OPTIMUM RUNWAY ORIENTATION RELATIVE TO CROSSWINDS

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Specific magnitudes of crosswinds may exist that could be constraints to the success of an aircraft mission such as the landing of the proposed space shuttle. A method is required to determine the orientation or azimuth of the proposed runway which will minimize the probability of certain critical crosswinds. Two procedures for obtaining the optimum runway orientation relative to minimizing a specified crosswind speed are described and illustrated with examples. The empirical procedure requires only hand calculations on an ordinary wind rose. The theoretical method utilizes wind statistics computed after the bivariate normal elliptical distribution is applied to a data sample of component winds. This method requires only the assumption that the wind components are bivariate normally distributed. This assumption seems to be reasonable. Studies are currently in progress for testing wind components for bivariate normality for various stations. The close agreement between the theoretical and empirical results for the example chosen substantiates the bivariate normal assumption.
TABLE OF CONTENTS

INTRODUCTION ........................................... 1

EMPIRICAL METHOD ........................................ 1

THEORETICAL METHOD ..................................... 6

CONCLUSIONS ............................................... 9

REFERENCES ................................................ 11

BIBLIOGRAPHY ............................................. 11

APPENDIX. COMPUTER PROGRAM FOR OPTIMUM RUNWAY
ORIENTATION RELATIVE TO CROSSWINDS ............. 12

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OPTIMUM RUNWAY ORIENTATION
RELATIVE TO CROSSWINDS

INTRODUCTION

Runway orientation is undoubtedly influenced by a number of factors—perhaps winds, terrain features, population interference, etc. In some cases, the frequency of occurrence of crosswind components of some significant speed may have received insufficient consideration. If, for example, a runway for the space shuttle vehicles is being planned, it may be prudent to consider the optimum runway orientation to minimize crosswind components of, for example, 20 knots. Aligning the runway with the prevailing wind will not ensure that crosswinds of this magnitude will be minimized. In fact, two common synoptic situations (one producing light easterly winds, and the other causing strong northerly winds) might exist in such a relationship that a runway oriented with the prevailing wind might be the least useful to an aircraft constrained by crosswind components ≥ 20 knots. Two methods (one empirical, the other theoretical) of determining the optimum runway orientation to minimize critical crosswind component speeds are described below. Both methods gave identical results for the Cape Kennedy, Florida, winds shown in Table 1.

EMPIRICAL METHOD

The following paragraphs outline a short procedure (one requiring only a desk calculator and a wind rose) for determining the best runway orientation relative to some specified crosswind component.

From the ordinary wind rose (Table 1), the percentage frequency of a number of wind speeds and directions can be obtained. For this procedure, the percentage frequency or number of cases in each class interval or "box" is assumed to be located at the class mark.

It is apparent that, if two of the reported wind directions (opposites) are chosen as a runway orientation, the crosswind component contributed by each box can be obtained from the product of the wind speed and the sine
TABLE 1. ANNUAL WIND ROSE AND STATISTICAL SUMMARY FOR CAPE KENNEDY.

<table>
<thead>
<tr>
<th>Speed Direction</th>
<th>1-9</th>
<th>10-19</th>
<th>20-29</th>
<th>30-39</th>
<th>40-49</th>
<th>50-59</th>
<th>60-74</th>
<th>Total All Obs.</th>
<th>Speed (Knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1605</td>
<td>2825</td>
<td>1339</td>
<td>73</td>
<td>3</td>
<td></td>
<td></td>
<td>5845</td>
<td>83239</td>
</tr>
<tr>
<td>NNE</td>
<td>1318</td>
<td>2323</td>
<td>446</td>
<td>44</td>
<td>2</td>
<td></td>
<td></td>
<td>4133</td>
<td>51145</td>
</tr>
<tr>
<td>NE</td>
<td>1271</td>
<td>2180</td>
<td>351</td>
<td>49</td>
<td>11</td>
<td></td>
<td></td>
<td>3862</td>
<td>47456</td>
</tr>
<tr>
<td>ENE</td>
<td>1708</td>
<td>3109</td>
<td>603</td>
<td>54</td>
<td>9</td>
<td></td>
<td></td>
<td>5483</td>
<td>68326</td>
</tr>
<tr>
<td>E</td>
<td>2342</td>
<td>4264</td>
<td>601</td>
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<td>5</td>
<td>1</td>
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<td>88086</td>
</tr>
<tr>
<td>ESE</td>
<td>2882</td>
<td>4879</td>
<td>598</td>
<td>29</td>
<td>11</td>
<td>2</td>
<td></td>
<td>8401</td>
<td>99428</td>
</tr>
<tr>
<td>SE</td>
<td>2637</td>
<td>3841</td>
<td>691</td>
<td>16</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>7212</td>
<td>85520</td>
</tr>
<tr>
<td>SSE</td>
<td>2203</td>
<td>3165</td>
<td>625</td>
<td>23</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>6023</td>
<td>71873</td>
</tr>
<tr>
<td>S</td>
<td>2313</td>
<td>2549</td>
<td>436</td>
<td>21</td>
<td>6</td>
<td>2</td>
<td></td>
<td>5332</td>
<td>59099</td>
</tr>
<tr>
<td>SSW</td>
<td>2197</td>
<td>1605</td>
<td>364</td>
<td>59</td>
<td>9</td>
<td>1</td>
<td></td>
<td>4235</td>
<td>45454</td>
</tr>
<tr>
<td>SW</td>
<td>1777</td>
<td>1548</td>
<td>288</td>
<td>39</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>3661</td>
<td>39733</td>
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<tr>
<td>WNW</td>
<td>1510</td>
<td>1527</td>
<td>514</td>
<td>50</td>
<td>2</td>
<td></td>
<td></td>
<td>3603</td>
<td>43621</td>
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<tr>
<td>NW</td>
<td>2428</td>
<td>2866</td>
<td>688</td>
<td>62</td>
<td>4</td>
<td></td>
<td></td>
<td>6048</td>
<td>72363</td>
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<tr>
<td>NNW</td>
<td>1949</td>
<td>2216</td>
<td>1111</td>
<td>101</td>
<td>3</td>
<td></td>
<td></td>
<td>5380</td>
<td>71943</td>
</tr>
<tr>
<td>Calm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>854</td>
<td>1000280</td>
</tr>
<tr>
<td>Totals</td>
<td>31174</td>
<td>41539</td>
<td>9279</td>
<td>770</td>
<td>91</td>
<td>16</td>
<td>6</td>
<td>83729</td>
<td></td>
</tr>
<tr>
<td>Percent</td>
<td>37.2</td>
<td>49.6</td>
<td>11.1</td>
<td>0.9</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>Mean Speed (Knots) By Group</td>
<td>6.1</td>
<td>13.8</td>
<td>22.6</td>
<td>32.6</td>
<td>42.5</td>
<td>52.4</td>
<td>63.5</td>
<td></td>
<td></td>
</tr>
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</table>
TABLE 1. (Concluded)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$V_r$</th>
<th>$S_v$</th>
<th>$\Sigma x_1 x_2$</th>
<th>$\Sigma v^2$</th>
<th>$\Sigma s_{x_1}^2$</th>
<th>$S_{x_1}/S_{x_2}$</th>
<th>$S_{v}/V_r$</th>
<th>$\sigma_a$</th>
<th>$\sigma_b$</th>
<th>$\sigma_s$</th>
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<tbody>
<tr>
<td>83</td>
<td>1.787</td>
<td>13.359</td>
<td>-1 048 423.600</td>
<td>15 210 144.000</td>
<td>-5.441</td>
<td>7.47</td>
<td>10.136</td>
<td>8.702</td>
<td>6.309</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>$\bar{x}_1$</td>
<td>$S_{x_1}$</td>
<td>Sum of E Components ($\Sigma x_1$)</td>
<td>$\Sigma x_1^2$</td>
<td>$nS_{x_1}/S_{x_1}$</td>
<td>$S_{x_1}/\sigma_s$</td>
<td>$r_{x_1 x_2}$</td>
<td>e</td>
<td></td>
<td></td>
</tr>
<tr>
<td>83 729</td>
<td>-1.775</td>
<td>9.230</td>
<td>-148 628.750</td>
<td>7 396 563.000</td>
<td>-5.200</td>
<td>2.12</td>
<td>-0.145</td>
<td>1.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\bar{x}_2$</td>
<td>$S_{x_2}$</td>
<td>Sum of N Components ($\Sigma x_2$)</td>
<td>$\Sigma x_2^2$</td>
<td>$nS_{x_2}/S_{x_2}$</td>
<td>$V_r\bar{v}$</td>
<td>$\Sigma v$</td>
<td>$\bar{v}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>126</td>
<td>-0.210</td>
<td>9.658</td>
<td>-17 563.027</td>
<td>7 813 581.000</td>
<td>-46.043</td>
<td>0.15</td>
<td>1 000 280</td>
<td>11.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

$S_{x_1} = \text{Standard deviation of east components.}$
$S_{x_2} = \text{Standard deviation of north components.}$
$S_v = \text{Standard vector deviation of wind velocity.}$
$r_{x_1 x_2} = \text{Correlation coefficient of north and east components.}$
$\bar{v} = \text{Average wind speed.}$
$V = \text{Scalar wind speed.}$
$\sigma_a = \text{Standard deviation of wind components along the major axis of the distribution.}$
$\sigma_b = \text{Standard deviation of wind components perpendicular to the major axis of the distribution.}$
$\psi = \text{Angle of rotation of the major axis of the wind distribution counterclockwise from EW direction.}$
$\theta = \text{Resultant wind direction.}$
$V_r = \text{Resultant wind speed.}$
$\sigma_s = \text{Standard deviation of wind speeds.}$
$e = \frac{\sigma_a}{\sigma_b}.$
of the angle between the wind direction and the runway orientation. Since wind directions are usually given to 16 points, the angles between the wind direction and the runway will be 22.5, 45, 67.5, and 90 deg. For example, if the runway orientation is chosen as EW, then all ENE, ESE, WNW, and WSW winds will be 22.5 deg off the runway. Likewise, NE, SE, NW, and SW will be 45 deg off, NNE, SSE, NNW, and SSW will be 67.5 deg off, and N and S will be 90 deg off. From Figure 1 it is apparent that each wind direction and wind speed will make a contribution to the total percent frequency of crosswind components. Since several wind directions in the same wind speed category

Figure 1. Runway crosswind component calculation.
will produce the same crosswind component, these boxes should be summed. For example, under the 10-19 category, sum NE, SE, NW, and SW (2180 + 3841 + 2866 + 1548 = 10 435) to obtain the frequency of crosswind components of 10.3 knots (14.5 sin 45 deg = 10.3). This process should be continued until all boxes except E-W have been used to compute a crosswind component.

In summary, the crosswind component computation procedure consists of the following steps:

1. Compute all possible crosswind components—the product of each wind speed and the sine of 22.5, 45, 67.5, and 90 deg.

2. Sum all boxes that contribute the same crosswind component for a specified runway orientation. Compute the frequency and percent frequency for each crosswind speed.

3. Order the crosswind component speeds from the largest to the smallest and tabulate the percent frequency of occurrence opposite each crosswind component.

4. Form the cumulative percentage frequency (CPF) from the values tabulated in step 3., starting with the highest wind speed. This CPF gives a description of the crosswind components for a single runway.

5. Interpolate the CPF for the desired wind speed. This interpolated value gives the probability in percent of equaling or exceeding the specified crosswind component for that runway orientation. From the few cases examined, it appears that the interpolation should be made assuming a normal distribution.

Of course, the procedure must be repeated for each pair of opposite wind directions (runway orientations) to determine the optimum runway orientation relative to a critical crosswind component.

This procedure was applied to the Cape Kennedy annual wind rose shown in Table 1, except that hurricane associated winds ≥ 50 knots were removed on the premise that landing operations would not be conducted during such periods. (Table 1 was prepared from hourly peak wind measurements made from September 1958 through June 1969.) Results of the analysis (Fig. 2), for which 20 knots was selected as the critical crosswind speed, indicate that the best runway orientation relative to speeds of this magnitude is about 150 to 330 deg true.
THEORETICAL METHOD

For Cape Kennedy, the assumption has been made that wind components are bivariate normally distributed; i.e., a vector wind data sample is resolved into wind components in a rectangular coordinate system and the bivariate normal elliptical distribution is applied to the data sample of component winds. For example, let $x_1$ and $x_2$ be normally distributed variables (the wind components at Cape Kennedy) with parameters $(\xi_1, \sigma_1)$ and $(\xi_2, \sigma_2)$. $\xi_1$ and $\xi_2$ are the respective means, while $\sigma_1$ and $\sigma_2$ are the respective standard deviations. Let $\rho$ be the correlation coefficient, which is a measure of the dependence between $x_1$ and $x_2$. Now, the equation of the bivariate normal density function is

\[
p(x_1, x_2) = \left[2\pi\sigma_1 \sigma_2 (1 - \rho^2)^{1/2}\right]^{-1} \exp\left\{ -\frac{1}{2(1 - \rho^2)} \left[ \left(\frac{x_1 - \xi_1}{\sigma_1}\right)^2 - 2\rho \left(\frac{x_1 - \xi_1}{\sigma_1}\right) \left(\frac{x_2 - \xi_2}{\sigma_2}\right) + \left(\frac{x_2 - \xi_2}{\sigma_2}\right)^2 \right] \right\}. \tag{1}
\]

Let $\alpha$ be any arbitrary angle in the rectangular coordinate system and let the sample estimates of $\xi_1$, $\xi_2$, $\sigma_1$, $\sigma_2$, and $\rho$ be denoted by $\bar{X}_1$, $\bar{X}_2$, $S_{x_1}$, $S_{x_2}$, and $r_{x_1x_2}$, respectively. From the statistics in the $(x_1, x_2)$ space, the statistics for any rotation of the axes of the bivariate normal distribution through any arbitrary angle $\alpha$ may be computed as given below [1].
Let \((y_1, y_2)\) be the new space after rotation with orthogonal axes \(y_1\) and \(y_2\). Let \(\bar{Y}_1, Y_2, S_{y_1}, S_{y_2}, \text{ and } r_{y_1y_2}\) be the means, standard deviations, and correlation coefficient in the \((y_1, y_2)\) space. From Reference 1,

\[
\begin{align*}
\bar{Y}_1 &= \bar{X}_1 \cos \alpha + \bar{X}_2 \sin \alpha \\
\bar{Y}_2 &= \bar{X}_2 \cos \alpha - \bar{X}_1 \sin \alpha \\
S_{y_1} &= \left( S^2_{x_1} \cos^2 \alpha + S^2_{x_2} \sin^2 \alpha + 2S_{x_1x_2} \cos \alpha \sin \alpha \right)^{1/2} \\
S_{y_2} &= \left( S^2_{x_2} \cos^2 \alpha + S^2_{x_1} \sin^2 \alpha - 2S_{x_1x_2} \cos \alpha \sin \alpha \right)^{1/2} \\
r_{y_1y_2} &= \frac{S_{y_1y_2}}{S_{y_1} S_{y_2}} ,
\end{align*}
\]

where \(S^2_{x_1}, S^2_{x_2}, \text{ and } S_{x_1x_2}\) are the variances and covariance in the \((x_1, x_2)\) space and \(S_{y_1y_2}\) is the covariance in the \((y_1, y_2)\) space.

In equations (2), all statistics on the right side are available for Cape Kennedy or may be computed from existing data (Table 1).

Equations (2) give the statistics in the \((y_1, y_2)\) space (this is the space after rotation through any angle \(\alpha\)) that defines the bivariate probability density function (1).

The existing wind statistics for Cape Kennedy (Table 1) are for the reference angle \(\alpha = 90\) deg; i.e., the \((x_1, x_2)\) space is for \(\alpha = 90\) deg.

Let \(\Delta \alpha\) denote the desired increments for which runway orientation accuracy is required; e.g., one may wish to minimize the probability of cross-winds with a runway orientation accuracy down to \(\Delta \alpha = 10\) deg. This means we must rotate the bivariate normal axes through every 10 deg beginning at \(\alpha = 90\) deg [the \((x_1, x_2)\) space]. It is only necessary to rotate the bivariate normal surface through 180 deg, since the distribution is symmetric in the
other two quadrants. This process will result in 18 sets of statistics in the 
\((y_1, y_2)\) space obtained from equations (2). \(y_1\) is the head wind component 
while \(y_2\) is the crosswind component. Since we are concerned with minimizing 
the probability of crosswinds \((y_2)\) only, we now examine the marginal distribu-
tions \(p(y_2)\) for the 18 orientations \((\alpha)\). Since \(p(y_1, y_2)\) is bivariate normal, 
18 marginal distributions \(p(y_2)\) must be univariate normal:

\[
p(y_2) = \left[\sigma_2 (2\pi)^{-1/2}\right]^{-1} \exp \left\{ -\frac{1}{2} \left[ (y_2 - \xi_2) / \sigma_2 \right]^2 \right\} . \quad (3)
\]

\(\xi_2\) and \(\sigma_2\) are replaced by their sample estimates \(\bar{Y}_2\) and \(S_{y_2}\). Now, let

\[
z = \frac{Y_2 - \bar{Y}_2}{S_{y_2}}, \quad (4)
\]

where \(y_2\) is the critical crosswind of interest. \(z\) is a standard normal variable 
and the probability of its exceedance is easily calculated from tables of the 
standard normal integral. Since a right or left crosswind \((y_2)\) is a constraint 
to an aircraft, the critical region (exceedance region) for the normal distribu-
tion is two-tailed; i.e., we are interested in twice the probability of exceeding 
\(|y_2|\). Let this probability of exceedance or risk = \(R\).

A computer program is available for computing the statistics in the 
\((y_1, y_2)\) space defined by equations (2), the integral of equation (3), and the 
risk \(R\) (see Appendix A). The Cape Kennedy data given in Table 1 are used as 
an example. The input to the program is \(\bar{X}_1 = -1.775, \bar{X}_2 = -0.210, \)
\(S_{x_1} = 9.230, S_{x_2} = 9.658, r_{x_1x_2} = -0.145,\) and \(\Delta \alpha = 10\) deg. Let the crit-
ical crosswind component \(x_2 = 20\) knots as in the empirical example in the 
preceding section. To obtain a valid comparison to the empirical probability, 
we will compute \(R = \) the probability of exceeding or equalling \(x_2 = 20\) knots. 
Table 2 summarizes the runway orientation angles \(\alpha\), the standard normal 
variate \(z\), and the risk \(R\) for Cape Kennedy.

For this example, the risk \(R = 5.63\) percent of exceeding or equalling 
a 20-knot crosswind is a minimum occurring at the runway orientation 
\(\alpha = 150 - 330\) deg. This theoretical result verifies the empirical conclusion 
arrived at in the previous section.
TABLE 2. SUMMARY OF RUNWAY ORIENTATION ($\alpha$) VERSUS RISK (R), CAPE KENNEDY.

<table>
<thead>
<tr>
<th>$\alpha$ (deg)</th>
<th>$z$</th>
<th>$R$ (%)</th>
<th>$\alpha$ (deg)</th>
<th>$z$</th>
<th>$R$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1.57</td>
<td>11.53</td>
<td>180</td>
<td>1.82</td>
<td>6.91</td>
</tr>
<tr>
<td>100</td>
<td>1.65</td>
<td>9.93</td>
<td>190</td>
<td>1.76</td>
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<td>110</td>
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<td>200</td>
<td>1.71</td>
<td>8.77</td>
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<td>120</td>
<td>1.80</td>
<td>7.25</td>
<td>210</td>
<td>1.66</td>
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<td>130</td>
<td>1.86</td>
<td>6.36</td>
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<td>1.61</td>
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<tr>
<td>140</td>
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<tr>
<td>150</td>
<td>1.91</td>
<td>5.63</td>
<td>240</td>
<td>1.55</td>
<td>12.07</td>
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<tr>
<td>160</td>
<td>1.90</td>
<td>5.78</td>
<td>250</td>
<td>1.54</td>
<td>12.45</td>
</tr>
<tr>
<td>170</td>
<td>1.86</td>
<td>6.22</td>
<td>260</td>
<td>1.53</td>
<td>12.61</td>
</tr>
</tbody>
</table>

The method described to determine the orientation of a runway which will minimize the probability of critical crosswinds is accurate and expedient. The procedure described may be used for any station. Only parameters estimated from the data are required as input. Consequently, many runways and locations may be examined rapidly.

CONCLUSIONS

Either the empirical or theoretical method described in this report may be used to determine an aircraft runway orientation that minimizes the probability of critical crosswinds. Again, it is emphasized that the wind components must be bivariate normally distributed to use the theoretical method. The Cape Kennedy wind component raw data were not available for the example used in this report. The bivariate normal assumption may or may not be a good assumption. However, the Cape Kennedy sample illustrates
the method. The agreement between the theoretical and empirical methods is very good. As shown by Figure 2 and Table 2, the minimum and maximum probabilities occur at the same azimuths for both methods. The minimum risk of 5.63 percent for the theoretical method occurs at 150 deg. The minimum risk for the empirical approach also occurs at 150 deg as shown by Figure 2. The empirical risk is approximately 4.7 percent. The maximum risks for the two methods also occur at approximately the same orientation. The maximum theoretical risk of 12.61 percent occurs at 260 deg, while the maximum empirical risk of about 7.6 percent occurs at 250 deg. One may view the differences in the theoretical and empirical probabilities as a measure of the departure of the data from normality.

In practical applications, the following steps are suggested:

1. Test the component wind samples for bivariate normality if these samples are available. See Reference 2 for bivariate normal goodness-of-fit tests.¹

2. If the component winds are available and cannot be rejected as bivariate normal using the bivariate normal goodness-of-fit test, use the theoretical method since it is more expedient and easily programmed.

3. If the component wind data samples are not available and there is doubt concerning the assumption of bivariate normality of the wind components, use the empirical method.

REFERENCES


BIBLIOGRAPHY


APPENDIX. COMPUTER PROGRAM FOR OPTIMUM RUNWAY ORIENTATION RELATIVE TO CROSSWINDS

Legend

$\alpha$  
Runway orientation

$\alpha (R)$  
Orientation or angle at which the wind component statistics are computed from the data sample

$\bar{X}_1$  
Mean of the head-tail winds at orientation $\alpha (R)$

$\bar{X}_2$  
Mean of the crosswinds at orientation $\alpha (R)$

$S_{X_1}$  
Standard deviation of head-tail winds at $\alpha (R)$

$S_{X_2}$  
Standard deviation of crosswinds at $\alpha (R)$

$S_{X_1 X_2}$  
Covariance at $\alpha (R)$

$R_{X_1 X_2}$  
Correlation coefficient at $\alpha (R)$

$\bar{Y}_1$  
Mean of the head-tail winds at orientation $\alpha$

$\bar{Y}_2$  
Mean of the crosswinds at orientation $\alpha$

$S^2_{Y_1}$  
Variance of the head-tail winds at $\alpha$

$S_{Y_1}$  
Standard deviation of the head-tail winds at $\alpha$

$S^2_{Y_2}$  
Variance of the crosswinds at $\alpha$

$S_{Y_2}$  
Standard deviation of the crosswinds at $\alpha$

$S_{Y_1 Y_2}$  
Covariance at orientation $\alpha$
**INPUT**

**SALP**  Starting $\alpha$

**ALPR**  $\alpha R$

**XB1**  $\bar{X}_1$

**XB2**  $\bar{X}_2$

**SX1**  $S_{x_1}$

**SX2**  $S_{x_2}$

**RX1X2**  $R_{x_1x_2}$

**DALP**  $\Delta \alpha$

**EALP**  End or Maximum $\alpha$

**NV**  No. of Y2

**Y2**  Table of Y2

The first Y2 is taken and the program is utilized with the starting $\alpha$. $\alpha$ is then increased by DALP and the program is utilized again. This is
repeated until EALP (maximum \( a \)) is reached. The next Y2 is taken and the process is repeated. This is continued until all of the Y2's have been used. A transfer is then made to STATEMENT to see if more data are to be read in. If not, the program terminates. Usually Y2 is the only data that vary.

Note: The STATEMENT following STATEMENT 10 is an end-of-file check. If there are no more data the program terminates. This end-of-file test may not be compatible with other computers.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812, June 23, 1972
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