Optimization of Structures on the Basis of Fracture Mechanics and Reliability Criteria

Obtaining the best possible performance, the least possible cost, or the least possible weight, etc., is an integral part of every structural design procedure; the optimization task is to find the values of controllable parameters, subject to various constraints, that make a desired objective function an extremum. Although the purposeful geometrical arrangement of the structural elements can be accomplished with adequate, better, or optimum results, in structural optimization it is usually assumed that the geometrical configuration of the structure is a constant and that the optimization will involve only the sizes of the individual structural elements as the generalized coordinates that are varied to determine within specified constraints those of their values which yield an extremum of a given objective.

The systematic summary of the factors which are involved in the optimization of a given structural configuration is part of a report resulting from a study of an analysis of an objective function; the objective function selected for optimization was the structural weight or the statistically expected cost, that is, the mean cost of coupon testing, proof testing, and mission degradation. The structural weight is expressible in terms of the physical parameters such as density and structural dimensions, and the cost items are expressible in terms of the proof-load test levels as well as the physical parameters. The optimization process developed during the study was intended to determine those proof-load test levels and physical parameters which yield minimum expected cost, or which yield minimum weight subject to an expected cost constraint.

The predicted reliability of performance of a finished structure is sharply dependent upon the results of coupon tests, that is, the statistical strength properties of the structural material. The efforts and costs which must be expended in order to establish, with sufficient confidence, the material strength distributions for one or more environmental conditions can be substantial. In particular, if the information is required at the tail end of the distributions, the number of coupon tests and the associated cost soon becomes intolerably high. Thus, in the overall cost analysis, the required expenditures for material characterization must be taken into account. Analysis of the statistics of coupon testing shows that the greatest contribution to the probability of failure during service stems from the region near the proof load. A detailed study of fracture mechanics concepts, the statistical aspects of fracture, and the effects of time on the growth of subcritical flaws was performed in order to be able to assess properly the costs of a material characterization program (coupon testing cost). Of course, if different structural subsystems are used with different types of materials, the total coupon testing cost is the sum of the characterization costs for each material.

The optimization analysis developed by the study also involves the expected cost of proof testing, that is, the statistically expected cost of structural testing in which one structure after another is tested at a certain proof level until a structure is obtained which passes the applied proof load. From a structural utilization viewpoint, it is not only important to consider the costs of coupon testing and proof testing,
but also the cost which will be incurred if the structure fails during the time of its use; for example, in space missions, failure of a spacecraft or any of its subsystems may result in loss of the entire mission or a substantial degradation of the mission. The statistically expected value of this cost, the mission degradation cost, is the product of the actual cost of mission degradation times the probability of occurrence of this degradation, which is the probability of structural failure.

As an example of the application of the methods developed during the study program, the report includes a detailed solution to a typical problem in structural design: A 51-cm (20 in) spherical pressure vessel is to be designed to sustain, after proof testing for 360 hours, a given internal pressure. The weight of the vessel is to be minimized for an appropriate choice of proof-load level and vessel wall thickness so that the total relative expected cost does not exceed a certain sum.

It is to be realized that although the report strongly reflects the requirements of structures for spacecraft missions, the results are broadly applicable to structures for any purpose. The clarity of presentation is such that the report can be used by individuals with only modest training in mathematics.

**Note:**
The following documentation may be obtained from:
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