STATISTICAL COMPLEX FATIGUE DATA FOR SAE 4340 STEEL AND ITS USE IN DESIGN BY RELIABILITY

by Dr. Dimitri Kececioglu and John L. Smith

Prepared under Grant No. 03-002-044 by The University of Arizona

Tucson, Arizona

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D. C.
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ABSTRACT

A brief description of the complex fatigue machines used in the test program is presented. The data generated from these machines are given and discussed. Two methods of obtaining strength distributions from the data are also discussed. Then follows a discussion of the construction of statistical fatigue diagrams and their use in designing by reliability. Finally, some of the problems encountered in the test equipment and a corrective modification are presented.
CHAPTER I
INTRODUCTION

1.1 Background on Testing Program

The purpose of the test program is to generate cycle-to-failure data by testing rotating specimens subjected to a bending moment and a constant torque. The test machines were designed and built at the University of Arizona.

The following is a brief discussion of the test machines. For a more complete discussion of the design and development of the test machines see Ref. 1. Presently there are three machines at the University of Arizona.

The test specimen is subjected to a bending moment by weights hung at the end of a lever arm. Fig. 1.1 shows a schematic diagram of the machine illustrating the loading geometry. The torque is applied by turning the torque coupling, which rotates shaft A with respect to shaft B and holds the relative position of the shafts. After the specimen is tightened in the holding collet, the adjusting nut on the torque coupling can be turned while holding shaft A from rotating. This will rotate shaft B with respect to shaft A. It requires a full 360 degree turn of the adjusting nut to rotate shaft B one degree with respect to shaft A. Once the play in the gear boxes is taken up
Fig. 1.1 Schematic Diagram of Test Machines Showing the Loading Configuration
then torque is exerted on the specimen.

The magnitudes of the shear stress and the normal stress that the specimen experiences are related to the stresses which the toolholder experiences. Two strain gage bridge circuits are mounted on the toolholder in the position shown in Fig. 1.1. Acting through an amplifier the output of one bridge is a measure of the normal stress and the output of the other bridge is a measure of the shear stress that the toolholder experiences. The outputs are in the form of a Visicorder record. The strain gage outputs and how they are used to determine the stresses in the specimen will be further discussed in Sect. 2.1.

Since the machines are capable of subjecting a specimen to combined stresses, tests can be conducted for any combination of alternating and mean stresses. The alternating stress is in the form of a normal stress caused by a constant bending moment on the rotating specimen. Any element of volume located at the surface of the specimen will experience an alternating tension and compression of equal magnitude. If torque is applied the element will also experience a constant shear stress perpendicular to the normal stress. Figure 1.2 show the stress element.

Early in the research program a study was made of the various failure theories. On the basis of this study it was concluded that the von Mises-Hencky theory most closely predicts fatigue failures in steel (1, p. 41).
Fig. 1.2 Stress Element on Surface of Test Specimen

Shigley (2, P. 185) makes the point that the von Mises-Hencky theory (also called the distortion-energy theory) was developed to predict yield under static loads. However, because of the good agreement between fatigue data and the theory it was accepted as the failure governing criterion for the research program.

In order to have some measure of the relative magnitudes of the alternating and mean stresses a stress ratio was defined in terms of the von Mises stresses (2, P. 188). For an ordinary element subjected to bi-axial stresses the von Mises stresses are given by:
\[ S_a = \left[ S_{xa}^2 - S_{xa} S_{ya} + S_{ya}^2 + 3 \tau_{xya}^2 \right]^{\frac{1}{2}} \\
S_m = \left[ S_{xm}^2 - S_{xm} S_{ym} + S_{ym}^2 + 3 \tau_{xym}^2 \right]^{\frac{1}{2}} \]

For the case of an alternating normal stress along the \( x \) axis and a constant shear stress perpendicular to the \( x \) axis, the above equations reduce to

\[ S_a = S_{xa} \quad (1) \]
\[ S_m = \sqrt{3} \tau_{xym} \quad (2) \]

The stress ratio is defined as

\[ R = \frac{S_a}{S_m} = \frac{S_a}{\sqrt{3} \tau_{xym}} \quad (3) \]

where

\[ R = \text{stress ratio} \]
\[ S_a = \text{alternating normal stress} \]
\[ S_m = \text{mean normal stress} \]
\[ \tau_{xym} = \text{mean shear stress}. \]

The specimens being used for the test program are made of SAE 4340 steel, condition C-4, heat treated to 35 - 40 R. The specimens were all manufactured from the same heat. Fig. 1.3 shows the specimen geometry.

The ultimate purpose of the test program is to develop statistical fatigue diagrams which can be used to design a specified reliability into a rotating shaft subjected to a bending moment and a constant torque for a specified life.
Fig. 1.2 Test Specimen

725 ± .005 DIA.  
.145 ± .005 RAD.

M

+ .005

- .005 DIA.

6

3

M

- .005

+.000

- .005
In order to secure data to construct the statistical fatigue surfaces the following test plan was proposed (1, pp. 132-138):

1. Tests are to be conducted at various alternating stress levels, holding the stress ratio constant, to determine the cycles-to-failure distributions at the various levels. Tests, of 18 specimens at each level, are to be conducted at 4 to 6 different stress levels for each stress ratio. This data can then be plotted on log-log paper with stress level on the ordinate and cycles-to-failure on the abscissa. The resulting diagram will look like fig. 1.4. Such a diagram can be obtained for each stress ratio except R = 0.

2. Once the cycles-to-failure diagrams are obtained then the strength distributions for specified lives would be obtained and these distributions used in constructing the fatigue diagrams to be used to design by reliability.

1.2 Data Reduction

The data as taken from the testing machines is in the form of a Visicorder record which contains two traces for each specimen. One trace records the amplitude of the alternating normal stress as seen by the strain gages on the
Fig. 1.4 S-N Diagram Showing Cycles-to-Failure Distributions for a Specific Non-Zero Stress Ratio.
toolholder and the other trace records the magnitude of the constant shear stress. From these records the nominal normal stress and average nominal shear stress can be obtained for all the specimens which were tested at a given level. The determination of these stress levels from the Visicorder records is discussed in Section 2.1.

The terms "nominal normal stress" and "nominal shear stress" refer to the stresses the specimen would experience at the outermost fiber of the cross section if it did not contain a stress riser; i.e., if the test section were of a constant diameter and that diameter were equal to the diameter across the base of the groove in the present specimen.

Section 2.2 discusses the methods employed in determining the endurance level at the various stress ratios.

Sections 2.3, 2.4 and 2.5 deal with the calculation of the mean and standard deviation, and the coefficients of skewness and kurtosis of both the cycles-to-failure data and the natural logarithms of the cycles-to-failure data.

Two computer programs were developed to aid in the reduction of the test data:

1. A program to reduce the Visicorder records to stresses and stress ratios.
2. A program to calculate the mean and standard deviation of the cycles-to-failure data and test how well the data fits the normal distribution. The program also makes these calculations for the log-normal distribution.

The programs are discussed in Appendices A and B. Included in the discussion is a complete description of the input data card formats for each deck and a flow chart for each program.

The first program mentioned above was developed by this author whereas the second was previously written but was modified to be used with the data of this research program.

1.3 Goodness-of-Fit Tests

Section 2.6.1 discusses the Chi-square goodness-of-fit test and its applicability to the test data. Section 2.6.2 discusses the Kolmogorov-Smirnov goodness-of-fit test and its applicability.

1.4 Generation of Statistical Fatigue Diagrams From Test Data

The generation of the statistical fatigue diagrams requires that the strength distributions at various cycles of life be known. The test plan that was proposed at the beginning of the research effort was set up with the objective of obtaining test data from which estimates could
be made of these strength distributions. Section 2.8 discusses the proposed method and its limitations. In the same section another test plan and method is proposed for obtaining the required distribution based on actual data.

Section 2.8.1 discusses how to use statistical fatigue diagrams to design a shaft to a specified reliability. The assumptions and limitations of the method are also discussed.

1.5 Modification of One Test Machine

With the approval of Dr. Kececioglu and Mr. Vincent R. Lalli of NASA-Lewis this author made a modification to one of the three test machines in December, 1969. It is believed that this modification will eliminate some of the problems encountered with the machines. None of the data presented in this report was obtained from the modified machine. Details of the modification are presented in Section 2.9.
CHAPTER II

DATA REDUCTION

2.1 Determination of Stress Levels and Stress Ratios

As stated in the introduction, the raw data is in the form of a Visicorder record containing two traces. Figure 2.1 is an illustration of what such a record looks like. The traces are a measure of the alternating normal stress and constant shear stress as seen by the strain gages which are mounted on the machine toolholder. The values of these traces, in terms of divisions, can be converted into values of nominal normal stress and nominal shear stress in the groove of the specimen. However, to be able to make this conversion several constants must be known which relate the output of the strain gages on the toolholder to the nominal stresses in the groove of the specimen. The calibrations required to obtain these constants are quite extensive and since the original calibrations were not conducted by this author only a brief discussion will be undertaken here. For a complete discussion of the calibration procedure see Ref. 3.

There are five constants required to make the conversion from divisions on the Visicorder record to nominal
Fig. 2.1 Illustration of a Visicorder Record Showing the Bending, Torque and Calibration Traces
stresses in the test specimen. They are:

\[ K_{bgr} = \frac{\sigma_{os}}{\sigma_{as}} = \text{constant to relate the output of the strain gages on the specimen to the actual stress in the specimen.} \]

\[ K_{gr-th} = \frac{\sigma_{ot}}{\sigma_{os}} = \text{constant to relate the output of the bending strain gages on the toolholder to the output of the strain gages on the specimen.} \]

\[ K_t = \frac{\tau_{as}}{\tau_{ot}} = \text{constant to relate the actual shear stress in the specimen to the output of the torque strain gages on the toolholder.} \]

\[ K_{t/b} = \frac{\sigma_{ot}}{\tau_{ot}} = \text{constant to relate the output of the bending gages on the toolholder to the output of the torque gages on the toolholder when the toolholder is subjected to a changing torque while the bending moment is held constant.} \]

\[ K_{b/t} = \frac{\tau_{ot}}{\sigma_{ot}} = \text{constant to relate the output of the torque gages on the toolholder to the output of the bending gages on the toolholder when the toolholder is subjected to a changing bending moment while the torque is held constant.} \]

where

\[ \sigma_{os} = \text{normal stress in the specimen as indicated by the output of strain gages on the test specimen.} \]
\( \sigma_{as} = \) actual normal stress in the specimen.

\( \sigma_{ot} = \) normal stress in the toolholder as indicated by the strain gages on the toolholder.

\( \tau_{as} = \) actual shear stress in the specimen.

\( \tau_{ot} = \) shear stress as indicated by the strain gages on the toolholder.

The last two constants listed are interaction constants. Due to misalignment, a bending gage may record an output when the shaft is subjected to pure torsion. Likewise, a misaligned torque gage may react to a pure bending on the shaft. These two constants take this interaction into account.

Whenever a strain gage on the toolholder is replaced or when a change is made to the machine that alters its loading characteristics some of the calibration constants may change. Therefore, whenever such changes are made, calibration tests must be performed to obtain corrected values for the constants that are affected by the change. Changes that affected the calibration constants were made at three different times throughout the test program. In order to know what constants were in effect during given time periods each time period was designated as a mode of operation. The values of these constants are given in Appendix C.
Figure 2.2 shows the sequence of calculations required to convert the strain gage outputs, as recorded by the Visicorder, to nominal stresses in the groove of the test specimen. The meanings of the symbols used in Fig. 2.2 which have not been defined earlier in this section are:

\[ N_{cb} = \] number of Visicorder record divisions used when adjusting the gain in the bending channel of the amplifier.

\[ N_{ct} = \] number of Visicorder record divisions used when adjusting the gain in the torque channel of the amplifier.

\[ R_{cb} = \] value in ohms of the calibrating resistance used when adjusting the gain in the bending channel amplifier.

\[ R_{ct} = \] value in ohms of calibrating resistance used when adjusting the gain in the torque channel amplifier.

\[ N_{vb} = \] amount of deflection, in Visicorder divisions, caused by the toolholder bending strain gage bridge output when load is applied to the specimen.

\[ N_{vt} = \] amount of deflection, in Visicorder divisions, caused by the toolholder torque strain gage bridge output when load is applied to specimen.
Output bending stress in toolholder from Visicorder divisions
\[ \sigma'_{ot} = \frac{N_{vb} E R_{rb}}{N_{cb} N_a G_b R_{cb}} \]

Output bending stress in toolholder corrected for torque interaction
\[ \sigma_{ot} = \sigma'_{ot} - K_t/b \tau'_{ot} \]

Output shear stress in toolholder from Visicorder divisions
\[ \tau'_{ot} = \frac{N_{vt} E R_{ct}}{N_{ct} N_a G_t R_{ct}} \]

Output shear stress in toolholder corrected for bending interaction
\[ \tau_{ot} = \tau'_{ot} - K_b/t \sigma_{ot} \]

Actual nominal bending stress in specimen groove
\[ \sigma_{as} = \frac{\sigma_{ot}}{K_{gr-th} K_{bgr}} \]

Actual nominal shear stress in specimen groove
\[ \tau_{as} = K_t \tau_{ot} \frac{J_{th} C_{rr}}{C_{th} J_{gr}} \]

Stress Ratio
\[ R = \frac{\sigma_{as}}{\sqrt{3} \tau_{as}} \]

Fig. 2.2 Flow Chart of Calculations to Obtain Nominal Stresses in the Specimen Groove From the Strain Gage Outputs.
\[ E = \text{modulus of elasticity for steel} \]
\[ = 30 \times 10^6 \text{ psi.} \]

\[ R_{gb} = \text{resistance of each bending gage} \]
\[ = 190 \text{ ohms.} \]

\[ R_{gt} = \text{resistance of each torque gage} \]
\[ = 120 \text{ ohms.} \]

\[ G_b = \text{bending gage factor} \]
\[ = 3.23 \]

\[ G_t = \text{torque gage factor} \]
\[ = 2.05 \]

\[ N_a = \text{number of active gages in each bridge} \]
\[ = 4 \]

\[ J_{th} = \text{polar moment of inertia of the toolholder cross section where the gages are mounted.} \]

\[ C_{th} = \text{radius of the toolholder where the gages are mounted.} \]
\[ = 1.000 \text{ inches.} \]

\[ J_{gr} = \text{polar moment of inertia of the specimen cross section at the base of the groove.} \]

\[ C_{gr} = \text{radius of the specimen at the base of the groove.} \]
\[ = .249 \text{ inches.} \]

In Fig. 2.2 the equation for \( \sigma_{ot} \), in the second
box on the left, is an approximation. The full equation is:

$$\sigma_{ot} = \frac{\sigma_{ot}^' - K_{t/b} \tau_{ot}^'}{1 + K_{t/b} K_{b/t}}$$  \hspace{1cm} (4)$$

Upon examination of the values of $K_{t/b}$ and $K_{b/t}$ in Appendix C, it is seen that they are small and the product is negligible compared to 1. Thus, Eq. (4) reduces to the one given in Fig. 2.2.

A program for the CDC 6400 computer at The University of Arizona using FORTRAN IV was developed to determine $\sigma_{as}$, $\tau_{as}$, and $R$, as per Fig. 2.2, from the data. The program, along with user instructions and flow chart, is presented in Appendix A.

Table 2.1 lists the average stress levels and ratios at which test data were collected. The computer outputs listing the stresses in the individual specimens are given in Appendix D.

2.2 Stress-to-Failure Data (Staircase Method)

The method used for testing specimens to determine the distribution of the endurance strength was the staircase method (4, p. 48; 5) sometimes called the "up and down method". Briefly, the method consists of testing a specimen subjected to a stress equal to the estimated endurance strength. If the specimen fails, the next specimen is subjected to a stress one increment lower than the failed specimen. If the specimen does not fail by a
TABLE 2.1

TABLE SHOWING THE AVERAGE STRESS LEVELS AND RATIOS AT WHICH SPECIMENS WERE TESTED

<table>
<thead>
<tr>
<th>Average Stress Ratio</th>
<th>Average Normal Stress* psi</th>
<th>Standard Dev. Normal Stress**</th>
<th>Number of Specimens Tested</th>
<th>Average Shear Stress***</th>
<th>Standard Dev. of Shear Stress**</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>144,000</td>
<td>1500</td>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>114,000</td>
<td>900</td>
<td>18</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>98,000</td>
<td>2700</td>
<td>18</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>81,000</td>
<td>900</td>
<td>18</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>73,000</td>
<td>1800</td>
<td>18</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.5</td>
<td>151,000</td>
<td>3850</td>
<td>12</td>
<td>25,000</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>115,000</td>
<td>1820</td>
<td>18</td>
<td>19,500</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>83,000</td>
<td>1200</td>
<td>18</td>
<td>13,500</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>74,000</td>
<td>750</td>
<td>18</td>
<td>12,500</td>
<td>600</td>
</tr>
<tr>
<td>0.825</td>
<td>111,000</td>
<td>1250</td>
<td>12</td>
<td>73,500</td>
<td>2050</td>
</tr>
<tr>
<td></td>
<td>92,000</td>
<td>6700</td>
<td>18</td>
<td>66,500</td>
<td>3400</td>
</tr>
<tr>
<td></td>
<td>76,000</td>
<td>3150</td>
<td>18</td>
<td>53,500</td>
<td>3450</td>
</tr>
<tr>
<td></td>
<td>65,000</td>
<td>3850</td>
<td>18</td>
<td>47,000</td>
<td>1300</td>
</tr>
<tr>
<td>0.44</td>
<td>69,000</td>
<td>1400</td>
<td>18</td>
<td>90,000</td>
<td>1600</td>
</tr>
<tr>
<td></td>
<td>60,000</td>
<td>700</td>
<td>18</td>
<td>78,500</td>
<td>1400</td>
</tr>
</tbody>
</table>

* Rounded off to nearest 1,000 psi.  ** Rounded off to nearest 50 psi.

*** Rounded off to nearest 500 psi.
predetermined time the test is terminated and the next specimen is subjected to a stress one increment higher. This procedure continues until the desired sample size is obtained. It should be noted that only approximately 50% of the tested specimens are used in the calculations of mean and standard deviation. The calculations are based on either the successes or failures, whichever has occurred the least number of times. The specimen just preceding the first change of mode is considered as the beginning of the test. A change of mode is a success followed by a failure or a failure followed by a success.

The equations for calculating the estimates of the mean and standard deviation of the endurance strength are given by (5, p. 114).

\[ m = y + d \left( \frac{A}{N} \pm \frac{1}{2} \right) \]  \hspace{1cm} (5)

and

\[ s = 1.620 \, d \left( \frac{NB-A^2}{N^2} + 0.029 \right) \]  \hspace{1cm} (6)

where

- \( m \) = mean
- \( s \) = standard deviation
- \( y \) = the lowest stress level at which a success or failure (whichever the analysis is based on) occurred.
- \( d \) = stress increment
- Sample size is large when
N = effective sample size; i.e., the number of specimens used in the calculations.

\[ A = \sum_{i=0}^{n} i \, n_i \]

\[ B = \sum_{i=0}^{n} i^2 \, n_i \]

n = number of successes (or failures) which occurred at the \( i^{th} \) level.

The lowest level is considered the zeroth level, the next the 1st level, etc. In Eq. (5) above the (+) is used if the calculations are based on successes and the (-) if based on failures.

The above analysis requires that the variate being tested is assumed to be normally distributed or can be transformed to a normal distribution (5, p. 111). Also, the stress increment should be in the range of \( .5\sigma \) to \( 2\sigma \), where \( \sigma \) is the standard deviation of the distribution. Therefore, some prior knowledge of the variance is helpful for good results.

The staircase method is very good for the determination of the mean of the variate being tested. However, the estimate of the variance can be poor if the sample size is not large. Mood and Dixon state that (5, p. 112), "Measures of reliability may well be very misleading if the sample size is less than forty or fifty." It is not clear,
however, whether they are referring to effective or actual sample size, the effective sample size being approximately 50\% of the actual sample size. Because of research program limitations the effective sample sizes of the endurance tests range from 16 to 18.

Endurance tests were conducted at $R = \infty$, 3.5, 1.0 and 0.44. As stated before, the tests of specimens which do not fail are terminated at a predetermined time. For stress ratio of $\infty$ the tests were terminated after 90 hours which is equivalent to more than 9.5 million cycles. In conducting the endurance tests at stress ratio of $\infty$, of the eighteen specimens which failed, sixteen failed before 48 hours of running time.

For stress ratio of 1.0 the tests were terminated at 48 hours which is equivalent to over 5 million cycles. Of the eighteen specimens which failed during the endurance tests at $R = 1.0$ all failed before 24 hours of running time. The endurance tests at $R = 3.5$ were terminated at 24 hours which represents more than 2.5 million cycles.

The endurance tests at $R = 0.44$ have not been completed as of the date of this report. They are also being terminated at 24 hours of running time.

The calculation of the means and standard deviations from the staircase data will be discussed separately in the next two sections. The results of those calculations are
given in Table 2.2

<table>
<thead>
<tr>
<th>Stress Ratio</th>
<th>Mean* (psi)</th>
<th>Standard Deviation**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>57,000</td>
<td>3,800</td>
</tr>
<tr>
<td>3.5</td>
<td>55,000</td>
<td>3,700</td>
</tr>
<tr>
<td>1.0</td>
<td>57,000</td>
<td>3,300</td>
</tr>
</tbody>
</table>

Estimates are based on the calculations discussed in Sections 2.2.1 and 2.2.2.

* Rounded off to nearest 1,000 psi.

** Rounded off to nearest 100 psi.
2.2.1 Endurance Tests for $R = \infty$ and $R = 1.0$

The tests were based on the amount of weight placed on the loading arm. An increment of one pound in the pan was used to obtain the staircase. The calculations for mean and standard deviation were also based on weight and these values were then converted to psi.

Figures 2.3 and 2.4 show the staircase plots for the tests. They were taken from an earlier report (6, pp. 48-49). Tables 2.3 and 2.4 list the specimens that were used in the endurance level calculations for $R = \infty$ and $R = 1.0$ respectively.

To obtain a better estimate of the endurance strength distribution the average of the stresses in all the specimens for a given pan weight was found and then the increment between these averages for the different pan weights was calculated. In order to obtain a uniform increment an average was found for the increments.

For stress ratio of $\infty$ the calculations are based on successes. The average stress at each pan weight and the number of specimens which succeeded at each level is:

<table>
<thead>
<tr>
<th>Pan Weight pounds</th>
<th>Stress psi</th>
<th>Number of Successes</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>61,857</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>59,999</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>54,430</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>51,915</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>59,421</td>
<td>1</td>
</tr>
</tbody>
</table>
Legend:  
- success, if specimen survived $10^7$ cycles
- failure, if specimen broke before $10^7$ cycles

Fig. 2.3 Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of $\infty$ for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.
Fig. 2.4 Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of 1.0 for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.
**TABLE 2.3**

LIST OF SPECIMENS AND THE CORRESPONDING STRESSES AND PAN WEIGHTS FOR ENDURANCE TESTS AT STRESS RATIO = ∞

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Pan Weight pounds</th>
<th>Nominal Normal Stress in Specimen psi.</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>25.0</td>
<td>60,519</td>
</tr>
<tr>
<td>17</td>
<td>25.0</td>
<td>37,476</td>
</tr>
<tr>
<td>74</td>
<td>25.0</td>
<td>46,827</td>
</tr>
<tr>
<td>202</td>
<td>25.0</td>
<td>51,125</td>
</tr>
<tr>
<td>182</td>
<td>25.0</td>
<td>57,353</td>
</tr>
<tr>
<td>124</td>
<td>25.0</td>
<td>60,411</td>
</tr>
<tr>
<td>216</td>
<td>26.0</td>
<td>58,017</td>
</tr>
<tr>
<td>205</td>
<td>24.0</td>
<td>49,580</td>
</tr>
<tr>
<td>199</td>
<td>25.0</td>
<td>54,036</td>
</tr>
<tr>
<td>209</td>
<td>25.0</td>
<td>53,903</td>
</tr>
<tr>
<td>177</td>
<td>24.0</td>
<td>54,249</td>
</tr>
<tr>
<td>168</td>
<td>25.0</td>
<td>57,132</td>
</tr>
<tr>
<td>190</td>
<td>26.0</td>
<td>61,981</td>
</tr>
<tr>
<td>207</td>
<td>27.0</td>
<td>61,857</td>
</tr>
<tr>
<td>142</td>
<td>25.0</td>
<td>65,528</td>
</tr>
<tr>
<td>208</td>
<td>23.0</td>
<td>59,422</td>
</tr>
</tbody>
</table>
Note that the average stress for both 3:2 and 2:3 stress ratios was determined for both specimens.

**TABLE 2.4**

**LIST OF SPECIMENS AND THE CORRESPONDING STRESSES AND PAN WEIGHTS FOR THE ENDURANCE TESTS AT STRESS RATIO = 1.0**

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Pan Weight psi.</th>
<th>Nominal Normal Stress in Specimen psi.</th>
<th>Stress Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>248</td>
<td>27.0</td>
<td>48,979</td>
<td>0.721</td>
</tr>
<tr>
<td>288</td>
<td>27.0</td>
<td>61,723</td>
<td>1.072</td>
</tr>
<tr>
<td>267</td>
<td>26.0</td>
<td>58,350</td>
<td>1.045</td>
</tr>
<tr>
<td>300</td>
<td>27.0</td>
<td>55,537</td>
<td>1.005</td>
</tr>
<tr>
<td>275</td>
<td>26.0</td>
<td>59,810</td>
<td>1.169</td>
</tr>
<tr>
<td>409</td>
<td>25.0</td>
<td>57,067</td>
<td>1.066</td>
</tr>
<tr>
<td>364</td>
<td>24.0</td>
<td>52,863</td>
<td>1.040</td>
</tr>
<tr>
<td>372</td>
<td>26.0</td>
<td>58,885</td>
<td>1.042</td>
</tr>
<tr>
<td>362</td>
<td>28.0</td>
<td>61,561</td>
<td>1.031</td>
</tr>
<tr>
<td>429</td>
<td>27.0</td>
<td>58,918</td>
<td>1.019</td>
</tr>
<tr>
<td>359</td>
<td>27.0</td>
<td>58,727</td>
<td>1.055</td>
</tr>
<tr>
<td>417</td>
<td>26.0</td>
<td>55,581</td>
<td>0.996</td>
</tr>
<tr>
<td>336</td>
<td>26.0</td>
<td>54,156</td>
<td>0.909</td>
</tr>
<tr>
<td>435</td>
<td>26.0</td>
<td>56,298</td>
<td>1.114</td>
</tr>
<tr>
<td>401</td>
<td>25.0</td>
<td>57,266</td>
<td>1.081</td>
</tr>
<tr>
<td>400</td>
<td>29.0</td>
<td>60,166</td>
<td>1.064</td>
</tr>
<tr>
<td>386</td>
<td>28.0</td>
<td>58,619</td>
<td>1.063</td>
</tr>
<tr>
<td>402</td>
<td>27.0</td>
<td>58,288</td>
<td>1.087</td>
</tr>
</tbody>
</table>
Note that the average stress for the one specimen at a pan weight of 23 pounds is greater than the average stress for both 25 and 24 pounds. To eliminate this inconsistency, the one data point taken at 23 pounds was eliminated from the calculations. Then, taking the average of the increments between the other four levels the value of 3,314 psi. for the stress increment was obtained.

In order to get values for the number of specimens which succeeded at each level, it was assumed that each specimen tested at a given pan weight was subjected to a stress equal to the average stress calculated for that pan weight. Thus, 2 specimens were run at 24 pounds, 10 specimens at 25 pounds, 2 specimens at 26 pounds and 1 specimen at 27 pounds. Using Eqs. (5) and (6) where

\[ y = 51,915 \text{ psi.} \]
\[ d = 3,314 \text{ psi.} \]
\[ N = 15 \]
\[ A = \sum_{i=0}^{3} i n_i = 0(2) + 1(10) + 2(2) + 3(1) = 17 \]
\[ B = \sum_{i=0}^{3} i^2 n_i = 1^2(10) + 2^2(2) + 3^2(1) = 27 \]

then

\[ m = 51,915 + 3,314 \left( \frac{17}{15} + \frac{1}{2} \right) \]
\[ m = 57,317 \text{ psi.} \]

and

\[ s = 1.620 \ (3,314) \left[ \frac{15(27)-(17)^2}{(15)^2} + 0.029 \right] \]

\[ s = 3,818 \text{ psi.} \]

Thus the estimates for the mean and standard deviation of the endurance strength distribution for stress ratio of \( \infty \) are 57,317 psi. and 3,818 psi. respectively.

Similarly, the estimates for the mean and standard deviation of the endurance strength distribution for stress ratio of 1.0 were found to be 56,785 psi. and 3,290 psi., respectively.

2.2.2 Endurance Level for \( R = 3.5 \)

The staircase method was used to conduct the tests at this ratio also, but instead of basing the tests on pan weight they were based on stress in the specimen. The stress increment was chosen to be 3,000 psi. Figure 2.5 shows the staircase plot for these tests. Table 2.5 lists the specimens which were used in the calculation of the distribution parameters. Also listed are the stress as recorded by the strain gages, the target stress and actual stress ratio in each specimen.

Table 2.5 indicates that the stresses were not held as close as targeted when the tests were conducted. The scatter may be due to carelessness on the part of the operators and test machine operating inconsistencies.
Legend: 
○ success, if specimen survived $2.5 \times 10^6$ cycles
○ failure, if specimen broke before $2.5 \times 10^6$ cycles

![Graph showing endurance strength data](image)

**Fig. 2.5** Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of 3.5 for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Actual Normal Stress in Specimen (psi)</th>
<th>Intended Normal Stress in Specimen (psi)</th>
<th>Actual Stress Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>477</td>
<td>55,917</td>
<td>60,000</td>
<td>3.54</td>
</tr>
<tr>
<td>465</td>
<td>55,825</td>
<td>57,000</td>
<td>4.26</td>
</tr>
<tr>
<td>478</td>
<td>58,291</td>
<td>60,000</td>
<td>3.64</td>
</tr>
<tr>
<td>506</td>
<td>51,256</td>
<td>57,000</td>
<td>3.22</td>
</tr>
<tr>
<td>528</td>
<td>57,376</td>
<td>60,000</td>
<td>3.78</td>
</tr>
<tr>
<td>444</td>
<td>53,410</td>
<td>57,000</td>
<td>3.26</td>
</tr>
<tr>
<td>466</td>
<td>54,236</td>
<td>57,000</td>
<td>3.60</td>
</tr>
<tr>
<td>524</td>
<td>47,071</td>
<td>51,000</td>
<td>3.03</td>
</tr>
<tr>
<td>546</td>
<td>50,292</td>
<td>54,000</td>
<td>3.93</td>
</tr>
<tr>
<td>499</td>
<td>60,903</td>
<td>57,000</td>
<td>5.72</td>
</tr>
<tr>
<td>532</td>
<td>54,533</td>
<td>57,000</td>
<td>4.43</td>
</tr>
<tr>
<td>483</td>
<td>50,828</td>
<td>54,000</td>
<td>3.10</td>
</tr>
<tr>
<td>529</td>
<td>51,381</td>
<td>54,000</td>
<td>3.56</td>
</tr>
<tr>
<td>587</td>
<td>52,067</td>
<td>54,000</td>
<td>3.50</td>
</tr>
<tr>
<td>581</td>
<td>54,320</td>
<td>57,000</td>
<td>3.67</td>
</tr>
<tr>
<td>611</td>
<td>56,826</td>
<td>60,000</td>
<td>3.42</td>
</tr>
<tr>
<td>589</td>
<td>54,118</td>
<td>57,000</td>
<td>3.47</td>
</tr>
<tr>
<td>603</td>
<td>53,813</td>
<td>57,000</td>
<td>3.41</td>
</tr>
</tbody>
</table>
These are discussed in Section 2.9. Because of this the average actual stress at each target level was determined as follows: 1 are 55,139 psi.

<table>
<thead>
<tr>
<th>Average Actual Stress (psi)</th>
<th>Target Stress (psi)</th>
<th>Number of Successes</th>
</tr>
</thead>
<tbody>
<tr>
<td>57,103</td>
<td>desired to eq. 60,000</td>
<td>4</td>
</tr>
<tr>
<td>54,823</td>
<td>at each stress 57,000</td>
<td>9</td>
</tr>
<tr>
<td>51,142</td>
<td>54,000</td>
<td>4</td>
</tr>
<tr>
<td>trial 47,071</td>
<td>51,000</td>
<td>1</td>
</tr>
</tbody>
</table>

Based on a study most report on the average stress increment between these levels is 3,377 psi. Using Eqs. (5) and (6) where duct goodness

\[ y = 47,071 \text{ psi.} \]

The predicted

\[ d = 3,377 \text{ psi.} \]

is estimated using the listed

dead load:

\[ A = \sum_{i=1}^{3} n_i = 1(4) + 2(9) + 3(4) \]

failure is the index

\[ B = \sum_{i=1}^{3} i^2 n_i = 1(4) + 4(9) + 9(4) \]

then

\[ N = 18 \]

the values:

\[ m = 47,071 + 3,377 \left( \frac{24}{18} + \frac{1}{2} \right) \]

\[ = 55,139 \text{ psi.} \]

and

\[ s = 1.620 \left( 3,377 \right) \left( \frac{(18)(18) - (34)^2}{(18)^2} + 0.029 \right) \]

\[ = 3,738 \text{ psi.} \]

The program had been "fix."
Thus, the estimated mean and standard deviation of the endurance strength distribution for stress ratio of 3.5 are 55,139 psi. and 3,738 psi., respectively.

2.3 Cycles-to-Failure Data at Specific Stress Ratios and Levels

It was desired to determine how the cycles-to-failure data at each stress level and ratio was distributed.

Based on a study made by Broome (7) in an earlier report on this research program it was decided to fit the normal and log-normal distributions to the data and conduct goodness-of-fit tests.

The parameters for these two distributions were estimated using the unbiased estimates for mean and standard deviation. For the normal case the cycles-to-failure is the variate and for the log-normal case the natural logarithms of the cycles-to-failure is the variate.

The moment coefficients of skewness and kurtosis were also estimated from the data. For the normal distribution the values of these two parameters are 0 and 3, respectively.

The Kolmogorov-Smirnov goodness-of-fit test was used as a measure of how well the distributions fit the data.

A computer program was used to make the calculations. The program had been developed previously but it was modified.
by this author to include the Kolmogorov-Smirnov test. The original program, which was written by Patel (8), used only the Chi-square goodness-of-fit test. The program and user instructions are included in Appendix B.

Table 2.6 lists the results obtained by estimating the normal distribution parameters from the data and conducting the Kolmogorov-Smirnov goodness-of-fit test. Table 2.7 lists the results obtained by using the natural logarithms of the cycles-to-failure data.

2.4 Normal Distribution Fit to the Data

The probability density function for the normal distribution is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2}, \quad -\infty < x < \infty \quad (7)$$

where $\mu = \text{mean}$

$\sigma = \text{standard deviation}$

If $\mu$ and $\sigma$ are not known from prior knowledge they must be estimated from the data as discussed in the next section.

2.4.1 Determination of Mean, Standard Deviation, Skewness and Kurtosis

By definition the moment generating function, denoted by $m(t)$, is the expected value of $e^{tx}$ which is denoted by $\mathbb{E}[e^{tx}]$. For the normal distribution
# TABLE 2.6
NORMAL DISTRIBUTION PARAMETER ESTIMATES AND THE MAX D VALUES

<table>
<thead>
<tr>
<th>Stress Ratio Stress Level (psi)</th>
<th>Sample Size</th>
<th>Average</th>
<th>Alternating</th>
<th>Normal Dist. Parameters</th>
<th>Max. D Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Size</td>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>0.825</td>
<td>111,000</td>
<td>12</td>
<td>6,554</td>
<td>1,005</td>
<td>.893</td>
</tr>
<tr>
<td></td>
<td>92,000</td>
<td>18</td>
<td>20,467</td>
<td>5,595</td>
<td>.239</td>
</tr>
<tr>
<td></td>
<td>76,000</td>
<td>18</td>
<td>61,028</td>
<td>11,132</td>
<td>-.428</td>
</tr>
<tr>
<td></td>
<td>65,000</td>
<td>18</td>
<td>127,577</td>
<td>21,227</td>
<td>.171</td>
</tr>
<tr>
<td>0.44</td>
<td>69,000</td>
<td>18</td>
<td>53,573</td>
<td>15,470</td>
<td>1.667</td>
</tr>
<tr>
<td></td>
<td>60,000</td>
<td>18</td>
<td>142,550</td>
<td>41,585</td>
<td>.393</td>
</tr>
</tbody>
</table>

* Maximum D-Value From K-S Test
<table>
<thead>
<tr>
<th>Stress Ratio</th>
<th>Average Alternating Stress Level (psi)</th>
<th>Sample Size</th>
<th>Log-Normal Dist. Parameters</th>
<th>Maximum D-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sample Size</td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>8</td>
<td>144,000</td>
<td>12</td>
<td>7.906547</td>
<td>.227914</td>
</tr>
<tr>
<td></td>
<td>114,000</td>
<td>18</td>
<td>9.101961</td>
<td>.116130</td>
</tr>
<tr>
<td></td>
<td>98,000</td>
<td>18</td>
<td>9.992123</td>
<td>.176382</td>
</tr>
<tr>
<td></td>
<td>81,000</td>
<td>18</td>
<td>11.252667</td>
<td>.153954</td>
</tr>
<tr>
<td></td>
<td>73,000</td>
<td>18</td>
<td>11.970997</td>
<td>.234941</td>
</tr>
<tr>
<td>3.5</td>
<td>151,000</td>
<td>12</td>
<td>7.262676</td>
<td>.197789</td>
</tr>
<tr>
<td></td>
<td>115,000</td>
<td>18</td>
<td>8.720894</td>
<td>.157247</td>
</tr>
<tr>
<td></td>
<td>83,000</td>
<td>18</td>
<td>10.545160</td>
<td>.286664</td>
</tr>
<tr>
<td></td>
<td>74,000</td>
<td>18</td>
<td>11.180338</td>
<td>.274219</td>
</tr>
<tr>
<td>0.825</td>
<td>111,000</td>
<td>12</td>
<td>8.777704</td>
<td>.145966</td>
</tr>
<tr>
<td></td>
<td>92,000</td>
<td>18</td>
<td>9.890129</td>
<td>.280831</td>
</tr>
<tr>
<td></td>
<td>76,000</td>
<td>18</td>
<td>11.001940</td>
<td>.195062</td>
</tr>
<tr>
<td></td>
<td>65,000</td>
<td>18</td>
<td>11.743320</td>
<td>.167235</td>
</tr>
<tr>
<td>0.44</td>
<td>69,000</td>
<td>18</td>
<td>10.856736</td>
<td>.248421</td>
</tr>
<tr>
<td></td>
<td>60,000</td>
<td>18</td>
<td>11.825351</td>
<td>.305282</td>
</tr>
</tbody>
</table>

* Maximum D-Value From K-S Test
Carrying out the integration yields

\[ m(t) = E[e^{tx}] = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} e^{-\frac{1}{2} (\frac{x-\mu}{\sigma})^2} \, dx \]

which is the moment generating function for the normal distribution.

The procedure for obtaining the \( r \)th moment is to take the \( r \)th derivative of the moment generating function with respect to \( t \) and evaluate it at \( t = 0 \).

By definition the \( r \)th sample moment is given by

\[ \mu_r = \frac{1}{n} \sum_{i=1}^{n} x_i^r \]

Taking the first derivative of the moment generating function with respect to \( t \) and evaluating at \( t = 0 \) yields

\[ m_1 = \mu \]

Then equating the first distribution moment and the first sample moment yields the estimate for the mean

\[ \hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i \]  \hspace{1cm} (8)

It can be shown that this is an unbiased estimator.

Taking the second derivative of the moment generating function with respect to \( t \) and evaluating at
t = 0 yields
\[ m_2 = \sigma^2 + \mu^2 \]

Equating the second moment yields
\[ \hat{\sigma}^2 + \hat{\mu}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} \]

but
\[ \hat{\mu}^2 = \left[ \frac{\sum_{i=1}^{n} x_i}{n} \right]^2 \]

thus
\[ \sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left[ \frac{\sum_{i=1}^{n} x_i}{n} \right]^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \]

where
\[ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \text{sample mean}. \]

It can be shown that this is a biased estimator for \( \sigma^2 \).

For unbiasedness, the expected value of the parameter estimator must equal the parameter (9, p. 72) thus in this case for unbiasedness
\[ E \left[ \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \right] = \sigma^2. \]

Going through the computations yields
\[ E \left[ \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \right] = \frac{n-1}{n} \sigma^2. \]
Therefore the estimator
\[ \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]
is a biased estimate for \( \sigma^2 \). However, the unbiased estimator for \( \sigma^2 \) is
\[ s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \] (9)
The moment coefficients of skewness and kurtosis are given by
\[ \alpha_3 = \frac{m_3}{(m_2)^{3/2}} \quad \text{and} \quad \alpha_4 = \frac{m_4}{(m_2)^2} \]
respectively, where
\[ m_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]
\[ m_3 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3 \]
and
\[ m_4 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4 \] (See Ref. 10, p.48).
Evaluating the third and fourth derivatives of the moment generating function at \( t = 0 \) and substituting into the equations for \( \alpha_3 \) and \( \alpha_4 \) will lead to values of \( \alpha_3 = 0 \) and \( \alpha_4 = 3 \) for the normal distribution.
The estimates of the mean, standard deviation
and the moment coefficients of skewness and kurtosis have been calculated for each group of cycles-to-failure data. The estimates of the moment coefficients of skewness and kurtosis were calculated in order to give a further indication of how well the data fits the normal distribution. However, it is important to note that even though \( \alpha_3 \) of a symmetrical distribution is zero, obtaining a value of zero for the estimate of \( \alpha_3 \) from the data does not necessarily mean that the distribution is symmetrical. Mood and Graybill make this point (9, p. 109) and state that, "knowledge of the third moment gives almost no clue as to the shape of the distribution." Therefore, the moment coefficient of skewness is not a good measure of whether or not a distribution is symmetrical.

The moment coefficient of kurtosis is a measure of the peakedness of the distribution.

2.5 Log-Normal Distribution Fit to the Data

A variate is distributed log-normal if the logarithm of the variate is distributed normal. That is, by letting \( y = \log_e x \) then

\[
f(y) = \frac{1}{\alpha \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{y-\mu}{\alpha} \right)^2}
\]

where the unbiased estimates of \( \mu \) and \( \alpha \) are

\[
\hat{\mu} = \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}
\]
Determination of Mean, Standard Deviation, Skewness and Kurtosis

The determination of these parameters for the log-normal distribution is the same as for the normal distribution as covered in Section 2.4.1 except the log of the variate is used in the calculations.

2.6 Goodness-of-Fit Tests

In order to determine how well the normal and log-normal distributions fit the data, two goodness-of-fit tests were proposed. They are the Chi-square and Kolmogorov-Smirnov goodness-of-fit tests. However, both tests were not used in determining whether or not to reject a particular distribution. In all cases the Chi-square goodness-of-fit test was not valid because of the small sample sizes. Further reasons for it not being valid are given in the next section.

Since the computer program is written using both tests it should be pointed out that when analyzing data where the Chi-square test is valid, if both tests are used to determine rejection, the level of significance changes. In other words, if the critical values for rejection are based on a significance level of $\alpha$ for both tests and the criteria for rejection of the null hypothesis is if either
one or the other or both tests rejects the distribution then the level of significance in rejecting is not $\alpha$. It is not clear what the confidence level would be since the extent to which the two tests are correlated is not known. Therefore, only one test should be used in determining goodness-of-fit. For the data in this report only the Kolmogorov-Smirnov test was used.

2.6.1 Chi-Square Goodness-of-Fit Test

The Chi-square goodness-of-fit test can be used only with grouped data, that is, data divided into cells. The total Chi-square value is the sum of the Chi-square values of each cell. This can be written as

$$V_{k-1} = \sum_{i=1}^{k} \frac{(E_i - O_i)^2}{E_i}$$

where

- $E_i$ = the expected frequency of the $i^{th}$ cell
- $O_i$ = the observed frequency of the $i^{th}$ cell
- $k$ = total number of cells
- $V_{k-1}$ = total Chi-square value

$V_{k-1}$ can be shown to be distributed $\chi^2_{k-1}$ hence the name Chi-square test, where $k-1$ is the number of degrees of freedom. However, if the parameters of the hypothesized distribution must be estimated from the data the degrees of freedom must be decreased by the number of parameters estimated. In the case where the fit of the
normal distribution to the data is being tested, if the mean and standard deviation are estimated from the data the number of degrees of freedom is \( k-1-2 \) or \( k-3 \).

The derivation of the Chi-square test is based on the law of large numbers in such a way that in order for the test to be valid the number of observations in each cell must be large. In actual practice it is generally accepted that the observed value of each cell must be greater than five.

Since the two parameters of the normal distribution were estimated from the data there must be at least four cells in order for the degrees of freedom to be greater than zero. Therefore, there would have to be at least 24 observations to have greater than five in each cell. In each case in this test program there were either twelve or eighteen specimens tested. Hence, the Chi-square test would be invalid for the sample sizes tested.

2.6.2 Kolmogorov-Smirnov Goodness-of-Fit Test

Briefly, the Kolmogorov-Smirnov test is a comparison of a hypothesized cumulative frequency distribution \( F_n(x) \) and the observed cumulative distribution \( S_n(x) \). Rejection or non-rejection is based on the absolute value of the maximum difference between the two functions. In mathematical terms

\[
D_n = \left| F_n(x) - S_n(x) \right| \quad (10)
\]
where

\[ D_n = \text{Kolmogorov statistic} \]

\[ F_n(x) = \int_{-\infty}^{x} f(x) \, dx \]

\[ f(x) = \text{probability density function of the hypothesized distribution} \]

\[ x_1 = \text{any specific value of the variate } X \]

\[ S_n(x) = \frac{X}{N} \]  \hspace{1cm} (11) \]

\[ X = \text{number of observations less than or equal to } x_1 \]

\[ N = \text{sample size} \]

If \( D_n \) is greater than some critical value \( (D_c) \), the hypothesized distribution is rejected at some level of significance \((\alpha)\). The probability statement is \( P(D_n > D_c) = \alpha \). Table 2.8 lists the critical values for sample sizes of 12 and 18 at various levels of significance.

**TABLE 2.8**

**TABLES OF CRITICAL VALUES OF D TO USE IN THE KOLMOGOROV-SHIRNOV GOODNESS-OF-FIT TEST**

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Level of Significance ((\alpha))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td>12</td>
<td>0.338</td>
</tr>
<tr>
<td>18</td>
<td>0.278</td>
</tr>
</tbody>
</table>
In using the Kolmogorov-Smirnov test the D-value is found at each data point. If at any point the calculated D-value is greater than the critical value at the desired significance level, the hypothesis is rejected. If not, the hypothesis is not rejected.

2.7 Discussion of Results

In 8 of the 15 stress levels tested the D-value (Kolmogorov Statistic) was less for the log-normal distribution than for the normal distribution. The moment coefficient of kurtosis was closer to 3.0 for the log-normal distribution in 9 of the 15 cases. The moment coefficient of skewness was closer to zero for the log-normal distribution in 7 of the 15 cases. However, as was mentioned in Section 2.4.1, obtaining a value close to zero for the moment coefficient of skewness does not necessarily mean that the distribution is symmetrical or normal.

Based on the Kolmogorov-Smirnov test, in no case can either the normal or the log-normal distribution be rejected with 90% confidence. For stress ratio of 0.44 at stress level 69,000 psi, the normal distribution can be rejected with 85% confidence.

From the above results it appears that the log-normal distribution is favored.

The cycles-to-failure data for stress ratios of 0
and 0.625 were previously analyzed and reported by Broome (7). He also concluded that the log-normal distribution was favored. He also points out that from a phenomenological viewpoint the log-normal distribution is justified.

Figures 2.6 through 2.9 show plots on log scales at the average stress levels of the cycles-to-failure distributions for stress ratios of \( \infty \), 3.5, 0.825 and 0.44. The distributions shown are log-normal. The mean line shown on each figure was obtained by fitting a straight line, by the method of least squares, to the estimates of the means of the distributions on each figure. The 3 standard deviation limits for each distribution are also shown. To obtain a smooth envelope the dashed lines were drawn in by sight.

Specimens were tested at only two stress levels for the stress ratio of 0.44. They were 69,000 psi. and 60,000 psi. An attempt was made to test at a level of 75,000 psi., but at a stress ratio of 0.44 this requires a shear stress of 98,500 psi. The specimen began yielding and finally broke as the torque was applied. Thus, for stress ratio of 0.44 the cycles-to-failure tests were restricted to two stress levels. The endurance tests for stress ratio of 0.44 have not yet been completed but it appears as though the mean endurance limit is about 51,000 psi.
Fig. 2.6 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of \( \infty \)
Fig. 2.7 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 3.5.
Fig. 2.8 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 0.825.
Fig. 2.9 Cycles-to-Failure Distributions at Stress Ratio of 0.44.
2.8 Construction of Statistical Fatigue Diagrams at Various Numbers of Cycles of Life

The ultimate objective of the research program is to construct statistical fatigue diagrams from which shafts can be designed for a specified cycle life and reliability.

One kind of conventional fatigue diagram is one using the modified Goodman line as shown in Fig. 2.10.

![Fatigue Diagram Showing the Modified Goodman Line and the Soderberg Line](image)

Fig. 2.10 Fatigue Diagram Showing the Modified Goodman Line and the Soderberg Line.

The diagram is a plot of alternating stress on the ordinate versus mean stress on the abscissa. The modified Goodman line is a line connecting the endurance strength and ultimate tensile strength. Another line, which
connects the endurance strength and yield strength, is called the Soderberg line. Both the modified Goodman line and Soderberg line are conservative (2, p. 178).

The modified Goodman line is used to design for an infinite life. Other lines can be constructed connecting the ultimate strength to some alternating stress corresponding to a specified life. This line would be used to design for that finite life. However, these types of lines are deterministic and do not account for variability in the strength of a material. If the strength distributions can be determined for various stress ratios and cycles of life then fatigue diagrams can be defined in terms of a mean line and standard deviation about the line.

The method of determining the strength distributions which has been proposed for this research program has been discussed in earlier reports (1, pp. 134-135; 11, pp. 24-26). Briefly, once the mean line and $3\sigma$ envelopes ($\sigma$ = standard deviation) have been established on the S-N diagrams as discussed in Section 2.7 then distribution parameters can be interpolated at evenly spaced stress levels. Suppose the strength distribution is desired at $N$ cycles. A histogram can be constructed along the $N$ cycle line such that the midpoints of each histogram cell is one of the interpolated stress levels, as shown in Fig. 2.11. The ordinate of each cell is the area, to the left of $N_1$, under the
distribution curve corresponding to that cell. For example, denoting the bottom distribution function on Fig. 2.11 as \( f(N|S_1) \) then the ordinate of the bottom histogram cell is

\[
F(N|S_1) = \int_{-\infty}^{N_1} f(N|S_1) \, dN. \tag{12}
\]

Likewise, the ordinate of the next cell will be

\[
F(N|S_2) = \int_{-\infty}^{N_1} f(N|S_2) \, dN. \tag{13}
\]

In general the ordinate of the \( i \)th cell will be

\[
F(N|S_i) = \int_{-\infty}^{N_1} f(N|S_i) \, dN \tag{14}
\]

and the total histogram will look like that shown in Fig. 2.11. This total histogram, as a first approximation, is taken to be cumulative strength histogram of specimens failing by \( N_1 \) cycles. The probability density histogram can be obtained from this cumulative histogram. Denoting the strength random variable along the \( N_1 \) axis as \( S \) then the value of the \( i \)th cell of the probability density histogram is \( f(S_i) = F(N|S_i) - F(N|S_{i-1}) \). After the probability density histogram is found then the normal distribution parameters can be estimated using statistical methods.
Fig. 2.11 Histogram Obtained From Cycles-to-Failure Distributions
A computer program was developed by R. E. Smith (11) using this method of estimating the strength distribution parameters. A printout of the program is given in Appendix F. The estimates of the strength distribution parameters at cycles of 10,000, 50,000 and 100,000 for a stress ratio of 3.5 are listed in Table 2.9. These distributions are plotted on Fig. 2.12. Note that the ± 3σ limits are inside the ± 3σ envelope for the cycles-to-failure distributions; hence, for this case the method appears to yield unconservative estimates of the strength distribution parameters. The reason for this will be investigated.

<table>
<thead>
<tr>
<th>Cycles of Life N</th>
<th>Parameter Estimates of Normal Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (psi)</td>
</tr>
<tr>
<td>10,000</td>
<td>106,639</td>
</tr>
<tr>
<td>50,000</td>
<td>79,021</td>
</tr>
<tr>
<td>100,000</td>
<td>70,972</td>
</tr>
</tbody>
</table>
Fig. 2.12 Plot of Estimated Strength Distributions at Various Cycles of Life and Estimated Cycles-to-Failure Distributions at Various Stress Levels for Stress Ratio of 3.5
Another approach might be to base the strength distribution estimates on actual data so that some measure of accuracy can be obtained through the use of an appropriate goodness-of-fit test. The approximate distributions of strength at a specified number of cycles is wanted, then the distribution of the stresses to which the specimens failing at N cycles are subjected might be obtained as follows. N is a random variable and the probability of even one specimen failing at exactly N cycles is zero. Therefore, the strength distribution at a discrete N is unattainable, but a distribution of specimens failing within a band of N is possible to obtain.

\[ S_0 \]

CONVENTIONAL
\[ S-N \] LINE

STRESS-LOG SCALE

\[ S_e \]

\[ N_1 \]

**Fig. 2.13** S-N Diagram Showing Random Data Points and Cell Structure for Testing and Obtaining Distributions.
Suppose the strength distribution is desired at $N_1$ cycles. From existing data an estimate of the conventional S-N curve can be obtained and a range of stress corresponding to $10^3$ cycles and the endurance limit can be obtained; $S_o$ and $S_e$ on Fig. 2.13. Then specimens could be tested at random levels within this range. The stress range could be divided into cells, as shown in Fig. 2.13 and specimens tested at random levels within each cell. This would insure a more even distribution of data points.

Once the random data is obtained then the parameter estimates can be calculated and goodness-of-fit tests conducted. A cell with $N_1$ as its midpoint can be constructed as shown in Fig. 2.13. For a conservative estimate the cell may be constructed so that $N_1$ is the lower boundary. The estimate of the mean strength at $N_1$ cycles would be the average of the stresses at which the specimens were tested that failed within the $N_1$ cell (the darkened data points on Fig. 2.13). The width of the cell would depend on how good an estimate was desired. Using these data points the estimates of any distribution parameters could be obtained and goodness-of-fit tests conducted to determine which distributions to reject. As an example, Fig. 2.14 is the S-N diagram for stress ratio of $\infty$ showing the estimated distributions of the cycles-to-failure data. Suppose it is desired to find the strength distribution...
for 70,000 cycles. If a cell of width 30,000 cycles is constructed on the conservative side of 70,000 cycles it passes through two distributions. The actual data points which fall within the cell can be obtained from Table 2.10 which lists the cycles to failure data at these two stress levels for stress ratio of $\infty$. For stress level 73,000 psi, one point falls in the cell and for stress level 81,000 psi, 13 points fall in the cell for a total of 14 points. Calculating the mean and standard deviation of the strength for these points yields

$$S = 80,516 \text{ psi}, \text{ and } \sigma_s = 2,632 \text{ psi}.$$  

Then, using the Kolmogorov-Smirnov goodness-of-fit test it can be determined whether or not the normal distribution is a good estimate for the strength distribution at 70,000 cycles.

The D statistic, as given by Eq. (10), is

$$D = \left| F_n(x) - S_n(x) \right|$$

where

$$F_n(x) = \frac{1}{\sqrt{2\pi}} \int_{-z_1}^{z_1} e^{-\frac{z^2}{2}} dz$$

and

$$z_1 = \frac{S - S}{\sigma S}$$
Fig. 2.14 S-N Diagram for Stress Ratio of ∞ Showing the Estimated Cycles-to-Failure Distributions and Test Band.
TABLE 2.10
CYCLES-TO-FAILURE AND STRESS DATA FOR
STRESS RATIO OF ~ AT STRESS LEVELS
OF 73,000 PSI. AND 81,000 PSI.

<table>
<thead>
<tr>
<th>Average Stress Level of 73,000 psi.</th>
<th>Average Stress Level of 81,000 psi.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Stress-psi.</td>
<td>Cycles-to-Failure</td>
</tr>
<tr>
<td>73,796</td>
<td>125,950</td>
</tr>
<tr>
<td>73,886</td>
<td>103,413</td>
</tr>
<tr>
<td>70,969</td>
<td>145,783</td>
</tr>
<tr>
<td>72,029</td>
<td>195,824</td>
</tr>
<tr>
<td>71,973</td>
<td>93,303*</td>
</tr>
<tr>
<td>73,442</td>
<td>178,192</td>
</tr>
<tr>
<td>73,404</td>
<td>196,894</td>
</tr>
<tr>
<td>72,572</td>
<td>183,306</td>
</tr>
<tr>
<td>74,931</td>
<td>177,508</td>
</tr>
<tr>
<td>72,374</td>
<td>203,019</td>
</tr>
<tr>
<td>72,925</td>
<td>155,178</td>
</tr>
<tr>
<td>74,819</td>
<td>205,041</td>
</tr>
<tr>
<td>75,982</td>
<td>172,661</td>
</tr>
<tr>
<td>73,647</td>
<td>124,137</td>
</tr>
<tr>
<td>72,961</td>
<td>127,794</td>
</tr>
<tr>
<td>74,629</td>
<td>172,572</td>
</tr>
<tr>
<td>68,029</td>
<td>172,275</td>
</tr>
<tr>
<td>72,164</td>
<td>182,860</td>
</tr>
</tbody>
</table>

* Indicates Specimens Whose Cycles-to-Failure Fall Within the Range of 70,000 to 100,000 Cycles.
S is the value of the stress of the data point being examined and

\[ S_n(x) = \frac{S}{N} \]

where

- \( S = \) the number of points which have a stress less than or equal to \( S \).

and

\( N = \) total number of points.

For example, using the last data point under the 81,000 psi. stress level column in Table 2.10;

\( S = 81,784 \) psi.

\( N = 14 \)

Since 81,784 psi. is the second highest stress value of the data points there are 13 points less than or equal to it, so

\[ S_n(x) = \frac{13}{14} = 0.929 \]

The value of \( z \) is

\[ z_1 = \frac{81,784 - 80,516}{2,632} = 0.482 \]

From standard normal tables this value of \( z_1 \) corresponds to

\( F_n(x) = 0.620 \)

Therefore

\( D = 0.309 \)

From a table of critical D-values (which can be found in most statistics books which discuss the Kolmogorov-Smirnov
test) it is seen that for a sample size of 14
\[ P(D > 0.292) = 0.15 \]
Thus, since the D obtained here is greater than 0.292—this normal distribution can be rejected with 85% confidence that the distribution does not fit the data, but can not be rejected with 90% confidence.

Past studies by others indicate that for a given cycle life the fatigue strength distribution is normal (12, p. 351). One reason the normal distribution can be rejected here with 85% confidence may be that the sample used to estimate the mean and standard deviation is not random and therefore the estimates may be biased. Definite stress levels were aimed for and although there is some scatter about those levels, the sample cannot be considered a random one. The properties of the estimators for the mean and standard deviation are known to be good if the sample is random. If the sample is not random the estimates may be biased or insufficient. It is for this reason, if this method is used, that the data can give little more than gross approximations of the strength distribution parameters. The present data might be useful if more tests were conducted randomly throughout the stress range to give an even distribution of points.

It must be pointed out that a very large number of specimens may have to be tested to obtain enough specimens,
so that preferably more than 35 fail within the narrow cycle life range desired for sufficient accuracy, particularly for strength distributions at lower cycles of life.

The staircase method should also be tried for finite life to see how the results from the three methods compare.

Once the random data is obtained and the strength distribution parameters have been calculated at the desired cycles of life, construction of the fatigue diagrams is relatively simple.

Assuming the stress ratio is held constant for the data points on each S-N diagram then the alternating strength distribution can be transformed to the mean strength distribution through the constant R. When either distribution and the stress ratio, R, is known then the other distribution is completely defined, so only the alternating strength distributions will be worked with.

The means and variances are related by

\[ S_a = R S_m \quad \text{and} \quad \sigma_{S_a}^2 = R^2 \sigma_{S_m}^2. \]

where

- \( S_a \) = mean of the alternating strength distribution
- \( S_m \) = mean of the mean strength distribution
- \( \sigma_{S_a} \) = standard deviation of the alternating strength distribution
\[ \sigma_{Sm} = \text{standard deviation of the mean strength distribution} \]

As an example, a fatigue diagram using the endurance strength distributions calculated in Sections 2.2.1 and 2.2.2 will be constructed. The endurance strength distribution parameters are listed in Table 2.11.

The distribution for stress ratio of 0 is the ultimate strength distribution for unnotched specimens as obtained from tensile tests. The details and results of these tests were reported in an earlier report (6) and therefore will not be discussed here. The results are presented in Tables 2.12 and 2.13. Twenty specimens were tested. Ten were identical to those used in the fatigue tests (Fig. 1.3) and ten were unnotched specimens. The unnotched specimens exhibited a yield point, which was recorded, and the notched specimens did not have a noticeable yield point.

The reasons for using the ultimate strength distribution obtained from the unnotched specimens rather than that for the notched specimens is as follows. The mean or constant stress that the specimens are subjected to in the test program is a shear stress. However, a mean normal stress is desired for the mean stress axis of the fatigue diagram. If the specimens would have been tested to fracture in static torsion the shear stress distribution would have been converted to normal stress by the relationship \[ S_m = \sqrt{3} \tau. \]
### TABLE 2.11

**ALTERNATING STRENGTH DISTRIBUTION PARAMETERS AND THE CORRESPONDING DISTRIBUTION PARAMETERS ALONG THE VARIOUS STRESS RATIO LINES FOR INFINITE LIFE**

<table>
<thead>
<tr>
<th>Stress Ratio R</th>
<th>Endurance Strength Distribution Parameters</th>
<th>Corresponding Dist. Parameters Along R Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean psi** $\bar{S}_a$</td>
<td>Standard Dev. psi*** $\sigma_{S_a}$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>57,000</td>
<td>3,800</td>
</tr>
<tr>
<td>3.5</td>
<td>55,000</td>
<td>3,700</td>
</tr>
<tr>
<td>1.0</td>
<td>57,000</td>
<td>3,300</td>
</tr>
<tr>
<td>0</td>
<td>178,000</td>
<td>2,500</td>
</tr>
</tbody>
</table>

* These values are the distribution parameters of the ultimate strength of unnotched specimens obtained from tensile tests.

** Rounded off to nearest 1,000 psi.

*** Rounded off to nearest 100 psi.
TABLE 2.12
DATA AND RESULTS FROM STATIC TESTS
ON NOTCHED SPECIMENS*
(Stress Ratio = 0)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Ultimate Load 1,000 lbs.</th>
<th>Breaking Load 1,000 lbs.</th>
<th>Ultimate Strength psi. **</th>
<th>True Breaking Strength psi. **</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.3</td>
<td>47.0</td>
<td>253,500</td>
<td>305,000</td>
</tr>
<tr>
<td>2</td>
<td>49.6</td>
<td>47.0</td>
<td>255,000</td>
<td>305,000</td>
</tr>
<tr>
<td>3</td>
<td>49.4</td>
<td>46.3</td>
<td>254,000</td>
<td>299,500</td>
</tr>
<tr>
<td>4</td>
<td>50.3</td>
<td>47.4</td>
<td>259,000</td>
<td>299,500</td>
</tr>
<tr>
<td>5</td>
<td>48.8</td>
<td>46.0</td>
<td>251,000</td>
<td>306,500</td>
</tr>
<tr>
<td>6</td>
<td>49.2</td>
<td>46.0</td>
<td>253,000</td>
<td>302,500</td>
</tr>
<tr>
<td>7</td>
<td>49.6</td>
<td>46.8</td>
<td>255,000</td>
<td>304,500</td>
</tr>
<tr>
<td>8</td>
<td>49.8</td>
<td>47.1</td>
<td>256,000</td>
<td>305,500</td>
</tr>
<tr>
<td>9</td>
<td>50.5</td>
<td>47.7</td>
<td>260,000</td>
<td>309,500</td>
</tr>
<tr>
<td>10</td>
<td>49.9</td>
<td>47.5</td>
<td>256,500</td>
<td>302,000</td>
</tr>
</tbody>
</table>

Mean 255,500 304,000

Standard Deviation 2,500 3,000

* Average specimen diameter at the base of the notch is 0.4975 inches.

** All strengths rounded to nearest 500 psi.
## TABLE 2.13
DATA AND RESULTS FROM STATIC TESTS ON UNNOTCHED SPECIMENS
(Stress Ratio = 0)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Yield Load 1000 lb</th>
<th>Ultimate Load 1000 lb</th>
<th>Breaking Load 1000 lb</th>
<th>Diameter Average In.</th>
<th>Area Aver. In²</th>
<th>Yield Strength psi*</th>
<th>Ultimate Strength psi*</th>
<th>True Breaking Strength psi*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.5</td>
<td>32.5</td>
<td>24.3</td>
<td>0.4753</td>
<td>0.1774</td>
<td>177,500</td>
<td>183,000</td>
<td>264,000</td>
</tr>
<tr>
<td>2</td>
<td>30.6</td>
<td>31.5</td>
<td>23.5</td>
<td>0.4764</td>
<td>0.1784</td>
<td>171,500</td>
<td>176,500</td>
<td>254,000</td>
</tr>
<tr>
<td>3</td>
<td>30.6</td>
<td>31.6</td>
<td>23.3</td>
<td>0.4754</td>
<td>0.1775</td>
<td>172,500</td>
<td>178,000</td>
<td>249,000</td>
</tr>
<tr>
<td>4</td>
<td>20.8</td>
<td>31.0</td>
<td>22.8</td>
<td>0.4755</td>
<td>0.1776</td>
<td>168,000</td>
<td>174,500</td>
<td>251,500</td>
</tr>
<tr>
<td>5</td>
<td>29.9</td>
<td>31.1</td>
<td>22.9</td>
<td>0.4758</td>
<td>0.1778</td>
<td>168,000</td>
<td>175,000</td>
<td>254,500</td>
</tr>
<tr>
<td>6</td>
<td>30.1</td>
<td>31.3</td>
<td>23.2</td>
<td>0.4722</td>
<td>0.1752</td>
<td>172,000</td>
<td>178,500</td>
<td>256,500</td>
</tr>
<tr>
<td>7</td>
<td>30.4</td>
<td>32.5</td>
<td>25.0</td>
<td>0.4787</td>
<td>0.1800</td>
<td>169,000</td>
<td>181,000</td>
<td>256,000</td>
</tr>
<tr>
<td>8</td>
<td>30.2</td>
<td>31.4</td>
<td>24.2</td>
<td>0.4757</td>
<td>0.1777</td>
<td>170,000</td>
<td>178,000</td>
<td>253,000</td>
</tr>
<tr>
<td>9</td>
<td>30.6</td>
<td>31.6</td>
<td>23.6</td>
<td>0.4763</td>
<td>0.1782</td>
<td>172,000</td>
<td>177,500</td>
<td>259,000</td>
</tr>
<tr>
<td>10</td>
<td>30.3</td>
<td>31.2</td>
<td>24.2</td>
<td>0.4755</td>
<td>0.1766</td>
<td>171,000</td>
<td>176,500</td>
<td>250,500</td>
</tr>
</tbody>
</table>

| Mean     | 171,000           | 178,000               | 255,000               |
| Standard Deviation | 3,000           | 2,500                  | 4,500                  |

* All Strengths Rounded to the Nearest 500 psi.
It is generally accepted that the static strength of ductile steel is not affected by a stress concentration. Therefore, theoretically, the same shear strength distribution should be obtained whether the specimens are notched or not.

However, the mean normal strength distribution was obtained directly from the tensile tests. In the case of tensile tests a notch does have an affect on the strength. The notched specimen has a higher ultimate strength due to a radial stress being introduced into the specimen at the root of the groove (6, p. 19). A grooved specimen subjected to a static torque load would not experience this radial stress. Therefore, the strength distribution used at stress ratio of 0 is the ultimate strength distribution for the unnotched specimens.

In order to construct the fatigue diagram the distributions along the various stress ratio lines must be calculated. The required relationships can be derived from Fig. 2.15. They are:

\[ \tan \theta = \frac{S_a}{S_m} = \bar{R} = \text{stress ratio} \]

\[ S_R = \frac{S_a}{\sin^2} \quad (15) \]

but
Fig. 2.15 Relationship Between the Stress Ratio Lines
\[
\sin \theta = \frac{\tan \theta}{\left[1 + (\tan \theta)^2\right]^{\frac{1}{2}}} = \frac{R}{\left[1 + R^2\right]^{\frac{1}{2}}},
\]

thus

\[
S_R = \frac{S_a (1 + R^2)^{\frac{1}{2}}}{R}
\]

(16)

Using the same derivation the expressions for the standard deviation along the \(R\) axis are

\[
\sigma_{S_R} = \frac{\sigma_{S_a}}{\sin \theta}
\]

(17)

and

\[
\sigma_{S_R} = \frac{\sigma_{S_a} (1 + R^2)^{\frac{1}{2}}}{R}
\]

(18)

Referring to Fig. 2.16 the distribution along the alternating stress axis \((R = \infty)\) can be plotted directly. For this axis \(\theta = 90^\circ\) and Eq. (15) yields \(S_{R, \infty} = S_a, \infty\)
and Eq. (17) yields \(\sigma_{S_{R, \infty}} = \sigma_{S_a, \infty}\).

For the distribution along the stress ratio line \(3.5\) Eqs. (16) and (18) give

\[
S_{R, 3.5} = \frac{S_{a, 3.5} (1 + R^2)^{\frac{1}{2}}}{R}
\]

\[
= \frac{55,000 \left[1 + (3.5)^2\right]^{\frac{1}{2}}}{3.5} = 57,201 \text{ psi}.
\]
for the mean and

\[ \sigma_{S, \text{mean}} = 3.5 \frac{(1 + R^2)^{1/2}}{R} \]

\[ = 3,700 \left[ \frac{1 + (3.5)^2}{R} \right]^{1/2} = 3,848 \text{ psi.} \]

for the standard deviation.

The same procedure would be used along any other stress ratio axis except the appropriate values of \( S_a \) and \( \sigma_{S_a} \) would be used. The results of the calculations for the parameters for endurance are listed in Table 2.11 and the resulting fatigue diagram is Fig. 2.16. The endurance tests for \( R = 0.44 \) have not yet been completed but from the tests that have been run it appears as though the endurance level will be in the range of 49,000 to 53,000 psi, and the scatter will be wide. The actual distribution will probably be very close to what the distribution shown on Fig. 2.16, in dashed lines, looks like.

It probably will not be possible to conduct tests at ratios much lower than 0.44, especially at high stress levels. The stress ratio is \( R = \frac{S_a}{S_m} \) and the lower limit on \( S_a \) is the endurance level. The upper limit on \( S_m \) is the yield strength which is 171,000 psi. Thus the lower limit on stress ratio, assuming the endurance does not fall much below 51,000 psi, is

\[ R = \frac{51,000}{171,000} = 0.28 \]
Fig. 2.16 Statistical Fatigue Diagram For Infinite Life.
This may not be a very good assumption since the fatigue diagram shows that it does fall off rapidly at low stress ratios. At these low ratios yielding will be the predominant failure mode. Unless the capability exists to monitor the torque on the specimen continuously, it will not be known whether or not the initial torque load was constant throughout the test.

2.8.1 Use of Statistical Fatigue Diagrams in Design

The conventional use of fatigue diagrams is to determine what combinations of mean and alternating stress are safe for a given number of cycles of operation. For example, in Fig. 2.17 the stress combination of $S_{a_1}$ and $S_{m_1}$ would be considered safe for a design life of $10^5$ cycles but not for infinite life. The combination of $S_{a_1}$ and $S_{m_2}$ is not safe for $10^5$ cycles of life. The question arises, "With what confidence can one say that a combination is safe?" To answer this question the distributions of the alternating and mean stresses to which the component in question is subjected and the distribution of the limiting fatigue boundary must be considered. A method of obtaining the distribution of the fatigue boundary at various cycles of life has been previously discussed.
Fig. 2.17 Conventional Fatigue Diagram Showing the Modified Goodman Line

If the distributions of the alternating and mean stresses to which a shaft is subjected are known or can be estimated then they can be plotted on the fatigue diagram as shown in Fig. 2.18. If the shaft stress distributions are normal and the stress ratio can be assumed to be constant then the distribution along the stress ratio axis ($R_2$ axis in Fig. 2.18) can be obtained from the relationships

$$
\bar{s}_{R_2} = \frac{s_a}{\sin \theta_2} \quad \text{and} \quad \sigma_{s_{R_2}} = \frac{\sigma_s}{\sin \theta_2}
$$

where
Fig. 2.18 Statistical Fatigue Diagram

\[ \bar{\sigma}_{R_2} = \text{mean of the stress distribution along the } R_2 \text{ axis.} \]
\[ \sigma_{\sigma_{R_2}} = \text{standard deviation of the stress distribution along the } R_2 \text{ axis.} \]
\[ \bar{s}_a = \text{mean of the alternating stress of the shaft.} \]
\[ \sigma_{s_a} = \text{standard deviation of the alternating stress of the shaft.} \]

Figure 2.19 shows the stress and strength distributions along the \( R_2 \) axis. Once the parameters of the distributions are known the probability of failure can be calculated. Defining the random variable \( Z \) as

\[ Z = S - s \]
where

\[ s = \text{the random variable, stress.} \]

then the mean and standard deviation of \( Z \) can be shown to be normally distributed with parameters (13, p. 113)

\[ Z = S - \overline{S} \]

(19)

\[ \sigma_Z = \sqrt{\sigma_S^2 + \sigma_s^2} \]

(20)

Failure occurs when \( s > S \) or when \( S - s < 0 \).

In Fig. 2.20 the shaded area represents the probability of failure. Stated in mathematical terms

\[
P(Z < 0) = \int_{-\infty}^{0} \frac{1}{\sigma_Z \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{Z-\overline{Z}}{\sigma_Z} \right)^2} \, dz
\]
Putting into the standard normal form by letting

$$x_1 = \frac{0 - \bar{z}}{\sigma_z} = - \frac{S - \bar{s}}{\sqrt{\sigma_s^2 + \sigma_z^2}}$$  \hspace{1cm} (21)$$

then

$$P(X < x_1) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2} \, dx$$

For a known $x_1$ the value of $P(X < x_1)$, which is the probability of failure, can be obtained from standard normal distribution area tables. The reliability $R_e$ is defined as

$$R_e = 1 - P(\text{failure}) = 1 - P(X < x_1)$$

which can be expressed as

$$R_e = \int_{x_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2} \, dx$$ \hspace{1cm} (22)$$
Thus the reliability of the shaft can be determined.

In most cases the designer must start with a level of reliability and determine the shaft diameter such that for the imposed loads the specified reliability will be met. The procedure in this case is to work backwards through the preceding derivation. The alternating stress in the shaft can be expressed in terms of the diameter by

\[ s = \frac{MD}{2I} = \frac{2\pi M}{\pi D^3} \]  

(23)

where

- \( M \) = the bending moment on the section in question.
- \( D \) = diameter of the shaft.
- \( I = \frac{\pi D^4}{64} \) = moment of inertia of the shaft cross section.

The standard deviation of the stress can be expressed in terms of the mean and the standard deviation of the diameter of the shaft. Using the approximate partial derivative method (13, p. 90) for the standard deviation of the alternating stress yields

\[ \sigma_s \approx \left\{ \left[ \frac{32}{\pi D^3} \sigma_M \right]^2 + \left[ -\frac{96}{\pi} \frac{M}{D^4} \sigma_D \right]^2 \right\}^{1/2} \]  

(24)

The expression obtained using the approximate partial derivative method may not be valid in certain cases.
For example, in the case of a product \( Z = XY \) the standard deviation of \( Z \) is given by (13, p. 123)

\[
\sigma_Z = \sqrt{(X \sigma_Y)^2 + (Y \sigma_X)^2 + (\sigma_X \sigma_Y)^2}
\]  

(25)

whereas the expression given by the partial derivative method is

\[
\sigma_Z = \sqrt{(X \sigma_Y)^2 + (Y \sigma_X)^2}
\]  

(26)

When the term \((\sigma_X \sigma_Y)^2\) is small compared to other terms on the right side of Eq. (25) then the approximation will yield good answers. If the values of \( \sigma_X \) and \( \sigma_Y \) differ from each other by 3 or 4 orders of magnitude then the term is no longer negligible. Then exact expressions for the standard deviations must be used. They can be found in Chapter 3 of Ref. 13.

The standard deviation of the shaft diameter is a function of the tolerance on the diameter. Assuming the distribution of the shaft diameter is normal then 99.73% of the shaft diameters will fall within \( \pm 3 \sigma_D \).

Denoting the shaft diameter as \( D \pm t \), where in most cases the tolerance \( 2t \) is known, then \( 3\sigma_D = t \).

The mean and standard deviation of the moment can be determined from the loading distribution. The only unknown in equations 21, 23, and 24 is \( D \) the diameter of the shaft and it can be solved for.
This, then, is a method of designing a rotating shaft, subjected to a bending moment and a torque, for a specified number of cycles of life with a predetermined reliability.

However it is important to recall the limitations and assumptions.

1. The amount of data used in estimating the distribution parameters can have a substantial effect on the shape of the distribution. The most pronounced effect is in the tails. Since the calculation of reliability is based on the overlap of the tails of two distributions it is necessary to have good estimates of the distributions in order to obtain accurate values of reliability.

2. The tests were conducted on specimens of given geometry, hardness, and material at room temperature. The fatigue surfaces generated from these tests are valid only for this type of specimen, environment and loading. The loading was a bending moment and a torque causing an alternating normal stress acting parallel to the centerline of the shaft and a constant shear stress acting perpendicular to the centerline.
3. It was assumed that along any given stress ratio axis the strength distribution is normal.

4. It was also assumed that the ratio of the alternating and mean stresses, to which the shaft being designed is subjected, is constant.

Obviously, it would be an almost impossible task to generate statistical fatigue surfaces to cover every type of material and configuration. However, it is feasible that enough tests could be conducted to allow empirical equations to be developed relating different stress concentration factors and conventional physical properties for the more common steels. This would make it possible to design from a diagram which was not generated for that particular configuration and material.

2.9 Modification of One Test Machine

This section will discuss some of the problems encountered in working with the test machines and the measures taken to overcome some of the problems. Two of the most troublesome areas are the instrumentation and the gear couplings. The gear couplings are the means by which the toolholder arms are allowed to swing downward slightly when the bending load is applied. Referring to Fig. 1.1 it is seen that the bending load configuration also applies a shear load to the couplings. However, the couplings
were not designed to withstand radial shear loading; their purpose being to transmit torsional loads only. The couplings consist of three major parts. Two of the parts are pressed onto the shafts that are being coupled and have an external gear. The third part is an internal gear sleeve which slides over with the other two parts with the gears meshing as shown in Fig. 2.21.

A radial shear load causes the sleeve to cock allowing the centerline of one shaft to fall below the centerline of the other. This misalignment, along with the angular deflection downward caused by the bending load, causes varying degrees of vibration in the toolholders and specimens. When the gear couplings are new the misalignment is small and there is very little vibration, but as the teeth in the couplings wear the misalignment and vibration become greater.

Note, also that there is no definite pivot point in the couplings, thus the length of the moment arm between the loading bearing and the pivot point can change while the machine is running causing a change in stress applied to the specimen. However, from observing tests in progress it appears as though the magnitude of the alternating stress remains quite constant for runs of up to about two hours. This investigator has never monitored a run longer than about two hours. The reason for this is that it is necessary
Fig. 2.21 Gear Coupling Used on Fatigue Testing Machine.
to have the contact points off the slip rings when the specimen breaks. Just prior to the specimen breaking, if the toolholder arms vibrate quite a bit. If the points are down on the slip rings when this happens they can become damaged or broken. Usually the endurance tests are monitored for about 20 minutes.

Another problem is that there is a difference in the number of divisions, as recorded by the Visicorder, between the static and dynamic outputs. In other words, after the specimen is loaded, the machine is rotated by hand and the Visicorder trace is observed. Then the loads are adjusted until the desired output is attained. However, when the machine is rotated under power the output is a different amount, sometimes as much as four divisions. This can represent as high as 8,000 psi. depending upon the gain setting of the amplifier. The change can be easily corrected for the bending stress simply by changing the weight on the loading arm. This can be done while the machine is running. To make the correction for torque load the machine must be shut down and the torque coupling adjusted. However, the amount of change between static and dynamic outputs is not constant. After adjusting the torque to allow for the change, when the machine is restarted the change may be a different amount so the machine must again be shut down and readjusted.
There is another complication entering the problem. After about 30 seconds of running time the torque trace begins to drift quite a bit. The amount of drift depends upon which machine is being used and the torque load applied to the specimen. The higher the torque load the less the drift. Tests were made to determine if the torque load was changing or if the drift was due to amplifier drift. Several factors led to the conclusion that the drift is caused by drift in the amplifier; one being the fact that the torque output drifts upward in two machines. It is hard to believe that if the torque were changing, it would become greater. The tests seemed to indicate that the zero datum point was drifting. However, it drifted only when the machine was running. The drift, when the machine was not running would amount to one division, at the most, in about 2 hours whereas the drift amounted to as high as 3 divisions in 2 minutes with the machine running. One result of the drift is that if, when adjusting the torque load, the proper number of divisions is not obtained in one or two tries the torque will start to drift and no accurate measure of the amount of torque on the specimen can be obtained. When this happens the loads must be removed from the specimen, one holding collet must be loosened and the amplifier recalibrated. This is very time consuming, especially if this procedure must
be gone through two or three times.

The drift problem could probably be eliminated by obtaining solid state amplification equipment. The present equipment has been in use since the start of the research program and is not solid state. Obtaining solid state equipment is planned for the near future.

It is not quite clear what is causing the difference between the static and dynamic outputs. For the bending stress it seems feasible that such a difference may be caused by the difference in the rate of elastic deformation of the toolholder. When the machine is rotated by hand the rate of deformation is slow but under power the machine rotates at about 1,750 rpm and the deformation of an element on the surface of the toolholder may lag behind the applied load. Using this line of reasoning it would seem that as the peak load is reached, if the corresponding deformation is lagging, then as the load begins to decrease the deformation will continue to increase only to that point where the deformation corresponds to the decreasing load. In other words, the toolholder never reaches the amount of deformation corresponding to the peak load. This reasoning would mean that the divisions of Visicorder output should be greater for the static case than for the dynamic case. However, the opposite is true. In all cases, where a difference in the bending output has been experienced, the
bending gage output has been greater with the machine running under power than when turned by hand.

The above reasoning cannot explain the difference in outputs in the torque gage bridge either, because the torque load is not alternating but is constant.

The torque gage bridge output for the static case has been greater than for the dynamic case in most tests. This can be easily explained. Torque is exerted on the specimen through the torque coupling on the backshaft (Fig. 1.1). The torque is transmitted from the backshaft to the toolholder shafts through the gear boxes. When torque is applied through the torque coupling the rate at which it is applied is very slow. When torque is applied the play in the gearboxes is taken up along with any other play in the system. Then when the machine is started the initial impact causes more play to be taken up and the torque decreases. At very high torque levels all the play is taken up in the initial application of the torque.

The initial starting impact causes very little decrease in torque gage bridge output, but the lower the torque level the greater is the amount of decrease. For the lower levels it is more difficult to take the decrease into account by setting the torque higher because at any given torque level the amount of decrease from specimen to specimen is not the same. Several trials enable one to determine the change
and set the divisions accordingly.

Late in November, 1969 one machine was partially dismantled for the purpose of modifying the gear couplings. The modification consisted of placing a spherical bearing inside of each of the two couplings. The bearings do several things to eliminate some of the problems. They absorb the radial shear load and create a definite point about which the toolholder arms can pivot. The bearings have not been in use long enough to determine whether or not the rate of gear tooth wear will decrease in the couplings, but it is believed that it will. Since there is now a definite pivot point, even if there is excessive tooth wear the outer ring will not be able to misalign and cause vibration. In Fig. 2.22 note that the inner diameter of the spherical bearing has a sliding fit with the shaft through it. This is true of both bearings. This eliminates the possibility of subjecting the specimen to an axial load when tightening the specimen in the collets.

Since the spherical bearings have been installed, about 34 endurance tests at a stress ratio of 0.44 have been run on the modified machine. No vibrational problems have been encountered and it appears as though the difference between the static and dynamic outputs is minimal.
Fig. 2. Modified Gear Coupling.
CHAPTER III
OVERALL CONCLUSIONS

1. Estimates for the normal distribution parameters were obtained for the endurance level at stress ratios of \( \infty \), 3.5 and 1.0. The tests at stress ratio of 0.44 are in progress.

2. Estimates for the normal and log-normal distribution parameters were obtained from the cycles-to-failure data. By comparing the D-values and the coefficients of skewness and kurtosis it was concluded that the data tends towards the log-normal distribution. It was also concluded that in order to make a more conclusive decision a larger sample size at each stress level would be required.

3. The method previously proposed to obtain strength distribution parameter estimates yields optimistic values and should be studied further.

4. If random data is obtained by the method proposed in this report estimates which might have favorable properties may be obtained.

5. Fatigue surface was generated using the endurance data which is among the first few attempts to obtain such design data. Efforts to generate such data based on larger samples, with solid state instrumentation, and with
greater operator care, should be continued.

6. On the basis of the 34 specimens tested on the machine with the modified couplings it is concluded that the spherical bearings improved the running characteristics of the machine. It is hoped that the long range performance would also be satisfactory.
CHAPTER IV
RECOMMENDATIONS

1. New solid-state instrumentation should be purchased. The new instrumentation should be drift free and should have the capability of constantly monitoring the strain gage outputs.

2. The gear couplings should be modified on the remaining two machines.

3. The endurance tests for stress ratios of 0 and 1.0 should be checked.

4. Before the next phase of the test program is embarked upon the machines should be recalibrated to see if the calibration constants are the same.

5. Larger sample sizes should be used in future tests, preferably 30 or more at each stress level to obtain the much needed cycles-to-failure design data.

6. The feasibility of incorporating random stress levels to produce data from which better strength distribution parameters might be estimated should be investigated.
APPENDIX A:

COMPUTER PROGRAM TO CALCULATE
STRESS LEVELS AND RATIOS

The purpose of this program is to convert the visi­
corder records into normal stresses, shear stresses and
stress ratios. The program also calculates the means and
standard deviations of the stresses and ratio of each group
of data. It also calculates the cycles-to-failure from
times to failure data, but the program will also accept
cycles-to-failure data. The program distinguishes be­
tween the two through the use of a code number. The code
also tells the program whether or not a group of data are
endurance test data. This discrimination is necessary
because of the calculation of mean and standard deviation
of the stresses. The program will not calculate the endur­
ance level distribution parameters. The discrimination
code is fed in as data and is as follows:

0 if the failure data is in times to failure
1 if the data is an endurance test
2 if the failure data is in cycles-to-failure.

The input format for the code will be discussed shortly.

The program will accept as many sets of data as
desired and the groups may be mixed; ie, endurance test,
group with cycles-to-failure data and group with times
to failure data. A group is all the data for one stress level. A group of data consists of the following. The first card contains in this order, the number of specimens in the level, the mode of the run and the code. The mode is dependent upon the date the run was made. For a further discussion on mode see Section 2.1.

The fields on the data card are as follows:

spaces 1 to 5 - number of specimens
spaces 6 to 10 - mode
spaces 11 to 15 - code

The number of specimens, mode and code are fixed point numbers and have no decimals but the numbers must be placed to the right in each field.

The next sequence of cards reads in the cycles-to-failure or times to failure in hours, minutes and seconds. If the data is in times to failure there are ten groups on a card, so the number of cards required will depend upon how many specimens are in the level. The format across the card is:

spaces 1 and 2 - blank
spaces 3 and 4 - hours
spaces 5 and 6 - minutes
spaces 7 and 8 - seconds
spaces 9 and 10 - blank

and the sequence continues in this manner. If the failure data is in cycles-to-failure the format is 8 fields of 10 spaces each and the decimals appear in the last space of
each field, i.e., spaces 10, 20, 30, etc. If the group of data is for an endurance test there is no failure data and these cards are left out. The program will automatically handle it if the proper code number is put on the first card.

Following the cycles-to-failure cards are the cards containing the information for each specimen in the stress level. The information must be placed on each card as follows:

spaces 1 to 5 - test number
spaces 6 to 10 - specimen number
spaces 11 to 15 - machine number
spaces 16 to 20 with a
decimal in space 20 - pan weight

spaces 21 to 30 with a
decimal in space 28 - bending calibration resistance

spaces 31 to 40 with a
decimal in space 38 - number of bending calibration divisions

spaces 41 to 50 with a
decimal in space 48 - number of divisions of bending

spaces 51 to 60 with a
decimal in space 58 - torque calibration resistance

spaces 61 to 70 with a
decimal in space 68 - number of torque calibration divisions

spaces 71 to 80 with a
decimal in space 78 - number of divisions of torque
The test number, specimen number and machine number are fixed point numbers and must be placed to the right in each field. There is one card for each test specimen and the cards must be placed in the same order as the failure data is placed on the cards preceding these cards. For data at stress ratio of ∞ there will be no torque stress data. In this case these fields can be left blank. The computer reads blanks on data cards as zeros.

This makes up one group of data at a given stress level and ratio. As many groups may be run as desired by simply placing the groups one behind the other.

Following is a listing of important variables in the program and Fig. A - 1 is a flow chart of the program. Figure A - 2 is a program listing. The outputs from this program are given in Appendixes D and E.
List of Definitions for Program to Find Stress Levels and Ratios (PROGRAM STRESS)

NCARDS = number of specimens tested at given level.
MODE = number of mode depending on date of test.
NCODE = 0 if failure data is in times to failure.
= 1 if data is from an endurance level.
= 2 if failure data is in cycles-to-failure.

XHOURS(I) = times to failure in hours, minutes and seconds.
XMIN(I) = cycles-to-failure.
SECS(I) = test number.
NOSPEC = specimen number.
MACHNO = machine number.
PANWT = amount of weight on loading arm.
RCALB = calibration resistance used in bending channel.
ENCALB = number of visicorder divisions used when calibrating bending channel.
ENVISB = number of divisions during actual test.
RCALT = calibration resistance used in torque channel.
ENCALT = number of visicorder divisions used when calibrating torque channel.
ENVIST = number of divisions during actual test.
ENA = number of active arms in strain gage bridge.
RGAGEB = resistance of bending strain gages.
RGAGET = resistance of torque strain gages.
GB = bending gage factor.
GT = torque gage factor.

CBGR = calibration constant $K_{BGR}$.

CGRTH = calibration constant $K_{GR\cdot TH}$.

CT = calibration constant $K_{T}$.

CTB = calibration constant $K_{T/B}$.

CBT = calibration constant $K_{B/T}$.

RPM = revolutions per minute of machine.

SOUTH = output normal stress corrected for interaction.

TAUTH = output shear stress corrected for interaction.

STRGR(I) = normal stress in specimen groove.

TAUGR(I) = shear stress in specimen groove.

SOUTHP = output stress not corrected for interaction.

TAUTHP = output stress not corrected for interaction.
Program to Calculate Stress Levels and Ratios

MAIN PROGRAM (STRESS)

START

READ NCARDS, MODE, NCODE

YES

END OF DATA ?

STOP

NO

DOES NCODE = 1

PRINT HEADINGS FOR NON ENDURANCE TESTS AND MODE

NO

DOES NCODE = 2

PRINT HEADINGS FOR ENDURANCE TESTS AND MODE

YES

READ CYCLES TO FAILURE

SPDIA = .4975
TOD = 2.0
TID = 1.3125
SPJC = \( \frac{(3.14159)(SPDIA)^3}{16} \)
THJC = \( \frac{(3.14159)(TOD^h - TID^h)}{(16)(TOD)} \)
CONST = \( \frac{THJC}{SPJC} \)

READ TIMES TO FAILURE

Fig. A - 1
Logical if statements to choose correct calibration constants for MACHNO and MODE

Fig. A - 1 (continued)
Fig. A - 1 (continued)
Fig. A-1 (continued)
\[
\begin{align*}
CYHR &= \text{xHOURS}(I)(60.)(\text{RPM}) \\
CYMIN &= \text{xMIN}(I)(\text{RPM}) \\
CYSEC &= \text{SECS}(I)(\text{RPM}) \\
TCY &= CYHR + CYMIN + CYSEC
\end{align*}
\]

PRINT NOTEST, NOSPEC, MACHNO, PANWT, TCY, RCALB, ENCALB, ENVISB, RCALT, ENCATL, ENVIST, STRGR(I), TAUGR(I), R(I)

Fig. A - 1 (continued)
Calculate mean and standard deviation of normal stress
SUBROUTINE MEAN

PRINT XMEAN, DEV

YES

DOES
NCODE = 1
AND
ENVIST = 0.0

NO

DOES
NCODE = 1

YES

Calculate mean and standard deviation of shear stress
SUBROUTINE MEAN

PRINT XMEAN, DEV

Calculate mean and standard deviation of stress ratio
SUBROUTINE MEAN

PRINT XMEAN, DEV

Fig. A-1 (continued)
SUBROUTINE MEAN

START

SIGMA = 0.
TOP2 = 0.

I = 1

SIGMA = SIGMA + X(I)
TOP2 = TOP2 + (X(I) - MEAN)^2

YES  I > NDATA NO  I = I + 1

XMEAN = SIGMA / DATA

DEV = \sqrt{\frac{TOP2}{DATA-1.0}}

PRINT XMEAN, DEV

RETURN

Fig. A-1 (continued)
APPENDIX A (continued)

Fig. A-2  Computer Printout of PROGRAM (STRESS)
PROGRAM STRESS (INPUT, OUTPUT, TAPE1 = INPUT)
DIMENSION STGK(50), TAU6R(50), T(50), XHOURS(50), XMIN(50),
ISECS(50), TOTCY(50)
C READ IN THE NUMBER OF CARDS IN THE STRESS LEVEL AND THE MODE
C OF OPERATION OF THE MACHINE
C---------NCODE = 1 FOR ENDURANCE LEVEL, NCODE = 0 IF FAILURES IN TIMES TO FAILURE.
C---------NCODE = 2 IF FAILURES IN CYCLES TO FAILURE.
000003 35 READ 100, NCARDS, MODE, NCODE
000015 100 FORMAT (I15)
000015 171 PRINT 91
000024 91 FORMAT (I15//)
000024 170 IF (NCODE.EQ.1) GO TO 175
C---------FORMAT FOR OUTPUT HEADINGS INCLUDING TIMES TO FAILURE.
000034 310 FORMAT (62X, 11HTEST MODE = /12)
000034 61 PRINT 61
000040 61 FORMAT (2X, 34HTEST SPEC. MACH. PAN CYCLES+7X+4HRCAL+7X
1+4HNCAL+7X+4HNVIS+6X+4HRCAL+6X+4HNCAL+6X+4HNVIS+5X+7BHENDING+4X+1
26HSHEAR STRESS/3X+35HNO. NO. WT. TO FAILURE/3X
37BHENDING+4X+7BHENDING+4X+7BHENDING+4X+6MTORQUE+4X+6MTORQUE+4X
46HTORQUE+5X+6HSTRESS+4X+6HSTRESS+4X+5HNO. NO. WT. TO FAILURE/)
000040 61 IF (NCODE.EQ.2) GO TO 320
C---------ROUTINE TO READ IN TIMES TO FAILURE.
000042 401 FORMAT (8(I2X, 3F2.0, 2XI)
000042 401 READ *01, (XHOURS(I), XMIN(I), SECS(I), I = 1, NCARDS)
000061 401 FORMAT (16(2X, 3F2.0, 2XI)
000061 401 GO TO 300
C---------HEAD IN CYCLES TO FAILURE.
000062 320 READ *02, (TOTCY(I), I = 1, NCARDS)
000075 402 FORMAT (6F10.0)
000075 402 GO TO 300
C---------FORMAT FOR OUTPUT HEADINGS WITHOUT TIMES TO FAILURE.
000076 90 PRINT 90, MODE
000094 50 FORMAT (50X, 14HENDUANCE TEST*10X+11HTEST MODE = /12)
000104 90 FORMAT (50X, 14HENDUANCE TEST*10X+11HTEST MODE = /12)
000110 60 FORMAT (3X+4HTEST, 3X+8HSPECIMEN, 3X+7MACHINE, 5X+3SPAN, 7X+4HRCAL+7X
1+4HNCAL+7X+4HNVIS+6X+4HRCAL+6X+4HNCAL+6X+4HNVIS+5X+7BHENDING+4X+1
25HSHEAR STRESS/3X+35HNO. NO. WT. TO FAILURE/3X
37BHENDING+4X+7BHENDING+4X+7BHENDING+4X+6MTORQUE+4X+6MTORQUE+4X
46HTORQUE+5X+6HSTRESS+4X+6HSTRESS+4X+5HNO. NO. WT. TO FAILURE/)
000110 60 C---------ROUTINE TO CALCULATE POLAR MOMENTS OF INERTIA.
C---------SPDIA = SPECIMEN DIA. T0D = TOOLHOLDER O. D. TID = TOOLHOLDER I. O.
000110 300 SPDIA = 4975
000111 300 SPDIA = 4975
000113 300 T0D = 2, 0
000113 300 TID = 3, 3125
000114 300 SPJC = (3.14159*SPDIA**3)/16, 0
000117 300 TMJC = (3.14159*6*(TID**3=10**4))/16, 0
000126 35 DO 120 I = 1, NCARDS
000130 120 READ 10, NO, TES, MACHNO, PAN, T, R, CALB, ENCAB, ENVIS, HCAL, IENCAT, ENVIS
000130 120 READ 10, NO, TES, MACHNO, PAN, T, R, CALB, ENCAB, ENVIS, HCAL, IENCAT, ENVIS
000132 10 FORMAT (315F5.0, 6F10.2)
000132 10 FORMAT (315F5.0, 6F10.2)
000132 10 READ *10, TES, MACHNO, PAN, T, R, CALB, ENCAB, ENVIS, HCAL, IENCAT, ENVIS
000162 10 FORMAT (315F5.0, 6F10.2)
000162 10 FORMAT (315F5.0, 6F10.2)
000162 10 C DEFINE THE ELASTIC MODULUS, NO. OF ACTIVE ARMS OF THE BRIDGES, THE
RESISTANCES OF THE BENDING AND TORQUE GAUGES, AND THE BENDING AND
TORQUE GAUGE FACTORS

E = 30000000

GAGE8 = 190
RGAGE8 = 120
GB = 3.23
GT = 2.05

SELECTION OF MACHINE AND MODE

IF (MACHNO. EQ. 3. AND. MODE. EQ. 2) GO TO 24
IF (MACHNO. EQ. 2. AND. MODE. EQ. 3) GO TO 24
IF (MACHNO. EQ. 3. AND. MODE. EQ. 4) GO TO 34

CALIBRATION PARAMETERS FOR GIVEN NODE AND MACHINE

C8GR = 0.08
RPM = 1760
GO TO 50

C8GR = 0.08
RPM = 1780
GO TO 50

C8GR = 0.08
RPM = 1780
GO TO 50

IF (ENVIST. EQ. 1.0) GO TO 160
GO TO 51
C  CALCULATION OF BENDING STRESS LEVEL FOR INFINITY RATIO
000377  160 SOUTH(ENVIS=ENGAGE)*/(ENCAL=ENGAGE)/RACAL
000405  STRGR(1) = SOUTH / (CGRTH = CHGR)
000411  TAUGR(1) = 0.3
000412  IF (NCODE,EQ,1) GO TO 301
000414  IF (NCODE,EQ,2) GO TO 330
C----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
000415  CYHR = XHOURS(I)*1500/RPH
000417  CYMIN = XM1N(I)*RPM
000421  CYSEC = (SECS(I)*RPM)/60,0
000423  TCY = CYHR*CYMIN*CYSEC
000427  GO TO 335
000427  330 TCY = TOTCY(I)
000431  335 PRINT 72,NOTES,NOSPEC,MACHNO,PAKNT,TCY,HCALT,ENCALT,ENVISH,RCALT,
000467  72 FORMAT((I16,F14,2F11,0,F11,2,F11,0,F10,2));
000467  72 IF11,2,F11,0,F10,2*;
000467  GO TO 120
000470  301 PRINT 71,NOTES,NOSPEC,MACHNO,PAKNT,HCALT,ENCALT,ENVISH,RCALT,
000524  71 FORMAT((I12,13,0,13,8,1,16,4,5,1,F12,0,F11,2,F11,0,F10,2*;
000524  IF11,2,F11,0,F10,0,3X,6,1NFINI(1,1);
000524  GO TO 120
C  CALCULATION OF BENDING STRESS, SHEAR STRESS AND STRESS RATIO FOR
C  ALL FINITE RATIOS
000525  51 SOUTH=ENVISH=SHEAR/*/ENCALT=ENGAGE+HCALT*(1)
000533  51 TAUGP=ENVISH=SHEAR/*/ENCALT=ENGAGE+HCALT*(1)
000541  SOUTH=SOUTH+15*1AUHP
000544  TAUGP=TAUHP+75*SOUTH
000547  STRGR(1) = SOUTH / (CGRTH = CHGR)
000553  TAUGR(1) = CT*TAUHP CONST
000555  RI(I) = STRGR(1) / (TAUGR(1) = 1,732)
000561  IF (NCODE,EQ,1) GO TO 302
000563  IF (NCODE,EQ,2) GO TO 340
C----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
000564  CYHR = XHOURS(I)*60,RPM
000566  CYMIN = XM1N(I)*RPM
000570  CYSEC = (SECS(I)*RPM)/60,0
000572  TCY = CYHR*CYMIN*CYSEC
000576  GO TO 345
000576  340 TCY = TOTCY(I)
000600  345 PRINT 73,NOTES,NOSPEC,MACHNO,PAKNT,TCY,HCALT,ENCALT,ENVISH,RCALT,
000640  73 FORMAT((I16,F8,1,2F11,0,F11,2,F11,0,F10,2*;
000640  IF11,2,F11,0,F10,0,F9,3)/
000640  GO TO 120
000641  302 PRINT 70,NOTES,NOSPEC,MACHNO,PAKNT,HCALT,ENCALT,ENVISH,RCALT,
000677  70 FORMAT((I14,13,0,13,8,1,16,4,5,1,F12,0,F11,2,F11,0,F10,2*;
000677  IF11,2,F11,0,F10,0,F9,3)/
000677  120 CONTINUE
000702  IF (NCODE,=E,1,AND,ENVISH,EQ,6,0) GO TO 35
000711  IF (NCODE,=E,1) GO TO 200
C  CALCULATION OF MEAN AND STANDARD DEVIATION OF BENDING STRESS, SHEAR
C  STRESS, AND STRESS RATIO
000713  CALL MEAN (STGR, CARDS,NCARDS, XMEAN, DEV)
000716  PRINT 3
000722  3 FORMAT (TH1)
PRINT 13, XHAN, DEV
000732 130 FORMAT (19X,3F10.5, 1, 5H 15 PSI//11X, 139HSTU, 15H OF BENDING STRESS IN GROOVE = F10,2+5H PSI/)
000732 132 IF(ENVISTR,EQ.,40) GO TO 35
000732 133 IF (NCODE.EQ.1) 51 TO 260
000732 134 CALL MEAN (TAUGH, CARUS, NCARS, XMEAN, DEV)
000741 PRINT 3
000745 PRINT 140, XMEAN, DEV
000755 140 FORMAT (19X,16HMEAN TORSION STRESS IN GROOVE = F10,3H PSI//, 139HSTU, 15H OF TORSION STRESS IN GROOVE = F10,2+5H PSI/)
000755 200 CALL MEAN (N, CARDS, NCARS, XMEAN, DEV)
000761 PRINT 3
000765 PRINT 150, XMEAN, DEV
000775 150 FORMAT (19X,19HMEAN STRESS RATIO = F10,5/ 234,27HSTU, 15H OF STRENGTH RATIO = F10,5)
000775 37 GO TO 35
000776 201 STOP
000776 204 END
SUBROUTINE MEAN (X, DATA, NDATA, XMEAN, DEV)
C---SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
000010   DIMENSION X(NDATA)
000010   SIGMA= 0.0
000011   DO 8 I=1, NDATA
000012   8 SIGMA=SIGMA+ X(I)
000013   XMEAN = SIGMA/DATA
000017   TOP2 = 0.0
000020   DO 9 I=1, NDATA
000021   9 TOP2 = TOP2 + (X(I) - XMEAN)**2
000026   DEV=SQRT(TOP2/(DATA - 1.0))
000036   RETURN
000036   END
APPENDIX B

Program to Calculate the Parameters of the Normal and Log-Normal Distributions and Conducts Goodness-of-Fit Tests

This program calculates the mean and standard deviation of the cycles-to-failure data for both the normal and log-normal distributions and calculates the moment coefficients of skewness and kurtosis. It also performs the Chi-square and Kolmogorov-Smirnov goodness-of-fit tests. The input consists of:

1. a card containing the number of data points at that stress level, the accuracy of the data, the stress level and the stress ratio.

2. a card or series of cards containing the cycles-to-failure data listed in descending order.

3. a card or series of cards containing the cumulative frequency of the cycles-to-failure up to that point. These must be listed in the same order as the cycles-to-failure data.

The input format for the first card is:

spaces 1 to 3 - number of data points in fixed point format.

spaces 4 to 8 with a

decimal in space 7 - number of data points in floating point format
spaces 9 to 17 with a
decimal in space 13  - accuracy of the cycles-to-
failure data.

spaces 18 to 27 with a
decimal in space 26  - stress level at which the data
was taken.

spaces 28 to 35 with a
decimal in space 30  - stress ratio at which the data
was taken.

The cards containing the cycles-to-failure data have
eight fields of ten spaces each with the decimal at the
right of each field. In other words the first data point
is in spaces 1-10 with the decimal in space 10, the second
data point in spaces 11-20 with the decimal in space 20, etc.

The format of the cards containing the cumulative
frequency data is 26 fields of three spaces each with the
decimal to the right of each field. The first value is in
spaces 1-3 with the decimal in space 3, the second value
is in spaces 4-6 with the decimal in space 6, etc.

The program is set up to accommodate as many sets
of data as desires.

Following is a list of important variables used in the
program. Figure B-1 is a flow diagram of the program and
Fig. B-2 is a printout of the program. Figure B-3 is
an example of the output for one set of data. The output
shown is for data tested at a stress level 114,000 psi. and
stress ratio of $\infty$. 
List of Definitions for Program to Fit Normal and Log-Normal Distributions to Cycles-to-Failure Data (PROGRAM CYTOFR)

Main Program:

\[ \text{NDATA} = \text{DATA} = \text{number of observations.} \]
\[ \text{STRLV} = \text{stress level in psi.} \]
\[ \text{AKURCY} = \text{accuracy to which cycles-to-failure data are known.} \]
\[ \text{RATIO} = \text{stress ratio} \]
\[ \text{X}(I) = \text{cycles-to-failure data} \]
\[ \text{CUMFREQ}(I) = \text{cumulative frequency of each } X(I); \text{ ie, number of } X \text{'s less than or equal to } X(I). \]
\[ \text{PCAREA}(I) = \frac{\text{CUMFREQ}(I)}{\text{NDATA}} \]

Subroutine to calculate the mean and standard deviation of the cycles-to-failure data (SUBROUTINE MEAN)

\[ \text{SIGMA} = \text{sum of the } X(I)'s \]
\[ \text{XMEAN} = \text{average of the } X(I)'s \]
\[ \text{TOP2} = \frac{1}{n} \sum_{i=1}^{n} (X(I)-X\text{MEAN})^2 \]
\[ \text{DEV} = \text{standard deviation of the } X(I)'s \]

Function subroutine to find the area under the normal curve (FUNCTION PROB(X)).

\[ X = \text{abscissa value for which corresponding area is desired.} \]
\[ \text{PROB} = \text{desired area.} \]
Subroutine for Chi-square goodness-of-fit test (SUBROUTINE CHISQA).

\[ K \quad = \quad \text{number cells.} \]
\[ X\text{MAX} \quad = \quad \text{largest value of cycles-to-failure.} \]
\[ X\text{MIN} \quad = \quad \text{smallest value of cycles-to-failure.} \]
\[ \text{CSV} \quad = \quad \text{cell starting value.} \]
\[ \text{CEV} \quad = \quad \text{cell end value.} \]
\[ \text{CLB} \quad = \quad \text{cell lower bound.} \]
\[ \text{CUB} \quad = \quad \text{cell upper bound.} \]
\[ \text{FREQ(J)} \quad = \quad \text{number of observations in } J^{th} \text{ cell.} \]
\[ \text{REQAREA(J)} \quad = \quad \text{expected value of } J^{th} \text{ cell.} \]
\[ \text{CHISQR} \quad = \quad \text{total Chi-square value.} \]
\[ U(I) \quad = \quad \text{Chi-square value of } I^{th} \text{ cell.} \]

Subroutine for Kolmogorov-Smirnov test (SUBROUTINE DTEST).

\[ Z(I) \quad = \quad \text{abscissa value on standard normal curve for a given } X(I). \]
\[ \text{ARUNCN} \quad = \quad \text{area under standard normal curve from } -Z(I) \text{ to } Z(I). \]
\[ \text{DSTAT}(I) \quad = \quad \text{absolute difference between the data cumulative frequency and the hypothesized cumulative frequency.} \]
\[ \text{XMEAN} \quad = \quad \text{average of the } X(I)'s. \]
\[ \text{DEV} \quad = \quad \text{standard deviation of the } X(I)'s \]
\[ \text{PROB(T)} \quad = \quad \text{area under the standard normal curve from } -T \text{ to } +T. \]
Subroutine to calculate the moment coefficients of skewness and kurtosis (SUBROUTINE ALPHA).

\[ \text{ALPHA}_3 = \text{moment coefficient of skewness}. \]
\[ \text{ALPHA}_4 = \text{moment coefficient of skewness}. \]
\[ \text{VAR} = \sum_{i=1}^{n} (X(i) - \bar{X})^2 \]
\[ \text{TOP3} = \sum_{i=1}^{n} (X(i) - \bar{X})^3 \]
\[ \text{SKEW} = \text{third moment of the data}. \]
\[ \text{STDEV} = \text{biased estimator for standard deviation}. \]
\[ \text{TOP4} = \sum_{i=1}^{n} (X(i) - \bar{X})^4 \]
\[ \text{TKURT} = \text{fourth moment of the data}. \]
Program to Calculate Parameter Estimates for the Normal and Log-Normal Distributions and Conduct Goodness-of-Fit Tests

MAIN PROGRAM (CYTOFR)

START

READ NDATA, DATA, AKURCY, STRLEV, RATIO

NO END OF DATA ?

YES STOP

READ X(I)'s

READ CUMFRQ(I)'s

I = 1

PCAREA(I) = CUMFRQ(I) / NDATA

YES I > NDATA NO

I = I+1

PRINT STRLV, RATIO, X(I)'s

Calculate mean and standard deviation of X(I)'s

SUBROUTINE MEAN

Do Chi-square goodness-of-fit test

SUBROUTINE CHISQA

Fig. B-1
Do Kolmogorov-Smirnov Goodness-of-Fit Test
SUBROUTINE DTEST

Calculate Moment Coefficients of Skewness and Kurtosis
SUBROUTINE ALPHA

AKURCY = .00001

I = 1

NX(I) = \left[ \left( \frac{\log X(I)}{20} + \log(20) \right)(10000.) \right] + .5

X(I) = NX(I)

X(I) = \frac{X(I)}{10000.}

YES

I > NDATA

NO

PRINT STRLEV, RATIO, X(I)'s

IS

Calculate mean and standard deviation of X(I)'s
SUBROUTINE MEAN

I = I + 1

Fig. B-1 (continued)
Do Chi-square
goodness-of-fit test
SUBROUTINE CHISQA

Do Kolmogorov-Smirnov
goodness-of-fit test
SUBROUTINE DTEST

Calculate moment coefficients of skewness and kurtosis
SUBROUTINE ALPHA

Fig. B-1 (continued)
Subroutine to Find Mean and Standard Deviation
SUBROUTINE MEAN

START

SIGMA = 0.
TOP2 = 0.

I = 1

SIGMA = SIGMA + X(I)
TOP2 = TOP2 + (X(I) - MEAN)^2

YES

I > NDATA

NO

I = I + 1

XMEAN = SIGMA / DATA

DEV = \sqrt{TOP2 / DATA - 1.0}

PRINT XMEAN, DEV

RETURN

Fig. B-1 (continued)
Subroutine to Find Area Under Standard Normal Curve

FUNCTION PROB(X)

START

IS (X-1.2) >0

≤ 0

XSQ = (X)^2

PROB = (.79788455)(X) .99999764- XSQ[-.16659433-
XSQ(.024638310-XSQ(.0023974867))]

RETURN

IS X-2.9 ≥ 0

<0

XSQ = X^2

PROB = 1.0

PTERM = 1.0

FACTOR = 1.0

ODDINT = 3.0

RETURN

RECXSQ = \frac{1}{X^2}

PROB = 1.0-(.79788453) EXP[
\left(-\frac{X^2}{2}\right)(1.0-RECXSQ(1.0-RECXSQ
(3.0-RECXSQ(15.0-RECXSQ(105))))])

RETURN

Fig. B-1 (continued)
Fig. B-1 (continued)
Subroutine to Conduct Chi-Square Goodness-of-Fit Test

SUBROUTINE CHISQA

START

CHISQR = 0.0

K = 1.5 + (3.322) \log_{10}\text{DATA}

REALK = K

XMAX = X(1)

XMIN = X(1)

I = 1

NO

IS

X(I) > XMAX

YES

XMAX = X(I)

YES

XMIN = X(I)

IS

X(I) < XMIN

NO

NO

IS

I > NDATA

YES

NO

I = I + 1

Fig. B-1 (continued)
RANGE = XMAX - XMIN

DIVIDE = 1.0 /

AKURCY

KW = \left[ \frac{RANGE + AKURCY}{REALK} + (0.5)AKURCY \right] / \text{DIVIDE}

RK1 = KW

W = RK1 / \text{DIVIDE}

PRINT XMAX, XMIN, W

I = 1

A = I

B = (0.5)AKURCY

CSV(I) = XMIN + (A-1.0)(W)

CEV(I) = CSV(I) + W - AKURCY

CLB(I) = CSV(I) - B

CUB(I) = CEV(I) + B

\text{NO}

\text{IS}

\text{YES}

I > K

CUB(I) = CEV(I) + B

CEV(K) = XMAX

CUB(K) = CEV(K) + B

Fig. B-1 (continued)
Fig. B-1 (continued)
\[ Z(I) = \frac{CUB(I) - XMEAN}{DEV} \]
\[ T = Z(I) \]
\[ AREA(I) = \frac{\text{PROB}(T)}{2.0} \]

\[ \text{YES} \quad I > K \quad \Rightarrow \]

\[ \text{REQAREA}(I) = 0.5 \times \text{AREA}(1) \]
\[ \text{MANU} = K - 1 \]

\[ \text{NO} \quad I > K \quad \Rightarrow \]

\[ I = I + 1 \]

\[ \text{YES} \quad I = 2 \quad \Rightarrow \]

\[ M = I - 1 \]

\[ \text{YES} \quad Z(I) \geq 0 \quad \text{AND} \quad Z(M) \geq 0 \quad \Rightarrow \]

\[ \text{REQAREA}(I) = \text{AREA}(I) - \text{AREA}(M) \]

\[ \text{NO} \quad \Rightarrow \]

\[ \text{YES} \quad Z(I) \leq 0 \quad \text{AND} \quad Z(M) \leq 0 \quad \Rightarrow \]

\[ \text{REQAREA}(I) = \text{AREA}(I) + \text{AREA}(M) \]

\[ \text{NO} \quad I > \text{MANU} \quad \Rightarrow \]

\[ I = I + 1 \]

Fig. D-1 (continued)
\begin{align*}
\text{REQAREA} &= 0.5 - \text{AREA}(K-1) \\
M &= 1 \\
\text{EXFREQ}(M) &= \text{DATA} \times \text{REQAREA}(M) \\
U(M) &= \frac{\text{EXFREQ}(M) - \text{FREQ}(M)^2}{\text{EXFREQ}(M)} \\
\text{CHISQR} &= \text{CHISQR} + U(M) \\
\text{IS} & \quad M > K \\
\text{IF} & \quad \text{YES} \\
I &= 1 \\
\text{PRINT} I, \text{CLB}(I), \text{CUB}(I), \text{EXFREQ}(I), \text{FREQ}(I), U(I) \\
I &= I + 1 \quad \text{NO} \\
\text{IS} & \quad I > K \\
\text{IF} & \quad \text{YES} \\
\text{PRINT CHISQR} \\
\text{RETURN} \\
\text{IF} & \quad \text{NO} \\
\text{IS} & \quad M = M + 1 \\
\text{IF} & \quad \text{YES} \\
\text{END} \\
\end{align*}

Fig. B-1 (continued)
Subroutine to Conduct Kolmogorov-Smirnov Goodness-of-Fit Test

SUBROUTINE DTEST

START

I = 1

\[
Z(I) = \frac{X(I) - X_{\text{mean}}}{\text{DEV}}
\]

\[ T = -Z(I) \]

IS \( Z(I) \) > 0 \( \rightarrow T = Z(I) \)

\[ D\text{STAT}(I) = 0.5 - \text{PCAREA} \]

ARUNCN = \( \frac{1 - \text{PROB}(T)}{2} \)

DSTAT = ARUNC - PCAREA(I)

ARUNCP = \( \frac{\text{PROB}(T)}{2} + 0.5 \)

DSTAT(I) = ARUNCP - PCAREA(I)

IS

I > NDATA

YES

PRINT DSTAT(I)'s

NO

I = I + 1

RETURN

Fig. B-1 (continued)
**Subroutine to Find the Moment Coefficients of Skewness and Kurtosis**

**SUBROUTINE ALPHA**

```
START

TOP3 = 0
TOP4 = 0
VAR = 0

I = 1

VAR = VAR + (X(I) - XMEAN)^2
TOP3 = TOP3 + (X(I) - XMEAN)^3
TOP4 = TOP4 + (X(I) - XMEAN)^4

YES  I > NDATA  NO  I = I + 1

SK\textit{E}W = \frac{TOP3}{DATA}
ST\textit{DE}V = \sqrt{\frac{VAR}{DATA}}
ALPHA3 = \frac{SK\textit{E}W}{(ST\textit{DE}V)^3}
TK\textit{UR}T = \frac{TOP4}{DATA}
ALPHA4 = \frac{TK\textit{UR}T}{(ST\textit{DE}V)^4}

PRINT ALPHA3, ALPHA4

RETURN
```

Fig. B-1 (continued)
APPENDIX B (continued)

Fig. B-2  Computer Printout of PROGRAM (CYTOFF)
PROGRAM CYTOHM (INPUT, OUTPUT, TAPE = INPUT)
C----- PROGRAM TO FIT NORMAL AND LOG-NORMAL CURVE TO DATA AND CHECK
C----- GOODNESS OF FIT.
C
000003 DIMENSION X(100), Y(100), X(200) + CUMF(100)

000003 SPLEAN(100) + STAT(100) + FJEW(100) + ARE(9) + REG(9) + TAF(9) + U(9)
000003 Z(100) + K(I) + RANK(100)
000003 EXTERNAL PLO"R
000003 710 PRINT 1
C----- MAXIMUM NUMBER OF OBSERVATIONS
C----- STRESS = STRESS LEVEL IN PSI.
000003 READ 100, NDATA, AKURCY, STRESS, RATIO
C----- AS NUMBER OF CYCLES TO FAILURE
000025 0 FORMAT (13, 5F9, 1, 10, 1, 5F9, 1)
000025 1 IF (EUP(I) > 50) GOSUB 410
000030 55 HEAD 7; X(I), I = 1 + NDATA
000043 7 FORMAT (4F10, 1)
C----- ROUTINE TO CALCULATE CUMULATIVE DISTRIBUTION FOR DATA.
C----- HEAD CUMULATIVE VALUE FOR EACH POINT.
000043 READ 7; (CUMF(A(I)) = 1 + NDATA)
000056 701 FORMAT (26F1.0)
C----- PCARE = F(N) OF OBSERVATIONS
000050 DO 759 I = 1 + NDATA
000059 759 PCARE(I) = CUMF(A(I)) / DATA
000054 PRINT 4: S
000070 405 FORMAT (45X, 5H NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA)
1A///
000070 IF (RATIO = 0.6) GO TO 414
000071 PRINT 4, 2, STRESS, RATIO
000010 402 FORMAT (5H STRESS LEVEL = 10.1, 5H PSI, 1A)
000010 414 STRESS RATIO = 10.3

000010 410 GO TO 415
000010 415 PRINT 4, 16, STRESS
000010 414 print 4, 16, STRESS
000010 411 FORMATION (5X, 12H CYCLES TO FAILURE DATA)
000011 PRINT 4, 14, (A(I)) = DATA
000012 463 FORMAT (50X, 10F14, 1)

000012 PRINT 3
000013 3 FORMAT (16H)
000013 CALL MEAN(A(I), NDATA, MEAN, MEAN, DEV.
000013 3 CALL CHISQ(A(I), NDATA, PROB, AKURCY, MEAN, MEAN, DEV.
000013 3 CALL DIEST (PCAREA + NDATA, DATA, X(I) = DATA + NDATA)
000015 CALL ALPHA(A(I), NDATA, DATA, MEAN, DEV, ALPHA1, ALPHA2)
000015 CALL AKURCY = 0.0001
000017 0 DATA
000017 10 DATA
000017 NA(I) = LOG((A(I)/26) + LOG(26)) * 10000. + .5
000020 X(I) = NA(I)
000020 X(I) = X(I)/10000.
000020 PRINT 1
000021 1 FORMAT (1H1+3H50///)

000021 PRINT 4, 1
000022 401 FORMAT (3X, 5H LOG-NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA)
000022 IF (RATIO = 0.6) GO TO 417
000022 402 PRINT 4, 2, STRESS, RATIO
000023 417 GO TO 418
PRINT 16, SIMLEV
PRINT 2
PRINT 13, (X(I)+1)/2*DATA)
FORMAT (5(5x,F12.5))
PRINT 3
CALL MEAN(X,DATA,NDATA,AMN,DEV)
CALL CHISQ(X,DATA,THUP,PHU,AMN,DEV,7)
CALL UTEST(PCASE,NDATA,AMN,DEV,USTAT,PHU,AMN),2)
CALL ALPHA(X,NDATA,DATA,XAMN,DEV,ALPHA,ALPHA)
GO TO 710
STOP
70323 END
SUBROUTINE MEAN (A, DATA, NDATA, XMEAN, DEV)
C-----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
DIMENSION A(NDATA)
DO 10 I=1,NDATA
10 SIGMA = SIGMA + A(I)
XMEAN = SIGMA/DATA
TOP2 = 0.0
DO 20 I=1,NDATA
20 TOP2 = TOP2 + (A(I) - XMEAN)**2
DEV = SQRT(TOP2/(DATA - 1.0))
PRINT 14, XMEAN
PRINT 15, DEV
14 FORMAT (13X, 12HSAMPLE MEAN=, F17.6)
15 FORMAT (13X, 15HSTANDARD DEVIATION=, F14.6)
RETURN
END
FUNCTION PROB(x)
C------THIS SUBROUTINE GIVES AREA UNDER NORMAL CURVE FROM -Z TO +Z
C WITH AN ACCURACY OF 0.00005
C------Z VALUE GIVEN BY CALLING PROGRAM MUST BE A POSITIVE NUMBER.
000003 11 ASUM=ASUM
000005 10 PHNUM = J.797538354X*(0.9999774+X5*3.16669433+X5*(0.92463831-15
000003 11 +0.2397667))
000016 RETURN;
000017 12 IF (x<2.4) 13+14,14
000022 13 ASUM=ASUM
000023 14 PHNUM=PHNUM
000024 15 FACTOR=1.0
000025 16 QUIT=1.0
000026 17 PTEN=PTEN*1.0
000028 970 PTEN=PTEN*X5/12.5*FACTOR)
000029 TCRN=TCRN/(Q5*QUIT)
000035 18 PHNUM=PHNUM+PTEN
000037 19 IF (ASUM (TEHN)= J.00007 ) 83+94,98
000042 90 FACTOR = FACTOR+1.0
000044 QUIT=QUIT*2.0
000046 80 QUIT=QUIT*2.0
000048 RETURN
000051 14 RECASU = 1.0 / (1+4)
000075 RETURN
000076 END
SUBROUTINE CHISQ (X, DATA, IORDATA, PROB, AKURCY, XMEAN, DEV, Z)
C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE CHI-SQUARE
C-----TEST FOR THE NORMAL OR LOG-NORMAL DISTRIBUTIONS.
000013  DIMENSION X(IORDATA), Z(IORDATA), CSV(9), CEV(9), CLCR(9), CMUR(9),
         IREA(9), AREA(9), EAFREG(9), FRED(9), U(9)
000043  CHISQ = .0
C-----TO DETERMINE THE NUMBER OF CLASS INTERVALS,K
000053  K = .5 * 3.322 * ALOG10 (DATA)
000024  REAL K
C-----IN ORDER TO DETERMINE THE RANGE, FIND X(MAX) AND X(MIN)
000036  X(MAX) = .0
000067  X(MIN) = X(1)
000039  DO 17 I = 1, IORDATA
000042  IF (X(I) .GT. XMAX) XMAX = X(I)
000037  17 IF (X(I) .LT. XMIN) XMIN = X(I)
000040  RANGE = XMAX - XMIN
C-----TO DETERMINE THE CLASS INTERVAL Width TO SAME NUMBER OF PLACES AS THE ACCURACY
000050  DIVIDE = 1.0 / AKURCY
000051  KW = ((RANGE + AKURCY + REALK) .5) / AKURCY * DIVIDE
000057  PK = 1./K
000066  N = INT (PK)
000062  PRINT 03, XMAX
000067  PRINT 03, XMIN
000007  PRINT 05, N
000103  DO 22 I = 1, N
000110  A = 1
000111  B = 1.0 / AKURCY
000112  CSV(1) = XMIN + (A - 1.0) * B
000117  CEV(1) = CSV(1) + AKURCY
000123  CLU(I) = CSV(I) + AKURCY
000127  22 CUB(I) = CEV(I) - B
000135  CEV(K) = XMAX
000137  CUW(K) = CEV(K) - B
000143  DO 21 J = 1, K
000144  PRINT 006, J
000150  DO 24 I = 1, IORDATA
000152  DO 26 I = 1, N
000153  IF (X(I) .GE. CLW(J) AND, X(I) .LE. CEV(J)) FRED(J) = FRED(J) + 1.0
000172  24 CONTINUE
C-----CHI-SQUARE TEST
000177  PRINT 04
000202  PRINT 05
000026  DO 30 J = 1, K
000206  Z(IJ) = (CUW(I) - XMEAN) / DEV
000213  T = ABS(Z(IJ))
000220  AREA(IJ) = PROB(T) / 2.0
000243  RETURN = 0.5 - AREA(I)
000244  RAVJURK = 1
000246  DO 32 I = 1, N
000247  AREA(IJ) = AREA(IJ) * AREA(N)
000250  IF (Z(IJ) .LE. 0.0 AND T(M) .LE. 0.0) OK(OK, Z(IJ) .LE. 0.0 AND T(M) .LE. 0.0)
000258  30 GO TO 31
000272  31 RETURN = AREA(I) * AREA(M)
000300  GO TO 32
000300  32 CONTINUE
HEUARE=(K)/.5-AREA(K-1)
00431 DO 34 K=1,K
00432 EXFREU=M=DATA(HEUARE(N))
00433 U(M)=(EXFREU(M)-FREU(M))*2)/EXFREU(N)
00434 C-----TO PRINT THE TABLE FOR CHI-SQUARE TEST
00435 33 PRINT 34*1+CL4(I)*CUB(I)* EXFREU(I)*FREU(I)*U(I)
00436 PRINT 35* CHISQ
00437 62 FORMAT(13X+14HMAXIMUM VALUE=*F15.6)
00438 43 FORMAT(13X+14HMIN. VALUE=*F15.6)
00439 55 FORMAT(16X+12HCLASS OF 10H=, F17.6)
00440 41 FORMAT(I40)
00441 40 FORMAT(1X+5H CELL+10X+10HLOWER CELL+10X+10HUPPER CELL+10X+10HNEPCEF
1TED+13H+3OBSERVED+13X+10HC1-SQUARE+10X+10HNUMHF+10X+10HOUNCART
13X+10BOUNDARY+13X+10HREQUENCY+12X+10HREQUENCY+13X+10HVALUE OF CE
3LL)
00442 34 FORMAT(10X+12,5F2.1)
00443 35 FORMAT(11X+9X+2S1OTAL CHI-SQUARE VALUE =*F10.6)
00444 20 RETURN
00445 END
SUBROUTINE DTEST (PCAREA, NDATA, X, DEVT, USTAT, PRMT, XMEAN, Z)
C-----SUBROUTINE TO CALCULATE THE KOLMOGOROV-SMIRNOV D-VALUES.
Ddimension PCAREA(ndata)x(x(ndata)+1(ndata),USTAT(ndata))
000013  DO 10 A=1, NDATA
000014  Z(A) = (X(A) - XMEAN)/DEV
000021  IF (Z(A) GT 2.0) GO TO 703
000024  Z(A) = ABS(Z(A))
000027  ARUNCH = 1.0 - PROBT(2.0)*2.0
000031  USTAT(A) = ARUNCH - PCAREA(A)
000044  GO TO 706
000047  T1 = 2.0
C-----ARUNCH AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR POSITIVE Z.
000050  ARUNCH = PROBT(A)/2.0 + .500
000053  USTAT(A) = (ARUNCH - PCAREA(A))
000066  706  CONTINUE
000071  PRINT 708 (USTAT(A),I=1,NDATA)
000075  FORMAT (F15.5)
000088  707  FORMAT (/40*A+53*4 VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT
000092  TEST/*41*52*LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA/)
000095  708  RETURN
000100  END
SUBROUTINE ALPHA (N, MEAN, VAR, ALPHA3, ALPHA4)

DIMENSION X(NDATA)

C------SUBROUTINE TO CALCULATE THE COEFFICIENTS OF SKEWNESS AND KURTOSIS

C------CALCULATE THE THIRD MOMENT OF THE DATA (SKEWNESS)

TOP3 = 0.0
DO 20 I = 1, NDATA
 VAR = VAR + (X(I) - MEAN)**2
 20 CONTINUE
SKEW = TOP3 / DATA
STDEV = SQRT(VAR / DATA)

C------ALPHA3 = MOMENT COEFFICIENT OF SKEWNESS,
ALPHA3 = SKEW / (STDEV**3)

C------CALCULATE THE FOURTH MOMENT OF THE DATA (KURTOSIS)

TOP4 = 0.0
DO 40 I = 1, NDATA
 TOP4 = TOP4 + (X(I) - MEAN)**4
 40 CONTINUE
KURT = TOP4 / DATA

C------ALPHA4 = MOMENT COEFFICIENT OF KURTOSIS,
ALPHA4 = KURT / (STDEV**4)

PRINT 712
PRINT 713
PRINT 714

FORMAT (1X, 3F12.4, 5X, 3F12.4) ALPHAS (ALPHA3, ALPHA4)

FORMAT (1X, 3F12.4, 5X, 3F12.4) NORMAL DISTRIBUTION ALPHAS = 0.23, 3.0

FORMAT (1X, 3F12.4, 5X, 3F12.4) ABOVE DATA

RETURN
END
APPENDIX B (continued)

Fig. B-3 Output of PROGRAM (CYTOFR)
NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA.

STRESS LEVEL = 114,000.0 PSI.  STRESS RATIO = INFINITY

<table>
<thead>
<tr>
<th>CYCLES TO FAILURE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1054.0</td>
</tr>
<tr>
<td>9362.0</td>
</tr>
<tr>
<td>8376.0</td>
</tr>
</tbody>
</table>

SAMPLE MEAN = 9622.27778
STD. DEVIATION = 1.324-216626
MAXIMUM VALUE = 10540.50000
MINIMUM VALUE = 7112.000000
CLASS WIDTH = 666.000000

<table>
<thead>
<tr>
<th>CELL NUMBER</th>
<th>LOWER CELL BOUNDARY</th>
<th>UPPER CELL BOUNDARY</th>
<th>EXPECTED FREQUENCY</th>
<th>OBSERVED FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7111.500000</td>
<td>7797.500000</td>
<td>2.966104</td>
<td>3.000000</td>
</tr>
<tr>
<td>2</td>
<td>7797.500000</td>
<td>8483.500000</td>
<td>3.260106</td>
<td>3.088000</td>
</tr>
<tr>
<td>3</td>
<td>8483.500000</td>
<td>9159.500000</td>
<td>4.639069</td>
<td>3.590000</td>
</tr>
<tr>
<td>4</td>
<td>9159.500000</td>
<td>9855.500000</td>
<td>4.741373</td>
<td>5.000000</td>
</tr>
<tr>
<td>5</td>
<td>9855.500000</td>
<td>10540.500000</td>
<td>3.774112</td>
<td>5.000000</td>
</tr>
</tbody>
</table>

TOTAL CHI-SQUARED VALUE = 1.175703

U VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
(LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)

<table>
<thead>
<tr>
<th>U</th>
<th>.067111</th>
<th>.04255</th>
<th>.01129</th>
<th>.007494</th>
<th>.00159</th>
<th>.03605</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.009313</td>
<td>.006613</td>
<td>.00424</td>
<td>.004242</td>
<td>.001501</td>
<td>.00547</td>
</tr>
<tr>
<td></td>
<td>.007134</td>
<td>.009874</td>
<td>.001622</td>
<td>.006662</td>
<td>.002639</td>
<td>.002495</td>
</tr>
</tbody>
</table>

MOMENT COEFFICIENT OF SKEWNESS (ALPHA3)
FOR NORMAL DISTRIBUTION ALPHA3 = 0
FOR ABOVE DATA --- ALPHA3 = -.247

MOMENT COEFFICIENT OF KURTOSIS (ALPHA4)
FOR NORMAL DISTRIBUTION ALPHA4 = 3.0
FOR ABOVE DATA --- ALPHA4 = 1.995
LOG-NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA.

STRESS LEVEL = 114000.0 PSI, STRESS RATIO = INFINITY

LOGS OF THE CYCLES TO FAILURE DATA

<table>
<thead>
<tr>
<th>CELL NUMBER</th>
<th>LOWER CELL BOUNDARY</th>
<th>UPPER CELL BOUNDARY</th>
<th>EXPECTED FREQUENCY</th>
<th>OBSERVED FREQUENCY</th>
<th>CHI-SQUARED VALUE OF CELL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.967535</td>
<td>9.404215</td>
<td>1.667972</td>
<td>2.088000</td>
<td>0.00320</td>
</tr>
<tr>
<td>2</td>
<td>9.105575</td>
<td>9.462646</td>
<td>2.927417</td>
<td>3.000000</td>
<td>0.00019</td>
</tr>
<tr>
<td>3</td>
<td>9.362695</td>
<td>9.755563</td>
<td>4.851237</td>
<td>3.000000</td>
<td>0.00009</td>
</tr>
<tr>
<td>4</td>
<td>9.103575</td>
<td>9.454255</td>
<td>4.453420</td>
<td>4.000000</td>
<td>0.00043</td>
</tr>
<tr>
<td>5</td>
<td>9.104255</td>
<td>9.452975</td>
<td>4.306947</td>
<td>6.000000</td>
<td>0.00034</td>
</tr>
</tbody>
</table>

TOTAL CHI-SQUARED VALUE = 1.31461

U VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST (LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)

<table>
<thead>
<tr>
<th>U VALUE</th>
<th>U VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04236</td>
<td>0.05342</td>
</tr>
<tr>
<td>0.0126</td>
<td>0.01644</td>
</tr>
</tbody>
</table>

MOMENT COEFFICIENT OF SKEWNESS (\(\alpha_3\))

FOR NORMAL DISTRIBUTION \(\alpha_3 = 0\)

FOR ABOVE DATA \(\alpha_3 = 0.407\)

MOMENT COEFFICIENT OF KURTOSIS (\(\alpha_4\))

FOR NORMAL DISTRIBUTION \(\alpha_4 = 3.0\)

FOR ABOVE DATA \(\alpha_4 = 2.123\)
APPENDIX C

CALIBRATION CONSTANTS OF EACH MACHINE FOR THE DIFFERENT MODES

Mode 1: 26 July, 1966 to 10 August, 1967

Mode 2: 11 August, 1967 to 1 June, 1969

<table>
<thead>
<tr>
<th>Machine</th>
<th>$K_{bgr}$</th>
<th>$K_{gr-th}$</th>
<th>$K_t$</th>
<th>$K_{t/b}$</th>
<th>$K_{b/t}$</th>
</tr>
</thead>
<tbody>
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APPENDIX C (continued)

CALIBRATION CONSTANTS OF EACH MACHINE
FOR THE DIFFERENT MODES

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**MEAN BENDING STRESS IN GROOVE = 113877.1 PSI.**

**STD. DEV. OF BENDING STRESS IN GROOVE = 886.70 PSI.**
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Std. Dev. of Bending Stress in Groove = 902.19 PSI.
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**Test Mode = 2**

**Mean Bending Stress in Groove = 150977.5 PSI.**

**Std. Dev. of Bending Stress in Groove = 3841.69 PSI.**

**Mean Torque Stress in Groove = 24832.0 PSI.**

**Std. Dev. of Torque Stress in Groove = 794.91 PSI.**

**Mean Stress Ratio = 3.51372**

**Std. Dev. of Stress Ratio = 0.14394**
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Std. Dev. of Bending Stress in Groove = 1820.89 PSI.

Mean Torque Stress in Groove = 19304.5 PSI.
Std. Dev. of Torque Stress in Groove = 716.79 PSI.

Mean Stress Ratio = 3.43179
Std. Dev. of Stress Ratio = 0.14174
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**Std. Dev. of Bending Stress in Groove** = 1178.33 PSI.

**Mean Torque Stress in Groove** = 13685.5 PSI.
**Std. Dev. of Torque Stress in Groove** = 1740.78 PSI.

**Mean Stress Ratio** = 3.51963
**Std. Dev. of Stress Ratio** = 0.18021
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Mean Bending Stress in Groove = 74222.7 PSI.
Mean Torque Stress in Groove = 12443.9 PSI.
Mean Stress Ratio = 3.45139
Std. Dev. of Stress Ratio = 0.16903
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**Std. Dev. of Bending Stress in Groove** = 1257.68 PSI.

**Mean Torque Stress in Groove** = 73329.6 PSI.

**Std. Dev. of Torque Stress in Groove** = 2051.25 PSI.

**Mean Stress Ratio** = 0.87577

**Std. Dev. of Stress Ratio** = 0.02569
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**Std. Dev. of Bending Stress in Groove = 3145.0 PSI.**

**Mean Torque Stress in Groove = 53720.2 PSI.**

**Std. Dev. of Torque Stress in Groove = 3437.55 PSI.**

**Mean Stress Ratio = .81769**

**Std. Dev. of Stress Ratio = .06975**
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Mean Bending Stress in Groove = 65187.0 PSI.
Std. Dev. of Bending Stress in Groove = 3869.69 PSI.

Mean Torque Stress in Groove = 46803.8 PSI.
Std. Dev. of Torque Stress in Groove = 1288.33 PSI.

Mean Stress Ratio = .80488
Std. Dev. of Stress Ratio = .05562
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Std. Dev. of Bending Stress in Groove = 700.25 PSI.

Mean Torque Stress in Groove = 78439.7 PSI.

Std. Dev. of Torque Stress in Groove = 1399.67 PSI.

Mean Stress Ratio = .43899

Std. Dev. of Stress Ratio = .01008
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**Mean Bending Stress in Groove** = 68868.3 PSI.

**Std. Dev. of Bending Stress in Groove** = 1422.63 PSI.

**Mean Torque Stress in Groove** = 89770.6 PSI.

**Std. Dev. of Torque Stress in Groove** = 761.94 PSI.

**Mean Stress Ratio** = .44295

**Std. Dev. of Stress Ratio** = .00948
APPENDIX E

COMPUTER OUTPUTS LISTING STRESS LEVELS AND RATIOS
OF INDIVIDUAL SPECIMENS FOR THE ENDURANCE TESTS
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**STD. DEV. OF STRESS RATIO** = 0.61987
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Mean Stress Ratio = 1.03214
Std. Dev. of Stress Ratio = 0.09408
APPENDIX F

PRINTOUT OF PROGRAM TO CALCULATE
THE STRENGTH DISTRIBUTION ESTIMATES
FOR MEAN AND STANDARD DEVIATION
PROGRAM STRENG (INPUT,OUTPUT,TAPE) = INPUT
C----PROGRAM FINDS NORMAL STRENGTH DISTRIBUTION FROM LOGNORMAL
C----CYCLES-TO-FAILURE DISTRIBUTIONS.
DIMENSION ASTP(10), AMLOG(10), ASIGL(10), SLAM(4), SIG3(A23),
1 XMLOG(301), XSIGL(301), AREA(301,23), FREQ(301,23), SIG3P(23),
2 CYC(23), SMLOG(23), SIGL(23), SIG4(23), CYTOF(15,25),
3 SUM(10), INDEX(1), FREQ(10,23), UMAX(10,23), AN2(10),
4 CHISQ(10,23), ST(10), SLSIG(9), EFREO(10,23), INDEX2(10), SUM(10),
5 CYCLES(301), ACYCLE(25), STLON(10), STAENAN(23), XCYCLE(23),

000003 EQUIVALENCE (AREA,FREQ),
1 (ASTP,ASIGL), (XMLOG,FREQ), (AREA(500),FREQ),
2 (AREA(250),FREQ)
000003 INT =
000004 5 HEAD(I) = JNAX, XJNAX, SINT, MN = RATIO
000024 10 FORMAT (2F10.6,F25*15*.5)
000025 50 C FAILURE DISTRIBUTION VALUES ARE HEAD IN FROM
C LOWEST TO HIGHEST STRESSES AND STRESSES ARE
C INTEGER VALUES OF SINT
000027 29 HEAD 30* (ASTP(I),AMLOG(I)), ASIJL(I), J=1,N
000058 30 FORMAT (F20.6,F6*15)
000050 HEAD 36* (ACYCLE(J), J=1,25)
000063 36 FORMAT (F9.4)
000065 30 UO = 3 + INT
000065 34 ACYCLE(I) = EXP (AMLOG(I))
000074 NNI = N - 1
000075 L = 1
000076 INDEX(I) =
000077 UO AS J=1,NNI
0000100 DIUM = ASTP(J)+AMLOG(J)+SUM(J) = J=1,N
0000103 DMEAN = AMLOG(J)+AMLOG(J)
0000105 DJSIG = ASIJL(J) - ASIJL(J)
0000107 DSTM(J) = DSTM/DMEAN
0000111 IF (DSIG) .GT.0,54,60
0000112 60 DSTM(J) = DSTM/DSIG
0000115 50 CONTINUE
0000115 40 SLSIG(J) = DSTM/DSIG
0000115 50 CONTINUE
0000125 JPI = J+
0000122 M = DSTM/SINT
0000128 INDEX(JPI) = L*M
0000125 SMIN = ASTP(I)
0000125 AMLOG(I) = AMLOG(I)
0000127 XSIGL(I) = ASIGL(I)
0000131 UO AS K=1,M
0000132 LAM = L + K
0000134 XMLOG(LX) = XMLOG(LX-1)+SINT/DSTM(J)
0000140 IF (DSIG) .GT.0,60,70
0000141 70 XSIGL(LX) = XSIGL(LX-1)
0000143 GO TO 80
0000144 60 XSIGL(LX) = XSIGL(LX-1)+SINT/SLSIG(J)
0000151 80 CONTINUE
0000154 L = LX
0000155 85 CONTINUE
0000160 PRINT 87
0000163 87 FORMAT (1H1//44X, 44*NORMAL STRENGTH DISTRIBUTIONS FROM LOGNORMAL
1L/44X,32CYCLES TO FAILURE DISTRIBUTIONS.///)
0000163 IF (RATIO = 0.00) GO TO 91
0000164 PRINT 88, RATIO
FORMAT (44X,17X'EXPERIMENTAL DATA',8X,14HSTRESS RATIO = $6.3$/)

GO TO 93

PRINT 92

FORMAT(44X,17X'EXPERIMENTAL DATA',8X,23HSTRESS RATIO = INFINITY$/)

PRINT 94

FORMAT(8X,13HSTRENGTH PSI.,9X,32HMEAN-CYCLES LOG MEAN-CYCLES, 18X,11HLOG STD DEV$/)

NP = N-1

PRINT 95. (ASTR(I), ACYCLE(I), XMLOG(I), ASIGL(I), I=2,NP)

PRINT 96

FORMAT(44X,17X'INTERPOLATED STRESS LEVELS$/)

PRINT 90

FORMAT(8X,13HSTRENGTH PSI.,9X,32HMEAN-CYCLES LOG MEAN-CYCLES, 18X,11HLOG STD DEV+4X,10HINTEGER(I)$/)

XINT = INT

STR = SMIN

DO 110 I=1,LX

110 CONTINUE

CONVERTING LOGNORMAL FAILURE DIST. PARAMETERS AT N

STRESS LEVELS TO CUMULATIVE LOGNORMAL FAILURE

DISTRIBUTION

UO 180 I=1,N

DO 180 J=1,N

CYC(J) = ALOG(CYCLE(J))

Z = (CYC(J)-XMLOG(J))/XSIGL(J)

IF(Z.126.140)160.130.130

130 Z = -Z

CALL NORMAL(Z,PROB)

AREA(I,J) = (1.0-PROB)/2.0

GO TO 180

140 IF(Z.3.5)150.150+170

150 CALL NORMAL(Z,PROH)

AREA(I,J) = PROH/2.6+0.5

GO TO 180

160 AREA(I,J) = 0.0

GO TO 180

170 AREA(I,J) = 1.0

180 CONTINUE

PRINT 225

INT2 = INT+4

STR = SMIN

PRINT 185

185 FORMAT(//46X,12HCUMULATIVE LOGNORMAL FAILURE DISTRIBUTIONS$)

186 FORMAT(3X,13HDATA BELOW IS J, AND CUMMULATIVE $)

187 CONTINUE

PRINT 180

PRINT 200

FORMAT (1X, 33MDATA BELOW IS J, AND CUMMULATIVE $)
C FREQUENCY DIST AND NORMAL DISTRIBUTION PARAMETERS
XMIN = XJMIN+XJINT
C PRINT 225
PRINT 225 FORMAT (1H11-
C PRINT 230
PRINT 230 FORMAT(5/I5,33/HCUMULATIVE STRENGTH DISTRIBUTIONS)
I = INDEX(J)
ANUM = NUM(I)
DO 224 K=1,N
FREQ(J,K) = ANUM*AREA(I,K)
C CONTINUE
C CONTINUE
C MEAN AND NORMAL DISTRIBUTION PARAMETERS OF HISTOGRAM
DO 300 J=1,M
FI = 0.
FIUI = 0.
UJ = SMIN+SMINT*0.5
DO 290 I=1,N
UJ = UI+SMINT
FI = FI+FREQ(I,J)
FIUI = FIUI+FREQ(I,J)*UI
C CONTINUE
SLOG(J) = FIUI/FI
STMEAN(J) = ALOG(SLOG(J))
SM = 0.
SM2 = 0.
SM3 = 0.
SM4 = 0.
UJ = SMIN+SMINT*0.5
DO 290 I=1,N
UJ = UI+SMINT
SJ = (UI-SLOG(J))*(UI-SLOG(J))
SM2 = SM2+SJ*FREQ(I,J)
SM3 = SM3+FREQ(I,J)*(UI-SLOG(J))*SJ
SM4 = SM4+50*SJ*FREQ(I,J)
C CONTINUE
C CONTINUE
SM2 = SM2/FI
290 CONTINUE
S3 = SM3/FI
S4 = SM4/FI
SSIGL(J) = SMRTF(SM2)
SIGP(J) = SMLOG(J) + 3.0*SSIGL(J)
SIGM(J) = SMLOG(J) - 3.0*SSIGL(J)
SK3(J) = SM3/SSIGL(J) + 3.0
SK4(J) = SM4/(SM2*SM2)
300 CONTINUE

PRINT 225
PRINT 305
FORMAT(///30X*7EHPARAMETERS OF NORMAL STRESS DISTRIBUTIONS AT SPECIFIED CYCLES TO FAILURE///)
PRINT 310
FORMAT(///30X*25HLOU CYCLES ///)
1 6X*MCYCLES*10X*13HMEAN STRENGTH*10X*9HST. LEV.*7X*5HLIMIT ///
2 Tx*34HLIMIT SKEWNESS KURTOSIS ///)
PRINT 330 (J=CYC(J)*CYCLE(J)*SMLOG(J) + SSIGL(J))
1 SIGM(J)+SIGP(J)+SK3(J)+SK4(J) + J =1+M
1 SIGM(J)+SIGP(J)+SK3(J)+SK4(J) + J =1+M
320 FORMAT(///4X*13+F15.6,F13.6,F20.0,F12.0,F12.4
PRINT 225
PRINT 322
FORMAT(///10H-3 SIGMA ///)
PRINT 323
FORMAT(///10HNUM. LOG CYCLES ///)
PRINT 324
FORMAT(///10HMEAN STRENGTH ///)
PRINT 325
FORMAT(///10HSTRAIN LIMIT ///)
PRINT 326
FORMAT(///10HLIMIT SKEWNESS ///)
PRINT 327
FORMAT(///10HUPTOSIS ///)
DO 600 I=2,NP
DATA = INT(I)
DO 600 (CYTOFR(I,J),J=1,NDATA)
605 FORMAT(///10F10.0)
600 CONTINUE
DO 500 J=1,M
ANUM(J) = 0.0
500 TOTAL = 6.0
DO 605 I=2,NP
605 NDAT = INT(I)
606 ANL = 0.0
600 NDATA = INT(I)
DO 665 K = 1,NDATA
IF (CYTOFR(I,K),XCYCLE(J)) AN1 = AN1 + 1.0
660 CONTINUE
AN2 = AN1
665 CONTINUE
DO 440 I=2,NP
440 Z = (ASTR(I) - SMLOG(J))/SSIGL(J)
IF(Z) 330,350,350
330 Z = -Z
340 CALL NORMAL (Z,PRB)
640 PROB = 0.5 - PRB*0.5
644 GO TO 370
350 IF(Z) 355,365,365
355 CALL NORMAL (Z,PRB)
655 PROB = PRB*0.5*0.5
654 GO TO 370
360 PROB = 0.0
60154 GO TO 370
60155 GO TO 370

c---Routine for Kolmogorov-Smirnov Goodness of Fit Test
C
370 FORMAT(///10HVALUES FOR KOLMGOOROV-SMIRNOV GOODNESS OF FIT TEST ///)
DO 600 I=2,NP
DATA = INT(I)
DO 600 (CYTOFR(I,J),J=1,NDATA)
605 FORMAT(///10F10.0)
600 CONTINUE
DO 500 J=1,M
ANUM(J) = 0.0
500 TOTAL = 6.0
DO 605 I=2,NP
605 NDAT = INT(I)
606 ANL = 0.0
600 NDATA = INT(I)
DO 665 K = 1,NDATA
IF (CYTOFR(I,K),XCYCLE(J)) AN1 = AN1 + 1.0
660 CONTINUE
AN2 = AN1
665 CONTINUE
DO 440 I=2,NP
440 Z = (ASTR(I) - SMLOG(J))/SSIGL(J)
IF(Z) 330,350,350
330 Z = -Z
340 CALL NORMAL (Z,PRB)
640 PROB = 0.5 - PRB*0.5
644 GO TO 370
350 IF(Z) 355,365,365
355 CALL NORMAL (Z,PRB)
655 PROB = PRB*0.5*0.5
654 GO TO 370
360 PROB = 0.0
60154 GO TO 370
60155 GO TO 370

PROBA = 1.0
GFREO(I,J) = PROBA
IF (TOTAL.EQ.0.0) GO TO 375
DArea = #NUM(I)/TOTAL
GO TO 380
DArea = 0.0
DMAX(I,J) = APS(DArea-GFREO(I,J))
CONTINUE
PRINT 490, XCYCLE(J), TOTAL*(AN2(I)+Dmax(I,J)+ I=2+NP)
FORMAT (A8,F8.0,21H CYCLES TOTAL N =F3.0/(4(F9.3,F6.3)))
PRINT 322
CONTINUE
GO TO 5
STOP
END
SUBROUTINE NORMALIZ(PROB)
C PROB = THE AREA UNDER NORMAL DISTRIBUTION BETWEEN
C PLUS AND MINUS 2 STANDARD DEVIATIONS
C
000005 IF(Z=2) 1000+1000+1019
000007 1000 ZSIG = Z
000010 1000 PROB = 0.74788455*ZSIG*(0.99999774-ZSIG*(0.16659431
1 =ZSIG*(0.02463831+ZSIG*(0.02387867)))
000020 GO TO 1070
000021 1010 IF(Z=2) 1020+1060+1060
000024 1020 ZSIG = Z
000025 1020 PROB = 1.0
000026 1020 PTERM = 1.0
000027 1020 FACT = 1.0
000028 1020 ODDIN = 3.9
000031 1030 PTERM = -PTERM*ZSIG/(2.0*FACT)
000035 1030 TERM = PTERM/ODDIN
000036 1030 PROB = PROB + TERM
000037 IF (ABS(TERM) < 0.00007) 1050+1040+1040
000040 1040 FACT = FACT + 1.0
000045 1040 ODDIN = ODDIN + 2.0
000047 GO TO 1030
000047 1050 PROB = 0.74788455*ZSIG*PROB
000051 GO TO 1070
000052 1060 REC = 1.0/17777
000054 1060 PROB = 1.0-0.74788453*EXP(-ZSIG**2/2)/2
1 = (1. - REC*(1. - REC**(15. - REC**105.)))
000076 1070 CONTINUE
000076 RETURN
000077 END
LIST OF REFERENCES


