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ANALYTICAL SOLUTION FOR HEAT TRANSFER IN THREE-DIMENSIONAL POROUS MEDIA INCLUDING VARIABLE FLUID PROPERTIES

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16. Abstract

An analytical solution is obtained for flow and heat transfer in a three-dimensional porous medium. Coolant from a reservoir at constant pressure and temperature enters one portion of the boundary of the medium and exits through another portion of the boundary which is at a specified uniform temperature and uniform pressure. The variation with temperature of coolant density and viscosity are both taken into account. A general solution is found that provides the temperature distribution in the medium and the mass and heat fluxes along the portion of the surface through which the coolant is exiting.

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SUMMARY

An analytical solution is derived for the flow and heat transfer in a three-dimensional porous cooled medium of arbitrary shape. The coolant and porous matrix material are assumed to be in sufficiently good thermal contact that their local temperatures are equal, and as a result a single energy equation governs the process. The coolant reservoir is at constant pressure and temperature. The portion of the surface through which the coolant exits from the porous medium is exposed to a constant pressure and has a specified uniform temperature that is higher than the reservoir temperature. By introducing a suitably defined potential function, the solution is reduced to solving Laplace's equation in the geometry of the porous material with simple constant boundary conditions at the coolant inlet and exit surfaces. This geometrical aspect is separated from the variable property effects. The viscosity variation with temperature is accounted for by performing a simple integration with respect to temperature. The integration relates the potential function obtained from Laplace's equation to the temperature variation in the medium. Relations are obtained for the temperature distribution in the medium and for the local mass flow and heat flux distributions along the coolant exit boundary. To illustrate the application of the solution and show the effect of fluid property variations, some results are carried out for cases where air and oil are the coolants.

INTRODUCTION

The desire to increase efficiency in power producing devices requires the practical utilization of working fluids at higher temperature levels. However, these fluids must be contained in some manner and the result is that the bounding walls must be able to withstand high imposed heat fluxes. Thus some form of efficient cooling is required to

maintain the walls at a sufficiently low temperature so that they do not lose their structural integrity. If a solid wall is to be cooled by passing the coolant along one side while the other side is subjected to a heat flux, two of the factors that govern the maximum wall temperature are the wall thickness and the wall thermal conductivity. For a given material the maximum surface temperature can be reduced by going to a thinner wall, but it is not possible to reduce the wall thickness in many instances as high fluid pressures are involved.

In view of these types of considerations a method that may prove increasingly useful for advanced applications is to employ transpiration cooling. The metal wall is made in a porous form by such methods as sintering layers of rolled wire mesh or small metal particles. The coolant is forced through the wall and exits through the boundary exposed to the high temperature source. The coolant must be kept clean to avoid plugging of the pores in the material. Some possible applications are cooling components in fusion devices, rocket nozzles, leading edges of vehicles in high speed flight and arc electrodes. Additional applications of porous media involving heat transfer are porous solar collectors for obtaining heated air, and porous burners using premixed combustibles.

Because of the high surface temperatures involved, the coolant can go through a substantial temperature change when flowing from the reservoir to the exit surface of the plate. This can result in large variations in fluid properties. The fluid thermal conductivity is generally unimportant as the heat conduction is dominated by that in the solid material; hence, it is the fluid density and viscosity variations that must be accounted for. The objective of this report is to obtain an analytical solution that includes the effects of geometry as well as variable fluid properties. The porous material can have an arbitrary three-dimensional shape, and the effect of both variable density and viscosity will be included. Some of the previous analyses of porous media heat transfer have dealt with one-dimensional situations including variable properties (refs. 1 to 3), two-dimensional solutions with constant properties (refs. 4 and 5), a two-dimensional numerical solution for compressible flow (ref. 6), and an analytical solution for two-dimensional compressible flow with constant viscosity (ref. 7).

The three-dimensional porous medium can have various shapes such as a wall of varying thickness along both its length and width, or an annular region with variable thickness along its length. One part of the porous region is in contact with the coolant reservoir which is at constant pressure and temperature. The coolant exits through another part of the porous region into a region at a constant pressure lower than the reservoir pressure. The part of the boundary through which the coolant exits is specified as being at constant temperature. The remaining parts of the boundary are either at infinity, or are both insulated and impervious to the flow. The constant temperature at the coolant exit face is determined by external conditions imposed on the porous medium. For example in a design calculation it may be taken to be the maximum safe tem-

perature to which the material can be subjected. The calculation would then yield the imposed heat flux that can be tolerated.

The restrictive assumptions in the analysis are that the mass flow through the porous material is assumed to be at a sufficiently low pore Reynolds number so that Darcy's law applies. Also it is assumed that the thermal resistance between the fluid and the porous matrix material is small enough so that the local fluid and matrix temperature are equal. As a consequence, a single energy equation can be written that includes the heat transport by conduction in the matrix and by convection of the coolant. The other governing equations are continuity, and for the compressible case, the perfect gas law.

Because of the fluid property variations, the flow and energy equations are coupled. These equations are solved simultaneously by using a suitably defined potential function. The potential accounts for the integrated effect of the variable viscosity along a flow path and is found by solving Laplace's equation in the porous region subject to a simple set of boundary conditions. The temperature distribution in the porous material can be expressed as a simple function of this potential. The pressure distribution can then be found from an integral involving the temperature and the variable viscosity. Thus the main part of the solution is reduced to solving Laplace's equation for which many analytical techniques are available, or the solution to Laplace's equation can be done numerically if the geometry is irregular.

To illustrate the effects of geometry, compressibility, and viscosity variation some illustrative results are given for a step porous wall with air as the coolant (compressible, variable viscosity) and with oil as the coolant (incompressible, variable viscosity).

SYMBOLS

A	thickness ratio of step porous wall
a, b	constants in linear viscosity variation, eq. (48)
C_p	specific heat at constant pressure
f	temperature ratio, $(t_r - t_\infty)/(t_s - t_\infty)$
h_r	reference dimension of porous region
$\hat{i}, \hat{j}, \hat{k}$	unit vectors in x-, y-, and z-directions, respectively
k_m	effective thermal conductivity of porous region
M	for incompressible case, $M = (1/2)(\mu/\mu_\infty)(t_\infty/t)$; for compressible case, $M = \mu/\mu_\infty$
$\hat{n}$	unit outward drawn normal

P	for incompressible case, $P = p/p_\infty$; for compressible case, $P = (p/p_\infty)^2$
p	pressure
$\vec{Q}$	dimensionless energy flux vector, $\vec{q}h_r/k_m t_\infty$
$\vec{q}$	energy flux vector, $-k_m \nabla t + \rho \vec{u} C_p t$
q_s	heat flux by conduction only at surface s
R	gas constant
S	exit surface of porous medium in dimensionless coordinates
S_0	inlet surface of porous medium in dimensionless coordinates
s	coolant exit surface of porous medium
s_0	coolant inlet surface of porous medium
T	temperature ratio, t/t_∞
t	absolute temperature
$\vec{u}$	velocity vector
X, Y, Z	dimensionless coordinates: x/h_r, y/h_r, z/h_r, respectively
x, y, z	coordinates in physical plane
κ	permeability of porous material
λ	parameter, $\rho_\infty C_p \kappa p_\infty / 2\mu_\infty k_m$
μ	fluid viscosity
ξ	intermediate mapping variable for step porous wall
ρ	fluid density
Φ	potential defined by eq. (22)
Φ_s	potential along coolant exit surface
φ	dimensionless potential, Φ/Φ_s
$\tilde{\nabla}$	dimensionless gradient, $\hat{i}(\partial/\partial X) + \hat{j}(\partial/\partial Y) + \hat{k}(\partial/\partial Z)$

Subscripts:

i	insulated impervious surface or surface at infinity
r	reference temperature
s	on surface where coolant exits from porous medium
0	on surface where coolant enters porous medium
∞	in coolant reservoir

Superscripts:

a	arithmetic average between reservoir and exit surface
r	at a reference value
s	exit surface
∞	inlet reservoir

ANALYSIS

Geometry and Imposed Physical Conditions

Consider the porous region shown in figure 1. The coolant reservoir at pressure and temperature p_∞, t_∞ is below the porous region; hence the coolant enters the porous

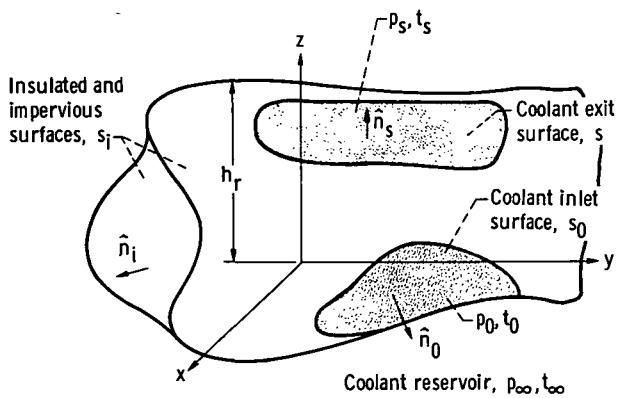


Figure 1. - Porous region $p_\infty > p_s$, $t_s > t_\infty$.

matrix through surface s_0 and exits through the part of the surface denoted by s which is at constant pressure p_s. The remainder of the boundary s_i is either impervious to both heat and coolant flow or is at infinity. Acceleration effects are neglected in the fluid as it approaches the reservoir boundary s_0 so that the pressure at this boundary is equated to the reservoir pressure $p_0 = p_\infty$. In this analysis we will always have $p_\infty > p_s$. Since p_0 and p_s are both constant, the fluid velocity at the inlet and exit surfaces will be locally perpendicular to these surfaces; these directions are indicated by the unit normal vectors $\hat{n}_0$ and $\hat{n}_s$. The normal to the remainder of the boundary is denoted by $\hat{n}_i$. The temperature t_s along the coolant exit surface is specified as constant and is higher than the coolant reservoir temperature. Hence heat is conducted into

the porous matrix at the coolant exit surface and the energy is transferred to the coolant and carried back out through s.

The effective thermal conductivity of the matrix material k_m is based on the entire cross-sectional area rather than on the area of only the solid material. Since the conductivity of the matrix material is generally much larger than that of the coolant, the heat conduction in the coolant is neglected. The Darcy velocity $\vec{u}$ will be used throughout the analysis. This velocity is the local volume flow divided by the entire cross-sectional area rather than by the pore cross-sectional area.

Governing Equations

The pores are assumed small compared with the size of the porous region; hence, volume averaged equations can be used. For the conditions discussed, the following equations apply for compressible and incompressible coolants:

Conservation of mass

$$\nabla \cdot (\rho\vec{u}) = 0 \text{ (compressible)} \tag{1a}$$

$$\nabla \cdot \vec{u} = 0 \text{ (incompressible)} \tag{1b}$$

Darcy's law

$$\vec{u} = -\frac{\kappa}{\mu(t)} \nabla p \tag{2}$$

Conservation of energy

$$\nabla \cdot \vec{q} = 0 \tag{3}$$

where

$$\vec{q} \equiv -k_m \nabla t + \rho\vec{u}C_p t \tag{4}$$

Perfect gas law (for compressible case)

$$p = \rho R t \tag{5}$$

Darcy's law as given by equation (2) applies for both the compressible and incompressible cases with temperature dependent viscosity (refs. 8 and 9). Kinetic energy and inertia effects have been neglected.

Boundary Conditions

As the fluid in the reservoir approaches the porous medium, there is some heat conduction from the coolant inlet face s_0 back into the fluid. This raises the coolant temperature (and, hence, matrix temperature, since they are equal in this analysis) to t_0 which is an unknown quantity along the surface s_0 and will be obtained later in the analysis. The thermal conductivity of the fluid is generally much less than that of the porous matrix material. Hence, as the fluid approaches the wall, the temperature rise from t_∞ to t_0 takes place in a thin region compared with the thickness of the porous material. In addition, the pressure p_0 is essentially equal to p_∞ since the pressure change associated with accelerating the fluid to the inlet of the porous material is negligible compared with the large drop of pressure through the porous region. This constant pressure condition results in the inlet velocity vector being everywhere locally normal to s_0 with no flow along this surface. These conditions permit the boundary conditions to be applied in a locally one-dimensional fashion along s_0. By balancing the heat conducted out of the wall and into the reservoir, with the energy carried back into the matrix by convection, we obtain

$$\left.\begin{aligned} k_m \hat{n}_0 \cdot \nabla t &= \rho C_p (t - t_\infty) \hat{n}_0 \cdot \vec{u} \\ p &= p_0 = p_\infty = \text{constant} \end{aligned}\right\} \quad \text{for } (x,y,z) \text{ on } s_0 \tag{6}$$

On the surface s through which the fluid leaves the porous material, the temperature is specified as constant so that the boundary conditions are

$$\left.\begin{aligned} t &= t_s = \text{constant} \\ p &= p_s = \text{constant} \end{aligned}\right\} \quad \text{for } (x,y,z) \text{ on } s \tag{7a}$$

And on the impervious surface the boundary conditions are

$$\left.\begin{aligned} \hat{n}_i \cdot \vec{u} &= 0 \\ \hat{n}_i \cdot \vec{q} &= 0 \end{aligned}\right\} \quad \text{for } (x,y,z) \text{ on } s_i \tag{7b}$$

Equations in Dimensionless Form

The governing equations and boundary conditions will now be placed in dimensionless form. The functions P and M (defined in eqs. (8) and (9)) are given different definitions for the compressible and incompressible flow cases. These definitions will result in the two cases reducing to the same set of relations. The dimensionless variables are

$$X = \frac{x}{h_r} \qquad Y = \frac{y}{h_r} \qquad Z = \frac{z}{h_r} \qquad T = \frac{t}{t_\infty}$$

$$P = \begin{cases} \dfrac{p}{p_\infty} & \text{incompressible} \\[2ex] \left(\dfrac{p}{p_\infty}\right)^2 & \text{compressible} \end{cases} \tag{8}$$

$$M(T) = \begin{cases} \dfrac{1}{2}\,\dfrac{\mu(t)}{\mu_\infty}\,\dfrac{t_\infty}{t} & \text{incompressible} \\[2ex] \dfrac{\mu(t)}{\mu_\infty} & \text{compressible} \end{cases} \tag{9}$$

$$\vec{Q} = \frac{\vec{q}h_r}{k_m t_\infty} \tag{10}$$

$$\lambda = \frac{\rho_\infty C_p \kappa p_\infty}{2\mu_\infty k_m} \tag{11}$$

$$\tilde{\nabla} = \hat{i}\,\frac{\partial}{\partial X} + \hat{j}\,\frac{\partial}{\partial Y} + \hat{k}\,\frac{\partial}{\partial Z}$$

For the compressible case, equation (5) is first used to eliminate ρ from equations (1a) and (4). Then for both the compressible and incompressible cases, Darcy's law is used to eliminate $\vec{u}$. The result is that equations (1), (3), and (4) reduce to

$$\tilde{\nabla}^2 P = \frac{1}{TM} \tilde{\nabla} TM \cdot \tilde{\nabla} P \tag{12}$$

$$\tilde{\nabla} \cdot \vec{Q} = 0 \tag{13}$$

$$\vec{Q} = -\tilde{\nabla} T - \frac{\lambda}{M} \tilde{\nabla} P \tag{14}$$

By a similar manipulation boundary conditions (6), (7a), and (7b) become

$$\left.\begin{aligned} \frac{T}{T-1} \hat{n}_0 \cdot \tilde{\nabla} T + \frac{\lambda}{M} \hat{n}_0 \cdot \tilde{\nabla} P &= 0 \\ P &= 1 \end{aligned}\right\} \quad \text{for } (X,Y,Z) \text{ on } S_0 \tag{15}$$

$$\left.\begin{aligned} T &= T_s \\ P &= P_s \end{aligned}\right\} \quad \text{for } (X,Y,Z) \text{ on } S \tag{16a}$$

$$\left.\begin{aligned} \hat{n}_i \cdot \vec{Q} &= 0 \\ \hat{n}_i \cdot \tilde{\nabla} T &= 0 \end{aligned}\right\} \quad \text{for } (X,Y,Z) \text{ on } S_i \tag{16b}$$

Formulation in Terms of a Potential Function

At this point we shall postulate, pending later verification, that P is a function of T only. It will be found that, with these conditions, a solution satisfying the governing equations and boundary conditions can be obtained; and, hence, this is the required solution. We shall now show that this assumption implies that the energy flux is the negative gradient of a potential Φ. That is,

$$\vec{Q} = -\tilde{\nabla} \Phi \tag{17}$$

Hence, equation (13) shows that

$$\tilde{\nabla}^2\Phi = 0 \tag{18}$$

which shows that this potential function can be found as a solution to Laplace's equation with boundary conditions to be prescribed subsequently.

Since P is only a function of T,

$$\frac{1}{M(T)}\tilde{\nabla}P = \frac{1}{M(T)}\frac{dP}{dT}\tilde{\nabla}T$$

Hence, if we put

$$h(T) = \int_{T_0}^{T} \frac{1}{M(T)}\frac{dP}{dT}dT \tag{19}$$

it follows that

$$\tilde{\nabla}h(T) = \frac{dh}{dT}\tilde{\nabla}T = \frac{1}{M(T)}\frac{dP}{dT}\tilde{\nabla}T = \frac{1}{M(T)}\tilde{\nabla}P \tag{20}$$

so that equation (14) can be written as

$$\vec{Q} = -\tilde{\nabla}T - \lambda\tilde{\nabla}h(T) = -\tilde{\nabla}\left[T + \lambda h(T)\right] \tag{21}$$

This gives

$$\vec{Q} = -\tilde{\nabla}\Phi$$

with the potential function Φ defined as

$$\Phi = T + \lambda\int_{T_0}^{T} \frac{1}{M(T)}\frac{dP}{dT}dT + C_o \tag{22}$$

where C_o is an arbitrary constant that can be used to fix the level of the potential. Note

that, since P is only a function of T, Φ is also a function of T only. Hence, P and T are functions of Φ only.

Equations (13) and (14) have been expressed in terms of a potential; now we return to equation (12). This can be written as

$$\tilde{\nabla}^2 P = \frac{1}{MT} \frac{d(MT)}{d\Phi} \tilde{\nabla}\Phi \cdot \frac{dP}{d\Phi} \tilde{\nabla}\Phi = \frac{1}{MT} \frac{d(MT)}{d\Phi} \frac{dP}{d\Phi} |\tilde{\nabla}\Phi|^2 \tag{23}$$

Now

$$\tilde{\nabla}^2 P = \tilde{\nabla} \cdot \tilde{\nabla} P = \tilde{\nabla} \cdot \frac{dP}{d\Phi} \tilde{\nabla}\Phi$$

In view of equation (18), this becomes[1]

$$\tilde{\nabla}^2 P = \frac{d^2P}{d\Phi^2} |\tilde{\nabla}\Phi|^2 \tag{24}$$

and it follows from equation (22) that

$$\frac{d\Phi}{dT} = 1 + \lambda \frac{1}{M} \frac{dP}{dT} = 1 + \frac{\lambda}{M} \frac{dP}{d\Phi} \frac{d\Phi}{dT} \tag{25}$$

Equations (24) and (25) can now be used to eliminate P from equation (23) giving the following equation which is a restatement of equation (12) in a form which determines T as a function of Φ:

$$\frac{d^2T}{d\Phi^2} + \frac{1}{T} \frac{dT}{d\Phi} \left(1 - \frac{dT}{d\Phi}\right) = 0$$

---

[1] Note that $\tilde{\nabla} \cdot \frac{dP}{d\Phi} \tilde{\nabla}\Phi = \frac{dP}{d\Phi} \tilde{\nabla} \cdot \tilde{\nabla}\Phi + \tilde{\nabla}\Phi \cdot \tilde{\nabla} \frac{dP}{d\Phi} = \frac{dP}{d\Phi} \tilde{\nabla}^2\Phi + \tilde{\nabla}\Phi \cdot \frac{d^2P}{d\Phi^2} \tilde{\nabla}\Phi$

$$= \frac{dP}{d\Phi} \tilde{\nabla}^2\Phi + \frac{d^2P}{d\Phi^2} |\tilde{\nabla}\Phi|^2 .$$

or equivalently

$$\frac{d}{d\Phi}\left[\frac{1}{T}\left(\frac{dT}{d\Phi} - 1\right)\right] = 0 \tag{26}$$

Before solving equation (26) the first boundary condition of equation (15) will also be expressed in terms of Φ. It follows from equations (21) and (20) that

$$\tilde{\nabla}\Phi - \tilde{\nabla}T = \frac{\lambda}{M}\tilde{\nabla}P$$

Upon solving this for $\tilde{\nabla}P$ and substituting it into the boundary condition, the latter becomes

$$\frac{T}{T - 1}\hat{n}_0 \cdot \tilde{\nabla}T + \hat{n}_0 \cdot (\tilde{\nabla}\Phi - \tilde{\nabla}T) = 0$$

After using the relation $\tilde{\nabla}T = (dT/d\Phi)\tilde{\nabla}\Phi$, the boundary condition becomes

$$\left(1 + \frac{1}{T - 1}\frac{dT}{d\Phi}\right)\hat{n}_0 \cdot \tilde{\nabla}\Phi = 0 \qquad \text{for } (X, Y, Z) \text{ on } S_0 \tag{27}$$

Solution for T and P in Terms of Φ

Equation (26) can be integrated twice to obtain

$$T = \frac{1}{C_1} + C_2 e^{-C_1\Phi} \tag{28}$$

The constants C_1 and C_2 are to be evaluated from the boundary conditions. Thus inserting solution (28) into equation (27) gives

$$(1 - C_1)\frac{T}{T - 1}\hat{n}_0 \cdot \tilde{\nabla}\Phi = 0 \qquad \text{for } (X, Y, Z) \text{ on } S_0$$

This boundary condition will be satisfied if $C_1 = 1$. Then equation (28) becomes

$$T = 1 + C_2 e^{-\Phi} \tag{29}$$

Since the second of boundary conditions (15) involves P, we use equations (25) and (29) to relate P and T. Thus equation (25) implies that

$$\frac{dP}{dT} = \left(\frac{d\Phi}{dT} - 1\right)\frac{M}{\lambda} \tag{30}$$

and equation (29) shows that

$$dT = -C_2 e^{-\Phi}\, d\Phi = (1 - T)d\Phi$$

These relations can be combined to obtain

$$\frac{dP}{dT} = \left(\frac{1}{1 - T} - 1\right)\frac{M}{\lambda} = \frac{MT}{1 - T}\,\frac{1}{\lambda} \tag{31}$$

Integrating this from S_0 to an arbitrary position in the medium and taking into account the second boundary condition (15) result in the following:

$$P - 1 = \frac{1}{\lambda}\int_{T_0}^{T} \frac{MT}{1 - T}\, dT \tag{32}$$

The value T_0 is unknown in this integral. To evaluate it, the specified boundary conditions of equation (16) are imposed to give

$$P_s - 1 = \frac{1}{\lambda}\int_{T_0}^{T_s} \frac{MT}{1 - T}\, dT \tag{33}$$

This integral can be carried out once the viscosity variation is specified so that $M(T)$ is known. The quantity T_0 is thereby related to the specified quantities P_s and T_s.

To obtain the simplest boundary conditions for the potential Φ which is governed by Laplace's equation, the constant C_o in equation (22) can be set equal to $-T_0$ to give

$$\Phi = T - T_0 + \lambda \int_{T_0}^{T} \frac{1}{M}\,\frac{dP}{dT}\, dT \tag{34}$$

and hence, since $T = T_0$ along S_0, we obtain the boundary condition

$$\Phi = 0 \qquad \text{for } (X,Y,Z) \text{ on } S_0 \tag{35}$$

It now follows from equations (35) and (29) that the constant C_2 is compatible with fixing $\Phi = 0$ on S_0 only if

$$T_0 = 1 + C_2$$

Then equation (29) becomes

$$T = 1 + (T_0 - 1)e^{-\Phi}$$

which can be solved for Φ, to obtain

$$\Phi = -\ln\left(\frac{T - 1}{T_0 - 1}\right) \tag{36}$$

This is evaluated at the coolant exit face to obtain the boundary condition

$$\Phi = \Phi_s \equiv -\ln\left(\frac{T_s - 1}{T_0 - 1}\right) \qquad \text{for } (X,Y,Z) \text{ on } S \tag{37a}$$

The surface temperature T_s is specified and T_0 can be calculated from equation (33). Hence Φ_s is known. Finally it follows from equations (16b) and (17) that Φ must satisfy the boundary condition on the impervious surfaces

$$\hat{n}_i \cdot \tilde{\nabla}\Phi = 0 \qquad \text{for } (X,Y,Z) \text{ on } S_i \tag{37b}$$

And since T is a function only of Φ, the second boundary condition (16b) is automatically satisfied. Equations (35), (37a), and (37b) provide the required boundary conditions to solve equation (18) for Φ. In the solution of Laplace's equation for the potential function it would be more convenient to have the boundary conditions go from 0 to 1 rather than from 0 to Φ_s. To this end the potential can be normalized as

$$\varphi(X,Y,Z) = \frac{\Phi(X,Y,Z)}{\Phi_s} \tag{38}$$

Then φ is obtained from

$$\tilde{\nabla}^2\varphi = 0 \tag{39}$$

$$\varphi = 0 \qquad \text{for } (X,Y,Z) \text{ on } S_0$$

$$\varphi = 1 \qquad \text{for } (X,Y,Z) \text{ on } S$$

$$\hat{n}_i \cdot \tilde{\nabla}\varphi = 0 \qquad \text{for } (X,Y,Z) \text{ on } S_i$$

By eliminating T_0 from equations (36) and (37a) the temperature distribution is found to be

$$T = 1 + (T_s - 1)e^{\Phi_s - \Phi} = 1 + (T_s - 1)e^{\Phi_s(1-\varphi)} \tag{40}$$

Heat and Mass Flux at Coolant Exit Surface

A quantity of practical interest is the heat flux that is being transferred to the surface of the porous material. This relates the flux to the surface temperature. Also of interest is the mass flux distribution blowing out of the exit face as this will influence any external flow past the surface.

The local heat flux conducted into the porous surface at the boundary S is

$$q_s = k_m \hat{n}_s \cdot \nabla t \qquad \text{for } (x,y,z) \text{ on } s \tag{41}$$

In dimensionless form this becomes

$$\frac{q_s h_r}{k_m(t_s - t_\infty)} = \frac{1}{T_s - 1}\,\hat{n}_s \cdot \tilde{\nabla}T = \frac{1}{T_s - 1}\,\hat{n}_s \cdot \frac{dT}{d\Phi}\tilde{\nabla}\Phi \qquad \text{for } (X,Y,Z) \text{ on } S \tag{42}$$

The term $dT/d\Phi\,|_{\Phi=\Phi_s}$ is obtained from equation (40) Since Φ is constant on S, the normal vector $\hat{n}_s$ is in the direction of the gradient of Φ at the boundary. Hence

$$\hat{n}_s = -\frac{\tilde{\nabla}\Phi}{|\tilde{\nabla}\Phi|} \qquad \text{for } (X,Y,Z) \text{ on } S \tag{43}$$

The negative sign accounts for the fact that Φ decreases in going from S_0 to S (note that Φ_s is negative). Equation (42) can therefore be written as

$$\frac{q_s h_r}{k_m(t_s - t_\infty)} = \frac{\tilde{\nabla}\Phi \cdot \tilde{\nabla}\Phi}{\tilde{\nabla}\Phi} = |\tilde{\nabla}\Phi|$$

or using the normalized potential

$$\frac{q_s h_r}{k_m(t_s - t_\infty)} = |\Phi_s|\,|\tilde{\nabla}\varphi| \qquad \text{for } (X,Y,Z) \text{ on } S \tag{44}$$

The local coolant flux distribution leaving the porous medium is given by

$$\rho\vec{u} \cdot \hat{n}_s = -\frac{\kappa}{\mu(t)}\,\rho\nabla p \cdot \hat{n}_s \qquad \text{for } (x,y,z) \text{ on } s \tag{45}$$

First consider the case of a compressible flow. By using equation (5) and introducing dimensionless variables this becomes

$$\rho\vec{u} \cdot \hat{n}_s = -\frac{\kappa}{\mu(t)}\,\frac{p}{Rt}\,\nabla p \cdot \hat{n}_s = -\frac{k_m}{C_p h_r}\,\frac{1}{MT}\,\tilde{\nabla}(\lambda P) \cdot \hat{n}_s \tag{46}$$

But upon noting that $\tilde{\nabla}(\lambda P) = \left[d(\lambda P)/dT\right](dT/d\Phi)\tilde{\nabla}\Phi$, we find from equation (31) and the line preceding it that

$$\tilde{\nabla}(\lambda P) = \left(\frac{MT}{1 - T}\right)(1 - T)\tilde{\nabla}\Phi = MT\,\tilde{\nabla}\Phi$$

Inserting this in equation (46) and using equations (43) and (38) give

$$\frac{h_r C_p}{k_m}\,\rho\vec{u}_s \cdot \hat{n}_s = \frac{\tilde{\nabla}\Phi \cdot \tilde{\nabla}\Phi}{|\tilde{\nabla}\Phi|} = |\Phi_s|\,|\tilde{\nabla}\varphi| \qquad \text{for } (X,Y,Z) \text{ on } S \tag{47}$$

A similar manipulation for the incompressible case shows that equation (47) also applies in this instance. Upon comparing equations (44) and (47), it becomes evident that the dimensionless heat and mass fluxes are equal.

Summary of Analytical Solution

The main results of the analysis are the temperature distribution in the porous material (eq. (40)), the local distribution of heat flux transferred to the coolant exit surface (eq. (44)), and the mass flux distribution exiting from the porous material (eq. (47)). The quantities needed to evaluate these expressions are the distributions of φ and Φ_s. The potential φ is found from equation (39); that is, by solving Laplace's equation in the domain of the porous region with the simple boundary conditions $\varphi = 0$ and 1 on the coolant inlet and exit surfaces, respectively, and $\hat{n}_i \cdot \tilde{\nabla}\varphi = 0$ on the remainder of the boundary. Equation (37) is used to obtain the quantity Φ_s needed in the final results. But this requires that T_0 be known. However, T_0 is determined by equation (33) where P_s and T_s are given and where the specified function M(T) accounts for the law of viscosity variation with temperature.

EXAMPLES ILLUSTRATING APPLICATION OF GENERAL SOLUTION

Geometry Effect

It follows from equations (44) and (47) that the dimensionless heat and mass fluxes along the coolant exit boundary both depend on $|\tilde{\nabla}\varphi|$ along that boundary. To demonstrate the geometric effect which is given by the $|\tilde{\nabla}\varphi|$ term, consider for example the two-dimensional wall with a step shown in figure 2. All of the lengths have been made dimensionless by dividing by the smaller thickness. It is necessary to evaluate $|\tilde{\nabla}\varphi|$ along the boundary $Y = 1$ where φ is determined from the boundary value problem $\tilde{\nabla}^2\varphi = 0$ in the region and $\varphi = 0$ and 1, respectively, along the lower and upper boundaries.

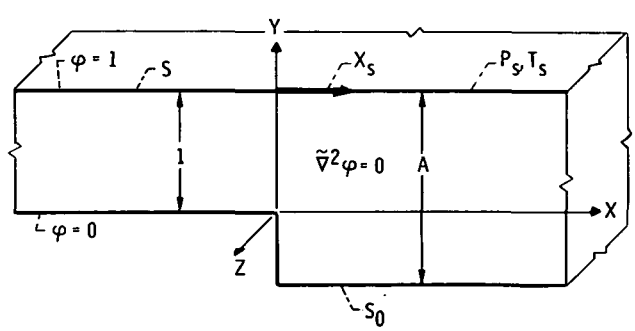


Figure 2. - Step porous wall in dimensionless physical plane.

This has been done in reference 7 where it is shown that

$$|\tilde{\nabla}\varphi|_{Y=1} = \frac{\xi}{A}$$

where ξ is a dummy variable related to the distance X_s along the upper surface by

$$X_s = \frac{1}{\pi} A \left[\ln\left(\frac{\xi + 1}{\xi - 1}\right) - \ln\left(\frac{A + \xi}{A - \xi}\right) \right] \qquad 1 \leq \xi \leq A$$

The variation with X_s of $|\tilde{\nabla}\varphi|_{Y=1}$ is shown in figure 3 for various thickness ratios. Since the heat and mass fluxes are directly proportional to this quantity, the plot shows how these fluxes decrease in the thicker portion of the medium.

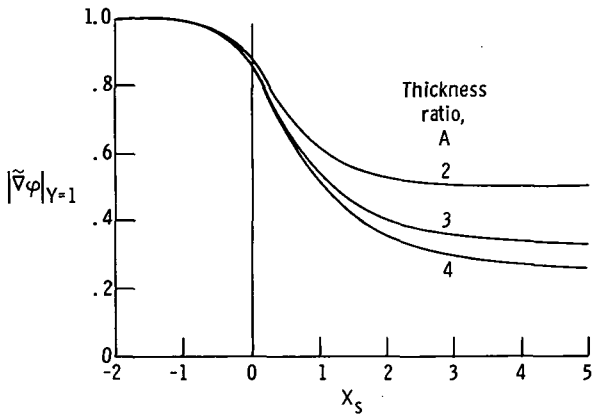


Figure 3. - Gradient of normalized potential along coolant exit surface of step porous wall.

Fluid Properties Effect

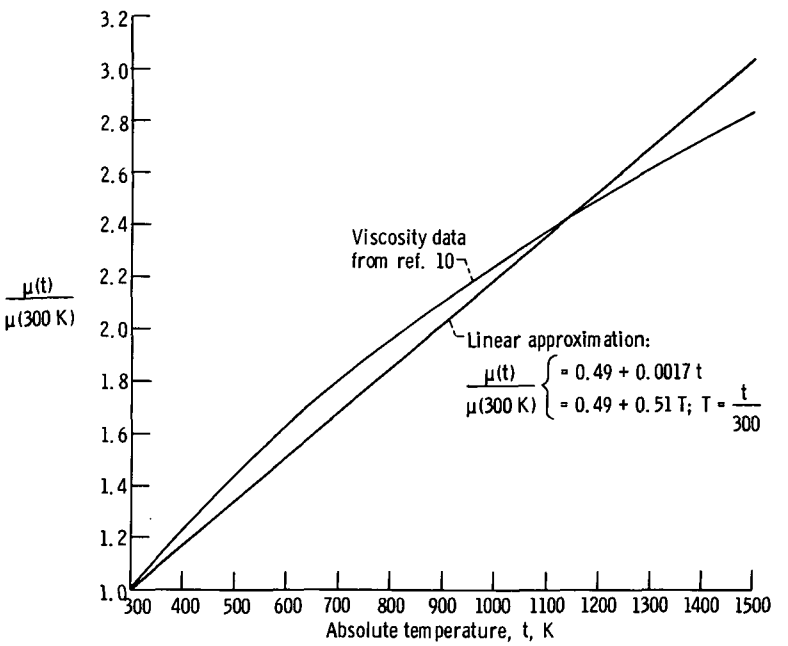
The dimensionless heat and mass fluxes are also proportional to Φ_s which depends on the viscosity variation with temperature. To illustrate the effect of this variation, Φ_s will be determined for two example fluids: air (compressible case), and oil (incompressible case).

Determination of Φ_s for air. - As an illustrative example, let the coolant reservoir be at $t_\infty = 300$ K and the exit face of the porous material be at $t_s = 1500$ K. Since the viscosity of air is insensitive to pressure, the variation of $\mu(t)/\mu_\infty$ is plotted at 1 atmosphere in figure 4(a) (from ref. 10) even though the pressure varies through the porous medium. As a simple approximation, the variation is taken as linear so that

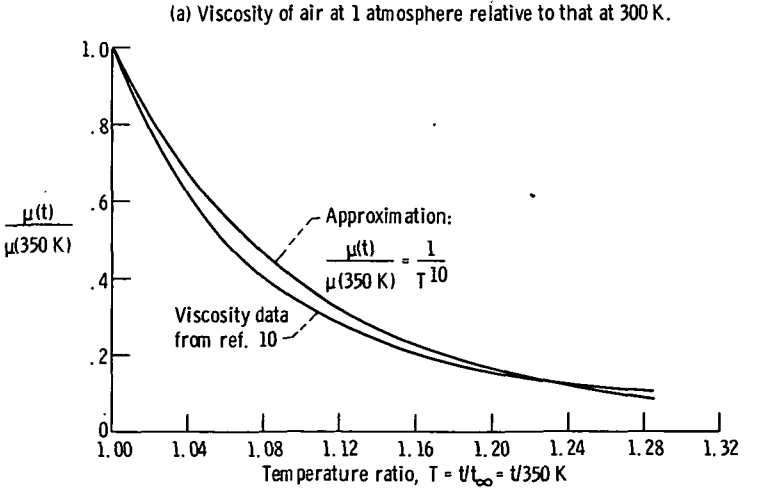
$$\frac{\mu}{\mu_\infty} = M(T) = a + bT = 0.49 + 0.51T \tag{48}$$

If $M = a + bT$ is inserted into equation (33) and the integration performed, we find that

$$(P_s - 1)\lambda = (a + b)(T_0 - T_s) + \frac{b}{2}(T_0^2 - T_s^2) + (a + b)\ln\frac{T_0 - 1}{T_s - 1} \tag{49}$$

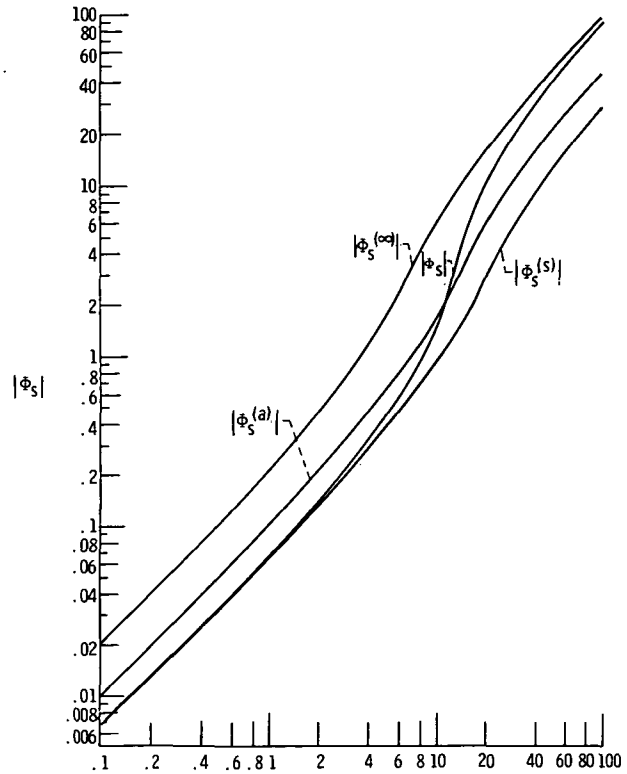


(a) Viscosity of air at 1 atmosphere relative to that at 300 K.

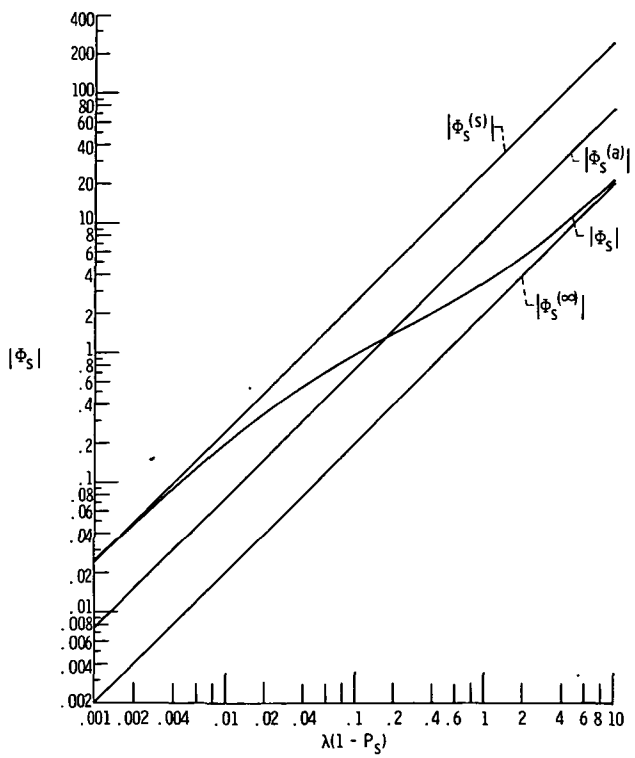


(b) Viscosity of oil relative to that at 350 K.

Figure 4. - Viscosity variations used in illustrative calculations.



(a) Results for air.



(b) Results for oil.

Figure 5. - Values of $|\Phi_s|$ for variable viscosity and various reference viscosities; $|\Phi_s|$ is directly proportional to mass and heat fluxes at coolant exit surface.

Upon using equation (37) to eliminate T_0 from equation (49), we obtain the relation between P_s and Φ_s as

$$\lambda(1 - P_s) = (T_s - 1)\left(1 - e^{\Phi_s}\right) - \Phi_s + \frac{b}{2}(T_s - 1)\left[(T_s + 1) - 2e^{\Phi_s} - (T_s - 1)e^{2\Phi_s}\right] \tag{50}$$

Note that this relation is independent of any geometric considerations. The factor $|\Phi_s|$ multiplies the geometrically dependent quantity $|\tilde{\nabla}\varphi|_s$ whose calculation was illustrated previously for a step porous wall. The factor $|\Phi_s|$ accounts for all the variable property effects.

The factor $|\Phi_s|$ is given as a function of $\lambda(1 - P_s)$ in figure 5(a) for the specific viscosity variation shown in figure 4(a) where $T_s = 1500/300 = 5$ and $b = 0.51$. This illustrates the variation of heat and mass flux as a function of λ and P_s for an arbitrary geometry.

In general $M(T)$ would not be a simple function of T that would permit equation (33) to be integrated analytically. In that instance a numerical integration can be done to relate T_0 to $\lambda(P_s - 1)$ and then this value of T_0 inserted into equation (37) to obtain Φ_s.

Determination of Φ_s for oil. - To provide a second illustration let the coolant be engine oil for which the viscosity variation with temperature is tabulated in reference 10. The reservoir temperature is chosen as $t_\infty = 350$ K, and the surface temperature at the coolant exit is $t_s = 450$ K. It can be seen from figure 4(b) that a reasonable approximation for μ/μ_∞ is (note also the definition for $M(T)$ in eq. (9))

$$\frac{\mu}{\mu_\infty} = 2TM = \frac{1}{T^{10}} \tag{51}$$

This is inserted in equation (33) to give

$$\lambda(P_s - 1) = \frac{1}{2}\int_{T_0}^{T_s} \frac{1}{T^{10}(1 - T)}\,dT$$

which can be integrated to obtain

$$\lambda(1 - P_s) = \frac{1}{2}\left[\ln\left(\frac{T_s - 1}{T_0 - 1}\frac{T_0}{T_s}\right) - \sum_{n=1}^{9}\frac{1}{n}\left(\frac{1}{T_0^n} - \frac{1}{T_s^n}\right)\right] \tag{52}$$

With $T_s = 450/350$ for this example, $\lambda(1 - P_s)$ can be found for various values of T_0 between 1 and T_s. Then these values of T_0 are inserted in equation (37) to calculate Φ_s. The corresponding values of $|\Phi_s|$ and $\lambda(1 - P_s)$ are plotted in figure 5(b) showing how the heat and mass flux vary with the parameter λ and with the imposed pressure ratio $P_s = p_s/p_\infty$.

Use of a Reference Temperature

It was shown that, with regard to the surface heat and mass fluxes, the geometry effect separates from the property variation effect. The latter is embodied in the quantity Φ_s which is related to the variation of viscosity with temperature. It is worthwhile to examine whether the constant viscosity solution could be used with the viscosity evaluated at a convenient reference temperature such as t_∞, t_s, or $(t_\infty + t_s)/2$.

Compressible case (for air). - For constant viscosity equation (33) becomes

$$P_s - 1 = \frac{M}{\lambda}\int_{T_0}^{T_s}\frac{T}{T - 1}\,dT = \frac{M}{\lambda}\left[(T_0 - T_s) - \ln\frac{1 - T_s}{1 - T_0}\right]$$

Then use of equation (37) to eliminate T_0 gives

$$\lambda(1 - P_s) = M\left[(T_s - 1)\left(1 - e^{\Phi_s}\right) - \Phi_s\right] \tag{53}$$

For the compressible case M is the reference viscosity divided by μ_∞. This will be written as $\mu^{(i)}/\mu_\infty$ where $i = \infty$, a, s designating that the reference value is evaluated at the reservoir temperature, arithmetic average temperature between the reservoir and exit surface, and exit surface temperature, respectively. The corresponding Φ_s for these cases are denoted by $\Phi_s^{(i)}$. This yields the following relations for $\Phi_s^{(i)}$:

Viscosity evaluated at t_∞

$$\lambda(1 - P_s) = (T_s - 1)\left(1 - e^{\Phi_s^{(\infty)}}\right) - \Phi_s^{(\infty)} \tag{54}$$

Viscosity evaluated at $(t_\infty + t_s)/2$ (for a linear viscosity-temperature dependence)

$$\lambda(1 - P_s) = \left[1 + \frac{b}{2}(T_s - 1)\right]\left[(T_s - 1)\left(1 - e^{\Phi_s^{(a)}}\right) - \Phi_s^{(a)}\right] \tag{55}$$

Viscosity evaluated at t_s

$$\lambda(1 - P_s) = \left[1 + b(T_s - 1)\right]\left[(T_s - 1)\left(1 - e^{\Phi_s^{(s)}}\right) - \Phi_s^{(s)}\right] \tag{56}$$

Note that it follows from equation (37) that the $\Phi_s^{(i)}$ are negative. The quantities $|\Phi_s^{(i)}|$ have been evaluated for various values of $\lambda(1 - P_s)$ and are shown in figure 5(a). The ratios of $\Phi_s/\Phi_s^{(i)}$ are given in figure 6(a).

Incompressible case (for oil). - By using the definition for M in equation (9) and following the same procedure as for the compressible case, the following relations are obtained for Φ_s:

Viscosity evaluated at t_∞

$$\Phi_s^{(\infty)} = -2\lambda(1 - P_s) \tag{57}$$

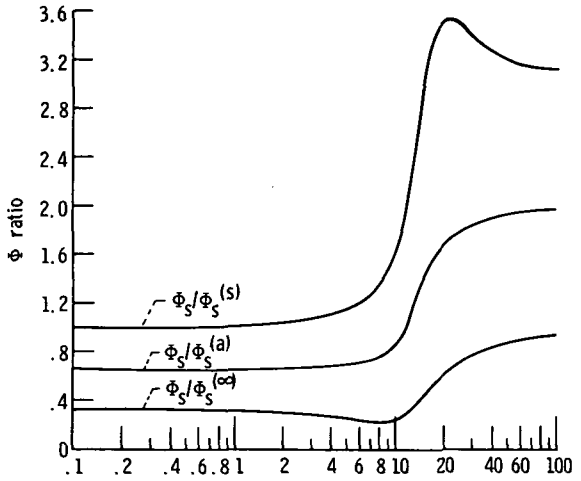
Viscosity evaluated at $(t_\infty + t_s)/2$

$$\Phi_s^{(a)} = -2\left(\frac{1 + T_s}{2}\right)^{10} \lambda(1 - P_s) \tag{58}$$

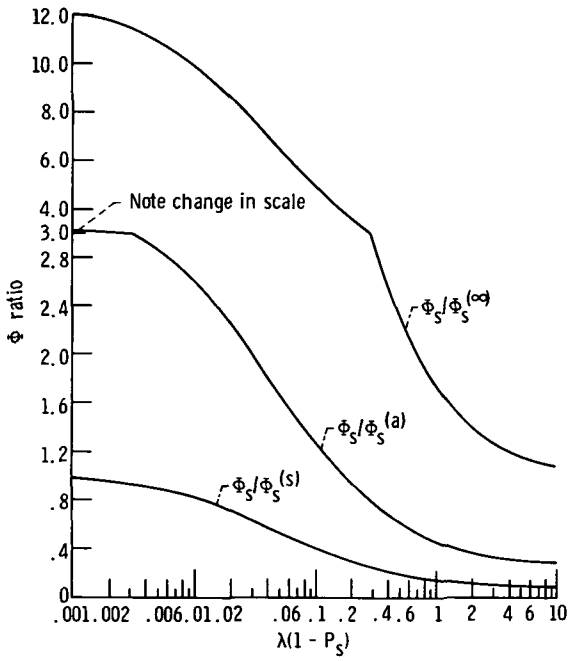
Viscosity evaluated at t_s

$$\Phi_s^{(s)} = -2T^{10} \lambda(1 - P_s) \tag{59}$$

The quantities $|\Phi_s^{(i)}|$ are plotted in figure 5(b) and the ratio $\Phi_s/\Phi_s^{(i)}$ in figure 6(b).



(a) Results for air.



(b) Results for oil.

Figure 6. - Ratios of Φ from variable properties solution to those using constant reference properties.

Proper Reference Temperature to Obtain Agreement with Exact Solution

If we let

$$f = \frac{t_r - t_\infty}{t_s - t_\infty} = \frac{T_r - 1}{T_s - 1} \tag{60}$$

where t_r is a reference temperature, then for the example with air (note that $a + b = 1$)

$$\frac{\mu^{(r)}}{\mu_\infty} = a + bT_r = 1 + fb(T_s - 1)$$

We insert this for M into equation (53) and then equate the result to equation (50) to obtain the f value that will cause the approximate equation to yield the same value of Φ_s as the exact equation

$$f = \frac{\frac{1}{2}\left[(T_s + 1) - 2e^{\Phi_s} - (T_s - 1)e^{2\Phi_s}\right]}{(T_s - 1)\left(1 - e^{\Phi_s}\right) - \Phi_s} \quad \text{(for air)} \tag{61}$$

where Φ_s is related to $\lambda(1 - P_s)$ by equation (50).

Similarly for oil

$$f = \frac{-1 + \left[\frac{-\frac{1}{2}\Phi_s}{(1 - P_s)\lambda}\right]^{1/10}}{T_s - 1} \quad \text{(for oil)} \tag{62}$$

where in this instance the $(1 - P_s)\lambda$ and Φ_s are corresponding values obtained from the exact solution equation (52). The values of f are plotted as a function of $\lambda(1 - P_s)$ in figure 7.

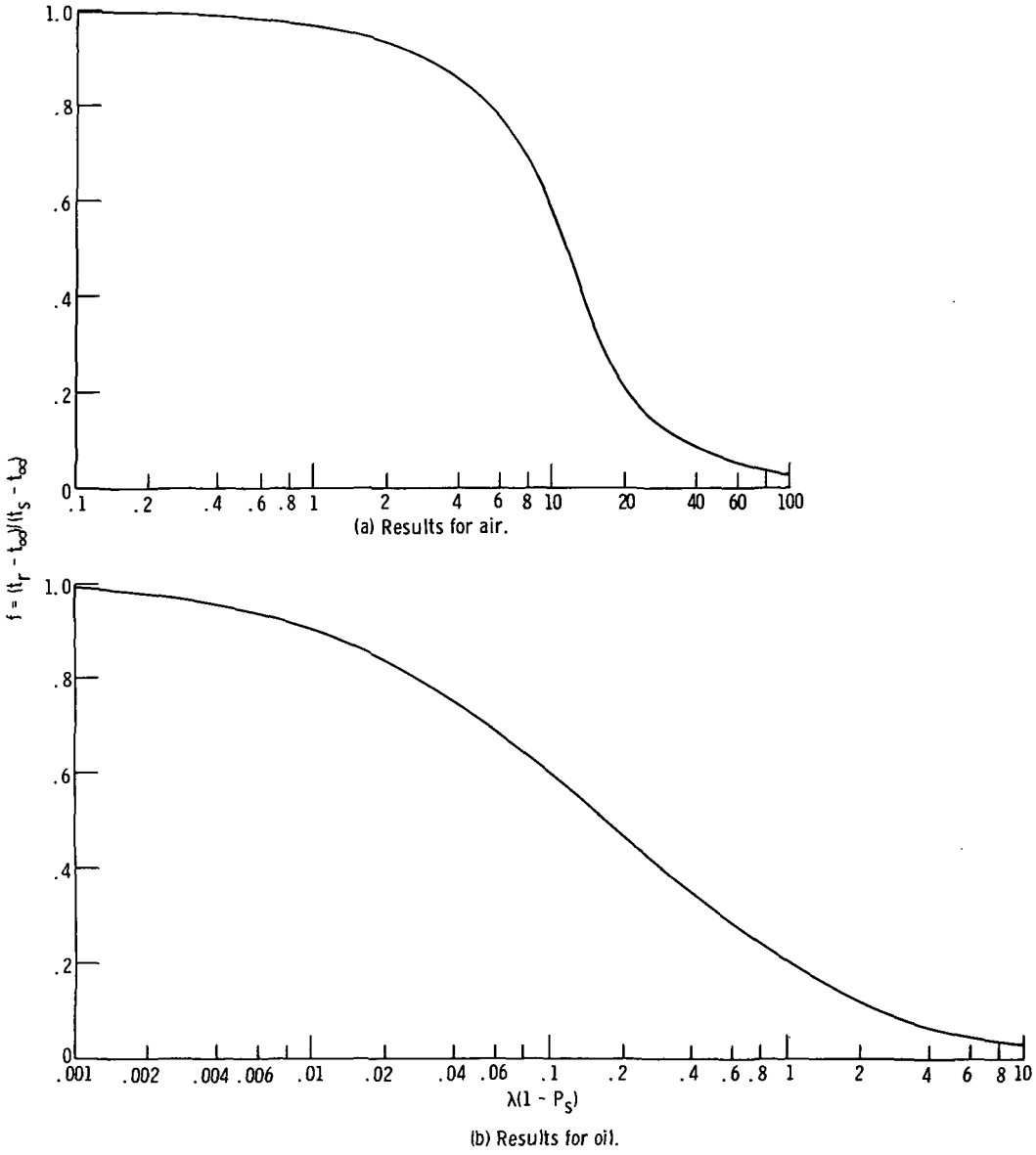


Figure 7. - Reference temperature to give correct mass and heat flow results when using constant properties solution.

DISCUSSION

An analytical solution has been obtained for the heat transfer behavior of a three-dimensional porous medium with the inlet and exit faces each at specified constant pressure and with the exit face at a specified uniform temperature. Compressible and incompressible cases are considered, and the fluid viscosity variation with temperature is taken into account.

It was found that the solution could be reduced to two independent portions, one de-

pending on the geometry and the other on the viscosity variation with temperature. The geometric portion involves solving Laplace's equation in the porous region with constant boundary conditions on the coolant inlet and exit surfaces. The heat flux and coolant mass flux along the coolant exit surface are proportional to the gradient in the solution to Laplace's equation evaluated along that surface. The effect of the shape of the porous material is illustrated by results for the porous wall with a step shown in figure 2. As a consequence of the change in thickness, the heat and mass flux vary in proportion to the function $|\tilde{\nabla}\varphi|$ shown in figure 3.

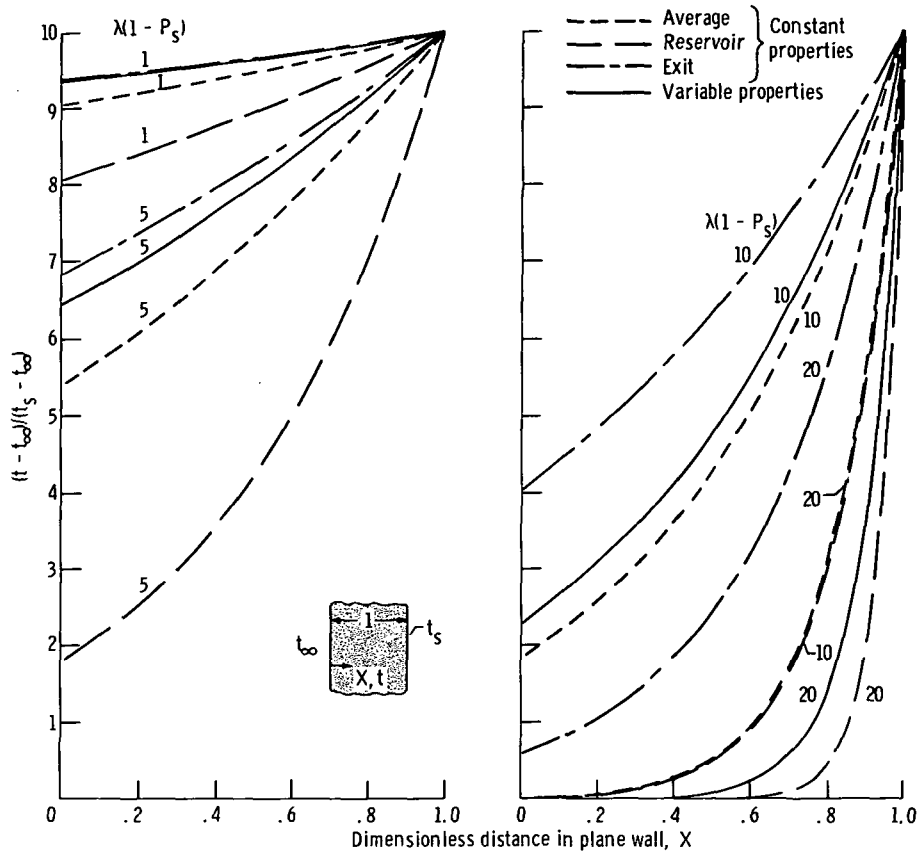
The exit surface heat and mass fluxes are also proportional to the function $|\Phi_s|$ which accounts for the variation of fluid viscosity with temperature as the fluid moves through the porous medium. Since the results depend on the nature of this temperature dependence, the analytical results were evaluated here for two illustrative fluids, air and oil, whose viscosities vary with temperature in opposite directions. The resulting $|\Phi_s|$ are shown as functions of $\lambda(1 - P_s)$ in figure 5. Note that, for the incompressible case, the quantity P_s on the abscissa is the dimensionless pressure ratio p_s/p_∞, while for compressible flow P_s is $(p_s/p_\infty)^2$. As $(1 - P_s)$ increases from 0 to 1, the pressure on the exit face of the porous medium is decreasing from the reservoir pressure toward zero. Figure 5 shows the corresponding increases in $|\Phi_s|$ which is proportional to the heat and mass flux. The detailed shapes of these curves are related to the fluid viscosity variations and can be better appreciated by looking at some typical temperature variations within the porous medium.

For simplicity the temperature variation will be examined for a plane layer for compressible flow; the same ideas will apply for a two- or three-dimensional shape and for the incompressible case. The solution for φ as obtained from $\tilde{\nabla}^2\varphi = 0$ in a plane layer with $\varphi = 0$ at $X = 0$ and $\varphi = 1$ at $X = 1$ gives the result $\varphi = X$ where X is the local position in the layer divided by the total thickness. Then equation (40) shows that the temperature distribution in the wall is

$$\frac{t - t_\infty}{t_s - t_\infty} = e^{(1-X)\Phi_s}$$

The quantity Φ_s is found as a function of $\lambda(1 - P_s)$ from figure 5(a) and the temperature distribution evaluated. Typical curves are shown in figure 8.

Consider the solid curves in figure 8; these correspond to the variable property solution. For a low flow or small λ (i.e., small $\lambda(1 - P_s)$), the conduction through the porous material and then into the fluid as it approaches the surface exposed to the reservoir, significantly preheats the flow before it enters the plate. Hence all the fluid in the plate has a viscosity close to that at the fluid exit temperature. For a high flow or large



(a) Low $\lambda(1 - P_s)$.

(b) High $\lambda(1 - P_s)$.

Figure 8. - Temperature distributions in porous plane wall for air.

λ (i.e., large $\lambda(1 - P_s)$), conduction into the plate cannot carry heat very far before the large amount of convection carries the energy out. In this instance the fluid is entering the plate at close to the reservoir temperature and most of the plate is near this temperature with a rapid temperature increase near the exit face.

These temperature curves help interpret the behavior of the $|\Phi_s|$ curves in figure 5. For small $\lambda(1 - P_s)$ the Φ_s is close to $\Phi_s^{(s)}$ where $\Phi_s^{(s)}$ is computed from the constant properties solution with properties evaluated at the exit surface. This is the proper behavior since all the coolant in the plate is near the exit temperature when $\lambda(1 - P_s)$ is small. On the other hand, when $\lambda(1 - P_s)$ is large, most of the coolant is near the inlet reservoir temperature; hence, Φ_s tends toward $\Phi_s^{(\infty)}$ which was computed using the inlet reservoir value.

The function $|\Phi_s|$ goes through a rapid increase in the midrange of $\lambda(1 - P_s)$ in figure 5(a). As evidenced by the corresponding temperature profiles in figure 8, this is the range where the flow within the porous wall is changing from being predominantly at the exit temperature to being predominantly at the inlet reservoir temperature. For air,

as the temperature decreases the viscosity decreases; hence, the increase of mass flow rate as $\lambda(1 - P_s)$ increases is accentuated by the decrease in the average fluid viscosity within the porous medium. (Recall that the $|\Phi_s|$ is directly proportional to the mass flow rate.) The opposite effect occurs in figure 5(b) since the viscosity of oil increases as the temperature in the porous medium is decreased. The results show that a constant viscosity analysis with the viscosity evaluated at a single reference temperature will not yield good predictions for all values of the parameters.

CONCLUDING REMARKS

The flow and heat transfer behavior have been considered for a three-dimensional porous medium. Fluid from a reservoir at constant temperature and pressure flows through the medium and exits through a surface at a specified uniform temperature that is higher than the reservoir temperature. The local fluid and matrix temperatures are assumed to be equal. For these conditions a potential function was obtained which allowed the solution to be reduced to solving Laplace's equation in the interior of the porous region with constant boundary conditions along the coolant inlet and exit surfaces.

Both compressible and incompressible fluids were treated and the variation of fluid viscosity with temperature was taken into account. As the pressure drop across the porous medium is increased thereby increasing the flow, the viscosity of the fluid entering the medium tends to be closer to the reservoir value. For air the fluid viscosity is lowest in the reservoir where the temperature is lowest. As a result, compared with the constant viscosity case, an increased pressure drop produces a larger flow increase as a result of lowering the average viscosity for the fluid in the medium. For oil on the other hand, the viscosity is highest at the reservoir condition. When the pressure drop is increased, the viscosity in the medium is closer to the reservoir value. The increase in viscosity tends to counter to some extent the flow increase resulting from the increased pressure drop. These trends also apply to the heat flux that can be imposed along the exit surface corresponding to the specified temperature at that surface.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 22, 1972,
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