A SHOCK SURFACE GEOMETRY:
THE FEBRUARY 15-16, 1967 EVENT

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Abstract

The flare associated interplanetary (IP) shock of February 15-16, 1967 observed by Explorer 33 and Pioneer 7 is analyzed to yield an estimation of the ecliptic plane geometry of the shock surface near 1 AU. These spacecraft were separated by 23° in heliocentric longitude and Pioneer 7 was at a distance of 1.12 AU from the sun. There was an 18.9 hour delay between the two observations. The estimated shock normal, using a least squares shock parameter fitting procedure for the Explorer 33 data, is found to be $\theta_{SE} = -53^\circ$ and $\phi_{SE} = 198^\circ$ which is close to the estimate of Hirshberg et al. (1970) obtained by using the magnetic coplanarity theorem and the Ames Research Center magnetometer data from Explorer 33. The (95% certainty) error cone angle for the shock normal of the Explorer 33 observation was approximately $7^\circ$. This severely inclined shock normal is not typical for IP shocks. The shock normal at Pioneer 7 position is found to be $\theta_{PC} = -14^\circ$ and $\phi_{PC} = -161^\circ$ using the magnetic coplanarity theorem and GSFC magnetic field data. However, the uncertainty is large ($\approx 25^\circ$ for 1° cone angle). Although a data gap occurred at the apparent time of passage of the disturbance at Pioneer 6, which was 85° in heliocentric longitude from Pioneer 7 and at 0.83 AU, the recovered data did suggest such a passage. A consistent picture of the shock propagation is given to explain the difference in arrival times at Pioneers 6, 7, and Explorer 33 and the difference of the shock normals observed by Pioneer 7 and Explorer 33. The average shock speed from the sun to each spacecraft and the local speed at Explorer 33 and their relations to the position of the initiating solar flare are obtained and discussed. In the region of space between the Earth and Pioneer 7 the shock surface radius of curvature in the ecliptic plane was 0.4 AU or less.
Introduction

Earlier in this conference we heard about statistical and theoretical estimates of interplanetary (IP) shock curvature (Dryor et al.; Siscoe; Chao and Lepping; also see deYoung and Hundhausen, 1971). In this paper we present an estimated shape of the February 15-16, 1967 flare-shock surface as seen in ecliptic plane cross-section, based on observations from Explorer 33, Pioneers 6 and 7, and an SSC. This shock, with regard to the near-earth observations and its flare association, has been studied by Hirshberg et al. (1970), henceforth referred to as HACBH. They used the A.R.C. magnetic field data from Explorer 33 and the plasma data from Vela 3A. Using Explorer 33’s magnetic field (GSFC) and plasma (M.I.T.) data we compare our near-earth results with the HACBH results, which indicated a severely “southward” (w.r.t. ecliptic plane) tipped shock normal (-59°). We then extend the study through added spacecraft (S/C) coverage, i.e., by also using data from the distant Pioneers. According to HACBH evidence for this being a flare-associated shock lies in the high helium to proton number density ratio of 22% of the shock driver-piston observed ∼9 hours behind the shock as seen in the Vela 3A data (see Hirshberg, 1971). The particular flare association made by them was based on its impressive 3B-4B importance designation. We assume the same flare association for the same reasons, and because of the apparently associated type IV radio emission observed, and also because of the reasonable mean shock-speeds (between the flare onset and the S/C) that result, according to the remarks made earlier at this conference (Chao and Lopping, 1972).

After first describing the positions of the S/C, we will utilize the available plasma and magnetic field data at each position to study the local conditions to eventually yield a consistent large scale picture of the event. These positions will be treated according to increased completeness of data.

Spacecraft Locations and Mean Shock Speeds

Figure 1 shows where the S/C were located relative to the associated flare during the shock passage. The mean speeds of 640 and 770 km/sec for the sun-to-Pioneer 7 and the sun-to-earth, respectively, are indeed typical values for this quantity. The value of 290 km/sec was computed assuming a simple spherical shell, centered at the sun, moving between the earth and Pioneer 7 in the observed 18.9 hours transit interval. This value is approximately 200 km/sec lower than characteristic local (ecliptic plane) shock speeds at 1 AU, and suggests that the large scale (1 AU) spherical shock assumption is a poor one in this region.
Pioneer 6

The shock jump was not actually observed at Pioneer 6 due to a data gap. Unfortunately, the (shock- or disturbance-) discontinuity probably occurred during a gap of 40 hours' duration. However, after a data recovery at 1216 UT on February 15 an enhanced magnetic field (11-13 $\mu T$) and plasma density (13-17/cm$^3$), w.r.t. quiescent values previous to the gap, were observed. These are typical post-shock parameter values. The post-gap velocity was only 360 km/sec, but that is not unusually low for a post-shock value. We conclude that a shock-like disturbance passed Pioneer 6 and the front of the disturbance probably passed the S/C shortly before (assume immediately before, for definiteness) data recovery. The latter assumption is based on the reasonable mean speed of 830 km/sec that results; this speed is consistent with the 770 km/sec value for the sun-to-earth speed ($\approx 7\%$ difference). If we assume the front occurred earlier, then higher, and less reasonable, mean speeds would result.

Pioneer 7: Observation and Analysis

In Figure 2 we show the magnetic field data from Pioneer 7; the plasma was sampled too infrequently, due to low bit rate telemetry at this large distance, to be useful in this context. The field data indicates a typical IP shock signature. Using the magnetic coplanarity theorem the normal of $\theta = -14^\circ$, $\phi = 161^\circ$ is obtained ($\phi = 0^\circ$ from S/C to sun). Due to large fluctuations in the field direction around the shock a large (1 sigma) error cone angle of $\approx 25^\circ$ results. Also the jump occurs at a reasonable delay interval w.r.t. the SSC's time. For these three reasons—typical shock signature, reasonable shock normal, and approximately expected onset time—we identify the discontinuity as the shock previously seen at Explorer 33.

We now proceed with an investigation of the near-earth region using Explorer 33's data.

Analysis for the Near-Earth Region

Since both plasma and magnetic field data were available from Explorer 33 around the shock jump, then use of a least-squares scheme to "best-fit" the shock parameters to a subset of the MHD shock conservation equations was possible (Lepping and Argentiore, 1971). The S/C was clearly in IP space ($X_{SE} = 9.1$, $Y_{SE} = -29.1$, $Z_{SE} = -13.0$, and $R = 33.1$, in earth radii—See Figure 1) at the time of shock passage. Such a procedure was carried out and the results are shown in Table 1. The first eleven parameters are those used in the fitting scheme, the normal $n_{33} = (n_x, n_y, n_z)$ and the change in total plasma kinetic pressure, $\Delta P$, are subsequently calculated via the magnetic coplanarity theorem and the normal momentum flux equation, respectively. Quantities are given in solar ecliptic coordinates centered at the earth with positive $X_{SE}$ in the direction of the sun, 1 and 2 subscripts refer to before and after the shock, $\vec{B}$ is the magnetic field, $\vec{V}(\vec{v}_2 - \vec{v}_1)$ is the
plasma bulk velocity difference, and \( N \) refers to the plasma (proton) number density. The data analysis interval was \( \pm 3.5 \text{ min} \), which allowed one plasma sample and two 82-sec field averages (16 points per average), on either side of the shock. The interval was limited by a slight change in field direction \( \approx 3.5 \text{ min} \) after the shock; the plasma was reasonably steady for \( \pm 40 \text{ min} \), however. The quantity \( q \) in the table was based on the rms estimates, \( \pm 3.5 \text{ min} \) for the field and \( \pm 40 \text{ min} \) for the plasma, with a conservative rounding-off in the case of \( \Phi \). The process was therefore weighted slightly in favor of the field quantities. The calculation was repeated using the same field data and the full \( \pm 40 \text{ min} \) of plasma data (i.e. \( \pm 8 \) points), which heavily weighted the process in favor of the plasma data. This deflected the estimate of the normal by only about \( 2^\circ \) from the previous estimate, and, in fact, changed all parameter estimates inconsequentially.

In the table a quality index for the fitting process is also shown. This index is defined as the square root of the ratio of the total number of points of all shock parameters used in the analysis to the standard \( q \)-weighted least-squares loss function at convergence (Lepping and Argentiero, 1971). This index is commonly very near to unity for characteristic IP shocks provided reasonable \( q \)-weights (usually rms deviations) are used in the loss function for all parameters.

The table shows that the average and best fit values agree very well considering their \( q \)-values; the average and best fit normals differ by \( 11^\circ \). As good as the agreement between the two sets of parameters is, the average set gives an unreasonable \( \Delta P = -8.0 \times 10^{-10} \) dyne/cm\(^2\), but the best-fit set gives a reasonable, although somewhat high, value of \( 3.3 \times 10^{-10} \) dyne/cm\(^2\). The quantity \( \Delta P \) is commonly a very sensitive function of the parameter values. Notice also the quality improvement from the average to the best-fit set. The (95% certainty) error cone is calculated via a standard Monte Carlo process (Lepping and Argentiero, 1971) and is shown in the table. The value of \( 16^\circ \) is misleadingly high due to the small number of magnetic field points (actually \( \pm 2 \) averages) used in the calculation. A value of about \( 7^\circ \) is more reasonable; this angle is justified below. The best-fit density ratio of 2.8 agrees very well with the value of 2.5 predicted by HACBH. The best-fit field magnitudes were \( |E_1| = 7.0 \gamma \) and \( |E_2| = 18.5 \gamma \), giving a ratio of 2.6, and there was \( \approx 17^\circ \) between \( E_1 \) and \( E_2 \).

The possible influence of thermal anisotropy of the plasma in the vicinity of the shock on the estimated parameters and normal was investigated (Lepping, 1972), and was found to be negligible in this case.

Figure 3 shows a comparison of our least-squares and mean field
normals, both from Table 1, with the normal estimated by HACBH; they used the magnetic coplanarity theorem and the magnetic field data from the Explorer 33 A.R.C. experiment. Our least-squares normal and the HACBH normal differ by only 6°.

Using the least-squares normal and shock parameters and the pre-shock plasma bulk velocity \( V \), we are able, via the conservation of mass flux equation, to obtain the local shock speed (w.r.t. an inertial coordinate system fixed with the sun) at Explorer 33. The local shock speed, defined along the shock normal, was 430 km/sec, where \( V \) was (-336.3, -13.5, -30.6) km/sec, in the same inertial system. By using the best-fit normal and the onset delay, 3.5 min, between the SSC (2348 UT) and the Explorer 33 observation (2351.5 UT--See Ness, 1970, Figure 25, for a discussion of the difference between this time indication and that of the A.R.C. experiment which was 2351.7 UT), we can check the local shock speed estimate. Such a kinematic check, which assumes a plane local shock surface, yielded \( V(\text{check}) = 320 \pm 130 \) km/sec, where a ±1 min error was assumed for the SSC time. By considering, first, the somewhat inflated Monte Carlo calculation of the shock normal error cone angle (16°, from Table 1); second, the reasonable agreement of the local shock speed 430 km/sec with \( V(\text{check}) \) considering the small delay time 3.5 min; and, third, the agreement, within 11°, of all four estimated normals (three from Figure 3 plus the best-fit result using ±40 min of plasma data), we obtain a refined error cone angle of about 7°.

The Alfven and fast mode Mach numbers, calculated w.r.t. the solar wind along the shock normal direction, were \( M_A = 9.9 \) and \( M_F = 3.1 \) preshock, and \( M_A^' = 5.9 \) and \( M_F^' = 0.8 \) postshock. These values refer to the standard calculation carried out in the shock frame of reference, i.e., for \( V = 430 \) km/sec. We repeat the calculation for the assumed shock piston frame (helium front) taking into consideration the shock normal projection of the piston speed. The front moved along the radial (\(-x_{SS}\)) direction at 559 km/sec (HACBH) and the projected speed was \( V = 320 \) km/sec. This yields the following Mach numbers; \( m_A = 4.5 \) and \( m_F = 1.4 \) preshock, and \( m_A = -3.1 \) and \( m_F = 0.4 \) postshock; \( m_F \) was 5 according to HACBH. In calculating the fast mode Mach numbers the electron temperature, before and after the shock, was assumed to be \( 1.5 \times 10^5 \) °K and the proton temperatures were \( 0.5 \times 10^5 \) °K (preshock) and \( 1.6 \times 10^5 \) °K (postshock). In HACBH the electron temperature was not included nor was the -53° inclination of the "apparent piston front". However, the inappropriate negative values for \( m_A \) and \( m_F \) indicate that using the piston speed approximation for obtaining the Mach number estimates was not applicable in this case. This indicates that the local shock speed is considerably faster than the local piston speed. (See below for estimation of 190 km/sec radial velocity difference.)
Gross Interplanetary Shock Surface Geometry

In Figure 4 we show the resulting estimate of the gross interplanetary shock surface geometry as it appears to cut the ecliptic plane at the time of the earth's observation. At the earth's position the surface is constrained to be orthogonal to the ecliptic plane projection of the best-fit normal direction, from Figure 3, designated $\hat{n}_{33}$. The local speed along this direction was determined to be 710 km/sec. This corresponds to a radial (sun-earth direction) speed at this location of 750 km/sec, which is 190 km/sec faster than the piston speed ($HACBH$).

We now use this rough estimate of 750 km/sec as a reasonable first-guess of the local radial speed at Pioneer 6 for the ecliptic-plane component of the disturbance speed. Also we assume that the disturbance first passed the S/C shortly before 1216 UT on 15 February. These assumptions give a minimum distance of the disturbance from the sun along the sun-S/C line at the time of the earth's observation, as shown in the figure.

Concerning the region near Pioneer 7 we estimate the shock position in two ways, each again for the ecliptic plane. First, we consider the possibility that the local speed along $\hat{n}_{7}$ (the ecliptic plane projection of the shock normal as given in figure 2; $\phi = 161^\circ$) was equal to the mean speed from the sun to Pioneer 7 of $640 \text{ km/sec}$, (see Figure 1) corrected by $\cos 19^\circ$ for the non-radial direction of $\hat{n}_{7}$, giving 600 km/sec. This determines a maximum-curvature surface, since we expect some deceleration over 1 AU (Chao and Lepping, 1972). For a reasonable minimum-curvature surface we simply assume that the shock propagates in such a way that it satisfies a circular fit between the earth and Pioneer 7, i.e., that circle which is locally normal to $\hat{n}_{33}$ at the earth and to $\hat{n}_{7}$ near Pioneer 7. This demands a speed along $\hat{n}_{7}$ of 320 km/sec; this unusually low speed, especially w.r.t. the value of 710 km/sec at Earth, lends credibility to the surface being an estimation of minimum-curvature. The radius of the circle then is approximately 0.4 AU centered at about 0.7 AU from the sun. If this is indeed the minimum-curvature estimate, then this indicates that a more probable radius of curvature near the earth-Pioneer 7 region is even smaller than 0.4 AU—this result, however, depends somewhat on the accuracy of the estimated normal $\hat{n}_{7}$ (see comments below on the angle between $\hat{n}_{1}$ and $\hat{n}_{33}$).

In the figure the portion between the earth and Pioneer 6 is shown as a dashed line since it is not clear that a true shock existed in that region much less what its shape was. Also we should stress that the actual location of the disturbance near Pioneer 6 could have been beyond, and possibly well beyond, the dashed line position.
Discussion

Before further comment on Figure 4 we should point out that one could attempt to dismiss the implication that the curvature along the shock surface varies substantially by using the following argument. Recall that relatively large uncertainty exists in the quantity $n_3'$ (25°) and $n_2$, although assumed very accurate, could be more an indication of a near-earth transient deformation, due to a shock-discontinuity (or inhomogeneity) interaction say, than an indication of a smooth fit to a gross surface. And since the present estimated surface appears to intersect the earth's orbit near all three relevant locations, then a circle of 1 AU is apparently another likely candidate for the estimated geometry. This argument appears correct as far as it goes, but a circle of 1 AU is unlikely for the following reasons.

Firstly, the tipping at Explorer 33 was probably not a local phenomenon, as HACBH pointed out, because terrestrial magnetic events closely reflected changes in interplanetary space throughout approximately a nine hour period after shock passage with about a 4 min time difference to Explorer 33 (Hirshberg and Colburn, 1969). That is, during the passage of a mass of plasma about 0.1 AU in length moving behind the shock at 560 km/sec, terrestrial magnetic features correlated with observations at Explorer 33 $\approx$ 4 min later. This is about the same delay (3.5 min) between these positions observed for the tilted shock itself [see discussion on V(check)]. These observations strongly suggest that the tipping was not simply a local, transient feature. Secondly, both normals, i.e. at Explorer 33 ($\theta_{33} = -53^\circ$) and at Pioneer 7 ($\theta \approx -14^\circ$), are consistent in showing a southward tilt, further strengthening our argument. Conversely, if $n_{33}$ does represent a large-scale indication of the ecliptic-plane-normal at the earth's position, then the displacement of $\hat{A}$ from it as indicated by $\hat{n}_2$ substantially exceeds the 25° error on $n_2$, plus the 23° solar-longitudinal difference between the earth and Pioneer 7, giving greater relevance to $\hat{n}_2$. This argument taken along with the fact of the 18.9 hours' delay between the earth's and Pioneer 7's shock passages argues further for the $\approx 0.4$ AU radius of curvature in the earth-Pioneer 7 region, regardless of the detailed technique used in estimating the Pioneer 7 local shock speed, [Recall the low speed of 290 km/sec derived via the spherical assumption, In Figure 1.]

Therefore, we conclude that the curve shown in Figure 4 is more probable than a near-circular one for this event and make the following comments about the observations and results in light of this interpretation:

1.) This example of an IP shock surface should not be considered typical. Our estimation of its severely inclined normal at the earth's position, consistent with HACBH, is interesting in
itself, and is evidence of the shock's unusual nature. Such severe inclinations are indeed uncommon.

2.) The curvature of the shock surface in the ecliptic plane near the earth-Pioneer 7 region is consistent with a radius of $\leq 0.4$ AU, which is smaller than previous estimates (Hirshborg, 1968; Bavassano et al., 1968; and Taylor, 1969), and differs markedly from the "typical" radius of curvature of 1 AU obtained statistically by Chao and Lepping (1972, these proceedings) and others.

3.) The radial shock speed at the earth's position appeared to be $\sim 190$ km/sec faster than the helium "shock piston", assumed driving the shock. This implies that the relative separation was increasing, and therefore the influence of the piston on the shock was decreasing.

4.) We have estimated a possible gross shock surface geometry over $85^\circ$ in solar-longitude. This estimation was based on the probable correspondence of observations of a definite shock occurrence at Pioneer 7 and Explorer 33 (over $23^\circ$) with a possible shock-like disturbance at Pioneer 6.

5.) If the disturbance was a shock over $85^\circ$ in solar longitude at $\approx 1$ AU, then the curvature along the surface changes severely, having possibly an exceedingly large radius of curvature for one or more regions between the earth and Pioneer 6 and becoming 0.4 AU or smaller between the earth and Pioneer 7.

Our estimated shock geometry is partially similar to that reported for the 25 February 1969 event by Mariani et al. (1969) using Pioneer 8 magnetic field data and the SSC at earth. They also show large curvature near the position of the S/C, which was $23^\circ$ eastward of the earth, for an assumed flare source $30^\circ$ west heliographic longitude. Earlier in this conference Dryer et al. (1972) also showed a similar shock geometry, based on similarity theory, whereby the greatest shock curvature occurred in a region distinctly eastward ($\approx 40^\circ$ heliographic) of the source site. We differ in a comparison of the westward regions, in that our geometry shows a much larger radius of curvature there.

What factors play a role in the development of a flare-induced non-spherical shock front in IP space? We list the following possible contributory factors that must be examined yet for this event (and in general):

1.) A possible non-spherical distribution, in density, velocity and temperature, of the solar source,

2.) The effect of the gross variation of the pre-shock solar wind velocity
w.r.t., the solar longitude angle.

3.) Anisotropic non-linear wave propagation with respect to the moving solar wind, with special attention given to the role of the magnetic field (see Greenstadt et al., 1972, this conference, and Whang and Ness, 1970).

4.) Results of the shock interacting with other abrupt (pre-existing) IP discontinuities, and/or

5.) With gross IP density and pressure inhomogeneities.

The effects of shock interaction with discontinuities and inhomogeneities in IP space can be appreciable as Neubauer (1972-#4) and Siscoe (1972-#5's 2 and 5) have shown, leading to more than 15° deflections over 1 AU in some cases. We plan to attempt to explain the February 15-16, 1967 shock surface shape in terms of 1 to 5 above, in light of these previous studies, and especially to sort out the important from the unimportant factors, at least for this single event.

Acknowledgements

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References


Neubauer, F., Nonlinear Interaction of MHD Shock Waves with Tangential Discontinuities in the Solar Wind (approx. title), to be submitted to the Physics of Fluids.


Figure Captions

Figure 1. The positions, in the ecliptic plane, of Pioneers 6 and 7 and Explorer 33 during the passage of the flare-shock of Feb. 13-16, 1967. The upper left-hand corner gives the transit times from the sun and the associated derived mean speeds. Both Pioneers are out of the ecliptic plane by less than 50 earth radii at this time.

Figure 2. The magnetic field data from Pioneer 7 at the time of the shock passage. F is field magnitude, $\phi$ is the azimuthal angle measured in the ecliptic plane ($\phi = 0^\circ$ in the direction of the sun, centered at the S/C), and $\theta$ is the inclination angle w.r.t. the ecliptic plane, positive "northward". The limit-arrows represent the analysis interval that was used to derive the shock normal direction, which is shown in the upper left.

Figure 3. A comparison of three estimates of the Feb. 15, 1967 shock normal at the earth's position. The normal $\theta_{SE} = -53^\circ$, $\phi_{SE} = 198^\circ$ refers to that one determined via the least-squares method.

Figure 4. An estimated shape for the February 13-16, 1967, shock surface as it intersects the ecliptic plane. The ideal spiral field line is shown only for reference. See text.
CONFIGURATION OF SPACECRAFT
AT THE TIME OF THE FEB. 13 - 16, 1967 EVENT

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SUN

FLARE 13 FEB 1816 U.T.
N: 22°  W: 10°

PIONEER 6
15 FEB <1216 U.T.

PIONEER 7
16 FEB 1848 U.T.

EARTH (EXP 33)
15 FEB 2351.5 U.T.

EARTH'S ORBIT

ECLIPTIC PLANE

FIGURE 1
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ESTIMATED SHOCK NORMAL AT EXPLORER 33 ON 15 FEB 1967

\( \theta_{SE} \)

\(-53^\circ\)

\( \phi_{SE} \)

198°

LEAST SQUARES NORMAL

HIRSHBERG ET AL NORMAL

MEAN FIELD NORMAL (GSFC)

FIGURE 3
POSSIBLE SHOCK SURFACE FOR
THE FEBRUARY 13-16, 1967 EVENT

FIGURE 4