \) the field from this specular point is not put into the sum. The scattered field with shadowing accounted for is called \( ESCWS \) (ELECTRIC FIELD SCattered WITH SHADOWING) and the scattered field calculated with only concave down non-shadowed specular points contributing is called \( ESCD \).
C. Using the Geometrical Optics Program

While the storage requirement is minimal, the running time of this program depends largely on the step sizes which have to be used during the search for the specular points, and the number of scattering angles. This means that as the length of the surface increases, the time per pass required to find the specular points goes up and the number of passes over the surface also increases, since to see detail in the scattered field pattern the scattering angle must be examined at a larger number of points (finer grain). The half-power beamwidth of a uniformly illuminated aperture of width $X\text{STOP}-X\text{STRT}$,

\begin{equation}
\text{beamwidth} \approx \frac{0.88 \lambda}{X\text{STOP} - X\text{STRT}} \text{ radians}
\end{equation}

affords a crude estimate of the fineness of the grain which must be taken. The increment in THS should be less than a fifth of this.

The program has been checked for several cases, two of which will now be mentioned. The simplest check was the comparison with hand calculations for a surface described by

\begin{equation}
H(x) = 50 \cos\left(\frac{2\pi x}{800}\right)
\end{equation}

with $x$ in the range $(-200,200)$. This surface has only one specular point or none at all depending upon THI and THS. Another check was performed for a sinusoidal surface like the one shown in Fig. 7.
Fig. 7.--Specular points on a sinusoidal surface.

In this case the specular return comes from a collection of regularly spaced points which look like a pair of linear arrays of point sources. The program found the specular points and calculated the total scattered field correctly.
CHAPTER III
THE PHYSICAL OPTICS METHOD

The next complexity of approximation to the scattered fields to be considered here is given by the physical optics method.

A. The Physical Optics Approximation

Physical optics (P.O.), (Ref. [14]), approximates the true surface currents on a perfectly conducting body by the currents

\[
J_s = \begin{cases} 
2\hat{n} \times \vec{H}^i & \text{on the portions of the surface which are illuminated} \\
0 & \text{on the portions of the surface which are shadowed}
\end{cases}
\]

(27)

where \( \hat{n} \) is the outward normal to the surface and \( \vec{H}^i \) is the incident magnetic field evaluated at the surface. These approximate currents are then used in the radiation integral to calculate the scattered fields. The P.O. surface current is exact if the scattering body is perfectly conducting half space and the incident field is a plane wave. As the surface curvature decreases the P.O. currents depart more and more from the true currents; as the curvature at some point on the surface goes to zero (a wedge), the method fails entirely. Nor do the scattered fields predicted by P.O. satisfy the reciprocity theorem except for backscattering. Nevertheless, the P.O. method has a significant advantage over G.O. in that the fields remain
bounded even if the radius of curvature of the surface becomes infinite. Hence the flat facets of a surface can be approximately analyzed.

Whether or not P.O. provides any more useful information than G.O. is a question of long standing, and the answer seems to depend upon the geometry of the scattering body (Ref. [15]). For the kind of surfaces considered here it will appear that P.O. gives a good approximation to the scattered fields over a significantly wider range of surface characteristics than G.O. It is important to note that in this work the far field radiation integral over the physical optics currents is evaluated numerically to give the scattered fields. Unlike a number of rough surface scattering theories (Ref. [16]), no stationary phase approximation to the far field radiation integral is used. It is well known (Ref. [17]) that when the stationary phase approximation must be made, one obtains the G.O. result and there is then no difference between the two approaches.

The far-zone scattered fields will now be calculated using the physical optics currents. In the T.M. case, (see Fig. 8) the incident electric field is a z polarized plane wave of unit magnitude and the incident magnetic field is

\[
\mathbf{H}^i = e^{+jk(x\cos(\Theta I) + H(x)\sin(\Theta I))} \left[-\sin(\Theta I)\hat{x} + \cos(\Theta I)\hat{y}\right]/\eta
\]

(28)

where \( \eta \) is the impedance of free space. Using Ref. [18] and the fact that the tangential electric field vanishes on the surface, the scattered electric field is given by

\[ \mathbf{E}_s \]
(29) \[ E^S (\vec{r}_0) = -\frac{j\omega \mu_0}{2\pi} \int \int_{c_{\| \| \| \| l l l l l l}} \frac{e^{-jk|\vec{r}-\vec{r}_0|}}{|\vec{r}-\vec{r}_0|} \] d\z d\c

where \( \vec{r}_0 \) is the position vector to the observation point, \( \vec{r} \) is the position vector of a point on the surface and \( \hat{n} \) is the unit outward normal to the surface. The notation \( c_{\| \| \| \| l l l l l l} \) indicates that the integration is to be carried out only over those portions of the contour which are optically illuminated.

Since \( H^i \) and \( \hat{n} \) are independent of \( z \) one can show, by using an appropriate integral representation for the Hankel function (Ref. [18]), that the scattered field is
(30) \[ E^S(\rho_o) = -\frac{k n}{4} \int (2\hat{n} \times \hat{H}) H_0^{(2)}(k|\rho - \rho_o|) \, dc \]

where all variables are confined to the x, y plane

(31) \[ \rho_o = x_o \hat{x} + y_o \hat{y} \]

(32) \[ \rho = x \hat{x} + y \hat{y} \]

and \( H_0^{(2)}(x) \) is the Hankel function of the second kind and zero order. Using the large argument approximation for \( H_0^{(2)}(x) \), the far field scattered electric field becomes

(33) \[ E^S(\rho_o) = -\left(\frac{2}{\pi k}\right)^{1/2} k \frac{\pi}{2} e^{\frac{jk}{\rho_o}} \int \frac{e^{-jk|\rho_o|}}{\sqrt{|\rho_o|}} \sin(THI - \tan^{-1}(\hat{H})) \, dx \]

where \( H(x) \) describes the surface height profile,

(34) \[ \dot{H} = \frac{dH}{dx}, \]

and \( Q(x) \) is given by Eq. (16). As before, the factor

\[ e^{-jk|\rho_o|/|\rho_o|} \]

23
has been suppressed in both the computed and reported values of the scattered electric field, so that the actual field $E_z^s(\vec{r}_0)$ is related to the print out value $E_z^s$ by

$$E_z^s = E_z^s(\vec{r}_0) \sqrt{|\vec{r}_0|} e^{+jk|\vec{r}_0|}.$$  

When the incident magnetic field is $\hat{z}$ directed (transverse electric case) it is convenient to work with the scattered magnetic field. The latter is found from Ref. [18]

$$4\pi \vec{H}^s(\vec{r}_0) = 2 \int_{\text{fill}} \int_{-\infty}^{\infty} (\hat{n} \times \vec{H}^i) \times \vec{v} \frac{-jk|r-r_0|}{|r-r_0|} \, dz \, dc$$

where $\vec{H}^i$ is the incident magnetic field (see Fig. 9). The two dimensional far field scattering becomes from Eq. (36)

![Diagram showing the geometry for T.E. physical optics.](image)

Fig. 9. -- Geometry for T.E. physical optics.
Again, the factor

\[ e^{-jk|\rho_0|} \sqrt{\frac{j\pi}{|\rho_0|}} \int_{C_{i11}} \sin(tan(A) - THS)e^{jkQ(X)} \sqrt{1+H^2} \, dx. \]

is suppressed in the programs of Appendix A, so that the plotted or tabulated field strengths, \( H^S_z \), are related to the true fields, \( H^S_z(\rho_0) \), by

\[ H^S_z = H^S_z(\rho_0) \sqrt{|\rho_0|} e^{jk|\rho_0|}. \]

There are two further considerations that may be discussed at this time. For bistatic scattering it may happen that not all of the currents set up on the surface by the incident field are optically visible to the observer (see Fig. 10). In the physical optics programs developed here no account was taken of this possibility. Obviously such considerations do not arise for backscattering.

So far, in this chapter a perfectly conducting surface has been assumed. Physical optics can be generalized to treat dielectric surfaces by using a pair of equivalent electric and magnetic surface
Fig. 10.--Optically invisible surface currents.

currents obtained from the fields of a plane wave incident on a dielectric half space (Ref. [19]). Since two integrations would be required to compute the scattered fields, it would seem that the running time should nearly double, but very little extra storage space would be required.

B. Discussion of the Physical Optics Computer Programs

For either polarization the physical optics program is divided into two parts. The first, and by far the most difficult, finds the shadow boundaries on the surface, since the integrations are to be performed only over the illuminated section of the contour. The second part performs the necessary integration to calculate the scattered far fields.
The program opens by considering the function $H(X)$ which describes the surface between the defined endpoints ALEP (Left End Point) and REP (Right End Point). The search for shadow boundaries begins at REP by determining whether or not the right endpoint casts a shadow on the surface and proceeds from right to left (see Fig. 11).

If $\theta_I$ (the incidence angle—required to be less than 90°) is greater than 80° it is assumed that no shadowing occurs. The starting point of the illuminated zone (either REP or A of Fig. 11) is stored in the first position of an array called SX (Shadow boundaries X coordinate). The value of $x$ is decremented until either a point on the surface is reached where the tangent-slope condition

$$\frac{dH}{dx} = \tan(\theta_I)$$

is satisfied, at which point a shadow zone begins, or $x$ becomes less than ALEP, in which case the second entry in SX is ALEP. On the other hand if Eq. (39) is satisfied for some $x$ between $S_{X1}$ and ALEP then this value of $x$ is stored in $S_{X2}$, a line with slope $\tan(\theta_I)$. 

Fig. 11.—Shadowing at the right end point.
is passed thru the point, and its intersection (if any) with \( H(x) \) is found. If there are no such intersections, then all of the surface to the left of the point is shadowed. If an intersection does exist then the search for a point where the tangent-slope condition is satisfied begins again. This process continues until \( x \) is decremented past ALEP. The array SX thus stores the positions of points with an illuminated zone on their left in oddly subscripted locations and the points with an illuminated zone on their right in evenly subscripted locations (see Fig. 12). The size of the decrement used to locate the boundaries should be small enough to catch the surface features, and to locate the ends of the shadow zones within a fraction of a wavelength.

![Diagram of shadowed and illuminated zones](image)

Fig. 12.--Illustration of shadowed and illuminated zones.

The integration over the illuminated sections of the surface to find the scattered fields is performed in a subroutine called \( \text{BINT}(XX,YY) \) (Bistatic radiation Integral) the arguments of which are the initial and final coordinates of one of the illuminated zones in...
The integration is repeated for each zone until all illuminated zones have been considered. The total scattered field (called $S$) for a particular THI and TNS is the sum of the zone fields. Except for normalization, the programs for the two polarizations differ only in the subroutine called FTBI(X) (Function To Be Integrated); the factor $\sin(\text{THI}-\tan^{-1}(\hat{A}))$ for the T.M. polarization is replaced in the T.E. case by $\sin(\text{THS}-\tan^{-1}(\hat{A}))$. The actual integration over the physical optics surface currents is performed by a five point Gaussian integration. In choosing the interval on the x axis over which the five point Gaussian integration is to be applied, two conditions must be met. The first is that the number of sample points along the contour must exceed five per wavelength. Presuming surface slopes of less than 60°, this means that ten sample points should be taken per electrical wavelength on the x axis. The second condition is that, if the surface were to be represented by a Fourier series, there should be 8-10 sample points per minimum mechanical wavelength along the x axis. Presuming, for example, that the first of the above conditions is the most stringent, each section of illuminated surface (i.e., between $x = SX_{j+1}$ and $x = SX_j$, j odd) would be divided into half electrical wavelength intervals plus a fractional interval, and the five point Gaussian integration would be applied to each of the half electrical wavelength intervals, and to the last, fractional, interval.
C. Comments on the Use of the Physical Optics Programs

As in the case of G.O., the storage requirements are minimal, while running time depends upon the length of the surface and number of incidence and scattering angles which are investigated. For each THI the search for illumination boundaries is performed only once, but the integration must be repeated for each scattering angle considered. For many of the scattered field computations considered here the angle of incidence was held fixed and the scattering angle was varied between 0 and 180°. For such cases the time required to find the illuminated zones on the surface is small compared to the time required to do the integrations for the scattered field.

As the surface length is increased the time required goes up rapidly since the integration for each scattering angle takes longer and THS must be incremented with a finer grain to get an accurate reproduction of the structure in the scattered field pattern. The size of the increment for THS has already been discussed in connection with the geometrical optics program. For example, the time required to run a surface 16 electrical wavelengths long, with THS incremented by 0.5° from 0 to 180°, was 1.8 min. By comparison, 21 min. were required for a surface 100 electrical wavelengths long with increments in THS of 0.25° from 30° to 170°, i.e., 560 values of THS. The value of the increment in the last case appears to have been just adequate to see the detail in the pattern.

Among the checks of the P.O. program is a computation for a flat strip with no tapering of the illumination, for which a closed form
physical optics result is easily obtained. The agreement was excellent for both polarizations. In Chapter V, P.O. will be compared with the other two methods of computing the scattered fields. Special attention will be given to the range of surface parameters over which the P.O. approximation is valid.
CHAPTER IV

THE INTEGRAL EQUATION METHOD

In this chapter the third and most accurate method for calculating the scattering will be examined. Here the scattered field is obtained from the exact surface current, which is found from a moment method solution of an integral equation (see, e.g., Refs. [20], [21]). There are no restrictions on the curvature or form of the surface, but because of machine storage limitations only surfaces of rather short length (30 \( \lambda_e \) to 60 \( \lambda_e \)) can be handled.

A. Moment Methods

This section contains a brief introduction to the method of moments. For more information and other applications of this method refer to Ref. [22], on which the following is based.

The objective of the moment method is to determine, numerically, the function \( F \) which is a solution of the inhomogeneous operator equation

\[
(40) \quad C(F) = G
\]

where \( C(\ ) \) is a given linear operator and \( G \) is a given function.

Suppose that \( F \) can be expanded in a set of basis functions \( b_n \)

\[
(41) \quad F = \sum_{n=1}^{N} F_n b_n .
\]
where $F_n$ is the $n$-th unknown coefficient of the expansion of $F$ in that basis. Note that if a computer is to be used, $N$ will have to be finite. Using the linearity property of $C$

$$C(F) = C \left( \sum_{n=1}^{N} F_n b_n \right) = \sum_{n=1}^{N} F_n C(b_n) = G.$$  

To convert the operator equation to a set of simultaneous equations an inner product, a scalar, $\langle h, g \rangle$ is defined for functions $h, g$ and $s$ and scalars $\alpha, \beta$ such that

$$\langle h, g \rangle = \langle g, h \rangle$$

$$\langle \alpha h + \beta g, s \rangle = \alpha \langle h, s \rangle + \beta \langle g, s \rangle$$

$$\langle h, h^* \rangle = 0, \text{ if } h \equiv 0.$$

Let $\{w_i\}$ be a set of weighting functions and take the inner product of both sides of Eq. (42) with $w_m$. Using the properties of the inner product, the original operator equation is converted to

$$\sum_{n=1}^{N} \langle w_m, C(b_n) \rangle F_n = \langle w_m, G \rangle$$

which is exactly the familiar matrix equation

$$\sum_{n=1}^{N} C_{mn} f_n = G_m$$

where
(48) \[ C_{mn} = \langle W_m, C(b_n) \rangle \]

and

(49) \[ G_n = \langle W_n, G \rangle. \]

The solution, \( F_i \), to this system of equations can be found by any one of several methods, two of which are discussed in Appendix B. The solution may be exact or approximate depending upon \( N, b_n \), and \( W_n \).

For the integral equations to be solved here, the current is expanded in a basis of non-overlapping pulses of unit amplitude, while the weighting functions are chosen to be delta functions whose singularities occur at the centers of the pulses. The inner product is chosen to be

(50) \[ \langle g,h \rangle = \int_c g h \, dc \]

where \( c \) is the contour of the scattering surface. This choice of basis and weight functions amounts to enforcing the integral equation at the centerpoints of the pulses, and is usually called "point-matching." For the operator equations considered in this work the system of simultaneous equations which result from point matching are well conditioned, i.e., suitable for computer solution (see Ref. [23]).
B. **Integral Equation for Transverse Magnetic Polarization**

In order to apply the point matching technique to the rough surface scattering problem, it is first necessary to find an appropriate linear operator. For this purpose the integral equation relating the unknown surface current to the known incident field has been chosen.

The incident electric field is $\hat{z}$ directed, the incident magnetic field is transverse (T.M. polarization) to the generators of the surface with contour $c$ as shown in Fig. 13. If the total electric field is written as the sum of the incident field $\vec{E}^i$ and the scattered field $\vec{E}^s$, the boundary condition

\begin{equation}
\vec{E}^i + \vec{E}^s = 0
\end{equation}

must be satisfied on $c$. The scattered field is given in terms of the $\hat{z}$ directed surface currents, $J_z(\rho')$, by (see Ref. [24])

![Fig. 13.--Geometry for T.M. scattering.](image)
for the two dimensional case, where \( H_0^{(2)} \) is the Hankel function of the second kind and order zero, \( \eta \) is the impedance of free space and \( k \) is the wave number, \( 2\pi/\lambda_e \). Combining this with the boundary condition (Eq. (51)) gives the integral equation for the unknown surface current

(53) \[ E_z^i(\rho) = \frac{k_n}{4} \int_{c} J_z(\rho') H_0^{(2)}(k|\rho-\rho'|) \, dl' \]

where \( \rho, \rho' \) are now both confined to the contour \( c \). Equation (53) can now be identified with Eq. (42) as follows:

\[ E_z^i(\rho) \text{ corresponds to } G, \]

\[ J_z(\rho') \text{ corresponds to } F, \]

and the operator

\[ \frac{k_n}{4} \int_{c} \left( \right) H_0^{(2)}(k|\rho-\rho'|) \, dl' \text{ corresponds to } C(\cdot). \]

As it stands the integral equation requires the consideration of the current on the entire boundary \( c \); if the entire contour of a two dimensional earth were to be included, the storage requirements for a moment method solution would be astronomical. It seems reasonable to assume that for standard radar wavelengths and with directive antennas, the surface current is appreciable over only a very small
portion of this contour. Thus it will be presumed that the surface current outside a certain illuminated region, which extends from -EP (End Point) to +EP, can be neglected (see Fig. 14). To simulate the illumination of the surface by a directive antenna, an amplitude taper \( t(x) \) is introduced* in the following way. The amplitude of the incident field is taken as unity to within two electrical wavelengths from each end point. Between one and two electrical wavelengths from each end the field is sinusoidally tapered to zero. Over the last wavelength the incident field is taken to be zero. The incident field with tapering included, \( E_z^i(\rho) \), is thus

\[ E_z^i(\rho) \]

---

*The use of amplitude tapering on a plane wave amounts to independently specifying the amplitude and phase, which makes such a field differ from a Maxwellian field. Changing the specification of the incident field would require no fundamental change in the methods and programs employed.
The neglect of the surface currents beyond the endpoints (±EP) has been checked by lengthening the dead zone at each end of the region under consideration and noting the change in the surface currents and scattered fields. The results of this test are presented in Section D of the chapter and do indeed justify the assumption of negligible currents beyond the illuminated region.

Although tapering of the incident field is not needed in the P.O. or G.O. formulations, it has usually been included in the calculations so that the results of all the techniques can be fairly compared. The only cases in which tapering is not used are special tests of the individual methods.

The integral equation becomes

$$E_z^i(\overline{\rho}) = t(x) e^{-j\hat{k} \cdot \overline{\rho}} e^{+j\frac{2\pi}{\lambda}(\cos(\Theta_1) x + \sin(\Theta_1) H(x))}$$

$$= t(x) e^{j\frac{2\pi}{\lambda}(\cos(\Theta_1) x + \sin(\Theta_1) H(x))}$$

with \(\overline{\rho}, \overline{\rho}'\) both confined to the section of the contour for which \(-EP < x < EP\).

The method of moments can now be applied. The surface is divided into segments of equal arclength \(DC\), and the current, \(J_z\), is expanded in a basis of non-overlapping pulse functions as
where \( \vec{r}_n \) is the position vector of the midpoint of the \( n \)-th segment of the surface, \( F_n \) is a complex number representing the magnitude and phase of the current over the \( n \)-th segment of the contour, and the \( n \)-th basis function \( P_{\alpha \phi} (\vec{r}'-\vec{r}_n) \) is a pulse of unit amplitude and width \( DC \) along the contour \( c \). Thus the actual surface current is to be approximated as shown in Fig. 15. For a reasonable representation of the surface current, the pulse width, \( DC \), must be a fraction of an electrical wavelength; \( \lambda_e/10 \) has been found to be satisfactory. The shape of the surface must also be considered in choosing \( DC \), since the surface must be accurately modeled by strips of width \( DC \). Hence, if \( \lambda_m \) is the shortest mechanical wavelength in the Fourier spectrum of the surface, then \( DC \) should also satisfy \( DC \leq \lambda_m/10 \). Of course the more restrictive of the two conditions should be met.

Applying the method of Section A of this chapter to Eq. (55)
\( E_z^i(\rho) = \frac{kn}{4} \int_{EP} \sum_{n=1}^{N} F_n P_{\mathbb{R}^2}(\rho', -\rho_n) H_0^{(2)}(k|\rho - \rho'|) \, d\lambda' \)

\( = \frac{kn}{4} \sum_{n=1}^{N} F_n \int_{EP} P_{\mathbb{R}^2}(\rho', -\rho_n) H_0^{(2)}(k|\rho - \rho'|) \, d\lambda' \)

\( = \frac{kn}{4} \sum_{n=1}^{N} F_n \int_{\mathcal{D}_n} H_0^{(2)}(k|\rho - \rho'|) \, d\lambda' \)

where \( \int_{\mathcal{D}_n} \) means "integrate over the \( n \)-th segment of the contour".

Taking the inner product of Eq. (57) with the weighting functions,

\( \langle \delta(\rho - \rho_m), E_z^i(\rho) \rangle = \frac{kn}{4} \sum_{n=1}^{N} F_n \langle \delta(\rho - \rho_m), \int_{\mathcal{D}_n} H_0^{(2)}(k|\rho - \rho'|) \, d\lambda' \rangle \)

so

\( E_z^i(\rho_m) = \frac{kn}{4} \sum_{n=1}^{N} F_n \int_{\mathcal{D}_n} H_0^{(2)}(k|\rho_m - \rho'|) \, d\lambda' \)

which is the same as the NXN matrix form

\( \mathbf{C} \mathbf{F} = \mathbf{E} \)

where

\( C_{mn} = \frac{kn}{4} \int_{\mathcal{D}_n} H_0^{(2)}(k|\rho_m - \rho'|) \, d\lambda' \),
and $F_n$ is the unknown amplitude and phase of the current in the $n$-th contour segment. Once Eq. (60) is solved, the surface current is known.

The far field scattering from the surface is found from the surface currents and Eq. (52) to be

$$E^S_z(\phi) = E^S_z(\phi) = \frac{k_n}{4} \sqrt{\frac{2}{\pi k}} e^{\frac{5\pi}{4}} e^{-jk|\phi|} \left[ \int_{EP}^{EP} J_z(\phi) e^{jk(\phi_\perp \cdot \hat{\phi})} d\phi \right]$$

$$= \frac{k_n}{4} \sqrt{\frac{2}{\pi k}} e^{\frac{5\pi}{4}} e^{-jk|\phi|} \left[ \sum_{n=1}^{N} F_n P_{eq}(\phi_\perp \cdot \hat{\phi}_n) e^{jk(\phi_\perp \cdot \hat{\phi})} d\phi \right]$$

$$= \frac{k_n}{4} \sqrt{\frac{2}{\pi k}} e^{\frac{5\pi}{4}} e^{-jk|\phi|} DC \sum_{n=1}^{N} F_n e^{jk(\phi_\perp \cdot \hat{\phi})} .$$

The output of the computer programs is a normalized scattered field, $E^S_z$, which is related to the true scattered field, Eq. (63), by

$$E^S_z = E^S_z(\phi) \sqrt{|\phi|} e^{jk|\phi|}.$$

C. Discussion of the Computer Program for Transverse Magnetic Polarization

Several different programs were written using the above formulation of the problem. In the first part of this section the common
features of the programs will be discussed and later their differences and relative merits.

All of the T.M.I.E. (transverse magnetic integral equation) programs require that the surface have its arclength subdivided into segments of width DC, and have the endpoints and midpoints of these segments stored. The surface breakdown is shown in Fig. 16. The  

Fig. 16.--Breakdown of surface into segments of length DC.

j-th segment lies between \(x_i\) and \(x_{j+1}\), while the \(j\)-th midpoint \((XM_j)\) is such that \(x_j < XM_j < x_{j+1}\). The surface is segmented by using the arclength formula and rectangular rule integration. After the surface subdivision is completed the programs differ somewhat depending on how the matrix elements are calculated.

Once the matrix elements have been calculated the first part of a two part solution of the system of equations begins. In all of the solution methods used the matrix is factored into an upper
and a lower triangular matrix, see Appendix B. The matrix elements depend only upon the surface profile \( H(x) \), and are independent of the incident field, THI or THS so that the factorization need be done only once for a given profile. In the second part of the solution the array \([F]\) is loaded with the tapered incident electric field at each of the \( \text{XM}_j \); the back substitutions (described in Appendix B) are then carried out to find the current coefficients, \( F_n \). The scattered fields are then calculated from Eqs. (63) and (64).

The differences in the several programs for the T.M.I.E. lie mainly in the calculation of the matrix elements (Eq. (61)). The simplest way to evaluate Eq. (61) for \( m \neq n \) is to presume that \( H_0^{(2)}(k|\vec{p}_m-\vec{p}_n|) \) is constant over the \( n \)-th interval; then

\[
(65) \quad C_{mn} \sim \frac{k n}{4} H_0^{(2)}(k|\vec{p}_m-\vec{p}_n|) \text{ DC}
\]

If \( m=n \), a small argument approximation to \( H_0^{(2)}(x) \) is made and integrated analytically, giving

\[
(66) \quad C_{mm} \sim \frac{k n}{4} \text{ DC} H_0^{(2)}(\frac{k \text{ DC}}{2e})
\]

where \( e \) is the base of the natural logarithm. In practice the matrix elements are simply the Hankel function and the \( \frac{k n}{4} \cdot \text{ DC} \) is accounted for when the fields are printed out. This approximation results in a symmetric matrix which, if efficiently stored, requires
only $N(N+1)/2$ storage locations. The length of surface which can be treated is increased by a factor of $\sqrt{2}$ over that which can be treated by methods requiring the storage of the full matrix. Appendix B gives the details of the storage and solution methods.

In another program, 5 point Gaussian integration, Ref. [25], is used to evaluate the $C_{mn}$ for $m \neq n$, and when $m = n$ Eq. (66) is used. The matrix is no longer symmetric so all $N^2$ terms must be stored.

A third program was written which takes advantage of the fact that the currents are continuous on the surface except at sharp edges (Ref. [26]). Since the column vector $[F]$ of Eq. (60) represents the current, continuity requires that adjacent entries be similar. Hence it is possible to interpolate. The currents at the even numbered stations may be approximated in terms of the adjacent currents by

\begin{equation}
F_{2n} = \frac{(F_{2n-1} + F_{2n+1})}{2}.
\end{equation}

For simplicity, the original matrix will be assumed to be of odd order

\begin{equation}
N = 2 kk + 1.
\end{equation}

If, for example, $N=7$ then, using Eq. (67) in Eq. (60), one obtains the reduced system
\[ E_1 = C_{11} F_1 + \frac{C_{12}}{2} (F_1 + F_3) + C_{13} F_3 + \frac{C_{14}}{2} (F_3 + F_5) + C_{15} F_5 + \frac{C_{16}}{2} (F_5 + F_7) + C_{17} F_7 \]

\[ E_3 = C_{31} F_1 + \frac{C_{32}}{2} (F_1 + F_3) + C_{33} F_3 + \frac{C_{34}}{2} (F_3 + F_5) + C_{35} F_5 + \frac{C_{36}}{2} (F_5 + F_7) + C_{37} F_7 \]

\[ E_5 = C_{51} F_1 + \frac{C_{52}}{2} (F_1 + F_3) + C_{53} F_3 + \frac{C_{54}}{2} (F_3 + F_5) + C_{55} F_5 + \frac{C_{56}}{2} (F_5 + F_7) + C_{57} F_7 \]

\[ E_7 = C_{71} F_1 + \frac{C_{72}}{2} (F_1 + F_3) + C_{73} F_3 + \frac{C_{74}}{2} (F_3 + F_5) + C_{75} F_5 + \frac{C_{76}}{2} (F_5 + F_7) + C_{77} F_7 \]

where only odd rows have been retained, i.e., \( F_2, F_4, F_6 \) are considered known. Collecting terms,

\[ E_k = (C_{k1} + \frac{C_{k2}}{2}) F_1 + (\frac{C_{k2}}{2} + C_{k3} + \frac{C_{k4}}{2}) F_3 + (\frac{C_{k4}}{2} + C_{k5} + \frac{C_{k6}}{2}) F_5 + (\frac{C_{k6}}{2} + C_{k7}) F_7 \]

for \( k = 1, 3, 5, 7, \)

and the number of unknowns has been reduced to \( k_k \). Since matrix manipulations are made using regular subscripts in the machine, it is very desirable to relabel the coefficients in the reduced system as follows

\[ C_m'_{i} = \frac{C((2m-1),(2i-2))}{2} + C((2m-1),(2i-1)) + \frac{C((2m-1),(2i))}{2} \]

for the "interior" columns where \( m=1, 2, 3, \ldots, k_k \) and \( i=2, 3, \ldots, k_k-1 \). The first and last columns of the reduced matrix are

\[ C_m'_{m2} = C((2m-1),1) + \frac{C((2m-1),2)}{2} \quad m=1, 2, 3, \ldots, k_k \]
\( C'_{m, kk} = \frac{C_{(2m-1), (2kk-2)}}{2} + C_{(2m-1), (2kk-1)}. \)

The \( C_{ij} \) are the elements of the original \( NXN \) matrix while \( C'_{ij} \) are elements of the \( kkXkk \) reduced matrix. In the computer program the \( C'_{ij} \) are called \( C_{ij} \) while the original matrix elements \( C_{ij} \) are labeled \( C_{ij} \).

When using the interpolation technique the surface is subdivided as usual except that, if an even number of segments is produced, then the last segment is dropped to make \( N \) odd. The system of equations is now

\[
[C][FP] = [E]
\]

where \([E]\) is filled with the incident electric field at the midpoints of the segments with odd subscripts and the matrix \([C]\) is loaded according to Eqs. (71), (72) and (73). After the solution has been found the column vector \( FP(j) \) contains the currents on the segments with odd subscripts. The complete set of surface currents \([F]\) is obtained by interpolation with

\[
\begin{align*}
F_{2j-1} &= FP_j \quad \text{for } j = 1, 2, \ldots, kk \\
F_{2j} &= (FP_j + FP_{j+1})/2 \quad \text{for } j = 1, 2, \ldots, kk-1.
\end{align*}
\]

Once the column vector \([F]\) has been filled in, the calculation of the scattered field proceeds as in Eqs. (63) and (64). The interpolation technique has been applied to the program which uses Gaussian integration to calculate the matrix elements.
The big advantage of interpolation is the dramatic increase in the size of the surface which can be handled for a given storage capacity. If the machine can handle an arclength of \( L \) using the non-symmetric, non-interpolation program then the symmetric matrix program can handle an arclength of \( \sqrt{2} L \) while the interpolation technique will do an arclength of \( 2L \) with the same amount of storage. The interpolation program still requires that all of the original matrix elements be evaluated to fill in the reduced matrix (Eqs. (71), (72) and (73)).

The integral equation programs require large amounts of storage and fairly long running times compared to either the G.O. or P.O. programs. The IBM 360-75 used here can hold a 275 x 275 complex matrix in high speed storage so that surfaces of length 27 \( \lambda_e \), or 54 \( \lambda_e \) if interpolation is used, can be handled with \( DC = \lambda_e/10 \). As for the running time, consider the 16 \( \lambda_e \) long surface mentioned in Chapter III Section C, which took 1.8 minutes using the P.O. program. The scattering from the same surface was computed by the three T.M. integral equation methods. The symmetric formulation required 2.8 minutes and storage for 14,000 complex numbers. The program which uses Gaussian integration to evaluate the matrix coefficients required 5.0 minutes and twice as much storage, while the interpolation program required 3.3 minutes and storage for 7,000 complex numbers. Where speed is important the use of the symmetric I.E. program is indicated, while long surfaces are best handled by the two point interpolation program.
D. Tests of the Transverse Magnetic Integral Equation Programs

The shortened contour assumption is one of the most crucial in the construction of the integral equation programs (Fig. 14). The obvious way to test it is to extend the non-illuminated portion of the surface, which amounts to lengthening the contour without changing the non-zero portion of the illumination (see Fig. 17). If the approximation is indeed valid, then the current in the non-illuminated sections should fall off rapidly and the scattered fields should be the same in both cases. The assumption was tested on a sinusoidal surface, using the program with Gaussian integration. When regular tapering was used, the current at the outer ends of the dead zones was down by a factor of 30 from that in the central part of the contour. When the extended taper was used, the current at the new outer ends was down by a factor of 100. The scattered fields for the two cases are displayed in Fig. 18 and show clearly that the differences are insignificant. Thus it may be concluded that tapering of the incident field does permit the replacement of the true contour by the shortened contour.

The wedge, Fig. 19, for which asymptotic solutions are available, provides a test case for the integral equation programs. The angle of incidence, \( \theta_h \), was chosen to be 90°. In order to emphasize the corner contribution, a Gaussian tapering of the incident field was used, i.e.,

\[
(76) \quad t(x) = e^{-(x/2\lambda_e)^2}.
\]
Fig. 17.—Contour and tapering function used to test the shortened contour assumption.
Fig. 18. Scattered fields with and without extended boundaries, T.M. case.

\[ \lambda_e = 25 \text{ cm} \quad |\mathbf{E}^i| = 1 \]

\[ H(x) = 5 \sin \left( \frac{2\pi x}{200} \right) \]
Fig. 19. -- Geometry for wedge test.

The surface current, Fig. 20, shows the expected singularity at the corner. The computed scattered field is plotted in Fig. 21 along with the scattered field calculated independently using the geometrical theory of diffraction, Ref. [27]. Again, the agreement is seen to be excellent. All three T.M. integral equation programs produced essentially identical scattered fields. In a test of the self consistency of the three programs the scattering from the surface $H(X) = 5 \sin \frac{2\pi}{200} x$ was computed. The differences in the scattered fields are very minor and would not be perceptible on the scale of, e.g., Fig. 18.

In the light of the above tests, there seems to be no reason to prefer one T.M. integral equation program over the other two if numerical accuracy is the only criterion. If the running time or storage requirements must be considered then the preferred formulation can be determined by the comments at the end of Section C of this Chapter.
Fig. 20.--Computed $|J_s|$ on a wedge, T.M. case.

Fig. 21.--Wedge scattered fields, T.M. case
E. Integral Equation for Transverse Electric Polarization

For the T.E. polarization, the incident magnetic field $\mathbf{H}^i$ is $z$ directed and it will be convenient to work with the integral equation for the magnetic field given (Ref. [28]) by

\begin{equation}
\mathbf{J}_s(r) = \hat{n} \times \mathbf{H}^i(r) + \frac{1}{2\pi} \hat{n}(r) \times \oint_{s} \mathbf{J}_s(r') \times \nabla' \frac{-jk|\mathbf{r}-\mathbf{r}'|}{|\mathbf{r}-\mathbf{r}'|} \, ds
\end{equation}

where $\mathbf{r}$, $\mathbf{r}'$ are both position vectors of points on the surface, $\mathbf{H}^i(r)$ is the incident magnetic field, $\mathbf{J}_s(r)$ is the surface current, $\hat{n}$ is the outward normal to the surface and $\oint_{s}$ indicates that the region about $\mathbf{r}' = \mathbf{r}$ is to be deleted from the integration. See Fig. 22.

Fig. 22. --Three dimensional geometry for T.E. integral equation.

The two dimensional integral equation can be obtained by considering an infinitely long cylinder as shown in Fig. 23. When the incidence direction lies in the $x,y$ plane the fields and surface current have no $z$ dependence so that Eq. (77) can be reduced to
Fig. 23.—Two dimensional geometry for T.E. integral equation.

\[ J_s(\vec{\rho}) = 2 \hat{n}(\vec{\rho}) \times H^4(\vec{\rho}) + \frac{k}{2j} \hat{n}(\vec{\rho}) \times \int_c J_s(\vec{\rho}') x (\hat{\vec{\rho}} - \hat{\vec{\rho}}') \]

\[ H_1^{(2)}(k|\vec{\rho} - \vec{\rho}'|) \, dc' \]

where \((\hat{\vec{\rho}} - \hat{\vec{\rho}}')\) is the unit vector in the \(\vec{\rho} - \vec{\rho}'\) direction and \(H_1^{(2)}(x)\) is the Hankel function of the second kind and order 1.

Just as in the T.M. case, tapering is introduced to account for the directional properties of radar antennas, and to limit the size of the system of linear equations which will result from Eq. (78). One may now assume that the surface currents are zero except near the illuminated region and the closed contour can then
be replaced by the open contour of Fig. 24. For this polarization the current flows transverse to \( \hat{z} \) along the surface so

\[
\mathbf{J}_S(\rho') = (\hat{z} \times \hat{n}(\rho')) \mathbf{J}_S(\rho') = \mathbf{T}(\rho') \mathbf{J}_S(\rho')
\]

where \( \mathbf{T}(\rho') \) and \( \hat{n}(\rho') \) are the unit tangent vector and the unit normal vector to the surface, as shown in Fig. 24. \( \mathbf{T}(\rho') \) is given in terms of the profile, \( H(x) \), by

\[
\mathbf{T}(\rho') = -\frac{[\hat{x} + \hat{H}(x')\hat{y}]}{\sqrt{1 + (\hat{H}(x'))^2}}
\]

where \( \hat{H} \) has the meaning assigned by Eq. (34). Using

\[
dc' = (1 + (\hat{H}(x'))^2)^{1/2} \, dx'
\]
and Eqs. (78) and (79) with the tapered incident field

\[ H^i_Z(\rho) = t(x) e^{-jK_1 \cdot \rho} \]

the integral equation becomes

\[ -t(x) e^{-jK_1 \cdot \rho} = \frac{J_s(\rho)}{2} + \frac{jk}{4} \int_{-EP}^{EP} \frac{J_s(\rho')}{\sqrt{(x-x')^2 + (H(x)-H(x'))^2}} \cdot \left[(H(x)-H(x'))-\dot{H}(x)(x-x')\right] \cdot \text{dx'} \]

where the integration over \( x' \) excludes a small region in the contour about the point described by \( \bar{\rho} \).

The method of moments is applied to Eqs. (83) just as in the T.M. case. the current is expanded in a basis of non-overlapping pulse functions of width DC, delta functions are used as weighting functions and the scalar product is the same as in the T.M. case. The current is thus represented by

\[ J_s(\rho') = \sum_{n=1}^{N} F_n \; P_{\psi}(\rho' - \rho_n) \]

where, \( \rho' \), \( \rho_n \) lie on the contour \( c \) and \( \rho_n \) is the position vector of the midpoint of the \( n \)-th segment, the \( F_n \)'s are the unknown expansion coefficients and the pulse functions \( P_{\psi}(\rho' - \rho_n) \) have been described in connection with the T.M. case. Placing this current in Eq. (83), taking the scalar product of both sides with the weighting functions and using the non-overlapping property of the basis functions results in
Since it is necessary to avoid $\bar{\rho}' = \bar{\rho}_m$ in the integration of Eq. (85), the summation will be forced to skip $n=m$ giving as a system of equations

$$-t(x_m) e^{-jk_1 \cdot \bar{\rho}_m} = \sum_{n=1}^{N} C_{mn} F_n$$

where

$$C_{mn} = \begin{cases} \frac{1}{2} & \text{if } m=n \\ \frac{jk}{4} \int_{x_n}^{x_{n+1}} H_1^{(2)}(k|\bar{\rho}_m-\bar{\rho}'|) \frac{[(H(x_m)-H(x'))-\hat{H}(x_m)(x_m-x')]\ dx'}{(x_m-x')^2+H(x_m)-H(x')^2} & \text{if } m\neq n \end{cases}$$

and $x_{n+1}$, $x_n$ are the upper and lower $x$ coordinates of the endpoints of the $n$-th surface segment respectively.
Once Eq. (86) is solved for the coefficients of the surface
current, $F_m$, the scattered field may be found from Ref. [18]

\begin{equation}
\vec{H}^S(\vec{r}) = \frac{1}{4\pi} \int_{S} \overline{J}_s(\vec{r}') \times \hat{n}' \frac{-jk|\vec{r} - \vec{r}'|}{|\vec{r} - \vec{r}'|} \, ds'.
\end{equation}

Specializing this to the far field scattering from an infinite
cylinder and using the fact that $\overline{J}_s(\vec{r}')$ is independent of $z$ and
non zero only over a portion of the cylinder (see Fig. 25).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig25.png}
\caption{Geometry for calculation of far field scattering, T.E. case.}
\end{figure}

\begin{equation}
H_z^S(\rho) = \frac{-jk|\rho|}{\sqrt{|\rho|}} \left( \begin{array}{c}
\frac{3\pi}{4} \int_{-EP}^{EP} \overline{J}_s(\rho') \left[ \sin(\text{THS})-H(x')\cos(\text{THS}) \right] \\
\sqrt{1+(\hat{H}(x'))^2}
\end{array} \right) \, d\rho' \,
\end{equation}
Substituting Eq. (84) whose coefficients are now known into Eq. (89) and assuming that the integrand is nearly constant over a surface segment of length DC,

\[
H_z^S(\vec{\rho}) = \frac{-j\frac{3\pi}{4}}{2\sqrt{\lambda}} DC e^{-jk|\vec{\rho}|} \sum_{n=1}^{N} F_n \cos(\Theta S - \Theta N(x_{M_n}))
\]

\[
\times e^{jk(X_{M_n}\cos(\Theta S) + H_{n}\sin(\Theta S))}
\]

where \(\Theta N(x)\) (\textit{THETA NORMAL}) is given by

\[
\Theta N(x) = (\pi/2) + \tan^{-1}(H(x))
\]

as shown in Fig. 25. The computed and plotted value of the scattered field, \(H_z^S\), is given by

\[
H_z^S = H_z^S(\vec{\rho}) \sqrt{|\vec{\rho}|} e^{+jk|\vec{\rho}|}.
\]

F. Discussion of the Computer Program for the Transverse Electric Polarization

The programs for the T.E. polarization are very similar to those for the T.M. polarization. As in the T.M. case the contour is broken up into segments of equal length DC. The same notation is used for the endpoints (x) and midpoints (XM) of the segments (Fig. 16). The T.E. and T.M. programs differ mainly in the values of the elements of the matrix \([C]\), and in the driving side of the system of equations.
Also, for the integral equation used, the matrix is non-symmetric no matter how the coefficients are evaluated. Once again the system of equations, (Eq. (86)), is solved in such a way that different scattering and incidence angles do not require a completely new solution. Only the back substitution portion need be repeated (see Appendix B).

Several different programs have been written for the T.E. case, the major difference between them being the method used to evaluate the coefficients (Eq. (87)). The simplest way is to assume that the integrand is constant over the strip width so that

\[
C_{mn} = \begin{cases} 
\frac{1}{2} & \text{if } m=n \\
\frac{\sqrt{k}}{4} \left( \frac{H(2)}{H(1)} \right) \frac{(H(X_m^n) - H(X_m^n)) - \hat{H}(X_m^n)}{\sqrt{\rho_m - \rho_n}} & \text{if } m \neq n.
\end{cases}
\]

In practice, only the five point Gaussian integration was used to evaluate the off diagonal elements of \([C]\), since it did not require much more running time than the simpler method. However, the interpolation technique retains all of its advantages and goes exactly as in the T.M. case with the \(C_{ij}'\) given by Eqs. (71), (72), and (73). Thus surface lengths of 27\(\lambda_e\) (or 54\(\lambda_e\) with interpolation) can be handled. As an example of the running times required, consider again the surface of length 16\(\lambda_e\) mentioned in Chapter 3 Section C. The T.E. physical optics program required 1.8 minutes while an equivalent run using the T.E. integral equation program required 5.0 minutes.
The interpolation program for this polarization took 3.5 minutes. Thus the interpolation program is superior to the non-interpolation program both with respect to storage requirement and running time.

G. Tests of the Transverse Electric Integral Equation Programs

The shortened contour assumption plays the same role and is tested in the same way in the T.E. integral equation programs as in the T.M. case. The contour is extended as shown in Fig. 17. When the regular tapering was used, the current at the outer ends of the dead zones was down by a factor of 70 from that in the central portion of the contour. When the extended surface was considered the current at the new outer ends was down by slightly more. The nearly identical scattered fields for the two cases are shown in Fig. 26.

The wedge provides a test case for which an independent result is available. The test geometry is as shown in Fig. 19 except that here the incident magnetic field is parallel to the corner of the wedge. Gaussian tapering of the incident field, Eq. (76), is used. In contrast to the current singularity in the T.M. case, the surface current in the T.E. case, Fig. 27, shows the expected $r^{2/3}$ behavior at the corner. The excellent agreement between the scattered fields calculated by the integral equation method and the fields obtained from the geometrical theory of diffraction, Ref. [27], is illustrated in Fig. 28. Both the non-interpolation and the interpolation T.E. integral equation programs gave the same result in this test.
Fig. 26. -- Scattered field with and without extended boundaries, T.E. case.
Fig. 27.—Computed $|J_s|$ near corner of wedge, T.E. case.

Fig. 28.—Wedge scattered fields, T.E. case.
The consistency of the two T.E. integral equation programs was checked on a surface with a height profile \( H(x) = 5 \sin(2\pi x/200) \). The results were nearly identical.

The above tests indicate that so far as numerical accuracy is concerned the non-interpolation and interpolation T.E. integral equation programs do not differ. The interpolation program is preferred however because of the savings in storage.
In this chapter the previously developed computer programs will be used to check the applicability of the geometrical optics, physical optics and perturbation approximations to the calculation of the scattering from non-uniform surfaces. The integral equation programs, which are believed to be exact, are used as standards.

The first surface to be considered has been especially chosen so that it fulfills the requirements necessary in order that physical and geometrical optics both give a valid approximation to the true scattered fields. The surface, a single half-cycle of a sine wave, has a profile \( H(x) = 50 \cos(2\pi x/800) \) with \( x \) between 200.0 cm and -200.0 cm, and clearly has but one specular point. The incident field is tapered, and has an electrical wavelength of 25 cm. Unless otherwise noted, these conventions have been used throughout. The criteria for the successful application of G.O. and P.O. are met by this profile since the minimum radius of curvature is 12.8 \( \lambda_e \) and, having a maximum height of two \( \lambda_e \), there are several Fresnel zones on the surface. The scattered fields predicted by the G.O., P.O. and I.E. programs are shown in Figs. 29 and 30 for the T.M. and T.E. polarizations respectively.
Fig. 29. Scattering from $50 \cos(2\pi x/800)$ as calculated by I.E., P.O., and G.O.; T.M. polarization.
Fig. 30. - Scattering from $50 \cos \frac{2\pi x}{800}$ as calculated by I.E., P.O., and G.O.; T.E. polarization.

$H(x) = 50 \cos \left(2\pi x / 800\right)$

$\lambda_e = 25 \text{ cm} \quad |\vec{H}| = 1$

THS - THE SCATTERING ANGLE

G.O. - P.O. - I.E.
It is apparent that all methods give nearly the same result for THS between 87° and 155°. No scattered fields are predicted by G.O. for THS outside the range 78° and 163° since the normals to the surface have a limited range of directions as illustrated in Fig. 31. The rise in the value of scattered field predicted by G.O. near 78° and 163° is due to the movement of the specular point into a region of the surface of increasing radius of curvature. However, as the specular point gets within two wavelengths of either endpoint the tapering of the incident field suppresses the expected singularity in the scattered field.

It should also be noted that for the P.O. results, the T.M. fields differ slightly from the correct fields for THS near grazing.
For either polarization the ripple observed in the scattered field and correctly predicted by P.O. is probably a consequence of the finite length of the surface. G.O., being a purely local theory, will not predict effects of this nature.

As a further check of the programs, the above profile was multiplied by minus one, i.e., instead of being concave down the surface was concave up. The amplitudes of the scattered fields remained unchanged but they all showed a phase shift of 90° due to what in G.O. theory is termed the caustic correction factor.

In order to establish more quantitatively the limitations on the G.O. and P.O. approximations, the scattered fields have been computed for a set of surfaces with height profile

\[ H(X) = A \sin\left(\frac{2\pi X}{200}\right) \quad -200 \text{ cm.} \leq x \leq 200 \text{ cm.} , \]

i.e., the surfaces are two complete mechanical wavelengths long.

With THI fixed at 60°, the amplitude, A, was varied over a range of 5.0 cm. to 50.0 cm. so that the minimum radius of curvature, \( r_{cm} \), varied from 8.0 \( \lambda_e \) to 0.8 \( \lambda_e \). The important features of the scattered fields over this range of \( r_{cm} \) for each polarization are shown in Figs. 32-37 in order of decreasing \( r_{cm} \). Some general trends are worthy of mention.

In the first place, as \( r_{cm}/\lambda_e \) decreases from 8 to 0.8, the agreement between the P.O. results and the exact fields goes from excellent to poor. It would appear that as long as the surface always has \( r_{cm}/\lambda_e \) greater than, say, 2.5, the P.O. approximation will
Fig. 32.--Scattered fields predicted by P.O., G.O., and I.E. methods for \( H(x) = 5 \sin(2\pi x/200) \), T.M. polarization.
MINIMUM RADIUS OF CURVATURE = \( r_{cm} = 8 \lambda_e \)

Fig. 33.—Scattered fields predicted by P.O., G.O., and I.E. methods for \( H(x) = 5 \sin(2\pi x/200) \), T.E. polarization
Fig. 34.—Scattered fields predicted by P.O., G.O., and I.E. methods for $H(x) = 15 \sin(2\pi x/200)$, T.M. polarization.
Fig. 35.--Scattered fields predicted by P.O., G.O., and I.E. methods for \( H(x) = 15 \sin(2\pi x/200) \), T.E. polarization.
Fig. 36. Scattered fields predicted by P.O., G.O., and I.E. methods for $H(x) = 25 \sin(2\pi x/200)$, T.M. polarization.
Fig. 37.—Scattered fields predicted by P.O., G.O., and I.E. methods for $H(x) = 25 \sin(2\pi x/200)$, T.E. polarization.
give reliable values for the scattered field. Even for values of \( r_{cm}/\lambda_e = 1 \), P.O. may still be considered usable, that is, it will reproduce the general structure of the scattered fields although with significantly lower accuracy. This limitation on the radius of curvature necessary for the successful application of the P.O. approximation is in agreement with the results of Ref. [29] in which the current on a sinusoidal surface of infinite extent is found. Except for scattering and incidence angles for which no specular points occur or for which a specular point coincides with a point of infinite radius of curvature, the G.O. and P.O. approximations give scattered fields very similar to each other even when they are not correct, e.g., Fig. 38. It is interesting to note that where the I.E. and P.O. (and hence the G.O.) fields agree the T.E. and T.M. fields are nearly identical but as the radius of curvature decreases the exact fields, T.E. and T.M., not only differ from the respective P.O. fields but from each other. This behavior is not entirely unexpected since for bodies with large radius of curvature in terms of wavelength the polarization independent G.O. is known to be a good approximation. As the radius of curvature goes to zero, e.g. a wedge, G.O. and P.O. both fail and the scattering is polarization dependent (see the wedge tests in Chapter IV).

The failure of G.O. when no specular point occurs on the surface or when a specular point coincides with a point of infinite radius of curvature makes it far less attractive than P.O., especially when numerical methods are involved. For example, when \( A=5 \), (see Fig. 32)
Fig. 38. -- Agreement of G.O. and P.O. when they are incorrect.
G.O. predicts no scattered field outside the range \(102^\circ < \text{THS} < 138^\circ\), and gives fields which are singular at either end of the range. On the other hand, the P.O. approximation correctly predicts the scattered fields for a far wider range of THS, including backscatter, and the fields are always bounded.

It is also of interest to note that what might be called the "fine structure" of the scattering, particularly for \(\text{THS} < 80^\circ\), (see Fig. 32) is not due entirely to the finite length of the illuminated region as in Figs. 29 and 30 but is strongly controlled by the height profile.

Another approximate theory whose validity can be checked by the numerical methods developed here is the perturbation theory for the scattering from "slightly rough" surfaces as formulated in Refs. [30] and [31]. Perturbation theory predicts that if the amplitude of the surface profile is much less than the electrical wavelength of the incident fields, then the amplitude of the scattered field due to the perturbation of the surface is proportional to the surface height amplitude. This was checked by calculating, using the T.M. integral equation program, the scattering from a surface profile described by

\[
H(x) = c \left( \sin\left(\frac{2\pi x}{50}\right) + \frac{1}{2} \sin\left(\frac{2\pi x}{19.71}\right) \right)
\]

for various values of \(c\). The field scattered by slightly rough surfaces is dominated by the scattered field from the unperturbed surface \((c=0)\) which is quite complex for the finite strips considered.
here. Thus the behavior of the perturbed fields can best be illustrated by considering the difference between the actual field and the flat plate field. The perturbation in the scattered field, \( E_p \), due to the perturbation in the height profile of the originally flat strip is then given by

\[
(96) \quad E_p = E^S_z - E^S_{z0}
\]

where \( E^S_z \) is the total scattered field as predicted by the computer program, and \( E^S_{z0} \) is the field scattered when \( c \) is zero (i.e., a flat strip). In order to test the prediction that \( |E_p| \propto c \), a low value of \( c \) (\( c=0.01 \) cm.), was chosen as a reference surface amplitude with reference scattered field \( |E_p| \), so that for a fixed scattering angle

\[
(97) \quad \frac{|E_p|}{|E_p|} = \frac{c}{c_1}
\]

expresses the perturbation theory result. The exact fields are compared with perturbation theory in Fig. 39 for several values of \( c \). The theory appears to fail at about \( c/c_1 = 200 \) which corresponds to a root mean square surface amplitude of approximately \( \lambda_e/10 \).

In addition to permitting the examination of the applicability of various electromagnetic approximations to the ocean surface scattering problem, the programs permit direct calculation of the scattered fields from any appropriate surface. One such application is to the calculation of the expected value of the backscattered power from an ensemble of ocean-like surfaces. Such an ensemble may be constructed from the known height spectrum, Ref. [32]. For the sea surface, the
\[ H(x) = \frac{C}{C_1} \left( C_1 \sin \left( \frac{2\pi x}{50} \right) + \frac{C_1}{2} \sin \left( \frac{2\pi x}{19.71} \right) \right) \]

\[ \Theta H I = 60^\circ \]

\[ \Theta H S = 90^\circ \]

---

**Fig. 39.**--Perturbation theory test.
height spectrum, Fig. 40, decays approximately as $k_m^{-4}$ over the significant range of wave numbers, where $k_m$ is the mechanical wavenumber.

$W(k) = \text{cm}^4$

Thus a particular member of the ensemble can be chosen to be a finite sum of sinusoids with random phases whose amplitudes vary roughly as $k_m^{-2}$. If the $k_m$'s are not harmonically related, the surface, like the ocean, will be aperiodic. One example of a surface of this type is given by the series

(98) \[ H(x) = 2.5(0.4 \sin(2\pi x/200.0 + 0.78) \\
+ 0.8(10.0/20.0)^2 \sin(2\pi x/10.954 + 1.6) \\
+ 0.8(6.66/20.0)^2 \sin(2\pi x/6.28318 + 2.4) \\
+ 0.8(5.0/20.0)^2 \sin(2\pi x/4.795 + 0.4)) \]

illustrated in Fig. 41. An ensemble of surfaces of finite length can be generated by using successive non-overlapping sections of this surface.
Fig. 41.--Four component representation of the surface

Physical optics was used to calculate the expected value of the backscattered power and field strength from a 75 member ensemble made from the surface described by Eq. (98). Each member of the ensemble was 75 electrical wavelengths long. On a CDC 6600 computer,
the time required for the run was about 40 minutes. The expected values \( <|E_z|^2| > \) are shown in Fig. 42; the expected value of \( E_z^S \) was found to be extremely small compared to the root mean square field.

![Graph](image)

**Fig. 42.**--Expected value of backscattered \( <|E_z|^2| \) from ensemble.

Notice that no special form of the slope distribution or other statistical properties of the surface have to be assumed. It is also possible to use a point by point, i.e. discrete, representation of the surface, such as might be generated by the prescribed statistical properties of the surface.
CHAPTER VI
SUMMARY AND CONCLUSIONS

In this work the scattering properties of cylindrical rough surfaces have been investigated by several numerical techniques in order to test the validity of previous theoretical work. The results, using as checks the integral equation solutions, show that geometrical optics is not usable for surfaces with radius of curvature smaller than $2.5 \, \lambda_e$ and may give poor results even when this condition is satisfied should the scattering geometry be such that no specular point exists or a specular point coincides with a point of infinite radius of curvature. With the exception of these two cases, geometrical optics and physical optics give nearly the same scattered fields.

It was found that the numerical evaluation of the scattered fields from the physical optics currents gives good results for almost any geometry (except perhaps deeply shadowed configurations) as long as the radius of curvature condition, $r_{CM} > 2.5 \, \lambda_e$, is satisfied. Physical optics, although not always so accurate, has an advantage over the integral equation formulation in that the length of surface which can be treated is not limited by machine storage capacity.
The integral equation program has been used to check the perturbation theory prediction that the amplitude of the scattered field increases in proportion to the increase in the amplitude of the surface height profile. The numerical results confirm in a quantitative way the fact that the theory fails when the root mean square height is about one tenth of an electrical wavelength.

The physical optics program, because of its ability to handle long surfaces and its superiority to geometrical optics, has been applied to the direct calculation of the expected value of the scattered power from an ensemble of ocean-like surfaces which were constructed from a height spectrum similar to that of the sea. The computer time required, while lengthy, was not found to be prohibitive.

The extension of the programs to very long surfaces, to non-cylindrical surfaces or to dielectric surfaces appears feasible only for the G.O. and P.O. methods; the storage requirements for an I.E. solution in either case would be prohibitive. P.O. would probably be the easiest to modify to non-cylindrical surfaces, especially if shadowing were neglected. Since location of the specular points becomes much more complicated in the non-cylindrical case, the G.O. method would be more difficult to implement.
APPENDIX A

COMPUTER PROGRAMS

A listing of all the programs discussed in the text is presented here. To facilitate understanding of the programs, the symbols used in the programs have been used in the text whenever possible.

All programs require the plot subroutine listed at the end. The function subprograms AHAN20(x) and AHAN21(x) are required in the T.M. and T.E. integral equation programs respectively.
THIS PROGRAM IS FOR BISTATIC BACKSCATTERING

ESCNS IS THE RETURNED E FIELD WITH SHADOWING NOT ACCOUNTED FOR
ESCWS IS E SCATTERED WITH SHADOWING ACCOUNTED FOR

GEOMETRICAL OPTICS FOR THE OCEAN SURFACE

SPECULAR POINT SEARCH IS DONE IN TWO STEPS
#1 IS MECHANICAL WAVELENGTH DEPENDENT, #2 IS REFINENN MECHANICAL OR
ELECTRICAL WHICHEVER IS MORE STRINGENT.
DELSHA IS SHADOW TEST STEP SIZE
THIS PROGRAM CAN HANDLE 200 SPECULAR POINTS /PASS IE. ONE THIRDS

DIMENSION XN(200), ANGLE(200)

DIMENSION ACDNS(720), AWS(720), AhCS(720), ASNS(720), AOS(720)

REAL PI, PI2
REAL MTWO

COMPLEX ESCNS, ESCWS, ENS
COMMON CA, CB, CKA, CKB, PHA, PHB, CC, CKC, PHC

COMPLEX ESCNS, ESCWS
NAMELIST /CAT/ CA, CB, CKA, CKB, PHA, PHB, CC, CKC, PHC, WAVE, THID
NAMELIST /CT/ ENS, ASNS, ECDNS, ACDNS, EWCS, AWS, AOS

THE FUNCTION WHICH DESCRIBES THE SURFACE IS

\[ H(x) = CA \cdot \sin((CKA \cdot x) + PHA) + CB \cdot \sin((CKB \cdot x) + PHB) + CC \cdot \sin((CKC \cdot x) + PHC) \]

CA = 10.0
CKA = 6.28318/200.0
PHA = 0.0

CB = 0.0
CKB = 0.0

PHB = 0.0
CC = 0.0

CKC = 0.0

PHC = 0.0

HMAX = ABS(CA) + ABS(CB) + ABS(CC)

PI = 3.14159

PI2 = 1.5707963

WMMIN IS THE MINIMUM MECHANICAL WAVELENGTH

WAVE IS THE ELECTRICAL WAVELENGTH

WAVE = 25.0

AMIN = (WAVE/20.0)

DLTAX = WMMIN / 10.0

XSTOP = XSTART * 10**9

THI = 60.0 + 3.1415927/180.0

WRITE(6, TOM)

DO 93 IRE = 1, NAN

ACDNS(IRE) = 0.0

ASNS(IRE) = 0.0

AMES(IRE) = 0.0

ACWCS(IRE) = 0.0

ESNS(IRE) = 0.0

ECDNS(IRE) = 0.0

87
CONTINUE
DO 17 J=1,N
THS=FLOAT(1J)*.8726646 E-02
THSD=THS*57.29578
AOS(1J)=THSD
WRITE(6,356) THID,THSD
FORMAT(11H INC ANGLE=,E15.8,13H SCATT ANGLE=.,E15.8)
SUCOS=CTHI+CO.$(THS)
SUSI=SNTHI+Sin$(THS)
N=0
C FIRST FIND POSITIONNS CF SPECULAK RETURN AND STORE THEM
C THE FIRST POSITION CAN NOT BE A SPECULAK POINT
XP=XSTMT
SUMD2=(TH+THS)/2.0
E=SUMD2-(TH(XP)+PI2)
102 XP=XP+DLTAX
EO=E
E=SUMD2-(TH(XP)+PI2)
IF(E.EQ.0.0) GO TO 100
IF(((EO.GT.0.0).AND.(ELT.0.0)).OR.(EO.LT.0.0).AND.(E.GT.0.0))) 2 GO TO 100
GO TO 101
100 N=N+1
XN(J)=XP
ANGLE(J)=THS-(TH(XP)+PI2)
101 IF (XP.LE.XSTOP) GO TO 102
IF(N.EQ.0) GO TO 372
THIS IS TO REFINE THE POSITION OF THE SPECULAK POINT
DO 25 K=1,N
XSO=XN(K) -DLTAX
E=SUMD2-(TH(XSO)+PI2)
222 XSO=XSO+DLTAXGO
EO=E
E=SUMD2-(TH(XSO)+PI2)
IF(E.EQ.0.0) GO TO 252
IF(((EO.GT.0.0).AND.(ELT.0.0)).OR.(EO.LT.0.0).AND.(E.GT.0.0))) 2 GOT0252
GO TO 253
252 XN(K)=XSO
ANGLE(K)=THS-(TH(XSO)+PI2)
253 CONTINUE
IF (XSO.LT.XSTOP) GO TO 222
25 CONTINUE
ESCN=CMPLX(O.O,O.O)
ESCDNS=CMPLX(O.O,O.O)
DD 10 K=1,N
PHASE=(TI/WAVE)*((SUCOS*XN(K))+SUSI*H(XN(K)))
RC=RS(XN(K))*COS(ANGLE(K))
IF(RC.LT.0.0) PHASE=PHASE+(PI/2.0)
ENS=-( (SQR(T(AOS(RC/2.0)))*EXP(CMPLX(0.0,PHASE)))
C TAPPERING INCLUDED
XG=XN(K)
IF(XG.GT.(XSTOP-WAVE)) ENS=CMPLX(0.0,0.0)
IF(XG.LT.(XSTR+HAVE)) ENS=CMPLX(0.0,0.0)
IF(XG.WAVE) ENS=CMPLX(0.0,0.0)
252 N=ENS+(0.5+(0.5*SIN((3.14159/WAVE)+(XG-(XSTR+1.5*WAVE))))
ESCN=ESCN+ENS
IF(RS(XN(K))).LE.0.0) GO TO 10
ESCN=ESCN+ENS
10 CONTINUE
ACD=CABS(ESCDNS)
IF(ACD.LT.1.0 E-05) GO TO 59
ANACD=57.29578*AANZ(AIMG(ESCONS),REAL(ESCONS))
59 CONTINUE
IF(AACD.LT.1.0 E-05) ANACD=0.0
ESMAG = \text{CAB}(ESCNS)
ESANG = \text{ATAN2}(\text{IMAG}(ESCNS), \text{REAL}(ESCNS)) \times 180.0 / 3.1415927
WRITE(6,726) ESMAG, ESANG

726 FORMAT('*, MAG. OF SCATT. E FIELD=',E15.8,' PHASOR ANGLE=',E15.8,
23X,' WITHOUT SHADOWING')
WRITE(6,121) ACD, ANACC

121 FORMAT(* *, SCATT. FIELD NO SHADOW CONCAVE DOWN TIPS ONLY=',E15.8,
2* PHASOR ANGLE=',E15.8)
ESNS(IJ) = ESMAG
ECDNS(IJ) = ACD
ASNS(IJ) = ESANG
ACDNS(IJ) = ANACC

C NOW FIND THE SHADOWING EFFECT
C INBOUND SHADOWING
IF (ABS(THI-PI2.I.LT.0.05) GO TO 500
DO 327 K = 1, N
DI = H(XN(K)) - (TAN THS \times XN(K))
STEPI = DELSHA
IF (TAN THS.LT.0.0) STEPI = -DELSHA
XI = XN(K) + STEP I
GO TO 471

470 XI = XI + STEP I
471 YI = (TAN THS.XI) + DI
IF (YI.LE.HI) XN(K) = XSKIP
IF (ABS(XN(K)).GT.XSTOP) GO TO 499
IF (ABS(YI).GT.XSTOP) GO TO 499
IF (YI.LE.HMAX) GO TO 470
CONTINUE
327 CONTINUE
500 CONTINUE

C OUTBOUND SHADOWING
IF (ABS(THS-PI2.I.LT.0.05) GO TO 639
TANTHS = TAN THS
DO 633 KK = 1, N
IF (XN(KK).GT.XSTOP) GO TO 633
THE ABOVE CARD MAKES SURE THAT TIME IS NOT SPENT ON A PT, ALREADY
C KNOWN TO BE SHADOWED
BO = H(XN(KK)) - (TAN THS \times XN(KK))
STEPO = DELSHA
IF (TAN THS.LT.0.0) STEP O = -DELSHA
XO = XN(KK) + STEP O
GO TO 671

670 XO = XO + STEP O
671 YO = (TAN THS.XO) + BO
IF (YO.LE.HI) XN(KK) = XSKIP
IF (ABS(XN(KK)).GT.XSTOP) GO TO 699
IF (ABS(YO).GE.XSTOP) GO TO 699
IF (YO.LE.HMAX) GO TO 670
 CONTINUE
633 CONTINUE
639 CONTINUE
C END OF SHADOWING EFFECT
INININ = 0
ESGS = CMPLX(0.0,0.0)
ESCD = CMPLX(0.0, 0.0)

DO 19 K = 1, N

C NEXT CARD SKIPS THE SHADOWED SPECULAR POINTS
IF (XN(K).GT.XSTOP) GO TO 19
INININ = K

PHASE = (PI/WAVE) - (SUCOS*XN(K) + (SUSIN*H(XN(K)))
RC = S(90 - ANGLE(K))
IF (RC.LT.0.0) PHASE = PHASE + (PI/2.0)
ENS = -(SQRT(ABS(RC/2.0)))*CEXP(CMPLX(0.0, PHASE))

C TAPERING INCLUDED
XG = XN(K)

IF (XG.GT.(XSTOP-WAVE)) ENS = CMPLX(0.0, 0.0)
IF (XG.LT.(XSTART+2.0*WAVE)) ENS = CMPLX(0.0, 0.0)

2ENS = ENS*0.5 - 0.5*SQRT(3.14159/WAVE)*XGhecy(XSTOP-1.5*WAVE))
IF (XG.GE.(XSTART+2.0*WAVE)) AND (XG.LE.(XSTOP-2.0*WAVE))
2ENS = ENS*0.5 + 0.5*SQRT(3.14159/WAVE)*XGhecy(XSTART+(1.5*WAVE))

ESCWS = ESCWS + ENS
IF (INININ.LE.0.0) GC TO 19
 ESCU = ESCU + ENS

19 CONTINUE
IF (INININ.EQ.0) WRITE(6,3149)
IF (INININ.EQ.0) GO TO 23

ABESCD = CAB9(ESCD)

IF (ABESCD.LT.1.0E-05) GO TO 58

ANESCD = 57.29578*ATAN2(AIMAG(ESCD), REAL(ESCD))

58 CONTINUE
IF (ABESCD + LT.1.0E-05) ANESCD = 0.0

ESMAGS = CMPLX(AIMAG(ESCWS),REAL(ESCWS))

ESANGS = ATAN2(AIMAG(ESCWS), REAL(ESCWS))

3149 FORMAT('NO SCATTERED E FIELD WITH SHADOWING')

776 FORMAT('MAG OF SCATT. E FIELD WITH SHADOWING=', E15.8, ' PHASOR ANGLE=', E15.8)

2118 FORMAT('SCAT FIELD WITH SHAD. CONCAVE DOWN ONLY=', E15.8, ' PHASOR ANGLE=', E15.8)

GO TO 372

EWS(IJ) = ESMAGS
EWCS(IJ) = ABESCD
AWS(IJ) = ESANGS
AWCS(IJ) = ANESCD

GO TO 23

372 WRITE (6,3152) THID, THSD

3152 FORMAT('NO SPECULAR POINTS FOR THID=', E15.8, ' AND THSD=', E15.8)

23 WRITE(6,779)

779 FORMAT('CONTINUE')

17 CONTINUE

C FOR THE PLOTS
DO 536 IKO = 1, NANI
IND = IKO - 1
THSD = AOS(IKO)
Y(I) = ESHS(IKO)

536 CALL PLOT(THSD, Y(I), INDI, 50.0, 0.0)
DO 536 IKO = 1, NANI
IND = IKO - 1
THSD = AOS(IKO)
Y(I) = EWS(IKO)

537 CALL PLOT(THSD, Y(I), INDI, 50.0, 0.0)
DO 536 IKO = 1, NANI
IND = IKO - 1
THSD = AOS(IKO)
Y(I) = EWCS(IKO)

538 CALL PLOT(THSD, Y(I), INDI, 50.0, 0.0)
DO 536 IKO = 1, NANI
IND = IKO - 1
THSD = AOS(IKO)
Y(I) = EWCS(IKO)
CALL PLOT(THSD,Y,IND,50.0,0.0)
DO 936 KKRL=1,NAN1
   ANGOS=FLOAT(KKRL)/2.0
   IF(EWS(KKRL).LE.0.0001) GO TO 936
   DBNS=20.0*ALOG10(EWS(KKRL))
   WRITE(6,937) DBNS,ANGOS
   CONTINUE
DO736 KKRL=1,NAN1
   ANGOS=FLOAT(KKRL)/2.0
   IF(EWS(KKRL).LE.0.0001) GO TO 736
   DBS=20.0*ALOG10(EWS(KKRL))
   WRITE(6,737) DBS,ANGOS
   CONTINUE
STOP
END

FUNCTION RS(X)
   COMMON CA,CB,CKA,CKB,PHA,PHB,CC,CKC,PHC
   C THIS GIVES THE RADIUS OF CURVATURE AT X
   HP=(CA+CKA*COS((CKA*X)+PHA)+(CB+CKB*COS((CKB*X)+PHB))
   2+(CC+CKC*COS((CKC*X)+PHC))
   HPP=-(CA+CKA*COS((CKA*X)+PHA)+(CB+CKB*COS((CKB*X)+PHB))
   2+(CC+CKC*COS((CKC*X)+PHC)))
   RS=((1.0+(HP*HP)**1.5)/(-HPP))
RETURN
END

FUNCTION TH(X)
   COMMON CA,CB,CKA,CKB,PHA,PHB,CC,CKC,PHC
   TH=ATAN2((CA+CKA*COS((CKA*X)+PHA)+(CB+CKB*COS((CKB*X)+PHB))
   2+(CC+CKC*COS((CKC*X)+PHC))),1.0)
   C THIS FUNCTION GIVES THE ANGLE BETWEEN THE TANGENT TO H(X) AND THE
   C HORIZONTAL
RETURN
END

FUNCTION H(X)
   COMMON CA,CB,CKA,CKB,PHA,PHB,CC,CKC,PHC
   H=1*(CA+SID((CKA*X)+PHA)+(CB+SID((CKB*X)+PHB))+CC+SID((CKC*X)+PHC))
RETURN
END
DIMENSION Y(10), ESS(360)
C THIS IS THE TM CASE.
C THIS PROGRAM USES PHYSICAL OPTICS TO CALCULATE THE BACKSCATTERING
C FROM A SEA SURFACE BY DIVIDING SURFACE INTO LIT AND UNLIT REGIONS
C IN THE LIT REGIONS THE SURFACE CURRENT IN ZNXH
C GAUSSIAN INTEGRATION USED
C FOR THIS PROGRAM TO GIVE USEFUL RESULTS THE SURFACE MUST HAVE
C RADII OF CURVATURE NO LESS THAN 1*WE
C NSP IS THE NUMBER OF SHADOW POINTS
C SURFACE IS DESCRIBED BY AONE*SIN(CONE*X+PONE) *ATWO*SIN(CTWO*X
C *PTWO) +ATRE*SIN(CTRE*X+PTRE)
C SURFACE UNDER CONSIDERATION LIES BETWEEN ALEP AND REP
C SN IS THE STEP SIZE TAKEN TO DETERMINE SHADOWING
C IT MUST BE SMALLER THAN ANY SURFACE FEATURES AND MUST ALSO
C ALLOW THE LOCATION OF THE END POINTS OF INTEGRATION WITHIN
C A SMALL FRACTION OF A WAVELENGTH
C NANI IS THE NUMBER OF ANGLES (SCATTERING) TO BE EXAMINED
C MAKE DIMENSIONS OF ESS, SCANG, EFPA SMALL AS POSSIBLE TO AVOID
C LARGE # OF CARDS RETURNED
C NANI SHOULD BE THE DIMENSION OF ESS, SCANG, EFPA
NAMELIST/RON/AB, ANG, DTHS
DIMENSION SCANG(360), EFPA(360)
COMPLEX S, BINT
C SCATTER SHADOWING HAS NOT BEEN ACCOUNTED FOR
COMMON /DOG/AONE, CONE, PONE, ATWO, CTWO, PTHO, ATRE, CTRE, PTRE
COMMON /DOG/ G, THI, THS, WE
COMMON /PIG/ SECTOR, DX, REP, SEC10
COMMON /GNN1/ GW1, GW2, GW3, GW4, GW5, GUI, GU2, GU3, GU4, GU5
WE=25.0
C WE IS THE ELECTRICAL WAVELENGTH
G=2.0E3.1415927/WE
SRTWE=SQR(TL(WE)
CX=WE/15.0
AONE=50.0
CONE=2.0E3.1415927/800.0
PONE=3.1415927/2.0
ATWO=0.0
CTWO=0.0
PTWO=0.0
ATRE=0.0
CTRE=0.0
PTRE=0.0
NANI=360
SECTOR=WE/2.0
SEC10=SECTOR/10.0
C CONSTANTS FOR GAUSSIAN INTEGRATION
GW1=0.2369268
GW2=0.47862867
GW3=0.568889
GW4=GW2
GW5=GW1
GUI=-0.9061798
GU2=-0.53846931
GU3=-0.0
GU4=-GUI
GU5=-GUI
C THE ANGLE OF INCIDENCE SHOULD NOT BE GREATER THAN 90 DEG
THI=60.0E3.1415927/180.0
C IF THE INCIDENCE ANGLE IS WITHIN TEN DEGREES OF 90 NO SHADOWING
C TAKEN INTO ACCOUNT
IF(ABS(THI-1.5707)<.175) GO TO 563
TANTHI=TAN(THI)
DTHI=180.0E3THI/3.1415927
WRITE(6,1071) DTHI
1071 FORMAT(1X, ' ANG OF INC FROM POS X AXIS = ',E15.8)
REP=200.0
ALEP=-REP
SN=WE/10.0
NSP=1
DIMENSION SX(1000)
IF(DH(REP)+TANTH[I]) GO TO 106
SX(NSP)=REP
GO TO 105
106 SLOPE=TANTH[I]
B=M(REP)-(SLOPE*REP)
X=REP
GO TO 109
109 X=X-SN
IF((SLOPE*X)+B.GT.H(X)) GO TO 109
IF(X.LE.ALEP) GO TO 1000
SX(NSP)=X-(SN/2.0)
105 CONTINUE
C
THIS ABOVE TAKES CARE OF THE FIRST RIGHTENDPOINT
15 X=SX(NSP)
22 X=X-SN
XS=X-SX
IF((DH(X).LT.TANTH[I]).AND.(DH(XN).GT.TANTH[I])) GO TO 53
IF(X.GT.ALEP) GO TO 22
GO TO 92
53 NSP=NSP+1
SX(NSP)=XN
SLOPE=TANTH[I]
B=M(SX(NSP))-(SLOPE*SX(NSP))
X=SX(NSP)-SN
29 X=X-SN
IF((SLOPE*X)+B.LT.H(X)) GO TO 39
IF(X.GT.ALEP) GO TO 29
GO TO 92
39 NSP=NSP+1
SX(NSP)=X-(SN/2.0)
GO TO 15
92 NSP=NSP+1
SX(NSP)=ALEP
GO TO 564
563 SX(1)=REP
SX(2)=ALEP
NSP=2
564 CONTINUE
C
LAST VALUE IN SX(J) IS ALEP
WRITE (6,101) (K,SX(K),K=1,NSP)
101 FORMAT(' ',*','SXI(','14,*)='E15.8')
DO 317 JNX=1,NANI
THS=FLOAT(JNX)*(0.8726646 E-02)
DTHS=180.0*THS/3.1415927
SCAN(JNX)=DTHS
S=CMPLX(0.0,0.0)
KKN=1
CONTINUE
ALCLW=SX(KKN+1)
AUPP=SX(KKN)
S=S+BINT(ALCW,AUPP)
KKA=KKN+2
IF(KKN.LT.NSP).AND.(KKN+1.LT.NSP)) GO TO 10
C
TO CONVERT TO TRUE SCATTERED E FIELD FOR EINC OF UNITY MAG.
S=CMPLX(-0.70711,-0.70711)*S/SRTWE
AB=CABS(S)
ESSS(JNX)=AB
ANG=180.0*ATAN2(TAIMAG(S),REAL(S))/3.1415927
EFPJ(JNX)=ANG
317 CONTINUE
DO 531 JK=1,NANI
E = ESSSI(JK)
DB = 20.0 * ALOG10(E)
A = EFPAl JK)
AS = SCAhG(JK)
531 WRITE (6, 532) AS, E, A, DB
532 FORMAT(' ', 'SCAT ANG FROM HORIZ= ', E15.8, ', MAG OF E FIELD=', E15.8, ', PHASE ANG= ', E15.8, ', DB= ', E15.8)
DO 535 IKE = 1, NANI
IND = IKE - 1
THSD = FLOAT(IKE) / 2.00
Y(I) = ESSSI(IKE)
535 CALL PLOT (THSD, Y, 1, IND, 50.0, 0.0)
GO TO 1002
1000 WRITE (6, 1592)
1592 FORMAT('SURFACE IS NOT ILLUMINATED')
1002 CONTINUE
STOP
END
FUNCTION H(X)
COMMON /DOG/AONE, CONE, PONE, ATWO, CTHO, PTHO, ATRE, CTRA, PTRA
H = AONE + COS(CONE * X + PONE) + ATWO * COS(CTWO * X + PTHO) + ATRE * COS(CTRE * X + PTRA)
2
RETURN
END
FUNCTION DH(X)
COMMON /DG/AONE, CONE, PONE, ATWO, CTHO, PTHO, ATRE, CTRA, PTRA
DH = AONE + CONE * X + PONE J + ATWO * CTHO * X + PTHO) + ATRE * CTRA * X + PTRA
2 + ATRE * CTRA * COS(CTRE * X + PTRA)
RETURN
END
FUNCTION BINT(XX, YY)
XX IS LOWER LIMIT OF INTEGRATION, YY IS UPPER LIMIT
C PHYSICAL OPTICS RADIATION INTEGRAL WITH PLANE WAVE INCIDENT
C TM CASE
C COMPLEX S, BINT
C COMPLEX GASS5
C COMMON /HOG/ G, THI, THS, WE
C COMMON /PIG/ SECTOR, DX, REP, SECD10
C BREAK INTEGRAL FROM XX TO YY INTO SMALLER SEGMENTS OF LENGTH
C SECTOR AND INTEGRATE OVER EACH SEGMENT USING GAUSSIAN INTEGRATION
S = CMPLX(0.0, 0.0)
LDS = INT((YY - XX) / SECTOR)
IF(LDS .EQ. 0) GO TO 10
DO 100 INJ = 1, LDS
UL = XX + FLOAT(INJ) * SECTOR
ALL = XX + FLOAT(INJ - 1) * SECTOR
100 S = S + GASS5(ALL, UL)
C NOW TO GET LAST FRACTION OF SEGMENT LEFT OVER FROM SURFACE SEGMENTATION
S = S + GASS5(XX + FLOAT(LDS) * SECTOR), YY)
GO TO 50
10 S = GASS5(XX, YY)
50 CONTINUE
BINT = S
RETURN
END
FUNCTION GASS5 (XL, XU)
COMPLEX GASS5, FTBI
C FIFTH ORDER GAUSSIAN INTEGRATION
C XL IS LOWER LIMIT, XU IS UPPER LIMIT
C XU-XL IS LESS THAN OR EQUAL TO SECTOR
C COMMON/GSNN/GW1, GW2, GW3, GW4, GW5, GU1, GU2, GU3, GU4, GU5
DVSDFP=(XU-XL)/2.0
DVSMEP=(XU+XL)/2.0
XU5=GU5*DVSDFP+DVSMEP
XU4=GU4*DVSDFP+DVSMEP
XU3=GU3*DVSDFP+DVSMEP
XU2=GU2*DVSDFP+DVSMEP
XU1=GU1*DVSDFP+DVSMEP
GASS5=DVSDFP*GW1*FTBI(XU1)+GW2*FTBI(XU2)+GW3*FTBI(XU3)
2*GW4*FTBI(XU4)+GW5*FTBI(XU5)
RETURN
END

FUNCTION FTBI(X)
COMPLEX FTBI
C THIS IS THE FUNCTION TO BE INTEGRATED
C THIS IS FOR THE TM CASE
C COMMON/HOG/G.THI, THS, WE
C COMMON/PIG/ SECTOR, DX, REP, SECDL0
GCC=G*(COS(THI)+COS(THS))
GSS=G*(SIN(THI)+SIN(THS))
RCK=REP-(2.0*WE)
FTBI=SIN(THI)-ATAN(I*(I-X)*GCC+I*(I-X)*GSS)
2*CEXP(CMPLX(0.0,((I-GCC)+(H(I)*GSS)))).
C THE FOLLOWING ACCOUNTS FOR TAPERING
ABS(X)=ABS(X)
IF(ABS(X)=RCK)1500,1500,2000
2000 IF(X.LE.(WE-REP))FTBI=CMPLX(0.0,0.0)
IF(X.GT.(REP-WE)) FTBI=CMPLX(0.0,0.0)
IF((X.GT.(WE-REP)).AND.(X.LE.(2.0*WE)-REP))
2 FTBI=FTBI+(0.5+0.5*SIN((G/2.0)+I-(1.5*WE)-REP)))
IF((X.LE.(REP-WE)).AND.(X.GT.(REP-2.0*WE)))
2 FTBI=FTBI+(0.5-0.5*SIN((G/2.0)+(X-(REP-(1.5*WE))))
1500 CONTINUE
RETURN
END
THIS IS THE TEST CASE

THE PROGRAM USES PHYSICAL OPTICS TO CALCULATE THE BACKSCATTERING
FROM A SEA SURFACE BY DIVIDING SURFACE INTO LIT AND UNLIT REGIONS
IN THE LIT REGIONS THE SURFACE CURRENT IN 2NxH
GAUSSIAN INTEGRATION USED
FOR THIS PROGRAM TO GIVE USEFUL RESULTS THE SURFACE MUST HAVE
RADII OF CURVATURE NO LESS THAN 1*WE
NSP IS THE NUMBER OF SHADOW POINTS
SURFACE IS DESCRIBED BY AONE*SINCONE*X+PONE +ATWO*SINTWO*X
+PTWO*ATRE*SINTRE*X+PTRE
SURFACE UNDER CONSIDERATION LIES BETWEEN ALP AND REP
SN IS THE STEP SIZE TAKEN TO DETERMINE SHADOWING
IT MUST BE SMALLER THAN ANY SURFACE FEATURES AND MUST ALSO
ALLOW THE LOCATION OF THE ENDS POINTS OF INTEGRATION WITHIN
A SMALL FRACTION OF A WAVELENGTH
NSP IS THE NUMBER OF ANGLES (SCATTERING) TO BE EXAMINED
MAKE DIMENSIONS OF ESNN, SCANG, EFPN SMALL AS POSSIBLE TO AVOID
LARGE # OF CARDS RETURNED
NSP SHOULD BE THE DIMENSION OF ESNN, SCANG, EFPN
DIMENSION Y (10), ESNN(360)
NAME LIST/MON/ABANG/DTDS
DIMENSION SCANG(360), EFPN(360)
COMPLEX S,INT.

SCATTER SHADOWING HAS NOT BEEN ACCOUNTED FOR
COMMON /DG/ AONE, CONE, PONE, ATWO, CTWO, PTWO, ATRE, CTRE, PTRE
COMMON /HOG/ G, THI, THS, WE
COMMON /PIG/ SECTOR, DX, REP, SECO
COMMON /GSNN/GH1, GH2, GH3, GH4, GH5, GU1, GU2, GU3, GU4, GU5
WE = 25.0

WE IS THE ELECTRICAL WAVELENGTH
G = 2.0*3.1415927/WE
SRTE = SQRT(WE)
DX = WE/15.0
AONE = 40.0
CON = 2.0*3.1415927/200.0
PONE = 0.0
ATWO = 0.0
CTWO = 0.0
PTWO = 0.0
ATRE = 0.0
CTRE = 0.0
PTRE = 0.0
NANI = 360
SECTOR = WE/2.0
SECO = SECTOR/10.0

CONSTANTS FOR GAUSSIAN INTEGRATION
GH1 = 0.2369268
GH2 = 0.47862867
GH3 = 0.53846931
GH4 = 0.0
GH5 = GH1
GU1 = 0.9061798
GU2 = 0.53846931
GU3 = 0.0
GU4 = GU2
GU5 = GU1

THE ANGLE OF INCIDENCE SHOULD NOT BE GREATER THAN 90 DEG
THI = 0.0*3.1415927/180.0
IF THE INCIDENCE ANGLE IS WITHIN TEN DEGREES OF 90 NO SHADOWING
TAKEN INTO ACCOUNT
IF (ABS(THI - 1.5707), LT. 0.175) GO TO 563
TANH = TAN(THI)
DTHI = 180.0*THI/3.1415927
WRITE (6, 1071) DTHI

1071 FORMAT ('° ', '° ANG OF INC FROM POS X AXIS = ', E15.8)
REP=200.0
ALEP=-REP
SN=WE/10.0
NSP=1
DIMENSION SX(1000)
IF(DH(REP).GT.TANTHI) GO TO 106
SX(NSP)=REP
GO TO 105
106 SLOPE=TANTHI
B=H(REP)-(SLOPE*REP)
X=REP
109 X=X-SN
IF((SLOPE*X)+B.GT.H(X)) GO TO 109
IF(X.LE.ALEP) GO TO 1000
SX(NSP)=X-(SN/2.0)
CONTINUE
C THIS ABOVE TAKES CARE OF THE FIRST RIGHTENDPOINT
15 X=SX(NSP)
22 X=X-SN
XN=X-SN
IF((DH(X).LT.TANTHI).AND.(DH(XN).GT.TANTHI)) GO TO 53
IF((X.GT.ALEP) GO TO 22
GO TO 92
53 NSP=NSP+1
SX(NSP)=XN
SLOPE=TANTHI
B=H(SX(NSP))-(SLOPE*SX(NSP))
X=SX(NSP)-SN
29 X=X-SN
IF((SLOPE*X)+B.LT.H(X)) GO TO 39
IF((X.GT.ALEP) GO TO 29
GO TO 92
9 NSP=NSP+1
SX(NSP)=X-(SN/2.0)
GO TO 15
92 NSP=NSP+1
SX(NSP)=ALEP
GO TO 564
563 SX(1)=REP
SX(2)=ALEP
NSP=2
CONTINUE
C SURFACE IS NOW SEPERATED INTO LIT AND UNLIT ZONES
C LAST VALUE IN SX(J) IS ALEP
WRITE (6,101) (K,SX(K),K=L,NSP)
101 FORMAT(*' ',*SX(*,I4,*)='E15.8*)
C THE FOLLOWING FINDS THE SCATTERED FIELDS DUE TO THE LIT ZONES
DO 317 JNX=2,NSP
THS=FLOAT(JNX)*L0.8726646 E-02)
DTHS=1CJO.O*THS/3.1415927.
SCANG(JNX)=DTHS
S=CMPLX(0.0,0.0)
KKN=1
10 CONTINUE
ALOW=SX(KKN+1)
AUPP=SX(KKN)
S=S+BINT(ALOW,AUPP)
KKN=KKN+2
IF ((KKN.LT.NSP).AND.((KKN+1).LT.NSP)) GO TO 10
C TO CONVERT TO TRUE SCATTERED H FIELD FOR HINC OF UNITY MAG
S=S*CMPLX(0.70711,0.70711)/SRTWE
AB=CAR(S)
DB=2C.0*AALOG10(AB)
ESSI(JNX)=AB
ANG=180.0*AATAN2(AIMAG(S),REAL(S))/3.1415927
EFPA(JNX)=ANG
WRITE(6,143) DTHS,AB,ANG,DB
143 FORMAT(' SCATTERING ANG.',E15.8,' MAG=',E15.8,' PHASE ANGLE=',E15.8,' DB=',E15.8)
317 CONTINUE
331 DO 531 JK=1,NANI
333 E=ESSS(JK)
334 A=EPFAP(JK)
335 AS=SCANG(JK)
337 FORMAT(' ', SCATTERING ANG.=',E15.8,' MAG OF H FIELD=',E15.8,' PHASE ANGLE=',E15.8)
339 DO 535 IKE=1,NANI
340 INO=IKE-1
342 TTHS=FLOAT(IKE/2.0)
343 Y(I)=ESSS(IKE)
345 CALL PLOT(TTHS,Y,1,IND,50.0,0.0)
347 GO TO 1002
349 1000 WRITE(6,1592)
350 1592 FORMAT(' SURFACE IS NOT ILLUMINATED.'
352 1002 CONTINUE
354 STOP
355 END

FUNCTION H(X)  
COMMON /HOG/AONE,CONE,PONE,ATWO,CTWO,PTWO,ATRE,CTRE,PTRE
H=AONE*SIN(CONE*X+PONE)+ATWO*SIN(CTWO*X+PTWO)+ATRE*SIN(CTRE*X+PTRE)
END

FUNCTION DH(X)  
COMMON /HOG/AONE,CONE,PONE,ATWO,CTWO,PTWO,ATRE,CTRE,PTRE
DH=AONE*CONE*COSS(X+PONE)+ATWO*CTWO*COS(X+PTWO)+ATRE*CTRE*COS(X+PTRE)
RETURN
END

FUNCTION BINT(XX,YY)  
XX IS LOWER LIMIT OF INTEGRATION, YY IS UPPER LIMIT
PHYSICAL OPTICS RADIATION INTEGRAL WITH PLANE WAVE INCIDENT
TM CASE
COMPLEX S,BINT
COMMON /HOG/ G,THI,THS,WE
COMMON/PIG/ SECTOR,DX,REP,SECD010
BREAK INTEGRAL FROM XX TO YY INTO SMALLER SEGMENTS OF LENGTH
SECTOR AND INTEGRATE OVER EACH SEGMENT USING GAUSSIAN INTEGRATION
S=CMPLX(0,0)
LDS=INT((YY-XX)/SECTOR)
IF(LDS,EQ.0) GO TO 10
DO 100 INJ=1,LDS
UL=XX+(FLOAT(INJ)*SECTOR)
ALL=XX+(FLOAT(INJ-1)*SECTOR)
100 S=S+GASSS(ALL,UL)
S=S+GASSS(XX+(FLOAT(LDS)*SECTOR),YY)
GO TO 50
10 S=GASSS(XX,YY)
50 CONTINUE
BINT=S
RETURN
END
FUNCTION GASSS(XL, XU)
COMPLEX GASSS,FTBI
C FIFTH ORDER GAUSSIAN INTEGRATION
C XL IS LOWER LIMIT, XU IS UPPER LIMIT
C XU-XL IS LESS THAN OR EQUAL TO SECTOR
COMMON/GSNN/GW1, GW2, GW3, GW4, GW5, GUL, GU2, GU3, GU4, GU5
DVSDFEP=(XU-XL)/2.0
DVSMEP=(XU+XL)/2.0
XU5=GU5*DVSDFEP+DVSMEP
XU4=GU4*DVSDFEP+DVSMEP
XU3=GU3*DVSDFEP+DVSMEP
XU2=GU2*DVSDFEP+DVSMEP
XU1=GU1*DVSDFEP+DVSMEP
GASSS=DVSDFEP*(GW1*FTBI(XU1)+GW2*FTBI(XU2)+GW3*FTBI(XU3)+GW4*FTBI(XU4)+GU5*FTBI(XU5))
RETURN
END

FUNCTION FTBI(X)
COMPLEX FTBI
C THIS IS THE FUNCTION TO BE INTEGRATED
C THIS IS FOR THE TM CASE
COMMON/HOG/G, TH1, THS, WE
COMMON/PIG/ SECTOR, DX, REP, SECD10
GCC=G*COS(TH1)+COS(THS)
GSS=G*SIN(TH1)+SIN(THS)
RCK=REP-(Z.0*WE)
FTBI=SIN(THS)-ATAN(DH(X))**2*CEXP(CMPLX(0.0,0.0)*(G(X)+GSS))
C THE FOLLOWING ACCOUNTS FOR TAPERING
ABSX=ABS(X)
2000 IF(ABSX-RCK) 1500,1500,2000
2000 IF(X.LT.(REP-(1.5*WE)) OR (X.GT.(REP-(1.5*WE)))) CONTINUE
1500 RETURN
END
THIS IS A METHOD OF MOMENTS SOLUTION
TM POLARIZATION SYMMETRIC MATRIX
NSUB SEGMENTS HAVE N MIDPOINTS
NSUB IS THE SUBSCRIPT WHICH COUNTS THE END POINTS
N IS THE SUBSCRIPT WHICH COUNTS THE MIDPOINTS
WATCH MAX SLOPE SO THAT THE X INCREMENTS ARE SMALL ENOUGH
DIMENSION Y(101),CMC(36D)
COMMON /PI/ AONE,ATWO,CTWO,CONE,PCNE,TCNE,PDNE,N
COMPLEX AHANZO

DIMENSION X(300),S(45150),SS,T
COMPLEX STD
WE IS THE ELECTRICAL WAVELENGTH
WE=25.0
G=6.2831853 /WE
S1S=SQRT(WE)*CMPLX(1.0,1.0)+(-0.707107)/3.1415927
DC=WE/10.0
DX=DC/100.0
DC2=DC/2.0

API=3.1415927
C THE FGOLLWING CONSTANTS DEFINE THE SURFACE
ADNE=25.0
CCNE=2.0*3.1415927/25.0
PONE=0.0
ATWO=0.0
CTWO=0.0
PTWO=0.0
CALL SCLOKL
C THE FOLLOWING BREAKS THE SURFACE INTO SEGMENTS DC CENTIMETERS LONG
C BY LINE INTEGRATION USING STEPS OF LENGTH DX FOR THE INTEGRATION
NSUB=1
X(NSUB)=-EP
DCC=DBLE(DC)
DX=DBLE(DX)
DCC=DBLE(DC2)

1002 DCC=O.000 DCC DR=DSLE(X(NSUB))
1001 DR=DL+DCC
R=SNGL(DC)
DCC=DL
CAL=CMPLX(DCC*DSQRT(1.0000)+((DBLE(DH(R))**2)))
IF(DCC*DC2-LT.0.0000) AND ((DC2-DC0))GE.0.0000)
2 XMID(NSUB)=R
IF(DLRT.DCC)GO TO 1001
NSUB=NSUB+1
X(NSUB)=R
AL=SNGL(CAL)
WRITE (6,352) AL,NSUB
352 FORMAT(' **AL=',E15.8,'
IF(IR.LT.EP)GO TO 1002
TIME=RCLKK(I,C)
WRITE(3276,3276) TIME
3276 FORMAT(' **TIME=',F10.6,' SECONDS')
N=NSUB-1
DO 1004 J=1,NSUB
IF(J.EQ.NSUB)XMID(NSUB)=0.0
XXX=X(J)
XMID=XMID(J)
1004 CONTINUE
END
THIS ENDS THE SURFACE SUBDIVISION
NMD=N-1
NM3=N-3

DIMENSION OF S IS N(N+1)/2
DIMENSION OF FING,F IS N

UPIF=0.7853982.
EE=2.71828
GA=G/DC/(2.0**EE)

SN is the diagonal element of the input matrix
SN=AHAN20(GA)
WRITE (6,4000) SN

400 FORMAT (5H SNN=,ZE15.8)
DO 100 NJ=1,N
NJPD=NJ+1
SISUB(NJ,NJ)=SN

C THIS FINDS ELEMENTS ON THE DIAGNOL
IF (NJPD.NR.GT.N) GO TO 100
DO 100 NA=NJPD,N

C THIS FINDS OFF DIAGNOL ELEMENTS
XM=XMID(NA)

100 CONTINUE

C THIS COMPLETES THE FILLIN OF THE MATRIX
C
C THIS BEGINS THE CONVERSION TO UPPER TRIANGULAR MATRIX
S(1)=CSQRT(S(1))
DO 1 I=2,N
IMC=I-1
IDO=IMC+1

1 T=CMPLX(0.0,0.0)
DO 3 L=1,IMD
II=1+(M-1)*L/2+N-I
2 T=T*(S(LI)**2)

3 T=T*(S(LI)**2)
II=1+(M-1)*L/2+N-I
STH=CSQRT(S(II)-T)
IF (IDO.GT.N) GOTO 2
DO 5 J=IDO,N

5 T=T+IS(MJ)*SIMI I
IJ=(I-1)*M/2+N-J
S(IJ)=CSQRT(S(IJ)-T)
IF (IDO.GT.N) GOTO 2
CONTINUE

C THIS ENDS THE CONVERSION TO UPPER TRIANGULAR MATRIX.
WRITE (6,1222) N,WE

1222 FORMAT (3H N=,13,4H WE=,E15.8)
TH=60.0*3.1415927/180.C
THX=180.0+TH/3.1415927
WRITE (6,9333) THX0

9333 FORMAT (9H INC ANG=,E15.8)

C THIS IS THE ANGLE OF INCIDENCE FROM THE HORIZONTAL
SIN=STH
COSTH=STH

C THIS FINDS THE INCIDENT FIELD ON THE NJTH SEGMENT
DO 455 MJ=1,N
ENJ=FLOAT(NJ)
XM=XMID(NA)

F(NJ)=CEXP(CMPLX(0.0,0.0)(XM*CTH)+(H(X)*SIN))
```fortran
TAPERED ILLUMINATION

IF(XM.LE.(WE+1.0)-EP)) F(NJ)=CMPLX(0.0,0.0)
IF(XM.GE.(EP-1.0*WE))) F(NJ)=CMPLX(0.0,0.0)
IF((XM.GT.(1.0*WE)-EP)).AND.(XM.LE.(2.0*WE)-EP))
2 F(NJ)=F(NJ)*(0.5+(0.5*SIN((G/2.0)+(XM-((1.5*WE)-EP))))
IF((XM.GE.(EP-(2.0*WE))).AND.(XM.LE.(EP-(1.0*WE))))
2 F(NJ)=F(NJ)*(0.5-(0.5*SIN((G/2.0)*(XM-(EP-(1.5*WE))))))

CONTINUE
WRITE(*,2948) (NJ,F(NJ),NJ=1,N)
2948 FORMAT(' INC FIELD F(',I4,')=',E15.8)
C THIS BEGINS THE BACK SUBSTITUTION
F(1)=F(1)/S(1)
DO 10 I=2,N
IMO=I-1
T=CMPLX(0.0,0.0)
DO 11 L=1,IMO
II=(L+1)/2+(N-I)
II=II*(L/2)+(N-I)
T=T+(S(L)*F(L))
11 F(I)=F(I)-T/S(I)
10 NN=(N*(N+1))/2
F(1)=F(1)-T/S(1)
NMC=N-1
DC 25 I=1,NMC
K=N-1
MP=K+1
T=CMPLX(0.0,0.0)
DO 26 L=KPG,N
KL=(K*N)-(1+(N-L)/2)+(K-L)
KL=K*(K-1)/2+N-L
T=T+(S(KL)*F(L))
26 F(K)=(F(K)-T)/S(K)
25 CONTINUE
C THIS ENDS THE BACK SUBSTITUTIONS
C 491 K=1,N
STT=CABS(F(K))
STD=F(K)
ANNN=2*(AIMAG(F(K)),REAL(F(K)))*180.0/3.1415927
491 WRITE(*,492) K,STT,ANNN
492 FORMAT(' INC RANGE AT ANGLE=',E15.8)
DO 317 JNX=1,360
T=0.872664625E-02*FLOAT(JNX)
T=CMPLX(0.0,0.0)
DO 310 I=1,N
XN=XM+DI
310 T=T+(F(I)*CEXP(CMPLX(0.0,G*I(XN*COS(TH))+(H(XN)*SIN(TH))))))
C THIS CORRECTS T TO TRUE SCATTERED FIELD
T=STST
CM=CARST
CMC=CM
CANG=57.296*ATAN2(AIMAG(T),REAL(T))
THD=THC=57.296
DB=20.0*ALOG10(CM)
317 WRITE(*,312) CM,CMG,THR,CMG
312 FORMAT(18H RELATIVE E FIELD=,E15.8,7H ANGLE=,E15.8,
2 23H ANGLE FROM HOPIZUNTAL=,E15.8,7H DB=,E15.8)
```
DO 576 IKE=1,360
THSD=FLOAT(IKE)/2.0
IND=IKE-1
Y(1)=CMC(IKE)
576 CALL PLOT(THSD,Y,1,IND,50.0,0.0)
STOP
END

FUNCTION H(X)
C THIS DEFINES THE SURFACE
COMMON /PIG/ ACNE,CONE,PONE,ATWO,CTWO,P1WO,N
H=ACNE*SIN(CONE*X+PONE)+ATWO*SIN(CTWO*X+PTWO)
RETURN
END

FUNCTION DH(X)
C DH(X) IS THE DERIV. OF H(X)
COMMON /PIG/ ACNE,CONE,PONE,ATWO,CTWO,P1WO,N
DH=ACNE*CONE*COS(CONE*X+PONE)+ATWO*CTWO*COS(CTWO*X+PTWO)
RETURN
END

FUNCTION ISUB(J,K)
COMMON /PIG/ ACNE,CONE,PONE,ATWO,CTWO,P1WO,N
C THIS CONVERTS ELEMENTS OF UPPER TRIANGULAR MATRIX TO A LINEAR
ISUB=(N*J)-(((J-1)*J)/2)+N-K
C ARRAY COUNTING LEFT TO RIGHT STARTING WITH FIRST ROW
RETURN
END
THIS IS A METHOD OF MOMENTS SOLUTION FOR BISTATIC SCATTERING CASE
GAUSSIAN INTEGRATION IS USED TO CALCULATE THE MATRIX ELEMENTS
UNIT INCIDENT ELECTRIC FIELD IS ASSUMED, OF COURSE THIS IS MODIFIED
NEAR THE ENDPOINTS OF THE SURFACE BY ILLUMINATION TAPERING
NSUB SEGMENTS HAVE N MIDPOINTS
NSUB IS THE SUBSCRIPT WHICH COUNTS THE 'END POINTS'
N IS THE SUBSCRIPT WHICH COUNTS THE MIDPOINTS
WATCH MAX SLOPE SO THAT THE X INCREMENTS ARE SMALL ENOUGH
THE ARRAY XM(I) CONTAINS THE X COORDINATES OF THE MIDPOINTS OF THE
SEGMENTS, XM(I) IS THE MIDPOINT OF THE I'TH SEGMENT
THE ARRAY X(I) CONTAINS THE X COORDINATES OF THE ENDPOINTS OF THE
SURFACE SEGMENTS, X(I), X(I+1) ARE THE LOWER AND UPPER X COORDINATES
OF THE ENDPOINTS OF THE I'TH SEGMENT
PHASE REFERENCE IS AT THE ORIGIN OF THE COORDINATE SYSTEM
COMPLEX SNN, SST
COMPLEX S
DIMENSION Y(101), XM(560)
NAMELIST /D/ WE, EP, THXXD, AONE, CONE, PONE, ATWO, CTWO, PTWO, N
NAMELIST /E/ F, XMID
COMPLEX FSS
COMPLEX STS
COMMON /PIG/ AONE, CONE, PONE, ATWO, CTWO, PTWO, N
COMPLEX CD(236, 236)
COMPLEX FD(236), FY, T, CTEST

THAT IS C(I,L) ---- F(I)

THE DIMENSIONS OF C AND F MUST BE COMMENSURATE
COMPLEX FIN
COMPLEX HAN2
DIMENSION X(500)
DIMENSION XM(500)

THE FOLLOWING CONSTANTS DESCRIBE THE SURFACE
ACNE=-50.0
CONE=6.28318/800.0
PONE=3.1415927/2.0
ATWO=0.0
CTWO=0.0
PTWO=0.0
WE IS THE ELECTRICAL WAVELENGTH
WE=25.0
G=6.283185 /WE
DC=WE/10.0
DX=DC/1000.0
DC2=DC/2.0
EP=200.0

THE FOLLOWING BREAKS THE SURFACE INTO SEGMENTS DC CENTIMETERS LONG

BY LINE INTEGRATION USING STEPS OF LENGTH DX FOR THE INTEGRATION
NSUB=1
X(NSUB)=-EP

AL=0.000
R=X(NSUB)

AL=AL+DX
ALU=AL
IF ((DC2-AL) .LE. 0.0) .AND. ((DC2-ALU) .GT. 0.0) XM(NSUB)=R
IF (AL .LT. DC) GO TO 1001
WRITE (6, 352) AL, NSUB
352 FORMAT (4, 'AL=', (E15.8, ' NSUB=', (I4)
NSUB=NSUB+1
XM(NSUB)=R
IF (AL .LT. EP) GO TO 1002
NSUB=1
DO 1004 J=1, NSUB
IF (J .EQ. NSUB) XM(NSUB)=0.0
XXX=XM(J)
XMD= XM(J)

1002 AL=AL+DX
1001 R=R+DX
1004 WRITE (6,1003) XXX,XMD,J
1003 FORMAT (6H X(J),E15.8,9H X(M,J)=E15.8,3H J=,I3)
C THIS ENDS THE SURFACE SUBDIVISION
NMO=N-1
NM3=3
C DIMENSION OF FNCF IS N
DP1F=0.7853982
EE=2.71828
GA=(DC/(2.0*EE))
C SNN IS THE DIAGONAL ELEMENT OF THE INPUT MATRIX
SNN=HAN2(GA)*DC
WRITE (6,400) SNN
400 FORMAT (5H SNN=,2E15.8)
DO 100 NJ=1,N
C(NJ,NJ)=SNN
100 CONTINUE
C CONSTANTS FOR GAUSSIAN INTEGRATION 5 TH ORDER
GU1=-0.9661798
GU2=-0.53846931
GU3=0.0
GU4=-GU2
GU5=-GU1
GW1=0.2369268
GW2=GU1+GW1
GW3=0.5685888
DO 3361 MR=1,N
XWM=X(MA)
XMMH=H(XMM)
DO 3361 MC=1,N
IF (MC.EQ.MR) GO TO 3361
EPL=X(MC)
EPU=X(MC+1)
DVDFEP=(EPU-EPL)/2.0
DVSMEP=(EPU+EPL)/2.0
XU5=GU5*DVDFEP+DVSMEP
XU1=GU1*DVDFEP+DVSMEP
XU2=GU2*DVDFEP+DVSMEP
XU3=GU3*DVDFEP+DVSMEP
XU4=GU4*DVDFEP+DVSMEP
C(MR,MC)=DVDFEP*(1+GW1*HAN2(G*SQRT(((XU1-XMM)**2)+((H(XU1)-XMM)**2)))*SQR((1.0+D(H(2 XU1))**2))
+GW2*HAN2(G*SQRT(((XU2-XMM)**2)+((H(XU2)-XMM)**2)))*SQR((1.0+D(H(2 XU2))**2))
+GW3*HAN2(G*SQRT(((XU3-XMM)**2)+((H(XU3)-XMM)**2)))*SQR((1.0+D(H(2 XU3))**2))
+GW4*HAN2(H(SQRT(((XU4-XMM)**2)+((H(XU4)-XMM)**2)))*SQR((1.0+D(H(2 XU4))**2))
+GW5*HAN2(H(SQRT(((XU5-XMM)**2)+((H(XU5)-XMM)**2)))*SQR((1.0+D(H(2 XU5))**2))))
3361 CONTINUE
C THIS COMPLETES THE FILLING OF THE MATRIX
C NCSYM CROUT
C FIRST COLUMN OK
C T00 GET FIRST ROW
DO 10 J=2,N
10 C(J,J)=C(J,J)/C(J,1)
C NOW WORK ON ROW AND COLUMN SET <
DO 11 K=2,N
KMO=K-1
KPO=K+1
11 105
TO GET DIAGONAL ELEMENT
S=CMPLX(0.0,0.0)
DO 12 IK=1,KMO
12 S=S+C(K,IK)*C(1K,K)
C(K,K)=C(K,K)-S

TO GET ELEMENTS IN COLUMN K BELOW ROW K
IF (KPO,GT,N) GO TO 17
DO 13 IROW=KPO,N
S=CMPLX(0.0,0.0)
DO 14 JJ=1,JMO
14 S=S+C(IROW, JJ)*C(JJ,K)
13 C(IROW,K)=C(IROW,K)-S

TO GET ELEMENTS IN COLUMN K TO THE RIGHT OF COLUMN K
DO 15 ICOL=KPO,N
S=CMPLX(0.0,0.0)
DO 16 JR=1,KMO
16 S=S+C(K, JR)*C(JR,ICOL)
15 C(K,ICOL)=(C(K,ICOL)-S)/C(K,K)

WRITE (6,1222) N,WE
1222 FORMAT(3H N=,13,4H WE=,E15.8)
THI=3.1415927*60.0/180.0
THXD=THI*180.0/3.1415927
WRITE (6,9333) THXD
9333 FORMAT(9H INC ANG=,E15.8)

THIS IS THE ANGLE OF INCIDENCE MEASURED FROM THE HORIZONTAL
SIN=SIGN(IH)
COS=CSIN(IH)

THIS FINDS THE INCIDENT FIELD ON THE NJTH SEGMENT
DO 455 NJ=1,N
XG=XM(NJ)
F(NJ)=EXP(CMPLX(0.0,G*(XG*COS)+I*(XG)*SIN))
455 CONTINUE

TAPERED ILLUMINATION

IF(XG.LE.((WE*1.0)-EP)) F(NJ)=CMPLX(0.0,0.0)
IF(XG.GE.(EP-(1.0*WE))) F(NJ)=CMPLX(0.0,G.0)
IF((XG,LT,(1.0*WE)-EP)) AND.(XG,LT,(2.0*WE)-EP))
2 IF(NJ)=F(NJ)*(0.5+(0.5*SIN((G/2.0)*XG-(1.5*WE)-EP))))
2 I(F(NJ)=F(NJ)*((XG.GE.(EP-(1.0*WE)))) AND.(XG.Lt.(EP-(1.0*WE))))
ABSF=CABS(F(NJ))
WRITE(6,83) NJ,ABSF
83 FORMAT(' INC FIELD AT XM(',I4,')=',E15.8)
455 CONTINUE

THIS BEGINS THE BACK SUBSTITUTION
CONVERSION OF SOURCE SIDE
F(1)=F(1)/C(1,1)
DO 90 IJ=2,N
S=CMPLX(0.0,0.0)
IJMO=IJ-1
DO 91 IK=1,1JMO
91 S=S+C(IJ, IK)*F(IK)
90 F(IJ)=(F(IJ)-S)/C(IJ, IJ)

NOW FOR FINAL BACK SUBSTITUTION
NMO=N-1
DO 160 L=1,NMO
K=N-1
KP0=K+1
S=CMPLX(0.0,0.0)
DO 175 JO=KP0,N
175 S=S+C(K,JO)*F(JO)
160 F(K)=F(K)-S
DO 425 KCURR=1,N
ABF=CMABS(F(KCURR))
ANGF=180.0*ATAN2(AIMAG(F(KCURR)),REAL(F(KCURR)))/3.1415927
425 WRITE(6,553) KCURR,ABF,ANGF
553 FORMAT('F(',I4,'J=',F15.8,' AT ANGLE',E15.8)
C THIS ENDS THE BACK SUBSTITUTIONS
DC 439 KURR=1,N
IND=KURR-1
Y(I)=CMABS(REAL(F(KURR)))**4.0*WE/((6.28318*377.0)
XORD=FLOAT(KURR)
439 CALL PLOT(XORD,Y,1,IND,0.0200,0.0)
DO 440 KURR=1,N
IND=KURR-1
Y(I)=180.0*ATAN2(AIMAG(F(KURR)),REAL(F(KURR)))/3.1415927
XORD=FLOAT(KURR)
440 CALL PLOT(XORD,Y,1,IND,180.0,-180.0)
DC 317 JNX=1,360
TH=0.87266463 E-02*FLOAT(JNX)
T=CMPLX(0.0,0.0)
DO 310 I=1,N
X(I)=XM(I)
310 T=T+((F(I)*CEXP(CMPLX(0.0,G*(I*(XN*COS(TH)+PONE*ATW)*CTWO*COS(TH)+PTWO)))))
T=T*0.0*SQRT(WE)*CMPLX(-0.707107,-0.707107)/3.1415927
CM=CMABS(T)
DB=20.0*ALOG10(DB)
CMC(JNX)=CM
CANG=57.296*ATAN2(AIMAG(T),REAL(T))
THD=TH+57.296
317 WRITE(6,312) CMC,CANG,THD,DB
312 FORMAT(18H RELATIVE E FIELD=',E15.8,7H ANGLE=',E15.8,
2 23H ANGLE FROM HORIZONTAL=',E15.8,6H DB=',E15.8)
DO 441 IES=1,360
IND=IES-1
Y(I)=CMC(IES)
THS=FLOAT(IES)/2.0
441 CALL PLOT(THS,Y,1,IND,50.0,0.0)
STOP
END
FUNCTION H(X)
C THIS DEFINES THE SURFACE
COMM /PIG/ AONE,CONE,PONE,ATWO,CTWO,PTWO,N
H=AONE*SIN(CONE*X+PONE)+ATWO*SIN(CTWO*X+PTWO)
RETURN
END
FUNCTION HANG2(X)
C I DO THIS TO AVOID RETYPING THE WHOLE GAUSS INT. PART
CMPLEX HAN2
COMPLEX HANG2
HAN2=HANG20X)
RETURN
END
FUNCTION DH(X)
C DH(X) IS THE DERIV. OF H(X)
COMM /PIG/ AONE,CONE,PONE,ATWO,CTWO,PTWO,N
DH=AONE*CONE*COS(CONE*X+PONE)+ATWO*CTWO*COS(CTWO*X+PTWO)
RETURN
END
THIS IS A METHOD OF MOMENTS SOLUTION FOR BISTATIC SCATTERING CASE USING TWO POINT INTERPOLATION.

GAUSSIAN INTEGRATION IS USED TO CALCULATE THE MATRIX ELEMENTS.

NSUB SEGMENTS HAVE N MIDPOINTS.

NSUB IS THE SUBSCRIPT WHICH COUNTS THE END POINTS.

N IS THE SUBSCRIPT WHICH COUNTS THE MIDPOINTS.

WATCH MAX SLOPE SO THAT THE X INCREMENTS ARE SMALL ENOUGH.

THE SURFACE UNDER CONSIDERATION LIES BETWEEN -EP AND +EP.

COMPLEX S,Co
COMPLEX FSS
COMPLEX FINC(20),STS
COMMON /PI,G/ AONE,CON,PDONE,ATWO,CTWO,PTWO,N
COMMON /HOG/ XM(400),X(400),GA,G,DC
COMMON/GASSN/ GUI,GU2,GU3,GU4,GU5,GW1,GW2,GW3,GW4,GW5
COMPLEX C(150,150)
COMPLEX F(400),FP(400),SS,T,CI
COMPLEX F
COMPLEX AB

DIMENSION ABES(360),Y(10)

WE IS THE ELECTRICAL WAVELENGTH.

WE=25.0

THE FOLLOWING CONSTANTS DESCRIBE THE SURFACE.

AONE=15.0

CON=2.0/3.1415927/200.0

PDONE=0.0

ATWO=0.0

CTWO=0.0

PTWO=0.0

DC=WE/10.0

DX=DC/1000.0

DC2=DC/2.0

UPH=0.7853982

G=6.283185/WE

EE=2.71828

GA=G*DC/(2.0*EE)

EP IS THE END POINT.

Ep=200.0

CONSTANTS FOR GAUSSIAN INTEGRATION 5TH ORDER.

GU1=-0.9061798

GU2=-0.53846911

GU3=0.0

GU4=GU2

GU5=-GU1

GW1=0.2369268

GW2=0.2369268

GW3=0.47862867

GW4=0.5688888

CONSTANTS FOR GAUSSIAN INTEGRATION 5TH ORDER.

THE FOLLOWING BREAKS THE SURFACE INTO SEGMENTS DC CENTIMETERS LONG.

BY LINE INTEGRATION USING STEPS OF LENGTH DX FOR THE INTEGRATION.

NSUB=1

1002

AL=0.0QQ

R=X(NSUB)

1001

R=R+DX

AL=AL+DX*SORT(1.0+(DH(R)=2)))

IF((DC2-AL).LE.0.0).AND.((DC2-AL).GT.0.0)) XM(NSUB)=R

IF(AL.LT.DC) GO TO 1001

NSUB=NSUB+1

X(NSUB)=R

IF (R.LT.EP) GO TO 1002
DO 1004 J = 1, NSUB
IF (J .EQ. NSUB) XM(NSUB) = 0.0
XXX = X(J)
XMD = XM(J)
1004 WRITE (6, 1003) XXX, XMD, J
1003 FORMAT (6H X(J)=, E15.8, 9H XM(J)=, E15.8, 3H J=, I3)

C THIS ENDS THE SURFACE SUBDIVISION
C THIS INSURES THAT N IS ODD

5733 KK = KK + 1
IF (2 * KK - 1 .EQ. N) GO TO 5731
IF (2 * KK .EQ. N) GO TO 5732
GO TO 5733
5732 N = N - 1
5731 CONTINUE

WRITE (6, 3728) N, KK
3728 FORMAT (10H CORRECTED VALUE OF N=, I4, 'K=', I4, '2*KK-1=N*)
NMO = N - 1
NMM = N - 3
C DIMENSION OF FINC,F IS N
C MATRIX FILL IN
C DO BY COLUMNS
C FOR FIRST COLUMN
C
3661 C(I, 1) = CO(I, 1) + CD(I, 1, 1) + (CD(2*I-1, 1) / 2.0)
C FOR LAST COLUMN
C
3678 C(I, KK) = CO(2*I-1, KK-1) + CD(I, 2*KK-2) / 2.0
C FOR MIDDLE COLUMNS
C
56 C THIS COMPLETES THE FILLING OF THE MATRIX
C NONSYMMETRIC CROUT
C FIRST COLUMN OK
C TO GET THE FIRST ROW
C
10 C(I, J) = C(I, J) / C(I, 1)
C NOW WORK ON ROW AND COLUMN SET K
C
11 K = 2 * KK
KMO = K - 1
KPO = K + 1
C TO GET DIAGONAL ELEMENT
S = CMPLX(0.0, 0.0)
12 S = S + C(I, K) * C(I, K)
C(K, K) = C(K, K) / S
C TO GET ELEMENTS IN COLUMN K BELOW ROW K
IF (KPO > GT, KK) GO TO 17
DO 13 IROW = KPO, KK
S = CMPLX(0.0, 0.0)
13 J = 1, KMO
S = S + C(IROW, J) * C(J, K)
C(IROW, K) = C(IROW, K) / S
C TO GET ELEMENTS IN ROW K TO THE RIGHT OF COLUMN K
DO 15 ICOL = KPO, KK
S = CMPLX(0.0, 0.0)
15 JR = 1, KMO
S = S + C(I, JR) * C(JR, ICOL)
C(I, ICOL) = C(I, ICOL) / S
17 CONTINUE
11 CONTINUE
WRITE (6,1222) KK,WE
1222 FORMAT(' ',KK=' ',I4,' WE='E15.8)
TH=3.1415927*G0.0/180.0
THDEG=57.29578*TH
WRITE (6,9333) THDEG
9333 FORMAT('TH=INC ANG='E15.8)
C TH IS THE ANGLE OF INCIDENCE FROM THE HORIZONTAL
SIN=SU(NJ)
COTH=CD(S(NJ)
C THIS FINDS THE INCIDENT FIELD ON THE NJTH SEGMENT
DO 455 NJ=1,KK
XG=XM2+NJ-1
FP(NJ)=EXP(CMPLX(0.0,G*(((XG+COTH)+(KXG)*SIN))))
IF(XG.GT.(EPI+0.0)) FP(NJ)=CMPLX(0.0,0.0)
IF((XG.GT.(1.0+0.0)) ANO.(XG.LE.(2.0+0.0))
2 FP(NJ)=FP(NJ)+0.5-0.5*SIN((XG-(EPI+0.0)))
455 CONTINUE
WRITE(6,9410) (NJ,FP(NJ),NJ=1,KK)
9410 FORMAT('','INCIDENT FIELD FINC(',I4,')='E15.8)
C THIS BEGINS THE BACK SUBSTITUTION
C CONVERSION OF SOURCE SIDE
FP(I)=FP(I)/C(I,1,1)
DO 90 IJ=2,KK
S=CMPLX(0.0,0.0)
IJMO=IJ-1
DO 91 IK=1,IJMO
S=S+C(IJ,IK)*FP(IK)
FP(IJ)=(FP(IJ)-S)/C(IJ,1,1)
91 S=S+C(IJ,IK)*FP(IK)
90 FP(IJ)=(FP(IJ)-S)/C(IJ,1,1)
C NOW FOR FINAL BACK SUBSTITUTION
NMO=KK-1
DO 160 L=1,NMO
K=KK-L
KPO=K+1
S=CMPLX(0.0,0.0)
DO 175 JQ=KPO,KK
S=S+C(IJ,JQ)*FP(JQ)
FP(KJ)=FP(KJ)-S
175 S=S+C(K,JQ)*FP(JQ)
160 FP(KJ)=FP(KJ)-S
KKM1=KK-1
C TO RECONSTRUC T THE CURRENTS
DO 47 IRA=1,KKM1
47 F(2*IRA)=FP(IRA)+FP(IRA+1)/2.0
46 IRA=1,KK
48 F(2*IRA)=FP(IRA)
WRITE (6,4970) ((J,FP(J),J=1,KK)
4970 FORMAT(' ','FP(',I5,')='E15.6)
WRITE (6,553) (F(K),K=1,N)
553 FORMAT('6H F(',I5,')=2E15.8)
C THIS ENDS THE BACK SUBTITUTIONS
DO 439 KURR=1,N
439 FORMAT(' ','KURR=',I5)
IND=KURR-1
Y(1)=CABS(F(KURR))*4.0*WE/(6.28318*377.8)
XORD=FLOAT(KURR)
439 CALL PLQT(XORD,Y+1,IND,0.0200,0.0)
DO 440 KURR=1,N
440 FORMAT(' ','KURR=',I5)
IND=KURR-1
Y(1)=180.0*ATAN2(AIMAG(F(KURR)),REAL(F(KURR)))/3.1415927
110
CALL PLOT(XORD,Y1,IND,180.,-180.0)
DO 317 JNX=1,360
TH=0.017*329*FLCAT/JNX/2.0
T=CMPLX(0.0,0.0)
DO 310 I=1,N
XN=XM(I)
310 T=T+(EXP(CMPLX(0.0,G((XN*COS(TH))+(H(XN)*SIN(TH))))))*C
********** THIS CORRECTS THE OUTPUT TO TRUE ELE. FIELD
T=T*DC*SQRT(NE)*CMPLX(-0.707107,-0.707107)/3.1415927
CM=CMABS(T)
DB=70.0*ALOG10(CM)
CANG=57.296*ATAN2(IMAG(T),REAL(T))
THD=TH*57.296
ABES(JNX)=CM
WRITE (6,312) CM,CANG,THD,DB
312 FORMAT (10H RELATIVE E FIELD=,E15.8,6H ANGLE=,E15.8,6H ANGLE FROM HORIZONTAL=,E15.8,6H DH=,E15.8)
DO 9500 JC=1,360
Y(J)=ABES(JC)
E=FLOAT(JC)/2.0
IND=JC-1
9500 CALL PLOT(E,Y,1,IND,50.0,0.0)
STOP
END

FUNCTION CD(MR,MC)
COMPLEX CD
COMPLEX AHAN20
COMMON/GASSNY/GU1,GU2,GU3,OU4,OU5,GW1,GW2,GW3,GW4,GW5
COMMON /HDG/ XM(400),X(400),GA,G,DC
IF(MR.NE.MC) GO TO 100
CD=DC*AHAN20(GA)
GO TO 200
100 CONTINUE
XMM=X(MR)
HXM=H(XMM)
EPL=X(MC)
EPU=X(MC+1)
DVDFEP=(EPU-EPL)/2.0
DVSMEP=(EPU+EPPL)/2.0
XU5=GU5*DVDFEP+DVSMEP
XU1=GU1*DVDFEP+DVSMEP
XU2=GU2*DVDFEP+DVSMEP
XU3=GU3*DVDFEP+DVSMEP
XU4=GU4*DVDFEP+DVSMEP
CO=DVDFEP*(
2*GK1*AHAN20(G)*SQRT(((XU1-XMM)**2)+((HXU1-HXMM)**2))*SQRT(1.0
2+DH(XU1)**2))
2*GK2*AHAN20(G)*SQRT(((XU2-XMM)**2)+((HXU2-HXMM)**2))*SQRT(1.0
2+DH(XU2)**2))
2*GK3*AHAN20(G)*SQRT(((XU3-XMM)**2)+((HXU3-HXMM)**2))*SQRT(1.0
2+DH(XU3)**2))
2*GK4*AHAN20(G)*SQRT(((XU4-XMM)**2)+((HXU4-HXMM)**2))*SQRT(1.0
2+DH(XU4)**2))
2*GK5*AHAN20(G)*SQRT(((XU5-XMM)**2)+((HXU5-HXMM)**2))*SQRT(1.0
2+DH(XU5)**2)))
200 CONTINUE
RETURN
END
FUNCTION DH(X)
    DH(X) IS THE DERIV. OF H(X)
COMMON /PIG/ AONE, CONE, PONE, ATWO, CTWO, PTWO, N
    DH=AONE*CON*COX(CONE*X+PONE)+ATWO*CITWO*COS(CTWO*X+PTWO)
RETURN
END

FUNCTION H(X)
C
    THIS DEFINES THE SURFACE
COMMON /PIG/ AONE, CONE, PONE, ATWO, CTWO, PTWO, N
    H=AONE*SIN(CONE*X+PONE)+ATWO*SIN(CTWO*X+PTWO)
RETURN
END
TE CASE GAUSSIAN INTEGRATION USED TO FILL IN MATRIX INTEGRAL EQN.
NSUB SEGMENTS HAVE N MIDPOINTS
NSUB IS THE SUBSCRIPT WHICH COUNTS THE END POINTS
N IS THE SUBSCRIPT WHICH COUNTS THE MIDPOINTS
WATCH MAX SLCPE SO THAT THE X INCREMENTS ARE SMALL ENOUGH
THE ARRAY XM(IJ) CONTAINS THE X COORDINATES OF THE MIDPOINTS OF THE
SURFACE SEGMENTS, X(I),X(I+1) ARE THE LOWER AND UPPER X COORDINATES
OF THE ENDPOINTS OF THE I'TH SEGMENT

COMPLEX SNN,SST
COMPLEX S,CO
COMPLEX FSS
COMMON/GASSN/ GU1,Gu2,Gu3,Gu4,Gu5,GW1,GW2,GW3,GW4,GW5
COMPLEX FINC201,STS
COMMON /PIG/ ACNE,CONE,PONE,ATWO,CTWO,PTWO,N
COMMON (235,235)
COMMON/HOG/ XM(400),G,X(400)
COMMON /DG/ DJC
COMMON DJC
COMMON F(235),SS,T,CTEST
COMMON FIN
COMMON HAN2
DIMENSION ABES(360),Y(10)

THE FOLLOWING CONSTANTS DESCRIBE THE SURFACE
ACNE=40.0
CONE=6.28318/200.0
PONE=0.0
ATWO=0.0
CTWO=0.0
PTWO=0.0

WE IS THE ELECTRICAL WAVELENGTH
WE=25.0
G=C.2831853 /WE
DC=WE/10.0
DX=DC/1000.0
DC2=DC/2.0
EP=200.0
STS=-DC*CMPLX(0.707107,0.707107)/(2.0*SQR(T(WE))
DJC=CMPLX(1.0,1.0)+G/4.0

CONSTANTS FOR GAUSSIAN INTEGRATION 5TH ORDER
GU1=-C.9061798
GU2=-0.53846931
GU3=0.0
GU4=-GU2
GU5=-GU1
GW1=0.2369268
GW2=0.47862867
GW3=0.5688888
GW4=0.47862867
GW5=0.2369268

CONSTANTS FOR GAUSSIAN INTEGRATION 5TH ORDER
THE FOLLOWING BREAKS THE SURFACE INTO SEGMENTS DC CENTIMETERS LONG
BY LINE INTEGRATION USING STEPS OF LENGTH DX FOR THE INTEGRATION
NSUB=1

X(NSUB)=-EP
1002 AL=0.0
R=X(NSUB)
1001 R=R+DX
AL0=AL
AL=AL+(DX*SQR(T(1.0+(DH(R)**2))))
IF((DC2-AL).LE.0.0).AND.((DC2-ALO).GT.0.0)) XM(NSUB)=R
IF(AL.LT.DC)GO TO 1001
WRITE(6,352) AL,NSUB
352 FORMAT(' AL=',E15.8,' NSUB=',I4)
NSUB=NSUB+1
X(NSUB)=R
IF (R.LT.EP) GO TO 1002
N=NSUB-1
WRITE(6,251) N,NSUB
251 FORMAT(' N=',I4,' NSUB=',I4)
DO 1004 J=1,NSUB
IF (J.EQ.NSUB) XM(NSUB)=0.0
XXX=X(J)
XMD=XM(J)
1004 WRITE (6,1003) XXX,XMD,J
1003 FORMAT (' X(J)=',E15.8,9H XMD(J)=,E15.9,3H J=',I3)
C THIS ENDS THE SURFACE SUBDIVISION
NMC=N-1
NM3=N-3
C DIMENSION OF FINC,F IS N
DPF=0.7653982
C MATRIX FILL IN
DO 3661 IR=1,N
DO 3661 IC=1,N
C(IR,IC)=C(IR,ICI
C THIS COMPLETES THE FILLING OF THE MATRIX
C NCSYMMETRIC CROUT
C FIRST COLUMN OK
C TO GET THE FIRST ROW
DO 10 J=2,N
10 CI1,J)=C11,J)/C11,1)
C NOW WORK ON ROW AND COLUMN SET K
DO 11 K=2,N
KMC=K-1
KPO=K+1
C TO GET DIAGONAL ELEMENT
S=CMPLX(0.0,0.0)
DO 12 IK=1,KMD
S=S+C(K,K)*C(1K,K)
C(K,K)=C(K,K)-S
C TO GET ELEMENTS IN COLUMN K BELOW ROW K
IF (KPO.GT.N) GO TO 17
DO 13 IROW=KPO,N
S=CMPLX(0.0,0.0)
DO 14 JJ=1,KMC
S=S+C(IROW,JJ)*C(JJ,K)
13 C(IROW,K)=C(IROW,K)-S
C TO GET ELEMENTS IN ROW K TO THE RIGHT OF COLUMN K
DO 15 JCOL=KPO,N
S=CMPLX(0.0,0.0)
DO 16 JR=1,KMD
S=S+C(K,JR)*C(JR,ICOL)
15 C(K,ICOL)=(C(K,ICOL)-S)/C(K,K)
17 CONTINUE
11 CONTINUE
C THIS ENDS THE MATRIX FACTORIZATION
WRITE (6,1222) N,WE
1222 FORMAT(3H N=,I3,4H WE=,E15.8)
THI=0.0*3.14159/180.0
WRITE (6,9333) THI
9333 FORMAT(9H INC ANG=,E15.8)
C THIS IS THE ANGLE OF INC. MEAS. FROM THE +VE X AXIS
STH=SIN(THI)
CTH=COS(THI)
C THIS FINDS THE INCIDENT FIELD ON THE NJTH SEGMENT
DO 455 NJ=1,N
XG=XM(JN)
THE SIGN ON THE INCIDENT FIELD HAS BEEN ADJUSTED TO AGREE WITH
THE INTEGRAL EQUATION

\[ F(NJ) = C \exp(C \text{CMPLX}(0.0, G \times (XG \times C \text{TH}) + (H(XN) \times \text{STH})) \times \text{CMPLX}(-1.0, 0.0)) \]

TAPPED ILLUMINATION

\[ \text{IF}((XG \leq ((\text{WE} \times 2.0) - \text{EP}))) \times F(NJ) = \text{CMPLX}(0.0, 0.0) \]

\[ \text{IF}((XG \leq ((\text{WE} \times 2.0) - \text{EP}))) \times F(NJ) = \text{CMPLX}(0.0, 0.0) \]

\[ \text{IF}((XG \leq ((\text{WE} \times 2.0) - \text{EP}))) \times F(NJ) = \text{CMPLX}(0.0, 0.0) \]

CONTINUE

WRITE(6, 2948) (NJ, F(NJ), NJ=1, N)

THIS BEGINS THE BACK SUBSTITUTION

CONVERSION OF SOURCE SIDE

\[ F(1) = F(1) / C(1, 1) \]

DO 90 IJ = 2, N

\[ S = \text{CMPLX}(0.0, 0.0) \]

IJKO = IJ - 1

DO 91 IJ = 1, IJKO

\[ S = S + C(\text{IJ, IK}) \times F(\text{IK}) \]

91

\[ F(\text{IJ}) = (F(\text{IJ}) / S) / C(\text{IJ, IJ}) \]

NOW FOR FINAL BACK SUBSTITUTION

\[ \text{NMO} = N - 1 \]

DO 160 L = 1, NMO

\[ K = N - L \]

\[ \text{KPC} = K + 1 \]

DO 175 JD = KPO, N

\[ S = S + C(\text{K, JD}) \times F(\text{JD}) \]

175

\[ F(\text{K}) = F(\text{K}) \times S \]

THIS ENDS THE BACK SUBSTITUTION

DO 554 IKUR = 1, N

\[ \text{AAF} = \text{CABS}(F(\text{IKUR})) \]

\[ \text{ANF} = 57.296 \times \text{ATAN2}(\text{AIMAG}(F(\text{IKUR})), \text{REAL}(F(\text{IKUR}))) \]

554

WRITE(6, 553) IKUR, AAF, ANF

FORMAT (' ', 'F(1, 1)', ' = ', E15.8, ' AT ANGLE=', E15.8)

DO 9553 IRRD = 1, N

\[ \text{IND} = \text{IRRD} - 1 \]

\[ Y(1) = \text{CABS}(F(\text{IRRD})) \]

\[ \text{XRRD} = \text{FLOAT}(\text{IRRD}) \]

9553

CALL PLOT(XRRD, Y, 1, IND, 1.0, 0.0)

DO 9554 IRRD = 1, N

\[ \text{IND} = \text{IRRD} - 1 \]

\[ Y(1) = 57.2958 \times \text{ATAN2}(\text{AIMAG}(F(\text{IRRD})), \text{REAL}(F(\text{IRRD}))) \]

\[ \text{XRRD} = \text{FLOAT}(\text{IRRD}) \]

9554

CALL PLOT(XRRD, Y, 1, IND, 180.0, -180.0)

DO 317 JNX = 1, 360

\[ \text{THS} = 0.01745329 \times \text{FLOAT}(\text{JNX}) / 360 \]

\[ T = \text{CMPLX}(0.0, 0.0) \]

317

\[ T = T \times C \exp(C \text{CMPLX}(0.0, G \times (XN \times \text{COS}(\text{THS}) + (H(XN) \times \text{SIN}(\text{THS}))))) \]

2 \times \text{COS}((\text{T5} - \text{THS} - \text{TSH}))

**** THIS CORRECTS THE OUTPUT TO TRUE MAG. FIELD

\[ T = T \times \text{ST5} \]

\[ \text{CM} = \text{CABS}(T) \]

\[ \text{DB} = 20.0 \times \text{ALOG10}(\text{CM}) \]

\[ \text{TANG} = 57.296 \times \text{ATAN2}(\text{AIMAG}(T), \text{REAL}(T)) \]

\[ \text{THSD} = \text{T5} + 57.296 \]

\[ \text{ABE} = 0.01745329 \times \text{FLOAT}(\text{JNX}) \]

317

WRITE (6, 312) CM, TANG, THSD, DB

115
FUNCTION H(X)
C THIS DEFINES THE SURFACE
COMMON /PIG/ AONE,CONE,PONE,ATWO,CTWO,PTWO,N
H=AONE*Sin(CONE*X)+PONE+ATWO*Sin((CTWO*X)+PTWO)
RETURN
END

FUNCTION DH(X)
C DH(X) IS THE DERIV. OF H(X)
COMMON /PIG/ AONE,CONE,PONE,ATWO,CTWO,PTWO,N
DH=AONE*CONE*COS(CONE*X)+PONE+ATWO*CTWO*COS(CTWO*X)+PTWO)
RETURN
END

FUNCTION CO(MR,MC)
C THIS GIVES THE OLD MATRIX COEFFICIENTS
COMPLEX CO
COMPLEX DJC
COMMON/GASSN/ GI,G2,GI4,GI5,GI2,GI3,GI4,GI5
COMMON/HCG/ XM(400),G,X(400)
COMMON /DOG/ DJC
COMPLEX AHAN21
IF(MR.NE.MC) GO TO 100
CO=CMPLX(0.50D,0.01
GO TO 200
100 CONTINUE
XM=XM(MR)
HX=H(XMM)
EPL=X(MC)
EPU=X(MC+1)
DVDFEP=(EPU-EPL)/2.0
DVSEM=(EPU+EPL)/2.0
XU5=GI5*DVDFEP+DVSEM
XU4=GI4*DVDFEP+DVSEM
XU3=GI3*DVDFEP+DVSEM
XU2=GI2*DVDFEP+DVSEM
XU1=GI1*DVDFEP+DVSEM
HXU1=H(XU1)
HXU2=H(XU2)
HXU3=H(XU3)
HXU4=H(XU4)
HXU5=H(XU5)
DXU1=DXU1
DXU2=DXU2
DXU3=DXU3
DXU4=DXU4
DXU5=DXU5
CO=DVDFEP*
2*(GI1*AHAN21(G*SQR((XU1-XMM)**2)+(HXU1-HXMM)**2))*
2*(-DXU1*(XMM-XU1))+(HXMM-HXU1)
2/SQR((XMM-XU1)**2)+(HXMM-HXU1)**2)*
2*(GI2*AHAN21(G*SQR((XU2-XMM)**2)+(HXU2-HXMM)**2))*
2*(-DXU2*(XMM-XU2))+(HXMM-HXU2)
\[ \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U2}} \right)^2 + \left( H_{\text{XM}} - H_{\text{U2}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U3}} - X_{\text{MM}} \right)^2 + \left( H_{\text{XU3}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U4}} - X_{\text{MM}} \right)^2 + \left( H_{\text{XU4}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U5}} - X_{\text{MM}} \right)^2 + \left( H_{\text{XU5}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U3}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U3}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U4}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U4}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U5}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U5}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U2}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U2}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U3}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U3}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U4}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U4}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U5}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U5}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U2}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U2}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U3}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U3}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U4}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U4}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U5}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U5}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U2}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U2}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U3}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U3}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U4}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U4}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U5}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U5}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U2}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U2}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U3}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U3}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U4}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U4}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{MM}} - X_{\text{U5}} \right)^2 + \left( H_{\text{MM}} - H_{\text{U5}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U2}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U2}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U3}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U3}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U4}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U4}} - H_{\text{MM}} \right)^2 \right) \]
\[ = \frac{2}{\sqrt{2}} \left( \left( X_{\text{U5}} - X_{\text{MM}} \right)^2 + \left( H_{\text{U5}} - H_{\text{MM}} \right)^2 \right) \]
THIS PROGRAM USES GAUSSIAN INTEGRATION TO GET MATRIX ELEMENTS

NSUB SEGMENTS HAVE N MIDPOINTS.

NSUB IS THE SUBSCRIPT WHICH COUNTS THE END POINTS

N IS THE SUBSCRIPT WHICH COUNTS THE MIDPOINTS

WATCH MAX SLOPE SO THAT THE X INCREMENTS ARE SMALL ENOUGH

COMPLEX SN, SST
COMPLEX S, CD
COMPLEX FSS
COMMON/GASSN/ GU1,GU2, GU3, GU4, GU5, GW1, GW2, GW3, GW4, GW5
COMMON/FINC(20), STS
COMMON/PIG/ AONE, CONE, PUNE, ATWO, CTWO, PTWO, N
COMMON/C(100,150)
COMMON/HIG/ XM(400), G, X(400)
COMMON/DOG/ DJC
COMMON/DJC
COMMON/F(400), EP(400), SS, T, CTST
COMPLEX FI
COMPLEX HAN2
DIMENSION ABES(360), Y(10)

WE IS THE ELECTRICAL WAVELENGTH
WE=25. C
G=6.2831853 /WE
AONE=5.0
CONE=6.283189/200. C
PCONE=0.50
ATWO=0.0
CTWO=0.0
PTWO=0.0
DC=WE/15.0
DX=DC/150.0
DC2=DC/2.0
EP=200.0
STS=DC*G/2.0

GU1=-0.9061798
GU2=-0.5384693
GU3=0.0
GU4=-GU2
GU5=-GU1
GW1=0.2365268
GW2=0.2365268
GW3=0.4786286
GW4=0.4786286
GW5=0.0

THE FOLLOWING BREAKS THE SURFACE INTO SEGMENTS DC CENTIMETERS LONG
BY LINE INTEGRATION USING STEPS OF LENGTH DX FOR THE INTEGRATION

NSUB=1

X(NSUB)=EP

1002 AL=0.000
R=X(NSUB)

1001 R=R+DX
ALO=AL
AL=AL+(DX*SQRT(1.0+(DH(R)**2)))*.

IF((DC2-AL).LE.0.0).AND.((DC2-ALO).GT.0.0)) X(NSUB)=R
IF(AL1.DT.0.0) GO TO 1001
WRITE(6,352) AL, NSUB
352 FORMAT(1X, AL=",E15.8", NSUB=",I4")
NSUB=NSUB+1
XNSUB)=K
IF (R.LT.EP) GO TO 1002
N=NSUB-1
WRITE(6,251) N,NSUB
251 FORMAT(' N=',I4,' NSUB=',I4)
DO 1004 J=1,NSUB
   IF (J.EQ.NSUB) XM(NSUB)=0.0
   XXX=X(J)
   XMD=EH(J)
1004 WRITE (6,1003) XXX,XMD,J
1003 FORMAT (6H X(J)=,E15.8,9H XMD=,E15.8,3H J=,I3)
C THIS ENDS THE SURFACE SUBDIVISION
C THIS INSURES THAT N IS ODD
KK=O
5733 KK=KK+1
   IF ((2*KK-1).EQ.N) GO TO 5731
   IF (2*KK,EQ,N) GO TO 5732
GO TO 5733
5732 N=N-1
5731 CONTINUE
WRITE (6,3728) N,KK
3728 FORMA'I T Format, lCORRECTED VALUE OF N=*,14,*Kk=*,14,*2*KK-1=N*)
NM=NN-1
NM3=NN-3
C DIMENSION OF FINC,F IS N
DPIF=0.7853982
C MATRIX FILL IN
C DO BY COLUMNS
C FOR FIRST COLUMN
DO 3661 I=1,KK
   C21,2I=C22*I-1+1+2*C00/2.0)
3661 CONTINUE
C FOR LAST COLUMN
DO 3678 I=1,KK
   C(I,KK)=(C2*I-1,2*KK-2)/2.0+C2*I-1,2*KK-1
3678 CONTINUE
C FOR MIDDLE COLUMNS
DO 56 I=1,KK
   II=2*I-1
   KK1=KK-1
   DO 56 J=2,KK1
      JJ=2*J-1
56 CONTINUE
C THIS COMPLETES THE FILLING OF THE MATRIX
C NONSYM. CROUT
C FIRST COLUMN OK
C TWO GET FIRST ROW
DO 10 J=2,KK
10 C(I,J)=C(I,K)/C(I,1)
C NOW WORK ON ROW AND COLUMN SET K
DO 11 K=2,KK
   K0=K+1
   C TO GET DIAGONAL ELEMENT
   S=CMLX(0.0,0.0)
   DO 12 IK=1,KM0
      S=S+C(IK,IK)*C(IK,K)
   12 C(K,K)=C(K,K)-S
C TO GET ELEMENTS IN COLUMN K BELOW ROW K
   IF(K0,GT,KK) GO TO 17
   DO 13 IROW=K0,KK
      S=CMLX(0.0,0.0)
      DO 14 JJ=1,KM0
         S=S+C(IROW,JJ)*C(JJ,K)
      14 C(IROW,K)=C(IROW,K)-S
   13 CONTINUE
C TO GET ELEMENTS IN ROW K TO THE RIGHT OF COLUMN K
C 119
C 16.15 17 11 12 22 9333 C
C DO 15 ICOL=KPG, KK
S=CMPLX(0.0,0.0)
DO 16 JR=1, KMo
16 S=S+C(K, JR)*C(JR, ICOL)
15 C(K, ICOL)=C(K, ICOL)-S/CMPLX(C(K, K))
17 CONTINUE
11 CONTINUE
WRITE (6,1222) KK, WE
1222 FORMAT(*',* KK=', I4, ', WE=', E15.8)
TH=3.14159265/180.0
THDEG=57.29578*TH
WRITE (6, 9333) THDEG
9333 FORMAT(' TH IS THE ANGLE OF INCIDENCE FROM THE HORIZONTAL
STH=SINL TH)
C THIS FINDS THE INCIDENT FIELD ION THE NJTH SEGMENT
C TAPERED ILLUMINATION ************ ************ ************
C DO 455 NJ=1, KK
XG=XG(J=1, NJ-1)
Fp(NJ)=CEXP(CMPLX(0.0,0.0)+ICM(SIN(XG)*CTH)+ICH(XG)*STH)))*CMPLX(-1.0,0.0)
C INCIDENT FIELD HAS BEEN ADJUSTED TO AGREE WITH INTEGRAL EQTN.
IF(XG=0.0, 0.0)+IP-1.0, NE-IP)
FP(NJ)=CMPLX(0.0,0.0)
J=1, NJ
455 CONTINUE
WRITE (6, 9410) (NJ, FP(NJ), NJ=1, KK)
9410 FORMAT(*', * INCIDENT FIELD FINISH', I4, ' =', E15.8)
C THIS BEGINS THE BACK SUBSTUTUTION
C CONVERSION OF SOURCE SIDE
FP(I)-FP(I)/CMPLX(I, I)
DO 90 IJ=2, KK
S=CMPLX(I, 0, 0.0)
IJMU=IJ-IU
90 FP(IJ)=FP(IJ)*S/CMPLX(IJ, K)
C NOW FOR FINAL BACK SUBSTUTION
NMU=KK-1
DO 160 L=1, NMU
K=KK-L
KPO=K+1
S=CMPLX(0.0, 0.0)
DO 175 JD=KPO, KK
175 S=S+C(K, JD)*FP(JD)
160 FP(K)=FP(K)-S
KKM1=KK-1
C TO RECONSTRUCT THE CURRENTS
DO 47 IRA=1, KKM1
F2=FP(IRA)+FP(IRA+1))/2.0
DO 48 IRA=1, KK
47 F2=FP(IRA)+FP(IRA+1))/2.0
48 IRA=1, KK
WRITE (6, 4970) (J, FP(J)), J=1, KK)
4970 FORMAT(*', FP(', I5, ')=', E15.8)
WRITE (6, 4970) (FP(K), K=1, N)
553 FORMAT(*', FP(K)=, E15.8)
DO 9553 IRA=1, N
IND=IRRO-1
Y(I)=CABS(F(I))
XRPO=FLOAT(IND)
120
CALL PLOT(XRRO, Y, 1, IND, 5.0, 0.0)
DO 9554 IRRO = 1, N
IND = IRRO - 1
Y(1) = 57.2958 * ATAN2(AIMAG(F(IRRO)), REAL(F(IRRO)))
XRRO = FLOAT(IRRO)

CALL PLOT(XRRO, Y, 1, IND, 180.0, -180.0)

THIS ENDS THE BACK SUBSTITUTIONS

DO 317 JNX = 1, 360

THS = 0.01745329 * FLOAT(JNX) / 2.0
T = CMPLX(0.0, 0.0)
DO 310 I = 1, N

XN = X(JNX)

THN = 1.5707963 + ATAN(DH(XN))

DO 310 I = 1, N

XN = X(JNX)

THN = 1.5707963 + ATAN(DH(XN))

THIS CORRECTS THE OUTPUT TO TRUE ELECTRONIC FIELD

T = T + STS
CM = CABS(T)
DB = 20.0 * ALOG10(CM)
CANG = 57.296 * ATAN2(AIMAG(T), REAL(T))

THSD = THS * 57.296

ABES(JNX) = CM

WRITE (6, 312) Cfi, CANG, THSD, CB
312 FORMAT (LH RELATIVE E FIELD=, E15.8, LH ANGLE=, E15.8,
(2 ANGLE FROM MORPHISTIC=, E15.8, 6H D)=, E15.8)

DO 9500 JC = 1, 360

Y(JC) = ABES(JC)
U = FLOAT(JC) / 2.0
IND = JC - 1

CALL PLOT(U, Y, 1, IND, 50.0, 0.0)
STOP
END

FUNCTION H(X)
THIS DEFINES THE SURFACE

FUNCTION DH(X)

FUNCTION CO(MR, MC)

THIS GIVES THE OLD MATRIX COEFFICIENTS

CONTINUE

XMM = XM(MR)

HXM = H(XMM)
EPL = X(MC)
EPU = X(MC+1)
DVDFEP = (EPU-EPL)/2.0
DVSMEP = (EPU+EPL)/2.0
XU1 = GU5*DVDFEP+DVSMEP
XU2 = GU2*DVDFEP+DVSMEP
XU3 = GU3*DVDFEP+DVSMEP
XU4 = GU4*DVDFEP+DVSMEP
ATDH1 = ATAN(DH(XU1))
ATDH2 = ATAN(DH(XU2))
ATDH3 = ATAN(DH(XU3))
ATDH4 = ATAN(DH(XU4))
HXU1 = H(XU1)
HXU2 = H(XU2)
HXU3 = H(XU3)
HXU4 = H(XU4)
HXU5 = H(XU5)
CO = DVDFEP*(
2+G01*AHAN21(G=SQRTR((XU1-XMM)**2)+(H(XU1)-HXM**2)))*SQRTR(1.0+
2*DH(XU1)**2)*((-SIN(ATDH1)*(XMM-XU1))+(COS(ATDH1)*(HXM-HXU1)))
2*SQRTR((XMM-XU1)**2)+(HXM-HXU1)**2)
2+G02*AHAN21(G=SQRTR((XU2-XMM)**2)+(H(XU2)-HXM**2)))*SQRTR(1.0+
2*DH(XU2)**2)*((-SIN(ATDH2)*(XMM-XU2))+(COS(ATDH2)*(HXM-HXU2)))
2*SQRTR((XMM-XU2)**2)+(HXM-HXU2)**2)
2+G03*AHAN21(G=SQRTR((XU3-XMM)**2)+(H(XU3)-HXM**2)))*SQRTR(1.0+
2*DH(XU3)**2)*((-SIN(ATDH3)*(XMM-XU3))+(COS(ATDH3)*(HXM-HXU3)))
2*SQRTR((XMM-XU3)**2)+(HXM-HXU3)**2)
2+G04*AHAN21(G=SQRTR((XU4-XMM)**2)+(H(XU4)-HXM**2)))*SQRTR(1.0+
2*DH(XU4)**2)*((-SIN(ATDH4)*(XMM-XU4))+(COS(ATDH4)*(HXM-HXU4)))
2*SQRTR((XMM-XU4)**2)+(HXM-HXU4)**2)
2+G05*AHAN21(G=SQRTR((XU5-XMM)**2)+(H(XU5)-HXM**2)))*SQRTR(1.0+
2*DH(XU5)**2)*((-SIN(ATDH5)*(XMM-XU5))+(COS(ATDH5)*(HXM-HXU5)))
2*SQRTR((XMM-XU5)**2)+(HXM-HXU5)**2))
CO = DJC*CO
RETURN END
FUNCTION AHAN21(X)
C
THIS IS THE HANKEL FUNCTION OF TYPE 2 AND OF ORDER 1
DOUBLE PRECISION XD, DX, A1, A2, A3, A4, A5, A6, AHJ1, A1, B2, B3, B4, B5, AHJ1;
2
TDX = A1, A2, A3, A4, A5, A6, T1, T2, T3, T4, T5, T6, T7, DSQX, B6
C
COMPLEX AHAN21
D X = DBLE(X)
IF (X .GT. 3.0) GO TO 200
XU = XD * DX / 9.0 + 0.0
A1 = - 0.31761 D - 0.10 C - 0.115 C D + 0.45 * XD
A2 = 0.00443 D + 0.0 + A1 * XD
A3 = - 0.03954 D + 0.6 + A2 * XD
A4 = 0.21093 D + 0.0 + A3 * XD
A5 = - 0.5624 D + 0.0 + A4 * XD
A6 = 0.5 D + 0.0 + A5 * XD
HJ1 = A6 * DX
B1 = - 0.040 C 0.976 D + 0.0 + 0.027873 D + 0.0 * XD
B2 = 0.31235 D + 0.0 + B1 * XD
B3 = - 1.316 D + 0.0 + B2 * XD
B4 = 2.1682 D + 0.0 + B3 * XD
B5 = 0.2212 D + 0.0 + B4 * XD
B6 = - 0.0366198 D + 0.0 + B5 * XD
AHJ1 = (B6 / DX) + T1 * DLG(DX / 2.0) * 0.63661977
AHAN21 = CMPLX(SGML(HJ1), -SGML(AHJ1))
GO TO 300
200
TOX = 3.0 / DX
A1 = 0.01136 D + 0.0 + 0.0020 D + 0.033 * TDX
A2 = 0.00294 D + 0.0 + A1 * TDX
A3 = 0.01716 D + 0.0 + A2 * TDX
A4 = 0.01696 D + 0.0 + A3 * TDX
A5 = 0.156 D + 0.0 + A4 * TDX
A6 = 0.79788 D + 0.0 + A5 * TDX
T1 = 0.000795 D + 0.0 + 0.0009166 D + 0.0 * TDX
T2 = 0.00743 D4 D + 0.0 + T1 * TDX
T3 = 0.00637 D + 0.0 + T2 * TDX
T4 = 0.00056 D + 0.0 + T3 * TDX
T5 = 0.12499 D + 0.0 + T4 * TDX
T6 = 2.3519 D + 0.0 + T5 * TDX
T7 = DX + T6
DSQX = A6 / DSQRT(DX)
AHAN21 = CMPLX(SGML(DSQX * DCOS(T7)), -SGML(DSQX * DSIN(T7)))
GO TO 300
300
CONTINUE
RETURN
END
FUNCTION AHAN20(X)
C
THIS IS THE HANKEL FUNCTION OF ORDER 0 AND OF TYPE 2
DOUBLE PRECISION XSQ,B10,B8,B6,B4,B2,C10,C8,C6,C4,C2,D5,D4,D3,
 2D2,D1,E5,E4,E2,E1,E0,XD,DX,F0,E3,HJ,DSX
COMPLEX AHAN20
DX=DBLEL(X)
IF (X.GT.3.0) GO TO 100
XSQ=DX*DX/C.9D+Ol
B10=-0.394440D-02+XSQ*O.210-03
B8=0.0444479D+00+XSQ*B10
B6=-0.3163866D+00+XSQ*B8
B4=1.2656208D+00+XSQ*B6
B2=-2.2469970D+00+XSQ*R4
HJ=1.00+00+XSQ*B2
C10=0.427916D-02-XSQ*O.248460-03
C8=-0.4261214D-01+XSQ*C10
C6=0.2530117D+00+XSQ*C8
C4=-0.74350384D+00+XSQ*C6
C2=0.6059366D+00+XSQ*C4
HY=SNGL(0.3674669DD+00+0.6366198D*HJ*DLOG(DX/2.D)+XSQ*C2)
AHAN20=CMPLX(SNGL(HJT,-HYI
GO TO 200
100 YO=3.0/DX
L5=-0.72805D-03+XD*0.14476D-03
D4=0.137237D-02+D5*XD
D3=-0.95120D-04+D4*XD
D2=-0.55274CD-02+D3*XD
D1=-0.77D-06+02*XD
FC=0.7978456D+00+XD+D1
E3=0.29333D-03+XD*0.1358D-03
E4=-0.54125D-03+E5*XD
E3=0.262573D-02+E4*XD
E2=-0.3954D-04+E3*XD
E1=-0.4166397D-01+E2*XD
E0= (-0.78539816D+00+XD*E1)+DX
DSX=DSQRT(DX)
AHAN20=CMPLX(SNGL(F0*DCOS(E0)/DSX),-SNGL(F0*DSIN(E0)/DSX))
CONTINUE
RETURN
END
SUBROUTINE PLOT( X,Y,N,IND,YMAX,YMIN)
    DIMENSION MARK(10), YLABEL(6), Y(10), MARK(10)
    DATA MARK(1, MARK(2, MARK(3), MARK(5), MARK(6), MARK(7), MARK(8),
    MARK(9), MARK(10), MARK(4), 1H, 1H, 1H, 1H, 1H, 1H, 1H, 1H, 1H, 1H, 1H)
    DATA IBLANK, NOPT, IPLUS, 1H, 1H, 1H
    IF (IND) 1, 1, 11
    WRITE(6, 3)
    3 FORMAT(1H1/25X, 48H ORDER IN WHICH PLOT SYMBOLS ARE USED * IXONH1Z
    *-//30X, 39XH1NE SYMBOL ($) INDICATES OFF-SCALE DATA //)
    DO 7 J=9,119
    7 M(J) = MARK(10)
    NCOUNT = 1G
    SCALE = YMAX - YMIN
    LLL = (-YMIN SCALE) + 1.5
    DO 8 J=1,6
    8 YLABEL(J) = R*ZD.D/SCALE + YMIN
    WRITE(6, 9) (YLABEL(J), I=1,6)
    9 FORMAT(6X, 2E9.2, 5(1PE20.2) /)
    GOTO 132
    11 NCOUNT = NCOUNT + 1
    DO 99 J=1,119
    99 M(J) = IBLANK
    IF (LLL GE 11. AND. LLL LE 110) M(J) = MARK(10)
    IF (NCOUNT - 10) 133, 132, 133
    132 DO89 J=11, 111, 20
    89 M(J) = IPLUS
    133 DO32 J=1, N
    L = (Y(J) - YMIN) * SCALE + 0.5
    IF (L) 14, 17, 17
    14 IF (L LE 10) 15, 16, 16
    15 M(J) = NOPT
    GOTO 20
    16 LL = L + 1
    M( LL) = MARK( J)
    GOTO 20
    17 IF (L) 19, 19, 19
    18 LL = L + 11
    M( LL) = MARK( J)
    GOTO 20
    19 M(119) = NOPT
    20 CONTINUE
    IF (NCOUNT - 10) 21, 25, 21
    21 WRITE(6, 24) (M(J), J=1, 119)
    24 FORMAT(1X, 119A1)
    GOTO 27
    25 WRITE(6, 26) (X,(M(J), J=9, 119))
    26 FORMAT(1X, F7.3, 111A1)
    NCOUNT = 0
    27 CONTINUE
    RETURN
END
APPENDIX B

SOLUTION OF SYSTEMS OF SIMULTANEOUS LINEAR EQUATIONS

Several direct methods exist which find the solution vector, [X], when the system of equations

(99) [C] [X] = [B]

is given. The two methods used here were the square root (or Cholesky) method for symmetric systems, and the Crout method for non-symmetric systems (Ref. [33]). Both methods take advantage of the fact that a non-singular matrix [C] is equivalent to [L][U], where [L] is a lower triangular matrix and [U] is an upper triangular matrix. So

or

\[
\begin{bmatrix}
\varepsilon_{11} & 0 & 0 & \cdots & 0 \\
\varepsilon_{21} & \varepsilon_{22} & 0 & \cdots & 0 \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\varepsilon_{N1} & \cdots & \cdots & \cdots & \varepsilon_{NN}
\end{bmatrix}
\begin{bmatrix}
u_{11} & u_{12} & \cdots & u_{1N} \\
0 & u_{22} & \cdots & u_{2N} \\
0 & 0 & \cdots & u_{NN}
\end{bmatrix}
= \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1N} \\
c_{21} & c_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
c_{N1} & \cdots & \cdots & c_{NN}
\end{bmatrix}
\]
\[
\min(i,j)
\]

\[
\sum_{k=1}^{\min(i,j)} \lambda_{ik} u_{kj} = c_{ij}
\]

since

\(\lambda_{ik} = 0\) if \(k > i\) and

\(u_{kj} = 0\) if \(k > j\).

In order to specify \([L]\) and \([U]\), \(N^2+N\) unknowns must be determined. Since there are only \(N^2\) equations, (values of \(C_{ij}\)) \(N\) unknowns may be specified. In the square root method the diagonal elements are assumed equal, i.e.,

\[u_{ii} = \lambda_{ii}\] for \(i = 1, \ldots, N\)

which gives the \(N\) extra conditions; in the Crout method one set of diagonals is specified, namely

\(\lambda_{kk} = 1\) for \(k = 1, \ldots, N\).

Suppose that \([C]\) has been broken up into \([L][U]\), then

\([L][U][X] = [B]\)

whence by defining

\([R] = [U][X]\)

there results
\[(107) \quad [L][R] = [B]\]

which has the solution

\[(108) \quad r_i = (b_i - \sum_{k=1}^{i-1} \ell_{ik} x_k)/\ell_{ii} \quad \text{for } i = 1, \ldots, N\]

and the sum is omitted, if \(i\) equals 1. Once the \([R]\) vector is known the system

\[(109) \quad [U][X] = [R]\]

is solved by

\[(110) \quad x_i = (r_i - \sum_{k=i+1}^{N} u_{ik} x_k)/u_{ii} \quad \text{for } i = 1, \ldots, N\]

where the sum is omitted if \(i\) equals \(N\). Wilkinson (Ref. [34]) has shown that most of the error in a solution of Eq. (99) by triangularization methods comes from the decomposition of \([C]\) into \([L][U]\) and not in the double back substitution (Eqs. (108) and (110)).

The details of the decomposition of \([C]\) into \([L][U]\) will now be considered. For Crout factorization the diagonal elements of \([U]\) are set equal to unity leaving \(N^2\) equations and \(N^2\) unknowns in the set of Eqs. (101), (102) and (103), which can be solved as follows:

\[(111) \quad \ell_{ik} = c_{ik} - \sum_{m=1}^{k-1} \ell_{im} u_{mk} \quad \text{for } i = k, \ldots, N\]
\[(112) \quad u_{kj} = \frac{1}{\ell_{kk}} (c_{kj} - \sum_{m=1}^{k-1} \ell_{km} u_{mj}) \quad \text{for} \ j = k+1, \ldots, N\]

\[(113) \quad \ell_{ik} = 0 \quad \text{if} \ i < k\]

\[(114) \quad u_{kj} = 0 \quad \text{if} \ j < k.\]

These equations are used in the order: first column of \([L]\), first row of \([U]\); second column of \([L]\), second row of \([U]\); third column of \([L]\), etc. In a computer solution the elements of \([U]\) and \([L]\) may be written over the original matrix \([C]\) as they are generated. Once this is done the matrix becomes

\[
\begin{bmatrix}
\ell_{11} & u_{12} & u_{1N} \\
\vdots & \ddots & \vdots \\
\ell_{N1} & \cdots & \ell_{NN}
\end{bmatrix}
\]

and the fact that the diagonal elements of \([U]\) are unity is used only in the previously described back substitution portion of the solution.

If \([C]\) is symmetric then \([C]\) can be factored into

\[(115) \quad [C] = [U]^T [U]\]

where \([U]^T\) is the transpose of \([U]\). Equation (101) becomes
\[
\min_{i,j} \sum_{k=1}^{\min(i,j)} u_{ki} u_{kj} = c_{ij}.
\]

The \(u_{i,j}\)'s are found from

\[u_{11} = \sqrt{c_{11}}\]

\[u_{ij} = \frac{c_{ij}}{u_{11}} \text{ for } j=2,\ldots,N\]

\[u_{ii} = \left(\frac{c_{ii}}{\sum_{k=1}^{i-1} u_{k1}^2}\right)^{1/2} \text{ for } i=2,\ldots,N\]

\[u_{ij} = \left(\frac{c_{ij}}{\sum_{k=1}^{i-1} u_{ki} u_{kj}}\right)/u_{ii} \text{ for } \begin{cases} j=i+1,\ldots,N \\ i=2,\ldots,N \end{cases}\]

and

\[u_{ij} = 0 \text{ if } i > j.\]

The value of this method lies in the reduction of storage space required for a given \(N\). With the usual Crout method \(N^2\) storage locations are required, but the square root method requires \(N(N+1)/2\) storage locations since only the upper triangular portion of \([C]\) need be stored and \([U]\) can be found using only the upper triangular part of \([C]\).

A small trick is required if this saving is to be realized in practice, since in FORTRAN IV the use of the dimension statement "COMPLEX C(N,N)" would set aside \(N^2\) complex storage locations for
the elements of [C] even if only the upper triangular part of [C] were to be filled in and manipulated. To economize on storage a way was found to load the elements of the upper triangular part of [C] into a linear array \(N(N+1)/2\) positions long. It was convenient to preserve the double subscript notation for the matrix manipulations and use a simple formula to access the proper location in the singly subscripted linear array. A symmetric matrix [C] is shown in Fig. 43 with the elements of the linear array \(S\) inserted into the corresponding locations of [C]. The order of the matrix is chosen to be 6 for this example.

\[
\begin{array}{cccccc}
S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\
S_7 & S_8 & S_9 & S_{10} & S_{11} & \\
S_{12} & S_{13} & S_{14} & S_{15} & \\
S_{16} & S_{17} & S_{18} & \\
S_{19} & S_{20} & \\
S_{21} \\
\end{array}
\]

Fig. 43. --Storing a symmetric matrix in a linear array.

Element \(c_{11}\) is stored in position \(s_1\), \(a_{12}\) in \(c_2\), etc. The element \(c_{ij}\) \((i \leq j)\) can be accessed in the following way. The rows above the \(i\)-th row contain \(N(i-1) - ((i-1)(i-2)/2)\) elements and in the \(i\)-th row there are \(j - i+1\) elements up to and including the one to be accessed, hence
\[c_{ij} = s(N(i-1) - \frac{(i-1)(i-2)}{2} + j - i + 1) = (sN \cdot i - \left(\frac{i(i-1)}{2} + N - j\right)).\]

In the programs the subscript manipulations are performed directly in the subscript or accessed by calling a function named ISUB(i,j) [Integer Subscript corresponding to $i,j$]. If, for example, $c_{15}$ were needed in a computation the element $s(ISUB(1,5))$ is used. Once the factorization is completed, the back substitutions are performed.

Notice that in either the Crout method or the square root method there are two distinct steps. The first is factoring the matrix and the second is the back substitution. The first step is independent of the driving column $[B]$ and hence need be done only once for any given matrix $[C]$ so any number of driving columns may be considered without re-factorizing $[C]$. 
REFERENCES


