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ABSTRACT

A straightforward analytic approximation is illustrated to Stromgren's solution for HII regions which enables us to explicitly exhibit the effects of internal dust on the ionization structure. We interpret recent far-infrared observations of HII regions in terms of true absorption by internal dust of a significant fraction of the Lyman continuum photons.

The discovery of large far-infrared emission from HII regions (Harper and Low 1971), significantly in excess of the apparent Lyman continuum output of the exciting stars inferred from measurements of the thermal radio continuum, on the assumption that the Lyman continuum photons are absorbed by the gas has stimulated us to re-examine the effects of internal dust on Stromgren's classical solution to the HII region problem. It is the purpose of this note to indicate a simple analytic approximation, which leads to results in reasonable agreement with earlier numerical work (Mathis 1971). Our result moreover both clarifies and adds considerable insight into the problem, and is useful in indicating how one may extend Mathis' results over a wide range of optical depths for dust absorption, with no additional numerical computation. Moreover, we apply our results to interpret the observed far-infrared emission from HII regions, and demonstrate the observations to be consistent with a moderate amount of absorption of Lyman continuum photons by internal dust.

The standard equation of ionization equilibrium for a spherically symmetric HII region of uniform density n, containing a central exciting star emitting S ionizing quanta per second can be written in the form

$$\frac{1-x}{x^2} = \frac{3y^2 e^{\tau}}{\tau_s} , \qquad (1)$$

where x is the fractional degree of ionization, $y \equiv r/r_s$ is the dimensionless radial coordinate in units of the Stromgren radius, which is r_s, defined by

$$S = \frac{4\pi}{3} r_s^3 n^2 \alpha , \qquad (2)$$

and

$$\tau_s = n\,\sigma\,r_s \ . \tag{3}$$

In these equations α is the hydrogen recombination coefficient summed over all levels excluding the ground level, and σ is an appropriately averaged value of the photo-ionization cross-section for hydrogen. The optical depth τ is defined by

$$\frac{d\tau}{dy} = (1-x)\tau_s + \tau_d \ , \tag{4}$$

where

$$\tau_d = n\,\sigma_g x_g r_s \ , \tag{5}$$

σ_g is the effective absorption cross-section of the grains, and x_g is the number of dust grains per H atom. The dust and gas are assumed to be uniformly distributed in the ionized region.

We make the approximation $1-x \ll 1$, which should be well satisfied throughout the ionized region. It is then a straight-forward matter to eliminate τ from equations (1) and (4), and obtain the following solution for x as a function of radial coordinate,

$$\frac{1}{1-x} = \frac{\tau_s}{3y^2}\left[e^{-\tau_d y} + \frac{6}{\tau_d^{\,3}}\left(e^{-\tau_d y} - 1 + \tau_d y - \frac{1}{2}\tau_d^2 y^2\right)\right] \ . \tag{6}$$

For $\tau_d \ll 1$, equation (6) reduces to the anticipated limit

$$\frac{1}{1-x} \sim \frac{\tau_s}{3y^2}\left[1 - y^3 - \tau_d y(1 - y^{3/4}) + O(\tau_d^2)\right] \ . \tag{7}$$

The neutral fraction $1-x$ is plotted in Figure 1 as a function of y for various values of τ_d. The parameter τ_s is set equal to 5000. For comparison, the numerical results of Mathis (1971) are

also shown (dashed line) for one of the cases he computed, namely $\tau_d = 0.4$. The agreement for the ionization structure is remarkably good, despite the approximations of our analysis. The small disagreement at low values of y is due to scattering of photons by dust which we have ignored completely.

Another useful quantity that we can compare with Mathis' numerical work is the fraction of Lyman continuum photons absorbed by the gas. The analytic expression for this fraction as a function of y is found to be $y^3/(1-e^{-\tau})$. At the boundary of the HII region ($y \equiv y_o$) $(1-x)\tau_s \gg 1$, and $\tau \to \infty$. Consequently, we obtain the extent of the ionized region by setting the left hand side of equation (7) equal to zero. The fraction f of stellar ionizing photons absorbed by gas in the entire ionized region is therefore given by

$$f = y_o^3 = \frac{1}{3}\tau_o^3 e^{-\tau_o}/[\tau_o^2 - 2\tau_o + 2(1-e^{-\tau_o})] , \qquad (8)$$

where $\tau_o = \tau_d y_o$ is the optical depth for absorption by dust within the ionized region.

In Figure 2 we plot the fraction f versus τ_o. For comparison with our analytic curves, we show results computed numerically by Mathis. As is evident our solution is in perfect agreement with the computations of Mathis for $\tau_o \leq 1.2$. For larger amounts of dust absorption, our solution tends to overestimate the fraction of ionizing radiation absorbed by the gas.

Part of this discrepancy is due to our neglect of helium. Mathis (1971) shows that the He^+ and H^+ ionization zones tends to coincide in the presence of dust, because the H^+ ionization zone moves inward with increasing τ_d as H-ionizing photons are absorbed by dust. One of us has also given an analytic expression for this effect (Petrosian

1972). The principal effect of He on the hydrogen ionization structure is to act as a converter of relatively hard He-ionizing photons to softer photons (each He recombination liberting ~ 0.9 H-ionizing photons), for which the cross-section for ionizing H is much higher (Hummer and Seaton 1964). Consequently the ionization of H is increased, which implies that at $\tau_d \geqslant 1$ our neglect of He leads us to overestimate the neutral fraction, and therefore the fraction of ionizing photons absorbed by the gas.

Part of the different between the analytic and numerical solutions appears to be due also to our neglect of scattering by the dust, the effect of which is to provide a higher probability for absorption of photons by either gas or dust grains. We can account for the effect of scattering by redefining all our optical depths as $\tau(r) = \int_0^r \sigma n \, ds$, where ds is an element of length measured along the actual scattering path of the photon. With this new definition of optical depth our results remain unchanged, but the points in Figure (2) from Mathis' numerical work move to the right approximately by a factor $\int_0^{r_o} \frac{ds}{r_o}$, where $r_o = r_s y_o$ is the Strömgren radius of the dusty nebula. This factor increases with increasing values of the dust albedo.

In order to apply Figure (2) to observations of far-infrared emission from HII regions, we note that the expected infrared luminosity is

$$L(IR) = L(Ly\alpha) + (1-f)\langle h\nu\rangle_{Lyc} S + L_{\nu < Lyc}(1-e^{-\tau_o'}) \qquad (9)$$

where $\langle h\nu\rangle_{Lyc}$ is the average energy of stellar Lyman continuum photons, $L_{\nu < Lyc}$ is the stellar luminosity below the Lyman continuum and τ_o' is the effective absorption optical depth of dust for photons

with $h\nu < 13.6$ ev. For a nebula which is optically thick to Lyα photons (case B), the value of the Lyα luminosity of the nebula (assuming that the Lyman Alpha can eventually escape from the nebula), is given by $L(\mathrm{Ly}\alpha) = \frac{2}{3} h\nu_{\mathrm{Ly}-\alpha} Sf$, and can be inferred either from the observations of Balmer photons suitably corrected for reddening or from the radio continuum luminosity. If the spectrum of the ionizing stars and the observed infrared flux of HII regions are known, we can use equation (9) to determine the fraction f, and from Figure (2) the effective absorption optical depth of the dust. This procedure may allow one to empirically determined the far ultraviolet absorption properties of the dust. For example, in the case of the Orion nebula, the total extinction by dust near Hβ has been found to be $1^m.8$ (Mathis 1970); this value is consistent with other independent determinations at optical wavelengths (cf. Münch and Persson 1971). On the other hand, we estimate that, in our earlier notation $L(\mathrm{IR})/L(\mathrm{Ly}\alpha) = 7.5$. Hence if we neglect the last term in equation (8), as might be the case for a hot central star or if $\tau_o' \ll \tau_o$, and set $\langle h\nu \rangle_{\mathrm{Lyc}} = 15$ ev, we find that $f \approx 0.26$. We then deduce from Figure 2 that the true absorption by dust of Lyman continuum photons is about $1^m.7$. It follows from these numbers that, for a ratio of total absorption cross-section in the Lyman continuum to that at H_β of between 2:1 and 3:1, the albedo at H_β must be about 0.6 ± 0.1. In principle, one could now infer further physical characteristics of the dust from this result: however, the paucity of available data on optical constants at UV wavelengths does not appear to allow any definitive conclusion to be drawn at present. In the event that the last term in equation (8) is not negligible, similar calculations can be carried out if the spectrum of the central star is known.

In summary, our analytic approximation (6) to the ionization structure of HII regions with a uniform distribution of dust should be useful in aiding future studies of line and continuum emission from HII regions. In particular, we have obtained a simple expression for the fraction of ionizing photons absorbed by the dust [equation (8)], a quantity which is consistent with the infrared observations provided that the dust absorption optical depths are typically between 1.0 and 2.0 magnitudes. This approach offers an independent way of obtaining the total absorption optical depth to internal dust in HII regions, which can then be compared with the optical depth at Hβ. If the stellar spectrum is known, this approach can be used for determination of the albedo of the dust.

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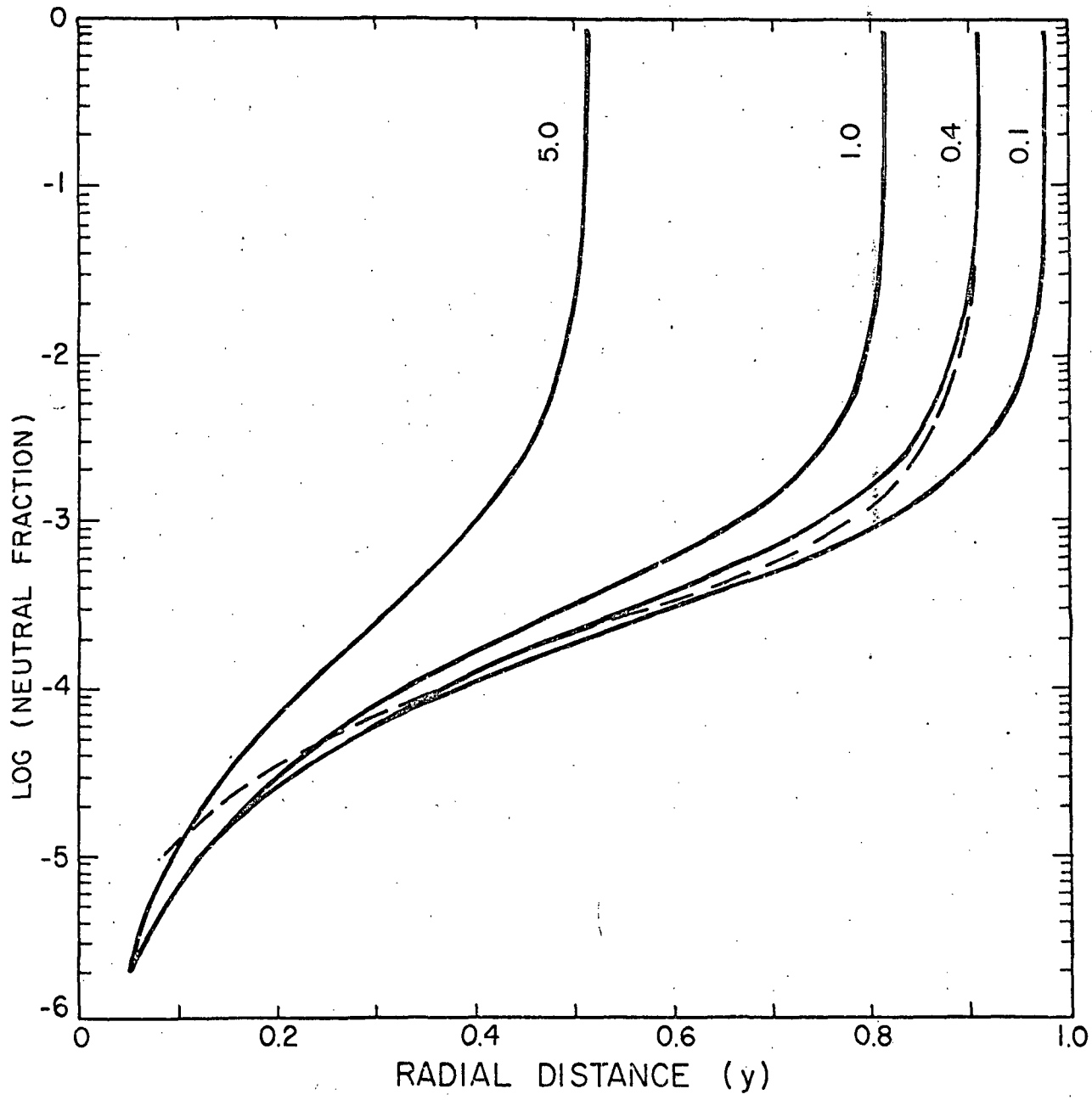
FIGURE CAPTIONS

Figure 1. Log_{10} (neutral fraction) plotted against radial distance in units of r_s. Each curve is labeled by the corresponding value of τ_d. For comparison, the numerical computation by Mathis (1971) is shown for the case $\tau_d = 0.4$ (dashed line).

Figure 2. Fraction of ionizing photons absorbed by gas plotted against optical depth for true absorption by dust within the ionized region. The numerical results of Mathis (1971) are indicated by crosses.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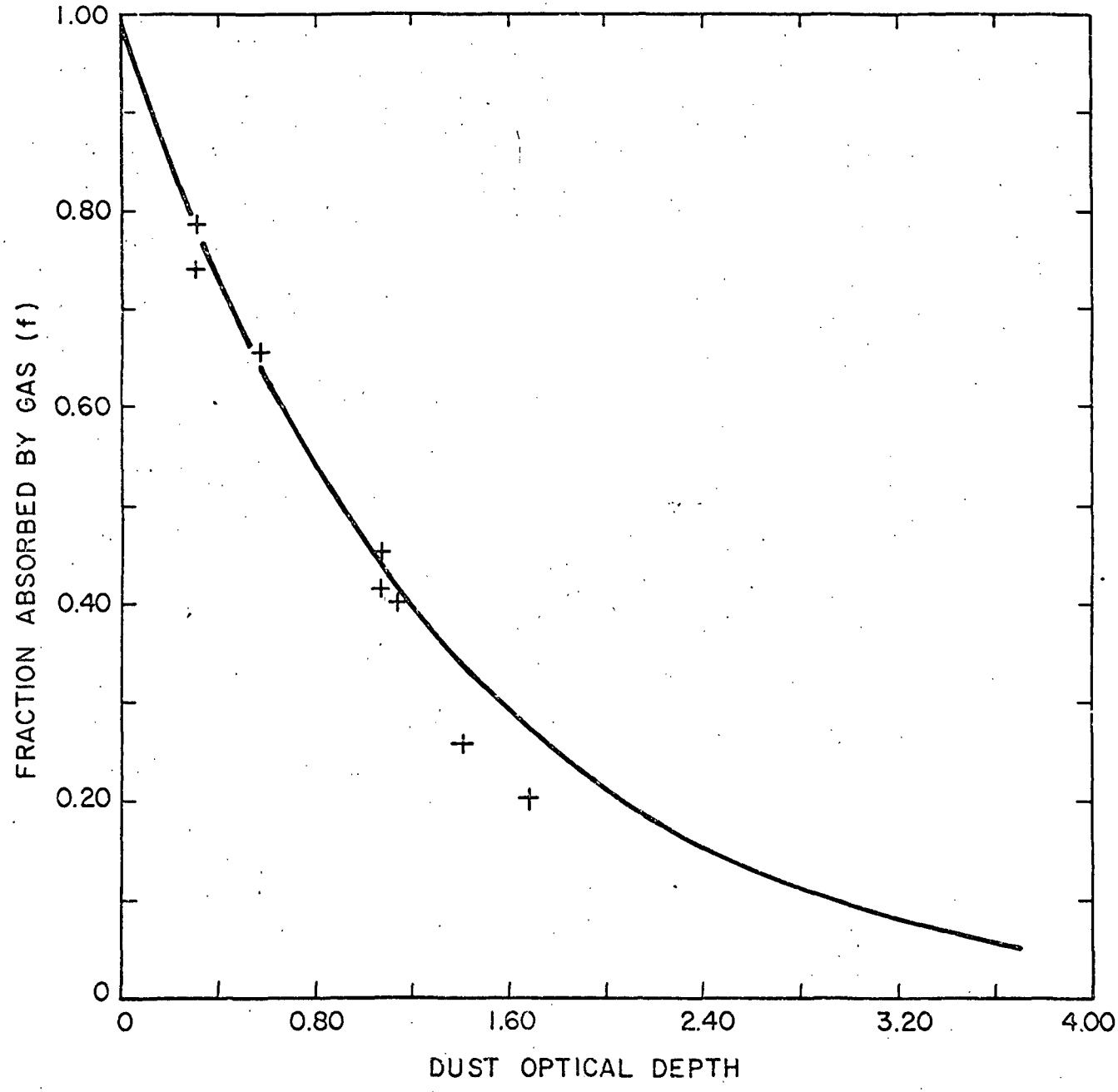
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