MINIMIZING THE AREA REQUIRED FOR TIME CONSTANTS IN INTEGRATED CIRCUITS

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When a medium- or large-scale integrated circuit is designed, efforts are usually made to avoid the use of resistor-capacitor time constant generators. The capacitor needed for this circuit usually takes up more surface area on the chip than several resistors and transistors. When the use of this network is unavoidable, the designer usually makes an effort to see that the choice of resistor and capacitor combinations is such that a minimum amount of surface area is consumed. The optimum ratio of resistance to capacitance which may be obtained from a unit of surface area for the particular process being used. The minimum area required is a function of the square root of the reciprocal of the products of the resistance and capacitance per unit area. This minimum occurs when the area required by the resistor is equal to the area required by the capacitor.
One of the most widely used networks in pulse and digital circuitry is the simple resistor-capacitor time constant generator. There are an infinite number of combinations of resistor and capacitor values that will provide the required time constant. When circuits are designed that use conventional, discrete components, the selection of these constants is determined by such factors as impedance matching, power dissipation, and the range of component values readily available. Most aerospace applications place a premium on small size and light weight. For this reason, the range of capacitance values used is usually restricted to those that are available in relatively small case sizes.

When designing a resistor-capacitor time constant for an integrated circuit, the designer must take into consideration the fact that the surface area needed for the components is of equal importance to power dissipation and impedance matching. The particular semiconductor fabrication process being used usually determines the amount of resistance or capacitance to be built into a unit area of integrated circuit surface. A capacitor value may be doubled by simply doubling the surface area of the component on the integrated circuit chip. In the case of a resistor, the resistance may be doubled by adding unit areas in series or may be halved by adding them in parallel. (See fig. 1.) For this reason, the standard terminology for resistivity of a resistor is ohms per square. The side dimension of the square is not important in that doubling or halving the length of the resistor accomplishes a corresponding dimensional change in the width, and the resistance remains constant.

The calculations in this document concern cases in which the value of the resistor increases rather than decreases as its area increases. The relationships between the required time constant $t$, the
resistance $R$, the capacitance $C$, and the areas of the resistance $A_r$ and capacitance $A_c$ components and their total $A_t$ are shown in the following equations:

\[ t = RC \quad (1) \]

\[ A_t = A_c + A_r \quad (2) \]

\[ A_c = \frac{C}{K_c} \quad (3) \]

\[ A_r = \frac{R}{K_r} \quad (4) \]

where $K_c$ and $K_r$ are sheet capacitance and sheet resistivity, respectively.

Substituting equations (3) and (4) into equation (2) yields

\[ A_t = \frac{C}{K_c} + \frac{R}{K_r} \quad (5) \]

and substituting $C = t/R$ from equation (1) into equation (5) yields

\[ A_t = \frac{t}{RK_c} + \frac{R}{K_r} \quad (6) \]

This equation shows that the value of $A_t$ required for $t$ can be described as a function of $t, R, K_r, \text{ and } K_c$. If $t, K_c, \text{ and } K_r$ are held constant, $A_t$ and $R$ are variables. Next, $A_t$ is differentiated with respect to $R$:

\[ \frac{dA_t}{dR} = \frac{-t}{R^2 K_c} + \frac{1}{K_r} \quad (7) \]

(Throughout this document, subscript $m$ is used to indicate minimum.) The resulting differential is set equal to zero to find $R_m$:

\[ \frac{-t}{R_m^2 K_c} + \frac{1}{K_r} = 0 \quad (8) \]

Therefore,

\[ R_m = \sqrt{\frac{tK_r}{K_c}} \quad (9) \]

Substituting $R = t/C$ from equation (1) into equation (5) and using the same method yields
The total minimum area required is the sum of the areas of each of the two minimum components:

\[ A_{tm} = A_{r_m} + A_{c_m} \]  

From equation (5),

\[ A_{tm} = \frac{C_m}{K_c} + \frac{R_m}{K_r} \]  

Substituting equations (9) and (10) into equation (12) yields

\[ A_{tm} = \sqrt{\frac{t}{K_c K_r}} + \sqrt{\frac{t}{K_c K_r}} \]

\[ = \sqrt{\frac{4t}{K_c K_r}} \]

This equation describes the minimum area required for a time constant when \( t, K_c, \) and \( K_r \) are known. It is interesting to note that \( A_{tm} \) occurs when \( A_c \) is equal to \( A_r \).

From equations (9) and (10),

\[ \frac{R_m}{C_m} = \frac{\sqrt{tK_r/K_c}}{\sqrt{tK_c/K_r}} \]

\[ = \frac{K_r}{K_c} \]

Thus, the ratio of \( R_m \) to \( C_m \) that yields the minimum area required is shown in equation (14) to be the ratio of \( K_r \) to \( K_c \).

A typical application of this relationship would be that of a metal-oxide-silicon field-effect transistor (MOSFET) integrated circuit. \( K_r \) provided by this process is 39 TΩ/m² (25 GΩ/in²), and \( K_c \) is 390 μF/m² (250 nF/in²). Values for \( A_r, A_c, A_t, \) and \( R/C \) are obtained by varying the values of \( R \) and \( C \) around the values of \( R_m \) and \( C_m \) predicted for the minimum area required for the time constant. These values are listed in table 1 and are plotted in figure 2. For this example, a value of \( t \) equal to 1 μs was chosen. The minimum \( A_t \) on the curve agrees with the value of \( A_{tm} \) found by inserting these constants into equation (13). The \( R/C \) ratio on the curve that corresponds to \( A_{tm} \) is \( K_r/K_c \), as was predicted by equation (14).
Table 1.—Determination of $A_{tlm}$

<table>
<thead>
<tr>
<th>$R$, $\text{M} \Omega$</th>
<th>$C$, $\text{pF}$</th>
<th>$A_r, \mu \text{m}^2$ (10^{-6} \text{ in}^2)</th>
<th>$A_c, \mu \text{m}^2$ (10^{-6} \text{ in}^2)</th>
<th>$A_t, \mu \text{m}^2$ (10^{-6} \text{ in}^2)</th>
<th>$R/C$, $\Omega/\text{F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>20</td>
<td>1 290 (2.00)</td>
<td>51 610 (80)</td>
<td>52 900 (82)</td>
<td>$0.25 \times 10^{15}$</td>
</tr>
<tr>
<td>.1</td>
<td>10.0</td>
<td>2 580 (4.0)</td>
<td>25 810 (40.0)</td>
<td>28 390 (44.0)</td>
<td>$1.0 \times 10^{16}$</td>
</tr>
<tr>
<td>.2</td>
<td>5.0</td>
<td>5 160 (8.0)</td>
<td>12 900 (20.0)</td>
<td>18 060 (28.0)</td>
<td>$4.0 \times 10^{16}$</td>
</tr>
<tr>
<td>.25</td>
<td>4.0</td>
<td>6 450 (10.0)</td>
<td>10 320 (16.0)</td>
<td>16 770 (26.0)</td>
<td>$6.25 \times 10^{16}$</td>
</tr>
<tr>
<td>.316</td>
<td>3.16</td>
<td>7 783 (12.64)</td>
<td>7 783 (12.64)</td>
<td>16 310 (25.28)</td>
<td>$1.0 \times 10^{17}$</td>
</tr>
<tr>
<td>.4</td>
<td>2.5</td>
<td>10 320 (16.0)</td>
<td>6 450 (10.0)</td>
<td>16 770 (26.0)</td>
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</tr>
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</table>

Figure 2.—$A_{tm}$ as a function of $R/C$. 

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