

X-551-72-285

PREPRINT

NASA TM X-66002

**TROPOSPHERIC RANGE  
ERROR PARAMETERS:  
FURTHER STUDIES**

**H. S. HOPFIELD**

(NASA-TM-X-66002) TROPOSPHERIC RANGE ERROR  
PARAMETERS: FURTHER STUDIES H.S. Hopfield  
(NASA) Aug. 1972 48 p CSCL 03B

N72-30346

Unclas  
G3/13 39479

**AUGUST 1972**

**GSFC**

**GODDARD SPACE FLIGHT CENTER  
GREENBELT, MARYLAND**

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H. S. Hopfield

August 1972

GODDARD SPACE FLIGHT CENTER  
Greenbelt, Maryland

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ABSTRACT

Improved parameters are presented for predicting the tropospheric effect on electromagnetic range measurements from surface meteorological data. More geographic locations have been added to the earlier list. Parameters are given for computing the dry component of the zenith radio range effect from surface pressure alone with an rms error of 1 to 2 mm, or the total range effect from the dry and wet components of the surface refractivity,  $N$ , and a two-part quartic profile model. ( $N \cong 10^6(n - 1)$ , where  $n$  is the index of refraction.) The new parameters are obtained, as before, from meteorological balloon data but with improved procedures, including the conversion of the "geopotential heights" of the balloon data to actual or "geometric" heights before using the data. The revised values of the parameter  $k$  (dry component of vertical radio range effect per unit pressure at the surface) show more latitude variation than is accounted for by the variation of  $g$ , the acceleration of gravity. This excess variation of  $k$  indicates a small latitude variation in the mean "molecular weight" of air and yields information about the latitude-varying water vapor content of air. Statistical values of the height integrals of both dry and wet components of  $N$  are given for all the locations studied. Some comparisons of the values of these integrals are shown for stations a few hundred miles apart.

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## ACKNOWLEDGMENTS

The meteorological balloon data used in this work were supplied by the U.S. National Climatic Center at Asheville, N. C. The assistance of H. K. Utterback of APL, who prepared the computing programs and handled the computations, is gratefully acknowledged.

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## 1. INTRODUCTION AND BACKGROUND

The lower, nonionized atmosphere (troposphere plus stratosphere) retards the passage of electromagnetic signals and thus introduces an error into any electromagnetic measurement of range or range rate. The observed range is the line integral, along the signal path, of the index of refraction,  $n$ , which at sea level differs from unity by 3 or 4 parts in  $10^4$  and which varies along the path. In an atmosphere that is radially symmetrical about the earth (no horizontal gradients of  $n$ ), a signal propagated vertically undergoes no path bending, and the observed range is  $\int n \, dh$ , where  $h$  is the height. The range error caused by refraction is then  $\int (n - 1) \, dh$ .

Range errors for other directions of propagation are related to the error at the zenith, but at low angles they depend on the profile shape also. At any angle, however, knowledge of the effect at the zenith is one of the requirements for predicting the correction for measurements extending through the atmosphere (e.g., to earth satellites and more distant objects). Much of the work reported here deals with predicting the correction at the zenith on the basis of local conditions at the earth's surface at the time of the range measurement. This work is a continuation of a long-term study.

The effect of the nonionized atmosphere is produced mostly by the troposphere proper. For brevity, the term "tropospheric range error" will be used here to include the stratosphere also, unless otherwise indicated.

The refractivity,  $N$ , of air (defined as  $10^6(n - 1)$ ) can be conveniently expressed as the sum of two parts, the so-called "dry" and "wet" components, here subscripted "d" and "w." In the radio range (up to at least 15 GHz), both components are significant and are independent of frequency. For optical signals, on the other hand, essentially the whole effect is due to the dry part, and there

is some wavelength dependence. The values of  $N_d$  computed from the Smith and Weintraub radio expression (Ref. 1) and from the Edlén optical expression (Ref. 2) are in very good agreement in the near-infrared region (wavelengths of a few micrometers). Values in the visible region differ from these by a few percent. The Smith and Weintraub expressions for the radio region will be used here. The work on the dry component, as reported below, can be applied to both wavelength regions, with a wavelength correction of a few percent for optical uses, or the addition of the wet component effect for radio uses.

The Smith and Weintraub expressions (Ref. 1) for the radio refractivity of air may be written:

$$\left. \begin{aligned} N_d &= \frac{77.6P}{T} \\ N_w &= \frac{3.73 \times 10^5 e}{T^2} \\ N &= N_d + N_w \end{aligned} \right\} \quad (1)$$

where  $T$  is the Kelvin temperature,  $P$  is the total pressure, and  $e$  is the partial pressure of water vapor, the latter two in millibars.

The refractivity components,  $N_d$  and  $N_w$ , decrease with height above the surface of the earth, but at different rates. Theoretically  $N_d$  would decrease exponentially with height in an isothermal atmosphere. Exponential expressions frequently have been used to approximate the  $N$  profile, both the total  $N$  and the separate components (Refs. 3 and 4).

It was previously shown (Ref. 5) that, in a dry atmosphere, if the air temperature varies linearly with height (constant lapse rate  $\alpha$ , where  $\alpha \equiv -dT/dh$ ), the  $N$  profile is theoretically a polynomial function of height, not exponential. The smaller the lapse rate, the higher

the degree of the polynomial (approaching an exponential as  $\alpha \rightarrow 0$ ). If heights are measured above the surface, the general form of the equation is:

$$N = N_s \left( \frac{h_d - h}{h_d} \right)^\mu, \quad h \leq h_d \quad (2)$$

where

$$h_d = \frac{T_s}{\alpha}, \quad (3)$$

$$\mu = \frac{g}{R\alpha} - 1, \quad (4)$$

and the subscript  $s$  refers to surface values. In Eq. (4)  $g$  is the acceleration of gravity and  $R$  is the gas constant per gram of dry air.

If  $\alpha = 6.8^\circ\text{C}/\text{km}$ , which is near its usual value in the troposphere, then  $\mu = 4$  for the  $N_d$  profile. Using the same form for the  $N_w$  profile, for convenience, the two-part expression becomes (Ref. 5):

$$\left. \begin{aligned} N_d &= \frac{N_{d_s}}{h_d^4} (h_d - h)^4, \quad h \leq h_d \\ N_w &= \frac{N_{w_s}}{h_w^4} (h_w - h)^4, \quad h \leq h_w \end{aligned} \right\} \quad (5)$$

It was also shown previously (Ref. 6), both theoretically and from observed data, that although  $N_d$  at any point is a function of both pressure and temperature, its zenith integral is a linear function of surface pressure

only. The tropospheric contribution to a vertical range measurement (electromagnetic) through the atmosphere is:

$$\Delta h_{\text{tro}_d} = 10^{-6} \int N_d dh = kP_s, \quad (6)$$

where  $k$  is a constant for a given location.

To reconcile Eqs. (5) and (6), it is necessary (Ref. 6) that

$$h_d = a_d T_s, \quad (7)$$

where  $h_d$  is height above the surface,  $a_d$  is a constant, and the surface temperature,  $T_s$ , is in degrees Kelvin. This linear variation is also implied in Eq. (3); thus theoretically:

$$a_d = \frac{1}{\alpha}. \quad (8)$$

There is yet no comparable expression for  $h_w$ .

The tropospheric contribution to a vertical range measurement on the basis of Eqs. (5) and (7) then becomes:

$$\left. \begin{aligned} \Delta h_{\text{tro}} &= \Delta h_{\text{tro}_d} + \Delta h_{\text{tro}_w} \\ \Delta h_{\text{tro}_d} &= 10^{-6} \int_0^{h_d} N_d dh = 10^{-6} \cdot \frac{1}{5} N_{d_s} a_d T_s \\ \Delta h_{\text{tro}_w} &= 10^{-6} \int_0^w N_w dh = 10^{-6} \cdot \frac{1}{5} N_w h_w \end{aligned} \right\} (9)$$

All the parameters for Eqs. (6) and (9) can be obtained by least squares procedures from observed zenith integrals of  $N$  that are obtained from balloon meteorological data. Preliminary values for several geographical locations were reported earlier (Refs. 6 and 7).

In the present study, the approximations made in the earlier work have been improved. Revised parameters are given for all the former stations and for some new ones. Further development of the theory leads to a new method of estimating the percent of water vapor in the atmosphere, and results are given. Some statistics on zenith range effects and some preliminary work on horizontal gradients are also presented.

## 2. THEORY

The theoretical expression for  $k$  of Eq. (6), derived earlier (Ref. 6), is independent of temperature or its lapse rate, and is:

$$k = \frac{77.6 R}{g} \times 10^{-9}, \quad (10)$$

where all quantities are in cgs units. The height variation of  $g$  within the nonionized atmosphere is small and may be neglected if a value of  $g$  at midatmosphere is used ( $\sim$  the 500-millibar level for a sea-level location). Since  $k$  is a function of  $g$ , its theoretical values show a latitude variation.

In order to deduce the value of  $k$  from observed data, reliable values of  $\int N_d dh$  are needed, to be used in Eq. (6). The following improvements have been made in the earlier assumptions and computing procedures for determining  $\int N_d dh$ :

1. Initially, the height integrals of  $N$  used the "geopotential heights" of the balloon data. These are essentially determinations of geopotential differences from meteorological data. Heights in geopotential meters are numerically equivalent to heights in real or geometric meters only when  $g = 980 \text{ cm/s}^2$ . A height of 10 "geopotential kilometers" is 10.036 km at the equator but 9.983 km at a pole. The initial "height" integrals were therefore an approximation to true or geometric height integrals, and although the initial integrals showed a latitude variation (Ref. 7), they did not show the theoretical amount. In the present work, geometric heights are derived from the "geopotential heights" of the balloon data (Ref. 8) and are used in the numerical integration.

2. The contribution of the unobserved top part of the atmosphere can now be approximated better than before, using the results of Refs. 6 and 7.

3. In the earlier numerical integration with height, a linear decrease of  $N_d$  was assumed between observed points. This would bias the integral slightly too high. An approximate correction has been developed on the basis of a nominal profile shape.

Details of these three improvements will be given in the Section "Data and Computations."

The empirical values of  $k$  are based on the real atmosphere, which contains a small percent of water vapor. If theoretical values are computed from Eq. (10) on the basis of the gas constant,  $R$ , per gram of dry air, a small discrepancy may be expected and may be used to provide a measure of the water vapor content of the atmosphere.

The gas constant,  $R$ , per gram of air (observed sample) is:

$$R = \frac{R^*}{M_{\text{obs}}}, \quad (11)$$

where  $M_{\text{obs}}$  is the mean "molecular weight" of the observed sample of air and  $R^*$  is the gas constant per mole. Combining Eqs. (10) and (11) and solving for  $M_{\text{obs}}$ , we obtain:

$$M_{\text{obs}} = \frac{10^{-9} \times 77.6 R^*}{gk}. \quad (12)$$

Thus a value of  $M_{\text{obs}}$  is obtained for each set of data that provides a value of  $k$ .

Now let:

$x$  = mean mole fraction of water vapor in the air sample,

$1 - x$  = mole fraction of dry air,

$M_w$  = molecular weight of water = 18.0160

$M_d$  = molecular weight of dry air = 28.966 (Ref. 8).

Then taking  $M_{obs}$  as a weighted mean of  $M_d$  and  $M_w$ :

$$M_w x + M_d(1 - x) = M_{obs} \quad , \quad (13)$$

and solving:

$$x = \frac{M_d - M_{obs}}{M_d - M_w} \quad . \quad (14)$$

Although the zenith integral of  $N_d$  can be predicted from surface pressure alone with Eq. (6), prediction at large zenith angles must also involve the temperature and its height variation. The quartic profile, which fits a temperature lapse rate of  $6.8^\circ\text{C}/\text{km}$  when  $g = 980 \text{ cm}/\text{s}^2$ , is suitable for most regions. For empirical study, the height,  $h_d$ , is written:

$$h_d = h_{d0} + a_d T_c \quad , \quad (15)$$

where  $h_{d0}$  is the "equivalent height" of the model at a surface temperature of  $0^\circ\text{C}$  and  $a_d$  is its temperature coefficient. For the wet component, there is as yet no comparable expression, and we assume that  $h_w$  is a constant for a

given data sample. Integrating Eqs. (5) with these values, these theoretical integrals are compared with the revised integrals obtained from the balloon data in order to get improved values of  $h_{d0}$ ,  $a_d$ , and  $h_w$  for each data set, by a least squares procedure. Prediction of the zenith integrals from these values will be reported below; its precision is independent of whether or not the shape of the model profile matches the observed profile well. This is not true at low elevation angles. The prediction errors to be expected at low angles have not yet been studied.

Equations (2), (3), and (4) would make it possible to write a set of mathematical expressions to correspond to any observed  $N_d$  profile. The profile would not generally be a continuous function but would consist of a sequence of curved segments, one segment for each air layer having the same lapse rate. The equations would have the form of Eq. (2) with a different value of  $\mu$  for each layer; they would be easy to write but not necessarily easy to handle, since  $\mu$  would generally not be an integer. The height parameter for each layer, however, would be deduced easily from the lapse rate in the layer and the temperature at the base of the layer. The agreement between Eqs. (3) and (4), and the empirical parameters found for Eq. (15), indicate that this procedure could give a close approximation to the observed  $N_d$  profile.

### 3. DATA AND COMPUTATIONS

The data were obtained from meteorological balloon ascents and were supplied by the U.S. National Climatic Center. The refractivity at each height of observation was computed from Eqs. (1), as in Ref. 6. The height integrals of  $N_d$  and  $N_w$  were then obtained by numerical integration. As before, data from a given balloon were used only if observations were complete up to the 30-millibar level or higher for pressure and temperature, and up to the 500-millibar level for water vapor.

The three improvements described below were made in computing the numerical integrals. The changes were made one at a time so that the individual effect of each could be noted.

1. The geometric heights were computed from the "geopotential heights" of the data, using the equations and parameters of Ref. 8. The procedure takes into account the ellipsoidal shape of the earth, its mean density distribution, centrifugal acceleration, and the inverse square law, but not local gravitational anomalies. The procedure is considered satisfactory even up to heights of several hundred kilometers. The heights involved in the balloon data are all less than 40 km. The resulting geometric, or true, height integrals differ from the earlier "geopotential height" integrals of Refs. 6 and 7 by latitude-dependent amounts up to 8 or 9 mm (out of a total integral of 2.3 meters). The change involves an increase near the equator and a decrease near the poles.

2. The new expression for the contribution of the unobserved top part of the atmosphere is:

$$\Delta h_{\text{tro(top)}} = 2.296 \times 10^{-3} P_{\text{top}} \text{ (meters),}$$

where  $P_{\text{top}}$  is the topmost observed pressure in millibars. This is intended to be used at heights not less than 23 km (30-millibar level), and the constant factor is based on a nominal value of  $g$  in that height region. This change produces a small but observable improvement in the precision of the least squares fit in getting parameters.

3. In the earlier numerical integration of  $N_d$  with height, it was assumed that the decrease of  $N_d$  is linear between successive observed heights. The resulting integrals all must be slightly too large. To improve this, it is here assumed that  $N_d$  decreases as a fourth-degree function up to a height of 5 km, and decreases exponentially above that (Appendix A). A small curvature correction is computed for each layer (height interval) and subtracted from the layer integral obtained by the earlier procedure. This improvement decreases the total integral  $\int N_d dh$  by approximately  $2.0 \pm 0.5$  mm. The larger of these curvature corrections occur, as expected, for profiles having fewer observed points. The rms fit of the derived parameters is slightly improved. This change has not been applied to the integration of  $N_w$ , where the observed values of  $N_w$  are not always monotonic with height.

All the data were in 1-year sets (two balloons per day, at 0<sup>h</sup> and 12<sup>h</sup> UT at all stations). Several data sets have been added to the list of Ref. 6. The results given here cover the complete set of stations used and come from the improved computing procedures.

## 4. RESULTS AND DISCUSSION

### COEFFICIENT $k$ FOR EQ. (6), AND INFERRED WATER VAPOR CONTENT

Equation (6) states a linear relation between  $\int N_d dh$  and the surface pressure. The coefficient,  $k$ , was determined for each 1-year set of data separately. The results for all the stations are listed in Table 1. The rms error in predicting the height integral from this value of  $k$  (also listed) is between 1 and 2 mm for each 1-year data set. Figure 1 shows representative samples of observed and predicted integrals,  $\int N_d dh$ , for comparison.

Figure 2 shows these observed values of  $k$  as a function of latitude; it also shows the theoretical  $k$  values for a dry atmosphere, calculated from Eq. (10), using the local value of  $g$  computed from geodetic parameters (Applied Physics Laboratory geodesy model No. 4.5) for the observed local height of the midpressure level. These calculated points do not fall on a smooth curve because of local field variations and the variety of station heights above the geoid (especially the latter). If the midatmosphere height were everywhere the same (e.g., 5, 6, or 7 km), and if the earth were an ellipsoid, one of the solid curves of Fig. 1 would represent the theoretical value of  $k$ . In fact, however, the usual height of the half-pressure level above a sea-level station is approximately 6 km near the equator and 5 km near a pole. If the station is much above sea level, the midpressure level is naturally higher also, about 1 km higher than would otherwise be expected if the station height is 1.5 km (see Albuquerque and Byrd Station).

The bias between observed and theoretical values of  $k$  for a dry atmosphere is conspicuous and is markedly dependent on latitude. As was discussed above, theory shows that this difference may be due to the moisture content of the real atmosphere. The average fractional content of water vapor molecules in the atmosphere, during

Table 1

Prediction of  $\int N_d dh$  from Surface Pressure:  $\int N_d dh = kP_s$ 

Station	Year	Latitude	Longitude	Height (meters)	k (meters/mb)	Prediction Error in $\int N_d dh$ , $\sigma$ (meters)
Weather Ship E	1963	35° N	48° W	10	0.002281504	0.0017736
Weather Ship E	1965	35° N	48° W	10	0.002281285	0.0016183
Weather Ship E	1967	35° N	48° W	10	0.002281130	0.0016839
Ascension Island	1967	7° 55' S	14° 24' W	79	0.002290524	0.0011532
Caribou, Maine	1967	46° 52' N	68° 01' W	191	0.002277725	0.0019329
Washington, D. C. (Dulles Airport)	1967	38° 59' N	77° 28' W	85	0.002280275	0.0020481
St. Cloud, Minn.	1967	45° 35' N	94° 11' W	318	0.002278233	0.0015620
Columbia, Mo.	1967	38° 58" N	92° 22' W	239	0.002280504	0.0019135
Albuquerque, New Mexico	1967	35° 03' N	106° 37' W	1620	0.002280765	0.0014815
El Paso, Texas	1967	31° 48' N	106° 24' W	1193	0.002282555	0.0016598
Vandenberg AFB, Cal.	1967	34° 44' N	120° 34' W	100	0.002280797	0.0015237
Pago Pago, Samoa	1967	14° 20' S	170° 43' W	5	0.002287643	0.0017371
Wake Island	1963	19° 17' N	166° 39' E	5	0.002286083	0.0015023
Wake Island	1965	19° 17' N	166° 39' E	5	0.002286238	0.0015791
Wake Island	1967	19° 17' N	166° 39' E	5	0.002286287	0.0017215
Majuro Island	1967	7° 05' N	171° 23' E	3	0.002289389	0.0015692
Point Barrow, Alaska	1967	71° 18' N	156° 47' W	8	0.002273335	0.0014273
Byrd Station, Antarctica	1967	80° 01' S	119° 32' W	1543	0.002272051	0.0011065

● OBSERVED  
○ PREDICTED FROM  $P_s$   
COLUMBIA, MISSOURI

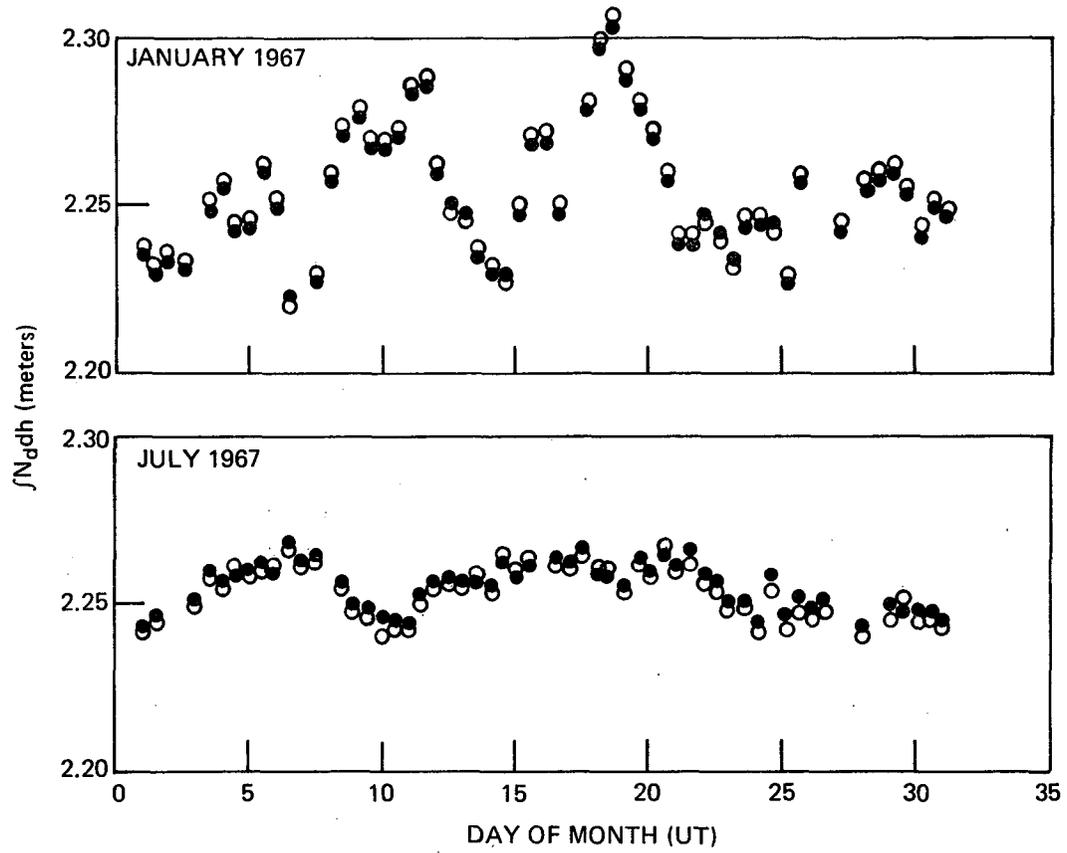
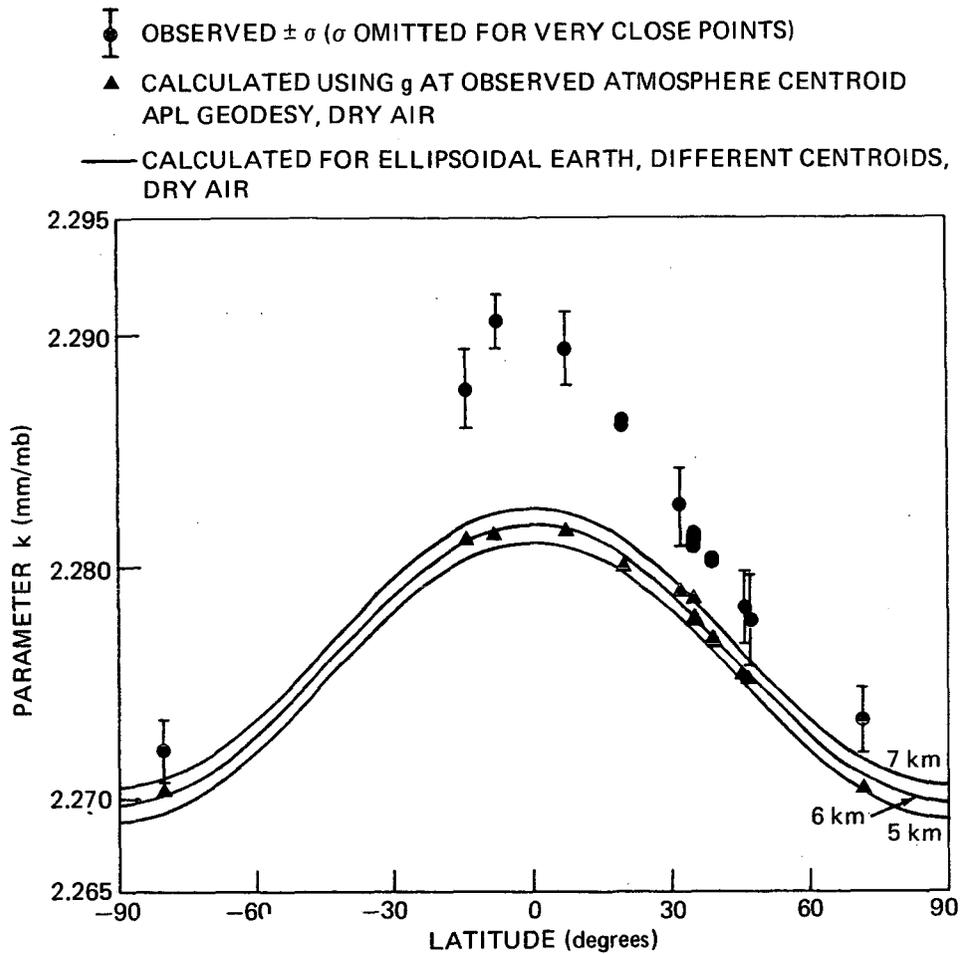


Fig. 1 OBSERVED AND PREDICTED VALUES OF  $fN_dh$  FOR COLUMBIA, MISSOURI, JANUARY AND JULY 1967



**Fig. 2 LATITUDE VARIATION OF TROPOSPHERIC RANGE  
 ERROR PARAMETER  $k$  (WHERE  $\int N_d dh = k P_s$ )**

the year, has been computed on this basis for each 1-year data set from Eqs. (12) and (14). The results are given in Table 2 and plotted in Fig. 3. The values must be considered approximate, since any data bias that leads to an error in  $M_{\text{obs}}$  will obviously have a large effect on the derived value of the water vapor content. The values, however, are of reasonable magnitude. Further evidence that they are at least qualitatively correct will be shown in a later figure.

### PARAMETERS FOR THE TWO-QUARTIC N MODEL

The revised computation procedures were also used to get revised height parameters  $h_{d_0}$ ,  $a_d$ , and  $h_w$  for the N model for all stations (Eq. (5)). These parameters are listed in Table 3.

The zenith integral,  $\int N_d dh$ , can be predicted from the model and from these parameters, at least as precisely as from the surface pressure and the  $k$  values, as will be seen from the residual errors of prediction in Tables 1 and 3.

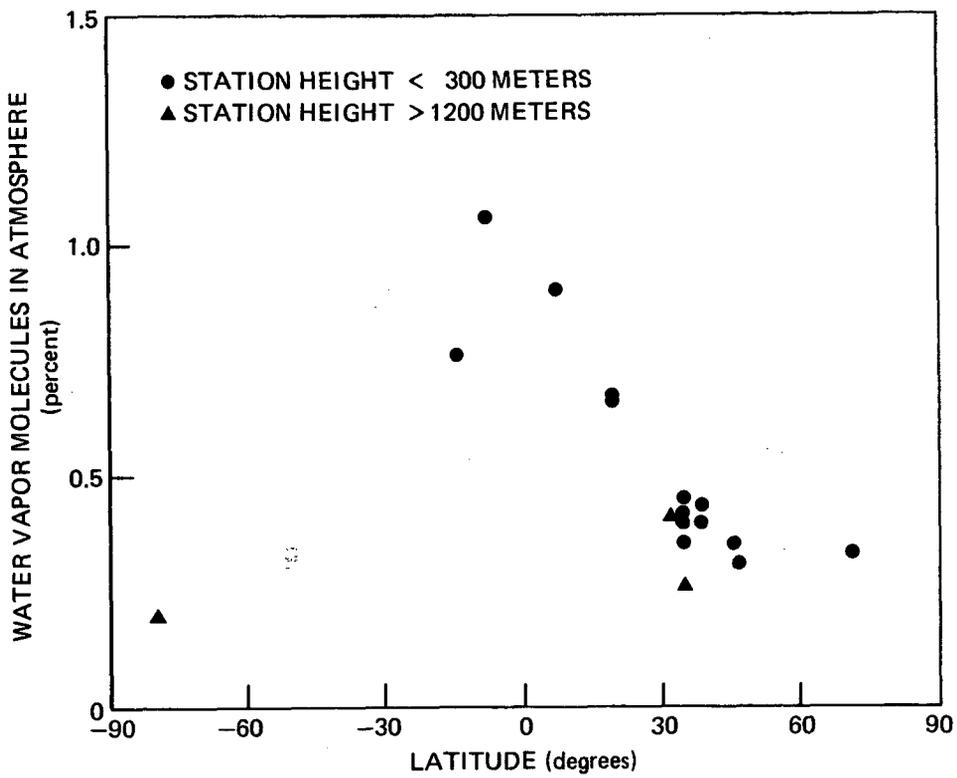
No detailed study has been made of the latitude effects and water vapor effects that might be expected in these "dry" parameters. However, some nominal theoretical values can be given for comparison with the mean values of Table 3.

Starting with Eq. (4), the use of a quartic  $N_d$  model implies a temperature lapse rate of  $6.814^\circ\text{C}/\text{km}$  if  $g = 978 \text{ cm}/\text{s}^2$  (midlatitude, midatmosphere value). From this, Eq. (3) gives  $h_{d_0} = 273.16/6.814$ , or  $40.088 \text{ km}$ , which is in good agreement with the values of Table 3. From Eq. (8),  $a_d = 0.14675 \text{ km}/^\circ\text{C}$ , whereas the mean of Table 3 is  $0.14872$ . Theoretical values based on dry air and nominal values of  $g$  can thus provide a good approximation to the observed parameters.

Table 2

Water Vapor Content,  $x$ , from  $k$  Values  
(Fraction of Total Air Molecules)

Station	Year	Height of Atmosphere Centroid (midpressure) (km)	$g$ at Centroid ( $\text{cm/s}^2$ )	Water Vapor Fraction, $x$
Weather Ship E	1963	5.83	977.95	0.0045
Weather Ship E	1965	5.83	977.95	0.0042
Weather Ship E	1967	5.83	977.95	0.0040
Ascension Island	1967	5.85	976.36	0.0106
Caribou, Maine	1967	5.52	979.06	0.0031
Washington, D. C.	1967	5.68	978.30	0.0040
St. Cloud, Minn.	1967	5.57	978.97	0.0035
Columbia, Mo.	1967	5.70	978.33	0.0044
Albuquerque, New Mexico	1967	7.05	977.56	0.0026
El Paso, Texas	1967	6.90	977.37	0.0041
Vandenberg AFB, Cal.	1967	5.82	977.90	0.0036
Pago Pago, Samoa	1967	5.86	976.47	0.0076
Wake Island	1963	5.80	976.87	0.0068
Wake Island	1965	5.80	976.87	0.0070
Wake Island	1967	5.80	976.87	0.0071
Majuro Island	1967	5.87	976.28	0.0090
Point Barrow, Alaska	1967	5.32	981.04	0.0034
Byrd Station, Antarctica	1967	6.63	981.10	0.0020



**Fig. 3 MEAN WATER VAPOR CONTENT OF ATMOSPHERE, VERSUS LATITUDE**

Table 3

## Parameters for Two-Quartic N Profile

Station	Year	$h_{d_0}$ (km)		$a_d$ (km/°C)	Prediction Error for $\int N_d dh$ , $\sigma$ (meters)	$h_w$ (km)		Prediction Error for $\int N_w dh$ , $\sigma$ (meters)
		Above Geoid	Above Station			Above Geoid	Above Station	
Weather Ship E	1963	40.038	40.028	0.15307	0.001372	11.333	11.323	0.028123
Weather Ship E	1965	40.082	40.072	0.15087	0.001423	10.715	10.705	0.027883
Weather Ship E	1967	40.077	40.067	0.15092	0.001509	9.708	9.698	0.032824
Ascension Island	1967	40.336	40.257	0.14991	0.001127	9.749	9.670	0.021204
Caribou, Maine	1967	40.275	40.084	0.14839	0.001548	11.255	11.064	0.027642
Washington, D.C.	1967	40.197	40.112	0.14918	0.001608	11.464	11.379	0.030072
St. Cloud, Minn.	1967	40.409	40.091	0.14795	0.001308	11.855	11.537	0.023500
Columbia, Mo.	1967	40.353	40.114	0.14878	0.001569	11.204	10.965	0.028744
Albuquerque, New Mexico	1967	41.749	40.129	0.14797	0.001393	14.434	12.814	0.016624
El Paso, Texas	1967	41.358	40.165	0.14765	0.001635	14.206	13.013	0.025084
Vandenberg, AFB, Cal.	1967	40.223	40.123	0.14855	0.001475	8.639	8.539	0.024808
Pago Pago, Samoa	1967	40.358	40.353	0.14409	0.001684	10.679	10.674	0.045408
Wake Island	1963	40.104	40.099	0.15239	0.001418	10.605	10.600	0.033192
Wake Island	1965	40.146	40.141	0.15099	0.001531	9.222	9.217	0.032717
Wake Island	1967	40.125	40.120	0.15173	0.001669	9.487	9.482	0.044369
Majuro Island	1967	40.498	40.495	0.14034	0.001489	11.268	11.265	0.049702
Point Barrow, Alaska	1967	40.030	40.022	0.14738	0.001261	11.515	11.507	0.017658
Byrd Station, Antarctica	1967	41.536	39.993	0.14675	0.001091	15.585	14.042	0.005542
Mean			40.136	0.14872				

Prediction of the wet component,  $\int N_w dh$ , is much poorer and leaves a residual error usually of a few centimeters instead of a millimeter or two. This problem will be discussed later.

## STATISTICS ON ZENITH INTEGRALS

Statistical values of the zenith integral are needed for range correction when no surface data are available (e.g., radar altimeter over ocean areas), and they are also of climatological interest. Table 4 gives 1-year mean values and standard deviations for the dry and wet component integrals and the totals for all the stations studied, based on the revised computations described above.

The rms error of using the annual mean to represent the height error at a specific time varies for different stations from a few millimeters to 2 cm for the dry component, and from 3 to 6 cm for the wet component, except at the Antarctica station (where it is less than 1 cm). The larger percent variations from the mean, in both components, are generally found for the higher-latitude stations, where seasonal changes are more marked. Statistical prediction can probably be improved by including a seasonal effect, but such parameters have not yet been obtained.

If there is no correlation between dry and wet component integrals on a day-by-day basis, the variance of the total integral should be equal to the sum of the variances of the components. On this basis, no conspicuous evidence of correlation between dry and wet integrals is seen in the present statistics; here, the variance of the total at one location is not markedly different from the sum of the component variances, and the deviation is not always in the same direction. However, this approach may not be suitable for detecting small effects. Some relations between the dry and wet parts have in fact been observed but have not yet been fully treated theoretically.

Table 4  
Annual Statistics on  $\int N$  dh and Components

Station	Year	$\int N_d$ dh (meters)		$\int N_w$ dh (meters)		Total $\int N$ dh (meters)	
		Mean	$\sigma$	Mean	$\sigma$	Mean	$\sigma$
Weather Ship E	1963	2.32546	0.01521	0.21370	0.04996	2.53916	0.05538
Weather Ship E	1965	2.32202	0.01694	0.18578	0.05174	2.50779	0.05888
Weather Ship E	1967	2.32843	0.01493	0.16647	0.05038	2.49490	0.05151
Ascension Island	1967	2.30176	0.00563	0.17516	0.03105	2.47692	0.02889
Caribou, Maine	1967	2.26051	0.01914	0.08675	0.05839	2.34727	0.05918
Washington, D. C.	1967	2.29975	0.01545	0.11671	0.06485	2.41646	0.06366
St. Cloud, Minn.	1967	2.22660	0.01800	0.08252	0.05224	2.30912	0.05140
Columbia, Mo.	1967	2.25361	0.01440	0.10922	0.06167	2.36283	0.05978
Albuquerque, New Mexico	1967	1.91237	0.01132	0.06752	0.04321	1.97989	0.04797
El Paso, Texas	1967	2.01287	0.00986	0.08923	0.05047	2.10211	0.05252
Vandenberg AFB, Cal.	1967	2.29101	0.00882	0.09298	0.03329	2.38399	0.03202
Pago Pago, Samoa	1967	2.31367	0.00492	0.25088	0.05089	2.56455	0.05101
Wake Island	1963	2.31700	0.00454	0.23929	0.05230	2.55628	0.05107
Wake Island	1965	2.31980	0.00431	0.19655	0.04698	2.51635	0.04651
Wake Island	1967	2.31756	0.00534	0.21938	0.05774	2.53694	0.05631
Majuro Island	1967	2.31261	0.00354	0.27775	0.05354	2.59036	0.05417
Point Barrow, Alaska	1967	2.30310	0.02024	0.04712	0.02862	2.35022	0.03037
Byrd Station, Antarctica	1967	1.85214	0.02115	0.01520	0.00696	1.86734	0.02461

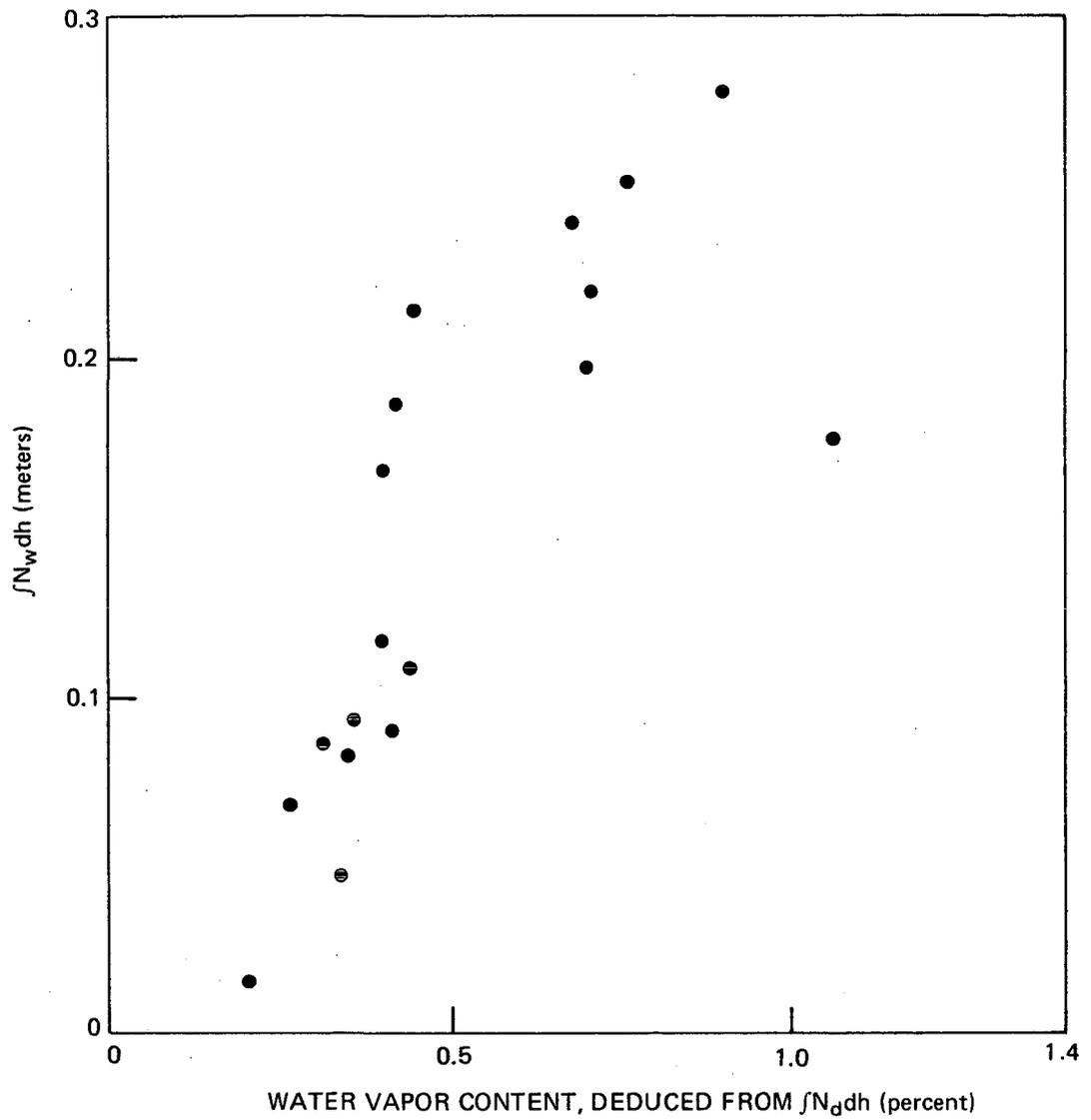
## OBSERVED RELATIONS BETWEEN DRY AND WET COMPONENTS

The mean percent of water vapor molecules in the atmosphere at each station was deduced above from the dry component integral,  $\int N_d dh$  (Table 2). The mean wet component integrals,  $\int N_w dh$ , for all stations are listed in Table 4. These two quantities are independently measured and are not measurements of the same thing; still, they both involve the water vapor content, and it is of interest to compare them. Figure 4 shows the comparison. The wet component integrals show a clear, though not precise, relation to the water vapor content as determined from the dry integrals. This corroborates the assumption made above that the systematic discrepancy between theoretical and observed values of the parameter  $k$  (Eq. (10)) is caused by the presence of water vapor in the atmosphere.

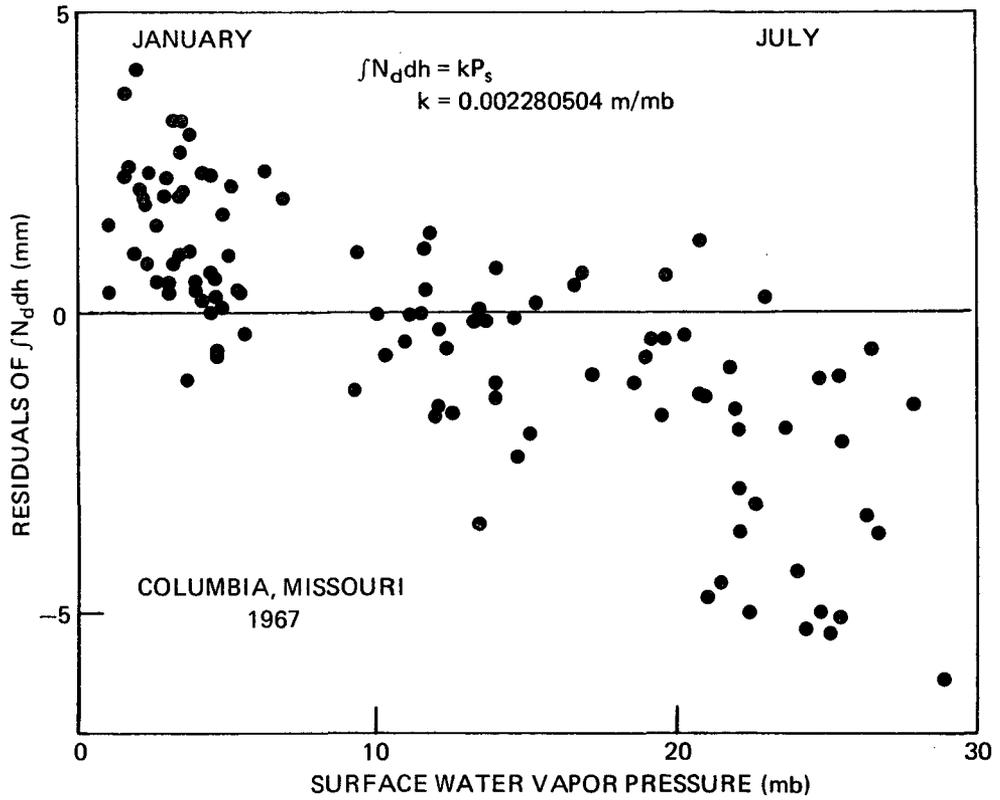
Another bit of corroborating evidence is found in the residual errors when  $\int N_d dh$  is predicted from surface pressure using Eq. (6) and the  $k$  values of Table 1. The observed and predicted values of  $\int N_d dh$  were shown in Fig. 1 for Columbia, Missouri, for one summer and one winter month. Each observed point pertains to one balloon flight. The prediction errors are small but systematic; predicted values are too high in January and too low in July. These residual errors on a much-enlarged scale are plotted in Fig. 5 as a function of observed water vapor pressure at the surface at the time of the balloon flight. The data scatter appears large on this large scale, but the trend is obvious. Data from all stations show the effect, but it is most observable where the seasonal variation of water vapor pressure is large.

The prediction of  $\int N_d dh$  can be improved on the basis of these findings. Empirically, we may rewrite Eq. (6) in the form:

$$\int N_d dh = k_1 P_s + k_2 e_{sur}, \quad (16)$$



**Fig. 4** COMPARISON OF  $\int N_w dh$  THROUGH ATMOSPHERE, WITH WATER VAPOR CONTENT DEDUCED FROM  $\int N_d dh$



**Fig. 5 WATER VAPOR DEPENDENCE OF RESIDUAL ERRORS OF PREDICTED  $\int N_d dh$**

where  $k_1$  and  $k_2$  are constants, and  $e_{\text{sur}}$  is the observed value of  $e$  at the surface. Preliminary values of  $k_1$  and  $k_2$  have been obtained from the balloon data, again by a least squares procedure. Equation (16) improves the prediction of  $\int N_d dh$ . Its use removes the seasonal bias from the prediction and results in smaller residual errors: generally an rms value of 1.5 mm or less at the zenith. Values of  $k_1$  are slightly smaller than those of  $k$  for the same location, and  $k_2$  has approximately one-tenth the value of  $k_1$ .

Equation (16) has the disadvantage that the measurement of  $e_{\text{sur}}$  is much less precise than that of  $P_s$ , and this lack of precision degrades the computed value of  $k_1$  as well as  $k_2$ . Further work is needed here, and the preliminary values of  $k_1$  and  $k_2$  will therefore not be listed. The theoretical aspect of Eq. (16) should be related to the approximations that were originally made in defining  $N_d$  (Ref. 1). This question has not been studied thoroughly and will not be discussed further at this time.

## HORIZONTAL GRADIENTS IN THE ATMOSPHERE

Up to this point, only vertical gradients of meteorological quantities have been considered; it has been assumed that horizontal gradients are negligible. Several figures will be presented here to give a qualitative idea of the relation between simultaneously observed zenith integrals at locations a few hundred miles apart.

Figure 6 shows the observed and calculated values of the dry zenith integral as a function of time for the months of January and July 1967 at Albuquerque, New Mexico, and El Paso, Texas, 200 miles to the south. For each month, the data for the two stations are plotted on the same scale, with no zero shift. The bias between Albuquerque and El Paso data is due to the difference in altitude of the stations. Albuquerque is 400 meters the higher of the two, with correspondingly less air above it:

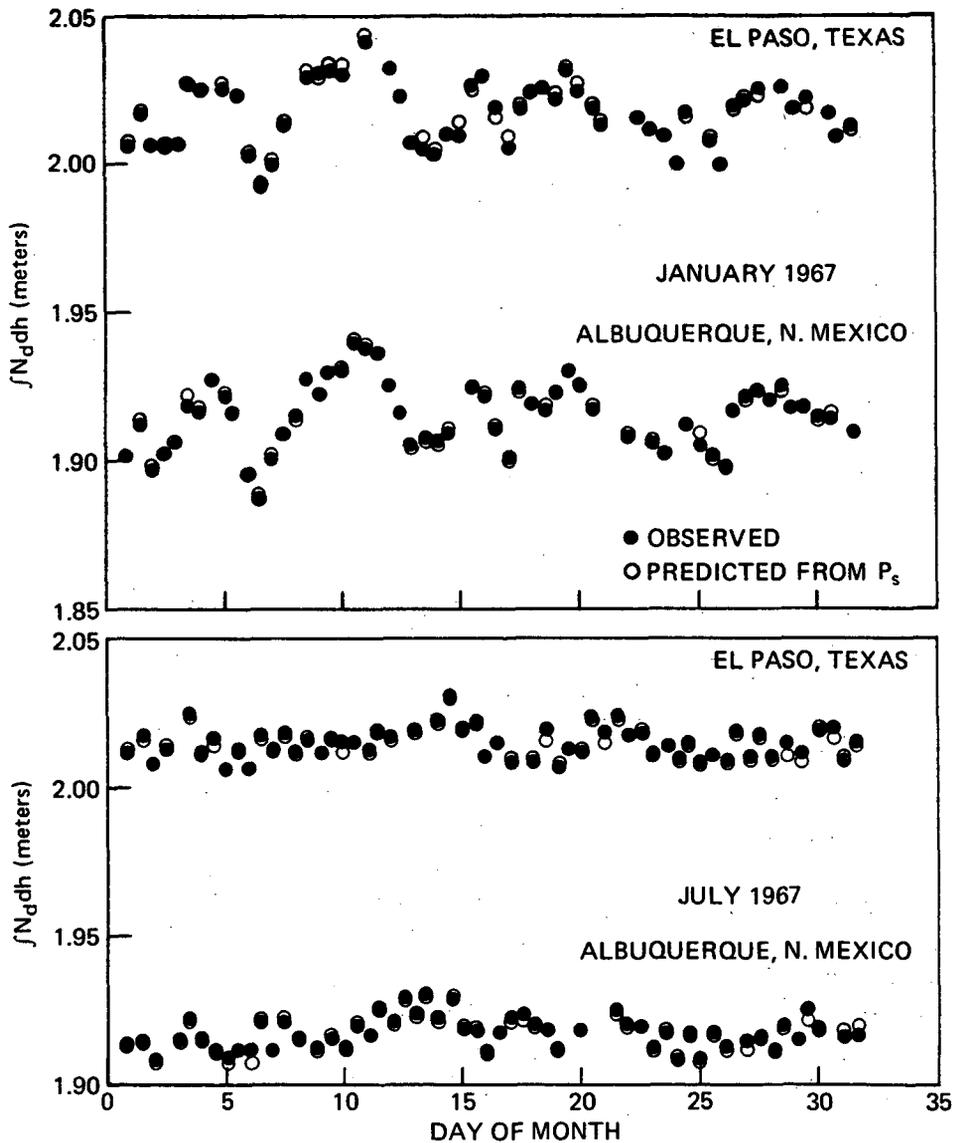


Fig. 6 COMPARISON OF VALUES OF  $\int N_dh$  AT EL PASO AND ALBUQUERQUE, WINTER AND SUMMER

the zenith range effect (dry component) at Albuquerque is 10 cm smaller than that at El Paso. The similarity of the pattern for the two stations is very marked at both seasons, the indication being that the horizontal variations are not very large over a few tens of miles. The consistency of the pattern for the two stations speaks well for the precision of the data. Each observed point was obtained from a different balloon flight, each with its own measuring instruments.

A diurnal variation is seen, especially in the summer samples (both observed and predicted). It is larger than the expected atmospheric tidal effect and is not yet explained.

Figure 7 shows corresponding observed data for January 1967 for three other U.S. stations: St. Cloud, Minnesota; Washington, D. C. (Dulles Airport); and Caribou, Maine. With Columbia (Fig. 1), these form the vertices of a quadrilateral that is roughly trapezoidal, and their separations are from 500 to about 1200 miles. Columbia is some 800 miles from Albuquerque. Over these distances, there is not the same close correlation as in Fig. 6, but marked similarities can often be seen even at distances greater than 500 miles, sometimes with a time delay (not always in the same direction). The deep minimum that appeared at Washington on 28 January and at Caribou a day later is a conspicuous example.

Figure 8 shows the same three stations as Fig. 7 but for the month of July, and Figs. 1 and 6 again supply further data. Correlations between stations are very noticeable in Fig. 8. The zenith integral (dry) was more variable in the January sample than in the July sample at all these stations. On a large-scale basis, it appears that the horizontal gradients are greater in winter, but this conclusion cannot be fully justified without more work on the apparent diurnal variation.

Less consistency is found between zenith integrals of the wet component at different stations, but some pattern

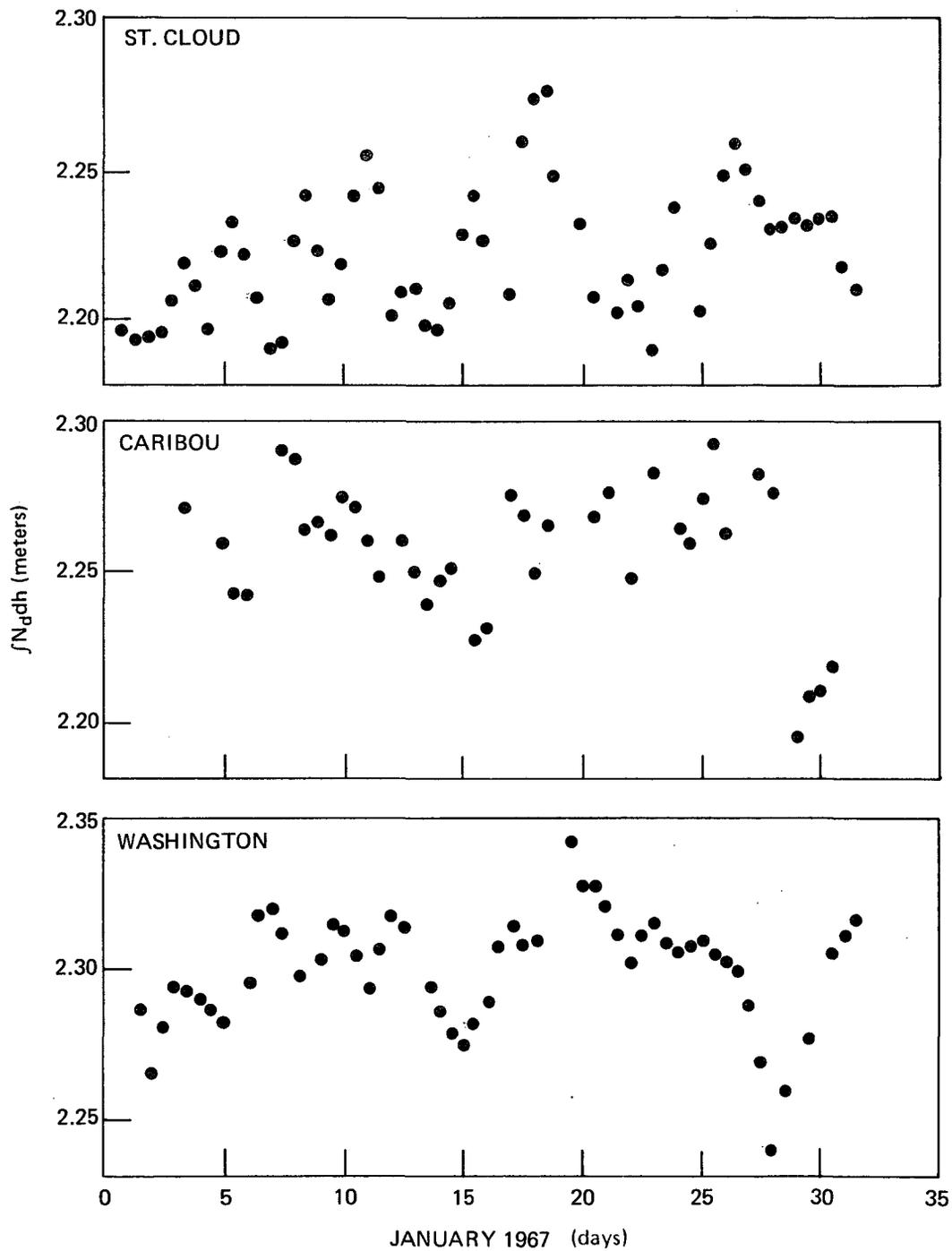


Fig. 7 COMPARISON OF WINTER SAMPLES OF  $fN_dh$  AT THREE LOCATIONS IN THE U.S.

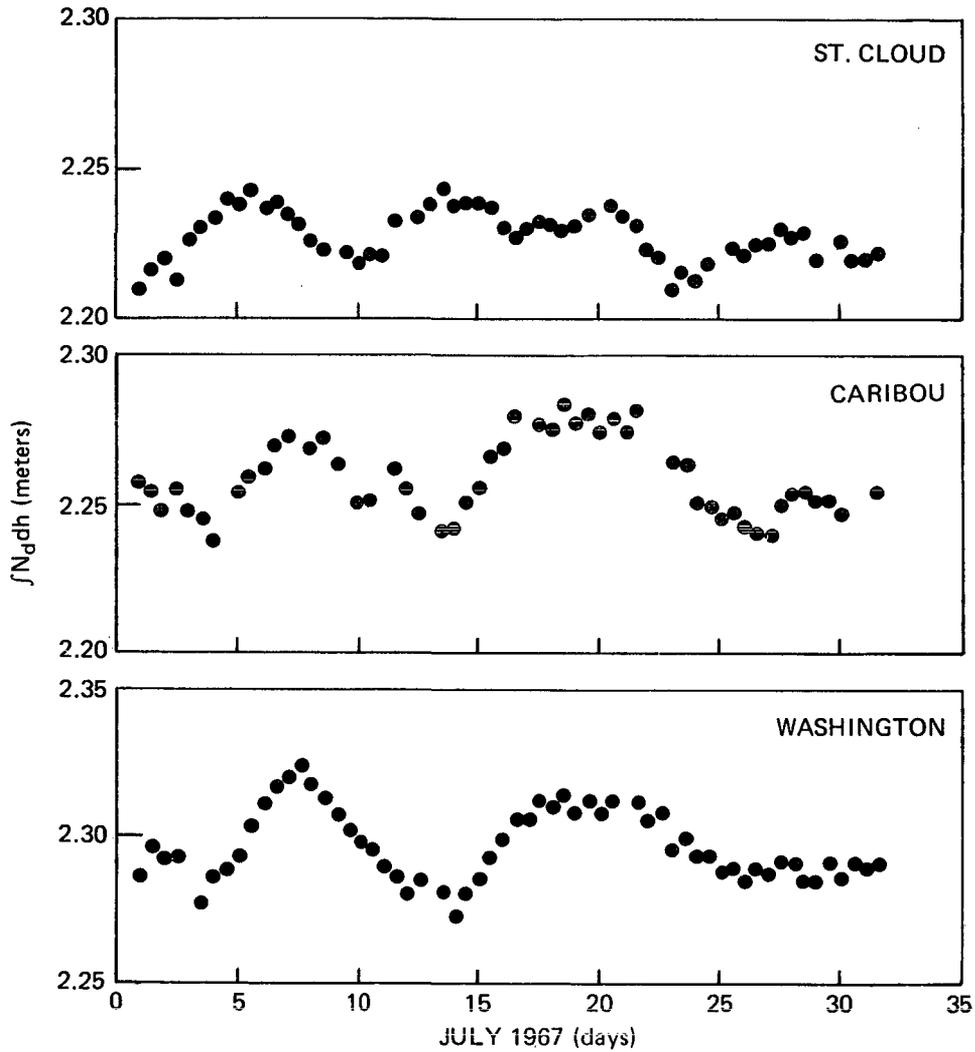


Fig. 8 COMPARISON OF SUMMER SAMPLES OF  $\int N_d dh$  AT THREE LOCATIONS IN THE U.S.

similarities can be observed. Figures 9 and 10 show a winter sample and a summer sample, respectively, for the St. Cloud, Caribou, and Washington stations. The difficulty of measuring water vapor content is a serious problem here, and improved methods are needed. It is impossible to be sure how much of the variation in these data is due to atmospheric variation and how much to the measurement. It would be difficult to get any reliable information about horizontal gradients from these data.

As expected, the wet zenith integral is considerably greater in summer than in winter at either station, and is greater on the average in the Washington area than in Maine at either season. There must be a systematic (but small) latitude gradient of the mean value of  $\int N_w dh$ , as well as systematic climatic gradients owing to geography (e.g., land versus ocean areas).

In recent work at the National Severe Storms Laboratory (Refs. 9 and 10), horizontal gradients of pressure, temperature, and water vapor pressure have been examined by the simultaneous launch of meteorological balloons about 15 nmi apart. The statistical results indicate that the pressure difference between stations at this separation is too small to be measured with present instruments. Detection of a temperature difference is marginal, but the statistics indicate that differences larger than 1° or 2°C are not commonly observed. Water vapor statistics are more variable and also more uncertain because of low instrumentation accuracy (Ref. 10).

Clearly it will be impossible to estimate  $\int N_w dh$  with accuracy comparable to the  $\int N_d dh$  prediction until the wet integral can itself be better measured than at present. It is not known how much of the variability now observed in the values of  $\int N_w dh$  can reasonably be attributed to the present measuring errors. This question will be studied further.

Additional instrumentation could help to solve this problem. If a refractometer were added to measure the

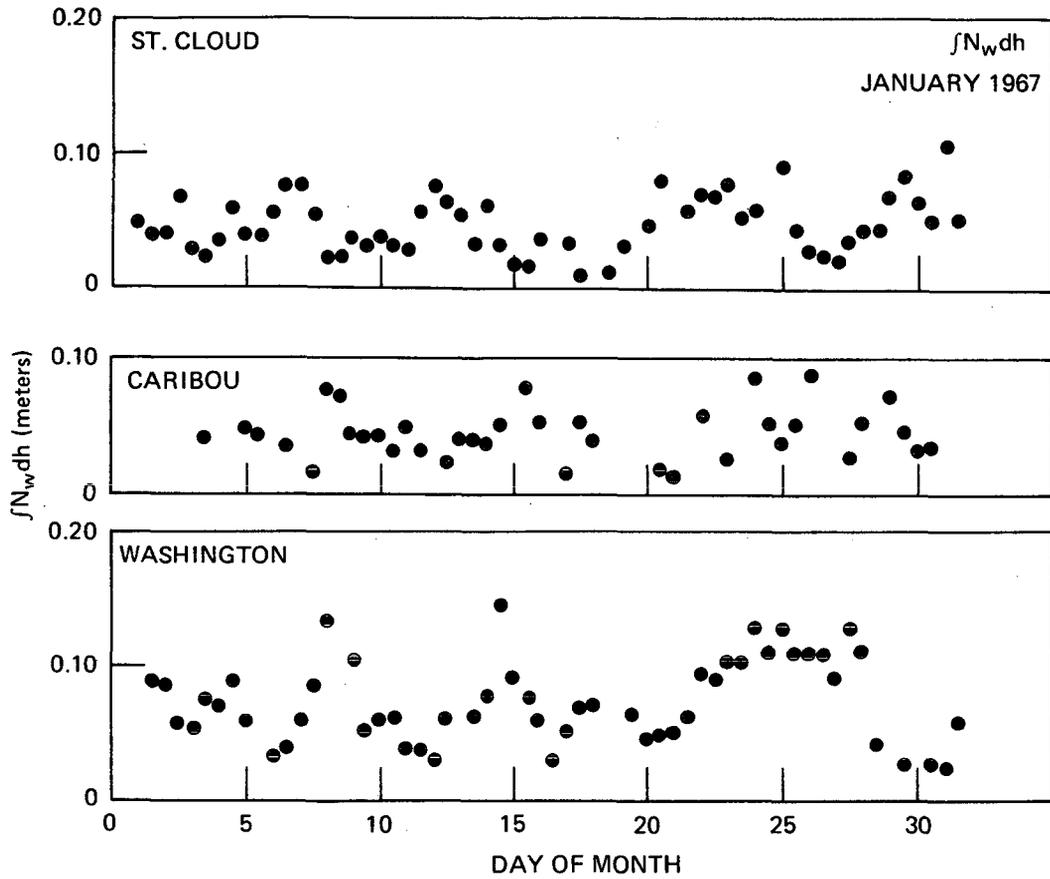


Fig. 9 COMPARISON OF WINTER SAMPLES OF  $\int N_w dh$  AT THREE LOCATIONS IN THE U.S.

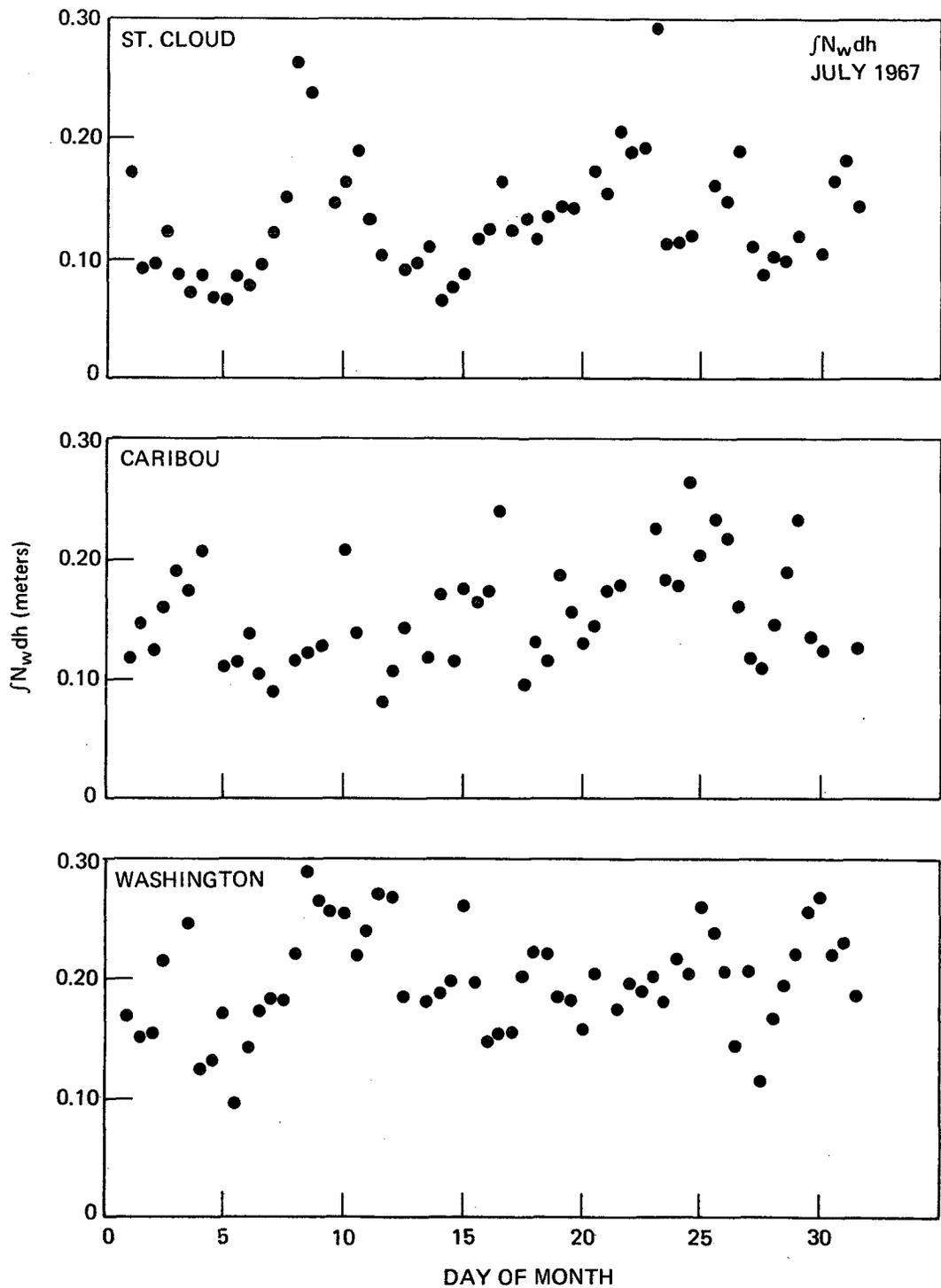
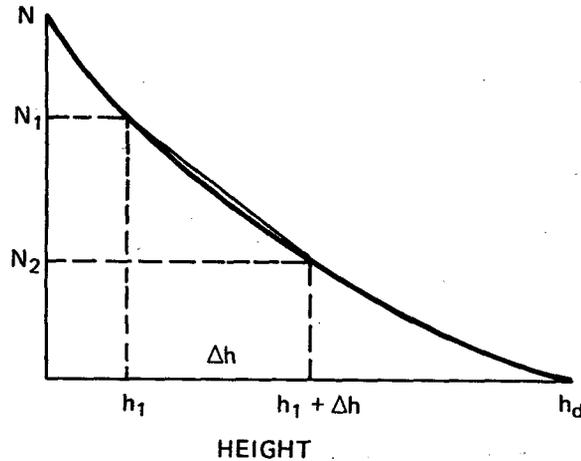


Fig. 10 COMPARISON OF SUMMER SAMPLES OF  $fN_{wdh}$  AT THREE LOCATIONS IN THE U.S.

total refractivity,  $N$ , along with the usual meteorological quantities, then the wet component,  $N_w$ , could be found from the difference,  $N - N_d$ , and compared with the value from humidity measurement. Experimentation with this redundant instrumentation at a surface site could determine whether its use would be justified in balloon flights.

APPENDIX A  
 NUMERICAL INTEGRATION ALONG A CURVE:  
 CORRECTION TO TRAPEZOIDAL RULE



Numerical integration of  $N$  with height from a set of observed points is most simply accomplished by assuming a linear change of  $N$  with height between successive points. For a function whose curvature is practically always in the same direction (e.g.,  $N_d$  versus height), the integral so computed will be slightly biased. The correction described here is not used in integrating  $N_w$ , which has a less consistent profile.

The separations,  $\Delta h$ , between observed points in these profiles varied widely (from a few meters to 2 km), and the theoretically correct functional form of the  $N_d$  curves varied also. Expressions for a correction were derived as given below, based (1) on a fourth-degree  $N_d$  profile (temperature lapse rate  $6.8^\circ\text{C}/\text{km}$ ), and (2) on an exponential profile (zero lapse rate). The first correction was applied to the lower and the second to the upper part of the atmosphere. The area under a chord joining two observed points is:

$$\text{chord: } \int_{h_1}^{h_2} N dh = \frac{N_1 + N_2}{2} \Delta h. \quad (\text{A-1})$$

For case (1), let both points lie on a fourth-degree function given by (Ref. 5):

$$N = \frac{N_1}{(h_d - h_1)^4} (h_d - h)^4, \quad (\text{A-2})$$

the vertex being at  $h_d$ . Let  $l = h_d - h_1$ . Then using values from Eq. (A-2) in Eq. (A-1), it can be shown that:

$$\text{chord: } \int_{h_1}^{h_2} N \, dh = \frac{N_1}{l^4} \left( l^4 \Delta h - 2l^3 \overline{\Delta h}^2 + 3l^2 \overline{\Delta h}^3 - 2l \overline{\Delta h}^4 + \frac{\overline{\Delta h}^5}{2} \right). \quad (\text{A-3})$$

Integrating the fourth-degree curve of Eq. (A-2) between the same limits yields:

$$\text{quartic: } \int_{h_1}^{h_2} N \, dh = \frac{N_1}{l^4} \left( l^4 \Delta h - 2l^3 \overline{\Delta h}^2 + 2l^2 \overline{\Delta h}^3 - l \overline{\Delta h}^4 + \frac{\overline{\Delta h}^5}{5} \right). \quad (\text{A-4})$$

The difference (chord - quartic) is the correction, and is:

$$\Delta \int_{h_1}^{h_2} N \, dh = \frac{N_1}{l^4} (l^2 \overline{\Delta h}^3 - l \overline{\Delta h}^4 + 0.3 \overline{\Delta h}^5). \quad (\text{A-5})$$

Since  $\Delta h$  is always small in comparison with  $l$ , the first term is by far the most significant.

For case (2), let two points lie on an exponential curve given by:

$$N = N_1 e^{-c(h - h_1)}. \quad (\text{A-6})$$

Then

$$N_2 = N_1 e^{-c \Delta h}, \quad (\text{A-7})$$

and using Eq. (A-1), the integral along the chord is:

$$\text{chord: } \int_{h_1}^{h_2} N \, dh = \frac{N_1}{2} (1 + e^{-c \Delta h}) \Delta h. \quad (\text{A-8})$$

Integrating Eq. (A-6) between the same two points, we get:

$$\text{exponential: } \int_{h_1}^{h_2} N \, dh = \frac{N_1}{c} (1 - e^{-c \Delta h}). \quad (\text{A-9})$$

The difference between Eqs. (A-8) and (A-9) then gives the correction. Expanding  $e^{-c \Delta h}$  in the usual infinite series in both equations and subtracting, the difference (linear - exponential) becomes:

$$\Delta \int_{h_1}^{h_2} N \, dh = \frac{N_1}{2} c^2 \overline{\Delta h}^3 \left( \frac{1}{6} - \frac{1}{2} c \Delta h + \dots \right). \quad (\text{A-10})$$

In order to evaluate and compare the quartic and the exponential corrections for curvature for a given interval,  $\Delta h$ , values for the parameters  $\ell$  in Eq. (A-5) and  $c$  in Eq (A-10) are needed. It follows from the results of Ref. 6 that in order to give the correct integral,  $\int N_d \, dh$ , through the atmosphere above any height,  $h_1$ , the parameter  $\ell$  for a quartic  $N_d$  profile is:

$$\ell = \ell_0 + aT_1, \quad (\text{A-11})$$

where  $T_1$  is the Celsius temperature at  $h_1$ ,  $l_0 = 40$  km (the equivalent height of the quartic model at  $0^\circ\text{C}$ ), and  $a = 0.147$  km/ $^\circ\text{C}$ . Thus  $l$  can be computed for any data point.

For the quartic model of Eq. (A-2):

$$\int_{h_1}^{h_1 + l} N \, dh = \frac{N_1 l}{5}. \quad (\text{A-12})$$

For the exponential model of Eq. (A-6):

$$\int_{h_1}^{\infty} N \, dh = \frac{N_1}{c}. \quad (\text{A-13})$$

If the exponential model is to yield the same integral above  $h_1$  as the quartic model, then:

$$c = \frac{5}{l}. \quad (\text{A-14})$$

The corrections of Eqs. (A-5) and (A-10) can thus be found. Since they are small and are approximations at best, it is not necessary to compute them precisely. It was estimated from observed profile data that the corrections for any interval,  $\Delta h$ , can be approximated adequately by the values:

$$\text{(linear-quartic curve)} \quad \Delta \int_{h_1}^{h_2} N \, dh = \frac{N_1}{l^2} \Delta h^3, \quad (\text{A-15})$$

$$\text{(linear-exponential curve)} \quad \Delta \int_{h_1}^{h_2} N \, dh = 1.9 \frac{N_1}{l^2} \Delta h^3. \quad (\text{A-16})$$

Either correction is to be subtracted from the corresponding integral obtained from the trapezoidal rule.

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