NUMERICAL METHOD AND FORTRAN PROGRAM
FOR THE SOLUTION OF AN AXISYMMETRIC
ELECTROSTATIC COLLECTOR DESIGN PROBLEM

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D.C. • SEPTEMBER 1972
This report describes the numerical calculation of the steady-state flow of electrons in an axisymmetric, spherical, electrostatic collector for a range of boundary conditions. The trajectory equations of motion are solved alternately with Poisson's equation for the potential field until convergence is achieved. A direct (noniterative) numerical technique is used to obtain the solution to Poisson's equation. Space charge effects are included for initial current densities as large as 100 A/cm$^2$. Ways of dealing successfully with the difficulties associated with these high densities are discussed. The report includes a description of the mathematical model, a discussion of numerical techniques, results from two typical runs, and the FORTRAN computer program.
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NUMERICAL METHOD AND FORTRAN PROGRAM FOR THE SOLUTION OF AN
AXISYMMETRIC ELECTROSTATIC COLLECTOR DESIGN PROBLEM

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SUMMARY

This report describes the determination of the steady-state flow of electrons in an
axisymmetric spherical collector under a variety of boundary conditions. The electron
trajectory equations of motion are solved alternately with Poisson's equation for the
potential field until convergence is achieved.

The bounding surfaces are a sphere below, a spherical cone above, and, optionally,
a thin axial spike suspended from the apex of the cone. Negative decelerating potentials
are prescribed on the cone-spike surface and on segmented zones of the spherical sur-
face. Polar angles of the cone may vary from $0^\circ$ to $160^\circ$.

The principal value of this report is that it pushes design research into the high-
current-density range, where space charge effects are strong. Initial attempts met
with convergence difficulties for current densities above 50 A/cm².

This report describes the method used to surmount these difficulties and handle
values as high as 100 A/cm². This was achieved in two ways: first, by using variable
mesh spacing to ensure accuracy in regions where the solution was changing rapidly;
and second, by using a fast, accurate subprogram for obtaining the potential field. This
subprogram uses a direct, noniterative numerical solution to Poisson's equation, the
details of which are given in NASA TN D-6438.

The present report includes a description of the mathematical model, a discussion
of numerical techniques, results from two typical runs, and the FORTRAN computer
programs.

INTRODUCTION

Satellite communication systems have been proposed in which the satellite can re-
ceive, amplify, and rebroadcast microwave signals. The high efficiency of the micro-
wave amplifiers to be used is of prime importance to the lifetime and reliability of such a system.

Microwave amplifiers, as a class, convert the energy of bunched electron beams into radiofrequency (rf) energy. Since this process is only partially efficient, the remaining beam energy is dissipated as heat in the electron collector. However, collectors can be designed and maintained at proper potentials to slow the electrons. The electron energy is then spent doing work against the collector potential and is recoverable as useful electrical power.

Reference 1 discusses how the results of the program described in this report are used in collector design. Reference 2 describes a computer program which solves Poisson's equation for a given space charge and which is required as a subroutine in the iterative solution of the current problem.

It is important to realize that there are two characteristics of electron beam trajectory patterns that facilitate efficient collector design:

1. Good, uniform, vertical and horizontal spreading (i.e., a sufficient rise away from the entry hole, and a pushing away from the collector axis)
2. Good, uniform separation permitting insertion of electrodes to collect electrons near the peak of their trajectories, where their energy is close to minimum

Both these characteristics imply no backstreaming, crossing, or impingement of beams on the underside of the electrodes.

The original computer program dealt with negative decelerating potentials prescribed only on segmented zones of the sphere. But this approach was ineffective because of the great distance between these surface potentials and the incoming stream of electrons.

A second stage of the design was to prescribe a decelerating potential on a cone of varying polar angle. This slowed down the incoming stream considerably, and in most cases the beam spread was sufficient.

In other instances where even more beam spread was desirable, a third and final design modification was made. Here a negative potential was prescribed on a thin spike suspended from the cone. This had the effect of spreading the flow even more as it entered, and allowing the other decelerating potentials to come into play more effectively.

The final FORTRAN program reported herein allows for the presence or absence of an axial spike of varying length, as well as for the choice of decelerating surface potentials on zonal segments of the sphere and on the cone-spike surface. The cone angle varies from 0° to 160°. Initial conditions also include variable velocity and injection positions of entering electron groups.

It was assumed that, prior to entry into the collector, the electron beam had been sorted into graded velocity groups, with the outermost being the slowest. This is physically more realistic than assuming a constant velocity only. The beam was then
injected through a small entry hole in the collector where the negative polar axis pierces the spherical surface. The potential distribution on the surface of the sphere was assumed to be a function of the polar angle only. This report describes the numerical solution to a problem used in the design of such a collector under varying conditions of surface potential, current density, and collector geometry.

A typical configuration is shown in figure 1. Notice that the electron groups reach minimum velocity at their peaks and start falling back. If electrodes of the proper potential are inserted to collect the electrons at these minimum velocity points, the efficiency of the collector will then be maximized.

The numerical technique and computer program described in reference 2 were used to determine the potential field inside the collector as required. The results were then used as part of the design criteria of the collector as described in reference 1.

In situations where current density is low, less than 10 A/cm², space charge effects were found to be negligible. But with higher current densities and associated higher space charge, certain problems arise near the entry hole that require special techniques to ensure numerical stability. This report describes these techniques. Current densities as high as 100 A/cm² were dealt with successfully. Initially, calculations were successful only with current densities half as large, before numerical difficulties precluded further computation.

A complete description of the mathematical model is given, along with a discussion of the assumptions made. The technique used in determining the space charge distribution is then described. Appendixes A to G contain the results of two typical computer
runs, a complete listing of the FORTRAN computer program, flow charts, magnetic field calculations, the derivation of the equations of motion, and the mathematical symbols.

**STATEMENT OF THE PROBLEM**

The problem is to determine the steady-state potential field and the associated steady-state electron beam trajectory patterns within a spherical collector (1) for any specified collector geometry (cone angle; collector radius; entry hole radius; and length of axial spike, if any), (2) for negative potential distribution on the surfaces of the collector, and (3) for magnetic field distribution and electron beam initial entry conditions (current, current density, perveance, injection angles, positioning, and velocity).

This involves the determination at every mesh point in the sphere of the potential \( V \) that satisfies Poisson's equation

\[
\nabla^2 V = \mathbf{f}(V)
\]

where

\[
\nabla^2 V = \frac{\partial^2 V}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial V}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{\cot \theta}{\rho^2} \frac{\partial V}{\partial \theta} + \frac{1}{\rho^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}
\]

The bars refer to dimensioned values. All mathematical symbols are defined in appendix G. For convenience, these expressions and all others following are normalized by the substitution

\[
\rho = \frac{\bar{\rho}}{R}
\]

We may now write

\[
f(V) = \frac{\rho_e(V)}{\varepsilon_0}
\]

The right-hand term in equation (1d) is related to space charge resulting from the beam trajectories. These trajectories are obtained from the solution to the differential equation of motion (eq. (3)). Note that the determination of \( V \) is an iterative process,
in that the right-hand side (or source term), which depends on the space charge, changes when the internal potential \( V \) changes.

The source term consists of the dielectric constant \( \epsilon_0 \) and \( \rho_e(V) \) where

\[
\rho_e(V) = \frac{|J|}{|u|} \tag{2}
\]

The velocity term \( u \) is easily calculated since it is simply a function of certain initial conditions and the potentials \( V \) and \( V_m \).

The current density term \( J \) is somewhat more complicated. In general, it is calculated from the current flow through annuli formed by adjacent trajectories of the various electron energy groups. This flow is shown later in figure 4. These trajectories are calculated from the differential equation of motion

\[
\frac{\partial^2 \rho}{\partial \theta^2} = \rho'' = \rho + \rho \sin^2 \theta \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\eta}{\partial^2 R} - \frac{1}{R} \frac{\partial V}{\partial \rho} \sin \theta \frac{\partial V_m}{\partial \theta} - \rho' \sin \theta \cos \theta \frac{\partial^2 \phi}{\partial \theta^2}
\]

\[
+ \frac{2 \rho'^2}{\rho} - \frac{\rho' \eta}{\rho^2 \partial^2 R^2} \left( \frac{\partial V}{\partial \theta} + \rho R \sin \theta \frac{\partial V_m}{\partial \rho} \right) \tag{3}
\]

where

- \( \rho \) normalized radius vector
- \( \theta \) polar angle
- \( \phi \) azimuthal angle
- \( R \) radius of sphere
- \( V_m \) magnetic field

Details of this iterative process are described in the next section.

METHOD OF SOLUTION

First, the steady-state flow of electrons with a given velocity distribution was determined within the collector. This was accomplished by taking the following four steps:

1. With the given collector geometry and a specified set of initial conditions, a first approximation to the potential field inside the collector was found by assuming zero space charge and solving Laplace's equation.
\[ \nabla^2 V = 0 \]  

(4)  

or  

\[ \frac{\partial^2 V}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial V}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{\cot \theta}{\rho^2} \frac{\partial V}{\partial \theta} = 0 \]  

(5)  

(2) An arbitrary number of equally spaced electron trajectories were then calculated from the equation of motion (eq. (3)). This equation is derived in appendix F. In general, nine electron groups were solved for in this problem.  

(3) From these trajectory patterns, the space charge was then calculated at each mesh point, and the results were used as source terms in Poisson's equation which was solved next,  

\[ \nabla^2 V = \frac{\rho e}{\epsilon_0} \]  

(6)  

Details of how the space charge was calculated are given in a later section.  

(4) Equation (3) was solved again to obtain new trajectories based on the new potential field obtained from equation (6). This iterative process was then continued until convergence was achieved. The entire process is shown in flow chart form in figure 2.  

Prior to solving the problem, the following four sets of normalized initial conditions must be specified. (The quantities listed in the first three sets are shown in figure 3 for the case of three electron classes.)  

(1) Collector geometry: radius of collector \( R \), initial polar angle \( \theta_0 \), and radius of entry hole \( a \)  

(2) Collector boundary values of electric potential: on a spherical surface \( V_1, V_2, \ldots, V_f \) (In general, the potential on the spherical surface decreases linearly to zero \( (V_f = 0) \) as the polar angle increases from \( \theta_0 \) to some prescribed \( \theta_f \); on a conical surface if \( \theta_0 \neq 0(V_1) \); and along the negative polar axial spike if \( \theta_0 \neq 0(V_1) \) (optional)  

(3) Electron beam entry conditions: type of flow (Brillouin or confined; confined flow means there is no initial rotational velocity of the beam, or \( \phi = 0 \)), values of beam energy classes \( v_i \), entry positions of electron classes in hole at injection \( \delta_i \) (\( \delta_i \) is the supplement of the polar angle \( \theta \)), and entry angles \( \alpha_i \). In general, \( i = 1, 2, \ldots, 9 \).
Figure 2. - Flow of operations in solution of complete problem.

Figure 3. - $\rho$-$\theta$ plane of a typical axisymmetric collector, showing various initial conditions.
(4) Electron beam parameters: magnetic flux density $B_0$, magnetic field potential $V_m$, electric potential $V_0$, current $I_0$, and current density $J_0$

Auxiliary Equations

We now discuss the auxiliary equations which are required in the solution of equation (3).

The first two are the magnetic field terms expressed in cylindrical coordinates.

\[
\frac{\partial V_m}{\partial \rho} = R \left( \frac{\partial V_m}{\partial r} \sin \theta + \frac{\partial V_m}{\partial z} \cos \theta \right) \quad (7a)
\]

\[
\frac{\partial V_m}{\partial \theta} = \rho R \left( \frac{\partial V_m}{\partial r} \cos \theta - \frac{\partial V_m}{\partial z} \sin \theta \right) \quad (7b)
\]

or letting $B_r = \partial V_m / \partial r$ and $B_z = \partial V_m / \partial z$, then

\[
\frac{\partial V_m}{\partial \rho} = R(B_r \sin \theta + B_z \cos \theta) \quad (8a)
\]

\[
\frac{\partial V_m}{\partial \theta} = \rho R(B_r \cos \theta - B_z \sin \theta) \quad (8b)
\]

Prior to entering the collector, the electron beam passes through a magnetic field which is used to confine the beam. Part of this field leaks into the collector through the entry hole and influences the flow of the electrons. It must be considered in the equations of motion.

Also used are the velocity terms, $\dot{\theta}^2$ and $\dot{\phi}$, and the magnetic flux $\psi$. These are expressed as

\[
\dot{\theta}^2 = \frac{1}{1 + (\rho' / \rho)^2} \left[ \frac{2\eta_e \gamma_1 (v_1 + V)}{\rho^2 R^2} - \dot{\phi}^2 \sin^2 \theta \right] \quad (9)
\]

\[
\dot{\phi} = \frac{\eta_e (\psi - \psi_0)}{2\pi \rho^2 R^2 \sin^2 \theta} \quad (10)
\]
\[ \psi = 2\pi \int_0^r B_z r \, dr \]  

which are equations (B16), (B8), and (B9) of reference 1, respectively.

Finally, the components \( B_r \) and \( B_z \) of the magnetic field \( \mathbf{V}_m \) (eqs. (D9) and (D10) of ref. 1) are expressed as

\[
B_r(r, z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} \frac{\partial^{2n+1} B(0, z)}{\partial z^{2n+1}} \left( \frac{r}{2} \right)^{2n+1}
\]

(12)

and

\[
B_z(r, z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n-1)!^2} \frac{\partial^{2n-2} B(0, z)}{\partial z^{2n-2}} \left( \frac{r}{2} \right)^{2n-2}
\]

(13)

where

\[
B(0, z) = B_0 \left[ 1 - \frac{1}{\pi} \tan^{-1} \left( \frac{a}{z} \right) - \frac{1}{\pi} \frac{z/a}{1 + (z^2/a^2)} \right]
\]

(14)

The approximations of \( B_r(r, z) \) and \( B_z(r, z) \) that were used are given in appendix E.

### The Equation of Motion

The second-order differential equation of motion (eq. (3)) was transformed into two first-order equations and solved by using a fourth-order Runge-Kutta method. As is typical in integration techniques of this type, the choice of step size is based largely on experience. In this problem, the most critical region is near the entry hole. For a large collector radius (\( R = 16 \) cm) and a small entry hole (\( a = 0.05 \) cm), an initial step size \( \Delta \theta \) of 0.01° was required. For a small collector radius (\( R = 8 \) cm) and a large entry hole (\( a = 0.10 \) cm), an initial \( \Delta \theta \) of 0.05° was sufficient. As the trajectory moved away from this critical region, the \( \Delta \theta \) step was increased to minimize computer execution time.

Initially, the solution was obtained in the \( \rho-\theta \) spherical plane. Then when any par-
ticular trajectory started to arc over ($\rho' \approx 0$), a change was made to the $r$-$z$ cylindrical coordinate system to avoid any possible discontinuities in the integration process.

**Poisson's Equation**

For any space charge field, the set of right-hand sides for Poisson's equation written at all interior grid points may be computed, and the subroutine of reference 2 may be applied to solve Poisson's equation for the internal potential field. The method of calculating the space charge field from the trajectory field is now presented.

**Calculation of Space Charge**

The source term, or space charge term, in equation (6) is

$$\frac{\rho_e}{\varepsilon_0}$$

where

- $\varepsilon_0$ dielectric constant, $8.86 \times 10^{-14}$ F/cm (farads per centimeter)
- $\rho_e$ volume charge density, C/cm$^3$ (coulombs per cubic centimeter)

Furthermore,

$$\rho_e = \frac{J}{u}$$

where

- $J$ current density, A/cm$^2$ (amperes per square centimeter)
- $u$ velocity, cm/sec

Figure 4 shows where a typical $\rho_e$ is computed; namely, at a point midway between two trajectories. To ensure accuracy, space charge is calculated at selected points along center lines midway between adjacent trajectories where the $\rho$-mesh lines are crossed.

Since $u$ is known at various points along a trajectory, one may interpolate to compute $u$ at any point, say at a position midway between two trajectories. Therefore, the
Figure 4. - Cross-sectional view of entry hole in spherical collector, showing typical injection pattern of electron energy groups 2 to 9 at radii \( r_2, r_3, \ldots, r_g \). First subscript refers to electron trajectory number; second subscript \( j \) refers to \( p \)-mesh line crossed by trajectory.

space charge at each mesh point may be determined once \( J \) is known. We now show how this is accomplished.

In general,

\[
J = \frac{I}{A} \quad (17a)
\]

where

I \quad current, A

A \quad any particular element of area through which current flows

Referring again to equation (6), we may now write

\[
J_i = \frac{I_{0,i}}{A_i} \quad (17b)
\]
where

\( I_0 \)  
initial current at injection

\( i \)  
ith electron energy group

Referring again to figure 4, it is seen that the equally spaced electron energy groups form eight annuli at the entrance hole. The areas \( dA \) of these annuli may be expressed

\[
\begin{align*}
    dA_1 &= \pi r_2^2 \\
    dA_2 &= \pi r_3^2 - dA_1 = \pi (2r_2)^2 - dA_1 = 3 \ dA_1 \\
    dA_3 &= \pi r_4^2 - \pi r_3^2 = \pi (3r_2)^2 - \pi (2r_2)^2 = 5 \ dA_1 \\
    \vdots \\
    dA_8 &= \pi r_9^2 - \pi r_8^2 = \pi (8r_2)^2 - \pi (7r_2)^2 = 15 \ dA_1
\end{align*}
\]

(18)

Since \( I = JA \), the current at injection through each annulus may be expressed

\[
\begin{align*}
    I_{0,1} &= J_0 \ dA_1 \\
    I_{0,2} &= J_0 \ dA_2 = 3J_0 \ dA_1 \\
    I_{0,3} &= J_0 \ dA_3 = 5J_0 \ dA_1 \\
    \vdots \\
    I_{0,8} &= J_0 \ dA_8 = 15J_0 \ dA_1 \\
    I_{0,i} &= (2i - 1)J_0 \ dA_1
\end{align*}
\]

(19)
In the example of figure 4, the point shown represents a typical space coordinate midway between two adjacent trajectories, in this case, trajectories 5 and 6. The normal distance between the trajectories at this point is \( d \). The area of the annulus is then

\[
A = 2\pi rd = 2\pi \rho_{5,j} \sin \vartheta_{5,j} d
\]  

(20)

The general expression for the current density, \( J = I/A \) (eq. (17a)), may now be written as

\[
J_{i,j} = \frac{(2i - 1)J_0 \, dA_i}{2\pi(\rho_{i,j} \sin \vartheta_{i,j}) \, d_{i,j}}
\]  

(21a)

or

\[
J_{i,j} = \frac{(2i - 1)2J_0}{2d_{i,j}(\rho_{i,j} \sin \vartheta_{i,j})}
\]  

(21b)

Notice that current is preserved in each annular region, namely

\[
I_{0,i} = (2i - 1)J_0 \, dA_i = J_{i,j} \, A_{i,j}
\]  

(22)

Since \( A_{i,j} \) can be computed at \((\rho_{i,j}, \vartheta_{i,j})\) by

\[
A_{i,j} = 2\pi \rho_{i,j} \sin \vartheta_{i,j} \, d_{i,j}
\]  

(23)

where \( d_{i,j} \) is the normal thickness of an annulus at this point.

Use of Irregular Mesh Spacing

The successful solution to this problem, especially those phases dealing with high current densities, was largely due to the use of irregular mesh spacing in both the \( \rho \) and \( \vartheta \) directions. From figure 5 it is clearly apparent that the region in which the solution varies most rapidly is immediately along and adjacent to the negative polar axis, where the entering electron beam is most dense and of greatest velocity. Consequently, a closely spaced mesh was used in this region. Conversely, a coarse spacing was used where the solution was varying very slowly and quite predictably.
Convergence Acceleration

Experience has shown that in solving this problem, the iterants are well behaved and ultimately converge to an acceptable approximation to the solution. But because of the steadily decaying oscillatory nature of the iterants, a considerable savings in computer execution time was achieved by halving the newly calculated changes in the variables at each step. Normally, a maximum of five iterations would provide an acceptable solution.

FORTRAN PROGRAM

General Description

The program consists of the following 16 subroutines:

1. MAIN is the executive routine for processing multiple cases and has primary control of logical flow throughout the complete program.

2. INPUT reads and prints initial conditions, parameters, and other data required for internal use by the program (printout frequency, looping indices, convergence criteria, etc.).
(3) INTGRN controls integration looping of equations of motion, step size, change-over of coordinate systems, and the number of iterations required for convergence.

(4) DE is the spherical coordinate version of a routine for computing trajectories by integrating the equations of motion.

(5) DERIV calculates $\partial V/\partial \rho$ and $\partial V/\partial \theta$ for any specified interior point.

(6) RK is the spherical coordinate version of a Runge-Kutta integration subroutine used to compute trajectories.

(7) MESH generates graduated mesh point arrays in both the $r$ and $\theta$ directions for any specified collector geometry.

(8) RZTRAJ is the cylindrical coordinate version of a routine for computing trajectories by integrating equations of motion.

(9) RZRK is the cylindrical coordinate version of a Runge-Kutta integration subroutine used to compute trajectories.

(10) RZDE contains the equations of motion in the cylindrical coordinate system. It is similar to subroutine DE.

(11) OUT controls the printing of results at specified intervals along the trajectories.

(12) VFIELD provides for the calculation of the potential field for any particular set of boundary values and source terms. It uses the program described in reference 2.

(13) EQLINE calculates the spherical coordinates of any specified number of equipotential lines. It is used only after convergence is achieved in the main portion of the program.

(14) RHSCAL generates space charge values along center lines of the various annuli formed by the trajectories. It interpolates for these values at each mesh point, as required in the solution of Poisson's equation.

(15) PSICAL calculates the magnetic flux $\psi$ of the magnetic field at each mesh point.

(16) BRBZ calculates $B_r$ and $B_z$, the $r$ and $z$ components of the magnetic field, at each mesh point.

All calculations were done on the IBM 7094 II/7044 Direct Couple System computer. The program will perform about 210 integration steps per minute. This figure is based on trajectory calculations for eight electron energy groups.

About 15,000 storages are required for the program. This does not include that required for the solution of Poisson's equation. An additional 10,000 storage locations are needed for a solution to Poisson's equation similar to that described in reference 2.

The FORTRAN program is listed in appendix B, FORTRAN symbols are defined in appendix C, and flow charts are presented in appendix D.
Using the Program

The program always starts by reading in a nine-card input data deck. The contents of each of these cards is now described.

Card 1 (Format 55H. . .)
Contains any desired information to identify the particular case being run (card columns 1 to 55 only). This will be printed out at the top of a listing.

Card 2 (Format 4E10.3)
- R  radius of collector, \( R \), cm
- \( B_0 \)  magnetic flux density, \( B_0 \), G
- DTHTD \( \theta \)  step size used in Runge-Kutta integration (spherical coordinate), \( \Delta \theta \), deg
- TH1 cone angle, \( \theta_1 \), deg

Card 3 (Format 8E10.3)
- \( Z_{I0} \)  initial current\(^1\), \( I_0 \), A
- \( Z_{JO} \)  current density\(^1\), \( J_0 \), A/cm\(^2\)
- V0  initial electric potential, \( V_0 \), V
- DELR  \( r \)  step size used in Runge-Kutta integration (cylindrical coordinates), \( \Delta r \)
- CONFLO indicates confined flow when equal to 1 and Brillouin flow when equal to 0
- RAD radius of collector entry hole, \( a \), cm
- FLOOP number of iterations for space charge calculations (If no space charge calculation is desired, this value should be set to 1.)

Card 4 (Format 40I2)
- NT  number of trajectories to be calculated
- KPOI printout frequency control, (e.g., 4 means to print results at every fourth integration step of a trajectory)
- N1  integrate trajectories N1 to N2 in steps of N3
- N2
- N3

\(^1\)These two values are used only in the calculation of the source term in Poisson's equation when space charge iteration is necessary.
IBAR when nonzero, indicates boundary values will be supplied along the negative polar axis
LOOP counter for number of space charge loops
KPOSC print control for interim space charge calculations

Card 5 (Format 4F5.1)
These values are used by the integration subroutine in the spherical coordinate system, which is the system used in the beginning portion of the program.

\[ \text{THTSAC} \] step-size controls; increase step size DTH by a factor of THTSAC
\[ \text{THTSDEL} \] every THTSDEL (deg) in the \( \theta_5 \) range from THTSWHI (deg) to THTSWLO (deg)

Card 6 (Format 16F5.3)
VSMI(I), electron energy classes for each of NT trajectories to be solved
I = 1, NT for

Card 7 (Format 12, 10F5.3)
This calculation is done only once, after convergence has been achieved in the main part of the program.
NEQL number of equipotential lines to be calculated
EQL(I), array of potential values for each equipotential line
I = 1, NEQL

Card 8 (Format 16F5.3)
XT(J, 16), boundary values on surface of spherical collector from initial J = 1, 16 cone angle \( \theta_1 \) to 180°, V

Card 9 (Format 16F5.3)
XT(J, 16) continuation of card 8
J = 17, JF

Two sample test runs are given in appendix A. These runs were chosen to more clearly illustrate the makeup of the input data deck and to show the various options in output format available to the user.

CONCLUDING REMARKS

We have described the numerical technique used to obtain a solution to a problem
associated with the design of an axisymmetric, spherical, depressed collector. Varying initial conditions include current density of electrons, surface potential on the collector, collector geometry, and the velocity and position of entering electron groups.

Of particular importance to the successful solution is the scheme used near the entry hole in the collector. Here the entering electron stream is of highest velocity and greatest density, giving rise to computational problems, especially in cases of high initial current densities.

Calculations by others in the past were successful in handling current densities to $50 \, \text{A/cm}^2$ before numerical difficulties precluded further computation. The numerical approach described in this report is used to solve systems with current densities to $100 \, \text{A/cm}^2$.

Computational stability was achieved primarily by using a graduated mesh in both the $r$ and $\theta$ directions, with greatest mesh point density in the hole region. In addition, computational efficiency was improved by taking advantage of the fact that all entering electron groups were distributed uniformly both in positioning and velocity grouping. That is, the fastest group entered closest to the axis, and the slowest group entered nearest the edge of the entry hole.

Space charge effects were found to be negligible for initial current densities less than $10 \, \text{A/cm}^2$. Consequently, the problem was reduced to requiring only one iteration and essentially solving Laplace's equation to obtain the potential distribution inside the collector.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 2, 1972,
111-05.
APPENDIX A

SAMPLE PROBLEMS

Short, partial, representative computer output listings of two sample runs are presented. Run 1, depicted graphically in figure 6, is for a 0.2 spike, no space charge iterations, confined flow, and a cone angle of 60°. Run 2, figure 7, is for no spike, space charge effects calculated, Brillouin flow, and a cone angle of 45°.

Figure 7 shows a poor pattern. Beam spread and separation is rather nonuniform and erratic. In contrast, figure 6 shows well-shaped and well-behaved beams with good spread and uniform spacing.

Figure 6. - Sample run 1. No iterations necessary (current density, 8 A/cm²); 0.2 spoke; cone angle, 60°.

Figure 7. - Sample run 2. Five iterations for space charge effect; no spike; cone angle, 60°.
### Sample Run 1

#### Sample Run No. 1

**NO SPACE CHANGE EFFECTS**

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**CLIPUP EVERY 4 STEPS.**

**INITIAL VALUES**

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| 4.0 | -0.903 |
| 5.0 | -0.411 |
| 3.0 | -0.277 |
| 4.0 | -0.198 |
| 5.0 | -0.148 |
| 6.0 | -0.108 |
| 7.0 | -0.076 |
| 8.0 | -0.048 |
| 9.0 | -0.028 |
| 10.0 | -0.014 |
| 11.0 | -0.005 |

**INPUT**

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| ETA | 7.590 |
| CMF | 9.590 |
| ETA | 11.590 |
| CMF | 13.590 |
| ETA | 15.590 |
| CMF | 17.590 |
| ETA | 19.590 |
| CMF | 0.0 |

**POTENTIAL FIELD**

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| Φ1 | 6.75E+03 |
| PHI1 | 1.50E+03 |
| C. | 8.00E+03 |
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| C. | 1.40E+03 |
| C. | 1.70E+03 |
| C. | 2.00E+03 |
| C. | 2.30E+03 |
| C. | 2.60E+03 |
| C. | 2.90E+03 |
| C. | 3.20E+03 |
| C. | 3.50E+03 |
| C. | 3.80E+03 |
| C. | 4.10E+03 |
| C. | 4.40E+03 |
| C. | 4.70E+03 |
| C. | 5.00E+03 |
| C. | 5.30E+03 |
| C. | 5.60E+03 |
| C. | 5.90E+03 |
| C. | 6.20E+03 |
| C. | 6.50E+03 |
| C. | 6.80E+03 |
| C. | 7.10E+03 |
| C. | 7.40E+03 |
| C. | 7.70E+03 |
| C. | 8.00E+03 |
| C. | 8.30E+03 |
| C. | 8.60E+03 |
| C. | 8.90E+03 |
| C. | 9.20E+03 |
| C. | 9.50E+03 |
| C. | 9.80E+03 |
| C. | 1.01E+04 |

**CLIPUP NO. 1**

| THETA | 0.200 |
| CMFCA | 0.300 |
| ETA | 0.400 |
| CMF | 0.500 |
| ETA | 0.600 |
| CMF | 0.700 |
| ETA | 0.800 |
| CMF | 0.900 |
| ETA | 1.000 |
| CMF | 1.100 |

**POTENTIAL FIELD**

| θ | 4.50E+03 |
| Φ1 | 6.75E+03 |
| PHI1 | 1.50E+03 |
| C. | 8.00E+03 |
| C. | 1.10E+03 |
| C. | 1.40E+03 |
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| C. | 2.30E+03 |
| C. | 2.60E+03 |
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| C. | 3.50E+03 |
| C. | 3.80E+03 |
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| C. | 4.40E+03 |
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| C. | 5.00E+03 |
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| C. | 6.50E+03 |
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| C. | 7.70E+03 |
| C. | 8.00E+03 |
| C. | 8.30E+03 |
| C. | 8.60E+03 |
| C. | 8.90E+03 |
| C. | 9.20E+03 |
| C. | 9.50E+03 |
| C. | 9.80E+03 |
| C. | 1.01E+04 |

**CLIPUP NO. 2**

| THETA | 0.200 |
| CMFCA | 0.300 |
| ETA | 0.400 |
| CMF | 0.500 |
| ETA | 0.600 |
| CMF | 0.700 |
| ETA | 0.800 |
| CMF | 0.900 |
| ETA | 1.000 |
| CMF | 1.100 |

**POTENTIAL FIELD**

| θ | 4.50E+03 |
| Φ1 | 6.75E+03 |
| PHI1 | 1.50E+03 |
| C. | 8.00E+03 |
| C. | 1.10E+03 |
| C. | 1.40E+03 |
| C. | 1.70E+03 |
| C. | 2.00E+03 |
| C. | 2.30E+03 |
| C. | 2.60E+03 |
| C. | 2.90E+03 |
| C. | 3.20E+03 |
| C. | 3.50E+03 |
| C. | 3.80E+03 |
| C. | 4.10E+03 |
| C. | 4.40E+03 |
| C. | 4.70E+03 |
| C. | 5.00E+03 |
| C. | 5.30E+03 |
| C. | 5.60E+03 |
| C. | 5.90E+03 |
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| C. | 6.50E+03 |
| C. | 6.80E+03 |
| C. | 7.10E+03 |
| C. | 7.40E+03 |
| C. | 7.70E+03 |
| C. | 8.00E+03 |
| C. | 8.30E+03 |
| C. | 8.60E+03 |
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| C. | 9.20E+03 |
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**TABLE OF Phi-VALUES FOR FOURTEENTH LINES**

**THETA** V. 0.0 90° V. 0.72° V. 0.50° V. 0.30° V. 0.20°

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**SWITCH TO R-J COORDINATE SYSTEM**
APPENDIX B

FORTRAN PROGRAM

C* PROGRAM FOR CALCULATING POTENTIAL FIELD IN AN AXISYMMETRIC
C* ELECTROSTATIC COLLECTOR, TAKING SPACE CHARGE INTO EFFECT.
C* EXECUTIVE ROUTINE FOR PROCESSING MULTIPLE CASES
C* AND MAIN CONTROL OF LOGICAL FLOW THROUGHOUT PROGRAM.
C*
C*** COMMON HO.CVF.DOHR(10),DTHD(16),DTTHD,DTHTR,DVDR(10),
1 UVUT(10),DVDFR,DVDRZ,EQL(10),ETA0IBAR,IT,JF,JK,KPO,KPOI,
2 KPOSK,UDQ,AEOL.N1,N2,N3,EPH2A(10),PI,P12,PSIA(10),PSIO(10),R,
3 RAN,HRH(17),RHOP(10),RHOA(10),RHT(10),RILV(10),SPCH,TAU,2,
4 THD(10),THMD(25),THSDEL,THSFAL,THSHEL,THSWL,
5 THTD(10),THTR(10),TH1,TH2,V(17,25),VO,VR,
6 VSM(10),V1I(10),W(16),XCTR,XTEM(15,16),Y(16),ZJO
EQUIVALENCE (DH1.Y(1),(TH.Y(2),(RH.Y(3),(RH2.RHP,Y(4)),
1 (RHI.P,Y(5))),(RH2P,Y(6)),(W(1),DELK),(W(2),AR),
1 (W(3),Z),(W(4),Z2,ZP),(W(5),Z1P),(W(16),Z2P)
C***
C SUBROUTINES-
C 1. MAIN
C 2. INPUT
C 3. INTGKC
C 4. DE
C 5. DERIV
C 6. RK
C 7. MESH
C 8. RZTRAJ
C 9. RZNERADZ
C 10. RZDE
C 11. OUT
C 12. VFIELD
C 13. EQLINES
C 14. RHSCAL (A.ZJO,VO,TH1,THFRH2),XV2,IBAR.RH,TH2)
C 15. PSICAL (RZ,PSI,BO,ASM)
C 16. HRH2Z (RZ,RH,BO,ASM)
C
10 FORMAT(5H0PASS.I3.1X.8HFOLLOW-)
C
C INITIALIZE SPACE CHARGE FIELD TO ZERO TO SET UP
C LAPLACE EQUATION.
DN I=1,16
DN J=1,25
1 XT(J,1)=0.0
C
C INITIALIZE MESH SIZE, BOUNDARY CONDITIONS, CONVERGENCE
C CRITERIA.
P1=0.0
LKTR=1
CALL INPUT
L=LOOP

28
COMPUTE POTENTIAL FIELD FOR GIVEN SPACE CHARGE FIELD.

CALL VFIELD
IF(SPCH.EQ.0.) GO TO 3

CONDITIONAL OUTPUT OF POTENTIAL FIELD.
WRITE(*,10) LKTR

CALL OUT

INTEGRATION OF ELECTRON CLASS TRAJECTORIES THROUGH THE COMPUTED POTENTIAL FIELD.

CALL INTGRN
L=1-1
L0CPL=LOOP-1
JT=1
LKTR=LKTR+1
CALL INPUT
IF(L.EQ.1) GO TO 2

N1=2
N2=8
N3=1
SPCH=0.0
KPO=0
IF(L.EQ.1) GO TO 2

STOP
END

SUBROUTINE INPUT

READS AND PRINTS INITIAL CONDITIONS, PARAMETERS AND MISCELLANEOUS DATA REQUIRED FOR INTERNAL USE BY PROGRAM (OUTPUT FREQUENCY, LOOPING INDICES, CONVERGENCE CRITERIA, ETC.).

***
COMMON 80.CVF,DRHT(10),DRHM(16),DTHTD,DTHTR,DVDRT(10),
1 DVDTI(10),DVDR,(10),EQL(10),ETA0,ITJ,ITJ,KPO,KPO1,
2 KPOSC,LOOP,REQL,N1,N2,N3,PM2A(10),PI,PI2,PSIA(10),PSID(10),R,
3 RAD,RHM(17),RHOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAU02,
4 TH0(10),THM2(25),THSDEL,THSFAC,THSWH1,THSWLO,
5 THTD(10),THTR(10),TH1,TH2,V(17,25),V0,VR,
6 VSM(10),VT(10),W(16),XCRK,XCT7(25,16),Y(16),ZJ0,
7 EQUIVALENCE (U,1),(1),(1),(1),(1),(1),(1),(1),(1),**
8 (RH1P,Y(5)) (RH2P,Y(6)),(W(1),DELK),(W(2),AR),
9 (W(3),Z1),(W(4),Z2P),(W(5),Z1P),(W(6),Z2P)

***
16 FORMAT(2X,3H1O=,1PE13.3)
17 FORMAT(2X,3HJO=,1PE13.3,4X,5HR==E10.3)
18 FORMAT(2X,3HVO=,1PE13.3,4X,5HB==E10.3)
19 FORMAT(2X,6HCMFGA=,1PE10.3,4X,5HETA0=,E10.3)
20 FORMAT(2X,6HFO==,1PE10.3,4X,5HKP==,E10.3)
21 FORMAT(1X,6HINPUT=)
22 FORMAT(40I2)
23 FORMAT(16H BRILLOUIN FLOW.).
C INITIALIZE TRAJECTORY CALCULATIONS.
C FIRST TIME SETTING OF INITIAL CONDITIONS, BOUNDARY
C PARAMETERS, CONVERGENCE CRITERIA.

JT=1
IF (PI NE 0.) GO TO 9

C CONSTANTS.
PI=3.14159265
PI2=2.*PI
CVF=180./PI
FO=8.*R60E-14
FTAO=1.753E+15
OM=1.414E+10

C READ/WRITE.
WRITE(6,31)
READ(5,32)
WRITE(6,32)
READ(5,33) R,RO,DTHTD,TH1
READ(5,33) ZU,J0,VO,DELR,CONFLO,RAD
READ(5,72) NT,KPOI,N1,N2,N3,IBAK,LOOP,KPOSC

C INCREASE STEP SIZE DTH, BY A FACTOR
C OF (THSFAC) EVERY (THSDEL) DEGREES
C IN THE THETA(5) RANGE FROM (THSWH1) TO (THSWL)

READ(5,41) THSFAC,THSDEL,THSWH1,THSWL
READ(5,76) (VSMI(I), I=1,NT)
READ(5,35) NFOL, (EWL(I), I=1,NEWL)
TH2=177.-TH1/60.
JF=25
KPO=0
ZKP=710/(VO*1.5)
TFST MAGNITUDE OF CURRENT DENSITY
TO SEE IF SPACE CHARGE CALCULATIONS SHOULD BE MADE.

\[ SPCH = \begin{cases} 0.0 & \text{IF}(700 \lt 10.0) \text{ GO TO 14} \\ 1.0 & \text{WRITE}(6.43) \text{ LOOP} \\ 0.0 & \text{GO TO 15} \end{cases} \]

\[ 14 \text{ WRITE}(6.42) \]

\[ 15 \text{ TAU} = (R \ast R) / (2 \ast \text{ETA0} \ast \text{VO}) \]

INDICATE TYPE OF FLOW.

\[ \text{IF}(\text{CONFLO} \lt 0.0) \text{ GO TO 1} \]

\[ \text{WRITE}(6.34) \]

\[ \text{GO TO 2} \]

\[ 1 \text{ WRITE}(6.23) \]

\[ 2 \text{ CONTINUE} \]

READ BOUNDARY CONDITIONS.

\[ \text{READ} \{5.26\} (X(J,16), J=1,JF) \]

\[ \text{DO} 3 J=1,JF \]

\[ 3 X(J,16) = -X(I,16) \]

\[ XCTR = X(I,16) \]

\[ \text{DO} 4 I=1,15 \]

\[ 4 X(I,1) = XCTR \]

INPUT DATA CHECK.

\[ \text{IF}(\text{TH}1 \geq 0.0 \text{ AND } \text{TH}1 \leq 174.0) \text{ GO TO 5} \]

\[ \text{WRITE}(6.36) \text{ TH1} \]

\[ \text{CALL EXIT} \]

\[ 5 \text{ IF}(\text{DTHTD} \lt 0.0 \text{ AND } \text{DTHTD} \geq -0.05) \text{ GO TO 6} \]

\[ \text{WRITE}(6.37) \text{ CTHTD} \]

\[ \text{CALL EXIT} \]

\[ 6 \text{ IF}(\text{VSM}(1) \leq \text{ABS}(XCTR)) \text{ GO TO 7} \]

\[ \text{TO} = \text{VSM}(\text{NT}) \]

\[ \text{WRITE}(6.38) \text{ XCTR} \]

\[ \text{WRITE}(6.40) \text{ TO} \]

\[ \text{CALL EXIT} \]

INITIAL OUTPUT.

\[ \text{WRITE}\{6.21\} \]

\[ \text{WRITE}\{6.20\} \text{ KO} \]

\[ \text{WRITE}\{6.19\} \text{ CM} \]

\[ \text{WRITE}\{6.16\} \text{ ZNO} \]

\[ \text{WRITE}\{6.17\} \text{ JNO} \]

\[ \text{WRITE}\{6.18\} \text{ VO} \]

\[ \text{WRITE}\{6.25\} \text{ TAU02} \]

\[ \text{WRITE}\{6.24\} \text{ KPNI} \]

\[ \text{DTHTR} = \text{DTHTD} / \text{CVF} \]

DETERMINE ENTRY CONDITIONS.

\[ \text{DIMENSION ALPH}(10), \text{DELI}(10) \]

\[ \text{TO} = \text{RAD} / \text{FLOAT}(\text{NT}-1) \]

\[ \text{DO} 8 I=1,\text{NT} \]

\[ 8 \text{ T1} = \text{FLOAT}(I-1) \ast \text{TO} \]

\[ \text{DELI}(I) = \text{ARSIN}(T1/R) \]
ALPH(I)=1.0*FLOAT(I)
T2=PI-DELI(I)
ST=SIN(T2)
CT=COS(T2)
ARF=R*ST
ZED=R*(1.0+CT)
CALL PSICAL(ARF,ZED,PSI,BO.RAD)
PSIO(I)=PSI
IF(CONFLO.EQ.0.) PSIO(I)=0.0
B CONTINUE

C SETTING OF RHO-THETA VARIABLE MESH.
CALL MESH

C SPREAD INITIAL TRAJECTORY VALUES.
C EVERY TIME INITIALIZATION OF TRAJECTORIES.
9 DO 10 IT=2,NT
  RHT(IT)=.9955
  ALPHA=ALPH(IT)/CVF
  GAMMA=PI-ALPHA-DELI(IT)+DTHTR
  T2=-RHT(IT)/(DTHTR*SIN(GAMMA))
  T3=SIN(ALPHA+DELI(IT))-SIN(GAMMA)
  RH2P=-T2*T3
  ORHT(IT)=RH2P
  RHOP(IT)=RH2P
10 THO(IT)=VSMI(IT)/(1.+RHOP(IT)**2)

C CALCULATE VT(I).
DO 11 IT=1,NT
  THTR(IT)=PI-DELI(IT)
  THTO(IT)=THTR(IT)*CVF
11 VT (IT)=VO*VSMI(IT)

C PRINT INITIAL TRAJECTORY VALUES.
WRITE(6,29)
WRITE(6,27)
WRITE(6,30)
DO 12 IT=2,8
  DELI0=DELI(IT)*CVF
  T1=180.-DELI0
12 WRITE(6,28)IT.,DELI(IT),DELI0,T1,VSMI(IT),VT(IT),ALPH(IT),PSIO(IT)

C
DO 13 IT=2,NT
  TH=THTR(IT)
  RH=RHT(IT)
  RHP=RHOP(IT)
  CALL DE
13 RHOPP(IT)=RH2P

C RETURN
END
SUBROUTINE IISTGRN

C* CONTROLS INTEGRATION LOOPING OF EQUATIONS OF MOTION.
C* STEP-SIZE, CHANGE-OVER OF COORDINATE SYSTEMS AND THE
C* NUMBER OF ITERATIONS REQUIRED FOR CONVERGENT SOLUTION.
.
C*** COMMON R0, CVF, DRHT(10), DRHM(16), DTHTD, DTHTR, DVO(D16),
1 DVO(10), DVOZ, EQL(10), ETA0, TBAR, IT, JF, JT, KPO, KPOl,
2 KPOS, LOP(10), N1, N2, N3, PHD2A(10), PI, PI2, PSIA(10), PSI0(10), R,
3 RAD, RHM(17), RHOM(10), RHOPP(10), RHT(10), RHV(10), SPCH, STHO2,
4 THD(10), THMC(25), THSDEL, THSFAC, THSWHI, THSWLO,
5 THTD(10), THTR(10), THI, TH2, V(17, 25), VO, VR,
6 VSMI(10), V01(10), W(16), XCTR, XT(25, 16), Y(16), ZJC
EQUIVALENCE (DTH, Y(1)), (TH, Y(2)), (RH, Y(3)), (RH2, RHP, Y(4)),
* (RH1P, Y(5)), (RH2P, Y(6)), (W(1), DELR), (V(2), AR),
* (W(3), Z), (W(4), Z2, ZP), (W(5), Z1P), (W(6), Z2P)
C***
10 FORMAT(7H0, DTHTD=1PE9.2)
EXTERNAL DE
DTH=OHTDTR

C CHECK FOR SPACE CHARGE ITERATION.
IF(SPCH.EQ.0.) GO TO 1
N1=5
N2=5
N3=1
IF(LS.W.EQ.9) GO TO 1
LSW=9
THO=THTD(5)

1 DO 2 IT=N1, N2, N3
TH =THTR(IT)
RH =RHT(10)
RHP=DRHT(10)
GOL=RH
CALL RK(DE)
RHOP (IT)=RHP
RHOPP (IT)=RH2P
THTR(IT)=TH
THTD(IT)=THTR(IT)*CVF
RHT(IT)=RH
2 ORHT(IT)=RHP

C INCREASE STEP SIZE DTH, BY A FACTOR
C OF (THSFAC) EVERY (THSDEL) DEGREES
C IN FROM (THSWHI) TO (THSWLO) DEGREES.
IF(THTD(IT).GT.THSMHI) GO TO 3
IF(THTD(IT).LT.THSMLO) THSWHI=0.0
THSWHI=THSWHI-THSDEL
THTD(IT)=THSFAC*DTH
T1=CVF*DTH
WRITE(6, 10) T1

3 CONTINUE
JT=JT+1
CALL OUT
IF(SPCH.EQ.0.) GO TO 4
IF(RH.GT.RHM(7)) GO TO 1
SUBROUTINE Cf

C CONTAINS EQUATIONS OF MOTION IN SPHERICAL
C COORDINATE SYSTEM.

C*** COMMON BO,CVF,DRHT(10),ORHM(16),DTHD,DTHTR,DVDR(10),
1 DVDT(10),DVDRD,DVDRZ,EQL(10),ETA0,IBAR,IT,JFJT,KPO,KPOI,
2 KPOSC,L00P,N1,N2,N3,PSI1A(10),PI,PI2,PSIA(10),PSIO(10),R,
3 RAD,PHD(17),RHO(10),RHOP(10),RHOP(10),RHV(10),SPCH,TAU02,
4 THD(16),THMC(25),THSDEL,THSDEL,THSDEL,THSDEL,THSDEL,
5 TTHD(10),THTR(10),TH1,TH2,TH1,TH2,TH1,TH2,TH1,TH2,
6 VSMI(10),VT(10),W(16),XCTR,XCTR,XCTR,XCTR,XCTR,
7 EQUIVALENCE (DTH,Y(1)),(DTH,Y(2)),(DTH,Y(3)),(DTH,Y(4)),
* (DTH,Y(5)),(DTH,Y(6)),(DTH,Y(7)),(W(1),DELK),(W(2),AR),
* (W(3),Z1),(W(4),Z2,ZP),(W(5),Z1P),(W(6),Z2P),
C***
ST=SN(TH)
ST2=ST**2
CT=CS(TH)

C COMPUTE DV/ORHO AND DV/OTHETA.
CALL DERIV
ARE=R*ST
ZED=R*(1.*R*CT)

C COMPUTE PSI.
CALL PSICAL(ARE,ZED,PSI,BO,RAD)

C COMPUTE BR AND BZ.
CALL BRHZ(ARE,ZED,BR,BZ,BO,RAD)
T1=-F1/ARE*ARF*PI2
PHD=T1*PSI-PSIO(1)
PHD2=PHD**2
PHD2A(1)=PHD
PSIA(IT)=PSI

C COMPUTES FIRST AND SECOND DERIVATIVES OF RHO WITH
C RESPECT TC THETA (SPHERICAL COORDINATES).
DVMDRH=R*BR*ST*Z*CT
DVMTHD=R*RH*R*CT-B*ST
T1=RH*R*R2**2*ST*PHD
PAR1=OVDR/(TI/R*ST*PD*OVMDTH
PAR2=OVOT/(TI/R-TI*OVMDRH
T2=(VSMI(TI)+VT/(TI))/TAUC2*RH/RH*(1.0+(RHP/RH)**2))
THD2=T2-(PD2*ST2)/(1.0+(RHP/RH)**2)
THD(TI)=THD2
T1=RH*ST*PD
T2=(T1*ST*PD)/THD2
T5=FAO/R
T3=T5/THD2
T2=-(RHP*ST*CT*PD2/THD2
T3=(2.0*RHP*PD2)/RH
T4=-<RHP*T5)/<RH*RH*THD2)
RHP=T1+T2+T3*PAR1
RHP=RH2
C
RETURN
END

SUBROUTINE DERIV

C
C* CALCULATES THE RATES OF CHANGE OF THE POTENTIAL
C* FIELDS IN SPHERICAL COORDINATES FOR ANY SPECIFIED
C* INTERIOR POINT.
C*
C***
COMMON 80,CVF,DRH(16),DRH(16),DHTR(16),DHTR(16),
1 DRDR(16),DRDOR,DRDU(16),DG(16),ETA0,IBAK,IT,IF,JI,KPO,KPOI,
2 KPOS0,LOOS*0.1,2.0.3,PH02A,PI12,PSIA(10),PSID(10),R,
3 RA0,TH(17),RHP(10),RHOP(10),RHT(10),RHP(10),SPCH,TAU02,
4 THD1(T10),THD(10),THSD(10),THSWAT,THSWHI,THSWLI,
5 THD(10),THTR(10),TH1,TH2,TH(17.17),THS,VR,
6 VSMI(10),VT(10),V(16),XC,XT(17.25),Y(16),ZJ0
C***
C EQUVALENCE TH(1),Y(2),TH(1),TH(1),TH(1),TH(1),TH(1),TH(1),TH(1),
* (RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),
* (RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),
* (RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),(RHP,Y(5)),
C***
T2=TH*CVF
T3=15.
C
C DETERMINE RHO INTERPOLATION FACTOR (FR).
DO 1 I=1,17
1 CONTINUE
2 IF(THD(I).GT.RH) GO TO 2
1 CONTINUE
2 IF(I.EQ.17) I=16
FR=(RH-RHM(I-1))/DRH(I-1)
C
C DETERMINE THETA INTERPOLATION FACTOR (FT).
DO 3 J=1,25
3 CONTINUE
4 IF(T2.GT.TH1) T3=.25
FT=(T2-THD(I-1))/T3
T3=T3/CVF
C

VP1=V(I-1,J-1)+FT*(V(I-1,J)-V(I-1,J-1))
VP2=V(I,J-1)+FT*(V(I,J)-V(I,J-1))
VP3=V(I-1,J) + FR*(V(I,J)-V(I-1,J))
VP4=V(I-1,J-1) + FR*(V(I,J-1)-V(I-1,J-1))
C
VT(IT)=VP1+FR*(VP2-VP1)
C EVA L U TA TES DV/D RH0 AND DV/D TH ETA AT (RH,T2) FROM CURRENT
C POTENTIAL FIELD.
DVDR(IT)=(VP2-VP1)/DRHM(I-1)*VG
DVDT(IT)=(VF3-VP4)/I3)*VO
IF(THTD(IT)<T1*TH2) DVDT(IT)=0.0
C
RETURN
END

SUBROUTINE RK(DE)
C*
C INTEGRATES ONE TRAJECTORY ONE STEP USING RUNGE-KUTTA
C FORMULAS IN SPHERICAL COORDINATES.
C*
C***
COMMON HO,CVF,DRHT(10),DRHM(16),DTHTD,DTHTR,DVDR(10),
1 DVDT(10),DRV0R,DVRD7,EQL(10),ETA0,IBAR,IT,JF,JI,KPO,KPO1,
2 KPOS,LOOP,NEQL,N1,N2,N3,PHD2A(10),PI,PI2,PSIA(10),PSI0(10),R3,
3 RAO,RHM(17),RAOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAUD2,
4 THD(10,THMC(25),THSDEL,THSFAC,THSWLC,THSWLU,
5 THTO(10),THTR(10),TH1,TH2,V(17,25),VO,VR,
6 VSM1(10),VT(10),W(16),XC TR,XT(25,16),Y(16),ZJO,
* EQUIVALENCE (DTH,Y(1)),(TH,Y(2)),(RH,Y(3)),(RH2,RHP,Y(4))
* (RH1P,Y(5)) , (RH2P,Y(6)),(W(1)+DEL), (W(2),AR),
* (W(3),7), (414), (W5,Z1P),(W6),Z2P)
C***
Y(15)=Y(3)
Y(16)=Y(4)
CALL DE
Y(7)=Y(1)*Y(5)
Y(8)=Y(1)*Y(6)
Y(9)=Y(2)+Y(1)/2
Y(10)=Y(15)+.5*Y(7)
Y(11)=Y(16)+.5*Y(8)
CALL DE
Y(12)=Y(1)*Y(5)
Y(13)=Y(1)*Y(6)
Y(14)=Y(2)+Y(1)/2
Y(15)=Y(19)+Y(11)
Y(16)=Y(16)+Y(12)
CALL DE
Y(17)=Y(1)*Y(5)
Y(18)=Y(1)*Y(6)
Y(19)=Y(2)+Y(1)/2
Y(20)=Y(15)+Y(11)
Y(21)=Y(16)+Y(12)
CALL DE
Y(13)=Y(1)*Y(5)
Y(14)=Y(1)*Y(6)
C
Y(3)=Y(15)+Y(7)+2*Y(9)+2*Y(11)+Y(13))/6.
Y(4)=Y(16)+(Y(8)+2*Y(10)+2*Y(12)+Y(14))/6.
36
SUBROUTINE MFSH

C* FOR ANY SPECIFIED COLLECTOR GEOMETRY, GENERATES
C* GRADUATED MESH POINT ARRAYS IN BOTH THE RHO AND
C* THETA DIRECTIONS.
C*
C*** COMMON 60,CVF,DRHT(16),DRHM(16),DTHTD,DTHTR,DVDR(10).
1 DVT(10),DVRDR,DVKDZ,EWL(10),ETA0,IBAR,IT,IF,JT,KFO,KPO1.
2 KPOSC.LOOP,NEQL,N1,N2,N3,P02A(IU),PI,PIZ,PSIA(10),PS10(10),R.
3 RAD,DRH(17),RHO(10),RHM(17),RHM(10),SPCH,TAU02.
4 THD(10),THMD(25),THSDEL,THSFAC,THSWEI,THSLU.
5 THD(10),THTR(10),TH1,TH2,VI(17),V0,VR.
6 VM1(10),VI(10),W(16),XCTR,XT(15,15),Y(16),ZJG
7 EQUIVALENCE (DRH,Y(1)),(TH,Y(2)),(RH,Y(3)),(RH2,RHP,Y(4)),
8 (RH1,Y(5)),(RH2,Y(6)),(W(1),DELK),(W(2),AR),
9 (W(3),Z),(W(4),Z2,ZP),(W(5),Z1P),(W(6),Z2P)
C***
C GENERATE RHO AND DELRHO ARRAYS.
T1=0.8/120.
RHM(17)=1.0
C GENERATE VARIABLE MESH RHO IN RHM ARRAY RANGING FROM
C RHM(1)=0 TO RHM(17)=1 WITH FINE STEP AT END.
DO 1 I=1,16
J=17-I
DRHM(J)=FLOAT(I)*T1
1 RHM(J)=RHM(J+1)-DRHM(J)
DRHM(1)=0.2
RHM(1)=0.0
C GENERATE THETA ARRAY.
JR=TH1/15.
JK=12-JK
C GENERATE VARIABLE MESH THETA DEGREES IN THMD ARRAY
C RANGING FROM THMD(1)=TH1 TO THMD(25)=180 WITH FINE STEP
C AT END.
DO 2 J=1,JK
2 THMD(J)=TH1+15.*FLOAT(J-1)
JK1=JK+1
JK2=JK+2
THMD(JK1)=177.*-25.*FLOAT(JK)
TH2=THMD(JK1)
DO 3 J=JK2,JK
3 THMD(J)=THMD(JK1)+.25.*FLOAT(J-JK1)
RETURN
END
SUBROUTINE RZTRAJ

C* CONTROLS INTEGRATION OF EQUATIONS OF MOTION IN THE
C* R-Z RECTILINEAR COORDINATE SYSTEM.
C COVERT FROM SPHERICAL COORDINATES (RHO,THETA) TO CYLINDRICAL
C COORDINATES (R,Z) AND CONTINUE INTEGRATION OF TRAJECTORY UNTIL
C IT REACHES SPHERICAL SURFACE.
C*

C*** COMMON RHO,CVF,DHT(10),DROH(16),DTH0,T,DTHTR,DDVR(10),
1 DDVR(10),DDVDR,DDVDRD,EUL(10),ETAO,IBAR,IT,JC,JE,JKP,KPO1,
2 KPOSC,LOOP,NEG,L1,N2,N3,PHOA(10),PI,P2,PSIA(10),PSIO(10),R,
3 RAD,RHMI(17),RHOH(10),RHOP(10),RHOP(10),RHQ(10),RHF(10),
4 RHQ(10),RHF(10),RHQ(10),SPCH,TAU02,
5 THD(10),THFQG(25),THSGF,THSWF,THSWO,
6 THD(10),THFQG(25),THSGF,THSWF,THSWO,
7 VSMI(10),Y(16),XCTR,XE(125,16),Y(16),Z10
EQUIVALENCE (DT,Y(1)),(TH,Y(2)),(RH,Y(3)),(RH2,RHP,Y(4)),
* (RHP,Y(5)),(RHP,Y(6)),(W(1),DELH),(W(2),AR),
* (W(3),Z1),(W(4),Z2),(W5),(W5),(W6),(W6),Z2P)
C***
10 FORMAT(33HSWITCH TO R-Z COORDINATE SYSTEM-)
11 FORMAT(15H TRAJECTORY NO. t I 3, 2Xt 5HDELR = t F 4. 3') ,
.
12 FORMAT(2X,F7.2>3F7.3,1P5E12.3)
C EXTERNAL RZCE
WRITE(6,10)
WRITE(6,11) N2,DELH
WRITE(6,12)
C CONCAT TO R-Z COORDINATES.
AR=RHT(N2)*SINTH
Z =RHT(N2)*CCS(TH)
C INITIALIZE R-Z INTEGRATION.
TO=RHT(N2)*Z-RHT(N2)*AR
T1=RHT(N2)*Z+RHT(N2)*AR
ZP=TO/T1
CALL RZDE
THRZ=90.-CVF*(ATAN(Z/AR))
RHRZ=SQR(AR+AR+Z*Z)
T2=DVDRD-Z*DVDRR
TO=VSMI(T1)+VR
T1=I1+ZP*2
T3=2.*(TN/T1)
ZP=T2/T3
WRITE(6,13) THRZ,RHRZ,AR,Z,DVDRR,DVDRD,ZP,Z2P,VR
C INTEGRATE TO SURFACE.
1 CALL RZDK(RZCE)
WRITE(6,13) THRZ,RHRZ,AR,Z,DVDRR,DVDRD,ZP,Z2P,VR
IF(RHRZ.LT.1.0) GO TO 1
IF(N1.EQ.N2) TO=TO/0.0
C N2=N2-1
RETURN
FND
38
SUBROUTINE RZRMRZDE
C
C INTEGRATES ONE TRAJECTORY ONE STEP USING RUNGE-KUTTA
C FORMULAS IN CYLINDRICAL COORDINATES.
C
C COMMON 80,CVF,DRHT(10);DRHM(16),DTHTD,DTHTR,DVDR(10),
1 DVDT(10),DVDDR,DVDOZ,EQL(10),ETAO,IBAR,IT,IF,JT,KPO,KPOI,
2 KPOSC,LOOP,NEGL,N1,N2,N3,PHD2A(10),PI,PI2,PSIA(10),PS10(10),R,
3 RAD,RHM(17),RHOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAU02,
4 THD(10),THMD(25),THSDFL,THSFAC,THSHL,THSLLG,
5 THTO(10),THTR(10),THII,TH2,V1,TH25,V0,VR,
6 YSML(10),YN(10),XCT,XT(25,16),Y(16),ZJC
EQUIVALENCE (DTH,Y(1),(TH,Y(2),(RH,Y(3),(RH2,RHP,Y(4)),
*(RH1P,Y(5),(RH2P,Y(6)),(W(1),DELK),(W(2),AR),
*(W(3),Z),(W(4),Z2,ZP),(W(5),Z1P),(W(6),Z2P)
C
W(15)=W(3)
W(16)=W(4)
CALL RZDE
W(7)=W(1)+W(5)
W(8)=W(1)+W(6)
W(2)=W(2)+W(11)/2.
W(3)=W(15)+5*W(7)
W(4)=W(16)+5*W(8)
CALL RZDE
W(9)=W(1)+W(5)
W(10)=W(1)+W(6)
W(2)=W(15)+5*W(9)
W(4)=W(16)+5*W(10)
CALL RZDE
W(11)=W(1)+W(5)
W(12)=W(1)+W(6)
W(2)=W(2)+W(11)/2.
W(3)=W(15)+W(11)
W(4)=W(16)+W(12)
CALL RZDE
W(13)=W(1)+W(5)
W(14)=W(1)+W(6)
CALL RZDE
RETURN
END

SUBROUTINE RZDF
C
C COMPUTES FIRST AND SECOND DERIVATIVES OF Z WITH
C RESPECT TO R (CYLINDRICAL COORDINATES).
C
C COMMON 80,CVF,DRHT(10),DRHM(16),DTHTD,DTHTR,DVDR(10),
1 DVDT(10),DVDDR,DVDOZ,EQL(10),ETAO,IBAR,IT,IF,JT,KPO,KPOI,
2 KPOSC,LOOP,NEGL,N1,N2,N3,PHD2A(10),PI,PI2,PSIA(10),PS10(10),R,
3 RAD,RHM(17),RHOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAU02,
SUBROUTINE OUT
C* PRINTS OUT SELECTED DATA ALONG A TRAJECTORY.
C*
C*** COMMON 80.CVF,DRHT(10),OHTHD,OHTHR,OVDR(10),
1 DVDT(10),DVDRD,DVDRZ,EQL(10),ETA0,IBAR,IT,IF,IT,KPO,KPUI,
2 KPOSC.LOOP,AEQL,N1,N2,N3,PHD2A(10),PL,PI2,PSIA(10),PS10(10),R,
3 RAd,RHM(17),RHOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAUD2,
4 THD(10),THMC(25),THSDEL,THSFAC,THSWHI,THSWLO,
5 THTR(10),THTR(10),TH2,V(17,25),VO,VK,
6 VSMI(10),VT(10),W(16),XCTR,XT(25,16),Y(16),ZJO
EQUIVALENCE (OTH:Y(1)),(TH:Y(2)),(RH,Y(3)),(RH2,RHP,Y(4)),
* (RH1P,Y(5)),(RH2P,Y(6)),(W(1),DELR),(W(2),AR),
* (W(3),Z),(W(4),Z2,ZP),(W(5),Z1P),(W(6),Z2P)
C***
C FORMAT STATEMENTS.
10 FORMAT(10HODELTHETA=,F7.4,5H OEG.)
11 FORMAT(5H STEP.,5F)
12 FORMAT(1X,3HTRJ,2X,5HTHETA,2X,3HRHO,6X,
A 2HVT,7X,5HDVRH,7X,5HDVTH,6X,5HRH-PR,6X,6HRH-PPR,6X,4HTHD2,
B 6X,4HPHD2,6X,3HPSI)
13 FORMAT(1X,I2,F8.2,F6.3,1P8E11.3)
C C PRINT TITLES.
IF(K9.EQ.9) GO TO 1
K9=9
WRITE(6,10) CTHTD
WRITE(6,12)
C
1 IF(KPO.GT.1) GO TO 4
JT1=JT-1
WRITE(6,11) JT1
DO 2 I=N1,N2,N3
2
SUBROUTINE VFIELO

COMPUTES THE POTENTIAL FIELD V FOR A GIVEN SPACE CHARGE
FIELD BY SOLVING POISSON'S EQUATION.

POISS1 AND POISS2 ARE SUBROUTINES DESCRIBED BY THE TN
OF REFERENCE 2. THEY WERE DEVELOPED TO SOLVE POISSON'S
EQUATION IN A SUBREGION OF A SPHERE IN AN EFFICIENT
NON-ITERATIVE FASHION, USING A BLOCK DIAGONAL MATRIX
TECHNIQUE.

COMMON/BLOCK1/NR,NT,RA(17),TA(25),X(25,16),X1,AVS
COMMON/BLOCK2/CV,DRH(10),DRHM(16),DTHTO,DTHTD,DVDR(10),
1 CV,DRH(10),DVDR,DRHM(16),DTHTO,DTHTD,DVDR(10),,
2 KPGSC,LOP,KURL,NEQL,N2,N3,PH02A(10),PI,P12,PSIA(10),PSID(10),R,
3 RA,RHR(17),RHOP(10),RHOPP(10),RHT(10),RHV(10),SPCH,TAUO2,
4 THD(10),THMD(25),THSD,THSFL,TSHW,TSHWL,
5 THD(10),THMD(25),THSFL,TSHW,TSHWL,
6 VSM(10),VT(10),W(16),X(25,16),Y(16),Z(16),
7 EQUIVALENCE (DTF,Y(1)),(TH,Y(2)),(RH,Y(3)),(RHT,Y(4)),
8 (RHY(5)),(RHY(6)),(W(1)),AR),
9 (W(3)),(W(4)),(Z2),(W(5)),(Z1),(W(6)),
*** LOGICAL AVS
DIMENSION XB(15),TD(25)
***
23 FORMAT(16F5.3)
24 FORMAT(1X,F6.2,1P12E10.2)
25 FORMAT(1H INPUT=-52X,3HRHO)
26 FORMAT(11H OUTPUT=-52X,3HRHO)
27 FORMAT(13H00 BAR**)
28 FORMAT(4X,HR,8X,1HV)
29 FORMAT(2X,F5.3,3X,F6.3)
30 FORMAT(AM OUTPUT=-52X,3HRHO)
31 FORMAT(6H THETA,3X,16(F5.3,2X))
32 FORMAT(6H THETA,3X,12(F5.3,5X))
33 FORMAT(4H X1=-F5.2)
34 FORMAT(1X,F6.2,16F7.4)
35 FORMAT(154X,15HPOTENTIAL FIELD)
C.
X1=XCTR
NR=17
NR1=NR-1
NR2=NR-2
DO 1 I=1,17
 1 RA(I)=RHM(I)
NT=JF
NT1=NT-1
DO 2 J=1,NT
 2 DO K=1,MR1
     2 X(J,K)=XT(J,K)
   2 END
C.
C. IF AXIAL SPIKE IS PRESENT IN THIS PARTICULAR PROBLEM,
C. READ IN THESE VALUES AND DISPLAY THEM.
1 IF(TRAR.EQ.0) GO TO 4
DO 3 I=1,MR2
 3 X(NT,I)=XR(I)
 4 NT1=NT-1
 5 TA(J)=THMD(J)/CVF
 6 IF(1BAR.EQ.0) GO TO 9
C.
C. READ BAR VALUES.
AVS=TRUE.
1 IF(LS=.EQ.9) GO TO 7
 7 LSW=9
 8 READ(5,23) (X(NT,I), I=1,MR1)
 9 DO 6 I=1,MR2
 10 XB(I)=-X(NT,I)
 11 X(NT,I)=-X(NT,I)
C.
C. PRINT BAR VALUES.
7 WRITE(6,27)
 27 WRITE(6,28)
 28 WRITE(6,29) RA(I),X1
 29 DO 8 I=1,MR1
 30 J=I+1
 31 WRITE(6,29) RA(J),X(NT,I)
 32 GO TO 10
 33 9 AVS=FALSE.
 34 WRITE(6,26)
C.
C. SET-UP AND FACTOR SUB-MATRICES IN BLOCK DIAGONAL
C. COEFFICIENT MATRIX.
10 CALL POISS1
C.
C. PRINT INPUT BLOCK.
WRITE(6,35)
35 WRITE(6,25)
36 WRITE(6,32) (RA(I), I=6,MR)
37 DO 11 J=1,NT
 38 11 WRITE(6,34) (X(J,K-1), K=6,MR)
 39 WRITE(6,33) X1
C.
C. CARRY OUT SOLUTION OF MATRIX PROBLEM TO YIELD V-FIELD.
CALL POISS2
C ADJUST EAR VALUES.
IF(ERA. EQ. 0) GO TO 17
DO 16 I=4,16
16 X(NT1.I)=X(NT1.I)
C PRINT OUTPUT BLOCK.
17 WRITE(6,35)
WRITE(6,30)
WRITE(6,31) (RA(I), I=2, NR)
DO 12 J=1, JF
12 WRITE(6,34) TD(J), (X(J,H-1), K=2, NR)
DO 13 J=2, JF
DO 13 K=1, 15
13 XT(J,K)=X(J,K)
C DISPLAY EQUIPOTENTIAL LINE DATA.
IF(LOOP.LE.1) CALL EQLINE
DO 14 J=1, JF
14 V(1,J)=XCTR
DO 15 I=2, 17
K=I-1
DO 15 J=1, JF
15 V(I,J)=XT(J,K)
C RETURN
END

SUBROUTINE ECLINF
C
C FIND ON EACH THETA MESH LINE THE RHO VALUES ASSOCIATED
C WITH NEOL PRE-SPECIFIED POTENTIAL VALUES. AN EQUIPOTENTIAL
C CURVE PASSES THROUGH SETS OF THESE POINTS.
C
C***
COMMON BO.CVF.ORHT(lG).DRHM(16).DTHTD.UTHTR.DVDR(10),
1 DVNT(10), DURDR,DVHDZ,EQL(10), ETAO, IBAR, IT, JF, JT, KPC, KPOI,
2 KPOS.LOOP.NFO.N2,N3,PHH2A(10),PI,PI2,PSIA(10),PSID(10),R,
3 RAO,RHMM(17),RHOP(10),RHUPP(10),KHTT(10),RV(10),SPCH,TAU02,
4 THD(10),THMD(25),THSDEL,THSFC,THSWHI,THSWLQ,
5 THTD(10),THTR(10),TH1,TH2,V(17,25),VR,
6 VSM1(10),VT(10),W(16),XCTR,XT(25,16),Y(16),ZJO
EQUIVALENCE (DTO,Y(1)),(TH,Y(2)),(RH,Y(3)),(RH2,RHP,Y(4)),
* (RHP,Y(5)),(RH2P,Y(6)),(W(1),DELR),(W(2),AR),
* (W(3),Z1),(W(4),Z2),(W(5),Z1P),(W(6),Z2P)
C***
10 FORMAT(/28X,23HTAIBLE OF RHO-VALUES FOR)
11 FORMAT(30X,15HEQUIPOTENTIAL LINES)
12 FORMAT(1X,F6.2,2X,10F10.3)
13 FORMAT(2X,9HTHETA,2X,10(F10.2))
C
C ZERO OUT RHV-ARRAY.
DO 6 I=1,10
6 RHV(I)=0.0
C
WRITE(6,10)
WRITE(6,11)
NR=17
DO 5 J=1, JF
DO 4 I=1, NEQL
DO 1 K1=1, NR
K=MR-K1
IF(EQL(I).GT.XT(J,K)) GO TO 2
CONTINUE
7 IF(K1.GT.1) GO TO 3
RHV(I)=0.0
GO TO 5

3 F=(XT(J,K)-ECL(I))/(XT(J,K)-XT(J,K+1))
IF(K,G.T.0) GO TO 4
F=(XCTR-ECL(I))/(XCTR-XT(J,K+1))
4 RHV(I)=RHM(K+1)+F*(RHM(K+2)-RHM(K+1))
5 WRITE(6,12) THMD(J), (RHV(I), I=1, NEQL)

RETURN
END

SUBROUTINE RHSCL(A,ZJO,VO,THO,THF,X,V2,IBAR,RHM,TH2,KPOSC)
C*  RIGHT-HAND SIDE CALCULATION FOR POISSON MATRIX EQUATION.
C*  EQUIVALENT TO CALCULATION OF SPACE CHARGE FIELD.
C*
DIMENSION V(11),C(9),V(10),RH(11),AD(11),RHS(11,9),ARG(16),
* THG(11,9),THK(11,9),RHSI(10,16),X(25,16),V2(10),RHM(17)
C
FORMAT STATEMENTS.
22 FORMAT(1X,F5.3,F7.3)
23 FORMAT(16F5.0)
24 FORMAT(11HORH5 ARRAY-I)
25 FORMAT(1X,16F8.1)
26 FORMAT(2X,16F8.2)
27 FORMAT(10F5.0)
28 FORMAT(3F8.0)
29 FORMAT(12HORH5 VALUES.1)
30 FORMAT(2X,3HROH,4X,1HC,2X,4HTUBE,4X,5HTHETA,6X,3HRHS)
31 FORMAT(1X,F5.3,2I4,1H-11,F10.3,F9.1)
32 FORMAT(5H TH1=,F7.3,7X,5HTHF =,F8.3)
33 FORMAT(4X,5HAXIAL/2X,9HPOTENTIAL/1X,12HDISTRIBUTION)
C
A=RADIUS OF ENTRY HOLE.
ZJO=JO (CURRENT DENSITY).
THO=INITIAL VALUE OF TRAJECTORY 5.
X=XT(25,16). SOURCE TERMS.
V2=ENERGY CLASSES.
TH2=CONF ANGLE.
TH1=THO
IF=11
JF= 7
JF1=JF+1
JF2=JF+2
EN1=8.
SEC = B.860F-18
ETA = 1.753F+15
C1 = 2.*EC*SORT(2.*ETA)*ENT*ENT
PI = 3.14159265
CVF = 1.80/PI
WRITE(6,29)
WRITE(6,32) IH, THF

SECTION FOR COMPUTING SPACE CHARGE ALONG TUBE CENTER LINES.
I = NIG LOOP FOR 11 RHO VALUES FROM 0.63 TO 1.00
J = SM LOOP FOR 7 TUBES (1-2, 2-3, ..., 7-8).
THF = RHO .63 CROSSING ANGLE OF TRAJ. NO. 5.
READ 11 AXIAL V-VALUES (RHO .63 TO 1.0) AND VSMM-ARRAY(9).

GENERATE ARG-ARRAY, C-ARRAY, AND VI-ARRAY.
ARG(K) = THETA MESH VALUES IN HOLE REGION.
C (J) = 1, 3, 5, ..., 13 (MULTIPLIERS).
VI (J) = 1.55, 1.40, ..., 0.40, 0 (ENERGY CLASSES IN TUBES).
DO 1 K = 1, 16
1 ARG(K) = 180. - FLOAT(K)*.25
DO 2 J = 1, JF
V(J) = (V2(J) + V2(J+1)) / 2
2 C(J) = 2*J-1

GENERATE AND PRINT RH-ARRAY AND V-ARRAY.
RH(I) ARRAY = 11 AXIAL RHO MESH VALUES
FROM .63 TO 1.00
V(I) = POTENTIAL DISTRIBUTION AT THESE 11 AXIAL MESH POINTS.
WRITE(6,33)
WRITE(6,28)
DO 3 I = 1, IF
RH(I) = RHM(I+6)
V(I) = -X(25, I+5)
3 WRITE(6,22) RH(I), V(I)

GENERATE THETA-ARRAY AND SPREAD THROUGHOUT SPACE CHARGE REGION (BASED ON TRAJECTORY NO. 5).
OTH = TH1 - THF
ORD = 1.0 - RHM(7)
TO = OTH/ORD
DO 5 I = 1, IF
I6 = I+6
DIFF = RHM(I6) - RHM(7)
T1 = THF + TO*DIFF
T2 = T1 + 180. - T1 / 8.
DEL = (180. - T2) / 4.
DO 4 J = 1, JF2
THD(I, J) = 180. - FLOAT(J)*DEL
4 THK(I, J) = THD(I, J)/CVF
5 CONTINUE

GENERATE 4D-ARRAY (DISTANCE BETWEEN TRAJECTORIES 4 AND 5 AT EACH OF 11 RHO MESH LINES).
DO 6 I = 1, IF
TX1 = RH(I)*TAN(THR(I, 5))
TX2 = RH(I)*TAN(THR(I, 4))
CALCULATE SOURCE TERMS FOR ALL TRAJECTORY CROSSINGS OF RHD MESH LINES.

\[ AJ = A^*A*2.70 \]

\[ DO 9 I=1,16 \]
\[ IF(KPOSC.NE.0) WRITE(6,30) \]
\[ DO 9 J=1,16 \]
\[ E1 = V1(J) - V1(I) \]
\[ IF(E1.GT.0.1) GO TO 7 \]
\[ RHS(I,J) = 0.0 \]
\[ GO TO 8 \]

\[ F2 = SQRT(E1) \]
\[ ST = SIN(THR(I,J)) \]
\[ DEN = C1*SQRT(V0**3)*E2*RH(I,J)*ST*AD(I) \]
\[ RHS(I,J) = (C(I)*AJ)/DEN \]
\[ J1 = J+1 \]
\[ IC = C(J) \]
\[ IF(KPOSC.NE.0) WRITE(6,31) RH(I,J),IC,J1,THD(I,J),RHS(I,J) \]
\[ 9 CONTINUE \]

SECTION FOR INTERPOLATING SPACE CHARGE FIELD AT MESH POINTS.

\[ DO 15 I=1,16 \]
\[ DO 15 K=1,16 \]
\[ DO 10 J=1,7 \]
\[ IF(ARG(K).GT.TH(I,J)) GO TO 11 \]
\[ 10 CONTINUE \]
\[ 11 IF(J.NE.1) GO TO 13 \]
\[ DELKHS = 0.0 - RHS(I,J) \]
\[ TL = 180.0 - THD(I,J) \]
\[ 12 IF(ARG(K1).LT.TH(I,J)) GO TO 14 \]
\[ F = 2* ARG(K) / TL \]
\[ IF(ARG(K1).LT.TH(I,J)) GO TO 14 \]
\[ TL = RHS(I,J) + F*DELKHS \]
\[ RHS(I,K) = (RHS(I,K) + TL)/2. \]
\[ GO TO 15 \]
\[ 13 TL = THD(I,J-1) - THD(I,J) \]
\[ UFLKHS = RHS(I,J-1) - RHS(I,J) \]
\[ GO TO 12 \]
\[ 14 RHS(I,K) = 0.0 \]
\[ 15 CONTINUE \]
\[ IF(KPOSC.EQ.0) GO TO 17 \]

PRINT.

\[ WRITE(6,24) \]
\[ WRITE(6,26) (ARG(K), K=1,16) \]
\[ DO 16 I=1,16 \]
\[ 16 WRITE(6,25) (RHS(I,K), K=1,16) \]

ZERO OUT RHS PREPATORY TO LOADING SOURCE TERMS.

\[ DO 18 J=1,7 \]
\[ DO 18 K=1,15 \]
\[ 18 X(J,K) = 0.0 \]
\[ IF(JBAR.NE.0) GO TO 20 \]
\[ DO 19 I=1,16 \]
\[ 19 X(25,I) = 0.0 \]

46
SUBROUTINE PSICAL(R,Z,PSI,BG,ASM)
C*
C* COMPUTES PSI, THE MAGNETIC FLUX, OF THE MAGNETIC
C* FIELD AT EACH MESH POINT IN THE REGION OF THE
C* ENTRY HOLE.
C*
PI=3.14159265
RF=R
RSQ=R*R
X= Z/ASM
X2= X*X
AT= ATAN2(ASM,-1)

T= 1.*X2
T0= PI-AT-X/T
T4= 3.-5.*X2
T6= 3.-7.*X2*(2.-X2)
T= 21.-5.*X2
T= 15.-X2*T
T8= 5.-3.*X2*T
T= 36.-5.*X2
T= 54.-X2*T
T= 70.-X2*T
T10=9.*(15.-11.*X2*T)
T= 11.-X2
T= 7.*X2*T
T= 198.-T
T= 154.-X2*T
T= 35.-X2*T
T12= 21.-13.*X2*T

X1= ASM*(1.*X2)
V= (RF/X1)**2

T= (T12+T10)/5.+(11.*T12+V)/4.
T= 7.*T8+0.5*V*T
T=(5.*T6)/3.*125*V*T
T= T4/4.+ 125*V*T
T= 1.+V*T
T= T0-(ASM*X*RSQ*T)/(X1**3)
PSI= R0*RSQ*T

RETURN
END
SUBROUTINE BFBZ(R,Z,BR,BZ,BO,ASM)

C*  CALCULATES BR AND BZ, THE R AND Z COMPONENTS OF THE
C*  MAGNETIC FIELD AT EACH MESH POINT IN THE REGION
C*  OF THE ENTRY HOLE.
C*

PI=3.14159265
IF(Z.FLESS0.0) Z=0.0004
X=Z/ASM
X2=X**2

C
T1=1.
T2=1.
T3=1.-5.*X2
T4=3.-5.*X2
T5=3.-7.*X2*(6.-5.*X2)
T6=3.-7.*X2*(2.-X2)
T7=1.-X2*(27.-7.*X2*(9.-3.*X2))
T8=5.-3.*X2*(15.-X2*(21.-5.*X2))
T9=30.-X2*(28.-5.*X2)
T9=5.-11.*X2*(20.-3.*X2*T9)
T10=54.-X2*(36.-5.*X2)
T10=20.-X2*T10
T10=9.*(15.-11.*T10*X2)

C
T11=18.--X2*(9.-X2)
T11=195.--143.*X2*(10.-X2*T11)
T11=3.--X2*T11
T12=--X2*(1001.-91.*X2)
T12=--X2*(2574.+T12)
T12=--X2*(2022.+T12)
T12=--X2*(455.+T12)
T12=11.*T21
T1B=(ASM*Z)/(ASM*ASM+Z*Z)

C
ZK=1.*X2
A1=ASM*ZK
A2=ASM*ZK*X

C
C1=(-2.*BO)/(PI*ASM*ZK*ZK)
C2=(-4.*C1*X)/A1
C3=C2/A2
C4=(-6.*C3*X)/A1
C5=C4/A2
C6=(-40.*C5*X)/A1
C7=(3.*C6)/A2
C8=(-14.*C7*X)/A1
C9=C8/A2
C10=(-4.*C9*X)/A1
C11=(45.*C10)/A2
C12=(-2.*C11*X)/A1

C
AT=ATAN2(ASM,-Z)
AB=(BO/PI)/(PI-AT-T1B)
B 1=C 1*T 1
B 2=C 2*T 2
B 3=C 3*T 3
B 4=C 4*T 4
B 5=C 5*T 5
B 6=C 6*T 6
\[ R_2 = R/2 \]
\[ RS = R_2 * R_2 \]
\[ T_1 = B_{10} - RS * B_{12}/36 \]
\[ T_1 = B_{11} - RS * T_1/25 \]
\[ T_1 = B_{12} - RS * T_1/16 \]
\[ T_1 = B_{13} - RS * T_1/5 \]
\[ T_1 = B_{14} - RS * T_1/4 \]
\[ T_1 = B_{15} - RS * T_1/12 \]
\[ T_1 = B_{16} - RS * T_1/6 \]
\[ T_1 = B_{17} - RS * T_1/2 \]
\[ B_0 = -R_2 * T_1 \]
\[
C
\]
\[ T_1 = B_9 - B_{11} * RS/30 \]
\[ T_1 = B_7 - RS * T_1/20 \]
\[ T_1 = B_5 - RS * T_1/12 \]
\[ T_1 = B_3 - RS * T_1/6 \]
\[ T_1 = B_1 - RS * T_1/2 \]
\[ B_0 = -R_2 * T_1 \]
\[
C
\]
RETURN
END
APPENDIX C

FORTRAN SYMBOLS

ALPH(10)  array of entry angles, $\alpha$, rad
ALPHA     entry angle, $\alpha$, deg
AR        rectilinear coordinate, $r$
ASM       radius of entry hole, $a$, cm
AVS       axial boundary value logical switch  TRUE indicates presence of an
           axial spike
B0        magnetic flux density, $B_0$
BR        $r$-component of magnetic field, $B_r$
BZ        $z$-component of magnetic field, $B_z$
CONFLO    switch for type of flow (confined flow when not equal to 0; Brillouin flow
           when equal to 0)
CVF       conversion factor, deg to rad
DELI(10)  $\delta_i$ array, rad
DElid      entry angles of electron classes, $\delta_i$, deg
DELR       step size $\Delta r$ for $r$-$z$ trajectory integration
DRHM(16)  $\Delta \rho$ of mesh
DRHT(10)  $\Delta \rho$ of trajectories
DTH       $\Delta \theta$ of trajectories (equivalent to DTHTR below), rad
DTHTD     $\Delta \theta$ of trajectories (step in $\theta$ for $\rho$-$\theta$ trajectory integration), deg
DTHTR     $\Delta \theta$ of trajectories, rad
DVDR(10)  $\partial V/\partial \rho$ array
DVDT(10)  $\partial V/\partial \theta$ array
DVMDRH    $\partial V_m/\partial \rho$ array
DVMDTH    $\partial V_m/\partial \theta$
DVRDR     $\partial V_r/\partial r$
DVRDZ     $\partial V_r/\partial z$
EQL(10)   array of values for which equipotential lines are to be calculated
**ETA0**  
initial electron charge-to-mass ratio

**E0**  
permittivity of free space, $\epsilon_0$

**FLOOP**  
storage for number of iteration loops in space charge calculation

**IBAR**  
indicator for axial boundary values

**IT**  
iteration counter

**J1, J2, JF, JT**  
looping indexes

**KPO, KPOI**  
main output frequency control of trajectory data (e.g., 5 means to print every fifth point); KPOI is constant; KPO may be varied

**KPOSC**  
output control for space charge values

**K9**  
initialization switch

**LKTR**  
counter for number of loops

**LOOP**  
indicator switch for space charge looping

**NEQL**  
number of equipotential lines to be calculated

**N1, N2, N3**  
control indexes for trajectory calculations

**NF**  
number of $\theta$-mesh lines, usually 26

**NR**  
number of $\rho$-mesh points, usually 17

**NT**  
number of trajectories, usually 9

**PHD**  
$\dot{\varphi}(\partial \varphi / \partial t)$, where $\varphi$ is azimuthal angle

**PHD2**  
$\dot{\varphi}^2$

**PHD2A(10)**  
$\dot{\varphi}^2$ array for output

**PI, PI2**  
$\pi$, $\pi^2$

**PSI**  
magnetic flux, $\psi$

**PSIA(10)**  
$\psi$ array

**PSIO**  
initial magnetic flux, $\psi_0$

**R**  
radius of collector, R, cm

**RA(17)**  
array of r-mesh values

**RAD**  
radius of entry hole, a, cm

**RH, RHP**  
$\rho$, $\rho'(\partial \rho / \partial \theta)$

**RH1P**  
$\rho_1$

**RH2, RH2P**  
$\rho_2$, $\rho_2'$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHM(17)</td>
<td>$\rho$-mesh values</td>
</tr>
<tr>
<td>RHOP(10)</td>
<td>$\rho'$</td>
</tr>
<tr>
<td>RHOPP(10)</td>
<td>$\rho''$</td>
</tr>
<tr>
<td>RHRZ</td>
<td>$\rho$ in r-z coordinate system</td>
</tr>
<tr>
<td>RHT(10)</td>
<td>$\rho$-values for each trajectory</td>
</tr>
<tr>
<td>RHV(10)</td>
<td>$\rho$-values for equipotential lines</td>
</tr>
<tr>
<td>SPCH</td>
<td>space charge indicator switch</td>
</tr>
<tr>
<td>ST, ST2</td>
<td>$\sin \theta, \sin^2 \theta$</td>
</tr>
<tr>
<td>T0, T1, T2</td>
<td>temporary storages</td>
</tr>
<tr>
<td>TA(25)</td>
<td>$\theta$-mesh values</td>
</tr>
<tr>
<td>TAU02</td>
<td>$\tau_0^2$</td>
</tr>
<tr>
<td>TH</td>
<td>variable of integration, $\theta$</td>
</tr>
<tr>
<td>TH0</td>
<td>initial cone angle, deg</td>
</tr>
<tr>
<td>TH1</td>
<td>initial cone angle, deg</td>
</tr>
<tr>
<td>TH2</td>
<td>collector angle at which surface charge is zero, deg</td>
</tr>
<tr>
<td>THD(10)</td>
<td>$\dot{\theta}$ array</td>
</tr>
<tr>
<td>THF</td>
<td>$\theta_f$</td>
</tr>
<tr>
<td>THMD(25)</td>
<td>$\theta$-mesh values, deg</td>
</tr>
<tr>
<td>THSDEL</td>
<td>$\theta$ step-size increment, deg</td>
</tr>
<tr>
<td>THSW</td>
<td>$\theta$ switch for step-size control</td>
</tr>
<tr>
<td>THTD(10)</td>
<td>$\theta$-coordinate of trajectory, deg</td>
</tr>
<tr>
<td>THD2</td>
<td>$\dot{\theta}^2$</td>
</tr>
<tr>
<td>THTR</td>
<td>$\theta$-coordinate of trajectory, rad</td>
</tr>
<tr>
<td>THRZ</td>
<td>$\theta$ in r-z coordinate system</td>
</tr>
<tr>
<td>V(17,25)</td>
<td>potential inside spherical collector</td>
</tr>
<tr>
<td>V0</td>
<td>initial normalized electrical potential, $V_0$</td>
</tr>
<tr>
<td>VR</td>
<td>interpolated potential in r-z system</td>
</tr>
<tr>
<td>VSMI(10)</td>
<td>$v_i$ array; beam velocities</td>
</tr>
<tr>
<td>VT(10)</td>
<td>array of initial voltages</td>
</tr>
<tr>
<td>W(16)</td>
<td>Runge-Kutta storage area in r-z system</td>
</tr>
</tbody>
</table>
XCTR value of source term at center of sphere

XT(25,16) source term needed for solution to Poisson's equation

Y(16) Runge-Kutta storage area in $\rho-\theta$ system

Z, ZP $z, z'(\partial z/\partial r)$

Z1P $z'_1$ miscellaneous storages for independent variables in cylindrical coordinate system

Z2, Z2P $z'_2, z'_2$ initial current density, $J_0$, A/cm$^2$

ZI0 initial current, $I_0$, A

ZKP permeance, $K_p$, A/V$^{3/2}$

ZED rectilinear coordinate, $z$
Flow charts are presented only for those five subroutines, INTGRN, DE, RZTRAJ, VFIELD, RHSCAL, which might not otherwise be easily understood. Diamond-shaped blocks indicate decision-making points in the subroutines. Numbers appearing on blocks refer to the corresponding FORTRAN statement numbers in the subroutines.

Subroutine INTGRN

Entry

1, 2

SPCH = 0 ?

1, 2

Integrate one step to obtain $p_i, \theta_{ij} (i = 1, NT)$ for NT electron velocity classes

Integrate one step to obtain $p_s, \theta_s$ for mid-electron velocity class

DTH = DTH + THSFAC

Change step-size $\Delta \theta$?

3

JT = JT + 1

Call RZTRAJ

Call OUT

4

Change to r-z coordinates?

Yes

No

5

Done? ($p > 17$)

Yes

No

Return

No

Return

Yes

$p_s \leq 0.633$ ?
Subroutine DE

Enter

Call subroutine
DERIV

to obtain
\( \delta V / \delta \theta \) and \( \delta V / \delta \phi \)

Call subroutine
PSICAL

to obtain \( \psi \)

Call subroutine
BRBZ

to obtain \( B_r \) and \( B_z \)

Call \( \phi, \psi, \phi^2, \delta V_m / \delta r, \delta V_m / \delta \theta, \theta \)

Calculate \( \rho^{**} \)

Return

Subroutine RZTRAJ

Entry

Write trajectory
number and \( \Delta r \)

Write \( r = \rho \sin \theta \)

Write \( z = \rho \cos \theta \)

Call subroutine
RZDE

Write \( r, z, z', z'', V_r, \delta V_r / \delta r, \delta V_r / \delta \theta \)

Write \( p\), \( \theta \)

1

Call subroutine
RZRK

Yes

No

Exit to
MAIN

Yes

Last trajectory?

No

Return

No

Return
APPENDIX E

MAGNETIC FIELD CALCULATIONS

\[ B_z^0(0, z) = \frac{B_0}{\pi} \left[ \pi - \tan^{-1}\left( \frac{a}{-2z} \right) - \frac{x}{1 + x^2} \right] \quad (E1) \]

where \( x = z/a \). Then,

\[ B_z^I(0, z) = -\frac{2B_0}{\pi a} \left( \frac{1}{(1 + x^2)^2} \right) \]

\[ B_z^{II}(0, z) = \frac{8B_0 x}{\pi a^2} \left( \frac{1}{(1 + x^2)^3} \right) \]

\[ B_z^{III}(0, z) = -\frac{8B_0 x}{\pi a^3} \left( 1 - 5x^2 \right) \left( \frac{1}{(1 + x^2)^4} \right) \]

\[ B_z^{IV}(0, z) = -\frac{48B_0 x}{\pi a^4} \left( 3 - 5x^2 \right) \left( \frac{1}{(1 + x^2)^5} \right) \]

\[ B_z^{V}(0, z) = -\frac{48B_0 x}{\pi a^5} \left( 3 - 42x^2 + 35x^4 \right) \left( \frac{1}{(1 + x^2)^6} \right) \]

\[ B_z^{VI}(0, z) = -\frac{4 \times 48B_0 x}{\pi a^6} \left( 3 - 14x^2 + 7x^4 \right) \left( \frac{1}{(1 + x^2)^7} \right) \]

\[ B_z^{VII}(0, z) = \frac{12 \times 48B_0 x}{\pi a^7} \left( 1 - 27x^2 + 63x^4 - 21x^6 \right) \left( \frac{1}{(1 + x^2)^8} \right) \]

\[ B_z^{VIII}(0, z) = \frac{12 \times 14 \times 48B_0 x}{\pi a^8} \left( 5 - 45x^2 + 63x^4 - 15x^6 \right) \left( \frac{1}{(1 + x^2)^9} \right) \]

\[ B_z^{IX}(0, z) = -\frac{12 \times 14 \times 48B_0 x}{\pi a^9} \left( 5 - 220x^2 + 990x^4 - 924x^6 + 165x^8 \right) \left( \frac{1}{(1 + x^2)^{10}} \right) \]

\[ B_z^{X}(0, z) = \frac{14 \times 48 \times 48B_0 x}{\pi a^{10}} \left( 135 - 1980x^2 + 5346x^4 - 3564x^6 + 495x^8 \right) \left( \frac{1}{(1 + x^2)^{11}} \right) \]

\[ B_z^{XI}(0, z) = \frac{14 \times 45 \times 48 \times 48B_0 x}{\pi a^{11}} \left( 3 - 195x^2 + 1430x^4 - 2574x^6 + 1287x^8 - 143x^{10} \right) \left( \frac{1}{(1 + x^2)^{12}} \right) \]

\[ B_z^{XII}(0, z) = -\frac{28 \times 45 \times 48 \times 48B_0 x}{\pi a^{12}} \left( 231 - 5005x^2 + 22022x^4 - 28314x^6 + 11011x^8 - 1001x^{10} \right) \]
To simplify these expressions, let

\[ T_3 = 1 - 5x^2 \]
\[ T_4 = 3 - 5x^2 \]
\[ T_5 = 3 - 42x^2 + 35x^4 = 3 - 7x^2(6 - 5x^2) \]
\[ T_6 = 3 - 14x^2 + 7x^4 = 3 - 7x^2(2 - x^2) \]
\[ T_7 = 1 - 27x^2 + 63x^4 - 21x^6 = 1 - 3x^2[9 - 7x^2(3 - x^2)] \]
\[ T_8 = 5 - 45x^2 + 63x^4 - 15x^6 = 5 - 3x^2[15 - x^2(21 - 5x^2)] \]
\[ T_9 = 5 - 220x^2 + 990x^4 - 924x^6 + 165x^8 = 5 - 11x^2\,[20 - 3x^2\,[30 - x^2(28 - 5x^2)]] \]
\[ T_{10} = 135 - 1980x^2 + 5346x^4 - 3564x^6 + 495x^8 \]
\[ = 9\,(15 - 11x^2\,[20 - x^2[54 - x^2(36 - 5x^2)]]) \]
\[ T_{11} = 3 - 195x^2 + 1430x^4 - 2574x^6 + 1287x^8 - 143x^{10} \]
\[ = 3 - 13x^2\,(15 - 11x^2\,[10 - x^2[18 - x^2(9 - x^2)]]) \]
\[ T_{12} = 231 - 5005x^2 + 22022x^4 - 28314x^6 + 11011x^8 - 1001x^{10} \]
\[ = 11\,[21 - 13x^2\,(35 - x^2[154 - x^2[198 - 7x^2(11 - x^2)]]]) \]

Then, substituting these expressions for those in (E2), we now have
\[
B^I_z(0, z) = -\frac{2B_0}{\pi a(1 + x^2)^2} \\
B^II_z(0, z) = \frac{8B_0^x}{\pi a^2(1 + x^2)^3} \\
B^III_z(0, z) = \frac{8B_0^T_3}{\pi a^3(1 + x^2)^4} \\
B^IV_z(0, z) = -\frac{48B_0^xT_4}{\pi a^4(1 + x^2)^5} \\
B^V_z(0, z) = -\frac{48B_0^T_5}{\pi a^5(1 + x^2)^6} \\
B^VI_z(0, z) = \frac{4 \times 480B_0^xT_6}{\pi a^6(1 + x^2)^7} \\
B^VII_z(0, z) = \frac{12 \times 480B_0^T_7}{\pi a^7(1 + x^2)^8} \\
B^VIII_z(0, z) = -\frac{12 \times 14 \times 480B_0^xT_8}{\pi a^8(1 + x^2)^9} \\
B^IX_z(0, z) = -\frac{12 \times 14 \times 480B_0^T_9}{\pi a^9(1 + x^2)^10} \\
B^{X}_z(0, z) = \frac{14 \times 48 \times 480B_0^xT_{10}}{\pi a^{10}(1 + x^2)^{11}} \\
B^{XI}_z(0, z) = \frac{14 \times 45 \times 48 \times 480B_0^T_{11}}{\pi a^{11}(1 + x^2)^{12}} \\
B^{XII}_z(0, z) = -\frac{28 \times 45 \times 48 \times 480B_0^xT_{12}}{\pi a^{12}(1 + x^2)^{13}} 
\]
Since

\[ \mathcal{B}_r(r, z) = -\mathcal{B}^I(0, z) \frac{r}{2} + \frac{1}{1! \cdot 2!} \mathcal{B}^{III}(0, z) \left( \frac{r}{2} \right)^3 - \frac{1}{2! \cdot 3!} \mathcal{B}^{V}(0, z) \left( \frac{r}{2} \right)^5 + \frac{1}{3! \cdot 4!} \mathcal{B}^{VII}(0, z) \left( \frac{r}{2} \right)^7 - \ldots \]  

(E5)

We make the substitution \( r_2 = r/2 \), and nest as follows,

\[ \mathcal{B}_r(r, z) = -r_2 \left[ \mathcal{B}^I - \frac{r_2^2}{1 \times 2} \left( \mathcal{B}^{III} - \frac{r_2^2}{2 \times 3} \left( \mathcal{B}^{V} - \frac{r_2^2}{3 \times 4} \left[ \mathcal{B}^{VII} - \frac{r_2^2}{4 \times 5} \left( \mathcal{B}^{IX} - \frac{B^{XI} r_2^2}{5 \times 6} - \ldots \right) \right] \right) \right) \right] \]  

(E6)

and similarly,

\[ \mathcal{B}_z(r, z) = \mathcal{B}^0 - \frac{r_2^2}{2^2} \left[ \mathcal{B}^{II} - \frac{r_2^2}{3^2} \left( \mathcal{B}^{IV} - \frac{r_2^2}{4^2} \left[ \mathcal{B}^{VI} - \frac{r_2^2}{5^2} \left( \mathcal{B}^{VIII} - \frac{r_2^2}{6^2} \mathcal{B}^{XII} - \ldots \right) \right] \right) \right] \]  

(E7)
APPENDIX F

DERIVATION OF EQUATIONS OF MOTION

The equations of motion as derived in reference 1, equations (B17) and (B18), are

\[ p' = \rho \dot{\theta}^2 + \rho \sin^2 \theta \dot{\phi}^2 - \rho \dot{\theta}^2 + \frac{\eta}{\rho R^2} \frac{\partial V}{\partial \rho} - \frac{\eta}{R} \sin \theta A \dot{\phi} - \frac{\eta}{\rho R} \rho \sin \theta \dot{\phi} \frac{\partial A}{\partial \rho} \]  

(F1)

\[ \ddot{\theta} = - \frac{2 \rho \dot{\theta}^2}{\rho} + \sin \theta \cos \theta \dot{\phi}^2 + \frac{\eta}{\rho^2 R^2} \frac{\partial V}{\partial \theta} - \frac{\eta}{\rho R} \cos \theta A \dot{\phi} - \frac{\eta}{\rho R} \sin \theta \dot{\phi} \frac{\partial A}{\partial \theta} \]  

(F2)

Solving these two equations simultaneously for \( \dot{p}'' \) yields

\[ \dot{p}'' = \rho + \frac{\rho \sin^2 \theta \dot{\phi}^2}{\dot{\theta}^2} + \frac{\eta}{R \dot{\theta}^2} f_1 + \frac{2 \rho \dot{\theta}^2}{\rho} - \frac{\rho \sin \theta \cos \theta \dot{\phi}^2}{\dot{\theta}^2} - \frac{\rho \eta}{\rho^2 R^2 \dot{\theta}^2} f_2 \]  

(F3)

where

\[ f_1 = \frac{1}{R} \frac{\partial V}{\partial \rho} - \sin \theta A \dot{\phi} - \rho \sin \theta \dot{\phi} \frac{\partial A}{\partial \rho} \]  

(F4a)

\[ f_2 = \frac{\partial V}{\partial \theta} - \rho R \cos \theta A \dot{\phi} - \rho R \sin \theta \dot{\phi} \frac{\partial A}{\partial \theta} \]  

(F4b)

or, using the relations described in appendix D of reference 1, these expressions may be rewritten

\[ f_1 = \frac{1}{R} \frac{\partial V}{\partial \rho} - \sin \theta \dot{\phi} \frac{\partial V_m}{\partial \theta} \]  

(F4b)

\[ f_2 = \frac{\partial V}{\partial \theta} + \rho^2 R \sin \theta \dot{\phi} \frac{\partial V_m}{\partial \rho} \]  

Away from the entry hole, the magnetic field terms \( \dot{\phi}, A \dot{\phi}, \) and \( V_m \) are negligible. Therefore, in this region equation (F3) may be expressed.
\[ \rho'' = \rho + \frac{\eta}{R^2 \delta^2} \times \frac{\partial V}{\partial \rho} + \frac{2 \rho'}{\rho} - \frac{\rho' \eta}{\rho^2 R^2 \delta^2} \times \frac{\partial V}{\partial \theta} \]  

(F5)

using equation (B16) of reference 1 as

\[ \dot{\theta}^2 = \frac{2\eta V \left( \epsilon_i + V \right)}{R^2 \rho^2 \left[ 1 + \left( \frac{\rho'}{\rho} \right)^2 \right]} \]  

(F6)

and, with the appropriate substitutions, we may express this equation in cylindrical coordinates

\[ z'' = \frac{\frac{\partial V}{\partial z} - z' \frac{\partial V}{\partial r}}{2 \left[ \frac{\epsilon_i + V}{1 + (z')^2} \right]} \]
APPENDIX G

MATHEMATICAL SYMBOLS

A  area of annulus, cm²
a  radius of collector entry hole, cm
B_r  r-component of potential field, \( \partial V_m/\partial r \), G/cm
B_z  z-component of potential field, \( \partial V_m/\partial z \), G/cm
B_0  magnetic flux density, G
b  radius of electron beam at injection, cm
E  \( v_i + V \)
I  current, A
J  current density, A/cm²
K_p  perveance, A/V̅³/²
R  unreduced radius of spherical collector, cm
r, z  cylindrical coordinates
u  electron energy
V  normalized electric potential
V_m  magnetic field potential
V_r  \( \partial V/\partial r \)
V_z  \( \partial V/\partial z \)
V_\rho  \( \partial V/\partial \rho \)
V_\theta  \( \partial V/\partial \theta \)
v_i  \( i^{th} \) normalized injection energy class
\( \epsilon_0 \)  permittivity of free space, \( 8.86 \times 10^{-14} \) F/cm
\( \eta_e \)  electron charge-mass ratio, charge/kg\times 10^{11}
\rho  normalized radius vector
\bar{\rho}  unnormalized radius vector, cm
\rho_e  space charge density, C/cm³
\( \tau_0 \)  \( R/(2\eta V_\rho)^{1/2} \)
\theta, \varphi  polar and azimuthal spherical coordinate angles
REFERENCES


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—National Aeronautics and Space Act of 1958

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