COROTATION OF AN INTERMITTENT SOLAR WIND SOURCE  

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The measured electron content of the solar wind in mid-1970 exhibited a region of relatively high electron density that reappeared at intervals of about 27.8 days. It is shown that the repeating event cannot be reconciled with the concept of a long-enduring steady flow, even though the recurrence period is close to the rotation period of the sun. This evidence of transients is inferred from the short duration of each appearance of the interval of higher density; each should last for roughly one corotation interval if it is caused by a steady stream. The radio path was approximately 0.8 AU long, and the corotation interval exceeded 3 days.

Other aspects of the content data patterns support the view that such transient events are common in the solar wind. The mid-1970 repeating event is an unusually good example of the intermittent character of flow regions in the solar wind that fluctuate on a time scale of days but endure as identifiable regions for many months. A sputtering corotating source of thin solar plasma streams could explain this series of events; it could also be explained in terms of a stream that is steady in density and speed but undulating north-south so that it passes into and out of the 0.8 AU radio path in a matter of a day or less.

INTRODUCTION

Observations of the solar wind and of its effects on the earth over the past six or seven decades have revealed a 27-day recurrence pattern, roughly in synchronism with the rotation of the sun. This “corotation” has been manifested in many parameters measured by spacecraft during the last decade, and consequently it is now generally believed that there are corotating flow regimes in the solar wind that endure for several solar rotations. A definitive early work on this subject was given by Bartels [1932] and more recent knowledge has been summarized by Wilcox [1968]. The 27-day repetition is clearly seen in interplanetary electron content measurements obtained by the radio propagation experiment on the Pioneer spacecraft [Croft, 1971].

When solar wind measurements are compared on successive rotations, it is seldom found that the detailed patterns in the data repeat; rather, it is found that the general character of the data patterns is repetitious. For example, a chaotic region will reappear on successive rotations, but individual fluctuations within that region are not observed to be faithfully reproduced. Similarly, quiet periods in the data will be observed for several successive rotations at 27-day intervals.

The Pioneer electron-content measurements permit the observer to discriminate in some cases between data fluctuations caused by temporal changes in the solar wind and those caused by corotation of the steady-state pattern through the volume of space in which the measurement takes place. In other words, temporal fluctuations can be separated from spatial fluctuations. From such analyses [Croft, 1971], the author has concluded that there are isolated sources of solar plasma corotating with the sun but that they typically sputter; that is, they eject plasma intermittently on a time scale of approximately a day. This general conclusion was supported by a number of observed patterns in the data.
A particularly convincing piece of evidence is presented here. It is shown that in mid-1970 in the vicinity of Carrington longitude 50° on the sun there was an intermittent source of plasma that endured for at least four solar rotations. The corotation of this region is inferred from the fact that it appears at very nearly the right time on each of several successive rotations. Its intermittent character is inferred from the fact that the duration of each content increase is too short to be explained in terms of a corotating steady stream.

**EXPERIMENTAL METHOD**

The electron content of the solar wind has been measured by a radio propagation technique employing special receivers placed aboard Pioneers 6, 7, 8, 9, and also aboard Mariner 5. Only the results obtained from Pioneers 8 and 9 will be employed here. The measurement technique has been explained most thoroughly by Koehler [1968] and most recently by Croft [1971].

The experiment can operate only when one of the Pioneers is above our horizon (once a day for each spacecraft) and only during times when the telemetry from the craft is being monitored by the NASA Deep Space Network. Two coherent signals at 49.8 and 423.3 MHz are transmitted to the craft from a 150-ft (46-mr) paraboloid antenna located near the Stanford campus. Both signals are phase modulated at about 8 kHz, and at the spacecraft the phase difference between the modulations is measured and encoded by a special receiver. Later, after the results of the measurement are telemetered to earth and analyzed, we derive the differential group delay at the two frequencies, and this is roughly proportional to the electron content from earth to the spacecraft along the radio path. The underlying physical mechanism is the decrease in group velocity brought about by free electrons in the solar wind; the phase velocity is increased by a nearly equal amount, and to take advantage of this fact we monitor changes in the phase delay by observing the relative radio frequency of the two carrier signals which reach the spacecraft.

To quantize this measurement, the higher frequency is multiplied by 2/17 and then subtracted from the lower frequency and the remaining signal is used to drive a counter. Fluctuations in the solar wind cause changes in the content on the path that are manifested in counter rates of approximately 1 Hz or less. The status of the counter is periodically telemetered to earth and subsequently the succession of counts is analyzed to determine the phase delay and, from this, the electron content along the path. This process involves integration and consequently the derived content includes an unknown additive constant, which is determined by comparing the phase delay to the group delay. The phase delay measurement provides finer resolution and it is less affected by nonlinearities in the earth's ionosphere; the group delay provides total content measurements (since there is no unknown constant) and also the group delay measuring system operates when the signal strength is too weak to permit reliable interpretation of the counter.

Both group and phase delay measurements thus yield electron content from the surface of the earth to the spacecraft, and this includes the content of the ionosphere and magnetosphere, which is unwanted. To eliminate this component, it is necessary to measure it and subtract it from the total, the Pioneer content. The measurement is accomplished by observing the Faraday rotation angle of signals from geostationary satellites on a continuous basis. Ionospheric and magnetospheric content is then inferred from the rotation angle; this yields the content along the line of sight to the geostationary satellite, not to the Pioneer where it is needed.

Two different computer algorithms have been devised for computing the content along the desired line of sight from measured content along geostationary lines of sight. These calculations work best when several geostationary lines are simultaneously monitored. If only one such line is monitored, then the calculation makes use of an assumed constant slab thickness of the ionosphere coupled with a geographic variation in the critical frequency following the numerical maps provided by the ionospheric predictions of NOAA. An alternative algorithm makes use of the assumption that the ionospheric structure is fixed relative to the sun, and that the earth revolves within this ionosphere without modifying it. Both algorithms are used because it has been found that each has strengths and weaknesses and the best result is achieved by human examination of the two derived solutions and comparison to the Pioneer-measured total content. The ionospheric processing is relevant to the following discussion because the errors involved in the calculation are the largest source of error in the entire experiment. The only comparable source of error is the nonlinear relation between ionospheric content and propagation delay, which is troublesome when the spacecraft are viewed near the horizon in the daytime.

Because the size of the ionospheric errors is independent of the distance from earth to the sun-orbiting spacecraft, it follows that the percentage accuracy of the interplanetary content measurement improves as the spacecraft recede from earth. For the time interval to be discussed here, Pioneer 8 was between 0.75 to 0.9 AU and Pioneer 9 between 1.5 to 1.8 AU from the earth.
Under these circumstances, the ionospheric error is typically less than 10 percent of the interplanetary content. However, the processing for these particular data is only of the manual “quick-look” variety, using a single geostationary satellite as an ionospheric monitor and ignoring the Pioneer phase delay measurement. Later, we hope to execute our computer processing of the full data set and recover much fine structure that is lost in the presentation given here. For the purpose of this paper, however, the “quick-look” data provide the necessary evidence.

**DISCRIMINATION BETWEEN TEMPORAL AND SPATIAL CHANGES**

The Pioneer spacecraft stay near the orbit of the earth; some move ahead and some fall behind the earth, and thus over a period of time the earth-spacecraft distances increase to a maximum value of roughly 2 AU when the spacecraft are at superior conjunction. For the time interval of interest here, Pioneer 8 was approximately 0.8 AU behind the earth and Pioneer 9 was twice that distance ahead. Figure 1 shows the positions of the spacecraft, sun, and earth, together with other features referred to later in discussing the results obtained. The most striking feature of the sputtering solar plasma source observed at 50° longitude was the timing of the appearance of its plasma along the Pioneer 8 path. Consequently, the method for discriminating between spatial and temporal effects is discussed here in relation to Pioneer 8 content measurements along the 0.8 AU path trailing the earth.

Electron content is the line integral of free electron number density along the radio path; when divided by the length of the path, the quotient is the spatial average of density on the path. If the solar wind consisted only of fully ionized hydrogen, then this quotient would represent the solar wind density. The presence of a few percent helium and the observed fluctuations in the percentage [Hundhausen, 1970] prevent us from relating electron density to solar wind density with an accuracy better than about 5 percent.

The test between spatial and temporal changes is based on the logic outlined below.

**Assume No Temporal Changes**

If we assume that there are no temporal changes, the flow pattern of the solar wind (its total distribution of velocity and density) never changes in a frame which rotates with the sun. Differential rotation is not of importance in this test and will thus be ignored. Plasma from a single source is then distributed along the familiar Archimedes spiral having a curvature that depends on the flow velocity.

**Calculate the Content-Time Function: The Steady Stream Response**

We calculate the recognizable characteristics of the content versus time pattern that must follow from the assumed steady-state flow. One consequence of the assumption is that the density measured at a point fixed in space relative to the sun would undergo exact repetition at the rotation rate. Therefore, at any given distance from the sun, one could determine density versus longitude as a single-valued function.

If there were an anomalously high-density plasma stream in the corotating flow structure, then the stream would follow the spiral configuration except for distortion due to interaction with the neighboring flow regimes if they have a different velocity. This distortion should be small since the anomalous stream is of relatively high density. If such a hypothetical stream rotated through a radio path whose content was being monitored, then the content would be observed to rise when the stream entered the path and fall when the stream left the path. This is illustrated in figure 1. For the duration of the increased content, the magnitude of the increase would be proportional to the stream density.
steady stream response at the point (or points) where it crosses the radio path immersed in the stream. The duration of the increase, the corotation delay, is easily calculated if the stream velocity is known; the author prefers the term corotation interval because in some applications the term delay is inappropriate.

The effect of such a hypothetical thin, corotating steady-state stream has been calculated once per solar rotation throughout the lifetime of Pioneers 8 and 9. This calculated content-time function will be called the steady stream response (SSR). It is analogous to impulse response in the sense that the steady stream would appear as an impulse in the density-longitude function that one could obtain from measurements at fixed heliocentric distance. If the velocity of the solar wind were the same everywhere, the SSR would be even more closely analogous to impulse response; one could obtain the spatially averaged density measurement of the Pioneer system by convolving the SSR with the density-longitude function, dividing by distance to convert content to density.

With the reservations noted above, and provided that the solar wind is in steady state, one can consider that the radio propagation experiment on Pioneer spacecraft is a measuring system having the SSR as its impulse response. The input function is density versus longitude, measured at 1 AU. Taking this view, one can see that a long radio path in interplanetary space should yield density-time curves that are smoothed in relation to the path length. Longer paths produce smoother data because the SSR becomes very long, and its effect, when it is convolved with the input function, is to smooth that function. As an extreme example, one can consider an SSR which is rectangular—that is, one that increases to a fixed value and remains at that level until its abrupt termination. The convolution of such a rectangle with the input function has a smoothing effect much like that which would be obtained by taking the running mean of the data over the duration of the rectangle. Short pulses in the input function then assume a broad form having a width comparable to the convolving rectangle. (However, in relation to Pioneer 9, we shall see, that paths exceeding roughly 1 AU have an SSR that does not produce the same degree of smoothing.)

**Compare Data for Irregularities**

We compare the real data to the character of hypothetical steady-state data to find features inconsistent with the steady-state restrictions. The most obvious of these inconsistent features are pulses that are short relative to the corotation interval and have a magnitude large relative to the content before and after; such pulses are inconsistent with the idea of prior convolution with the SSR. The appearance of a short pulse indicates some kind of temporal change in the solar wind; that is, as viewed in a corotating frame of reference, the distribution of density was undergoing change. From the content data alone, it cannot be decided which of several forms of temporal change were the cause of individual observed fluctuations. However, it appears credible to the author that these transient phenomena are outward traveling irregularities in the solar wind. The identification of the character of such irregularities is an extensive subject in itself, and is under study by J. A. Landt, who is presenting some of his conclusions at this symposium (p. 598). One particularly large event has been described in the literature [Landt and Croft, 1970].

**DISCRIMINATION TESTS ON MID-1970 CONTENT DATA**

Beginning in April 1970, there was a period of intensive operation of Pioneers 8 and 9 to gather background data as Pioneer 9 approached superior conjunction and the interesting events associated with the occultation of the radio path by the solar corona. This minimized the major source of ambiguity in our data, which is its intermittent character, a disadvantage that is most pronounced when operations are conducted on only a few of the daily opportunities. The most useful data for the purpose of steady-state testing are those taken at times when observations are conducted almost everyday. There was one such period enduring for roughly nine solar revolutions in 1968; these data have been presented by Croft [1971]. However, the 1968 measurements were taken with Pioneer 8 at a time when the corotation delay was only a few hours; consequently, the steady-state tests could not be conclusively applied in that interval. Beginning in 1970, however, the commencement of daily operation revealed great structural detail in the flow patterns and, in particular, it revealed the repetitive fluctuations that will be discussed here.

The average density measured during five rotations of the sun is shown in figure 2. The average along the path to Pioneer 8 is plotted upward and that to Pioneer 9 is plotted downward. It is readily observed that the Pioneer 9 density is the higher of the two, and this is attributable to the fact that the path to Pioneer 9 lies nearer the sun where the density is higher.

Note that figure 2 includes a scale showing the Carrington longitude of the central meridian of the sun. Because the earth’s orbit is not quite circular, the length of one Carrington rotation changes with the season from a minimum of roughly 27.19 to a maximum of about 27.34 days. The time scale is fixed throughout and
Figure 2. Average density of the solar wind along the line from earth to Pioneer 8 (plotted upward) and to Pioneer 9 (plotted downward). Short-line segments show plotted hourly points as measured, and the intervening shading is a visual aid spanning all data gaps of 4 days or less. The long vertical lines enclose pulses in the Pioneer 8 data.
consequently the longitude, as plotted, is a slight approximation. If used without compensation, errors of up to about 0.1 day can be found, but it should be pointed out that this is only a third the error that is inherently ignored when the Bartels 27-day rotation numbering system is used as a basis for plotting successive rotations. In the context of this discussion, such errors are negligible, but the variation can make a difference when the data are statistically analyzed in a search for long-term trends.

While there are several features on figure 2 that are inconsistent with the concept of steady flow, attention here is directed to the pulses shown in Pioneer 8 data on the second through fifth rotations in the longitude interval from 0° to 40°. If we assume that these bursts of plasma required 4 days to travel from the sun to reach the radio path, then their source near the sun must be located at a Carrington longitude of about 50°. All four peaks reached their maximum value at the Carrington synodic rotation period plus or minus 0.9 day. However, if we allow freedom to fit a period to the data, it happens that the four peaks recur each 27.8 days with a tolerance of only plus or minus 0.2 day. Even this slight tolerance could be eliminated if one were free to fill in the missing data without restriction.

Since the 27.8-day period provides the best fit to these four pulses, it will be used here in the test for the possibility of steady-state flow. An important distinction must be made between the rotation rate used for testing and the rotation rate believed to be correct. This seeming contradiction is due to the nature of the test, which is negative rather than positive; we are attempting to prove that temporal fluctuation occurred by showing that steady-state flow cannot have occurred. To make this negative proof convincing, we must illustrate the impossibility of steady-state flow using any rotation period reasonably close to 27 days. After the testing is concluded and it is found that steady-state flow did not occur, then we may revert to the Carrington period or to any other period thought to be likely.

There is one interesting sidelight to the choice of the 27.8-day period; it happens that there was a slight pulse during Carrington rotation 1564 and two other more marked pulses during rotations 1554 and 1555, all of which correlate with the four pulses under consideration if one assumes the 27.8-day period. Further, in mid-1969 during Carrington rotations 1547-9, regions of higher density occurred that also are consistent with recurrence of this feature at the 27.8-day rate. The physical reality of such alignments remains to be determined; at this time, we are in the process of extracting the statistics of these observations through objective mathematical processes.

For the five Carrington rotations that appear on figure 2, the SSRs are similar, varying somewhat in dimension but not enough to affect the tests discussed here. As examples, two SSRs for Pioneer 8 and one for Pioneer 9 are plotted on figure 3. At other times during the events depicted on figure 2, there was negligible change in the SSRs insofar as these steady-state tests are concerned. The variation in the Pioneer 8 SSR between the two examples plotted will serve to illustrate the magnitude and character of the changes that occur in an interval of three rotations, or about 82 days.

![Figure 3. The steady stream response (SSR) of the Pioneer 8-Pioneer 9 measuring system in mid-1970. This is the average electron number density versus time along the radio paths caused by a steady stream from a source rotating at the Carrington period. In this example, a flow speed of 400 km/sec was assumed and 1-hr integrations were performed.](image-url)

There is a qualitative difference between the SSRs of Pioneers 8 and 9; the Pioneer 8 SSR is roughly a rectangle, while that for Pioneer 9 starts with a sharp, short pulse containing more than 20 percent of the total area defined by the SSR. The sharp onset of the function is attributable to the fact that the corotating spiral stream first encountered the Pioneer 9 path broadside, before the stream hit either earth or Pioneer 9. This can be visualized on figure 1. Thus, at the beginning of the influence of the stream, the measured content is very high because the length of the path inside the stream is at a maximum just after the point of
tangency. There is a sharp drop in the Pioneer 9 SSR which coincides with the termination of the Pioneer 8 SSR; prior to the drop, the plasma stream crossed the Pioneer 9 path in two separate regions. At the moment of the drop, the stream strikes the earth; thereafter, the stream is not intercepted by the Pioneer 8 path, and also the single crossing of the Pioneer 9 path occurs at more nearly perpendicular incidence, with the attendant short immersed path length and low content. There are a number of interesting facets to this calculation of the SSR, which are described in a paper now in preparation.

Little has been said of the testing of Pioneer 9 data for possible steady-state causation. This is because the Pioneer 9 SSR contains the large, short initial feature, and when such a function is convolved with another function, sharp features in the second function are comparatively well preserved. Consequently, steady-state tests are more subtle. However, the Pioneer 8 SSR is much like a rectangle and so its convolving effect is similar to the action of a running mean. In the resulting content measurement, pulses should not occur having a duration less than the corotation interval.

The observed content variation on April 10, 1970, when the first of the four repeating pulses occurred, exhibits a high rate of decrease, which could only occur at the termination of the effect of a steady stream—a view inconsistent with the measurements of the preceding three days when the content was relatively low. If we neglect this sharp drop on April 10 and attribute it only to the chance coincidence of a transient event at the beginning of the passage of a steady stream, then it would be possible to reconcile the April 10 onset with the steady-state hypothesis since the density stays high for the following two (or three) days.

However, the second occurrence on May 8 is seen to be an isolated, high value occurring among a series of daily values which are relatively undisturbed, although exhibiting a slight rising tendency. The maximum permissible rate of decrease of content over a period of several days is limited by the SSR; the measured content can only decrease at the same proportional rate as the SSR in the unlikely event that the steady-state flow is followed by a region in which there is a vacuum. No such region has ever been detected in the solar wind, and it must be inferred that if there is steady-state flow, the content can never decrease at a proportional rate equal to (or faster than) the SSR. Yet, from May 8 to 9, the content decreases by a factor of 2. Clearly this is inconsistent with steady-state flow.

At the third occurrence on June 5, the rate of decrease for several hours was again too great to be explained in terms of steady flow, and the drop from June 5 to 6 is also excessive. Notice the slight increase on the following day; if the source of this family of pulses is a single sputtering source, then we should expect to see outbursts more than once in a single rotation on some occasions, and on other occasions we should not see it at all. Perhaps the slight increases observed on June 7 and on April 12 are reappearances of the same sputtering source on rotations 1559 and 1561. A trend that supports this view is the observation that the second appearance in each of these two rotations is less pronounced than the first appearance. This would be expected because the SSR decreases by roughly a factor of 2 during the corotation interval. The decrease is attributable to the rotation of the spiral stream so that it encounters the radio path at more nearly perpendicular incidence (fig. 1). The length of path inside the stream is then shorter, and consequently the content is decreased. This same mechanism would act to decrease the size of content pulses caused by sputtering plasma streams, so that reappearances would cause smaller content pulses.

The final appearance of the recurring pulse is on July 2, although it is possible that the upward trend on July 1 is an indication that the interplanetary content may have begun its increase on that earlier date. Again, steady-state flow can be ruled out because of the high rate of decrease from July 2 to July 3 and to July 4. The observed rate is too high even if one assumes the existence of a steady, thin plasma stream followed by pure vacuum.

A POSSIBLE CAUSE OF THE REPEATING PULSE

We have shown that the repetitious event on figure 2 cannot be reconciled with steady flow. There are no clues presently available to the author that permit definition of the character of the corotating region. It is possible that other spacecraft measurements will provide such clues and permit us to determine the details of these detected temporal changes.

These data are consistent with their possible causation by a single source corotating with the sun and near its surface at a Carrington longitude of roughly 40° to 50°. However, this source cannot be steady but must eject plasma at a rate that varies on a time scale of approximately one day. The flow from the source is not necessarily extinguished between outbursts, but its density must change, and if this occurs, it seems likely that the velocity of the stream also changes. Such a behavior would explain the variation of the location of the pulses observed over a time span of roughly one corotation interval. In fact, if the Carrington rotation rate is assumed, and if one wishes to include the primary and secondary pulses mentioned earlier, then it is
necessary to include all pulses over a period of 4 days; this amount of variation could easily be explained in terms of a stream width of $10^6$, or in terms of varying velocity, which could change the earth-sun transit time by $\pm 0.5$ day. See, for example, the velocity distributions of Gosling et al. [1970].

It is not necessary to consider that the sputtering source is ejecting plasma clouds that are discrete entities with sharply defined leading edges. Rather, one can explain these measurements in terms of a unending stream of varying density. Of course, if velocity varies also, then bunching will often occur and sharp leading edges may occur.

There is at least one alternative explanation, which is quite different: In principle, it is possible to explain these observations in terms of a flow of plasma that is steady in density and velocity, but which undulates north-south and east-west by approximately $10^6$, heliocentric. If the stream is sufficiently narrow, then it can pass in and out of our radio path in a north-south direction, and it would be visible only for a period of roughly 4 days on each rotation. The author finds this explanation to be less credible than that of the intermittent source, but the judgment is subjective and therefore it is suspect.

Other features in figure 2 are relevant to this discussion. Notice the faithful reproduction of the 27-day content pattern from rotation 1560 to 1561, except the pulse on May 17. We have made this kind of measurement continuously for over 5 years and this particular pair of rotations exhibits the best structural match that we have found.

On rotations 1560 through 1562, notice that there is a tendency for pulses to occur on the Pioneer 9 path about 2 days after the Pioneer 8 pulses under scrutiny here. The three Pioneer 9 pulses are more closely synchronized to the Carrington rotation period than are the Pioneer 8 pulses and, because of the impulsive character of the Pioneer 9 SSR, this may be a clue that the source region near the sun actually rotates at nearly the Carrington rate.

There are other Pioneer 8 pulses in figure 2 that exhibit rates of increase followed by rates of decrease at short intervals inconsistent with the steady-state flow. Some notable examples are those on February 25, March 5, March 31, May 17, and June 19. The interested reader can find more examples in figure 3 of Croft [1971].

**CONCLUSIONS**

It is seen that a recurring transient increase in the measured solar wind content cannot be attributed to a corotating flow having steady velocity and density. The sequence of events can be most readily explained if the causative mechanism is the corotation of an intermittent source that ejects a narrow plasma stream of variable density and, perhaps, of variable velocity. However, on the basis of the data available, we cannot rule out the possibility that these events are caused by a steady flow that undulates in the north-south direction so that it can enter and leave the Pioneer radio path on a time scale of approximately one day. The intermittent flow concept has been indicated by other patterns in the Pioneer content measurements, but this series of events in mid-1970 appears to offer the most conclusive evidence of the existence of sputtering, corotating regions.

**REFERENCES**


**ACKNOWLEDGMENTS**

The National Aeronautics and Space Administration supported this work under Contract NAS 2-4672 and Grant NGR 05-020-407. The data were gathered under the leadership of Professor V. R. Eshleman and the measurements were taken under the direction of H. T. Howard, supported by a series of NASA grants and contracts in addition to those cited above.
J. H. Binsack  I have two points. One, I'm just curious why most of your data points on your graphs seem to be vertical.

T. A. Croft  The data form that I am showing is what we call our interim process data, and still has a lot of ionospheric error in it. In the days for which I discussed this impulse the ionospheric error I could see was not very large, but there are on some rotations periods when the ionospheric error is large and it has a diurnal variation.

J. H. Binsack  Have you removed the effect of the earth's plasmasphere?

T. A. Croft  We measure the Faraday rotation to Geostationary Satellites and from that infer that content, and then map it over to the Pioneer line of sight.

J. H. Binsack  My point is this, that as I recall your heliocentric plot Pioneer 8 was coming out on the dusk side of the earth. That is just in the region where the plasmasphere has this notorious bulge which is very responsive to geomagnetic activity. It can increase its integrated electron content in a very short time period, perhaps shorter than what you can obtain from your geostationary satellite normalization factors. My point is that perhaps the increased density or response function that you do find along the ray path to Pioneer 8 is solely due to the plasma bulge in the plasmasphere. Could you comment on that?

T. A. Croft  Yes. The impulse response I showed was calculated without that effect in it. The amount of variation that can be attributed to this effect is roughly 5X10^16 electrons/m^2, and for the distance to Pioneer 8 that is not a very big effect. By the time we divide by the distance and express that in terms of average density on the path, that amount of error is not very big. The typical error in just the ionospheric portion can be quite a bit larger than that. I think that is important. But I think the ionospheric error in this data form is even bigger. But during the pulses that you saw here I think the errors are roughly about on the order of 10 percent, and not enough to invalidate these arguments about the intermittency of flow.

J. T. Gosling  I'd just like to repeat a point that I made several days ago. When you do a statistical analysis of a large amount of so-called corotating streams, you find out that this type of event in terms of repeating velocity structures is very very rare. I think you probably make the same point looking at your data, that a structure that comes back again and again is a fairly rare event.

T. A. Croft  I think I disagree with you, but it may be semantics. I see very very clear and consistent repetition of regions of differing character. There will be a chaotic region and a quiet region, and that will repeat consistently and very often. But individual fluctuations on the time scale of a day almost never repeat. Would you agree that if you see a chaotic 4 days of data on one rotation you will usually see relative chaos on the next rotation during those four days?

J. H. Binsack  Sometimes yes, sometimes no.

A. J. Lazarus  How do your intensity peaks compare with the spacecraft measurements of plasma density?

T. A. Croft  Well, the best comparison we've had is yours, and the correlation wasn't very good. The reason is that you are measuring at a point, and we are measuring the average over a long distance. Unless we analyze in terms of this steady state response, and consider that you're measuring the input function and we are measuring the output function, we can't do very well. The only data we have to work with to do the job here are your Mariner 5 and our Mariner 5 data. And it is not clean enough yet to do that.

COMMENTS

N. F. Ness  At the risk of causing some indigestion, I suppose, I'm going to mention power spectra again. I hope it won't upset lunch.

There was a discussion this morning about estimating slopes of power spectra in which I tried to point out that because of the manner in which power spectra are computed, there
are associated uncertainties with slopes that can be very quantitatively specified. What I have sketched in figure 1 is what one computes standardly from any time series or it could be a spatial parameter. It is the power spectral density, which is measured in units of amplitude squared per unit frequency as a function of frequency. But it could be

![Spectral Estimate Diagram]

**Figure 1.** Power spectra results yield estimates of amplitude squared over a finite frequency band Δf.

presented as a function of wavelength. The important point here is that the actual quantity computed in fact is an estimate averaged over an elementary frequency interval Δf centered at frequency \( f_l \). The value of Δf is determined by the number of estimates you make \( M \) and the Nyquist frequency \( f_c \) as \( Δf = f_c/M \).

Now, depending on how many of these estimated points you have—and, of course, the number of sample points—then each of those estimated averages has associated with it a statistical confidence limit determined by the degrees of freedom \( k \). This value depends on the number of sample points \( N \) and \( M \) as \( k = 2N/M \). As \( k \), the number of degrees of freedom, increases the confidence becomes higher, and these uncertainties become less. Usually \( k \) is assumed to relate to a chi-square distribution to determine the size of the confidence limits.

In lots of the graphs we've seen here today, no one presents these confidence limits, and no one tells us the degrees of freedom, and no one tells us what the resolution is in the frequency domain. As an example, I've sketched in figure 2 a very simple spectrum drawn through the averages. As you can see, two straight lines fit through all those points equally well. The question is, which one of those do you want to use? Well, I don't know how the people who presented these continuous straight lines or curves have constructed their power spectrum.

I suggest that in presenting experimental results on the power spectrum of fluctuations one be careful to include these kind of statistical estimate uncertainties with the confidence limits clearly indicated so that the reader can judge for himself whether or not the slope you have chosen is the maximum or minimum and how certain you are of that slope.
As a side comment, I didn’t quite understand Siscoe’s remark about the variability in the confidence limits, because these, if plotted on a log scale, have the same amplitude. So generally it is very convenient to present your spectra with log of the spectral density either linearly or logarithmically as a function of the frequency so that these confidence limits then don’t have to have error bars of different height associated with each point.