Use of Multivariable Asymptotic Expansions in a Satellite Theory

The principal test of a satellite theory is that it yields the position and velocity (or equivalent parameters) of a satellite to a sufficiently high accuracy even after many revolutions about the central mass. The only means of deciding whether a given theory yields a sufficiently accurate solution is to compare the resultant solution with a standard of much higher accuracy.

The theory of the variation of coordinates with multivariable asymptotic expansions has been applied to the differential equations governing the motion of a satellite. The initial conditions and the perturbative force of the satellite are restricted to yield the motion of an equatorial satellite about an oblate body (second harmonic only). In this manner, an exact analytic solution exists and can be used as the standard of comparison in numerical accuracy comparisons. The normalized differential equations that govern the restricted motion of a satellite also govern the relativistic motion of a point mass moving around a central point mass so massive as to dominate the system.

A complete set of exact equations has been developed for the restricted satellite motion, that is, equations that yield the position and velocity of the satellite. The right ascension of the satellite is found to consist of constants and a Legendre normal elliptic integral of the first kind, whereas the time associated with the position of the satellite is found to consist of constants, powers of the radius distance, and Legendre normal elliptic integrals of the first, second, and third kinds.

Detailed numerical accuracy studies of the uniformly valid asymptotic expansions obtained by using the concepts of multivariable asymptotic expansions were made during the study, and the effectiveness of these concepts in a satellite theory was established.

Note:
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