NASA CONTRACTOR REPORT

NASA CR-61382

METODOLOGY FOR THE SYSTEMS ENGINEERING PROCESS

Volume III: Operational Availability

By James H. Nelson
Martin Marietta Corporation
P.O. Box 179
Denver, Colorado 80201

March 1972

Final Report

Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812
FOREWORD

This report is submitted in accordance with the requirements of Contract NAS8-27567. Martin Marietta Corporation submits this report in three volumes as follows:

Volume I--System Functional Activities (NASA CR-61380)
Volume II--Technical Parameters (NASA CR-61381)
Volume III--Operational Availability (NASA CR-61382)
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>A.</td>
<td>Approach and Definition</td>
<td>2</td>
</tr>
<tr>
<td>B.</td>
<td>Analytic Development of Availability Parameters</td>
<td>3</td>
</tr>
<tr>
<td>C.</td>
<td>Detail Breakdown of Availability</td>
<td>5</td>
</tr>
<tr>
<td>D.</td>
<td>Availability Specification</td>
<td>10</td>
</tr>
<tr>
<td>II.</td>
<td>SCOPE</td>
<td>18</td>
</tr>
<tr>
<td>III.</td>
<td>ANALYTICAL DESCRIPTION OF OPERATIONAL AVAILABILITY</td>
<td>22</td>
</tr>
<tr>
<td>IV.</td>
<td>ILLUSTRATIVE EXAMPLES OF OPERATIONAL AVAILABILITY</td>
<td>26</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Considerations for Systems Performance Effectiveness Models</td>
<td>37</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Analytic Derivation of the Availability Function</td>
<td>54</td>
</tr>
<tr>
<td>Appendix C</td>
<td>Reliability and Maintainability Tradeoff Approach</td>
<td>76</td>
</tr>
<tr>
<td>Appendix D</td>
<td>Some Probability Background - Joint Probabilities</td>
<td>83</td>
</tr>
<tr>
<td>Appendix E</td>
<td>Launch On Time Analyses</td>
<td>92</td>
</tr>
<tr>
<td>Appendix F</td>
<td>Monte Carlo Simulation Model</td>
<td>115</td>
</tr>
<tr>
<td>Bibliography</td>
<td>119</td>
<td></td>
</tr>
</tbody>
</table>
Volume III is a detailed description and explanation of the operational availability parameter. The fundamental mathematical basis for operational availability is developed, and its relationship to a system's overall performance effectiveness is illustrated within the context of identifying specific availability requirements. Thus, in attempting to provide a general methodology for treating both hypothetical and existing availability requirements, the concept of an "availability state", in conjunction with the more conventional probability-time capability, is investigated. In this respect, emphasis is focused upon a balanced analytical and pragmatic treatment of operational availability within the system design process. For example, several applications of operational availability to typical aerospace systems are presented, encompassing the techniques of Monte Carlo Simulation, System Performance Availability Trade-Off Studies, Analytical Modeling of specific scenarios, as well as the determination of launch-on-Time probabilities. Finally, an extensive bibliography is provided to indicate further levels of depth and detail of the operational availability parameter.
Operational availability is a measure of the extent to which a system can be expected to be in a state or condition to perform its assigned function within an established time frame and under given environmental conditions. As such, a system's operational availability includes both a detailed description of the performance characteristics of individual system elements, as well as the specification of the overall system states. In this respect, the resultant complexity of large aerospace systems involves a broad set of requirements that express the multitude of mission objectives. This complexity brings with it the need for a quantitative means to measure the total effectiveness of a system, particularly where alternative approaches are to be considered. One fundamental approach to the measurement of total systems effectiveness (SE) is to formulate its effectiveness in terms of a figure of merit, such as:

$$SE = \sum P(A) \cdot P(C),$$

where,

- $P(A)$ = probability that the system will be operational during a specified time interval, or at a particular instant of time;
- $P(C)$ = probability of achieving the mission objectives, given the system is available and dependable.

The development of analytic models for total SE evaluation is treated in detail in Appendix A. The basic characteristics of an effectiveness model, along with the equations for specifying significant effectiveness functions are also given. The real value of the resultant SE index lies in the definition/design of systems where choices between alternatives are made, and the primary objective is to select that particular candidate having the greatest overall benefit. In such cases, indices of effectiveness aid the decision maker in terms of augmenting his experience and skill with a quantifiable measure of overall systems performance.
A. APPROACH AND DEFINITION

In this study, the specification and definition of operational availability is approached from two distinct points of view:

1) The contribution of operational availability to the determination of a total SE index or figure of merit within the overall systems context;

2) The quantification of the availability parameter in terms of specific performance characteristics, such as instantaneous availability, interval availability, steady-state availability, or a time-dependent probability.

Thus, in the real world, the availability parameter addresses the time usage of a system and deals with problems of failure of system elements (people, equipment, facilities, etc.), as well as with the areas of support requirements needed to correct and prevent failures. As such, the measure of availability is a determination of the degree to which a system can be expected to perform some particular function. It is a probabilistic-time capability of the system. The origin of these requirements lie in the prior mission analyses that are usually performed to quantify the overall objectives and that will ultimately drive the definition of a particular availability index. In the early concept phase, some of these factors are:

1) What is needed to satisfy the objectives?

2) What is achievable with the resources available?

3) What are the acceptable risks of success and failure?

The mission analyses in the concept phase examine these factors in detail for the major performance objectives and formulate specifications (initial) that will subsequently be used to select the "design to" availability parameters for the concept that best meets these total mission requirements. The degree of sophistication and complexity of such analyses can vary from very complex simulations of the mission scenarios to a cursory examination of only the principal factors. Much depends on the criticality of the particular problem being considered.

The scope of such studies depends on how much is known about the mission and types of systems that can accomplish it. It is obvious, for example, that a contemplated mission never before attempted, requiring systems never before produced, will be much more uncertain (in terms of a probability measure) than a modification to an
existing system for a previously performed mission. The former case is of greater significance, because it poses the problem of establishing availability requirements for new and untried systems. In attempting to provide a general methodology for treating both hypothetical and existing availability requirements, the concept of an "availability state" is introduced. This formulation of availability is entirely consistent with the probability-time capability previously discussed and further includes the dimension of status. The definition previously given for operational availability, i.e., a measure of the extent a system can be expected to be in a state or condition to perform its assigned function, is general; however, it does imply that a probability, a status, and a time capability make up the quantification of this parameter. Availability requirements can, therefore, be expressed in several ways--determined by the system and the specific manner in which it is used. The availability requirement may be expressed as:

1) A specific schedule time;
2) A time period or interval;
3) A random time within a time period;
4) A set of time frames, i.e., states;
5) An instantaneous time;
6) A steady-state condition;
7) A probability-time-state use;
8) Combinations of the above.

B. ANALYTIC DEVELOPMENT OF AVAILABILITY PARAMETERS

A complex system's usage may include one or more of these requirements distributed among several elements of the system and applying to various usage states (launch, payload delivery, orbit operation, recovery, etc). The probabilities associated with each state requirement represent the degree of assurance or probability of success desired. Thus, operational availability may be expressed as a probability of time requirement for any system state. The relationships are illustrated in Fig. 1, and bring together the concepts of probability-time-state to quantify the availability parameters.
In Fig. 1, \( A \) is the availability requirement, subscripts 1, 2, 3...indicate system elements, and superscripts a, b, c...indicate specific availability requirements for the system elements. Thus, the total operational availability for all states and all system elements may be written symbolically from Fig. 1 as:

\[
A_S = F \left\{ A_1^a, A_2^b, A_3^c; A_1^d, A_2^e, A_3^f; t (A) \right\},
\]

where \( t \) denotes the time dependency for each particular state.

One may, for the purpose of this illustrative example, consider \( X \) as the launch state, and \( Y \) as the orbit mission state. The elements of \( X \) are the launch vehicle, support equipment, launch site facilities, etc, while the elements of \( Y \) are the payload, experiment package, telemetry, data link, etc. Then, some typical availability specifications for these elements would be:

1) Launch on time probability (launch state);

2) Probability of payload availability (orbit state).

For each system element, its operational availability would be expressed symbolically as:

\[
A_1 = \sum A_1^a \cdot A_1^d \\
A_2 = \sum A_2^b \cdot A_2^e \\
A_3 = \sum A_3^c \cdot A_3^f
\]

System element level operational availability

For each system state, the operational availability is:

\[
A_{\text{Launch}} = \sum A_1^a \cdot A_2^b \cdot A_3^c \\
A_{\text{Orbit}} = \sum A_1^d \cdot A_2^c \cdot A_3^f
\]

System state level operational availability
The total system availability is given by:

\[ A_{\text{System}} = \sum A_{\text{Launch State}} \cdot A_{\text{Orbit State}} \]

where the availability of the orbit state is conditional on the launch state availabilities.

These symbolic equations illustrate the technique of dealing both with the system element and system state level of availabilities.

C. DETAIL BREAKDOWN OF AVAILABILITY

For each specific element and subelement, the measure of operational availability may be broken down still further as follows:

1) Instantaneous availability - The probability that the element will be available at any random time \( t \);

2) Interval availability - The proportion of time in an interval that the element is available for use;

3) Steady-state availability - The proportion of time that the element is available for use when the time interval considered is very large.

In the limit, these three measures of availability approach the steady-state availability. Which measure is most applicable depends on the element state and its conditions of use. For elements or subelements that are to be operated in continuous systems, e.g., a detection radar system, steady-state availability may be the satisfactory measure. For elements whose usage is defined by a duty cycle, e.g., a tracking radar system that is called on only after an object has been detected and is expected to track continuously during a given time period, interval availability may be the most satisfactory measure. Finally, for elements that are required to perform a function at any random time, e.g., a data processing system as part of a telemetry system that is to be employed to process orbital data and then remain idle for a length of time, instantaneous availability may be the most satisfactory measure. In general, the duty cycle of each element would determine the form of availability most appropriate as the measure of performance.
The expression for instantaneous availability can be determined by describing the failure and repair process in a system of linear differential and/or difference equations as with reliability, with the exception that one allows for repairs out of the failed state of the system. These equations would describe the transition from one operability state to another. The solution of the differential equations is the probability that the system is available at any random time $t$.

1. **Instantaneous Availability**

For a single equipment (single element), the measure of instantaneous availability is given by:

$$A(t) = \left( \frac{\mu}{\mu + \lambda} \right) + \left\{ \frac{\exp \left( -（\mu + \lambda)t \right)}{\mu + \lambda} \right\},$$

where

$\mu = \text{equipment repair rate; }$

$\lambda = \text{equipment failure rate; }$

$t = \text{a random time.}$

This result and the two- and three-equipment parallel and standby redundant system instantaneous availability equations are shown in Table 1. Both parallel and standby redundancy and single and multiple repair capability formulas are given for comparative purposes.

2. **Interval Availability**

The proportion of time that the element is available in an interval $t_1$ to $t_2$ can be computed by taking the average value of instantaneous availability over the time of usage.

$$A_m(t) = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} A(t)dt,$$

where

$t_1 = \text{start of mission time (usually } t_1 = 0 );$

$t_2 = \text{end of mission time}.$
<table>
<thead>
<tr>
<th>No. of Equipment</th>
<th>Type Redundancy</th>
<th>Repair</th>
<th>Instantaneous Availability Model</th>
<th>Definitions of Constants for Instantaneous Availability Model</th>
<th>Steady-State Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single</td>
<td></td>
<td>( A(t) = \frac{\mu}{\mu + \lambda} + \frac{1}{\mu + \lambda} e^{-(\mu + \lambda)t} )</td>
<td>( s_1 = -(1 + \mu) - \sqrt{\mu \lambda} ) ( s_2 = -(1 + \mu) + \sqrt{\mu \lambda} )</td>
<td>( \frac{\mu}{\mu + \lambda} ) 0.998</td>
</tr>
<tr>
<td>2</td>
<td>Single</td>
<td></td>
<td>( A(t) = \frac{\mu^2 + 2\mu \lambda}{\mu^2 + 2\mu \lambda + \lambda^2} - \frac{2\mu^2 (\mu^2 + \lambda^2)}{\mu^2 (\mu^2 + 2\mu \lambda + \lambda^2)} ) ( s_1 = -(\mu + \lambda) + \sqrt{\mu \lambda} ) ( s_2 = -(\mu + \lambda) - \sqrt{\mu \lambda} )</td>
<td>( \frac{\mu^2 + 2\mu \lambda}{\mu^2 + 2\mu \lambda + \lambda^2} ) 0.999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multiple</td>
<td></td>
<td>( A(t) = \frac{2\mu^2 + 2\mu \lambda}{2\mu^2 + 2\mu \lambda + \lambda^2} - \frac{2\mu^2 (\mu^2 + \lambda^2)}{2\mu^2 (2\mu^2 + 2\mu \lambda + \lambda^2)} ) ( s_1 = -(\mu + \lambda) + \sqrt{\mu \lambda} ) ( s_2 = -(\mu + \lambda) - \sqrt{\mu \lambda} )</td>
<td>( \frac{2\mu^2 + 2\mu \lambda}{2\mu^2 + 2\mu \lambda + \lambda^2} ) 0.999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parallel</td>
<td></td>
<td>( A(t) = \frac{\mu^2 + 2\mu \lambda}{\mu^2 + 2\mu \lambda + \lambda^2} - \frac{2\mu^2 (\mu^2 + \lambda^2)}{\mu^2 (\mu^2 + 2\mu \lambda + \lambda^2)} ) ( s_1 = -(\mu + \lambda) + \sqrt{\mu \lambda} ) ( s_2 = -(\mu + \lambda) - \sqrt{\mu \lambda} )</td>
<td>( \frac{\mu^2 + 2\mu \lambda}{\mu^2 + 2\mu \lambda + \lambda^2} ) 0.998</td>
<td></td>
</tr>
</tbody>
</table>

The availability \( A(t) \) of redundant systems is calculated using various models depending on the type of redundancy and repair. The constants \( s_1, s_2, s_3 \) correspond to the three roots of a specific polynomial, ensuring the steadiness of the system's availability. The steady-state availability is computed for different cases, with values ranging from 0.998 to 0.999, indicating high reliability and performance of the systems under consideration.
3. Steady-State Availability

The proportion of time that the element is available in an interval 
(0, t) as t becomes very large can be arrived at by taking the limit 
as t → ∞ of the above equation.

\[ A_{ss}(\infty) = \lim_{t \to \infty} \frac{1}{t} \left\{ \int_{0}^{t} A(t) dt \right\}. \]

Note that integrating the right-hand terms of the instantaneous 
availability equations of Table 1 over the interval (0, ∞) 
results in a solution leaving the first term on the right side of 
these equations. Thus, for example, the limiting expression for a 
single element (single equipment) system is:

\[ A_{ss}(\infty) = \frac{\mu}{(\mu + \lambda)}. \]

For most elements, the measure of steady-state availability is 
satisfactory. Note also that because the steady-state value is 
always less than the values of instantaneous and interval 
availability, the use of the former measure allows for conservation.

If one assumes that repairs cannot be made until complete element 
failure, the expression for a single element steady-state availability 
can be approximated by:

\[ A_{ss} = \frac{MTTF}{MTTF + MRT}, \]

where

\[ MRT = \text{mean repair time} = 1/\mu; \]

\[ MTTF \text{ is given by } 1/\lambda. \]

This rather familiar expression may be further expanded to 
illustrate in greater detail the pragmatic subelement considera-
tions. For example, the above formula may be reduced to:

\[ A_{ss} = \frac{\text{Total Up-Time for a Period}}{\text{Total Up-Time + Total Downtime for this Period}} = \frac{\text{Mean Up-Time for a Period}}{\text{Mean Up-Time + Mean Downtime for this period}}. \]

The above can be written:

\[ A_{ss} = \frac{MTBM}{MTBM + MDT} = \frac{MTBM}{MTBM + (MTT + MRT)}. \]
where

\* MTBM - mean time between maintenance actions involving both corrective and preventive maintenance;

\* MDT = mean downtime;

\* \( (Mct + Mpt) \) = mean of the time for the aggregate of corrective and preventive maintenance actions;

\* Mct = corrective maintenance downtime caused by failures and other causes such as accidents, human-induced maintenance, etc;

\* Mpt = preventive maintenance that is scheduled (usually includes daily, weekly, and monthly actions), which may include replacement of scheduled wearout parts, recycling, etc., depending on customer project rules.

**Note:** Operational administrative delay of maintenance is not considered.

The above expression can be reduced to:

\[ A_{ss} = \frac{Y_{BF}}{MTBF + Mct} \]

where

\* Mpt does not cause system downtime. This is the case when Mpt is eliminated or its need is restricted by design to off-duty periods such as during Mct or excused periods such as recycling or depot overhaul.

\* Mct considered is limited to maintenance correction of random hardware failures and does not include accidents, human induced failures, etc.

\[ Mct = (Mct_{Inherent} + Delay_{Supplier} + Delay_{Maintenance Administrative}) \]

where
MC Inherent = the mean of the controlled degree of maintainability designed into hardware. It is the responsibility of maintenance to define (by participation in system analysis and trade studies), specify, predict, review, assist designers, determine design corrections, and evaluate results in hardware for maintenance time and design characteristics that will enable economical maintenance on time to achieve A, assuming that the facilities, environment, manpower, skills, procedures, tools, test equipment, spares, and supplies, obligated by the maintenance concept are used.

Delay Supply = the mean of the lost time while a maintenance task is suspended awaiting spares or supplies. It starts on official supply demand by maintenance at the designated supply point. Such delay is a function of stock identification, planned stock layout, purchase and delivery, and receipt/storage/issue by launch sites. It is aggravated when sites do not follow the planned system.

Delay Maint Admin = mean of the lost time while a maintenance task is delayed awaiting the arrival of skills, tools, test equipment, and time out for personnel and official reasons. This can be aggravated when launch sites do not follow the plans identified for facilities, maintenance, environment, manpower, skills, procedures, tools, and test equipment.

D. AVAILABILITY SPECIFICATION

The detailed illustrative samples presented in the preceding sections furnish a general framework within which the specification of availability parameters may be carried out. In particular, these methods include modeling of the mission to provide the means for examining the given scenarios and contingencies for each mission state and apportioning the system availability parameters to the major elements and subelements. The techniques and procedures in this initial phase of activity are systems analyses, in which modeling and simulations based on estimates, similarities to previous systems, etc are developed. The definition/design phase proceeds with specific requirements for operational availability allocated to these major modules. The task faced in the definition/design phase is to establish system availability specifications.
1. Illustrative Example of Availability Specification

The above methods are illustrated in the following example, which specifies the availability requirements as they relate to an extended mission of a manned orbital vehicle. In particular, the analysis deals with the maintainability tradeoffs in terms of maximizing the total systems effectiveness and minimizing such cost factors as maintenance times and the weight and volume of spares. The methodology makes use of a computer simulation to determine:

1) Whether to use redundant parts, carry spare modules for easy replacement, make repairs in flight, or how best to combine these approaches;

2) Whether to use built-in automatic failure location devices or auxiliary test gear;

3) What the effect is of replacement times and the required weight and volume of spare modules on the equipment availability;

4) The applications for which commonality of parts can be used effectively;

5) The techniques that can be used to improve the maintainability per unit weight, volume, or other associated cost factors.

Almost every aspect of reliability/availability/maintainability design is a matter of compromise. If every conceivable requirement in the way of spare parts, tools, test equipment, and trained repairmen is foreseen and provided, the space vehicle would become a combination warehouse and factory and never get off the ground. Because of the long mission times and the large number of system parts involved, it is practically impossible to achieve acceptable dependability/availability through reliability alone, but it would be equally impossible to maintain an unreliable system. Furthermore, these factors must be considered from the standpoint of the value of the mission because, if dependability/availability were the primary mission goal, man would stay on the ground. The following discussion does not include all factors, but does indicate the scope of the problem.

Initially, there are the tradeoffs between reliability, maintainability and availability. The increase of reliability through the use of more reliable components, of parallel and standby redundancy, of higher standards of quality control, of more exacting requirements for assembly and checkout, etc., is limited by the law of diminishing returns, and, in the case of manned space mission with extended lifetimes, the value of maintainability is easily seen.
Next, there are the tradeoffs associated with "indenture" level or the size or amount of circuitry or equipment included in each replacement module. The lower the indenture level, the smaller the replacement modules and the less replacement of properly operating equipment; but locating the failure, gaining access to the failed part, and replacing it would be expected to take longer. The penalty, in terms of failure effect on the mission objectives, would depend on the importance of the objectives involved, the criticalness of their scheduling, etc. Also, the optimum indenture level would depend on whether the replaced modules are to be serviced. Using a higher indenture level has the advantages of shorter failure location and module replacement times, and, if the replaced module can be repaired, this would greatly reduce the required number of spares. There would, however, be a costly penalty associated with the test and repair equipment needed and providing a trained repairman for the crew.

Further tradeoffs are associated with the completeness of the failure location and test equipment, and the required crew training. Presumably, failure location equipment provided could be so complete that the module containing the failed part is indicated automatically without any further checking. However, this would involve a rather extensive system that would, itself, be subject to failure. For each type of mission and vehicle, there is some optimum level of completeness of the failure location equipment and the associated training of the crew members, etc.

To obtain the optimum compromise in these and other aspects, it is necessary to compare costs and the accomplishment of various mission objectives. This suggests the use of the concept of system cost effectiveness. Cost effectiveness is defined as the per unit cost of value received and system effectiveness as the probability of achieving mission objectives. If some commensurable value can be assigned to each mission objective, then it is more useful to define system effectiveness as the expected mission accomplishment in these value units and system cost effectiveness as the per unit cost of this expected accomplishment. This provides a single quantity for comparison and reduces the reliability, maintainability and availability, design problem to finding the approach that yields the maximum system cost effectiveness.
The approach is to use a computer program to simulate the performance of a set of mission objectives along with a history of the failure and maintenance actions performed on the various system elements based on failure and repair or module replacement times chosen randomly from specified probability distributions. The purpose of the simulation program, however, is not to show what can happen in one particular case, but to indicate the distribution of events along with their probabilities.

2. Application to a Sample Program

To gain a better understanding of some of the relations involved, the simulation program was used to study indenture level tradeoffs in a typical attitude control computer as it might be incorporated in the attitude control system of a manned orbital space station. This piece of equipment was selected because it is typical of spaceborne equipment and because reasonable estimates can be made of the various pertinent parameters such as the mean time to failure and the fault location and repair times. A mission time of 1000 hr was selected, during which time a set of idealized experiments were assigned a representative range of performance times, critical times, availability coefficients, etc. The importance of each branch to each mission is given in terms of a set of a priori objective coefficients.

Five representative mission objectives were selected. The aim was to have mission objectives with a range of performance times, allowed failure effects, critical downtimes, and availability coefficients. To prevent complication of the results, the availability coefficients were purposely kept low but were varied to have a basis for determining the effects and possible flexibility of this variation. The first objective has an objective performance time as long as the total mission. The equipment must operate for the entire time to have no failure effect (the allowed failure effect is zero), and a downtime longer than 1/2 hr would result in the failure of this objective (the critical downtime is 0.5). The second objective also has a performance time as long as the mission, but can be achieved in one-tenth of this time (the allowed failure effect is 0.9 of the performance time without taking away from the success of this objective). The critical downtime is also as long as the mission, which means that any operation of the equipment during the objective performance time will result in at least partial success in meeting the objective. The remaining objectives have performance times of 100, 10, and 1 hr, respectively, and a variety of allowed failure effects and critical downtimes to evaluate the effects of these quantities on the mission achievement. In this study, the mission achievement was either 1.000 or 0.999 for every run, and no further analysis of the failure effect was made.
Four different indenture levels or module arrangements (where modules are the parts to be replaced) were investigated with the attitude control system. It was assumed that each module was positioned with holddown screws and connected into the remainder of the system with connectors. The four indenture levels are:

1) Single-circuit modules (total, 35 modules);
2) Dual-circuit modules (total, 19 modules);
3) Multicircuit modules (total, 4 modules);
4) Complete system module (total, 1 module).

Twenty-five runs were made for each indenture level where a run is the simulation of a complete mission. Each run has a different sequence of random numbers so it represents a different possible mission history.

The mean time between failures (MTBF) for each circuit in the attitude control system was computed by summing the mean failure rates for each component. The mean times for maintenance operations were estimated for each indenture level on the basis of the module size and composition.

The computer simulation results for a typical indenture level is shown in Table 2. Listed for each run are the weight and volume of spares required, the combined weight of computer and spare modules, and the system cost effectiveness (computed using a weight cost factor of 10^4 to put the values in a convenient range, a volume cost factor of 0, and based on the combined weight of computer systems and spares). These runs are arranged in order of decreasing cost effectiveness.

The contribution of each run to the system cost effectiveness is given by the last column in Table 2. From these results, it is evident that, for this case, the optimum indenture level depends on the confidence required in meeting the mission objectives (although the relation between percentage of runs and the confidence of achieving a given system cost effectiveness is not discussed here).
<table>
<thead>
<tr>
<th>Run</th>
<th>Mission Achievement</th>
<th>Spares Weight, g</th>
<th>Spares Volume in. 3</th>
<th>Combined Weight, g</th>
<th>SCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.999</td>
<td>90.10</td>
<td>3.02</td>
<td>3394.00</td>
<td>2.943</td>
</tr>
<tr>
<td>21</td>
<td>0.999</td>
<td>270.20</td>
<td>9.04</td>
<td>3574.10</td>
<td>2.795</td>
</tr>
<tr>
<td>11</td>
<td>0.999</td>
<td>368.30</td>
<td>13.06</td>
<td>3672.20</td>
<td>2.720</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>458.40</td>
<td>15.08</td>
<td>3762.30</td>
<td>2.653</td>
</tr>
<tr>
<td>5</td>
<td>0.999</td>
<td>458.40</td>
<td>15.08</td>
<td>3762.30</td>
<td>2.655</td>
</tr>
<tr>
<td>18</td>
<td>0.999</td>
<td>639.40</td>
<td>21.10</td>
<td>3943.30</td>
<td>2.533</td>
</tr>
<tr>
<td>2</td>
<td>0.999</td>
<td>548.50</td>
<td>18.09</td>
<td>3852.40</td>
<td>2.593</td>
</tr>
<tr>
<td>1</td>
<td>0.999</td>
<td>639.40</td>
<td>21.10</td>
<td>3943.30</td>
<td>2.533</td>
</tr>
<tr>
<td>6</td>
<td>0.999</td>
<td>639.40</td>
<td>21.10</td>
<td>3943.30</td>
<td>2.533</td>
</tr>
<tr>
<td>8</td>
<td>0.999</td>
<td>639.40</td>
<td>21.10</td>
<td>3943.30</td>
<td>2.533</td>
</tr>
<tr>
<td>16</td>
<td>0.999</td>
<td>639.40</td>
<td>21.10</td>
<td>3943.30</td>
<td>2.533</td>
</tr>
<tr>
<td>7</td>
<td>0.999</td>
<td>729.50</td>
<td>24.12</td>
<td>4033.40</td>
<td>2.477</td>
</tr>
<tr>
<td>23</td>
<td>0.999</td>
<td>729.50</td>
<td>24.12</td>
<td>4033.40</td>
<td>2.477</td>
</tr>
<tr>
<td>14</td>
<td>0.999</td>
<td>290.70</td>
<td>30.15</td>
<td>4213.60</td>
<td>2.371</td>
</tr>
<tr>
<td>15</td>
<td>0.999</td>
<td>909.70</td>
<td>30.15</td>
<td>4213.60</td>
<td>2.371</td>
</tr>
<tr>
<td>3</td>
<td>0.999</td>
<td>1007.80</td>
<td>33.16</td>
<td>4311.70</td>
<td>2.317</td>
</tr>
<tr>
<td>12</td>
<td>0.999</td>
<td>1007.80</td>
<td>33.16</td>
<td>4311.70</td>
<td>2.317</td>
</tr>
<tr>
<td>17</td>
<td>0.999</td>
<td>1007.80</td>
<td>33.16</td>
<td>4311.70</td>
<td>2.317</td>
</tr>
<tr>
<td>19</td>
<td>0.999</td>
<td>1007.80</td>
<td>33.16</td>
<td>4311.70</td>
<td>2.317</td>
</tr>
<tr>
<td>10</td>
<td>0.999</td>
<td>1131.70</td>
<td>39.20</td>
<td>4435.60</td>
<td>2.252</td>
</tr>
<tr>
<td>13</td>
<td>0.999</td>
<td>1131.70</td>
<td>39.20</td>
<td>4435.60</td>
<td>2.252</td>
</tr>
<tr>
<td>25</td>
<td>0.999</td>
<td>1131.70</td>
<td>39.20</td>
<td>4435.60</td>
<td>2.252</td>
</tr>
<tr>
<td>22</td>
<td>0.999</td>
<td>1861.10</td>
<td>63.32</td>
<td>5165.00</td>
<td>1.934</td>
</tr>
<tr>
<td>9</td>
<td>0.999</td>
<td>2984.80</td>
<td>102.51</td>
<td>6288.70</td>
<td>1.588</td>
</tr>
</tbody>
</table>
The tradeoff between indenture level and downtime is not apparent because the allowed failure effects, critical downtimes, and the objective coefficients assigned to the various mission objectives were such that the mission achievement was equally high for all indenture levels. For more critical mission objectives, this would not be the case, and the additional time required to locate the failures and replace the failed modules at the lower indenture level would result in a significant decrease in the mission achievement; this would have to be weighed against the advantage of the small weight of spares.

Another factor contributing to the lower weight and volume of spares required for the lowest indenture level (A) is the high commonality. The 35 modules are of five types, which reduces the spares requirement considerably. For higher indenture levels, the ratio of the number of modules to the number of types is shown as follows:

1) Indenture Level A, 35 modules,
   5 types, commonality ratio = 7.0;

2) Indenture Level B, 19 modules,
   5 types, commonality ratio = 3.8;

3) Indenture Level C, 4 modules,
   2 types, commonality ratio = 1.0;

4) Indenture Level D, 1 module,
   1 type, commonality ratio = 1.0.

In the attitude control computer selected, this commonality occurs because many of the elements and branches perform similar tasks. In many other systems, the different elements would perform distinctive tasks, and commonality could be increased only at the expense of the efficiency or performance of the system.

3. Conclusion

The problem of determining the expected range of failure/maintenance/availability histories, spare parts and maintenance equipment requirements, and the resulting effects on the system cost effectiveness for manned space vehicles with extended mission times is ideally suited to the simulation approach. However, developing a simulation program is only part of the problem; an effective study of the maintainability design problem must also include an effort to determine the significant factors and how to incorporate these factors in the program to improve the program as a design tool to specify given requirements for maintainability, reliability, and availability.
In the following chapters, the analytical basis for the formulation of operational availability and examples of practical problems are discussed. Although the treatment of this subject is specifically aimed at the definition/design phase, the theoretical considerations and parameters are also present in the concept analyses areas.
I. SCOPE AND CRITERIA

The problem of specifying operational availability requirements, either at the system state, or at the system element level is a complex of analytical and design actions that involve a wide range of design variables and disciplines. As in the design of launch vehicle systems to achieve a given performance capability, the design for operational availability is a series of iterative actions in which sizing of system elements permits comparing what is obtainable versus what is needed. At each stage of design, choices exist between both design alternatives and the system performance parameters involved in meeting the mission requirements. The decision criteria must, therefore, be provided at each design maturity, and these criteria must be consistent with each level of detail. The respective design interaction, and the correlations existing among some of the availability parameters, provide both a rationale and a baseline for refining the design solution.

The subsequent implementation of a design protocol for the determination of system operational availability involves the delineation of those functional areas and associated analytical techniques that permit quantification of the availability parameters. However, within each given functional area, detailed analyses are required to determine, in depth, the respective parametric performance characteristics and their interactions on related subsystems. In this respect, the discipline of systems analysis furnishes the necessary methodologies to assess various alternatives as well as conduct optimization and suboptimization studies.

Design Approach - As in most design problems, sizing and optimization of design is an endeavor to strike a balance between several variables that are often conflicting. Sizing to achieve an operational availability is no exception. In a maintainable system, the balance is achieved between reliability of the system and the maintenance provisions. This balance is usually established on the basis of a criterion that describes the value of the system and program. The value criteria in an aerospace program will usually include the following factors:

1) Cost;
2) Development risk;
3) Weight;
4) Safety;
5) Size;
6) Schedule.

The implementation of a balanced design takes the form of a detailed examination of the requirements to achieve the objective (in this case, the degree and amount of operational availability) as well as the postulation of alternatives that will satisfy the program/system requirements. The closing on a compatible design configuration of operational and support elements is essentially a progressive process.

Figure 2 illustrates the interaction between the respective system design disciplines and the basic elements comprising each functional area. The analytical tools required, as well as the parameters generated at each point in the design process, yield an operational policy for implementing the design decisions in accordance with a set of criteria. The overall analysis uses simulation models and hypothetical scenarios for estimating the system performance characteristics. Subsequent verification of these system concepts provides a baseline from which further refinements are made. This iterative process converges rapidly in terms of satisfying the original requirements and objectives. Figure 3 is an example of some of the elements that enter into the maintainability/reliability design process. The resultant design solutions (limited by technology available) include an assessment of the critical or most sensitive design or performance parameters. The analytic formulation of some typical design problems is treated in the following chapters. The basic areas of redundancy, wearout, maintenance policies, failure and repair rates, as well as operational factors are considered. Both analytical approximations and simulation techniques are demonstrated as viable tools in the solution of large-scale system problems.
Fig. 2 -- System Design Disciplines, Criteria, and Functional Elements
Fig. 3 -- Maintainability/Reliability Support Parameters
III. ANALYTICAL DESCRIPTION OF OPERATIONAL AVAILABILITY

A full description of the availability function and the associated operational practice of a given system that can be maintained requires the specification of:

1) The equipment failure and repair processes;
2) The system configuration;
3) The repair and maintenance policy;
4) The state in which the system is to be defined as failed.

Within the past few years, much attention has been paid to an analytical technique known as Markov Processes. Mathematical models based on Markov Processes have found wide applicability in the fields of biology, engineering, physics, and the social sciences. In general, this technique uses the essential concept that the probability of obtaining a particular outcome on any trial (given a sequence of independent trials) depends only on the outcome of the directly preceding trial. This implies a knowledge of the conditional probability associated with every pair of outcomes. In addition, the space of all possible admissible states and how transitions are made over a sequence of trials must be adequately defined. Thus, for example, one may define the states of a piece of equipment as operating or failed, and then consider how transitions are made back and forth from each of the possible states. It is essentially this formulation that is applicable to the analytical description of availability. Furthermore, if the conditional transition probability is constant, the resultant Markov Process is stationary. In the discussion that follows, the development of an analytical model using the Markov Process is developed under these conditions.

To employ a Markov representation, it is assumed that the individual equipment fails in accordance with the negative exponential distribution, and the times-to-repair are also exponentially distributed.

If an item of equipment is designed so that those items that are expected to fail frequently have a relatively short repair time compared with those items that fail infrequently, an exponential distribution of repair times is observed. On the other hand, if every part in an item of equipment has the same failure rate, and each takes equally as long to repair, a rectangular or uniform distribution of repair times results. The induced
distribution of equipment repair times can be more easily controlled by the designer than can the distribution of times to failure. For example, if a few parts contribute the greatest percentage of equipment failures, the system designer can make sure that they are easily accessible for rapid replacement.

It is assumed that the equipment failure cumulative distribution is described by a negative exponential distribution, \( F(t) = 1 - e^{-\mu t} \), so that the conditional failure probability in the interval \( t, t + dt \) is \( \lambda dt \). It is also assumed that the major portion of failures can be repaired in a short time, while those items that fail infrequently take a long time to repair. Therefore, the equipment repair cumulative distribution is exponentially distributed, and given by \( G(t) = 1 - e^{-\mu dt} \). Also, the probability of completing a repair in the interval \( t, t + dt \), given that it was not completed at time \( t \), is \( \mu dt \).

In this formulation for maintained systems, it is necessary to develop the forward and backward differential equations that describe how transitions are made back and forth from state to state. If it is assumed that when an item of equipment fails it is immediately detected and repair is begun, and the times-to-failure and times-to-repair are each independently exponentially distributed, the resulting Markov Process is referred to as a "Birth and Death Process." The Birth and Death Process describes a system's availability in terms of transient and steady-state components. For systems that are to be operating continuously for a long time, the steady-state solutions are usually sufficient. The basic equations for this simple formulation are developed in Appendix B, as well as some significant variations, such as partial repair, the case of \( n \) equipments with \( r \) repair men \((r = n\) and \( r < n\)), as well as a two-equipment items redundant system. In addition, the problem of wearout, which involves a non-Markovian Process, is decomposed into a sequence of Markovian Processes by segmenting the input distribution into several exponential phases. This technique is also discussed in detail in Appendix B.

For systems that can be maintained, there are two figures of merit that are usually of interest. The extent to which a system can be expected to be in a state or condition to perform its assigned function within an established time frame and under given environmental conditions is referred to as the system's "availability." This approach has been treated in detail in the previous chapters. Still another figure of merit that is relevant to maintained systems is the "Mean Recurrence Time." This is the length of time to return to an acceptable state from a failed state. Sometimes this...
A figure of merit is called Mean Single Downtime. The importance of the Mean Recurrence Time is obvious, because availability is concerned with the total time the system spends in acceptable states and does not indicate how this time is distributed. For example, in a 10,000-hr period, the system may fail once and be down for a 10-hr duration, yielding an availability of 0.999. On the other hand, the system may fail 10 times in the same period and be down for 1 hr each time, also yielding an availability of 0.999.

Durations of single downtimes can also be related to a penalty cost that must be paid for each consecutive unit time the system is unavailable. An air traffic control system that establishes flight plans and directs the landing and takeoff of aircraft is an example. Long durations of single downtimes may mean queuing at takeoff, thereby delaying schedules, lost flights, and so on. In this sense, it would be preferable to have a higher system failure rate with shorter durations of single downtime incidences than a low system failure rate with long durations of single downtime incidences.

During the preliminary design phases, consideration is usually given to the expected repair time of all items in the system and which failures would require the system to be down. A good example is the case of bearing supports in a radar pedestal.

Although the failure rate of bearings may be relatively low, it may take several hours to replace one, and the radar set would have to be "off the air." Therefore, although the steady-state availability may not be significantly affected, the eventuality of a bearing failure may cause serious consequences---the radar would be unavailable for several hours. At the preliminary design phase, this effect must be considered and alternative approaches that would reduce the duration of single downtime without significantly increasing system cost evaluated. One approach is to use high-level redundancy and duplex the radar pedestal. However, this alternative is costly. Another approach is a lower level of redundancy where only the bearing gear supports are duplexed to permit repair of a single bearing failure without affecting system downtime.
It may be well at this point to distinguish between the concept of statistical expectation and the steady-state, or state of statistical equilibrium. The expected value is simply the average value of the availability function over all possible values of the variable. If we had a large number of equipment items that had been operating for some time, then at any particular time we would expect the number of equipment items that are in state 0 (available) to be NP. Thus, the ratio of the number of equipment items available to the total number of equipment items is simply NP/\(N = P_0\).

When we are concerned with steady-state solutions, we are postulating the existence of limits, i.e., the steady-state distribution will maintain itself ideally in an infinitely large ensemble. The Markovian formulation does not consider fluctuations in the individual items of equipment. For example, if a particular equipment item fails on the average every 100 hr of operation and takes 1 hr to repair, its steady-state availability is \(P_0 = 100/101\).

However, the range of the time of equipment failure is \((0, \infty)\). Thus, the steady-state availability \(100/101\) says nothing about the fluctuations of an individual equipment item's availabilities. It tells us that, in an infinitely large ensemble of equipment items, for each item that never fails, there is one that fails the instant it is "put-on." For a single-equipment item system (including some significant variations) it is not too difficult to develop the availability distribution and its moments. However, for complex configurations the analytic effort is intractable and Monte Carlo simulation techniques usually have to be employed. In this respect, it is essential to conduct a statistical analysis of the simulation results to obtain confidence limits and assess the effects of sampling. The question of how many samples to run is usually a compromise between computer costs involved and the accuracy of the results desired.
IV. ILLUSTRATIVE EXAMPLES OF OPERATIONAL AVAILABILITY

A. MINIMUM COST CRITERIA FOR A GIVEN AVAILABILITY

When operational availability is used to address a problem of meeting a time constraint, i.e., a specific schedule time, the analysis is concerned with the selection of operational modes in terms of time, spares, and expected failure rates. When availability is defined to mean the probability that the system will be in an acceptable condition in any given state, the analysis is concerned with either a particular solution (evaluation of initial conditions) or long-term (steady-state) solution of the basic availability equation. In this case, the sizing of MTBF and MTTF serve as goals or allocations from which system concept and definition design can be performed.

One example of the latter case is described in Appendix C. In this design problem, the steady-state, long-term availability of a system in a given mission state is of prime concern. This example describes a particular problem in which the given availability and minimum cost are the criteria for selection. In addition to the specification of the factors that contribute to MTBF and MTTF, this example problem also shows that the sensitivity of figures of merit require testing to verify that the indicated results are valid. In this case, a simple joint probability computation verifies the selection.

The analytical development of this technique for a uniform distribution is given in Appendix D. The results are quite straightforward, and the derivation may be applied to various other overlapping probability densities.

B. PROBABILITY OF LAUNCH ON TIME

The launch-on-time concept of availability can be addressed in a manner similar to the example given in Appendix C. The decision as to how to achieve a given level of probability of readiness is concerned with apportioning between MTBF and MTTF and then allocating these values to the technical parameters of the system.
There is, however, yet another approach to solving this same
generic type of problem. This is illustrated in Appendix E.
This example is an operational analysis aimed at selecting
mission state duration, as well as the generation of a concept
to meet a given probability of two successful launches. In this
case, the MTBF of the system is assumed fixed. This is a
reasonable condition because the vehicle design leads the ground
system design in the development process. This analysis has as
its objective to establish the launch operations strategy and the
reliability and logistics requirements that yield an acceptable
operational availability. The exact nature of this analysis
is described below:

It is assumed that the system concept is made up of a launch
vehicle and a mission payload module. The problem is then to
launch two payloads within a specified launch period. The
operational analysis provides a solution to establish the launch
operations strategy and the reliability and support requirements
that best meet the availability objectives. The variables in
this analysis are:

1) The probability of holds during operations on the launch pad
due to,
   a) Launch vehicle malfunction,
   b) Spacecraft malfunctions,
   c) All other causes;

2) Recycle of the launch vehicle in the event of a malfunction
   on the launch pad;

3) Recycle of the spacecraft in the event of a malfunction on
   the launch pad;

4) Spares provisioning and replacement time for,
   a) Launch vehicle,
   b) Spacecraft;

5) The duration of the launch period;

6) Turnaround time between launch attempts.
Dealing at this level of design, major system decisions are made such as:

1) What is the optimum time for launch operations as a function of reliability of the major system elements?

2) What are the system element spares that yield the greatest probability of mission success?

To illustrate this operational analysis, a hypothetical case is developed in which it is desired to perform two launches of a vehicle and its payload. In this example, the analyses resulted in:

1) Estimates of reliability for the system elements (launch vehicle and payload);

2) Recycle time for the system elements in the event of malfunction;

3) Limitation on the number of malfunctions.

Furthermore, previously determined mission analysis studies have selected three operational concepts for analysis:

1) Candidate 1,
   a) 22-day launch period,
   b) 10-day turnaround (launch-to-launch),
   c) Spare launch vehicle and payload;

2) Candidate 2,
   a) 30-day launch period,
   b) 16-day turnaround,  
   c) No launch vehicle with spare payload;

3) Candidate 3,
   a) 30-day launch period,
   b) 10-day turnaround (launch-to-launch),
   c) No launch vehicle with spare load.
Table 3 shows the initial conditions that apply to these three cases.

Table 3 Conditions and Assumptions

<table>
<thead>
<tr>
<th>Description</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Probability of a malfunction or delay during any one launch attempt</td>
<td></td>
</tr>
<tr>
<td>attempt (can reoccur):</td>
<td></td>
</tr>
<tr>
<td>Launch vehicle</td>
<td>0.05</td>
</tr>
<tr>
<td>Payload</td>
<td>0.05</td>
</tr>
<tr>
<td>All other causes</td>
<td>0.05</td>
</tr>
<tr>
<td>2. Recycle Times (malfunction-to-launch):</td>
<td></td>
</tr>
<tr>
<td>Payload recycle</td>
<td>19 days</td>
</tr>
<tr>
<td>Launch vehicle recycle</td>
<td>14 days</td>
</tr>
<tr>
<td>Replace integrated launch vehicle and payload</td>
<td>5 days</td>
</tr>
<tr>
<td>Remove payload from launch vehicle and replace</td>
<td>9 days</td>
</tr>
<tr>
<td>All other causes</td>
<td>3 days</td>
</tr>
<tr>
<td>3. The payload is limited to a total of 2 recycles.</td>
<td></td>
</tr>
<tr>
<td>4. The probability of the payload requiring recycle before</td>
<td></td>
</tr>
<tr>
<td>launch pad operations = 0.05 (can reoccur). This applies to both</td>
<td></td>
</tr>
<tr>
<td>prelaunch and recycle operations.</td>
<td></td>
</tr>
<tr>
<td>5. The probability of the launch vehicle sustaining a malfunction</td>
<td></td>
</tr>
<tr>
<td>during recycle that would preclude launching on time = 0.05.</td>
<td></td>
</tr>
<tr>
<td>6. The program requirement for probability of two launches</td>
<td></td>
</tr>
<tr>
<td>within a specified time period is 0.015.</td>
<td></td>
</tr>
</tbody>
</table>

29
The detailed analysis of these three candidates is included in Appendix E. For this sample problem, the analytical approach is relatively simple and straightforward. It would have required more complex methods and techniques had the numbers of variables entering into the decision been larger.

Table 4 compares the three approaches in terms of percentage of launch-on-time risk. The second candidate differs from the first in that the launch period was longer (22 to 30 days) and the turnaround time was longer (10 to 16 days). These changes essentially counteracted each other. The significant difference is that a launch vehicle and payload were considered in Candidate 1. Deletion of the spare set increased the launch vehicle risk from 0.30 to 5.46%.

The third candidate differs from the second in that the turnaround time has been reduced from 16 to 10 days. This reduces the total risk from 5.98 to 0.85%. Most of the risk in No. 3 is due to the launch vehicle, because no spare is provided. This candidate compares favorably with No. 1, although it is not quite as good.

Table 5 compares the approaches in terms of the number of malfunctions that can be tolerated on the launch pad during the launch period. The No. 2 approach is very restrictive; only one malfunction could occur in the launch vehicle and that had to occur in the first vehicle. The third candidate can tolerate more malfunctions than No. 1, depending on where they occur; however, there is an increased risk of a malfunction occurring during launch vehicle recycle. The first approach allows for one launch vehicle recycle, while No. 3 allows a maximum of three. The more recycles, the higher the risk.

The following conclusions were drawn from this study:

1) The launch-on-time risk for No. 3 is substantially less than for No. 2 and compares favorably with No. 1. This was achieved by reducing the turnaround time from 16 to 10 days;

2) Candidate No. 1 meets the allocated launch-on-time risk of 1.5%;

3) Most of the risk is caused by the launch vehicle because a spare is not available;

4) The launch-on-time risk cannot be reduced by further reduction of the turnaround time or extension of the launch period;
Table 4 Comparison of Launch-on-Time Risks

<table>
<thead>
<tr>
<th></th>
<th>Launch-on-Time Risk, %</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Candidate 1</td>
<td>Candidate 2</td>
<td>Candidate 3</td>
</tr>
<tr>
<td>Payload</td>
<td>0.32</td>
<td>0.54</td>
<td>0.09</td>
</tr>
<tr>
<td>Launch Vehicle</td>
<td>0.30</td>
<td>5.46</td>
<td>0.76</td>
</tr>
<tr>
<td>All Other Causes</td>
<td>0.05</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Total Risk, %</td>
<td>0.66</td>
<td>5.98</td>
<td>0.85</td>
</tr>
</tbody>
</table>

No. 1 - 22-day launch period
10-day turnaround
Spare launch vehicle and payload

No. 2 - 30-day launch period
16-day turnaround
No spare launch vehicle
With spare payload

No. 3 - 30-day launch period
10-day turnaround
No spare launch vehicle
With spare payload
Table 5 Maximum Number of Malfunctions That Can Be Tolerated (Two Launches)

<table>
<thead>
<tr>
<th>Malfunction</th>
<th>Candidate 1</th>
<th>Candidate 2</th>
<th>Candidate 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload</td>
<td>2</td>
<td>2*</td>
<td>2 or 3</td>
</tr>
<tr>
<td>Launch Vehicle</td>
<td>2</td>
<td>1</td>
<td>2 or 3</td>
</tr>
<tr>
<td>All Other Causes</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

*Both malfunctions must occur in the first vehicle; otherwise, we can tolerate one payload malfunction in Vehicle 1 or one payload malfunction in Vehicle 2.

Must occur in Vehicle 1. We cannot tolerate any launch vehicle malfunctions in Vehicle 2.
5) An increase in turnaround time or a decrease in the launch period would result in a significant increase in the launch-on-time risk;

6) The launch-on-time risk could be significantly reduced by providing a standby launch vehicle and payload. This would also provide flexibility to changes in turnaround time and launch period. However, a spare launch vehicle and payload is not warranted by this analysis. A more sophisticated analysis would have to be conducted to determine if such a selection should be made;

7) On the basis of this analysis, Candidate 1 is selected because it provides the most margin with respect to the system risk requirement.
C. MONTE CARLO AVAILABILITY SIMULATION

The Monte Carlo method may briefly be described as the device of studying a real-world phenomena by means of a simulated mathematical process, using random sampling to structure the solution and establish statistical inferences. The device is certainly not new. Moreover, the theory of mathematical simulation has been a subject of study for quite some time, and the novelty of the Monte Carlo method does not lie here. The novelty lies rather in the suggestion that where complex situations exist that demand numerical solutions not readily obtainable by standard analytical methods, there may exist a mathematical simulation process with statistical distributions or parameters that adequately approximate the real-world circumstances. In these instances it may actually be more efficient and economical to construct such a process and compute the statistics, rather than attempt to use deterministic methods to describe the system.

Historically, the original von Neumann-Ulam concept seems to have been that Monte Carlo specifically designated the use of random sampling procedures for treating complex deterministic mathematical problems not easily amenable to numerical solution. However, some define Monte Carlo to be the exclusive use of random sampling procedures to treat problems, whether of a deterministic or probabilistic sort. Others demand that the sampling be sophisticated (involve the use of some variance reducing technique) to qualify as Monte Carlo; they reserve the names straightforward sampling, experimental sampling, or model sampling for the cases where purely random sampling is used. However, the economics of computing changes so rapidly with the advent of faster and faster machines that Monte Carlo methods are being successfully applied to more and more sophisticated and complex problems. The situation now is that in common usage, Monte Carlo is synonymous with any use of random sampling in treatment of either deterministic or probabilistic problems.

The application of the Monte Carlo simulation technique to operational availability is given in Appendix F. This model computes the system availability of a launch vehicle, which involves a sequence of tests, and storage of specified system and subsystems. Figure 4 illustrates in schematic form the launch vehicle storage and test cycles.
Figure 4 -- Simplified Transition Model of Storage and Test Cycles
The development of the storage and test cycles is illustrated in Appendix F, along with the simulation results. It may be concluded that:

1) The most important factor affecting the availability of the launch vehicle is failure rate of parts;

2) Availability after each storage and test cycle drops off continuously;

3) Probability of detecting a failure is relatively (within normal ranges) insensitive to availability;

4) Length of launch vehicle storage (for example, 12 months or 36 months) will cause availability to fall off sharply.

In running this Monte Carlo simulation, the basic data were accumulated over a 36-month period, using Markov transition matrices for storage and test operations. Thus, the sample size was large, so that statistical variations were minimized.

The illustrative examples presented focus attention on the various applications and different types of problems encountered in determining operational availability. The analytical concepts developed provide a framework for structuring a broad class of problems, and furnish insight into the subtle mechanisms of obtaining real-world solutions that are useful to the engineer and the designer. The subsequent interpretation and integration of these results into a comprehensive methodological design tool is a continuing process, combining both the skill and analytical ability of the design team. Note that a total system approach involves a complete effectiveness analysis of the essential parameters of operational availability in terms of the performance characteristics of the overall system. Thus, although operational availability has been treated as a separate topic, its effective impact on related subsystems and other functional requirements must also be examined. The end result is to yield a design concept that is analytically sound, economically feasible, and capable of being implemented in a real-world application.

In summary then, operational availability may be approached from several apparently different and divergent viewpoints. However, it should be observed that this basic parameter must be analyzed both on a systems and individual basis to achieve an integrated and balanced design.
APPENDIX A - CONSIDERATIONS FOR SYSTEMS PERFORMANCE EFFECTIVENESS MODELS.

A. ANALYTIC MODELS FOR EFFECTIVENESS EVALUATION

Any meaningful application of the systems performance effectiveness concept to a particular project requires a quantitative methodology to evaluate the effectiveness of a proposed or actual system in terms of selected measures, requirements, and decision criteria. Until this is done, the concept of systems performance effectiveness for a project has little use—except perhaps as a rallying point for arguments about the advantages of System X over System Y. The need for a systems performance effectiveness evaluation methodology begins at the inception of the system life cycle and continues through the succeeding design, development, production, test, and operational phases. Despite the obvious differences in the depth of the analysis applicable to these phases, the need for a quantitative methodology applies throughout.

Evaluation methodologies for systems performance effectiveness characteristics can be broadly characterized in terms of two approaches—the empirical and the analytic (Ref 1).

An empirical methodology is one devoted to data collection and evaluation of existing systems. Thus, it is possible to evaluate systems performance effectiveness by means of performance observations of systems in the field. While this approach is undoubtedly the most accurate, it is feasible only for systems or projects that are very far advanced in their life cycles.

An analytic methodology, on the other hand, is one that derives its results by inference, and uses a set of assumptions and procedures as a framework to compute an effectiveness description of the system in question. This descriptive system framework is called an analytic model, and the description of System X in these terms is called the analytic model of System X.

Purely empirical or purely analytic methodologies are, of course, not very useful. The former yields highly authoritative data too late to be useful, while the latter yields answers unsupported by facts. In practice, a balance is sought. This balance will normally change during the life cycle of a system. As
data about the behavior of the system become more available, the analytic model gradually merges into an empirical model; as the data become more available and as confidence in their value increases, statistical sample data supplant the assumptions.

Analytic models, moreover, usually remain useful even with regard to the empirical data obtained from system samples taken during acceptance tests. Also, these data often require interpretation simply because it is impractical to conduct tests that are sufficiently elaborate to yield statistically significant effectiveness data directly.

The need for analytic models to predict systems performance effectiveness thus emerges from the need to evaluate the effectiveness before the system has been in use for many years. The following discussion considers the analytic models—with the understanding that empirical methods will always be required to provide inputs for the analysis.

B. CHARACTERISTICS OF AN EFFECTIVENESS MODEL (REF 2)

There are certain general characteristics that any mathematical model should have to be a useful tool to predict effectiveness. These are:

1) Independence from design assumptions - If the concept of effectiveness is to be applied as a technical management tool, there is a demand that the effectiveness-analysis technique, and consequently the analytic model, be capable of evaluating alternative (or modified) system designs with respect to a fixed set of mission models and variables. To whatever extent the analytic model presupposes system design configurations or characteristics, the model is not able to evaluate alternative designs that are not within these constraints, and hence it may not provide a basis for comparison or optimization. For example, if the analytic model is built in terms of a given system-design configuration, other design configurations may be inequitably treated if subjected to the same analysis.
2) Usefulness throughout the system life cycle - The analytic model should be one that can be used throughout the system life cycle. In the early stages of the cycle, relatively few data are available on the statistical or performance capability of the system, and a substantial number of assumptions must be made to permit the analysis. As the life cycle progresses through design, development, test, and implementation, additional design and sample test data ordinarily become available. The analytic model, therefore, should be designed to accommodate these changes in inputs, and yield successive systems performance effectiveness predictions throughout the life cycle, with increased confidence in the results.

3) Realism in the analytic assumptions - The physical and mathematical assumptions on which the model is founded must be realistic with respect to the expected characteristics of the mission and system operations. There is a great temptation to construct analytic models based more on mathematical elegance than on realism.

4) Tractability of the evaluation - For the model to be usable, it must give numerical answers when exercised. This implies the model must be quantitative even in the face of limited data and it must be amenable to computation. Clearly, this model characteristic must be traded off against the characteristic of realism. The art of modeling consists, in large measure, of establishing this balance.

C. SELECTING AN EFFECTIVENESS MODEL

There appear to be three fundamental classes of considerations that enter into the selection of an appropriate effectiveness model— the outputs required for system management and optimization, the nature of the systems to be analyzed, and the mission characteristics to be employed.

1. Output Considerations

The definitions of the variables, requirements, and decision criteria influence the selection of an appropriate effectiveness model. The following questions are typical of those that must be answered:
1) Can the system-oriented performance variables be identified with specific hardware, or are they more closely tied to overall system behavior, including software?

2) Was an iterative methodology employed to establish the requirements and decision criteria? (The requirements on the model themselves could change during the iterative procedure, and these changes must be incorporated.)

3) Is there one principal performance variable that corresponds to one principal system function, or is the system called upon to do many things?

4) Will the utility and tradeoff data permit the results of effectiveness analysis to be expressed in terms of discrete quantities, or will probability distributions be required to describe systems performance effectiveness adequately?

5) Are the variables binary (success/failure) or multivalued?

2. System Considerations

System considerations concerning the choice of an effectiveness model have their greatest effect on the statistical and logical assumptions that underlie the model. In a given system, it may be uniquely possible to identify subsystems with their corresponding functions, and in such a case the effectiveness evaluation is simplified. On the other hand, if interaction of subsystem functions is expected, particularly with degraded modes of operation, the model must incorporate this flexibility of interaction. Additionally, the analytic model often incorporates assumptions concerning the statistical behavior of the system. These assumptions may be valid for the system in question, and they may be consistent with the available data. Finally, the scope and complexity of the system must be considered. The delicate balance between tractability and realism discussed above must be resolved in terms of anticipated system size (size being expressed in such terms as the number of components).

3. Mission Considerations

In addition to mission effects, described in Subsection 1, a series of representative mission profiles also must be examined as part of the model-selection process.
The results of the studies discussed elsewhere in this report are closely involved in this examination. For example, is the system operating in a steady-state environment, or are the missions short compared with other statistical time parameters? In the former case, an equilibrium or steady-state model may be employed. In the latter case, a mission-sensitive model in whole or in part is required.

Again, is the mission function carried out over a time segment, or is a point mission involved? Are there one or several critical mission segments? Do the requirements and decision criteria for systems performance effectiveness change with mission mode? Do the reliability and maintainability characteristics of the subsystems change as a function of a mission segment?

a. Mission Analysis (Ref 3) - Before analyzing a well-defined system, let alone developing a new one, it is necessary to know what the system is supposed to do, i.e., what missions it must perform. It is relatively easy to establish performance envelopes for various subsystems that, in turn, enable the system to perform one task in one environment. The problem, however, becomes much more difficult if one must consider many tasks under many environments.

One scheme often used is to develop figures of merit. This scheme weights the effectiveness figure for each task by the frequency with which the system may be called upon to perform each task. However, systems are developed to satisfy specific mission requirements that have been formulated on the basis of specific tools. The best system selected in accordance with such a scheme may not, therefore, be capable of responding to a specific parameter value that of itself may be extremely serious, but also may occur very often. Thus, a system may not be satisfactory from the standpoint of a specific subtask.

It is possible, however, to alter the method by weighting the parameter by their severity and expected frequency of occurrence, and thus design the system for some other weighted average of values. However, this does not solve the problem either, because optimizing an answer to any average value does not optimize the answer to each particular value. Perhaps the solution is to develop a procedure to optimize the effectiveness of answering a suitable, chosen, average that is subject to the achievement of a given effectiveness figure against each specific value.
On one hand, it makes little sense to speak about the average effectiveness, or the average environment. On the other hand, one cannot optimize with respect to a single mission unless the system has only one task to perform. Such a procedure, therefore, leads to all the difficulties associated with suboptimization, which results when one tries to optimize a system by optimizing each subsystem.

b. Figure of Merit. Perhaps the difficulty associated with the formulation of an appropriate figure of merit can be best explained by a discussion of the analogous problem encountered with the parameters of a frequency distribution. For example, with a distribution a population can be characterized by its mean; but this does not indicate the variability about this mean. A measure of variability, called the standard deviation can also be added to this characterization.

Yet there will be other properties of the population that are not pictured by any of these characterizations, e.g., the lack of symmetry. If the 10th and 90th percentiles are also given, however, a more complete picture of the population begins to take shape. Nevertheless, no finite set of parameters can ever completely describe a real population or its frequency distribution.

Similarly, if one wants to know the system-effectiveness figures in all situations, something analogous to the frequency-distribution example, it can never give all the information about the system.

c. Degraded Performance. At the component level, degradation can be thought of as measured by the number of failed components. At the system level, however, degraded performance may refer to the probability of performing the mission. For example, the system might be designed to be capable of performing its mission 95% of the time in one environment. In another environment, however, it may have only a 90% capability, and this may be considered degraded performance.

On the other hand, consider a radar-weapons system. Its design performance consists of being able to pick up an object at a certain distance, and then being able to assign appropriate weapons once the object has been correctly classified. A probability is associated with each subsystem performance and, hence, also with the system's performance. If the probability associated with any subsystem is reduced, however, the probability of system performance will also be reduced; this, too, can be considered degraded performance.
Models of degraded performance can also be modified so that they are time-variant. There are, moreover, many other possible definitions of degraded performance. The one to use, of course, is the one that is useful in assessing a system. Much current work is misunderstood because one group does not use the same concept of system degradation as another.

d. Human Factors - Concern for human factors falls into three major areas—life support, personnel and training, and human engineering. It is the goal of life support to maintain and protect the human by controlling his environment; the goal of personnel and training is to select, train, and assign the human for operational tasks; and human engineering provides the design engineer with the basis for the most effective use of the human component of the system. In short, the discipline of human factors in systems performance effectiveness requires that the man module be considered just as a hardware component—to be evaluated for cost, reliability, maintainability, availability, and operability. In addition, the man module must be considered for trainability.

To achieve these goals in a disciplined fashion, certain procedures must be followed. These procedures are not one-time events to be accomplished early in the development phase, but rather are iterative procedures that must be reviewed and changed where necessary just as design is reviewed and changed during an equipment's development.

D. ANALYTIC FRAMEWORKS FOR EFFECTIVENESS MODELS

The development of a model that satisfied the conditions cited in the preceding section is at best a complicated task. However, even under the assumption that such a model is realizable, its range of application without some types of modification would be restricted. This is due to the nature of the requirements, diverse operating conditions, and use factors that generally are an integral part of the specific system. To help solve this problem, an approach that uses a system effectiveness framework has been developed. In this approach, the basic system performance effectiveness elements remain constant for different system missions and use functions, although the more detailed factors underlying the basic elements are subject to change, depending on the particular problem analyzed.
Several systems performance effectiveness models have been developed. Table 1 lists equations for the more general models used. All these equations concern systems performance effectiveness, but each approaches the subject in a different manner, reflecting the needs of the individual user.

The following general equations are derived (Ref 4):

$$E_S = E_A \approx E,$$

$$f(P_A, A, U) = f(P_C, P_T) \approx f(A, D, C).$$

Note that properly constructed models of the same system carrying out the same mission will give the same evaluation and may even be mathematically identical. The basic equations given in Table 1 can only be used for simple systems in simple missions, even if the equations are assumed to be in matrix form.

Several systems performance effectiveness models are given below. Emphasis has been placed on describing the framework of each model rather than on providing a detailed description.

1. The Effectiveness Model (Ref 5)

The first term, performance (P), in the effectiveness model (PAU) can be expressed within several frames of reference. In the single-mission system, the expression is derived from a variety of measurements, e.g., area destroyed, tons of cargo or number of passengers delivered, emitters located and identified. Two important conditions apply: (1) the measurement standard used must be applicable to the parameter used to determine the performance level, and (2) the answer derived from exercising the expression must be used with caution because, with other than extremely simple systems, the achieved performance capability is almost always less than the theoretical performance capability. This circumstance occurs because the design-optimization process requires that some tradeoffs be made to achieve optimization of the overall system. As a result, even for the relatively simple, single-mission system, P is expressed as an index representing the ratio of the achieved performance level to the theoretical desired level. In essence, it is a figure of merit even under the assumption of absolute availability and absolute use.
<table>
<thead>
<tr>
<th>TITLE</th>
<th>EQUATION</th>
<th>TERM</th>
<th>EXPLANATION OF TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systems Performance Effectiveness</td>
<td>( E_S = (PAU) )</td>
<td>( E_S )</td>
<td>Index of Systems Performance Effectiveness</td>
</tr>
<tr>
<td></td>
<td>( E_S = f(P,A,U) )</td>
<td>( P )</td>
<td>Index of System Performance - a numerical index expressing system capability assuring a hypothetical 100% availability and utilization of performance capability in actual operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( A )</td>
<td>Index of System Availability - a numerical index of extent to which a system is ready and capable of fully performing its assigned mission(s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( U )</td>
<td>Index of System Utilization - a numerical index of the extent to which the performance capability of the system is utilized during the mission</td>
</tr>
<tr>
<td>Analytic Systems Performance Effectiveness</td>
<td>( E_A = (P_P_C_T) )</td>
<td>( E_A )</td>
<td>Systems Performance Effectiveness</td>
</tr>
<tr>
<td></td>
<td>( E_A = f(P_P_C_T) )</td>
<td>( P_C )</td>
<td>Measure of Performance Capability - a measure of adequacy of design and system degradation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_T )</td>
<td>Measure of Detailed Time Dependency - a measure of availability with a given utilization</td>
</tr>
<tr>
<td>TITLE</td>
<td>EQUATION</td>
<td>TERM</td>
<td>EXPLANATION OF TERMS</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------------------------</td>
<td>------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>System Effectiveness</td>
<td>( E = (ADC) )</td>
<td>( E )</td>
<td>Quantitative measure of systems performance effectiveness</td>
</tr>
<tr>
<td></td>
<td>( E = f(A,D,C) )</td>
<td>( A )</td>
<td>Measure of Availability - a measure of the condition of a system at the start of a mission when the mission is called for at unknown (random) point in time</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D )</td>
<td>Measure of Dependability - a measure of the system condition during the performance of the mission given its condition (availability) at the start of the mission</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C )</td>
<td>Measure of Capability - a measure of the results of the mission given the condition of the system during the mission (dependability)</td>
</tr>
<tr>
<td>Cost Effectiveness</td>
<td>( E_C = \frac{ES}{C_a + C_o} )</td>
<td>( E_C )</td>
<td>Index of Cost Effectiveness</td>
</tr>
<tr>
<td></td>
<td>( = \frac{f(P,A,U)}{C_a + C_o} )</td>
<td>( C_a )</td>
<td>Cost of Acquisition - the aggregated costs of acquiring the system, including prorated development costs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C_o )</td>
<td>Cost of Ownership - the aggregated costs of operating and maintaining the system</td>
</tr>
<tr>
<td>TITLE</td>
<td>EQUATION</td>
<td>TERM</td>
<td>EXPLANATION OF TERMS</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------------------------------------------------------</td>
<td>-------</td>
<td>--------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Defense Effectiveness</td>
<td>$E_d = \frac{E}{E_t}$</td>
<td>$E_d$</td>
<td>Index of Defense Effectiveness</td>
</tr>
<tr>
<td></td>
<td>$E_c = \frac{W}{E_t(C + C)}$</td>
<td>$W$</td>
<td>Index of Military Worth</td>
</tr>
<tr>
<td></td>
<td>$z_t$</td>
<td>$z_t$</td>
<td>Index of Time Effectiveness</td>
</tr>
<tr>
<td>Cost of Acquisition</td>
<td>$C_a = f(C_{at}, C_{am}, C_{af}, C_{as})$</td>
<td>$C_{at}$</td>
<td>Acquisition Time Costs - the calendar time required to acquire the system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_{am}$</td>
<td>Acquisition Manpower Costs - the manpower and skill levels required to acquire the system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_{af}$</td>
<td>Acquisition Financial Costs (Dollar) - the dollar outlays required to acquire the system, including the dollar costs associated with manpower and its acquisition and supporting facilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_{as}$</td>
<td>Acquisition Supporting Facilities Costs - the penalty cost to other systems through use of support facilities by the system during acquisition of the system</td>
</tr>
<tr>
<td>TITLE</td>
<td>EQUATION</td>
<td>TERM</td>
<td>EXPLANATION OF TERMS</td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------</td>
<td>------</td>
<td>-------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Cost of Ownership</td>
<td>( C_o = f(C_{om}, C_{of}, C_{os}) )</td>
<td>( C_{om} )</td>
<td>Ownership Manpower Costs - the manpower and skill levels required to operate and maintain the system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C_{of} )</td>
<td>Ownership Financial Costs (Dollar) - the dollar outlays required to operate and maintain the system, including the dollar costs associated with manpower and its acquisition and supporting facilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C_{os} )</td>
<td>Ownership Supporting Forces and Facilities Costs - the penalty costs to other systems through requirement for and use of supporting facilities and forces in the operation and maintenance of the system</td>
</tr>
</tbody>
</table>
In the case of the multimission system, consideration must also be given to the interaction of two judgments. The first comprises the assignment of weighting (importance) factors to the several mission modes of the system in such a way that their sum is 1.0. The second comprises the determination of the fraction of the system's total mission time that will be devoted to each of the several mission modes.

It is in the multimission system that the compelling reason for using indexes becomes most apparent. Many such systems have completely disparate standards for measuring the performance of their various mission modes. A comparison or aggregation of performance indexes that use different measurement standards cannot be attempted validly. For example, tons of cargo delivered, area destroyed, personnel transported, and enemy radar sites located and identified cannot logically be compared or aggregated.

The second term, availability (A), is more complex than the first. Overall availability is relatively easy to measure, but separating the overall value into factors of reliability, maintainability, operability, and supportability remains a difficult task—particularly in regard to prediction of the effect of up-time or downtime.

The third system's performance effectiveness term, utilization (U), accounts for factors that are introduced by the tactical, functional, logistical, and environmental use of the system; all four are a function of the operational doctrine of the system.

Utilization factors represent the degradation in system performance caused by mission conditions. The following are some examples:
1) Loss of accuracy;
2) Increase in part failure rate due to high ambient temperature;
3) Reduction in repair-part availability due to remoteness of location from supply depot;
4) Infrequent use of search radar for security reasons.

The utilization factors, except for analytic exercise of the model, are relatively constant. However, the assigned values will change whenever operational goals and criteria are modified in the process of achieving consonance between them and technical goals and criteria.
The real significance of the utilization factors lies in their ability to be varied in both sensitivity and tradeoff analyses for optimizing the entire system and its use. The systems performance effectiveness model thus becomes a tool for bringing operational and technical goals and criteria into agreement with each other.

If the goals and criteria are not in agreement, technical managers can use the models to demonstrate to their operational counterparts the desirability of changing the operational goals and criteria. In such a demonstration, the utilization factors are varied to show the impact of the variances on the index of the system's performance effectiveness. If this exercise does not demonstrate the desirability of changing the operational goals and criteria, the technical manager can readily understand why he must revise his goals and criteria to coincide with those of the operational manager. In most cases it will become clear to both that revisions are necessary on both sides to achieve an optimum system.

As with the performance and availability indexes, the variances in utilization indexes must be evaluated in terms of cost considerations and overall worth considerations. Each variance of a factor affects the other factors and is, in turn, affected by variances in other factors. At the same time, each variance of a factor has an associated cost that must be considered. Only when all factors have been considered in terms of mission accomplishment will true performance effectiveness be achieved for the system.

Mission-Oriented System-Effectiveness Model

This subsection summarizes the generalized mission-oriented system-effectiveness model that was developed by Task Group II of WSEIAC.* In the simple case in which the system can only be in either a working state or a failed state, the measures of availability, dependability, and capability concern the following fundamental questions:

1) Is the system working at the start of the mission?

2) If the system is working at the start of the mission will it continue to work throughout the mission?

3) If the system worked throughout the mission, will it achieve mission success?

Although these questions represent the fundamental approach to be used in evaluating effectiveness on a mission-oriented basis, they are too simplified for purposes of model construction. Moreover, as the systems considered become more complex (e.g., there are more than two possible system states) such elements as degraded modes of operation, multimission requirements, enemy countermeasures, and natural environment must also be quantified in the model.

The basic effectiveness model can be divided into two major elements—the probability that the system will be in a particular state at mission-performance time, and the effectiveness of the system when it is in that state. Thus, if effectiveness is quantified by a probability that the system will successfully meet the mission objectives, each term in the product \( P \cdot D \) represents the probability that the system will be in a particular state, and the corresponding term in the \( \mathbf{G} \) vector is the effectiveness of the system, given that state. For example,

\[
E = \mathbf{G} \cdot D \cdot L = \sum \{ P[\text{system is in state } i] \cdot P[\text{mission objectives are met, given state } i] \}.
\]

For some types of systems and missions, it may be more desirable to quantify effectiveness by some performance parameter other than a probability. For example, the expected miss distance for a missile might be a more meaningful performance parameter than the probability of "hitting within a specified area. For a reconnaissance system, the average amount of usable information might be appropriate. Figures of merit for these forms are readily usable by the appropriate quantification of the \( \mathbf{G} \) vector.

The mission model proposed by the WSEIAC Task Group II is, in essence, more a model framework for effectiveness evaluation than a directly applicable set of equations. This generality is necessary because the range of possible systems, missions, and depth of analysis precludes the specification of any single model.
The model framework, based on the availability, dependability, and capability factors, allows for flexibility in application by an appropriate combination of the associated elements. In Volume 3 of the Task Group II report, detailed examples are presented for an airborne avionics system, an intercontinental missile system, a radar surveillance system, and a spacecraft system.

The level of detail at which an analysis is performed will depend on the information and data available and on the purpose of the evaluation. For one study, a mean repair time may be sufficient input for the availability evaluation, while for another study such factors as queuing theory, spare parts availability, maintenance efficiency, and periodic-checkout procedures may have to be incorporated.

There are still many different areas that will require further research. One major problem is to develop improved techniques to convert available data into the appropriate vector and matrix elements of A, D, and C. Better analytic and computational techniques are required to incorporate state changes and those associated capabilities that can occur over a continuous interval. Such factors as state occupancy times and steady-state behavior may be involved in such analyses. Study also is recommended on a means to obtain some measure of "confidence" in the results of the effectiveness evaluation, both in the probabilistic combination of estimates and in guiding the decision process associated with the evaluation. Computerized analytic and simulation methods are needed for complex systems that generate a very large number of system states.

The WSEIAC model framework, or similar approach, has been applied to several systems, and has generally been found to be a reasonable method for evaluating effectiveness on a mission-oriented basis. Because of the impetus provided by WSEIAC, a great deal of research is being sponsored by the military and private agencies in order to improve this first effort.

3. Data Problems

The quality of the data used to perform calculations during the course of a systems performance effectiveness analysis will have a significant effect on the accuracy and utility of the results. Unfortunately, the mathematical model that describes a system configuration and behavior often is far more precise than the input data available. If effectiveness values--obtained from an exercise of the system model--are used as relative rather
than absolute values, the quality of the data is usually found
to be adequate. As a general rule, such values are satisfactory
when the analysis is performed to obtain comparisons between
alternate designs, or to determine the effect of changes on a
specific configuration. If absolute values are required, however,
extreme care must be used in selecting the input data, and cau-
tion should be observed in interpreting the results.

E. REFERENCES

1. **Systems Effectiveness.** Compiled by Systems Effectiveness
   Branch, Office of Naval Material, January 1965. (AD 659-520)

2. **Proceedings of the NMSE Systems Performance Effectiveness
   Conference.** The NMSE Systems Performance Effectiveness
   Steering Committee, April 1965. (AD 629-145)

3. **Proceedings of the Second NMSE Systems Performance Effective-
   ness Conference.** The NMSE Systems Performance Effectiveness
   Steering Committee, April 1966. (AD 651-819)

4. **Proceedings of the NMC Third System Performance Effectiv-
   eness Conference.** The NMC System Performance Effectiveness
   Steering Committee, May 1967. (AD 660-422)

5. L. D. Whitelock and P. J. Giordano: **The Navy's Systems Per-
   formance Effectiveness Program.** Aeronautic and Space Engi-
   neering Manufacturing Meeting, Society of Automotive Engi-
   neers, October 1966.
APPENDIX B ANALYTIC DERIVATION OF THE AVAILABILITY FUNCTION

A rather definitive treatment of the rationale and equations leading to the derivation and specification of an availability function is given in G. H. Sandler's text, System Reliability Engineering, Prentice Hall, 1964. In particular, Chapter A5 presents several interesting and appropriate variations, such as multiple repairmen, series and parallel redundancy, as well as n equipment items that have failed. The intent of this appendix is to highlight the method for obtaining the basic equations for the simplest case of a single equipment system and interpret the results in terms of system design approaches. Also, the problem of wearout is treated from a design viewpoint, and a rather simple reduction to a Markovian Process is illustrated for the assumptions given.

For the simple single equipment system, we designate two states—State 0 (the system is operating) and State 1 (the system is failed and under repair). Now because the conditional probability of failure in t, t + dt is dt, and the conditional probability of completing a repair in t, t + dt is dt, we have the following transition matrix:

\[
P = \begin{bmatrix}
1-\lambda & \lambda \\
\mu & 1-\mu
\end{bmatrix}.
\]

The differential equations describing the stochastic behavior of this system can be formed by considering the following: the probability that the system is in State 0 at time t + dt is derived from the probability that it was in State 0 at time t and did not fail in t, t + dt, or that it was in State 1 at time t and returned to State 0 in t, t + dt. Thus, we have

\[
P_0(t+dt) = P_0(t)(1-\lambda dt) + P_1(t)\mu dt + O dt.
\]

Similarly, the probability of being in State 1 at time t + dt is derived from the probability that the system was in State 0 at time t and failed in t, t + dt, or it was in State 1 at time t, and the repair was not completed in t, t + dt. Therefore,
\[ p_1(t + dt) = p_0(t) \lambda dt + p_1(t)(1 - \mu dt) + 0(dt). \]

The term \(0(dt)\) in both equations represents the probability of two events taking place in \(t, t + dt\), which is negligible. Note that the coefficients of these equations represent the rows of the transition matrix. As before, we find the differential equations by defining the ratio of:

\[
\frac{p_1(t + dt) - p_1(t)}{dt}
\]

which yields,

\[
\begin{align*}
p'_0(t) &= -\lambda p_0(t) + \mu p_1(t) \\
p'_1(t) &= \lambda p_0(t) - \mu p_1(t)
\end{align*}
\]

If we say that at time \(t = 0\) the system was in operation, the initial conditions are \(p_0(0) = 1, p_1(0) = 0\). It is also of interest to consider the case where we begin when the system is down and under repair. In this case, the initial conditions are \(p_0(0) = 0, p_1(0) = 1\).

Transforming Equations (1) into Laplace transforms under the initial conditions that \(p_0(0) = 1, p_1(0) = 0\) we have,

\[
\begin{align*}
s p_0(s) - 1 + \lambda p_0(s) - \mu p_1(s) &= 0, \\
s p_1(s) - \lambda p_0(s) + \mu p_1(s) &= 0,
\end{align*}
\]

and simplifying,

\[
\begin{align*}
(s + \lambda)p_0(s) - \mu p_1(s) &= 1, \\
-\lambda p_0(s) + (s + \mu)p_1(s) &= 0.
\end{align*}
\]

Although the solution for \(p_0(s)\) and \(p_1(s)\) can be found easily in this case, we shall apply Cramer's rule because it will be useful in later examples. To solve this system of equations, we
Introduce the determinant $D$, whose elements are the coefficients of $P(s)$'s. We also introduce the determinant $D_{1}$, which is formed by substituting the solution vector for the $i$th coefficient column. Then the solution is $P_{i}(s) = D_{1}/D$. Therefore:

$$P_{o}(s) = \frac{1}{s(s + \lambda - \mu)} \begin{vmatrix} 1 & -\mu \\ 0 & s + \mu \end{vmatrix},$$

and

$$P_{o}(s) = \frac{s + \mu}{s(s + \lambda + \mu)}.$$

Now the availability function that shall designate at $A(t)$ will be the inverse transform of $P_{o}(s)$, that is, $A(t) = F^{-1}[P_{o}(s)]$. Solving

$$A(t) = P_{o}(t) = \frac{\mu}{\lambda + \mu} + \left[ \frac{\lambda}{\lambda + \mu} \right] e^{-(\lambda + \mu)t},$$

and

$$\left\{ 1 - A(t) \right\} = P_{1}(t) = \frac{\lambda}{\lambda + \mu} \left[ \frac{\lambda}{\lambda + \mu} \right] e^{-(\lambda + \mu)t}.$$  \hspace{1cm} (2)

If the system was initially failed, the initial conditions are $P_{o}(0) = 0, P_{1}(0) = 1$, and the solutions are

$$A(t) = P_{o}(t) = \frac{\mu}{\mu + \lambda} \left[ \frac{\lambda}{\mu + \lambda} \right] e^{-(\lambda + \mu)t},$$

and

$$\left\{ 1 - A(t) \right\} = P_{1}(t) = \frac{\lambda}{\lambda + \mu} \left[ \frac{\lambda}{\lambda + \mu} \right] e^{-(\lambda + \mu)t}. \hspace{1cm} (3)$$
We note that the components become very large, Equations (2 and (3) become equivalent. This indicates that after the system has been operating for some time, its behavior becomes independent of its starting state.

The availability function \( A(t) \) can be interpreted as the probability that at any time \( t \) the system is in an operating state. In many cases, we are interested in the average uptime for some definite period. This can be found simply by summing \( A(t) \) over the time interval of interest and dividing by the total time.

\[
A(T) = \frac{1}{T} \int_{0}^{T} A(t) \, dt \tag{4}
\]

In this instance we have

\[
A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2} - \frac{\lambda e^{-(\lambda + \mu)T}}{(\lambda + \mu)^2} \tag{5}
\]

If we are interested in the long-term availability of the system we can let \( t \to \infty \) and find,

\[
A(\infty) = \left( \frac{\mu}{\mu + \lambda} \right) \tag{6}
\]

This condition is usually referred to as the steady-state availability. Essentially, it implies that for a large ensemble of equipment items, the process will maintain itself in a state of statistical equilibrium.

It is this analytical expression that is commonly used to describe availability in the form:

\[
\text{Operational Availability} = \frac{\text{Mean Time Between Failures (MTBF)}}{\text{MTBF} + \text{Mean Time to Repair (MTTR)}}
\]

This parameter is a measure of an attribute of the system and, as such, is subject to two conditions:

1) The value that is attainable within the resources available;
2) The value that is needed to meet the objectives of the mission.
The initial design analysis objective is to establish that the attainable probability of system availability meets the mission needs. The mission requirement depends on the criticality of the mission and risk that is acceptable to management. It may be a rigorous requirement or flexible, i.e., the highest probability attainable within a given resource allocation. In either case, the analytical problem becomes one of optimizing the two operational availability parameters MTBF and MTTR for the operational modes of the system.

The above analysis is for a single item of equipment with constant repair and failure rates. Several variations of this basic approach to more complex situations are of interest. For example, consider the problem where the equipment is subjected to two types of repair. Thus, when the equipment fails for the first time, partial repair is performed, which restores the system to operation. However, this increased the probability of failure. After the equipment fails for the second time, a second repair is performed. Thus, the analytical procedure is as follows:

1) Let $\lambda_1$ designate the failure rate when the equipment has been through a complete repair;

2) Let $\lambda_2$ designate the failure rate when the equipment has been through a partial repair ($\lambda_2 > \lambda_1$);

3) Let $\mu_1$ be the repair rate for a partial repair;

4) Let $\mu_2$ be the repair rate for a complete repair, i.e., $\mu_2 < \mu_1$.

Consider the four states of the system as:

1) State 0 - System is failed and a partial repair is being performed;

2) State 1 - System is failed and a partial repair is being performed;

3) State 2 - System is operating after completion of partial repair;

4) State 3 - System is failed and a complete repair is being performed.
The transition matrix becomes:

\[
\begin{pmatrix}
(1 - \lambda_1) & \lambda_1 & 0 & 0 \\
0 & (1 - \mu_1) & 1 & 0 \\
0 & 0 & (1 - \lambda_2) & \lambda_2 \\
\mu_2 & 0 & 0 & (1 - \mu_2)
\end{pmatrix}
\]

The resulting proportion of time spent in an acceptable state,

\[A(\infty) = p_0 + p_2\]

is given by:

\[A(\infty) = \frac{\lambda_1 \mu_1}{\lambda_1 \mu_2 + \lambda_2 \mu_1} + \frac{\lambda_1 \mu_2}{\lambda_1 \mu_2 + \lambda_2 \mu_1} + \frac{\lambda_2}{\lambda_2 + \mu_2} + \frac{\mu_2}{\lambda_2 + \mu_2}.
\]

Thus, if \(\lambda_1 = \lambda_2\) and \(\mu_1 = \mu_2\), the above equation reduces to Equation (6).

There are several additional interesting variations of the repair or maintenance policies that can be considered analytically. The available function is given for each of these formulations. The respective derivations may be found in System Reliability Engineering by G. H. Sandler.

1) \(n\) equipment with \(r = n\) repairmen:

\[A(\infty) = \frac{\mu^n}{(\lambda + \mu)^n}\]  \(\text{(8)}\)

2) Two-equipment item series system with two repairmen:

\[A(\infty) = \frac{3 \mu}{3 \mu + \mu 4 \lambda + 2 \lambda^2}\]  \(\text{(9)}\)

3) \(n\) equipment items with \(R < n\) repairmen working independently:
$$A(\infty) = \sum_{K=0}^{n-m-1} P_k,$$  \hspace{1cm} (10a)

where

$$P_k = \frac{n! \theta^k}{(n-k)! k!} \begin{cases} \{p\} \quad \text{for } k<r \end{cases} \hspace{1cm} (10b)$$

$$P_k = \frac{n!}{(n-k)!} \begin{pmatrix} \theta \\ \frac{k-r}{r} \end{pmatrix} \begin{pmatrix} p \\ 0 \end{pmatrix} \quad \text{for } k\geq r,$$  \hspace{1cm} (10c)

$$P_0 = \left\{ \sum_{k=0}^{r-1} \frac{n! \theta^k}{(n-k)! k!} + \sum_{k=r}^{n} \frac{n! \theta^r}{(n-k)! k!} \begin{pmatrix} \theta \\ \frac{k-r}{r} \end{pmatrix} \right\}^{-1} \hspace{1cm} (10d)$$

$$\theta = \frac{\lambda}{\mu}.$$  \hspace{1cm} (10e)

4) Two-equipment item redundant system operating in parallel:

$$A(\infty) = \frac{(\mu^2 + \mu \lambda 2)}{[\mu + \lambda]^2}.$$  \hspace{1cm} (11)

5) Two-equipment item redundant system operating in parallel in which it is not possible to service a failed item of equipment until the complete system fails:

$$A(\infty) = \frac{3 \mu^2 + \mu \lambda 2}{3 \mu^2 + \mu 3 \lambda + \lambda^2}.$$  \hspace{1cm} (12)
A. DESIGN FOR WEAROUT

The expression of an availability probability in terms of variables MTBF and MTTR serves the purpose of establishing system objectives that can be assessed for feasibility and guidelines for conceptual design.

The total mission time can be compared to MTBF to assure compatibility. If the mission time in the order of 1/10 of the MTBF, the failure characteristic can be treated as an exponential decay function. If this is not the case, and the mission time is a larger percentage of the MTBF, then the problem of wearout (useful life) enters into the problem. Wearout is the case where the failure rate starts to increase after the steady-state failure performance has been reached, as in Fig. 1.

![Diagram of Wearout Characteristic](image)

**Fig. 1 -- Wearout Characteristic**

The analytical expression for operational availability comes into use in the concept and definition phases. In the case of wearout, this involves the formulation of a non-Markovian Process to obtain the steady-state availability function. Initially, nonlinear stochastic equations result, which, in many cases (but the simplest), prove to be intractable. However, there are many cases in which the equipment failure distributions are other than exponential, but the transition process can be treated as a
Markov Process by increasing the number of states, each being described by a constant transition rate. For example, consider the single equipment case in which \( F(t) \) is the gamma distribution (see Fig. 1). It is assumed that the equipment goes through two exponential phases, each of length \( 1/\lambda \). Three states are defined:

1) State 0 - System is operating in the first phase;
2) State 1 - System is operating in the second phase;
3) State 2 - System is failed.

The transition matrix becomes:

\[
\begin{bmatrix}
(1-\lambda) & 0 \\
0 & (1-\lambda) \\
\mu & 0 & (1-\mu)
\end{bmatrix}
\]

The steady-state equations are:

\[-\lambda P_0 + \mu P_2 = 0 \]
\[\lambda P_0 - \lambda P_1 = 0 \]
\[\lambda P_1 - \mu P_2 = 0 \]
\[P_0 + P_1 + P_2 = 1 \]

\[A(\infty) = 2P_0 = \frac{2\mu}{2\mu + \lambda} \]

This simple example illustrates the technique for solving these types of problems in terms of the Markov formulation. In general, the application to a specific situation involves the following steps:
1) On the mission level, the equipment performance characteristics are compared against the mission profile to provide an hypothetical operational scenario, for the purposes of sizing and to determine critical equipment usages. The respective sensitivities in margins as to the allocation of failure rates and repair rates are assessed to determine overall availability requirements and equipment compatibilities;

2) The identification of a subsequent maintainenac model, and repair regimen is delineated in accordance with the projected equipment use profile, and provision for wearout is made by the specification of a regimen in the operational cycle at which wearout results in a catastrophic failure;

3) The analysis procedure includes the formulation of either a simulation or, at first, a simple analytical model to indicate the appropriate solution to the availability function. This implies the estimation of failure rates, and the knowledge of the state of the system before wearout;

4) Refinement of the availability function to include the effects of interrupted states may be accomplished by the specification of successive transition matrices and solving for the composite availability function.

In terms of the design methodology, and its implications concerning the systems engineering process, each new state corresponds to an alternative solution in terms of specifying transition probabilities and failure and repair rates. The optimal combination of each transition matrix at each point in the design process involves a multistage decision process, which can be formulated as a dynamic programming problem. In a great many instances intuition and empirical experience are substitutes for formal analytical procedures. However, where a large number of cost tradeoffs are involved, and a great number of transition matrices need be computed, the automation of the decision process would prove useful.
B. SINGLE ELEMENT WEAROUT DESIGN FOR MAINTAINED SYSTEMS

In general, it has been observed that a large class of electronic and mechanical equipment and components exhibit failure distributions similar to those shown in Fig. 2 (Ref 1). This distribution can be considered most conveniently as the sum of three elementary distributions, i.e.,

\[ f(t) = a_\alpha f_\alpha(t) + a_\beta f_\beta(t) + a_\gamma f_\gamma(t), \]

where

\[ a_\alpha, a_\beta, \text{ and } a_\gamma = \text{weights for combining the distribution so that } f(t) \text{ will satisfy the conditions of a probability density distribution function. For the purpose here, } a_\alpha + a_\beta + a_\gamma = 1 \]

\[ f_\xi(t) = \text{failure distribution which dominates in period } \xi \]

Fig. 2 -- General Failure Distribution for Electronic and Mechanical Equipment and Components
Period 1 is called the early failure or debugging period where high failure rate items are uncovered and defects that escaped quality control inspection are found. The number of failures is expected to decrease rapidly with time in this period. A gamma distribution may be used to approximate the distribution that dominates in this period. Figure 3 illustrates the shapes of some common failure distributions and related functions.

Period 2 is called the normal failure or constant failure rate (to be more exact, constant hazard rate) period. It is the period in which equipment reliability is usually considered. In this period, the exponential law of failure dominates.

Period 3 is called the Gaussian or wearout failure period where some elements of the equipment fail from wear. The normal (Gaussian) distribution may be used to approximate the dominating distribution for this period. To the extent that such a period is known to exist during the useful life of the equipment, it is necessary to establish overhaul and maintenance policies. It is, therefore, the level or amount of equipment survivability that is the key to providing the designer with a methodology for achieving the required performance. The concept is that there exists some degree of survivability, p, between 0 and 1, which the designer can manipulate early in the design cycle to obtain alternative and meaningful solutions to the reliability and maintainability problems. This degree of survivability can be thought of in two ways:

1) It is a compilation of experience with respect to similar equipment in similar operational environments—-the percentage of failures that have been found to occur and are repairable.

2) It is a rough measure of maintainability—defined by spares availability, level of checkout, and system repair capability.

Thus, in case of a failure, a subsystem can be repaired with a certain probability. Generally, this probability will be a complex function of the above maintainability factors, availability of spares, the capability of the repairman, and the efficiency of the fault isolation equipment.
<table>
<thead>
<tr>
<th>TYPE OF DISTRIBUTION</th>
<th>FAILURE DISTRIBUTION ( f(t) )</th>
<th>RELIABILITY FUNCTION ( R(t) )</th>
<th>HAZARD RATE ( h(t) = \frac{f(t)}{R(t)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>( \lambda e^{-\lambda t} )</td>
<td>( e^{-\lambda t} )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( e^{-\lambda t} )</td>
<td>( e^{-\lambda t} )</td>
<td>( \lambda t )</td>
</tr>
<tr>
<td>Gamma</td>
<td>( \frac{1}{(a-1)b} \left( \frac{t}{b} \right)^{a-1} e^{-t/b} )</td>
<td>( \frac{1}{(a-1)b} \int_0^t \frac{r}{b} e^{-r/b} dr )</td>
<td>( \frac{1}{(a-1)b} \left( \frac{t}{b} \right)^{a-1} e^{-t/b} )</td>
</tr>
<tr>
<td>Normal</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} e^{-\left( t-\mu \right)^2/2\sigma^2} )</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} \int_0^t e^{-\left( t-\mu \right)^2/2\sigma^2} dt )</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} \int_0^t e^{-\left( t-\mu \right)^2/2\sigma^2} dt )</td>
</tr>
<tr>
<td>Log normal</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} e^{-\left( \log t - \mu \right)^2/2\sigma^2} )</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} \int_0^t e^{-\left( \log t - \mu \right)^2/2\sigma^2} dt )</td>
<td>( \frac{1}{\sigma \sqrt{2\pi} \int_0^t e^{-\left( \log t - \mu \right)^2/2\sigma^2} dt )</td>
</tr>
</tbody>
</table>

Fig. 3 Shapes of Common Failure Distributions and Related Functions

66
What follows is neither a very formalistic nor a mathematically rigorous approach. Rather, the emphasis is on how the designer plays his hand through the proper specification of the survivability factor, \( p \). In this, he is using \( p \) as a convenient and useful rule of thumb—a designer's index—which, however, can have a powerful effect on the design of other elements of the system. The basic premise is that the reliability of a maintained system is a function of four parameters, \( \psi \), \( \Omega \), \( t \), and \( p \). These are defined as follows.

\( t \) is the desired operating time for the equipment. It is assumed that repairs can be accomplished at any point within this time—the model ignores the periods during the mission where repair is impossible. Examples of such periods are rendezvous operations where the crew is fully occupied with the rendezvous maneuver; midcourse correction where the crew members are strapped to their seats and are unable to move to the areas of the spacecraft where repairs may be necessary; and the descent or ascent phases of the mission. Usually, these time periods account only for a very small fraction of the total equipment operating time, and, in this case, no significant error is introduced by ignoring them.

\( \psi \) is the failure rate, the reciprocal of the mean-time-between failures, for the system. The widely accepted Weibull failure distribution is assumed here. This means that it is possible to approximate failures by varying the distribution parameters during any of the three time periods delineated in Fig. 2.

\( \Omega \) is the repair rate, the reciprocal of the average time to restore the system to operating condition. Depending on the equipment design and maintenance concept, the repair-time distribution may take many forms. It is assumed to be exponentially distributed. The reason for this assumption is that since the model is insensitive to the exact form of the distribution (see Ref 1), a distribution that simplifies the mathematics as much as possible will be used.
The factor, $p$, is the keystone of this approach. It defines the probability of wearout, i.e., survivability. In effect, $p$ is equivalent in many respects to some of the current definitions of maintainability, at least with respect to a system in an operating environment. The functional relationship of $p$ to the factors previously given (spares availability, level of checkout, and system repair capability) is a complicated one, but since the emphasis here is toward the application of the designer's knowledge, experience, or the intuitive judgment of $p$, it is assumed that each of these factors is probabilistically independent. This is not entirely accurate. Indeed, much of the work being done today in maintainability is directed toward trying to establish a more formal relationship between, for example, test equipment and training, in order to structure and understand their interaction.

It is clear that if a piece of equipment is repaired and put back into operation, it will have a new failure distribution. If a group of equipment of various ages are kept operating by means of immediate repair after any failure, ultimately this mixed-age population will appear to have an exponential failure distribution.

1. **MATHEMATICAL MODEL FOR A SINGLE ELEMENT MAINTAINED SYSTEM**

Figure 4 illustrates a typical single element maintained configuration for which the designer desires to achieve a given level of survivability. There are many functions

**If the equipment being designed is similar to other equipment already in use, and if the operational environments are comparable, $p$ can be obtained from experience, i.e., the percentage of those failures that have occurred over a period of time that were found to be repairable. Furthermore, if spares are consumed during a mission, $p$ is really a function of time. However, it is assumed that spares consumption is low, thereby allowing the time dependence of $p$ to be ignored.**
aboard a spacecraft that can be fulfilled by such a system where a backup is not required. For example, the VHF communication system may very well be without backup, since being "down" for repairs does not necessarily imply a catastrophic event. For these systems one is primarily interested in availability; i.e., what percentage of the total mission time the communication system is in operating condition. The basic assumptions for this model are:

1) During standby operation or while undergoing repair, the failure rate of the element is zero. During operation, the failure distribution is of the Weibull type, i.e., the probability that a failure occurs between t and t + dt is $\lambda \psi dt$, where $\lambda \psi$ is the failure rate.

2) The repair distribution is also of the Weibull type, with repair rate $\Omega$.

3) The probability of survival is $p$ (wearout condition).

4) Each failure is repairable.

5) Failures are detected immediately, and repair action starts as soon as the failure is discovered.

At any one time, the system may be in one of three mutually exclusive states.
State 1 - operating
State 2 - down and in repair
State 3 - down and nonrepairable

Following the usual methods (Ref 2) one can relate the state probabilities at time "t" with those a short time "dt" later by the equations:

\[
P_1(t + dt) = P_1(t) - \psi P_1(t)dt + \Omega P_2(t)dt
\]

\[
P_2(t + dt) = p \psi P_1(t)dt + P_2(t) - \Omega P_2(t)dt
\]

\[
P_3(t + dt) = P_1(t) (1 - p) dt + P_3(t)
\]

These equations can be solved by first obtaining the Laplace transform, where \( P_1(0) = 1 \), and \( P_2(0) = P_3(0) = 0 \), and then finding the inverse. Thus, the differential transition matrix is given by:

\[
\begin{bmatrix}
-\psi & p\psi & (1-p)\psi \\
\Omega & -\Omega & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(14)

The probability that the system is operational at time t is given by

\[
P_1(t) = \mathcal{L}^{-1} \left[ \frac{S(S + \Omega)}{S(S + \psi)(S + \Omega) - \Omega(1 - \psi)} \right]
\]

(15)

\[
P_2(t) = \frac{1}{a-b} \left\{ (\Omega + a) e^{at} - (\Omega + b) e^{bt} \right\}
\]

(16)

where

\[
a = - (\psi + \Omega) + \left\{ (\psi - \Omega)^2 + 4\psi \Omega (1 - p) \right\}^{1/2}
\]

(17)

\[
b = - (\psi + \Omega) - \left\{ (\psi - \Omega)^2 + 4\psi \Omega (1 - p) \right\}^{1/2}
\]
The availability of the system, or the average time the system is operational, during an interval \( t \), is

\[
A(T) = \frac{1}{t} \int_0^t P_1(S) \, ds
\]  

(18)

\[
= \frac{1}{(a-b)t} \left\{ \left[ \frac{Q+a}{a} \right] \left( e^{at} - 1 \right) - \left[ \frac{Q-b}{b} \right] \left( e^{bt} - 1 \right) \right\}
\]  

(19)

The percentage availability improvement over a similar nonmaintained (i.e., nonsurvivable) system is defined as

\[
\left( \frac{\text{availability of a maintained system}}{\text{availability of nonmaintained system}} \right) - 1 \times 100
\]

This function is plotted in Fig. 5 for given parametric values of \( \Psi \) and \( Q \). The use of this tool by the designer is illustrated in the following section.

2. A DESIGN APPLICATION

Consider the designer who is in the prototype design stages of, say, the VHF communication system of a spacecraft. He has been told that the availability required of this particular system is 0.928. However, now that he has finished his prototype design, he does a simple and acceptable analysis by means of a parts count, and finds that his availability is 0.800. How does he get the 16% improvement to meet the requirement?

There are a few standard approaches he can use. First, he can reexamine his circuit design with the aid of a reliability expert and perhaps uncover some marginal component applications. He can, with the aid of a component parts expert, examine the parts in the equipment and determine whether he has, in fact, used the highest reliability components available to him. If, at the conclusion of this exercise he has improved the availability of the
Fig. 5 -- Availability Improvement of a Single-Element Maintained System
If he is guided by past experience, the designer will examine the possibility of incorporating some form of redundancy into his design. The equations that will give him additional availability for a given degree of redundancy are fairly straightforward. He applies them and notes that he can indeed achieve the additional availability in this manner. This, of course, means that he is not making the entire VHF system redundant but, rather, he is making redundant only those parts of the system that he has found to be the least reliable and/or most amenable to such redundancy. In so doing, however, he has probably increased the number of interconnections and also the system weight; and, since this is a space system and weight is extremely important, he may be in trouble.

This search for the proper level of redundancy is a valid one. In many cases the use of redundancy will solve his problem, especially if the system has a relatively short desired operating time as compared to its mean-time-between-failures (MTBF). However, it is at this point that the designer, if he has not yet achieved the required design availability, frequently shows his lack of appreciation for the role of maintainability. He will attempt a wholesale circuit redesign in order to achieve his reliability goal, rather than examine the logical next step—enhancing the survivability of the system by making it maintainable.

Making a system maintainable is not easy. For one thing the system design must be such that components or subsystems can be removed and replaced and, furthermore, spares must be made available for replacement. The designer must determine just how much and what kind of repair should be included in his system. He must do this in an environment of considerable uncertainty, for at this early stage of the project he does not know the characteristics of the checkout or fault-isolation equipment; he does not know what spares he will be allowed, and he has only a very general appreciation of the maintenance requirements.
But this air of uncertainty has some advantages for the designer. It allows him to place additional requirements on the system design, because, as the example given below will show, he can determine just how much improvement in system reliability a given level of survivability will give.

For the system shown in Fig. 5, let the MTBF be 500 hr, and assume the desired operating time is 250 hr (about 11 days). The availability of this system is 0.844. This number is obtained from:

\[ A(T) = (1 + \Psi t)e^{\Psi t} \]

Suppose the designer's availability goal, arrived at through apportionment, is 0.928. This implies that availability improvement of at least 10% is required. By referring to Fig. 5, we see that for a value of \( t = 0.002 \times 250 = 0.5 \), a \( p \) factor of approximately 0.6 gives an availability improvement of 10%. So a survivability of 0.6 will yield the desired boost in availability—the designer must now find ways of obtaining the 0.6 \( p \) factor.

Let us assume that the designer concludes that the system repair capability is 75%. Then, based on his experience with fault-isolation systems and knowledge of the fault isolation task inherent in his design, and from consultations with designers of in-flight test systems, he may reasonably require the fault isolation system to be 90% effective in locating the faulty part within his system. Then, there remains the problem of assessing the spares availability that can be obtained from a certain allowance of spares weight. Fortunately, there are techniques (Ref 3) for solving this problem but, for the moment, we will just apply the rule of thumb that for electronic equipment more than 90% of the failures are attributable to 10% of the total number of parts in the equipment. Further, assume that the designer can, in fact, stock this 10% of the parts and stay within the weight constraints. He can then consider his spares availability to be equal to 90%. Multiplying these three factors, the designer obtains a value

\[ p = 0.75 \times 0.90 \times 0.90 = 0.6075 \]
and has achieved the p factor required for the desired degree of availability improvement.

It is at this point that the designer should verify that this system does indeed meet the availability specifications more efficiently (say at a lower overall system weight) than that of his other alternatives.

REFERENCES


APPENDIX C  RELIABILITY AND MAINTAINABILITY TRADEOFF APPROACH

This example illustrates an approach to conducting a Reliability (MTBF) and Maintainability (MTTR) tradeoff when given a specified Inherent Availability. The desired alternative is based on initial and sustaining costs.

A. Design Problem

A requirement exists to design a radar receiver that will meet an Inherent Availability of 0.990, a minimum MTBF of 200 hours, and a MTTR not to exceed 4.0 hours. Existing design with the use of Military Standard parts meets an Availability of 0.97, a MTBF of 150 hours, and a MTTR of 4.64 hours.

B. Possible Solutions

Three different alternative design configurations are being considered to satisfy availability requirements:

<table>
<thead>
<tr>
<th>Design Configuration</th>
<th>A</th>
<th>MTBF*</th>
<th>MTTR (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. R – derating of military standard parts M – modularization and automatic testing</td>
<td>0.990</td>
<td>200</td>
<td>2.02</td>
</tr>
<tr>
<td>2. R – design includes high reliability parts/components M – limited modularization and semiautomatic testing</td>
<td>0.990</td>
<td>300</td>
<td>3.03</td>
</tr>
<tr>
<td>3. R – design includes partial redundancy M – manual testing and limited modularization</td>
<td>0.990</td>
<td>350</td>
<td>3.54</td>
</tr>
</tbody>
</table>

Design Configuration 1 emphasizes the Maintainability aspects in the design while Design Configuration 3 emphasizes Reliability improvement. The Reliability-Maintainability relationship is derived through the equation:

\[
\text{Inherent Availability (A_i) = \frac{MTBF}{MTBF + MTTR}}
\]

* Conservative Estimate - Lower 2σ Bound
+ Conservative Estimate - Upper 2σ Bound
The area for reliability-maintainability tradeoff is illustrated in Figure 1.

**Figure 1** — Reliability-Maintainability Tradeoff
C. Tradeoff Approach

1) Cost data are developed against each configuration. Such data include both initial costs (those associated with design and manufacture of the equipment) and sustaining costs (those associated with field operations - manpower, test equipment, spare parts, facilities, etc).

2) Initial cost factors are as follows (the values used are estimated and not necessarily representative of actual experience).

<table>
<thead>
<tr>
<th>Item</th>
<th>Existing Configuration (dollars)</th>
<th>Configuration 1 (dollars)</th>
<th>Configuration 2 (dollars)</th>
<th>Configuration 3 (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDT &amp; E Cost</td>
<td>300,000</td>
<td>324,937</td>
<td>319,125</td>
<td>321,500</td>
</tr>
<tr>
<td>Reliability</td>
<td>-</td>
<td>1,187</td>
<td>6,625</td>
<td>16,750</td>
</tr>
<tr>
<td>Maintainability</td>
<td>-</td>
<td>23,750</td>
<td>12,500</td>
<td>4,750</td>
</tr>
<tr>
<td>Manufacture Cost*</td>
<td>4,500,000</td>
<td>4,534,250</td>
<td>4,524,700</td>
<td>4,530,250</td>
</tr>
<tr>
<td>Reliability</td>
<td>-</td>
<td>1,750</td>
<td>9,200</td>
<td>22,500</td>
</tr>
<tr>
<td>Maintainability</td>
<td>-</td>
<td>32,500</td>
<td>15,500</td>
<td>7,750</td>
</tr>
</tbody>
</table>

* Manufacture Cost is total based on 300 units.

The following tabulation presents the net incremental value (cost) of Reliability and Maintainability in equipment design as derived from the above figures.

<table>
<thead>
<tr>
<th>Item</th>
<th>Configuration 1 (dollars)</th>
<th>Configuration 2 (dollars)</th>
<th>Configuration 3 (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability</td>
<td>2,937</td>
<td>15,825</td>
<td>39,250</td>
</tr>
<tr>
<td>Maintainability</td>
<td>56,250</td>
<td>28,000</td>
<td>12,500</td>
</tr>
<tr>
<td>Total</td>
<td>59,187</td>
<td>43,825</td>
<td>51,750</td>
</tr>
</tbody>
</table>

3) Sustaining cost factors represent estimated cost to support 300 units for 10 years.
## SUSTAINING COST FACTORS

<table>
<thead>
<tr>
<th>Item</th>
<th>Existing Configuration (dollars)</th>
<th>Configuration 1 (dollars)</th>
<th>Configuration 2 (dollars)</th>
<th>Configuration 3 (dollars)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Spares</td>
<td>210,000 (14@3,000)</td>
<td>150,970 (10@15,097)</td>
<td>105,420 (7@15,060)</td>
<td>90,402 (6@15,067)</td>
<td>Frequency of maintenance/10 yr.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Existing Conf--139:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 1 --104;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 2-- 69;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 3-- 59;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Assume 10% spares.</td>
</tr>
<tr>
<td>Cost of Repair</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Administrative</td>
<td>385,065</td>
<td>125,370</td>
<td>124,773</td>
<td>124,176</td>
<td>Assume $1.99/direct labor hour:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>One technician/maintenance action</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Existing Conf-E6 $3.62 per hour</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 1 - E3 $2.40/hr.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 2 - E4 $3.00/hr.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conf. 3 - E5 $3.40/hr.</td>
</tr>
<tr>
<td>Active Repair</td>
<td>700,470</td>
<td>151,200</td>
<td>188,100</td>
<td>212,160</td>
<td></td>
</tr>
<tr>
<td>Test Equip/Spares</td>
<td>174,150</td>
<td>56,700</td>
<td>56,430</td>
<td>56,160</td>
<td>Assume 3 items-standard equipment (burden-$0.90/direct labor hr)</td>
</tr>
<tr>
<td>Spares (piece parts)</td>
<td>1,390</td>
<td>1,040</td>
<td>600</td>
<td>590</td>
<td>Assume $10/repair action</td>
</tr>
<tr>
<td>Facilities</td>
<td>36,765</td>
<td>11,970</td>
<td>11,913</td>
<td>11,856</td>
<td>Use existing facilities (burden-$0.19/direct labor hour)</td>
</tr>
<tr>
<td>Cost of training</td>
<td>140</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>Based on $30/day</td>
</tr>
<tr>
<td>Item</td>
<td>Cost of Provisioning</td>
<td>Cost of Technical Manual</td>
<td>Provisioning—assume cost of $1000/line item.</td>
<td>Handling—assume $19/line item/year</td>
<td>Remarks</td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------------</td>
<td>--------------------------</td>
<td>---------------------------------------------</td>
<td>-----------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Existing Configuration (dollars)</td>
<td>20,000</td>
<td>14,000</td>
<td>18,000</td>
<td>5 moths.</td>
<td>Assume $200/page</td>
</tr>
<tr>
<td>Configuration 2 (dollars)</td>
<td>30,000</td>
<td>15,000</td>
<td>494,950</td>
<td>2655 parts</td>
<td>1,058,876</td>
</tr>
<tr>
<td>Configuration 3 (dollars)</td>
<td></td>
<td></td>
<td>487,450</td>
<td>2565 parts</td>
<td>1,005,876</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1,018.64</td>
</tr>
<tr>
<td>Total Cost</td>
<td>2,052,980</td>
<td>1,236,280</td>
<td></td>
<td></td>
<td>2,058,876</td>
</tr>
</tbody>
</table>
Referring to the sustaining cost table, a net sustaining cost savings is realized for each proposed design configuration. This saving is derived through the improvement in equipment availability, resulting in a reduction in required maintenance support. The following computation illustrates the cost savings against each design configuration when compared against existing design:

Configuration 1 - $966,700  
Configuration 2 - $997,104  
Configuration 3 - $984,816

4) Subsequent to deriving both initial and sustaining cost factors, the net effect is obtained through subtracting the initial added costs for Reliability and Maintainability from the overall sustaining cost savings:

Configuration 1 $966,700 - $59,187 = $907,513  
Configuration 2 $997,104 - $43,825 = $953,279  
Configuration 3 $984,816 - $51,750 = $933,066

Because the sustaining cost factors are estimated values, an uncertainty of ± 20% in the total costs yields the following overall sustaining cost savings:

± 20%  
Configuration 1 $759,444 - $59,187 = $700,257  
Configuration 2 $795,928.80 - $43,825 = $752,103.80  
Configuration 3 $781,182.20 - $51,750 = $729,433.20

- 20%  
Configuration 1 $1,173,956 - $59,187 = $1,114,769  
Configuration 2 $1,198,279.80 - $43,825 = $1,154,454.80  
Configuration 3 $1,188,448.80 - $51,750 = $1,136,698.80

D. Design Decision

The intent of this tradeoff is to generate and evaluate the alternative Reliability and Maintainability design features required to meet a specified Availability. In doing so, the basic evaluation criterion is cost. Referring to the above cost factors, Configuration 2 satisfies the required equipment Availability with maximum cost savings or minimum initial cost expenditure, even under the conditions of a ± 20% uncertainty in the total estimated costs. This lack of sensitivity indicates that the total sustaining cost savings has a broad maximum, over which perturbations in such factors as spares, facilities, and test equipment have little influence in changing the decision relative to the remaining alternatives. The reason for this is that
Configuration 2, which represents the most cost/effective condition, is a conservative position between the two extremes of too little reliability (R), and too much modularization (M) and testing. The combined effect is to produce a cost savings figure of merit based on the product of M and R that, as one factor is increased, the other factor decreases in approximately the same proportion. Thus, the essential tradeoff (for the same amount of availability) emphasizes a design solution that is a compromise and does not incorporate one overriding critical cost element that would result in a reversal of the sensitivity figures for Configurations 1 or 3.
APPENDIX D  SOME PROBABILITY BACKGROUND - JOINT PROBABILITIES

In the study of probability theory, it is shown that for two random continuous variables, x and y, which may or may not be independent random variables, one can define a joint probability density function, \( f(xy) \). The important property of this function is that if one takes a random sample from the sample space of xy, then the probability of finding a value of x that is somewhere in the region \( \Gamma \) and, at the same time, finding a value of y that is in the same region \( \Gamma \) is given by

\[
P \left\{ xy; x, y \in \Gamma \right\} = \int \int_{\Gamma} f(xy) \, dx \, dy
\]

where the integration is made over the region \( \Gamma \).

\[
P \left\{ xy; x_1 \leq x \leq x_2, y_1 \leq y \leq y_2 \right\} = \int \int f(xy) \, dx \, dy
\]

Furthermore, if x and y are independent random variables, then the joint probability density function \( f(xy) \) is simply the product of the probability density functions for each of the variables individually.

In particular, if x and y are assumed to be independent random variables, the joint probability density function for these variables is:

\[
f(xy) = p_x(x) p_y(y)
\]

where \( p_x(x) \) is the probability density function for x and \( p_y(y) \) is the probability density function for y.
A. PROBABILITY OF EVENT $y > x$

Let $A$ be the probabilistic event that the random variable $y$ exceeds $x$. Then the probability for the occurrence of event $A$ is the probability that, in any random sample taken from $x$ and $y$, one will find $y \geq x$ and that $x$ will be found any place at all,

$$P \{ A \} = P \{ xy ; y \geq x, -\infty \leq x \leq \infty \}$$

To calculate $P \{ A \}$, the joint probability density function $f(xy)$ must be integrated over the shaded region $\Gamma$ of the $x$, $y$ plane because that area is the region where $y \geq x$ and where $-\infty \leq x \leq \infty$.

This gives:

$$P \{ A \} = \int \int p_x(x) p_y(y) \, dx \, dy$$

or

$$P \{ A \} = \int_{x=\infty}^{x=\infty} \int_{y=\infty}^{y=\infty} p_x(x) p_y(y) \, dx \, dy$$
B. CALCULATION PROCEDURE

To evaluate this integral, perform the y integration first. Then, x will appear in the result when the limits are substituted. Then perform the x integration. This procedure can be shown in the following formula. Let

\[ J(x) = \int_{y=x}^{y=\infty} p_y(y) \, dy \]

Then

\[ P(A) = \int_{x=-\infty}^{x=\infty} p_x(x) \, J(x) \, dx \]

C. ALTERNATIVE INTEGRAL

Alternatively, we could cover the same shaded region \( \Gamma \) by another integration,

\[ P(A) = \int_{y=-\infty}^{y=\infty} \int_{x=-\infty}^{x=y} p_x(x) \, p_y(y) \, dx \, dy \quad (7) \]

Again to evaluate this integral, perform the x integration first, and y will appear as a parameter in the result. Then perform the y integration. The procedure can be shown by the following. Let

\[ I(y) = \int_{x=-\infty}^{x=y} p_x(x) \, dx \quad (8) \]

Then

\[ P(A) = \int_{y=-\infty}^{y=\infty} p_y(y) \, I(y) \, dy \quad (9) \]
D. STUDY OF SOME SPECIAL CASES

To check the validity of these formulae, we shall use them to calculate \( \mathbb{P}\{A\} \) for some simple cases where the computation is not difficult and the results can be checked with intuitive expectation.

1. Case 1 -- Two Nonoverlapping Rectangular Distributions

Consider two rectangular distributions as shown above.

\[
\begin{align*}
\mathbb{P}_x(x) &= \begin{cases} 
\frac{1}{d-c} & \text{for } c \leq x \leq d \\
0 & \text{otherwise}
\end{cases} \\
\mathbb{P}_y(y) &= \begin{cases} 
\frac{1}{b-a} & \text{for } a \leq y \leq b \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]

Using eq (9), we calculate directly that

\[
I(y) = \int_{-\infty}^{y = y} \mathbb{P}_x(x) \, dx = \begin{cases} 
y - c & \text{for } y < c \\
\frac{y - c}{d - c} & \text{for } c \leq y \leq d \\
1 & \text{for } y > d
\end{cases}
\]

and

\[
\mathbb{P}\{A\} = \int_{-\infty}^{y \to \infty} \mathbb{P}_y(y) \, I(y) \, dy
\]
But $I(y) = 0$ for $y < c$

and $P_y(y) = 0$ for $y < b$

and also $b < c$.

Therefore, $P\{A\} = 0$ as expected.
2. Case 2 -- Two Rectangular Distributions, Completely Overlapping

Consider:

\[
P_y(y) = \begin{cases} \frac{1}{b-a} & a \leq y \leq b \\ 0 & \text{otherwise} \end{cases}
\]

\[
P_x(x) = \begin{cases} \frac{1}{d-c} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}
\]

Using Eq (9) we calculate directly that:

\[
I(y) = \int_{x=y}^{x=\infty} P_x(x) \, dx = \begin{cases} 0 & \text{for } y < c \\ \frac{y-c}{d-c} & \text{for } c \leq y < d \\ 1 & \text{for } y > d \end{cases}
\]

and by a straightforward calculation,

\[
P\{A\} = \int_{x=y}^{x=\infty} P_y(y) I(y) \, dy = \frac{(b-d)}{(b-a)} + \frac{1}{2} \left( \frac{d-c}{b-a} \right)
\]

3. Special Case -- Identical Distributions

Suppose that \( d = b \) and \( c = a \). Then the result above reduces to:

\[
P\{A\} = \frac{1}{2}
\]
4. Special Case -- Symmetrically Located Distributions

Suppose that the two rectangular distributions are symmetrically located relative to each other. That is, $b-d = c-a$. Then the result for $P\{A\}$ in Case 2, again, reduces to

$$P\{A\} = \frac{1}{2}$$

5. Case 3 -- Two Rectangular Distributions, Partial Overlapping

Consider:

$$P_x(x) = \begin{cases} \frac{1}{d-c} & \text{for } c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$P_y(y) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq y \leq b \\ 0 & \text{otherwise} \end{cases}$$

Again, using Eq (9)

$$i(y) = \int_{x=-\infty}^{x=y} P_x(x) \, dx = \begin{cases} 0 & y < c \\ \frac{y-c}{d-c} & c \leq y \leq d \\ 0 & y > d \end{cases}$$

and

$$P_A = \int_{-\infty}^{\infty} P_y(y) \, I(y) = \left\{ \frac{(b-d)^2}{2(b-a)(d-c)} \right\}$$
or
\[
P\{A\} = \frac{1}{2} \left( \frac{b-c}{b-a} \right) \left( \frac{d-c}{d-c} \right) = \frac{1}{2} \left( \frac{\text{Length of Overlap}}{\text{Region Relative to Length of } b-a} \right) \left( \frac{\text{Length of Overlap}}{\text{Region Relative to Length of } d-c} \right)
\]

Using the approach described in the preceding section and assuming a fixed probability over the tolerance interval, we can obtain expressions for the mean values for the two alternate systems \( x \) and \( y \), and determine the probability that the central value of \( x > y \). The following figure describes this case.

\[
P\{A\} = \int_{y=-\infty}^{y=\infty} \int_{x=-\infty}^{x=\infty} P(y) P(x) \, dx \, dy
\]
evaluating \( x \) first.

Integrating by parts
\[
x < c \quad P(x) = 0
\]
\[
c < x < d \quad P(x) = \left( \frac{1}{d-c} \right)
\]
\[
x > d \quad P(x) = 0
\]

for \( y \)
\[
y < a \quad P(y) = 0
\]
\[
a < y < b \quad P(y) = \left( \frac{1}{b-a} \right)
\]
\[
b < y < d \quad P(y) = 0
\]
\[
y > d \quad P(y) = 0
\]
Now

\[ x = y \]

\[ I(y) = \int_{x=-\infty}^{x=+\infty} P(x) \, dx = \frac{y-c}{d-c} \]

\[ P\{A\} = \int_{y=\infty}^{y=+\infty} \frac{y-c}{d-c} \times \frac{1}{b-a} \, dy \]

From the evaluation of \( P(x) \) and \( P(y) \) by parts, the only interval in which both are real values other than zero, are \( b \) to \( c \)

\[ P\{A\} = \int_{y=c}^{y=b} \frac{y-c}{(d-c)(b-a)} \, dy = \left\{ \frac{(b-c)^2}{2(b-a)(d-c)} \right\} \]

An evaluation of this expression using the values in the figure

\[ p\{A\} = 0.35 \]

It is therefore concluded that because there is only a 35% chance that System \( x \) has a larger central value than System \( y \), we must treat them as essentially equal.
APPENDIX E  LAUNCH-ON-TIME ANALYSES

A. POST DELIVERY REQUIREMENTS

The postdelivery requirements are based on two recycles during prelaunch operations. Therefore, the probability of exceeding 2 recycles is considered in the launch-on-time analysis. This includes the following recycle conditions.

1) \( P_N \) = Probability of the payload surviving prelaunch operations with \( N \) recycles or less.

\[ P_{N-1} = \text{Probability of the payload surviving prelaunch operations with } N - 1 \text{ recycles or less (allow one recycle from launch pad).} \]

\[ P_0 = \text{Probability of no malfunctions in the payload during prelaunch and marriage tests during any one cycle (can recur) or recycle.} \]

2) \( P_N = P_0 + (1 - P_0) P_0 + (1 - P_0)^2 P_0 + \ldots + (1 - P_0)^N P_0 \)

\[ = 1 - (1 - P_0)^{N+1} \]

3) For \( N = 2 \), \( P_0 = 0.95 \):

\[ P_N = 1 - (0.05)^3 \times 0.999875 \]

\[ P_{N-1} = 1 - (0.05)^2 \times 0.9975 \]

4) Parametric variations of \( P_N \) are presented in Fig. 1. The above conditions are used in the following calculations for launch operations.

B. LAUNCH PAD OPERATIONS - CONFIGURATION 1

22-Day Launch Period

10-Day Turnaround

With Spare Spacecraft
Fig. 1 -- Probability of Recycle during Prelaunch and Marriage Tests

\[ N = \text{Number of Recycles} \]
1. **Payload**

Figure 2 presents the timeline showing all possible outcomes of the payload resulting in two launches within the launch period. Figure 3 presents the corresponding success/failure diagram. Let

\[ P = \text{the probability of achieving a launch on any one attempt (can recur)} \]

and

\[ (PLOT)_2 = \text{the probability of achieving two launches in the launch period} \]

The probability of achieving two launches because of the payload is the sum of the probabilities of the outcome shown in Fig. 2 and 3 as follows.

<table>
<thead>
<tr>
<th>Outcome No.</th>
<th>Probability of Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( P_N^2 P^2 )</td>
<td>( \text{Outcome No.} )</td>
</tr>
<tr>
<td>2 ( P_N^3 P^2 (1 - P) )</td>
<td>( \text{Probability of Outcome} )</td>
</tr>
<tr>
<td>3 ( P_N^3 P^2 (1 - P) )</td>
<td></td>
</tr>
<tr>
<td>4 ( P_N^3 P^2 (1 - P)^2 P_{N-1} P_O )</td>
<td></td>
</tr>
<tr>
<td>5 ( P_N^3 P^2 (1 - P)^2 P_{N-1} P_O )</td>
<td></td>
</tr>
</tbody>
</table>

\[
\sum = (PLOT)_2 = P_N^2 P^2 \left\{ 1 + 2P_N (1 - P) + 2P_N P_{N-1} P_O (1 - P)^2 \right\}
\]

Let \( N = 2, P_O = 0.95, \) and \( P = 0.95. \)

Then

\[
P_N = 0.999875
\]

and

\[
P_{N-1} = 0.9975
\]

See Section A
NOTE: 22-Day Launch Period,  
10-Day Turnaround,  
With Spare Launch Vehicle and Payload (LP)

<table>
<thead>
<tr>
<th>Outcome No.</th>
<th>Payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

- LAUNCH
- MALFUNCTION

Figure 2 -- Timeline for Configuration 1, Payload and Launch Vehicle
```
Figure 3 -- Configuration 1, Success/Failure Diagram, Payload and Launch Vehicle
\[(\text{PLOT})_2 = (0.999875)^2 (0.95)^2 \, 1 + 2(0.999875)(0.05) + 2(0.999875)(0.9975) (0.95)(0.0025)\]

\[(\text{PLOT})_2 = 0.996765\]

2. **Launch Vehicle**

Figures 2 and 3 also apply to the launch vehicle, except that there is no limitation on the number of recycles \((P_N\text{ and }P_{N-1}\text{ do not apply}).\)

<table>
<thead>
<tr>
<th>Outcome No.</th>
<th>Probability of Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(P^2)</td>
</tr>
<tr>
<td>2</td>
<td>(P^2 (1 - P))</td>
</tr>
<tr>
<td>3</td>
<td>(P^2 (1 - P))</td>
</tr>
<tr>
<td>4</td>
<td>(P^2 (1 - P)^2 P_0)</td>
</tr>
<tr>
<td>5</td>
<td>(P^2 (1 - P)^2 P_0)</td>
</tr>
</tbody>
</table>

\[(\text{PLOT})_2 = P^2 \left\{ 1 + 2(1 - P) + 2P_0 (1 - P)^2 \right\}\]

for \(P = 0.95, \quad P_0 = 0.95\)

\[(\text{PLOT})_2 = 0.997037\]

3. **All Other Causes**

Let \(P\) = probability of launching on any one attempt (can recur)

\(N_1\) = number of attempts required for the first launch

\(N_2\) = number of attempts available for the second launch

\[(\text{PLOT})_2 = \text{probability of achieving 2 launches in the launch period}\]
The number of attempts available for the second launch depends on the number of attempts used for the first launch. Therefore, the probability of achieving the second launch in \( N_2 \) attempts or less depends on the probability of the first launch being achieved in exactly \( N_1 \) attempts.

\[
\begin{align*}
P_N &= P_{N_1} \cdot P_{N_2} \\
\sum_{N} P_N &= P(1 - P)^{N-1} \\
\sum_{N} P_{N_2} &= 1 - (1 - P)^{N_2} \\
N_2 &= N_T - N_1
\end{align*}
\]

where \( N_T \) = total attempts available.

The probability of launching two on time is the sum of the probabilities of all outcomes

\[
(PLOT)_2 = \sum P_N
\]

The calculations are presented in Table 1 for \( P = 0.90 \).

From Table 1

\[
(PLOT)_2 = 0.999540
\]
### Table 1
Calculations for Probability of Two Launches, All Other Causes, Configuration 1

<table>
<thead>
<tr>
<th>Attempts at First Launch $N_1$</th>
<th>Attempts Available for Second Launch $N_2$</th>
<th>Probability of Achieving First Launch in $N_1$ Attempts</th>
<th>Probability of Two Launches in $N_1$ and $N_2$ Attempts</th>
<th>Probability of Achieving Second Launch in $N_2$ Attempts or Less</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.85</td>
<td>0.899910</td>
<td>0.000810</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.9999</td>
<td>0.000000</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.99</td>
<td>0.000000</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.99</td>
<td>0.000000</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Note: 22-Day Launch Period; 10-Day Turnaround

\[(PLOD)_2 = 0.995540\]
4. Results - Configuration #1

22-Day Launch Period

10-Day Turnaround (launch-to-launch)

With both Spare Launch Vehicle and Payload

<table>
<thead>
<tr>
<th></th>
<th>Probability of Launch-on-Time</th>
<th>Launch-on-Time Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload</td>
<td>0.996765</td>
<td>0.003235</td>
</tr>
<tr>
<td>Launch Vehicle</td>
<td>0.997037</td>
<td>0.002963</td>
</tr>
<tr>
<td>All Other Causes</td>
<td>0.999540</td>
<td>0.000460</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.993353</strong></td>
<td><strong>0.006647</strong></td>
</tr>
</tbody>
</table>
C. Launch Pad Operations, Configuration 2

30-Day Launch Period
16-Day Turnaround
No Spare Vehicle with Spare Payload

1. Payload

Figure 4 presents the timeline for all outcomes of the payload resulting in two launches. Figure 5 presents the corresponding success/failure diagram. Calculations are as follows.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability of Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P^2 P^2_N$</td>
</tr>
<tr>
<td>2</td>
<td>$P^2 P^3_N (1 - P)$</td>
</tr>
<tr>
<td>3</td>
<td>$P^2 P^3_N (1 - P)$</td>
</tr>
<tr>
<td>4</td>
<td>$P^2 P^3_N P_{N-1} P_0 (1 - P)^2$</td>
</tr>
</tbody>
</table>

$(PLOT)_2 = P^2 P^2_N \{ 1 + 2P_N (1 - P) + P_N P_{N-1} P_0 (1 - P)^2 \}$

For $P_0 = 0.95$, $P = 0.95$, $P_N = 0.999875$, $P_{N-1} = 0.9975$,

$(PLOT)_2 = 0.994627$

2. Launch Vehicle

Figure 6 presents the timeline and the success/failure diagram for the launch vehicle using Configuration 2 data. Calculations are as follows.
NOTE: 30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload (P)

Figure 4 -- Timeline for Configuration 2, Payload
Figure 5 — Success/Failure Diagram, Payload Configuration 2, Payload
NOTE: 30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload

Timeline

| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|    |    | TURNAROUND |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 2  |    | RECYCLE |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|    |    | RECYCLE LP |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

Success/Failure Diagram

Launch 1

Launch 2

Figure 6 -- Timeline and Success/Failure Diagram, Launch Vehicle
\[
\text{Outcome Pr \ babi lity of Outcome} \\
\text{1} \quad P^2 \\
\text{2} \quad P^2 \left\{ P_0 (1 - P) \right\} \\
\text{(FLOT)}_2 = P^2 \left\{ 1 + P_0 (1 - P) \right\} \\
\text{For } P = 0.95 \text{ and } P_0 = 0.95, \\
\text{(FLOT)}_2 = 0.945369 \\
\]

3. All Other Causes

Table 2 shows the number of launches available for the second launch as a function of the number of attempts used for the first launch and presents the calculations. From Table 2,

\[
\text{(FLOT)}_2 = 0.999945 \\
\]

4. Results, Configuration 2

30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability of Launch-on-Time</th>
<th>Launch-on-Time Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload</td>
<td>0.994627</td>
<td>0.005373</td>
</tr>
<tr>
<td>Launch Vehicle</td>
<td>0.945369</td>
<td>0.054631</td>
</tr>
<tr>
<td>All Other Causes</td>
<td>0.999945</td>
<td>0.000055</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.94038</td>
<td>0.059762</td>
</tr>
</tbody>
</table>
Calculations for Probability of Two Launches, All Other Causes, Configuration 2

<table>
<thead>
<tr>
<th>Attempts at First Launch ($N_1$)</th>
<th>Probability of Achieving First Launch in Exactly $N_1$ Attempts</th>
<th>Probability of Achieving Second Launch in $N_2$ Attempts or Less</th>
<th>Probability of Two Launches in $N_1$ and $N_2$ Attempts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.99999</td>
<td>0.899991</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.9999</td>
<td>0.089991</td>
</tr>
<tr>
<td>3</td>
<td>0.009</td>
<td>0.9998</td>
<td>0.008991</td>
</tr>
<tr>
<td>4</td>
<td>0.0009</td>
<td>0.999</td>
<td>0.000891</td>
</tr>
<tr>
<td>5</td>
<td>0.00009</td>
<td>0.99</td>
<td>0.000081</td>
</tr>
</tbody>
</table>

NOTE: 30-Day Launch Period, 16-Day Turnaround.

\[(P|L)|^2 = 0.99999\]
D. Launch Pad Operations, Configuration 3

30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle with Spare Payload

1. Payload

Figure 7 presents the timeline for Configuration 3. Figure 8 presents the corresponding success/failure diagram. The calculations are as follows.

<table>
<thead>
<tr>
<th>Outcome No.</th>
<th>Probability of Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p^2 p_N^2$</td>
</tr>
<tr>
<td>2</td>
<td>$p^2 p_N^3(1 - p)$</td>
</tr>
<tr>
<td>3</td>
<td>$p^2 p_N^3(p_{N-1} p_0)(1 - p)^2$</td>
</tr>
<tr>
<td>4</td>
<td>$p^2 p_N^3(1 - p)$</td>
</tr>
<tr>
<td>5</td>
<td>$p^2 p_N^3(p_{N-1} p_0)(1 - p)^2$</td>
</tr>
<tr>
<td>6</td>
<td>$p^2 p_N^3(p_{N-1} p_0)(1 - p)^2$</td>
</tr>
<tr>
<td>7</td>
<td>$p^2 p_N^3(p_{N-1} p_0)^2(1 - p)^3$</td>
</tr>
<tr>
<td>8</td>
<td>$p^2 p_N^3(p_{N-1} p_0)^2(1 - p)^3$</td>
</tr>
</tbody>
</table>
NOTE: 30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle with Spare Payload

Figure 7 -- Timeline for Configuration 3, Payload
Figure 8 -- Success/Failure Diagram, Payload
\[
(PLOT)_2 = P^2 P_N^2 \left\{ 1 + 2 P_N (1 - P) + 3 P_N (1 - P)^2 \right. \\
+ 2 P_N (P_{N-1} P_O)^2 (1 - P)^3 \right. \\
\]

For \( P = 0.95 \), \( P_N = 0.999875 \), \( P_{N-1} = 0.9975 \), and \( P_O = 0.95 \),
\[
(PLOT)_2 = 0.999104
\]

2. Launch Vehicle

Figure 9 presents the timeline for Configuration 3. Figure 10 presents the corresponding success/failure diagram. Calculations are as follows.

<table>
<thead>
<tr>
<th>Outcome No.</th>
<th>Probability of Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P^2 )</td>
</tr>
<tr>
<td>2</td>
<td>( P^2 P_O (1 - P) )</td>
</tr>
<tr>
<td>3</td>
<td>( P^2 P_O (1 - P) )</td>
</tr>
<tr>
<td>4</td>
<td>( P^2 P_O^2 (1 - P)^2 )</td>
</tr>
<tr>
<td>5</td>
<td>( P^2 P_O^2 (1 - P)^2 )</td>
</tr>
<tr>
<td>6</td>
<td>( P^2 P_O^3 (1 - P)^3 )</td>
</tr>
</tbody>
</table>

\[
(PLOT)_2 = P^2 \left\{ 1 + 2 P_O (1 - P) + 2 P_O^2 (1 - P)^2 + P_O^3 (1 - P)^3 \right. \\
\]

For \( P = 0.95 \) and \( P_O = 0.95 \),
\[
(PLOT)_2 = 0.992407
\]
Figure 9 -- Timeline for Configuration 3, Launch Vehicle

NOTE: 30-Day Launch Period
No Spare Launch Vehicle
NOTE: 30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle

Figure 10 -- Success/Failure Diagram, Launch Vehicle
3. All Other Causes

Table 3 shows the number of launch attempts available for the second launch as a function of the number used for the first launch. From Table 3,

\[(PLOT)_2 = 0.999999\]

4. Results, Current Baseline

30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle with Spare Payload

<table>
<thead>
<tr>
<th>Cause</th>
<th>Probability of Launch-on-Time</th>
<th>Launch-on-Time Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload</td>
<td>0.999104</td>
<td>0.000896</td>
</tr>
<tr>
<td>Launch Vehicle</td>
<td>0.992407</td>
<td>0.007593</td>
</tr>
<tr>
<td>All Other Causes</td>
<td>0.999999</td>
<td>0.0000001</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.991516</td>
<td>0.008484</td>
</tr>
</tbody>
</table>
Table 3
Calculation for All Other Causes

<table>
<thead>
<tr>
<th>Number of Attempts Used for First Launch ($N_1$)</th>
<th>Probability of Achieving First Launch in Exactly $N_1$ Attempts</th>
<th>Number of Attempts Available for Second Launch ($N_2$)</th>
<th>Probability of Achieving Second Launch in $N_2$ Attempts or Less</th>
<th>Probability of Achieving Two Launches in Exactly $N_1 + N_2$ or Less</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>7</td>
<td>0.9999999</td>
<td>0.89999999</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>6</td>
<td>0.999999</td>
<td>0.08999999</td>
</tr>
<tr>
<td>3</td>
<td>0.009</td>
<td>5</td>
<td>0.9999</td>
<td>0.00899999</td>
</tr>
<tr>
<td>4</td>
<td>0.0009</td>
<td>4</td>
<td>0.999</td>
<td>0.00089999</td>
</tr>
<tr>
<td>5</td>
<td>0.00009</td>
<td>3</td>
<td>0.99</td>
<td>0.00008999</td>
</tr>
<tr>
<td>6</td>
<td>0.000009</td>
<td>2</td>
<td>0.9</td>
<td>0.00000891</td>
</tr>
<tr>
<td>7</td>
<td>0.0000009</td>
<td>1</td>
<td>0.9</td>
<td>0.00000081</td>
</tr>
</tbody>
</table>

$$(PLOT)_2 = 0.999999927$$

NOTE: 30-Day Launch Period, 10-Day Turnaround.
## Appendix F  Monte Carlo Simulation Model

### Development of Storage Matrix

<table>
<thead>
<tr>
<th>State</th>
<th>UnTestable Parts a</th>
<th>Testable Parts b</th>
<th>BOTH GOOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>BOTH GOOD</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>a GOOD</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>b GOOD</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>NEITHER GOOD</td>
</tr>
</tbody>
</table>

\[
R_1 \text{ (Untestable Parts)} = e^{-\delta \tau_a}
\]

\[
R_2 \text{ (Testable Parts)} = e^{-\zeta \tau_b}
\]

<table>
<thead>
<tr>
<th>Finish in State</th>
<th>Start in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
| 2               | 0   | 0   | Q_2 = \epsilon^{-\zeta \tau_b}
| 3               | 0   | 0   | 0   | 1   |
| 4               | 0   | 0   | 0   | 1   |

\(\gamma\) = Storage Failure Rate

\(\delta\) = Untestable Parts

\(\zeta\) = Testable Parts

\(\tau\) = Time of Storage
DEVELOPMENT OF TEST MATRIX

\[ D = \text{PROBABILITY OF NOT CAUSING A DETECTABLE FAILURE} \]

\[ N = \text{PROBABILITY OF NOT CAUSING AN UNDETECTABLE FAILURE} \]

\[ P = \text{PROBABILITY OF DETECTING A DETECTABLE FAILURE} \]

<table>
<thead>
<tr>
<th>START IN STATE</th>
<th>FINISH IN STATE</th>
<th>GOOD</th>
<th>DETECTABLE</th>
<th>UNDETECTABLE</th>
<th>DET. &amp; UNDET.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GOOD</td>
<td>1 GOOD</td>
<td>DN</td>
<td>(1-D) N</td>
<td>(1-N) D</td>
<td>(1-\sum) = 1-D-N + ND</td>
</tr>
<tr>
<td>2 DETECT.</td>
<td>2 DETECT.</td>
<td>DNP</td>
<td>N(1-PD)</td>
<td>PD(1-N)</td>
<td>(1-\sum) = 1-N-DP-PDN</td>
</tr>
<tr>
<td>3 UNDET.</td>
<td>3 UNDET.</td>
<td>0</td>
<td>0</td>
<td>D</td>
<td>1-D</td>
</tr>
<tr>
<td>4 DET. &amp; UNDET.</td>
<td>4 DET. &amp; UNDET.</td>
<td>0</td>
<td>0</td>
<td>PD</td>
<td>1-PD</td>
</tr>
</tbody>
</table>

EXAMPLES OF MATRIX CONSTRUCTION

START IN STATE 1, FINISH IN STATE 3, OR GOOD AT START, UNDETECTABLE FAILURE AT FINISH

\[ D = \text{PROBABILITY OF NOT CAUSING A DETECTABLE FAILURE} \]

\[ 1-N = \text{PROBABILITY OF CAUSING AN UNDETECTABLE FAILURE} \]

STATE

\[ D \times (1-N) = 1-N \times D \]

116
START IN STATE 2, FINISH IN STATE 1 OR
DETECTABLE FAILURE AT START, GOOD AT FINISH.

\[
\text{STATE 2} \quad P \quad N \quad D
\]
\[
\text{DETECTABLE FAILURE} \quad \text{DETECT DET. FAILURE} \quad \text{DO NOT CAUSE UNDET. FAILURE} \quad \text{DO NOT CAUSE DET. FAILURE} \quad \text{STATE 1}
\]

\[= PND\]

START IN STATE 2, FINISH IN STATE 2 OR
DETECTABLE FAILURE AT START, DETECTABLE
FAILURE AT FINISH.

\[
\text{STATE 2} \quad P \quad N \quad \text{STATE}
\]
\[
\text{DETECTABLE FAILURE} \quad \text{DO NOT DET. DET. FAILURE} \quad \text{DO NOT CAUSE UNDET. FAILURE} \quad \text{STATE 2}
\]
\[
\text{DETECT. DET. FAILURE} \quad \text{DO NOT CAUSE UNDET. FAILURE} \quad \text{CAUSE NEW DET. FAIL.} \quad \text{STATE 2}
\]
\[
\quad \quad \text{1-P} \quad \text{N} \quad \text{1-D}
\]

\[= (1-P)N + PN(1-D) = N(1-PD)\]

NOTE: SINCE ALL ROW SUMS IN A STOCHASTIC MATRIX = 1,
THE VALUE OF THE ENTRY IN COLUMN 4 OF EACH ROW
IS CALCULATED BY SUBTRACTING THE SUM OF ENTRIES
1, 2, and 3 OF EACH ROW FROM ONE.
BIBLIOGRAPHY

1. Systems Reliability, Systems Engineering, and Measures of Reliability Effectiveness


Reliability Notebook, Rome Air Development Center, Supplement 2, December 31, 1961


Hald, A., Statistical Tables and Formulas, John Wiley and Sons, New York, 1958

II. Markov Processes, Probability Theory Optimization of Reliability
Parameters, Availability, and Allocation of Redundancy

Feller, W., An Introduction to Probability Theory and Its
New York, 1957

Parzen, E., Modern Probability Theory and Its Applications,

Bharucha-Ried, A. T., Elements of the Theory of Markov-
Processes and Their Applications, McGraw-Hill Book Co., Inc.,
New York, 1953

Doob, J. L., Stochastic Processes, John Wiley & Sons, Inc.,
New York, 1953

Loeve, M., Probability Theory, D. Van Nostrand Co., Inc.,
Princeton, N. J., 1955

Tables of the Binomial Probability Distribution, National
D.C., 1949

Cramer, H., Mathematical Methods of Statistics, Princeton
University Press, Princeton, N. J., 1946

Mood, A. M., Introduction to the Theory of Statistics,

Dynkin, E. B., Theory of Markov Processes translated from
Russian by Brown, D. D., Prentice-Hall, Inc., Englewood
Cliffs, N. J., 1961

Kemeny, J. G. and Snell, J. L., Finite Markov Chains, D.

Bellman, R., Introduction to Matrix Analysis, McGraw-Hill
Book Co., Inc., New York, 1960

Fraser, W. A., Duncan, N. J., and Collar, A. R., Elementary
Matrices and Some Applications to Dynamics and Differential
Equations, Cambridge University Press, London, 1938

Chernoff H. and Moses L. E., Elementary Decision Theory,
John Wiley & Sons, Inc., New York, 1959


Lipp, J. P., "Topology of Switching Elements Versus Reliability," IRE Transactions on Reliability and Quality Control, PGRQC-10, June, 1957


Balaban, H., "Some Effects of Redundancy on System Reliability," Sixth National Symposium on Reliability and Quality Control in Electronics, Wash., D. C., Jan., 1960


Derman, C., "Some Asymptotic Distribution Theory for Markov Chains with a Denumerable Number of States," Biometrika, Vol. 43, 1956


Morse, P. M. M., Queues, Inventories and Maintenance, John Wiley and Sons, Inc., New York, 1958


Rutenberg, Y. H., "Sequential Decision Models" Case Institute of Technology, April, 1961


Dennis, J. E., Mathematical Programming and Electrical Networks, John Wiley and Sons, 1960


Shewhart, W. A., Economic Control of Quality of Manufactured Product, Van Nostrand Co., Princeton, New Jersey, 1931


Reliability Notebook, Rome Air Development Center, Supplement 2, December 31, 1961


Wong, K., "Use of Redundancy," Hughes Aircraft Company Reliability Guide, Section 7.0, August 18, 1960

Myers, R. H., "Which Road to Satellite Reliability," Hughes Aircraft Company Report R-60-2, September 1, 1960


Reliability Models of Maintained and Non-Maintained Systems, and System Maintenance Policies


Morse, P. M. M., Queues, Inventories and Maintenance, John Wiley & Sons, Inc., New York, 1958


Barlow, R. E. and Proschan, F., "Planned Replacement," DI-82-0102, Boeing Scientific Research Laboratories, April, 1961


IV. Military Reliability and Availability Requirements


RADC Reliability Notebook, Section 8, Rome Air Development Command, December 31, 1961


Thomas, R. E., A. R. Fish, H. M. Braner, and W. E. Chapin, Development of a Methodology for Screening Electronic Parts by Using Linear Discriminants," Battelle Memorial Institute Electronic Component Reliability Center


Raytheon Company Industrial Components Division Report, "Reliability Assurance Program," Figure 1, p. 19


Hyatt Roller Bearings, Catalog 150, Hyatt Bearings Division, General Motors Corporation, Harrison, New Jersey


MIL-STD-441, "Reliability of Military Electronic Equipment," June 20, 1958


MIL-R-22256 (AER), "Reliability Requirements for Design of Electronic Equipment or Systems," November 5, 1959

MIL-R-22732 (Ships), "Reliability Requirements for Shipboard and Ground Electronic Equipment," March 10, 1961

MIL-R-26474 (USAF), "Reliability Requirements for Production Ground Electronic Equipment," June 10, 1959

MIL-R-26484 (USAF), "Reliability Requirements for Development of Electronic Subsystems or Equipment," June 2, 1958


MIL-R-27070 (USAF), "Reliability Requirements for Development of Ground Electronic Equipment," March 25, 1960

MIL-R-27173 (USAF), "Reliability Requirements for Electronic Ground Checkout Equipment," July 6, 1959

MIL-R-27542 (USAF), "Reliability Program Requirements for Aerospace Systems, Subsystems, and Equipment," June 28, 1961

Sylvania, Design Review Manual, Electronic Systems Division, Mountain View, California. Note: The bulk of the material in this manual was developed at Sylvania by R. S. Cazanjian with the invaluable support of David Ehrenpreis, Consulting Engineer, who provided much of the original thought and effort.


NAVEPS 16-1-519, "Handbook of Preferred Circuits, Navy Aeronautical Electronic Equipment," National Bureau of Standards, Department of Commerce for Bureau of NavWeps, Department of the Navy, September 1, 1955, Supplements 1, 2 and 3

Hurley, R., Junction Transistor Electronics, John Wiley and Sons, New York, 1958


Reliability Notebook, Rome Air Development Center, Supplement 2, December 31, 1961, Section 7

MIL-R-27542 (USAF), Reliability Program Requirements for Aerospace Systems, Subsystems and Equipments, June 28, 1961

MIL-STD-105C, Sampling Procedures and Tables for Inspection by Attributes, July 18, 1961
MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective, July 11, 1957


Document Number 2, Vol. 1, Battelle Memorial Institute, February 15, 1961, pp. 191 - 194


Ashby, R. M., "A Research and Development View of High Reliability," Transactions of the 1961 Reliability Symposium, Los Angeles, California, Sponsored by the American Society for Quality Control


MIL-R-26667A (USAF), Reliability and Longevity Requirements Electronic Equipment, General Specification, June 2, 1959


AMCP 74-1A, Reliability Evaluation Procedures for Pilot Production and Production, February 1, 1961
Reliability Practices, Reliability Production and Testing


Tong, K. N., Theory of Mechanical Vibrations, John Wiley and Sons, 1960


Office of Secretary of Defense, Bulletins of Symposium on Shock, Vibration and Associated Environments, Bulletins February 24, 1957; December 25, 1957; June 27, 1958; August 28, 1960; June 29, 1961


Yarwood, J., High Vacuum Technique, John Wiley and Sons, 2nd ed., revised 1955

Scott, R. B., Cryogenic Engineering, Van Nostrand, Princeton, New Jersey, 1959


Sissenwine, N. and A. Court, "Climatic Extremes for Military Equipment," Report No. 146 OQMO, Department of Army, November, 1951


Carrier Corporation, "Psychrometric Chart, Normal Temperatures," Copyright 1946

MIL-STD-441, "Reliability of Military Electronic Equipment," June 20, 1958

MIL-R-27542 (USAF), "Reliability Program Requirements for Aerospace Systems, Subsystems, and Equipments," June 28, 1961

MIL-R-22256 (aER), "Reliability Requirements for Design of Electronic Equipment or Systems," November 20, 1959

MIL-R-22732 (SHIPS), "Reliability Requirements for Shipboard and Ground Electronic Equipment," March 10, 1961

MIL-R-26474 (USAF), "Reliability Requirements for Production Ground Electronic Equipment," June 10, 1959

MIL-R-26484 (USAF), "Reliability Requirements for Development of Electronic Subsystems or Equipment," April 18, 1960

MIL-R-26667A, "Reliability and Longevity Requirements, Electronic Equipment, General Specification For, June 2, 1959

MIL-R-27173 (USAF), "Reliability Requirements for Electronic Ground Checkout Equipment," July 6, 1959


Molina, E. C., Poisson's Exponential Binomial Limit Tables, Van Nostrand, Princeton, New Jersey, 1942