A COMPARATIVE STUDY OF ELECTRICAL PROBE TECHNIQUES FOR PLASMA DIAGNOSTICS

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A COMPARATIVE STUDY OF ELECTRICAL PROBE TECHNIQUES FOR PLASMA DIAGNOSTICS

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I. INTRODUCTION

The experimental techniques of plasma diagnostics and the associated theoretical foundations have over the years seen considerable treatment in the literature. The techniques which seem to have received the broadest application are those involving electrical probes, generally being preferred over other methods because of their relative simplicity in implementation and analysis. Probe techniques are additionally attractive in that they result in local measurements of plasma density and temperature, whereas other approaches to plasma diagnostics yield values of the aforementioned parameters which are averaged over relatively large dimensions of the plasma under study.

The use of electrical probes for plasma diagnostics began primarily with the pioneering work of Irving Langmuir\(^1\) who made famous the technique of the single probe, the Langmuir probe. Since his initial investigations considerable advances have been made in understanding the details of single-probe response in the presence of a broad range of plasma parameters and other techniques have been introduced in an effort to
circumvent some of the possible difficulties and perturbations that could be brought about by a Langmuir probe measurement of plasma density and temperature. From the theoretical point of view the behavior of probes immersed in a collisionless plasma is well understood - particularly through the work of Laframboise
2, while the recent analysis of Kirchhoff, Peterson and Talbot
3 has contributed significantly to the understanding of probe response in the domain which spans the gap between the continuum and collisionless limits. These two theoretical works have recently been utilized to enhance the understanding of probe behavior in ionospheric applications and to establish the roles of plasma density, temperature, ionic species and collision frequencies on the body potential of a rocket or satellite. It is the object of this paper to review and extend some of this work and point out its importance with respect to on-board probe experiments. Specific consideration will be given to the simple Langmuir probe (LP), the symmetric double probe (DP) of Johnson and Malter
4, the variable-area probe (VAP) of Fetz and Oechsner
5 and a completely new approach to a floating probe technique
6 which will be described here as a fixed-bias double probe (FBDP). Emphasis will be placed on the response of the various methods in the collisionless and near collisionless limit with the treatment directed toward area influences, floating potentials, and the regions of the electron-energy distribution sampled by the various techniques. In addition, some commentary will be concerned with the possible
perturbation of the various measurement techniques by local oscillations in the plasma potential.

It should be noted that this study has been limited in scope by time and space limitations and consequently treats only those probe techniques with which the author has first-hand experience. Within this framework, the concept of a probe is taken to mean any electrode, generally of spherical, cylindrical or planar geometry, which is inserted in a plasma and actively collects current from that plasma as a function of an applied voltage. Devices like that of a retarding potential analyzer are considered outside the realm of this definition. The format for presentation of the material is intended to complement and extend the generally standard approach to review works on probes and their satellite applications by providing a novel point of view which readily lends itself to the development of an improved understanding of some the physical principles involved.

II. PROBE CONFIGURATIONS AND ANALYSIS PROCEDURES

Figures 1A-D schemetically present the probe configurations utilized in the four techniques which will be discussed in this comparative study. It can readily be seen in these figures that a common feature is a simple circuit arrangement involving two electrodes separated by a bias voltage and an electrometer for the measurement of the circuit current I. In the case of the LP and DP the bias voltage is variable and the electrode areas are fixed, while the techniques of
the VAP and FBDP employ bias voltages that are fixed in magnitude and require that one of the two electrodes be of variable area. In each of the four techniques there is the need to measure the differential voltage, \( V \) (with appropriate subscript), with the end points in the cases of the LP and DP being the illustrated electrodes. The voltage measurement in the cases of the VAP and FBDP is made between the probe and a fixed point in contact with the plasma which is different from that of the reference electrode.

In the techniques of the LP, DP and FBDP the object of the experimental configuration is the generation of a current-voltage characteristic which can under appropriate circumstances lead to the determination of the plasma density and its temperature. In the case of the VAP the technique is more complex since the experimental configuration is designed to generate a probe area \( A_p \) vs probe voltage \( V_p \) characteristic for the determination of plasma temperature and an \( I \) vs \( V_p \) characteristic for the corresponding determination of density.

The technique of the FBDP has an experimental configuration which in many respects resembles that of the VAP. However a major difference is that the roles of the variable-area electrode are changed. In the case of the FBDP the probe has a fixed area while the collecting surface of the reference electrode is variable. This leads to a significant difference in the analysis procedure for the determination of plasma density and temperature. In the case of the VAP
technique it is necessary to know the area of the probe (which is continually varied) for all values of current and voltage. In the FBDP technique the need to know the area of the variable-electrode is non-existent.

III. RELATIVE MERITS

Sampled Electron Energies and Floating Potentials

Quite often an important experimental objective is the determination of the energy distribution of plasma electrons which in the case of a Maxwellian distribution means the determination of the electron temperature. Whatever the case, it is to the experimenter's advantage to sample the widest possible range of electron energies.

In this connection, the current-voltage characteristic of an electrode immersed in a plasma (generally referred to as a LP characteristic) can be used as a convenient means for comparing the range of electron energies $E_e$ sampled by the various techniques. Such a characteristic, with the voltage coordinate normalized to $kT_e/e$, is shown in Fig. 2 where two important reference points on the abscissa are the plasma and floating potentials. For electrode potentials greater than that of the plasma, $V_\infty$, electrons are attracted while electrode potentials less than that of the plasma retard electrons. Within the context of this section the floating potential, defined as that potential at which no net current flows to the electrode, is important because it represents (in equivalent units) the approximate upper limit of electron energies sampled in the technique of the LP. This is indicated by the left-hand edge of the solid horizontal bar near the
top of the figure. In principle the lower limit of electron
energies sampled by the LP is zero but an experimental dis-
tortion generally referred to as "rounding of the knee" near
the plasma potential establishes a practical lower limit of
approximately 0.5 times the electron thermal energy, i.e.
$0.5kT_e$. This questionable regime of reliable energy sampling
is indicated by the toned region near the plasma potential
in the horizontal bar presentation. Since the I-V char-
acteristics of the FBDP are identical to those of the LP,
the preceding comments on electron energy sampling, are
equally applied.

The consideration of the energy range of electrons
collected by the symmetric DP technique is of major impor-
tance since the total number of electrons sampled by the
probes is very small, generally being in the high end of
the electron energy distribution in a region which brackets
a value equivalent to the floating potential of the probe
by approximately $\pm 0.5 kT_e$. (See the bottom-most horizontal
bar in Fig. 2.) This aspect of DP operation leaves the
technique subject to possible inaccuracies in the deter-
mination of the electron-energy distribution function if
the ambient electrons are not totally Maxwellian. In a
typical application of the DP technique to a thick-sheathed
collision-free plasma the technique of the DP would sample
only 1% of the entire distribution of electron energies in
the ambient plasma.
The discussion thus far has closely linked the energy sampling capabilities of the LP, DP and FBDP with the floating potential of the probes themselves. This coupling can also be extended to the VAP technique since its upper limit in energy sampling capabilities is set by practical considerations to a value approximately equal (in appropriate units) to one-half the floating potential. These aforementioned dependences of energy sampling on floating potential result in some very interesting consequences since the floating potential is determined not only by the plasma density, temperature and ionic specie but also by the geometry and size of the probe itself. (A nominal value of \(-4kT_e/e\) was selected for the floating potential in Fig. 2.) Some insight into this dependence and the importance of the roles of the various parameters can be achieved by a study of Fig. 3. The physics which is summarized in this figure is discussed elsewhere and only a few pertinent comments will be made here. The figure is a plot of the floating potential \(X_f\) of a spherical probe (referenced to the plasma potential) in units of \(kT_e/e\) as a function of the charge-normalized ion mass \(M(=m_i/Z^2)\), where \(m_i\) is the mass of the ion in amu and \(Z\) is its multiplicity of ionization. \(\tau\) is the ratio of ion-to-electron temperature, \(T_i/T_e\), and \(\beta\) is the ratio of the probe radius to the electron Debye length, \(R_p/\lambda_D\).

The results of this figure, which were generated by employing the Laframboise calculations, reveal two aspects of floating potentials not made known by earlier simplified models of single-probe response. These aspects are the
dependence on the ion-to-electron temperature ratio, $\tau$, as well as on the ratio of probe radius-to-Debye length, $\beta$. Of immediate consequence to energy sampling by probes is the dependence of $\chi_f$ on $\beta$. For any given plasma condition an increase in probe size is reflected in an increase in $\beta$ with a corresponding increase in $-\chi_f$. In more familiar terms this means that an increase in probe radius results in a more negative floating potential. Remembering the link with ranges of sampled electrons, this points out that a bigger spherical electrode can sample higher electron energies. As a specific illustration consider a plasma for which $\tau=0$, $M=1$ and $n_e/T_e = \text{const}$ (i.e. $\lambda_D = \text{const}$). By appropriate selection one can have a probe such that $\beta(=R_p/\lambda_D)=2$ or with a radius 50 times larger so that $\beta=100$. In the first case $E_{e}^{\text{max}} \approx -kT_e\chi_f = 2kT_e$ while the larger probe leads to $E_{e}^{\text{max}} \approx -kT_e\chi_f = 3.2kT_e$.

It is worthwhile to note that this figure can be easily used to determine to a first approximation the floating potential of a spherical satellite (alternately referred to as a body or skin potential) in an ionospheric or interplanetary plasma. As an illustration consider a spherical satellite in the F region of such a size that $\beta = 10$. With $\tau=0$ and $M=16$, Fig. 3 indicates that the floating potential is approximately $3.6kT_e/e$ volts negative with respect to the plasma. With $T_e = 1500^o\text{K}$ the corresponding body potential is $-46$ volts.
The results in Fig. 3 make it clear that the floating potential of a spherical probe is in general not the same as that of a spherical satellite of much larger radius. Assuming that the values of $\beta$ are 2 and 100 for the probe and satellite, respectively, this figure shows that the difference in floating potentials range from .5 to 1.5$kT_e$. For 1500$^0$K electrons this corresponds to a range of 65 to 195 mv. (Qualitatively, the difference in floating potentials as discussed here in connection with spherical geometry are also present in the cylindrical case\textsuperscript{10}.)

In simple applications of the LP technique these differences can manifest themselves in the I-V characteristic by requiring a non-zero bias voltage to establish a zero-current in the probe circuit. When surface effects play no role, the magnitude of this bias voltage is exactly equal to the difference in the probe and satellite (or rocket) floating potentials.

In ionospheric and interplanetary applications the results and implications of Fig. 3 are altered to one degree or another by photoemission, payload velocities and charged-particle mean free paths relative to specific body dimensions. The end result of the first two considerations is generally to shift the floating potential to values which are closer to the potential of the plasma than those indicated in Fig. 3. It should be noted that the effects associated with photoemission and payload velocities are grossly asymmetric with respect to body geometries and have only received approximate
treatments in the literature; and these treatments are considerability limited by uncertainties in the photoelectron yield function over the surface of the spacecraft, as well as the isotropy and distribution of energy of the emitted electrons. From this point of view, the results of these models which attempt to include the effects of photoemission and payload velocities cannot in general be considered any better than those presented in Fig. 3.

The influence of ion-atom collisions competes with the effects of photoemission and payload velocity in that it shifts the floating potential to more negative values than those indicated in Fig. 3. A quantitative measure of this influence can be achieved by a study of Fig. 4 which presents the dimensionless floating potential of a cylindrical probe as a function of the ratio $\frac{R_p}{\lambda_D}$ for various values of the ion-atom Knudsen number, $\frac{\lambda_{ia}}{R_p}$, where $\lambda_{ia}$ is the ion-atom mean free path. For any given values of $\frac{R_p}{\lambda_D}$ it can be seen that decreasing values of $\lambda_{ia}$ result in increasingly more negative values of the floating potential. Since the DP technique samples electron energies in the neighborhood of a value equivalent to the probe's floating potential, this figure makes it quite clear that the effect of ion-atom collisions is to drive the region of sampled electron energies to higher values and correspondingly smaller percentages of the total distribution. By contrast, ion-atom collisions broaden the range of electron energies sampled by the LP
and FBDP by extending the upper limit as measured by the probe's floating potential.

The discussion of the results in Fig. 3 pointed out that in general the floating potential of the vehicle or payload is different from that of the probe. Comments that follow will point out that ion-atom collisions enhance this difference. Consider for example a cylindrical probe operating in an ionospheric plasma under conditions for which \( \frac{R_p}{\lambda_D} = 0.1 \) and \( \frac{\lambda_{ia}}{R_p} = 100 \). (This value of ion-atom Knudsen number has been shown by Kirchoff, et al.\(^3\) to mark the minimum condition for collisionless ion-current response to probe potentials. It was the analysis scheme of Ref. 3 which was utilized to generate the results of Fig. 4.) If it is assumed that \( R_r = 10^3 R_p \), where \( R_r \) is the radius of the vehicle serving as a reference electrode, the conditions which determine the floating potential of the vehicle are \( \frac{\lambda_{ia}}{R_r} = 0.1 \) and \( \frac{R_r}{\lambda_D} = 100 \). If the vehicle, or payload, has cylindrical geometry, the results of Fig. 4 show that the difference is floating potential between the probe and the vehicle is \( \chi_p^f - \chi_r^f \approx -4.6 + 8.6 = 4.0 \). If the electron temperature is 1500 K, this corresponds to a difference of \( 4kT_e/e = .51 \) volts. (These conditions can easily be attained in the F-region.) There have been attempts in the past\(^1^2\) to explain experimentally observed differences of this nature in terms of contact potentials and surface layering. While these effects can be genuine, the results presented here make it clear that
the basic principles underlying the interaction of a plasma with a probe and its reference electrode can in themselves lead to detectable differences in floating potentials.

**Plasma Fluctuations**

In connection with the concept of floating potentials and the link with the regions of electron energies sampled by the various techniques it is important in some applications to consider modifications in what has been said as a result of plasma fluctuations. For example, the literature has shown both theoretically and experimentally that local oscillations in plasma potential can greatly influence the measurement of electron temperature and density by electrostatic probes. In this regard, a most important consideration is the region of operation of the probe with respect to the plasma potential. It is known that the slope of the time-averaged curve of a Langmuir probe characteristic is unaffected by fluctuations in plasma potential as long as the electron distribution is Maxwellian and the amplitude of the fluctuations do not extend the probe operation to potentials greater than that of the plasma. This means of course that the more negative the probe, the more reliable the determination of $T_e$. This has been borne out in the work of Boschi and Magistrelli who found that in the presence of fluctuations in the plasma potential $T_e$ could always be determined near the floating potential, but it was impossible to get information from the region near the potential of the plasma.
This brief treatment of plasma fluctuations is primarily intended to point out that the advantage of the LP and FBDP techniques as previously ascribed to their wide range of electron-energy sampling is in practice non-existent where relatively large plasma fluctuations are present. In such cases the technique of the DP is best applied since its region of operation is furthest from the plasma potential. When plasma fluctuations are present the technique of the VAP is the most severely affected with its region of operation being near the potential of the plasma.

Some Area Considerations

In rocket or satellite applications a practical consideration is the possibility that an active probe might have some perturbing effect on the vehicle potential and consequently result in a distortion of all or some of the on-board experiments. Neglecting such things as probe wakes, the technique most likely to alter the potential of the vehicle is that of the LP and its ability to do so is measured by two quantities: One is the differential voltage applied between the probe and the vehicle, and the second is the ratio of the vehicle area to the area of the probe.

Figure 5 provides a measure of the shift in vehicle potential that results when a positive bias voltage is applied to a Langmuir probe where it has been assumed that the bias voltage is sufficient to establish the probe at a potential equal to that of the plasma and that the vehicle is of spherical geometry and of such a size as to attain
the condition $R_\lambda/\lambda_D(=\beta_r) = 100$. (When $r$ is used as a subscript or a superscript the variable in question is associated with the reference electrode which in this case is the vehicle.) $\chi^r_\alpha$, which is the electrical potential (in units of $kT_e/e$) that the vehicle will attain when the probe is established at the plasma potential by an appropriate bias voltage, is plotted as a function of $\alpha(=A_r/A_p)$ for various values of the charge-normalized ion mass, $M(=m_i/Z^2)$ in amu, and for ratios of ion-to-electron temperature, $\tau = T_i/T_e$, equal to 0 and 1. (When $\beta_r=100$ there is no difference in the results for the two cases of $\tau$.) As $\alpha \to \infty$ all the curves in Fig. 5 approach an asymptotic value for $\chi^r_\alpha$, which is in fact the unperturbed floating potential of the reference vehicle. In an ideal application of the LP technique the vehicle will always remain at its floating potential. This can be guaranteed when the ratio of $\alpha$ can for all practical purposes be considered infinite. This figure shows that in application the condition of $\alpha \to \infty$ can readily be attained at $\alpha \gtrsim 10^4$. For values of $\alpha < 10^4$ the vehicle's potential will shift to more negative values with the degree of the shift being a function of $M$, $\tau$, and $\beta_r$.

Consider the case with $M=16$. The reference electrode will remain fixed at its floating potential, $-4.6$ dimensionless units, for $\alpha \gtrsim 10^4$. As $\alpha$ is decreased the reference electrode will begin shifting to more negative potentials with respect to the plasma and at a value of $\alpha = 10^2$ will be at $-6.8$
dimensionless units, a shift of \(-0.38\) volts from its potential when no bias voltage was applied to the probe. \((T_e = 2000^0K\) has been assumed in this illustration.) This shift in potential will in every case effect the measurement of electron temperature by the LP with the net result of the perturbation being an indicated value of temperature greater than that actually present in the ambient plasma. In addition, it appears quite clear that such a shift would have a perturbing influence on any grid-type plasma detector which is flush-mounted with the skin of the vehicle.

To extend the use of the results in Fig. 5 it can readily be shown that if the positive bias on the probe were such that the probe operation were in the electron saturation portion of its characteristic and collecting current equal to \(\gamma\) times the current collected at the plasma potential then the value of \(\alpha\) appropriate for purposes of this extension, defined by \(\alpha_e\), would be \(\alpha/\gamma\). To illustrate this use of Fig. 5 assume that \(\gamma = 10\) and the physical situation is such that \(\alpha = 10^3\). Then \(\alpha_e = 10^2\) and the results of the previous example apply when \(M = 16\). Values of \(\gamma\) larger than 10 will result in very large shifts in the reference electrode potential as evidenced by the steep slope of all curves in the region of their lowest plotted values of \(\alpha\).

IV. COMMENTS AND CONCLUSIONS

Of the four techniques which have been compared the Langmuir probe has had the longest history. It has also had
the largest application to ionospheric investigation\textsuperscript{10} and, perhaps as a result of this, is continually undergoing stages of revaluation which at times question the integrity of the method on both theoretical and experimental grounds. Recently a controversy has developed\textsuperscript{17-19} over the discrepancies in the values of electron temperature as measured by the techniques of the LP and radar backscatter. When the discrepancy exists, the values of $T_e$ as determined by the LP are higher than those determined by radar backscatter, with the ratio at times being as high as 2 over certain ionospheric regions. As yet there has been no satisfactory solution to the controversy with all proposals ultimately pointing to the need for an independent third measurement of $T_e$.

In laboratory studies of LP response there has been strong evidence that the most reliable method for generating a current-voltage characteristic is by applying the voltage in incremented pulses of widths less than 50\,$\mu$sec instead of in the continuous sweep mode. This pulse procedure has led to results which are considered truly representative of the test plasma and which differ significantly from those generated by the standard procedure of an applied voltage sweep\textsuperscript{20,21}. In connection with the aforementioned LP and radar backscatter discrepancy it is perhaps noteworthy that the value of $T_e$ as derived in the normal voltage sweep mode are higher than those in the pulsed-voltage mode, with the difference being attributed to perturbations which result from charged-particle depletion in the case of an applied voltage sweep.
The technique of the FBDP is a floated double-electrode method which permits a minimum electrode exposure to the plasma and consequently can be expected to impose a minimum influence upon the plasma parameters it is measuring. This technique can sample as wide an electron-energy range as that of the LP but requires no variable bias voltage. Instead, an electrode of variable area is needed. In this case the advantage or disadvantage of variable voltage or variable electrode area must be weighed according to the specific application. The current-voltage characteristic of the FBDP is identical to that of the LP and this equivalence permits the application of the standard Langmuir probe analysis procedure to the FBDP for the determination of plasma density and temperature.

Next only to the LP, the symmetric DP of Johnson and Malter has had widest use in laboratory plasmas and recently has been applied to the measurement of electron temperature in the ionospheric D-region. This technique is particularly attractive in lower ionospheric applications since it can lead to meaningful temperature determination under conditions of relatively high collision frequencies where the Langmuir probe has been observed to fail. The DP method can be used everywhere the LP finds application in Maxwellian plasmas but the reverse situation is not true. In general the method of the DP will introduce a far smaller perturbation (if any at all) on the plasma than the LP; it is less susceptible
to fluctuations in the ambient plasma than the LP, and in ionospheric applications it is less affected by varying pay-load or rocket potentials. Perhaps the greatest drawback of the DP technique is that it samples only a small percentage of the ambient distribution of electron energies and as a result would be subject to inaccuracies in non-Maxwellian cases.
REFERENCES

23. E.P. Szuszczewicz, NASA X-625-72-219 (July 1972); sub-
FIGURES

Fig. 1 - Experimental configurations of several electrical-probe techniques.

Fig. 2 - Current-voltage characteristic of an electrode immersed in a plasma. The horizontal bars represent (in equivalent units) the regions of electron energies sampled by the various techniques. The toned sections represent questionable reliability in energy sampling.

Fig. 3 - Dimensionless floating potential $\chi^f = e(V^f - V_\infty)/kT_e$ of a spherical body immersed in a collisionless Maxwellian plasma plotted as a function of the charge-normalized ion mass $M = m_i/Z^2$ (amu) for ratios of ion-to-electron temperature $\tau = T_i/T_e = 0, 1$. $\beta$ is the ratio of the body radius to the electron Debye length [Ref. 11].

Fig. 4 - Dimensionless floating potential $\chi^f = e(V^f - V_\infty)/kT_e$ of a cylindrical body immersed in a completely thermalized plasma plotted as a function of the ratio of probe radius to electron Debye length, $R_p/\lambda_D$. $\lambda_{ia}$ is the ion-atom mean free path.

Fig. 5 - Dimensionless potential $\chi_{\alpha}$ of a spherical reference electrode as a function of $\alpha(=A_r/A_p)$ for $\beta_r(R_r/\lambda_D) = 100$. $M$ is the charge-normalized ion mass, $m_i/Z^2$ (amu), and $\tau$ is the ratio of ion-to-electron temperature [Ref. 11].
(a) SIMPLE LANGMUIR PROBE (SLP)

(b) SYMMETRIC DOUBLE PROBE (DP)

(c) VARIABLE-AREA PROBE (VAP)

(d) FIXED-BIAS DOUBLE PROBE (FBDP)

FIG. 1
ELECTRONS RETARDED
LANGMUIR PROBE (LP) AND FIXED-BIAS DOUBLE PROBE (FBDP)
VARIABLE-AREA PROBE (VAP)
SYMmetric DOUBLE - PROBE (DP)

ELECTRONS ATTRACTED

\[ e(V_{pr} - V_a)/kT_e \]

FLOATING POTENTIAL
PLASMA POTENTIAL

Fig. 2
Fig. 3

SPHERE

- $\beta = 100$
- $\tau = 0$
- $\tau = 1$
FIG. 4

\[ \lambda a/R_p = 0.1 \]

\[ L/R = 200 \]
\[ \tau = 1 \]
\[ M_i = 30 \text{ amu} \]
Fig. 5