ELASTOSTATIC STRESS ANALYSIS OF ORTHOTROPIC RECTANGULAR CENTER-CRACKED PLATES

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A mapping-collocation method was developed for the elastostatic stress analysis of finite, anisotropic plates with centrally located traction-free cracks. The method essentially consists of mapping the crack into the unit circle and satisfying the crack boundary conditions exactly with the help of Muskhelishvili's function extension concept. The conditions on the outer boundary are satisfied approximately by applying the method of least-squares boundary collocation. A parametric study of finite-plate stress intensity factors, employing this mapping-collocation method, is presented. It shows the effects of varying material properties, orientation angle, and crack-length-to-plate-width and plate-height-to-plate-width ratios for rectangular orthotropic plates under constant tensile and shear loads.
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SUMMARY

A mapping-collocation method was developed for the elastostatic stress analysis of
finite, anisotropic plates with centrally located traction-free cracks. The method es-
sentially consists of mapping the crack into the unit circle and satisfying the crack
boundary conditions exactly with the help of Muskhelishvili's function extension concept.
The conditions on the outer boundary are satisfied approximately by applying the method
of least-squares boundary collocation.

A parametric study of finite-plate stress intensity factors, employing this mapping-
collocation method, is presented. It shows the effects of varying material properties,
orientation angle, and crack-length-to-plate-width and plate-height-to-plate-width ratios
for rectangular orthotropic plates under constant tensile and shear loads.

INTRODUCTION

Anisotropic materials are frequently applied in present-day structural design, as
exemplified in composite materials. Therefore, the problems pertaining to the stress
distribution in finite and quasi-infinite plates of such materials containing cracks are of
immediate importance. The infinite-plate problems present no difficulty (refs. 1 to 8);
however, the problems of finite anisotropic plates with cracks remained fairly neglected
until quite recently (refs. 9 to 11).

The mapping-collocation method as applied in this report was developed in refer-
ence 10 and parallels Bowie's mapping-collocation technique (ref. 12) for a finite iso-
tropic plate with a central traction-free crack.

This report analyzes the effects of varying material properties, orientation angle,
and crack-length-to-plate-width and plate-height-to-plate-width ratios on both the
opening- and sliding-mode stress intensity factors in a rectangular orthotropic plate
with a centrally located traction-free crack under constant tensile and shear loads.

In addition to the parametric investigation, a brief summary of the development of the mapping-collocation method is presented; however, for a detailed exposition the reader is referred to reference 10.

SYMBOLS

The following list contains the more commonly used symbols and their representative meaning unless otherwise defined in the text. In general, all symbols are defined when first introduced.

\[ A_{1n} \] \text{nth complex constant in Laurent series associated with positive exponent terms}

\[ a \] \text{half-crack length}

\[ B_{1n} \] \text{nth complex constant in Laurent series associated with negative exponent terms}

\[ C_{ij} \] \text{ijth element of elastic compliance matrix}

\[ \gamma_{1n} \] \text{nth modified complex constant associated with positive exponent terms in Laurent series}

\[ \gamma_{2n} \] \text{nth modified complex constant associated with negative exponent terms in Laurent series}

\[ \bar{\gamma} \] \text{complex constant depending on material properties}

\[ \bar{\gamma}_s, \bar{\gamma}_d \] \text{modified complex constants depending on sum and difference of } A_{11} \text{ and } B_{11}, \text{ respectively}

\[ c \] \text{half-width of rectangular plate}

\[ E_{11}, E_{22} \] \text{Young's moduli of elasticity with respect to coordinate directions}

\[ F_{1n}, F_{2n}, G_{1n}, G_{2n} \] \text{complex quantities utilized in construction of coefficient matrix in least-squares boundary collocation method}

\[ G_{12} \] \text{shear modulus of an orthotropic material}

\[ H_1, H_2 \] \text{nondimensional opening- and sliding-mode stress intensity factors, respectively}

\[ h \] \text{half-height of a rectangular plate}

\[ K_1, K_2 \] \text{opening- and sliding-mode stress intensity factors, respectively}
M
total number of collocation points on outer boundary of region with
central crack

NN, NP
truncation numbers on infinite sums associated with negative and posi-
tive exponent terms in Laurent series, respectively

Ts
constant shear stress applied to boundary of square plate

Tt
constant tensile stress applied to boundary of rectangular plate

x, y
rectangular coordinates

z
conventional complex variable

z1, z2
complex variables modified by material properties

δ
material orientation angle

ζ, ζk
complex variables in parametric planes

ν12
Poisson's ratio in an orthotropic material

μ1, μ2
Lekhnitskii's material parameters

σxx, σyy, σxy
components of stress tensor, rectangular coordinates

φ
Airy's stress function

φ1(z1), φ2(z2)
complex stress potential functions

BRIEF DESCRIPTION OF MAPPING-COLLOCATION METHOD

Consider a doubly connected orthotropic region L defined by a crack Γc and the
region's outer boundary Γ, as shown in figure 1(a). The crack and its exterior can be
mapped, as shown, into the unit circle γ and its exterior L* by the mapping function

\[ \xi = \frac{z + \sqrt{z^2 - a^2}}{a} \]  

where \( z \) is the usual complex variable, \( x + iy \).

We introduce two additional complex variables, \( z_1 \) and \( z_2 \), defined by

\[ z_k = \frac{1 - i\mu_k}{2} z + \frac{1 + i\mu_k}{2} \bar{z} \quad k = 1, 2 \]  

where \( \mu_k \) and \( \overline{\mu_k} \) are the roots of the characteristic equation
\[ C_{22} - 2C_{26}\mu + (C_{66} + 2C_{12})\mu^2 - 2C_{16}\mu^3 + C_{11}\mu^4 = 0 \]  

(3)

and the \( C_{ij} \) are the material compliances appearing in the generalized Hooke's law for an orthotropic material.

The regions \( L_k \) with the boundaries \( \Gamma_{ck} \) and \( \Gamma_k \) obtained by the transformation (2) can also be mapped into their respective \( \xi_k \)-planes by the mapping functions

\[ \xi_k = \frac{z_k + \sqrt{z_k^2 - a^2}}{a} \quad k = 1, 2 \]  

(4)

as illustrated in figures 1(b) and (c).
The Airy stress function $\varphi(x, y)$ can be written in terms of two analytic functions of the complex variables $z_1$ and $z_2$:

$$\varphi(x, y) = R\text{e}\left[F_1(z_1) + F_2(z_2)\right]$$  \hspace{1cm} (5)

With the following definitions:

$$\varphi_1(z_1) = \frac{dF_1}{dz_1}$$

and

$$\varphi_2(z_2) = \frac{dF_2}{dz_2}$$

the zero traction conditions on the crack can be satisfied by applying Muskhelishvili's function extension concept across the unit circle (ref. 13). This is accomplished by expressing one of the unknown complex stress potentials in terms of the other. If $\varphi_2(\zeta_2)$ is chosen as

$$\varphi_2(\zeta_2) = \frac{\mu_2 - \mu_1}{\mu_2 - \mu_2} \varphi_1(\zeta_2) + \frac{\mu_2 - \mu_1}{\mu_2 - \mu_2} \varphi_1(\zeta_2)$$  \hspace{1cm} (6)

the crack boundary conditions will be satisfied for any choice of $\varphi_1(\zeta_1)$, provided it is analytic.

The substitution of expression (6) into the boundary conditions for the outer boundary of the region results in two conditions given completely in terms of $\varphi_1(\zeta_1)$, $\varphi_1(\zeta_2)$, and $\varphi_1(1/\zeta_2)$.

The determination of $\varphi_1(\zeta_1)$ depends on its form of representation and the satisfaction of the boundary conditions on the outer boundary. Specifically, it is assumed that $\varphi_1(\zeta_1)$ can be represented in the form of a truncated Laurent series, such that

$$\varphi_1(\zeta_1) = A_{10} + \sum_{n=1}^{NP} A_{1n} \zeta_1^n + \sum_{n=1}^{NN} B_{1n} \zeta_1^{-n}$$  \hspace{1cm} (7)
Because of the symmetry conditions for the orthotropic plate problems in question (figs. 2 and 3), \( n \) becomes necessarily odd and the final forms of the boundary conditions for the outer boundary are obtained as

\[
\begin{bmatrix}
0 & \text{Re} F_{11} - \text{Im} F_{11} & \ldots & \text{Re} F_{1NP} - \text{Im} F_{1NP} & \ldots & \text{Re} F_{2NN} - \text{Im} F_{2NN} \\
y \text{Re} G_{11} - \text{Im} G_{11} & \ldots & \text{Re} G_{1NP} - \text{Im} G_{1NP} & \ldots & \text{Re} G_{2NN} - \text{Im} G_{2NN}
\end{bmatrix}
\begin{bmatrix}
\text{Re} [c_0] \\
\text{Re} [c_2] \\
\text{Re} [c_4] \\
\vdots \\
\text{Re} [c_{2NN}] \\
\text{Im} [c_0] \\
\text{Im} [c_2] \\
\text{Im} [c_4] \\
\vdots \\
\text{Im} [c_{2NN}]
\end{bmatrix}
\]

\[
= \frac{1}{2} \left[ T_t x - T_s y \right]
\]

where \( NN \) and \( NP \) are odd. The terms in equations (8) are defined as
Equations (8) must now be solved for the unknowns, \( \varphi_{1n} \) and \( \varphi_{2n} \). The applicability of the least-squares boundary collocation method (refs. 14 and 15) to the problems at hand is readily obvious. Equations (8) are to be approximately satisfied at \( M \) points on the outer boundary, while the number of unknowns is given by \( NN + NP + 1 \). Then the resulting matrix equation can be written as
For the least-squares boundary collocation method, the problem becomes

$$\begin{align*}
G^T G^* &= \mathbf{A}^T \mathbf{A} \\
(10)
\end{align*}$$

where $G^*$ is the desired solution vector.

The solution of equation (11) was carried out on a digital computer by declaring all quantities and operations in double precision and utilizing the Gauss-elimination technique with complete pivoting. It should also be mentioned that the elements of the coefficient matrix $G$ were scaled by the divisor

$$R^n = \left( \sqrt{c^2 + h^2} \right)^n$$

After the approximate solution vector $G^*$ is obtained, the computation of the stress intensity factors and the stress components is a matter of substitution of the elements of $G^*$ into the following expressions:

$$K_1 = \frac{4}{\sqrt{a}} \text{Re} \left[ (\mu_2 - \mu_1) \left( a_{D} + \sum_{n=3,5,7}^{NP} n^a n_{1n} - \sum_{n=3}^{NN} n^a n_{2n} \right) \right]$$

$$K_2 = -\frac{4}{\sqrt{a}} \text{Re} \left[ \overline{\mu}_2 (\mu_2 - \mu_1) \left( a_{D} + \sum_{n=3,5,7}^{NP} n^a n_{1n} - \sum_{n=3}^{NN} n^a n_{2n} \right) \right]$$

(12) (13)
\[
\sigma_{xx} = 2 \text{Re} \left\{ \varphi_{d} \right\} + 2 \text{Re} \left\{ \varphi_{d} \left[ \mu_{2} - \overline{\mu_{2}} \right] \frac{z_{1}}{z_{1}^{2} - a^{2}} + \mu_{2}^{2} \left( \overline{\mu_{2}} - \mu_{1} \right) \frac{z_{2}}{z_{2}^{2} - a^{2}} + \mu_{2}^{2} \left( \mu_{2} - \mu_{1} \right) \frac{z_{1}}{z_{2}^{2} - a^{2}} \right\}
\]

\[
+ 2 \text{Re} \sum_{n=3, 5, 7}^{\text{NP}} \left\{ n \varphi_{1n} \left[ \mu_{2}^{2} \left( \overline{\mu_{2}} - \mu_{1} \right) \left( z_{1} + \sqrt{z_{1}^{2} - a^{2}} \right) + \mu_{2}^{2} \left( \overline{\mu_{2}} - \mu_{1} \right) \left( z_{2} + \sqrt{z_{2}^{2} - a^{2}} \right) + \mu_{2}^{2} \left( \mu_{2} - \mu_{1} \right) \left( \overline{z}_{1} + \sqrt{\overline{z}_{1}^{2} - a^{2}} \right) \right] \right\}
\]

\[
- 2 \text{Re} \sum_{n=3, 5, 7}^{\text{NN}} \left\{ n \varphi_{2n} \left[ \mu_{2}^{2} \left( \overline{\mu_{2}} - \mu_{1} \right) \left( z_{1} + \sqrt{z_{1}^{2} - a^{2}} \right) + \mu_{2}^{2} \left( \overline{\mu_{2}} - \mu_{1} \right) \left( z_{2} + \sqrt{z_{2}^{2} - a^{2}} \right) + \mu_{2}^{2} \left( \mu_{2} - \mu_{1} \right) \left( \overline{z}_{1} + \sqrt{\overline{z}_{1}^{2} - a^{2}} \right) \right] \right\}
\]

\[
(14)
\]

\[
\sigma_{yy} = 2 \text{Re} \left\{ \varphi_{d} \left[ \mu_{2} - \overline{\mu_{2}} \right] \frac{z_{1}}{z_{1}^{2} - a^{2}} + \left( \overline{\mu_{2}} - \mu_{1} \right) \frac{z_{2}}{z_{2}^{2} - a^{2}} + \left( \mu_{2} - \mu_{1} \right) \frac{z_{1}}{z_{2}^{2} - a^{2}} \right\}
\]

\[
+ 2 \text{Re} \sum_{n=3, 5, 7}^{\text{NP}} \left\{ n \varphi_{1n} \left[ \mu_{2} - \overline{\mu_{2}} \left( z_{1} + \sqrt{z_{1}^{2} - a^{2}} \right) + \mu_{2} - \mu_{1} \left( z_{2} + \sqrt{z_{2}^{2} - a^{2}} \right) + \mu_{2} - \mu_{1} \left( \overline{z}_{1} + \sqrt{\overline{z}_{1}^{2} - a^{2}} \right) \right] \right\}
\]

\[
- 2 \text{Re} \sum_{n=3, 5, 7}^{\text{NN}} \left\{ n \varphi_{2n} \left[ \mu_{2} - \overline{\mu_{2}} \left( z_{1} + \sqrt{z_{1}^{2} - a^{2}} \right) + \mu_{2} - \mu_{1} \left( z_{2} + \sqrt{z_{2}^{2} - a^{2}} \right) + \mu_{2} - \mu_{1} \left( \overline{z}_{1} + \sqrt{\overline{z}_{1}^{2} - a^{2}} \right) \right] \right\}
\]

\[
(15)
\]
For a detailed discussion and complete derivation of the above method, refer to reference 10.

RESULTS AND DISCUSSION OF SOLUTIONS OF TENSION AND SHEAR PROBLEMS

Specifications of Parameters

The solutions of the two problems shown in figures 2 and 3 are based on the following specifications:

(1) The nondimensional stress intensity factors $H_K$ are defined as $H_K = K_k/\sqrt{a} T_t$ for tension and $H_K = K_k/\sqrt{a} T_s$ for shear ($k = 1, 2$).

(2) With the exception of the investigation of the effects of material properties on the stress intensity factors, the ratios

$$\frac{E_{22}}{E_{11}} = 0.366$$

and
were used throughout the entire analysis. These ratios correspond to the properties of fiberglass.

(3) In studying the effects of varying material properties on the stress intensity factors in both the tension and shear problems, the following parameter ranges were considered:

\[
\frac{E_{11}}{E_{22}} = 0.257
\]

\[
\left(\frac{E_{11}}{2G_{12}} - \nu_{12}\right)^2 = 0.001
\]

(4) In studying the effects of varying plate aspect ratio \( h/c \) and crack-length-to-plate-width ratio \( a/c \) on the stress intensity factors in the tension problem, the parameter ranges were chosen as follows:

\[
0.05 \leq \frac{E_{22}}{E_{11}} \leq 1
\]

\[
0.001 \leq \frac{E_{11}}{E_{22}} \leq \infty
\]

\[
\frac{a}{c} = \frac{2}{3}, \quad \frac{h}{c} = 1, \quad \delta = 45^0
\]

(5) The effects of variation of the orientation angle \( \delta \) were also analyzed for both the tension and shear problems, considering the following parameter ranges:

\[
0 < \frac{a}{c} \leq \frac{2}{3}
\]

\[
0.25 \leq \frac{h}{c} \leq 133
\]

\[
\delta = 45^0
\]
\[
0 < \frac{a}{c} \leq \frac{2}{3}
\]
\[
0^\circ \leq \delta \leq 90^\circ
\]
\[
\frac{h}{c} = 1
\]

**Examination of Mapping-Collocation Method**

The available results which can be used to check the accuracy of this method are those for centrally cracked infinite plates (refs. 1 and 3) and the results of Kobayashi, Isida, and Sawyer (refs. 16 to 18) for finite isotropic rectangular plates. In order to gain a certain degree of confidence in the mapping-collocation method, both the results for the infinite orthotropic and finite isotropic plates were verified.

The first verification is obtained by making the plate dimensions large compared to the crack length.

This resulted in \( H_1 = 1.0000 \) and \( H_2 = 0.0000 \). The corresponding analytical results for an infinite plate are \( H_1 = 1 \) and \( H_2 = 0 \). It was, however, also noted that the satisfaction of the stress boundary conditions markedly improved with the use of more and more terms in the Laurent series expansion of \( \varphi_1(\zeta) \). Thus, the results of the infinite plates were verified for both the tension and shear problems.

The verification of Kobayashi's, Isida's, and Sawyer's opening-mode stress intensity factors for various finite isotropic rectangular plates in tension was carried out by setting the ratios

\[
\frac{E_{22}}{E_{11}} = 1
\]

and

\[
\frac{E_{11}}{E_{22}} = \left( \frac{E_{11}}{2G_{12} - \nu_{12}} \right)^2 = 1
\]
The value of the orientation angle had no influence on the results. The cases considered are shown in Table I, where the various stress intensity factors are tabulated and can readily be compared with each other. As can be seen from Table I, agreement with the known results is excellent.

In order to economize the mapping-collocation method with regard to the use of the number of terms in the Laurent series expansion of \( \varphi_1(\zeta) \), two different truncation numbers \( NN \) and \( NP \) were assigned as limits of the sums in the various expressions. The numbers \( NN \) and \( NP \) are odd for orthotropic materials and correspond to the number of the negative and positive exponent terms in the Laurent series expansion of the stress potential function \( \varphi_1(\zeta) \). Upon investigating various \( NN/NP \) ratios for the finite plate problems, it was found that a \( NN/NP \) ratio of 5/3 resulted in fairly fast convergence with rather well-satisfied boundary conditions. This \( NN/NP \) ratio was retained throughout the whole investigation of the problems given in Figures 2 and 3.

One of the most important indications of the degree of accuracy of the solutions obtained by the mapping-collocation method comes from the examination of the boundary stress between collocation points. Since the least-squares method of collocation results in the satisfaction of the boundary conditions only in the approximate sense, the magnitude and frequency of oscillations of the boundary stresses serve as an indicator to the "exactness" of the solution of the stress boundary-value problem. Thus, in each case, the boundary stresses were also examined. For example, in the case of a square plate in tension with \( a/c = 2/3 \) and \( \delta = 45^\circ \), the largest error in the boundary stresses was found to be about 4 percent and it occurred in a sudden manner at the corners of the plate.

In addition, the pattern of convergence of the stress intensity factors \( H_1 \) and \( H_2 \) was also investigated. The results are shown in Table II and Figures 4 and 5. It was ob-

![Figure 4. Typical pattern of convergence of opening-mode stress intensity factor for tensile loading. Plate size, \( h/c = 1 \); crack length, \( a/c = 1/2 \); orientation angle, \( \delta = 45^\circ \).](image-url)
served that the insensitiveness of $H_1$ and $H_2$ to a change in the number of terms of the Laurent series expansion of the stress potential $\varphi_1(\zeta_1)$ is an excellent indicator to how well the boundary conditions are satisfied by the least-squares approximation.

Upon considering various materials, plate sizes (aspect ratios), $a/c$ ratios, and orientation angles under the loading conditions shown in figures 2 and 3, two conclusions became obvious regarding the convergence of the stress intensity factors:

(1) Convergence of the stress intensity factors is definitely affected by the material parameters. It was found, for example, that for a square plate in tension with $a/c = 2/3$ and $\delta = 45^\circ$, the use of the dimensionless material constant

$$\frac{E_{11}}{E_{22}} = 0.0001$$

with any $E_{22}/E_{11}$ ratio resulted in oscillatory values of $H_1$ and $H_2$. In this case, the number of unknowns was taken as 83, which gave excellent results for higher values of the dimensionless material constant.
(2) Convergence of the stress intensity factors is also affected by the a/c ratios. For a square plate in tension with

\[
\frac{E_{11}}{E_{22}} = 0.257
\]

\[
\left( \frac{E_{11}}{2G_{12}} - \nu_{12} \right)^2
\]

and with \( \frac{E_{22}}{E_{11}} = 0.366, \) a/c = 0.9, and \( \delta = 45^\circ \) with the use of 83 unknowns, it was found that the boundary conditions were rather badly satisfied. The method probably resulted in unconverged stress intensity factors. For smaller a/c ratios, well-converged values of \( H_1 \) and \( H_2 \) were obtained.

SOLUTION OF TENSION PROBLEMS OF ORTHOTROPIC RECTANGULAR PLATE WITH CENTRAL CRACK

The consideration of the tension problem, that is, when the constant boundary stress is applied in the y-direction (fig. 2), resulted in various parametric studies involving material properties, a/c ratios, h/c ratios, and orientation angles.

Effects of Material Properties

An orthotropic material can be characterized by two dimensionless ratios constructed from the engineering material constants \( E_{11}, E_{22}, G_{12}, \) and \( \nu_{12} \) (ref. 10). These dimensionless ratios are

\[
\frac{E_{22}}{E_{11}} \quad \text{and} \quad \frac{E_{11}}{E_{22}} \left( \frac{E_{11}}{2G_{12}} - \nu_{12} \right)^2
\]

The effects of these two dimensionless ratios on the stress intensity factors are shown in table III and figures 6 and 7.
Figure 6 demonstrates the effects of the dimensionless material ratios on the opening-mode stress intensity factor, while figure 7 shows the sliding-mode stress intensity factor as a function of these two ratios. Figure 6 shows that, as the $\frac{E_{22}}{E_{11}}$ ratio decreases, the opening-mode dimensionless stress intensity factor $K_1$ increases in value and that, for a fixed value of this ratio, the stress intensity factor exhibits a minimum value.

Figure 7 shows the interesting result that there exists a sliding-mode stress intensity factor for the case of a pure tensile load, contrary to the results for an infinite or isotropic plate. This results strictly from the orthotropy of the material and the finiteness of the plate dimensions. It may be noted that the maximum magnitude of the sliding-mode stress intensity factor shown in figure 7 is about 14 percent of the opening-mode stress intensity factor for the same $\frac{E_{22}}{E_{11}}$ ratio.

**Effects of Plate Dimensions**

In order to investigate the effects of various plate widths and plate lengths on the stress intensity factors $K_1$ and $K_2$, the dimensionless material ratios were set at
Figure 8. - Effects of plate width and plate size on opening-mode stress intensity factor for tensile loading. Orientation angle, $\delta = 45^\circ$.

Figure 9. - Effects of plate width and plate size on sliding-mode stress intensity factor for tensile loading. Orientation angle, $\delta = 45^\circ$. 
\[
\frac{E_{22}}{E_{11}} = 0.366 \quad \text{and} \quad \frac{E_{11}}{E_{22}} = 0.257 \quad \left(\frac{E_{11}}{2G_{12} - \nu_{12}}\right)^2
\]

with an orientation angle \( \delta = 45^\circ \). These ratios correspond approximately to fiberglass properties. The results are shown in table IV and figures 8 and 9.

**Effects of Orientation Angle and Crack Length**

For the investigation of the effects of the orientation angle and the crack length on the stress intensity factors, the \( h/c \) ratio was taken as 1 (square plate) and the dimensionless material ratios were held constant at the values specified previously.

The variations of the orientation angle \( \delta \) and the \( a/c \) ratio resulted in table V. The graphs constructed from table V are presented in figures 10 and 11. Figure 10 shows the effects of various orientation angles \( \delta \) on the opening-mode stress intensity factor \( H_1 \) for constant \( a/c \) ratios. Figure 11 shows the sliding-mode stress intensity factor.
factor $H_2$ as affected by changes in $\delta$ and $a/c$.

As an illustration of the typical stress distribution on the line $y = 0$ for the range $1 < x/a \leq 3/2$, the values of the stress components were tabulated in table VI for a plate with $h/c = 1$, $a/c = 2/3$, $\delta = 45^\circ$ and plotted in figures 12 and 13.

The presence of $\sigma_{xy}$ results from the general orthotropy of the material. It should also be observed that $\sigma_{xy}$ becomes effectively zero on the boundary, which it should be for proper satisfaction of the boundary conditions.

For each $H_1$ and $H_2$ value obtained, the complete boundary-value problem of an orthotropic rectangular plate containing a central crack had to be solved. In each case, in addition to the computation of the stress components on the boundary, the stress components on the $y = 0$ line were also recorded. It gives one confidence in the method to know that for isotropic and specially orthotropic materials, the shear stress component $\sigma_{xy}$ was effectively zero on the $y = 0$ line. Also for problems with $E_{22}/E_{11} = 1$, the shear stress component $\sigma_{xy}$ on the $y = 0$ line was practically zero for any value of

$$\frac{E_{11}}{E_{22}} \left( \frac{E_{11}}{2G_{12}} - \nu_{12} \right)^2$$
SOLUTION OF SHEAR PROBLEM OF ORTHOTROPIC SQUARE PLATE WITH CENTRAL CRACK

In addition to the tension problem discussed in the preceding section, the problem of an orthotropic square plate containing a central crack and loaded by unit shear stress (fig. 3) was also considered. As in the case of the tension problem, the shear problem was also solved for various dimensionless material ratios, a/c ratios, and orientation angles. The h/c ratio was held constant (h/c = 1) throughout the entire analysis of the shear problem.

The results of the parametric study involving variations of the dimensionless material ratios while we held a/c = 2/3 and δ = 45° are summarized in table VII and figures 14 and 15.
The effects of variation of the orientation angle $\delta$ and the $a/c$ ratio are shown in table VIII and figures 16 and 17. There is an important observation which concerns the effects of the variation of the orientation angle in the shear problem. As far as $\delta$-dependence is concerned, the orthotropic plate behaves differently in tension and in shear. In the tension problem it was found that the maximum $H_1$ values always occurred at $\delta = 90^\circ$ (specially orthotropic material), while in the shear problem it is obvious that the maximum $H_2$ is $\delta$-dependent. However, in both the tension and shear problems the minimum values of the significant stress intensity factors occurred at $\delta = 0^\circ$, which designates the other type of special orthotropy.

As an illustration of the typical stress distribution on the line $y = 0$, $1 < x/a \leq 3/2$, the values of the stress components were recorded in table IX and depicted in figures 18 and 19 for a plate with $a/c = 2/3$ and $\delta = 45^\circ$. Both $\sigma_{xx}$ and $\sigma_{yy}$ were found to be zero on the line $y = 0$, $1 < x/a \leq 3/2$ with $E_{22}/E_{11} = 1$ and for any value of

$$
\frac{E_{11}}{E_{22}} \left( \frac{E_{11}}{2G_{12}} - \nu_{12} \right)^2
$$
Figure 16. - Effects of orientation angle and plate width on opening-mode stress intensity factor for shear loading. Plate size, wh/c = 1.

Figure 17. - Effects of orientation angle and plate width on sliding-mode stress intensity factor for shear loading. Plate size, wh/c = 1.
Figure 18. Normal stress distribution on $y=0$, $x>a$ line for shear loading. Plate size, $h/c=1$; crack length, $a/c=2/3$; orientation angle, $\delta=45^\circ$; constant shear stress, $T_s=1$.

Figure 19. Shear stress distribution on $y=0$, $x>a$ line for shear loading. Plate size, $h/c=1$; crack length, $a/c=2/3$; orientation angle, $\delta=45^\circ$; constant shear stress, $T_s=1$. 
The special cases of isotropy and special orthotropy resulted also in zero values for the normal stresses $\sigma_{xx}$ and $\sigma_{yy}$ on the $y = 0, 1 < x/a \leq 3/2$ line.

CONCLUDING REMARKS

The mapping-collocation method as developed in reference 10 was applied to a large number of rectangular orthotropic plate problems in order to study the effects of varying material properties, orientation angles, and crack-length-to-plate-width $a/c$ and plate-height-to-plate-width $h/c$ ratios on the stress intensity factors for both tensile and shear loadings.

A natural extension of the mapping-collocation method as applied to finite orthotropic regions with centrally located traction-free cracks would be to consider various shapes for the inner boundary. For example, triangular, rectangular, and elliptical boundaries could be specified by a single general Schwarz-Christoffel transformation and mapped into the unit circle. This possibility would require further research as to the accuracy of the method when it is applied to various types of doubly connected regions.

Another direction of research presents itself in the consideration of a more detailed parametric study of the effects of dimensionless material constants, orientation angles, $a/c$ ratios, and $h/c$ ratios on the stress intensity factors. It is a possibility that, for certain parameter ranges, "practical forms" of expressions could be obtained for the approximation of the stress intensity factors. These forms would result from curve fitting the various parametric data.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 21, 1972,
501-22.

REFERENCES


### TABLE I. COMPARISON OF OPENING-MODE STRESS INTENSITY FACTORS FOR RECTANGULAR ISOTROPIC PLATES WITH CENTRAL CRACKS

[Plate size, h/c = 2.]

<table>
<thead>
<tr>
<th>Crack length, a/c</th>
<th>Opening-mode nondimensional stress intensity factor, $H_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kobayashi (ref. 16)</td>
</tr>
<tr>
<td>1/12</td>
<td>1.0048</td>
</tr>
<tr>
<td>1/6</td>
<td>1.0191</td>
</tr>
<tr>
<td>1/4</td>
<td>1.0448</td>
</tr>
<tr>
<td>1/3</td>
<td>1.0830</td>
</tr>
<tr>
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<td>1.2215</td>
</tr>
<tr>
<td>2/3</td>
<td>1.4665</td>
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### TABLE II. TYPICAL PATTERN OF CONVERGENCE OF STRESS INTENSITY FACTORS FOR TENSILE LOADING

[Plate size, h/c = 1; crack length, a/c = 1/2; orientation angle, $\delta = 45^\circ$.]

<table>
<thead>
<tr>
<th>Number of equations</th>
<th>Number of unknowns</th>
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<td></td>
<td>15</td>
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<tr>
<td></td>
<td>$H_1$</td>
</tr>
<tr>
<td>16</td>
<td>1.3548</td>
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<tr>
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<td>1.3466</td>
</tr>
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<td>------</td>
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<td>64</td>
<td>1.3451</td>
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<tr>
<td>96</td>
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</tr>
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<td>128</td>
<td>1.3451</td>
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<tr>
<td>176</td>
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### TABLE III. - EFFECTS OF MATERIAL PROPERTIES ON STRESS INTENSITY FACTORS FOR TENSILE LOADING

- Plate size, h/c = 1; crack length, a/c = 2/3; orientation angle, $\delta = 45^\circ$

<table>
<thead>
<tr>
<th>$E_{11}/E_{22}$</th>
<th>$E_{22}/E_{11}$</th>
<th>0.05</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<tr>
<td>$\left(\frac{E_{11} - \nu_{12}}{2G_{12}}\right)^2$</td>
<td></td>
<td>H₁</td>
<td>H₂</td>
<td>H₁</td>
<td>H₂</td>
<td>H₁</td>
<td>H₂</td>
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<td>-0.189</td>
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<td>-0.153</td>
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<td>-0.084</td>
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<td>0.750</td>
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<td>-0.140</td>
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<td>1.655</td>
<td>-0.069</td>
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### TABLE IV. - EFFECTS OF CRACK LENGTH AND PLATE SIZE ON STRESS INTENSITY FACTORS FOR TENSILE LOADING

- Orientation angle, $\delta = 45^\circ$

<table>
<thead>
<tr>
<th>Crack length, a/c</th>
<th>Plate size, h/c</th>
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<tbody>
<tr>
<td></td>
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<td>H₁</td>
<td>H₂</td>
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</tr>
<tr>
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<td>1/3</td>
<td>2.331</td>
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<td>1/2</td>
<td>3.465</td>
</tr>
<tr>
<td>2/3</td>
<td>4.946</td>
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### TABLE V. - EFFECTS OF CRACK LENGTH AND ORIENTATION ANGLE ON STRESS INTENSITY FACTORS FOR TENSILE LOADING

(Plate size, \( h/c = 1 \))

<table>
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<tr>
<th>Orientation angle, ( \delta ), deg</th>
<th>Crack length, ( a/c )</th>
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<td></td>
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</tr>
<tr>
<td></td>
<td>( H_1 )</td>
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<td>1.028</td>
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<tr>
<td>40</td>
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<tr>
<td>45</td>
<td>1.052</td>
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<tr>
<td>50</td>
<td>1.071</td>
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<td>60</td>
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<td>64</td>
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<tr>
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<td>1.071</td>
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</table>
TABLE VI. - STRESS DISTRIBUTION ON y = 0, x > a
LINE FOR TENSILE LOADING

[Plate size, h/c = 1; crack length, a/c = 2/3; orientation angle, δ = 45°; constant tensile stress, T = 1]

<table>
<thead>
<tr>
<th>x/a</th>
<th>Components of stress tensor (rectangular coordinates)</th>
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<tbody>
<tr>
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<td>σ(_{yy})</td>
<td>σ(_{xy})</td>
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<td>1.046</td>
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<td>2.26</td>
<td>.085</td>
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<td>.153</td>
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<td>1.500</td>
<td>0</td>
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<td>0</td>
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</table>

TABLE VII. - EFFECTS OF MATERIAL PROPERTIES ON STRESS INTENSITY FACTORS FOR SHEAR LOADING

[Plate size, h/c = 1; crack length, a/c = 2/3; orientation angle, δ = 45°]

<table>
<thead>
<tr>
<th>E(<em>{11}) / E(</em>{22})</th>
<th>E(<em>{22}/E</em>{11}) 0.05</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
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<tbody>
<tr>
<td></td>
<td>H(_1)</td>
<td>H(_2)</td>
<td>H(_1)</td>
<td>H(_2)</td>
<td>H(_1)</td>
<td>H(_2)</td>
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TABLE VIII. - EFFECTS OF PLATE WIDTH AND ORIENTATION ANGLE ON STRESS INTENSITY FACTORS FOR SHEAR LOADING

[Plate size, $h/c = 1$]
TABLE IX. - STRESS DISTRIBUTION ON \( y = 0, \ x > a \)

LINE FOR SHEAR LOADING

[Plate size, \( h/c = 1 \); crack length, \( a/c = 2/3 \); orientation angle, \( \delta = 45^\circ \); constant shear stress, \( T_s = 1 \)]

<table>
<thead>
<tr>
<th>( x/a )</th>
<th>Components of stress tensor (rectangular coordinates)</th>
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</thead>
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<td>( \sigma_{xx} )</td>
</tr>
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"The aeronautical and space activities of the United States shall be conducted so as to contribute ... to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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