

X-553-72-378

PREPRINT

NASA TM X

66078

THE GEOCENTRIC ORIENTATION VECTOR FROM LIMITED ASTRO-GEODETTIC DATA

R. S. MATHER

OCTOBER 1972



GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

(NASA-TM-X-66078) THE GEOCENTRIC
ORIENTATION VECTOR FROM LIMITED
ASTRO-GEODETTIC DATA R.S. Mather (NASA)
Oct. 1972 16 p

CSSL 08N

N73-11342

Unclas

G3/13

46616

X-553-72-378

THE GEOCENTRIC ORIENTATION VECTOR
FROM LIMITED ASTRO-GEODETTIC DATA

R. S. Mather
Geodynamics Branch

October 1972

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

PRECEDING PAGE BLANK NOT FILMED

THE GEOCENTRIC ORIENTATION VECTOR
FROM LIMITED ASTRO-GEODETTIC DATA

R. S. Mather*
Geodynamics Branch

SUMMARY

The establishment of a world geodetic system from gravity data available at the present time, can be based on the corrections necessary to convert the separation vectors, as determined from astro-geodetic information, to equivalent values which could be directly compared with those determined from an imperfect knowledge of the Earth's gravitational field.

Test computations using the comprehensive sets of both astro-geodetic and gravimetric data available for the region covered by the Australian Geodetic Datum indicate that the accuracy of the geocentric orientation vector so determined from incomplete representations of the Earth's gravitational field, is dependent on the overall extent of the datum over which the comparisons are made.

The lack of an astro-geodetic determination of the "geoid" due to an inadequate density of astro-geodetic stations over certain parts of the datum does not materially affect these determinations. The effect is only marginal if the station density of the area referred to does not markedly differ from the average for the region. These tests indicate that sufficient astro-geodetic data exists for the connection of the North American Datum to a world geodetic system assembled from gravimetric considerations and referred to the geocenter with an accuracy of 0.2 or its linear equivalent in each component of the geocentric orientation vector.

*On leave of absence from the University of New South Wales, Sydney, Australia.

Preceding page blank

CONTENTS

	<u>Page</u>
1. INTRODUCTION	1
2. THE MATCHING OF SOLUTIONS	4
3. APPLICATIONS TO THE NORTH AMERICAN DATUM.	8
4. CONCLUSION	10
5. ACKNOWLEDGMENTS	11
6. REFERENCES	11

TABLES

<u>Table</u>	<u>Page</u>
1 The Effect of Excluding the Geoid Solution from Determinations of the Geocentric Orientation Parameters	10

ILLUSTRATIONS

<u>Figure</u>	<u>Page</u>
1 Elevation Above Ellipsoid	2
2 The Geocentric Orientation Vector	6
3 Orientation Errors from Limited Fixes	8

Preceding page blank

THE GEOCENTRIC ORIENTATION VECTOR FROM LIMITED ASTRO-GEODETTIC DATA

1. INTRODUCTION

The establishment of a world geodetic system based on gravimetric considerations has been mooted as a distinct possibility (Mather 1971b). The geodetic reference frame established in this manner has the considerable advantage of being directly related to the Earth's center of mass or geocenter, as a consequence of conditions implicit in the solution (ibid, p. 83). A number of technical problems which have to be coped with in the implementation of such a scheme are elaborated on by Mather (1972a). While the accuracy attainable from this type of determination is, in the short term, estimated as only being equivalent to that from the geometrical use of satellites using optical techniques, it is nevertheless of fundamental importance in the definition of the observation space as it provides a relationship between the geocenter and the observing stations at the surface of the Earth. The significance of this aspect is discussed in detail by Mather (1972b).

The basic techniques can be described briefly as follows. The separation vector d is defined as the separation in Earth space between equivalent points P on the physically defined surface (e.g., the Earth's surface; the geoid) and those on a system of reference used to relate the former, and defined with respect to the geocenter (e.g., the telluroid; the reference ellipsoid). Its component d_i along the axes x_i of a local Cartesian system of reference whose x_3 axis is coincident with the local normal to the reference surface, while the x_1 and x_2 axes are oriented in the north and east directions respectively, defining the horizontal plane, are given by

$$d_1 = R \xi; d_2 = R \eta; d_3 = h_s \quad (1)$$

where R is the mean radius of curvature of the equipotential surface of the reference system passing through the point of reference. ξ and η are components of the deflections of the vertical in the meridian and prime vertical directions respectively, and h_s is the evaluation above ellipsoid.

h_s is related to the height anomaly h_d , the normal elevation h_n , the orthometric elevation h_o and the results of astro-geodetic levelling as shown in Figure 1, by the following set of relations.

$$h_s = h_d + h_n = N_a + \sum_{\text{geoid}}^P \Delta h_o + N_{\text{geoid}} = N + h_o \quad (2)$$

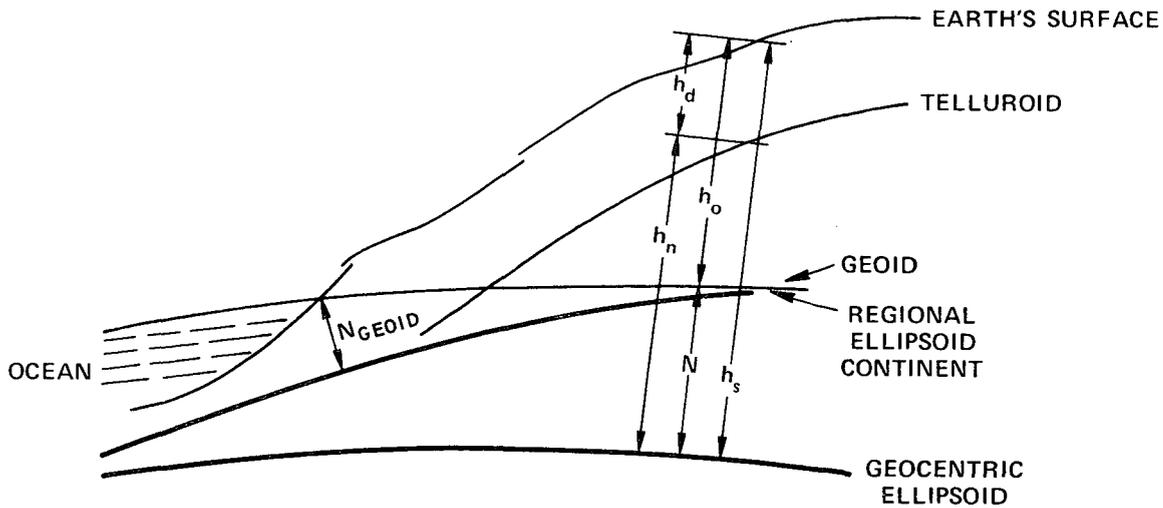


Figure 1. Elevation Above Ellipsoid

where N is the elevation of geoid above ellipsoid, the subscript $_{\text{geoid}}$ referring to the value at a point on the geoid from which the astro-geodetic levelling defining N_a is commenced. The summation which appears after the second equality in equation 2, is taken over the route of the astro-geodetic levelling, h_o being the increments in orthometric elevation. The results of astro-geodetic levelling, given as N_a in equation 2, are defined by the relation

$$N_a = - \sum_{\text{geoid}}^P \zeta d\ell \quad (3)$$

where ζ is the component of the deflection of the vertical along the element of length $d\ell$ whose azimuth is a , it being assumed that the geoid has a linear gradient between the terminals of the element of length. ζ is defined by the relation

$$\zeta = \xi \cos a + \eta \sin a \quad (4)$$

Both h_n and h_o are linear interpretations of the difference in geopotential ΔW between the geoid and P which is an observed quantity. h_n is defined on the reference system, being a displacement in space unoccupied by matter, in the gravitational field exterior to an ellipsoid of revolution whose bounding surface is an equipotential and with mass characteristics similar to that of the Earth (e.g., Heiskanen & Moritz 1967, p. 171). h_o on the other hand is a displacement along the local vertical in space occupied by the topography exterior to the geoid and within the Earth's surface. It follows that while Δh_o is an observed quantity, h_o is dependent on the model adopted for the Earth's topography exterior to the geoid. This has to be specified before h_o and N have a unique definition for locations in land areas.

Most of these distinctions are conveniently blurred in practice due to limitations on the accuracy of solutions as a consequence of the incomplete definition of the requisite data sets. Practical solutions are usually called representations of the "geoid" irrespective of whether they are merely the results of the astrogeodetic levelling operation which merely defines N_a on its own, or if the determination is gravimetric, obtained by the use of free air anomalies in Stokes' integral, given by

$$N_f = \frac{W_o - U_o}{\gamma} - R \frac{M \{ \Delta g \}}{\gamma} + \frac{R}{4\pi\gamma} \iint f(\psi) \Delta g \, d\sigma \quad (5)$$

where Δg is the value of the free air anomaly representing the element of surface area $d\sigma$ which is at an angular distance ψ from the point P at which N_f is determined. $f(\psi)$ is Stokes' function whose formulation is well known (e.g., *ibid* 1967, p. 94). W_o is the potential of the geoid, U_o that assigned to the surface of the reference ellipsoid and $M \{ \Delta g \}$ the global mean of the free air anomaly.

The free air geoid N_f so defined is a convenient quantity to work with as it embodies the total effect of observed gravity and further, N_f is the major contributor to both N and h_d . While having significant magnitude which is approximately 10% that of N_f , the correction terms could exhibit variations which cannot be ignored in certain instances. The expressions for these correction terms have been investigated and the required relations are

$$N_f = h_d - N_c = N - N_i \quad (6)$$

N_c is given by (Mather 1971b, p. 85)

$$N_c = \frac{R^2}{2\pi} \iint \frac{1}{r} \left(\left(R \sin \psi \frac{dh}{dr} + (h_p - h) \right) \frac{N_f}{r^2} - \xi_a \tan \beta_a \right) d\sigma + o \{ fN_c \}$$

if

$$\frac{1}{2} \left(\frac{h_p - h}{r} \right)^2 < f \quad (7)$$

r being the distance between the element $d\sigma$ and the point of computation P. The terms to be integrated in equation 7 should be evaluated at $d\sigma$, h referring to the orthometric elevation, the subscript p indicating the value at P. $\tan \beta_1$ and $\tan \beta_2$ are the ground slopes in the north and east directions, while

$$\frac{dh}{dr} = \cos a \tan \beta_1 + \sin a \tan \beta_2 \quad (8)$$

Also note that the use of the repeated greek subscript in equation 7 refers to summation over two indices, while R and γ in equations 5 and 7 refer to the mean radius and the mean value of gravity for the Earth model. For a detailed description of N_i , see Mather 1971c, p. 84. The solutions described above are correct to the order of the flattening f and should suffice until such time as improvements in the definition of the Earth's gravitational field warrant extensions of computation techniques.

Fryer (1970, p. 155) has computed N_i using an approximate technique and his results indicate that its magnitude is generally small with maxima over the Himalayas and the Andes, but with rather slow attenuation. This calls for the exercise of caution when using the free air geoid alone for the representation of the gravimetric determination of the separation vector as has been done for an investigation covering the Australian Geodetic Datum (Mather 1971a). This is necessary as a significant distorting effect could be produced on the computed value of the geocentric orientation vector if any significant correlation were to exist between topographic variations and position. The region covered by the Indian Geodetic Datum would be an example of such an area.

Such considerations would also apply, though to a lesser degree, to the region covered by the North American Datum (NAD) which has greater topographical variations than the Australian region. Consequently, it is possible to compare astro-geodetic solutions directly with the free air geoid over the latter datum without errors much in excess of ± 1 m due to this cause and terrain variations produce insignificant effects on the computation of the geocentric orientation vector (Mather, Barlow & Fryer 1971, p. 24). The NAD is therefore an excellent test region for the investigation of topographical effects in view of the quality of both the gravity and elevation data available in the area and the variety of solutions available for geocentric determinations from geometrical considerations, which would afford independent checks.

2. THE MATCHING OF SOLUTIONS

Solutions for the geocentric orientation vector can be visualized as the matching of imperfect determinations of the same surface using two different techniques. Such a description is somewhat simplistic in the light of the reasons given in the last section. The necessary equations are obtained by attributing the difference in the values for the separation vector as obtained from gravimetry and astro-geodetic methods in turn, as being due to the latter being referred to an ellipsoid whose center is not at the geocenter, though its minor axis is parallel to that of the Earth's rotation for the epoch of observation. The solutions can be referred without difficulty to a reference figure of the same dimensions by

the standard relations (e.g., Mather 1968, pp. 290 et seq)

$$\delta\phi = df \sin 2\phi + o\{f^3\}; \quad (9)$$

$$\delta\lambda = 0 \quad (10)$$

and

$$\delta h_s = da + a df \sin^2\phi - v \delta\phi \tan\phi + o\{f^3 a\} \quad (11)$$

which link changes da in the equatorial radius a , df in the flattening f of the reference ellipsoid with changes $\delta\phi$ in the latitude ϕ , $\delta\lambda$ in the longitude λ , and δh_s in the ellipsoidal elevation h_s . v is the radius of curvature of the reference ellipsoid in the prime vertical section at the point considered.

The geocentric orientation vector $\underline{0}$, which is the space vector defining the displacement of the center of the regional ellipsoid with respect to the geocenter, as illustrated in Figure 2, can be represented by the components $\Delta\xi_i$ in the Laplacian triad at the origin of the regional datum. The transformation between components d_{ia} of the separation vector in the local Laplacian triad at any point of the regional datum, as determined from astro-geodetic considerations, can be related to those evaluated from gravimetry (d_{ig}) by the equations (Mather 1971a, p. 63 et seq)

$$d_{ig} = d_{ia} + \frac{1}{h_i} A_{ij} (h_o)_j \Delta\xi_j, \quad (12)$$

where the summation convention applies. A_{ij} are the elements of the array

$$A = \begin{vmatrix} \cos\phi_o \cos\phi & \sin\phi \sin\Delta\lambda & \sin\phi_o \cos\phi \\ +\sin\phi_o \sin\phi \cos\Delta\lambda & \cos\Delta\lambda & -\cos\phi_o \sin\phi \cos\Delta\lambda \\ -\sin\phi_o \sin\Delta\lambda & \sin\phi \cos\phi_o & 0 \\ \sin\phi \cos\phi_o & -\cos\phi \sin\Delta\lambda & \sin\phi \sin\phi_o \\ -\sin\phi_o \cos\phi \cos\Delta\lambda & +\cos\phi_o \cos\phi \cos\Delta\lambda & \end{vmatrix} \quad (13)$$

where

$$\Delta\lambda = \lambda_o - \lambda,$$

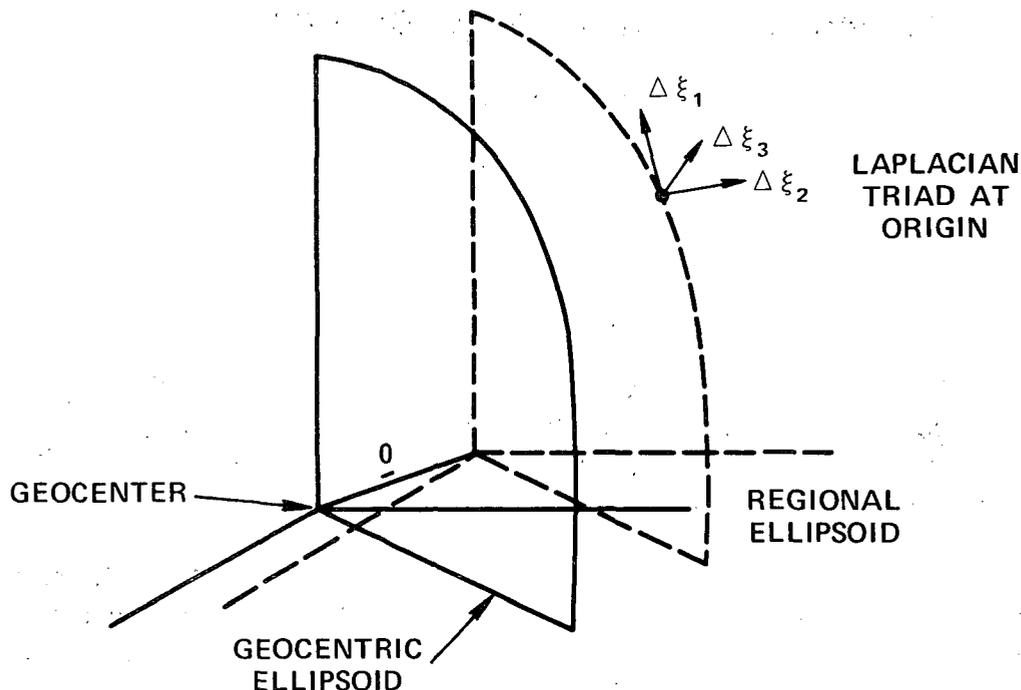


Figure 2. The Geocentric Orientation Vector

the subscript $_0$ referring to values at the origin of the regional datum at which the geocentric orientation parameters $\Delta \xi_i$ are computed. The quantities h_i are given by

$$h_1 = -(\rho + h_s); \quad h_2 = -(v + h_s); \quad h_3 = 1, \quad (14)$$

where ρ is the radius of curvature of the reference ellipsoid in the meridian.

Three types of solutions are possible for the quantities $\Delta \xi_1$. The first compares values of h_s only which is equivalent to restricting the block of equations at 12 to those obtained when $i = 3$. The second is based on the comparison of values of ξ and η which restricts the equations at 12 to those obtained when $i = 1$ and $i = 2$. A third solution uses all three quantities ξ , η and h_s . Expressions of gravimetric values of ξ and η which are directly comparable with astro-geodetic values are given in Mather 1971b, p. 87.

Experiences based on tests carried out on the Australian Geodetic Datum indicate that solutions of type 1 and 3 give identical results. Solutions of type 2 give slightly different results for $\Delta \xi_1$ and $\Delta \xi_2$ (Mather 1971a, p. 72). These have been attributed to possible errors in the process used to deduce N_a and hence h_s from a density of astro-geodetic stations which is less than ideal. This conclusion is strengthened by the fact that the discrepancies between the values

of $\Delta\xi_1$ and $\Delta\xi_2$ as determined by either one of types 1 and 3 or type 2, are smaller when the density of astro-geodetic stations is doubled.

If the definition of the global gravity anomaly field is free from error, it follows that $\Delta\xi_i$ can be obtained by a direct comparison of d_{gi} and d_{ai} at the origin alone. Such a technique cannot be resorted to in practice without significantly affecting the accuracy of the result, because the accuracy of the astro-geodetic determinations of ϕ and λ is restricted to ± 0.3 arcsec unless the direction of the vertical at the origin has been established by techniques more akin to those used at an observatory. A second difficulty which would be complicating is the effect any large scale prediction of gravity values has on the final results.

Australia is a region where the gravity anomaly field was based on a station density which was adequate for geodetic calculations, affording representation on a 10 km grid with 80% coverage. Earlier tests indicated that serious errors are obtained when determining the geocentric orientation parameters $\Delta\xi_i$ for an area this size if the values of $\Delta\xi_i$ were obtained by a comparison of the components of the separation vector d_i at locations which were restricted to a small part of the datum (Mather & Fryer 1970, p. 278).

This is due to two causes. The first is the significant effect errors of prediction have on the final result. Such errors could be minimized in areas where no observed gravity is available over extensive regions by constraining predicted values to fit estimates based on those from satellite orbital analysis, as strengthened by combination with surface gravimetry (Mather 1971a, pp. 65 et seq). However this merely minimizes, rather than eliminates the error.

The second is due to errors in the representation of the largely unsurveyed ocean areas by low degree harmonics of the Earth's gravitational field obtained by the technique referred to in the previous paragraph, as initially proposed by Kaula (1966). Errors in the higher harmonic coefficients which are determined with greater uncertainty, could seriously distort the geocentric orientation parameters $\Delta\xi_i$ as determined from small areas, as illustrated in Figure 3.

Tests on the orientation of the Australian Geodetic Datum (AGD) which covers an area equivalent to that of the United States, indicated that the best results are obtained by comparing gravimetric and astro-geodetic values of the separation vector \underline{d} at points which would provide the widest possible representation of the entire datum whose geocentric orientation vector is to be defined. The precision of such determinations is not dependent on the number of points at which comparisons are made but rather on the extent of datum represented. Thus the use of 38 well chosen points for a determination of the AGD on this basis, gave essentially the same result as that obtained from comparisons at 693 points on an evenly spaced grid (Mather 1971a, p. 72).

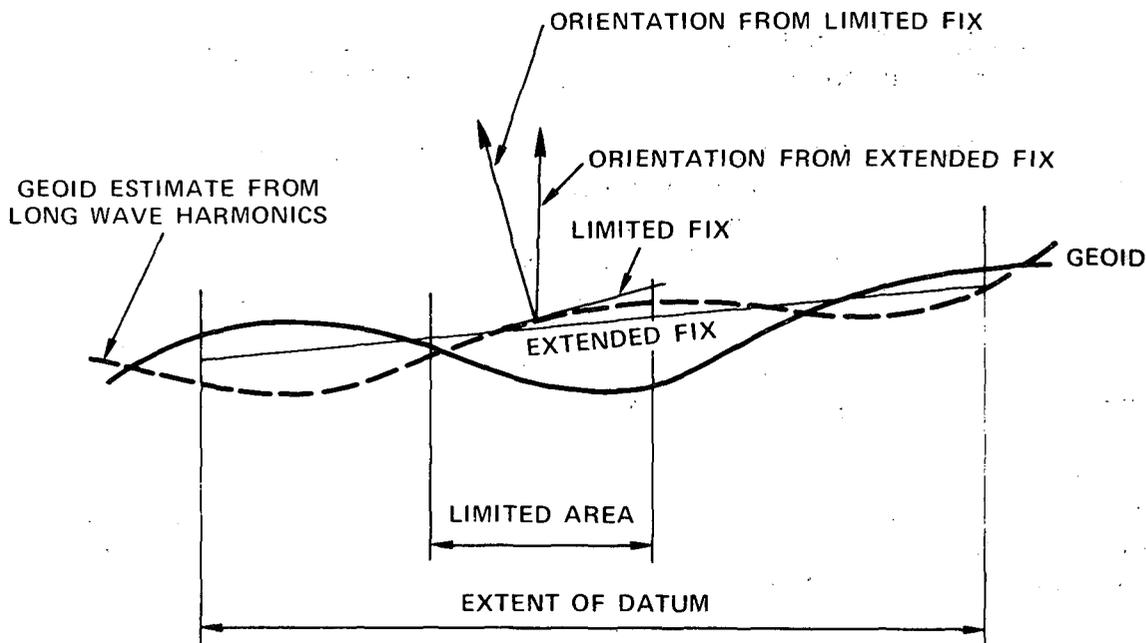


Figure 3. Orientation Errors from Limited Fixes

The value of the geocentric orientation vector obtained from such comparisons, effectively allows for errors in the determination of individual values of d . It can be concluded that the errors in the final value of the geocentric orientation vector are a consequence of those harmonics in the global representation of the gravity field whose departures from linearity over the region covered by the datum are less than the errors inherent in the technique used for the determination.

3. APPLICATIONS TO THE NORTH AMERICAN DATUM

The North American Datum (NAD) is taken as the geodetic datum for the United States, Canada and Mexico (NAS Report 1971, p. 10). The surface area covered is approximately two and a half times that of the AGD. Determinations of the separation vector as defined in equation 1 are available at a widely distributed network of points in the United States (Rice 1971) and southern Canada (Ney 1951; Corcoran 1967). While the density of the astro-geodetic stations over the rest of the datum is low, the U.S. Defense Mapping Agency's Topographic Command has produced a geoid map (i.e., N_a values) of North and Central America (Fischer et al 1967). Without detracting from the ability of the group working at this organization on the preparation of geoid maps from astro-geodetic data, which is second to none, it is always possible that the paucity of data can give either an over-smoothed result in these sparsely surveyed

regions or else a slight bias in the values of the geocentric orientation parameters $\Delta\xi_i$ determined, in view of the dominating influence of d_3 on the solution, as mentioned in Section 2.

It is therefore preferable to provide the vital coverage for the large areas in northern Canada and Mexico by the deflections of the vertical alone. This is considered to be necessary as the determination of the quantities $\Delta\xi_i$ by restricting comparisons to the central region alone could bias the results for the reasons given in Section 2.

The optimum technique for the determination of the geocentric orientation parameters $\Delta\xi_i$, and hence the geocentric orientation vector $\underline{0}$ is the following.

Equation 13 is applied in two different ways depending on whether sufficient data were available for a reliable astro-geodetic determination of the ellipsoidal elevations h_s (i. e., d_3).

If it were, then three observation equations would be formed at each point, in terms of equation 13. Else, there would only be two, with the last row in the array on the right of equation 14 not being used.

In this manner, the final solution for the geocentric orientation parameters $\Delta\xi_i$ will be representative of the comparisons over the entire datum.

The use of this representation of the separation vector as determined from astro-geodesy was put to the test on the Australian Geodetic Datum where an astro-geodetic determination of h_s is available for the entire region, with no significant (i. e., $< 0!2$) bias between the deduced geoid solution and the observed deflections of the vertical (Mather et al 1971, p. 24). The results of a comparison made over all the available astro-geodetic stations on the AGD is given in row 1 of Table 1. The Australian continental area extends in latitude between -10°N and -40°N , if Tasmania were excluded.

It was decided to exclude the geoid solution from the determination of the geocentric orientation parameters $\Delta\xi_i$ at latitudes above a certain minimum value ϕ_m , and restrict the comparison of components of the separation vector at latitudes $\phi (> \phi_m)$ to those of the deflections of the vertical only (i. e., d_1, d_2). Comparisons at points where $\phi > \phi_m$ would still be represented by the three observation equations implied in equation 13.

The adoption of the procedure outlined in the previous paragraph creates a situation akin to that existing at the present time for the NAD. The solutions obtained for the geocentric orientation parameters $\Delta\xi_i$ as ϕ_m varied between -10°N and -34°N are set out in Table 1.

Table 1

The Effect of Excluding the Geoid Solution
from Determinations of the Geocentric Orientation Parameters

ϕ_m	No. of Astro-Geodetic Stations		Geocentric Orientation Parameters		
	$\phi < \phi_m^*$	$\phi > \phi_m^{**}$	$\Delta \xi_1$ (sec)	$\Delta \xi_2$ (sec)	$\Delta \xi_3$ (m)
-10° N	1084	0	-3.959	-4.126	8.30
-13° N	1071	13	-3.964	-4.125	8.31
-16° N	1022	62	-3.958	-4.119	8.29
-19° N	926	158	-3.961	-4.113	8.30
-22° N	750	334	-3.986	-4.103	8.38
-25° N	624	460	-3.988	-4.112	8.34
-28° N	444	640	-3.975	-4.107	8.24
-31° N	242	842	-4.012	-4.088	8.45
-34° N	85	999	-3.984	-4.086	7.92

*Three observation equations per point

**Two observation equations per point

The resulting values of the orientation parameters show a stability which is certainly an order of magnitude better than the final accuracy attainable from the gravity data available at present. It should be pointed out that the density of astro-geodetic stations at latitudes greater than ϕ_m is no different to those at latitudes less than ϕ_m as the astro-geodetic stations are evenly distributed over the Australian region. It may be reasoned that the results would therefore not be directly applicable to the North American situation. However, earlier tests on the AGD indicate that results of equivalent accuracy can be obtained from a few well distributed stations as is possible from a much higher density, though the errors in $\Delta \xi_i$ are likely to be greater than those implied in the results from the Australian tests. This would still not bias the final values of the geocentric orientation parameters $\Delta \xi_i$ in excess of the accuracy estimates given in the summary.

4. CONCLUSION

The relatively sparse distribution of astro-geodetic stations in the regions covered by the North American Datum, and lying outside the United States and southern Canada, need not be an inhibiting factor in defining the geocentric

orientation vector for the NAD with an accuracy equivalent to that attainable by the geometrical use of artificial Earth satellites based on optical techniques. A solution of adequate accuracy could be obtained if those regions not covered by networks of astro-geodetic levelling were represented by individual astro-geodetic stations at which the deflections of the vertical had been established, with the proviso that these stations afforded an evenly distributed representation of the regions.

When considering the consequences of the relatively low density of astro-geodetic stations in these regions, on the values of the geocentric orientation parameters, it is instructive to recall that the Australian Geodetic Datum was established by setting the reference ellipsoid parallel to an estimate of the mean geoid slope across the region using only 150, but well spaced astro-geodetic stations (Bomford 1967). The additional information provided by nearly eight times as many stations indicate that the consequent error between the desired condition and that achieved with this sparse but well distributed sample was only 0.2 in each of the orientation parameters at the origin (Mather et al 1971, p. 24). It is confidently estimated that the use of the technique proposed will introduce errors which are well within these figures.

5. ACKNOWLEDGMENTS

This research was done while the author was the holder of a National Academy of Sciences Resident Research Associateship at National Aeronautics & Space Administration's Goddard Space Flight Center, Greenbelt, Maryland while on leave of absence from the University of New South Wales, Sydney, Australia.

6. REFERENCES

1. Bomford, A. G., 1967. The Geodetic Adjustment of Australia 1963-1966, Surv. Rev., 144, 52-71, 1967.
2. Corcoran, G. A., 1967. Astro-geodetic Deflections of the Vertical in Canada, I. U. G. G. General Assembly, Lucerne, 1967.
3. Fischer, I., Slutsky, M., Shirley, R. and Wyatt, P., 1967. Geoid Charts of North & Central America, U. S. Army Map Service Tech. Rep. 62, 1967.
4. Fryer, J. G., The Effect of the Geoid on the Australian Geodetic Network, UNISURV Rep., 20, Univ. of NSW, Kensington, NSW, Australia, 1970.
5. Heiskanen, W. A. & Moritz, H., Physical Geodesy, Freeman, San Francisco, 1967.

6. Kaula, W. M., Tests and Combination of Satellite Determinations of the Gravity Field with Gravimetry, J. geophys. Res., 71, 5303-5313, 1966.
7. Mather, R. S., The Free Air Geoid for South Australia and its relation to the Equipotential Surfaces of the Earth's Gravitational Field, UNISURV Rep., 6, Univ. of NSW, Kensington, NSW, Australia.
8. _____, The Geocentric Orientation Vector for the Australian Geodetic Datum, Geophys, J. R. astr. Soc., 22, 55-81, 1971a.
9. _____, A World Geodetic System from Gravimetry, Geophys, J. R. astr. Soc., 23, 75-99, 1971b.
10. _____, The Harmonic Representation of the Disturbing Potential, Bull. geodes., 99, 65-88, 1971c.
11. _____, Practical Techniques for the Establishment of a World Geodetic System from Gravity Data, in Extra Collection of Papers Contributed to I.A. U. Symposium No. 48, edited by S. Yumi, 105-121, 1972a.
12. _____, Earth Space, UNISURV Rep., G 17, 1-41, 1972b.
13. Mather, R. S. & Fryer, J. G., Geoidal Studies in Australia, Surv. Rev., 156, 268-282, 1970.
14. Mather, R. S., Barlow, B. C. & Fryer, J. G., A Study of the Earth's Gravitational Field in the Australian Region, UNISURV Rep. 22, 1-37, 1971.
15. NAS Report, North American Datum, National Academy of Sciences, Washington, D.C., 1971.
16. Ney, C. H., Contours of the Geoid for South Eastern Canada, Geodetic Survey of Canada, Ottawa, Canada, 1951.
17. Rice, D. A., Private Communication, 1971.