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X-640-72-425
PREPRINT

NASA TM X- 66105

ON GREENSTADT'S BINARY INDEX CRITERION

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NOVEMBER 1972

GSFC

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On Greenstadt's Binary Index Criterion

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ABSTRACT

The Greenstadt-type criterion for distinguishing between laminar and "pulsating" (i.e., irregular) bow shock crossings can be expressed as a critical angle between the shock normal and the magnetic field. When the field is more tangent to the shock plane than this critical angle, the shock is laminar. More importantly, the requirement of Galilean invariance of the physics underlying such a criterion reveals that the Greenstadt p index must vary inversely as the solar wind velocity.

Greenstadt [1972] has observed that bow shock crossings can be separated into roughly two types, laminar and "pulsating." A crossing of the laminar type shows the magnetic field $\vec{B}$ varying in a smooth, monotonic fashion from the interplanetary value to that of the magnetosheath (or vice versa). In addition, there usually are the regular wave trains (~ 1 Hz upstream and 0.3 Hz downstream) present at these smooth crossings [Holzer et al., 1972]. With a pulsating type crossing the magnetic field varies in such an irregular fashion that it is often difficult even to ascertain whether the spacecraft is in the solar wind or the magnetosheath. It has been recognized for some time that the laminar type crossings tend to occur when $\vec{B}$ is nearly parallel to the shock surface (magnetosonic shock), while the pulsating type occur for $\vec{B}$ more normal to the shock. Greenstadt, however, has developed a more quantitative criterion. His procedure is to construct a vector $\vec{w}$ which is the sum of the solar wind velocity $\vec{u}$ and another vector directed <u>upstream</u> parallel (or anti-parallel, if required) to the upstream $\vec{B}$. The upstream directed vector along $\pm \vec{B}$ is of magnitude p times the magnitude u of the solar wind. If $\vec{w} = \vec{u} \pm pu\hat{B}$ lies outside of the plane tangent to the shock at the point of crossing, he observes the irregular pulsating behavior. If $\vec{w}$ lies inside he observes a laminar crossing. In short, the sign* of $\vec{w} \cdot \hat{n}$ determines which type of crossing will be observed,

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*Greenstadt uses the sign of $\vec{w} \cdot \hat{n}_G$ as the criterion, where $\hat{n}_G$ is the normal to the curve produced by the intersection of the bow shock with the plane determined by $\vec{B}$ and $\vec{u}$. However, one can show that $\vec{w} \cdot \hat{n} =$ (positive factor) $\vec{w} \cdot \hat{n}_G$, so that either criterion may be used equally well.

$\hat{n}$ being the outward directed normal. Greenstadt has determined empirically that $p = 1.6$ gives the best separation between the two types of crossing. This value of p seems to be constant over the shock surface or at least over as much of it as has been surveyed via this criterion. His observations have also been restricted to a solar wind speed $u \cong 400$ km/sec.

The purpose of this paper is to show that:

1. The separation criterion between the two types of crossing can be written in terms of a critical angle θ_c between the magnetic field and the shock normal. $\cos \theta_c = \cos \alpha/1.6$, where α is the angle between the outward drawn normal and $-\vec{u}$, as shown in Fig. 1. Because the separation criterion can be formulated purely geometrically, it relieves one of ascribing the laminar or pulsating character to reflected protons as Greenstadt does or to any other particular agent at this time.
2. Galilean invariance requires that if there is a unique p applicable to the entire shock at one solar wind velocity, then at any lower velocity there is another unique p.
3. p varies as $1/u$.

The present work neither supports nor refutes the existence of Greenstadt's criterion; it only says that if there is such a criterion then (1) - (3) above must also hold. The basic physics underlying such a criterion remains to be uncovered.

Let Fig. 1 represent Greenstadt's situation where $u = 400$ km/sec. To be definite we have assumed that $\vec{B}$ points generally downstream into the shock. Then $\vec{w} = -pu\hat{B} + \vec{u}$ and $\vec{w} \cdot \hat{n} = (p \cos\theta - \cos\alpha)u$. Thus θ_c at any point on this bow shock is given by $\cos\theta_c = \cos\alpha/p$ where p is Greenstadt's empirically determined 1.6. At the subsolar point, $\alpha = 0$, making $\theta_c = 51^{\circ}$.

Proof of (2) and (3) requires the assumption that the physical processes which determine the laminar or pulsating nature of the crossing are essentially local to the vicinity of the crossing. To state it differently, if the causal factors are the same in two shock crossings (call them Cases I and II) then the effects (i.e. whether the shock is laminar or pulsating, or alternatively the value of θ_c) must be the same. Let us take the Greenstadt case as Case I and accept the fact that $\cos\theta_c = \cos\alpha/1.6$ over the entire bow shock.

As Case II, consider a situation with $u' < u$. Since both a proton temperature-flow speed correlation and a density-flow speed correlation appear to exist in the solar wind, proton temperature and density may be different in the two cases. Let us assume, however, that solar wind proton temperature and density are not causal factors or else take them to be the same in Cases I and II. At an arbitrary point X' on the shock of Case II all vectors will look like Fig. 1, except they will be labeled by primes. Is it possible to find a point X on the shock in Case I where the causal factors are identical to those at X', i.e., $\vec{u}$, $\hat{n}$, and $\vec{B}$

have the same magnitudes and relative orientations as $\vec{u}'$, $\hat{n}'$, and $\vec{B}'$? If the answer is "yes," then θ_c at X is identical to θ_c' at X'.

It is generally impossible to find such a point X. Because $u < u'$, there is however a point X where the normal flow speeds are the same ($u \cos \alpha = u' \cos \alpha'$). One can also consider the situation where $|\vec{B}| = |\vec{B}'|$, $\theta = \theta'$, but the tangential flow velocities $\vec{u}_t$ and $\vec{u}_t'$ will still be different. If one compensates for this difference in $\vec{u}_t$ by a Galilean transformation of Case I to a frame moving at velocity $\vec{u}_t' - \vec{u}_t$, the two situations become identical. This follows because $\vec{B}$ is changed only in order u^2/c^2 by such a transformation. (The electric fields are small of order uB/c already.) Because $\vec{B}$ does not change under the frame transformation, θ_c does not change either, so that $\theta_c' = \theta_c$. But $\cos \theta_c = \cos \alpha/p$. If in Case II the critical angle is θ_c', define p' by $\cos \theta_c' = \cos \alpha'/p'$. Then $\cos \alpha'/p' = \cos \alpha/p$ and

$$p' = \frac{\cos \alpha'}{\cos \alpha}\, p = \frac{u}{u'}\;\frac{u' \cos \alpha'}{u \cos \alpha}\, p = \frac{up}{u'} \tag{1}$$

$$\cos \theta_c' = \frac{u_n'}{pu}$$

pu is about 1.6 x 400 or 640 km/sec, according to Greenstadt.

Since $u' < u$ we can carry out the procedure described in the previous paragraph for any point X' with the same result $p' = 640/u'$. It follows then that p' is the same for all points on bow shock II.

When $u' > u$, there are points near the subsolar point of bow shock II for which our proofs fail; one cannot find points X such that $u' \cos \alpha' = u \cos \alpha$. Our logic here only shows that a constant $p' = 640/u'$ exists for all X' at which $\cos \alpha' < 400/u'$. The failure of our logic does not preclude the possibility that $p' = 640/u'$ at all X' even in this case. Indeed, we see no physical reason why this criterion should break down abruptly at $\cos \alpha' = 400/u'$.

The fact that $u'p' = 640$ km/sec, independent of u', permits a simple geometric picture of the Greenstadt criterion under any conditions of $\vec{B}$ and $\vec{u}'$ when $\cos \alpha' < u/u'$. As in Fig. 2, one need only construct a vector 640 km/sec long directed upstream along $\pm \vec{B}$. To this vector add $\vec{u}$. The sum is $\vec{w}$, whose position outside or inside the shock determines its pulsating or laminar character respectively.

References

Greenstadt, E. W., Binary index for assessing local bow shock obliquity, J. Geophys. Res., 77, 5467-5479, 1972.

Holzer, R. E., T. G. Northrop, J. V. Olson and C. T. Russell, Study of waves in the earth's bow shock, J. Geophys. Res., 77, 2264-2273, 1972.

Figure Legends

Fig. 1: Definition of angles used in text

Fig. 2: Construction of $\vec{w}$ for any solar wind velocity $\vec{u}$. The figure as drawn is for a pulsating shock.

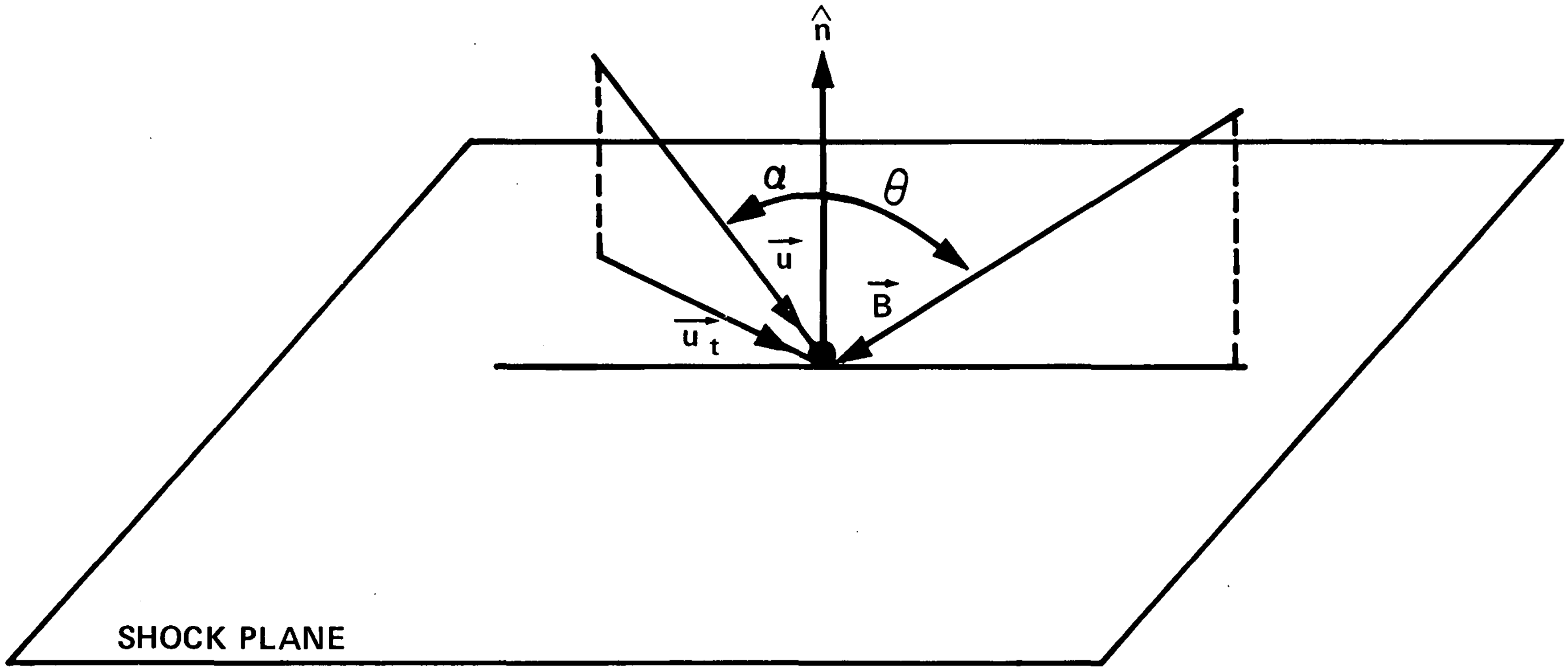

FIGURE 1

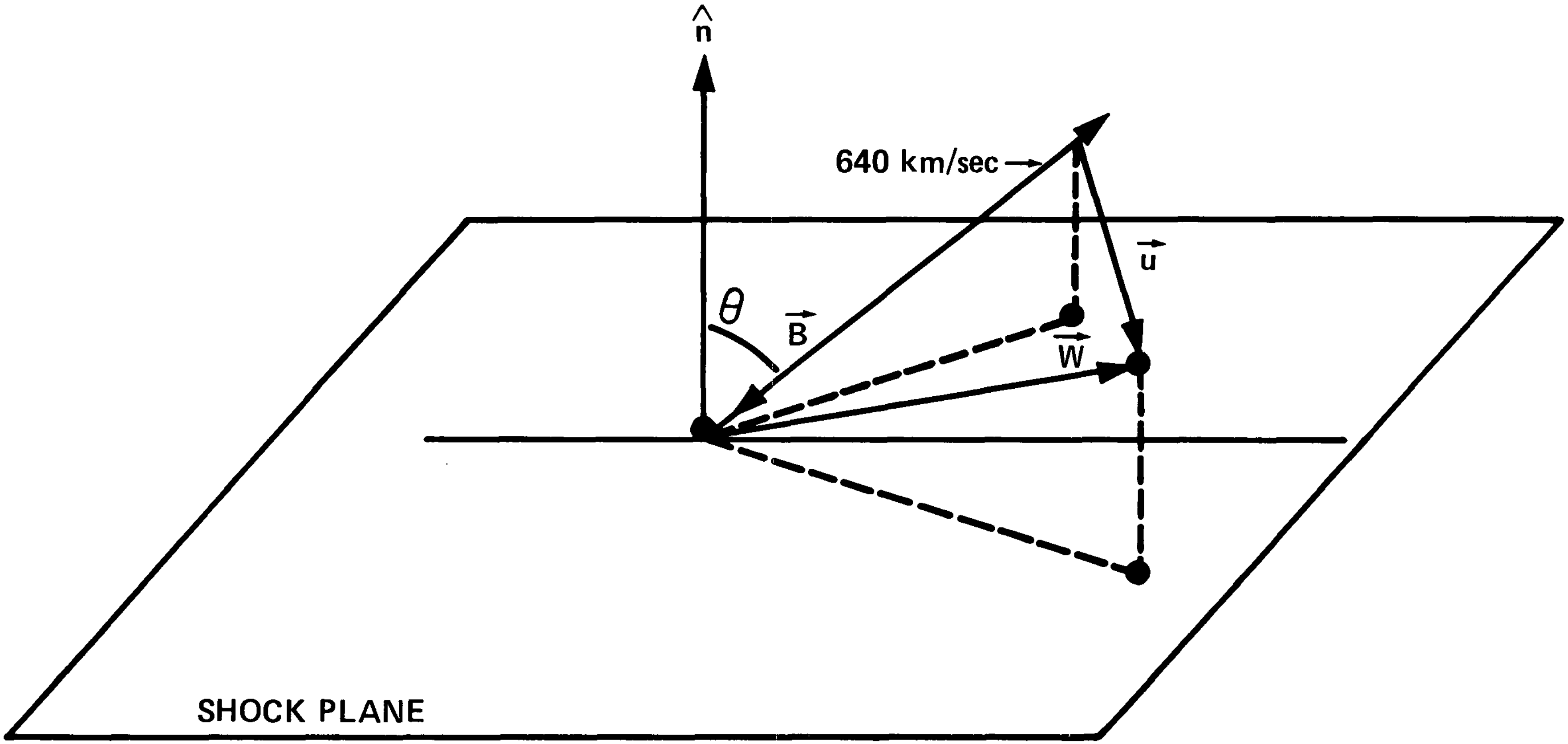

FIGURE 2

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