INTERACTION OF THE SPACE SHUTTLE
CONTROL SYSTEM WITH POGO

by A. Stewart Hopkins and William F. Davis

Prepared by
MCDONNELL DOUGLAS ASTRONAUTICS COMPANY
Huntington Beach, Calif. 92647
for Langley Research Center

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The asymmetric configuration of the Space Shuttle results in coupled pitch and axial vibration modes. This coupling results in interaction between the pogo and control systems. A detailed model of representative Space Shuttle structure, feedline, control, and engine systems revealed the coupled system may be unstable even though the pogo and control systems are stable individually. A method is developed for predicting the coupled system stability in terms of the stability of the separate systems.
CONTENTS

SUMMARY ........................................... 1
INTRODUCTION ...................................... 2
SYMBOLS ............................................ 4
ANALYTICAL MODELS ................................. 9
  Structure ....................................... 10
  Feedline ....................................... 19
  Control ....................................... 26
  Engine ....................................... 29
  Coupled System ................................ 32
STABILITY INVESTIGATION ......................... 39
  Method ....................................... 40
  Baseline ..................................... 41
  Sensitivity ................................... 43
  Interaction ................................... 57
CONCLUSIONS ....................................... 62
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McDonnell Douglas Astronautics Company

SUMMARY

Pogo and control instabilities have been of major concern in the design of aerospace vehicles; however, the symmetry of past vehicles and the associated absence of axial-lateral coupling have precluded the interaction of the pogo and control systems.

Currently proposed Space Shuttle Systems consist of an orbiter riding in parallel with various combinations of propellant tanks and boosters. These asymmetric configurations result in coupled axial and pitch modes of vibration. The pogo system, sensitive to axial motion, and the control system, sensitive to pitch motion, will interact because of the coupled modes. To investigate this phenomenon, detailed models of a representative Space Shuttle structure and the associated feedline, control, and engine systems were acquired or developed. A model generated by coupling the subsystems in an appropriate manner was evaluated for stability by calculation of the complex eigenvalues. Eigenvalues of the coupled system were compared with those of systems with the engine or control system inactive. It was determined that even though a vehicle is stable with respect to both the pogo and control systems individually, the coupled system may have a strong instability. To allow the traditional independent analysis and design of the pogo and control systems, a method was developed for determining the stability of the coupled system in terms of the stability of the pogo and control systems separately. Although the method becomes imprecise for the case of very close roots, a simple extension provides upper and lower bounds for the stability.
INTRODUCTION

Oscillations due to coupled propulsion-structure instabilities, pogo, and control-structure instabilities have been experienced on numerous aerospace vehicles. The source of the pogo instability may be envisioned as follows. Propellant oscillations induced by axial structural oscillations produce variations in engine inlet pressure and propellant flow. These result in thrust perturbations which reinforce the structural motion. Similarly, control signal oscillations induced by lateral structural oscillations produce control forces which reinforce the structural motion. The axial and lateral modes are almost entirely uncoupled for traditional axisymmetric vehicles. Methods have been developed to evaluate pogo stability with respect to axial modes and control stability with respect to lateral modes. Requirements will exist to ensure the stability of the Space Shuttle in this conventional sense.

The Space Shuttle configuration, however, introduces the possibility of a new type of instability produced by pogo-control interaction. Currently proposed Space Shuttle Systems, such as illustrated in fig. 1, exhibit a high degree of asymmetry, which results from coupling an orbiter with various combinations of boosters and propellant tanks. As has been confirmed by preliminary analysis, the structural asymmetry results in strongly coupled axial and lateral modes. Structural oscillations, therefore, induce responses in both the propulsion and control systems. Additionally, the long lateral feedlines characteristic of many Space Shuttle designs allow propellant oscillations to be induced by lateral structural motion. The combined regenerative forces may be of substantially greater magnitude and different phase, introducing the possibility of instabilities that would not be predicted by conventional methods.

Although the study conclusions are relevant to any Space Shuttle configuration exhibiting structural asymmetry with attendant axial-lateral coupling or lateral feedline runs, the McDonnell Douglas Corporation (MDC) "recommended Phase B fully reusable baseline configuration" was selected for detailed analysis. This representative configuration was selected so the existing structural idealization, control and engine system models, and feedline design could be utilized. Analysis was limited to the pitch plane since symmetry precludes coupling with the remaining degrees of freedom, and to the LOX line because the fuel feedline runs are relatively short and the engine is insensitive to fuel side perturbations. The coupled system model was developed to accurately reflect the system behavior from 0.07 to 7 Hz. This includes the dominant frequency ranges of the control system (0.2 to 3 Hz), engine system (0.1 Hz and up), and the fundamental feedline and structural modes (0.7 to 7 Hz).

The purpose of this study was to determine the nature of the pogo-control system interaction and to develop evaluative stability analysis techniques. The stability of the coupled system was evaluated with closed-loop analysis procedures. These methods were found to be substantially more effective than open-loop or time-domain procedures. After the stability of a baseline configuration was evaluated at selected burn times, the period of maximum control participation, 25 percent burn, was selected for evaluating sensitivity to parametric variations of the structural, feedline, control, and engine systems.
# SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>control conditioned pitch rate</td>
</tr>
<tr>
<td>a</td>
<td>real part of system eigenvalue</td>
</tr>
<tr>
<td>(A_E)</td>
<td>area of feedline at engine</td>
</tr>
<tr>
<td>(A_F)</td>
<td>area of feedline segment</td>
</tr>
<tr>
<td>(A_{FT})</td>
<td>area of feedline at tank</td>
</tr>
<tr>
<td>(A_G)</td>
<td>area of tank at liquid surface</td>
</tr>
<tr>
<td>(A_R)</td>
<td>effective area ratio defined in equation (4)</td>
</tr>
<tr>
<td>B</td>
<td>control conditioned attitude error</td>
</tr>
<tr>
<td>b</td>
<td>imaginary part of system eigenvalue</td>
</tr>
<tr>
<td>C</td>
<td>control command signal</td>
</tr>
<tr>
<td>(C_{L\alpha})</td>
<td>aerodynamic lift coefficient gradient</td>
</tr>
<tr>
<td>D</td>
<td>coupled system damping matrix</td>
</tr>
<tr>
<td>d</td>
<td>feedline segment diameter</td>
</tr>
<tr>
<td>(D_C)</td>
<td>control system damping matrix</td>
</tr>
<tr>
<td>(D_{EE})</td>
<td>engine system damping matrix</td>
</tr>
<tr>
<td>(D_S)</td>
<td>structure-feedline damping matrix</td>
</tr>
<tr>
<td>(D_{SS})</td>
<td>augmented structure damping matrix partition</td>
</tr>
<tr>
<td>E</td>
<td>feedline wall Young's modulus</td>
</tr>
<tr>
<td>(F_A)</td>
<td>actuator force</td>
</tr>
<tr>
<td>(F_C)</td>
<td>control force</td>
</tr>
<tr>
<td>(F_L)</td>
<td>aerodynamic lift force</td>
</tr>
</tbody>
</table>
\( F_{XG} \) axial force at gimbal
\( F_{ZG} \) lateral force at gimbal
\( F_{\theta G} \) moment at gimbal
\( g \) acceleration of gravity
\( H_F \) feedline modal force vector
\( h_L \) height of liquid surface above center of mass
\( H_S \) structure modal force vector
\( I \) identity matrix
\( i \) \( \sqrt{-1} \)
\( I_E \) engine moment of inertia x 12
\( I_C \) interaction coefficient
\( K \) coupled system stiffness matrix
\( K_A \) aerodynamic spring defined in equation (2)
\( K_C \) control system stiffness matrix
\( K_{EE} \) engine system stiffness matrix
\( K_{\alpha\beta} \) \( \alpha, \beta = R \) (for reaction), \( C \) (for cavitation), \( RC \) (for both reaction and cavitation), and \( F \) (for free); feedline stiffness matrix partitions defined by equations (10) and (11)
\( K_S \) structure-feedline stiffness matrix
\( K_{SS} \) augmented structure stiffness matrix partition
\( K_U \) ullage spring rate
\( K_\theta \) control attitude gain
\( K_{\dot{\theta}} \) control attitude rate gain
\( L \) feedline segment length
\( \ell \) center of pressure to center of mass distance
\( M \) coupled system mass matrix
\( M_A \) actuator moment
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$M_C$</td>
<td>control system mass matrix</td>
</tr>
<tr>
<td>$M_E$</td>
<td>engine mass x 12</td>
</tr>
<tr>
<td>$M_{EE}$</td>
<td>engine system mass matrix</td>
</tr>
<tr>
<td>$M_{FF}$</td>
<td>feedline mass matrix partition</td>
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<tr>
<td>$M_L$</td>
<td>tank fluid mass</td>
</tr>
<tr>
<td>$M_{L\alpha\beta}$</td>
<td>$\alpha, \beta = S$ (for structure), $F$ (for fluid); structure-tank mass matrix partitions defined by equation (6)</td>
</tr>
<tr>
<td>$M_R$</td>
<td>mass flow from tank</td>
</tr>
<tr>
<td>$M_S$</td>
<td>structure-feedline mass matrix</td>
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<tr>
<td>$M_{SL}$</td>
<td>slosh mass</td>
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<tr>
<td>$M_{\alpha\beta}$</td>
<td>$\alpha, \beta = S$ (for structure), $T$ (for tank fluid); augmented structure mass matrix partitions defined by equations (7) and (8)</td>
</tr>
<tr>
<td>$M_\infty$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$P_G$</td>
<td>ullage gas pressure</td>
</tr>
<tr>
<td>$P_S$</td>
<td>suction pressure at engine inlet</td>
</tr>
<tr>
<td>$P_T$</td>
<td>pressure at tank sump</td>
</tr>
<tr>
<td>$q$</td>
<td>dynamic pressure</td>
</tr>
<tr>
<td>$Q_C$</td>
<td>engine fluid inertial force</td>
</tr>
<tr>
<td>$q_C$</td>
<td>engine fluid inertial displacement</td>
</tr>
<tr>
<td>$Q_F$</td>
<td>feedline force vector</td>
</tr>
<tr>
<td>$q_F$</td>
<td>feedline displacement vector</td>
</tr>
<tr>
<td>$Q_{FT}$</td>
<td>feedline force at tank</td>
</tr>
<tr>
<td>$q_{FT}$</td>
<td>feedline displacement at tank</td>
</tr>
<tr>
<td>$q_L$</td>
<td>total displacement tank fluid center of mass</td>
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<td>$Q_R$</td>
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<tr>
<td>$q_R$</td>
<td>feedline reaction displacement vector</td>
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<tr>
<td>$Q_{RC}$</td>
<td>vector whose partitions are $Q_R$ and $Q_C$ defined by equation (11)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$q_{RC}$</td>
<td>vector whose partitions are $q_R$ and $q_C$ defined by equation (11)</td>
</tr>
<tr>
<td>$q_{RE}$</td>
<td>vector whose partitions are $q_R$ and $X_E$ defined by equation (12)</td>
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<td>$Q_S$</td>
<td>augmented structure force vector</td>
</tr>
<tr>
<td>$q_S$</td>
<td>augmented structure displacement vector</td>
</tr>
<tr>
<td>$q_{SL}$</td>
<td>slosh generalized displacement</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>$S_{REF}$</td>
<td>aerodynamic reference area</td>
</tr>
<tr>
<td>$T$</td>
<td>engine thrust x 12</td>
</tr>
<tr>
<td>$t$</td>
<td>feedline wall thickness</td>
</tr>
<tr>
<td>$V_G$</td>
<td>tank ullage gas volume</td>
</tr>
<tr>
<td>$V_L$</td>
<td>tank fluid volume</td>
</tr>
<tr>
<td>$W$</td>
<td>weight flow through engine</td>
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<tr>
<td>$X$</td>
<td>global state vector</td>
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<tr>
<td>$X_E$</td>
<td>engine axial displacement</td>
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<td>$X_L$</td>
<td>displacement structural tank mode</td>
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<tr>
<td>$X_R$</td>
<td>tank fluid center of mass motion due to flow</td>
</tr>
<tr>
<td>$X_T$</td>
<td>axial displacement of tank bottom</td>
</tr>
<tr>
<td>$Z_E$</td>
<td>lateral displacement at engine center of mass</td>
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<tr>
<td>$Z_G$</td>
<td>lateral displacement at gimbal</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>aerodynamic angle of attack</td>
</tr>
<tr>
<td>$\beta$</td>
<td>LOX bulk modulus</td>
</tr>
<tr>
<td>$\chi$</td>
<td>control gain parameter</td>
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<tr>
<td>$\delta$</td>
<td>engine gimbal angle</td>
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<tr>
<td>$\epsilon$</td>
<td>control frequency parameter</td>
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<tr>
<td>$\eta_F$</td>
<td>feedline modal amplitude vector</td>
</tr>
<tr>
<td>$\eta_S$</td>
<td>structure modal amplitude vector</td>
</tr>
</tbody>
</table>
\[ \gamma_G \quad \text{ullage gas specific heat ratio} \]
\[ \lambda \quad \text{gimbal to engine center of mass distance} \]
\[ \mu \quad \text{metric scale factor} \]
\[ \omega \quad \text{system natural frequency} \]
\[ \omega_A \quad \text{aerodynamic frequency} \]
\[ \omega_F \quad \text{feedline frequency diagonal matrix} \]
\[ \omega_S \quad \text{structural frequency diagonal matrix} \]
\[ \omega_{SL} \quad \text{slosh frequency} \]
\[ \phi_F \quad \text{feedline mode shape matrix} \]
\[ \phi_{T \text{SARU}} \quad \text{vector of structure rotational mode shapes at attitude reference unit} \]
\[ \phi_{SR} \quad \text{structure mode shape matrix at reaction coordinates} \]
\[ \phi_{SRC} \quad \text{structure mode shape matrix at reaction coordinates and engine} \]
\[ \phi_{T \text{SRG}} \quad \text{vector of structure rotational mode shapes at rate gyro} \]
\[ \phi_{T \text{SXG}} \quad \text{vector of structure axial mode shapes at gimbal} \]
\[ \phi_{T \text{SZG}} \quad \text{vector of structure lateral mode shapes at gimbal} \]
\[ \phi_{T \text{SGG}} \quad \text{vector of structure rotational mode shapes at gimbal} \]
\[ \phi_{T \text{XL}} \quad \text{vector of structure mode shapes for fluid tank mode} \]
\[ \phi_{T \text{XT}} \quad \text{vector of structure mode shapes at tank bottom} \]
\[ \phi_{Z,2} \quad \text{lateral mode shape in rigid-body plunge} \]
\[ \phi_{T \text{ZT}} \quad \text{vector of structure lateral mode shapes at tank bottom} \]
\[ \phi_{\theta,3} \quad \text{rotational mode shape in rigid-body pitch} \]
\( \rho \)  
LOX mass density

\( \theta_{\text{ARU}} \)  
pitch rotation at attitude reference unit

\( \theta_{E} \)  
engine rotation

\( \theta_{G} \)  
rotation at gimbal

\( \theta_{\text{RG}} \)  
pitch rotation at rate gyro

\( \zeta \)  
system fraction of critical damping

\( \zeta_{A} \)  
aerodynamic fraction of critical damping

\( \zeta_{C} \)  
system fraction of critical damping, control only

\( \zeta_{F} \)  
feedline fraction of critical damping diagonal matrix

\( \zeta_{P} \)  
system fraction of critical damping, pogo only

\( \zeta_{PC} \)  
system fraction of critical damping, pogo and control

\( \zeta_{S} \)  
structure fraction of critical damping diagonal matrix

\( \zeta_{SL} \)  
slosh fraction of critical damping

\( \zeta_{O} \)  
system fraction of critical damping, neither pogo nor control active

**ANALYTICAL MODELS**

To investigate the interaction of the pogo and control systems, the representative Phase B fully reusable baseline configuration was modeled in detail. The total system model was a composite of structural, fluid, control, and engine system analytical models.

The available structural model (representing a somewhat shorter version of the Shuttle) was scaled to the baseline configuration length, aerodynamic and slosh effects were incorporated, and fluid motion into the tank was added to facilitate coupling to the top of the feedline. The feedline model was developed from the feedline design and includes structural coordinates at bends for coupling to the structure, and weight flow for coupling to the engine model. Springs were incorporated to reflect pump cavitation and the design accumulator. The Phase B baseline control system was incorporated, including inertial effects of the gimbaled engine. A preliminary analytical engine model was obtained from an engine manufacturer. These four analytical models
were incorporated in a composite model by transforming each to a common set of variables and applying the appropriate compatibility and equilibrium relationships. The common set of variables includes pressures and flow as well as displacements and is therefore called a state vector.

**Structure**

The structural model was generated by making three enhancements to the available Phase B baseline idealization. Certain aerodynamic effects were included because of their potential effect on the control system. A slosh mode was added, also because of potential effect on the control system. Relative motion of fluid through the tank sump was introduced to facilitate later coupling with the feedlines.

The structural idealization used in this study was developed during the Phase B Space Shuttle study by McDonnell Douglas Astronautics Company-East for booster Configuration 14 and high cross-range Orbiter 0050. The lumped mass and beam element idealization used is illustrated in fig. 2. Three degrees of freedom, axial, plunge, and pitch rotation were allowed at each mass. The fluid mass in each tank was elastically connected to the tank aft bulkhead so that its frequency corresponded to the first tank mode frequency predicted by a simplified hydroelastic model. The Phase B fully reusable baseline configuration, booster Model 256-20B, is illustrated in fig. 3. The existing data were extrapolated to the baseline design by scaling the Model 14 length approximately 15 percent to make it conform with the Model 20B length. Natural frequencies and modal deflections remained unaltered. The natural frequencies for the model are presented in table 1. Modal damping of one percent was assumed for all modes, and all modes were mass normalized. Rigid-body axial, plunge, and pitch rotation modes were generated based on the mass properties generated during Phase B studies.

The aerodynamic force at the center of pressure was determined from the relationship:

\[ F_L = S_{REF} \cdot q \cdot C_{L\alpha}(M_\infty) \cdot \alpha \]  \hspace{1cm} (1)

where \( F_L \) is the aerodynamic lift force, \( S_{REF} \) is the reference area, \( q \) is the dynamic pressure, \( C_{L\alpha} \) is the lift coefficient gradient, \( M_\infty \) is the Mach number, and \( \alpha \) is the pitch angle of attack. Numerical values for these parameters, and the location of the center of pressure were generated during Phase B studies. Aerodynamic damping forces associated with pitch rate ranged from 0.03 to 11.0 percent of critical damping. The fraction of critical damping was conservatively set at 0.01 percent, \( \zeta_A = 0.0001 \). No other aerodynamic forces were considered relevant to the study. The aerodynamic forces of equation (1) are incorporated in the mathematical model as additions to the rigid-body partition (upper left) of the damping and stiffness matrices;
<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Frequency (Hz)</th>
<th>L/O</th>
<th>25% Burn</th>
<th>50% Burn</th>
<th>75% Burn</th>
<th>MECO</th>
</tr>
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<tbody>
<tr>
<td>S4</td>
<td>0.9083</td>
<td>1.0453</td>
<td>1.0453</td>
<td>1.0453</td>
<td>1.0453</td>
<td>1.0453</td>
</tr>
<tr>
<td>S5</td>
<td>1.314</td>
<td>1.663</td>
<td>1.663</td>
<td>1.663</td>
<td>1.663</td>
<td>1.663</td>
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<tr>
<td>S6</td>
<td>2.1490</td>
<td>2.5427</td>
<td>2.5427</td>
<td>2.5427</td>
<td>2.5427</td>
<td>2.5427</td>
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<tr>
<td>S7</td>
<td>2.693</td>
<td>3.171</td>
<td>3.171</td>
<td>3.171</td>
<td>3.171</td>
<td>3.171</td>
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<td>S9</td>
<td>3.8008</td>
<td>4.315</td>
<td>4.315</td>
<td>4.315</td>
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<tr>
<td>S11</td>
<td>4.5945</td>
<td>5.195</td>
<td>5.195</td>
<td>5.195</td>
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<td>S13</td>
<td>7.2343</td>
<td>7.718</td>
<td>7.718</td>
<td>7.718</td>
<td>7.718</td>
<td>7.718</td>
</tr>
</tbody>
</table>

*Rigid-body axial, plunge, and pitch rotation structural modes (S1, S2, S3) omitted*
and

\[
\begin{bmatrix}
\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 2 \xi \omega_A \\
0 & 0 & 0
\end{array}
\end{bmatrix}
\begin{bmatrix}
\eta_{S,1} \\
\eta_{S,2} \\
\eta_{S,3}
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & K_A \\
0 & 0 & 0
\end{array}
\end{bmatrix}
\begin{bmatrix}
\eta_{S,1} \\
\eta_{S,2} \\
\eta_{S,3}
\end{bmatrix}
\]

\[
\omega_A^2 = \phi_{\theta,3}^2 \frac{F_L}{\alpha} \ell
\]

and

\[
K_A = -\phi_{\theta,3} \phi_{Z,2} \frac{F_L}{\alpha}
\]

and where \( \eta_{S,1}, \eta_{S,2}, \eta_{S,3} \) are the rigid-body axial, plunge, and pitch modal amplitudes; \( \phi_{Z,2}, \phi_{\theta,3} \) are the plunge and pitch deflections of the rigid-body plunge and pitch mode shapes, and \( \ell \) is the magnitude of the distance from center of mass to the center of pressure.
An attempt was made to incorporate slosh mass because of potential control system coupling. Only the booster LOX slosh mass was incorporated since it is much larger than the LH$_2$ or orbiter slosh masses. It can be shown that the addition of a relative slosh degree of freedom, $q_{SL}$, can be accomplished by augmenting the structural equations as follows:

\[
\begin{bmatrix}
0 & \sqrt{M_{SL}} \phi_{ZT}^T \\
-\frac{1}{\sqrt{M_{SL}} \phi_{ZT}} & 0 \\
\end{bmatrix}
\begin{bmatrix}
\eta_S \\
q_{SL} \\
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & 2\zeta_{SL} \omega_{SL} \\
\end{bmatrix}
\begin{bmatrix}
\eta_S \\
q_{SL} \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\end{bmatrix}
\]  

where $M_{SL}$ is the slosh mass, $\zeta_{SL}$ is the slosh fraction of critical damping, $\omega_{SL}$ is the slosh frequency, $\eta_S$ is the structure modal amplitudes, and $\phi_{ZT}$ is the lateral structural mode shapes at the attach point. Slosh parameters were generated during Phase B studies. The slosh mass was attached at the bottom of the LOX tank since mode shapes at that point were already being used for feedline coupling. This choice of attach point led to a minor control instability. For reasons discussed in the baseline stability investigation section, slosh was omitted in subsequent runs.

The structural modes as provided assumed no fluid motion out of the LOX tank; it was added since coupling the feedlines required that degree of freedom. The displacement of the tank mode degree of freedom, $X_L$, is approximately the same as the motion of the fluid center of mass (c.m.) with no flow. The total motion of the c.m., $q_L$, is the sum of the tank mode motion and an additional motion of the fluid c.m. due to a small flow out of the tank, $X_R$. For small relative flow, these relationships have the form:

\[q_L = X_L + X_R\]
where

\[
X_R = \frac{M_R}{M_L} h_L = \frac{2 A_F T \rho h_L (q_{FT} - X_T)}{\rho V_L} = A_R (q_{FT} - X_T)
\]

so

\[
q_L = X_L + A_R (q_{FT} - X_T)
\]

or

\[
q_L = (\phi_{XL} - A_R \phi_{XT}) \eta_S + A_R q_{FT} \tag{4}
\]

where \(M_R\) is the relative mass flow, \(M_L\) is the LOX tank fluid mass, \(h_L\) is the distance from the c.m. to liquid surface, \(2 A_F T\) is the area of the two feedlines, \(\rho\) is the density, \(V_L\) is the LOX tank volume, \(q_{FT}\) is the fluid displacement at the top of the feedline, \(X_T\) is the axial displacement at the tank bottom, \(A_R\) is an effective area ratio defined by the equation, and \(\phi_{XL}\) and \(\phi_{XT}\) are the axial structural mode shapes of the tank mode, and the tank bottom degrees of freedom. The tank fluid mass is removed from the structural coordinate and added to the total motion coordinate, and the spring due to ullage gas, \(K_U\), is added. The structural equations are augmented as follows:

\[
\begin{bmatrix}
-M_L & O \\ 0 & -M_L
\end{bmatrix}
\begin{bmatrix}
\dot{X}_L \\ \dot{q}_L
\end{bmatrix}
+
\begin{bmatrix}
-K_U & -K_U \\ -K_U & K_U
\end{bmatrix}
\begin{bmatrix}
X_L \\ q_L
\end{bmatrix}
\]

where

\[
K_U = \frac{Y_G \rho_G A_G^2}{V_G}
\]
Expressing \( q_L \) in terms of \( X_L \), \( q_{FT} \) and \( X_T \) yields:

\[
\begin{bmatrix}
M_{LR}^2 A_R & -M_{LR}^2 A_R & -M_{LR}^2 A_R \\
-M_{LR}^2 A_R & 0 & M_{LR}^2 A_R \\
-M_{LR}^2 A_R & M_{LR}^2 A_R & M_{LR}^2 A_R
\end{bmatrix}
\begin{bmatrix}
X_T \\
X_L \\
q_{FT}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
Q_{FT}
\end{bmatrix}
\tag{5}
\]

where \( Q_{FT} \) is the associated force, \( \gamma_G \) is the gas specific heat ratio, \( P_G \) the ullage gas pressure, \( A_G \) the tank area at the liquid surface, and \( V_G \) the ullage gas volume. Comparison of the stiffness additions, \( K_{UR} A_R \), with the accumulator and other system stiffnesses showed them to be two orders of magnitude smaller. Since the springs are effectively parallel, the ullage spring can be neglected. Transforming the mass additions to structural modal coordinates yields:

\[
\begin{bmatrix}
M_{LR}^2 A_R^2 & \phi_{XT}^T & \phi_{XT} \\
-M_{LR}^2 A_R^2 & \phi_{XL}^T & \phi_{XL} \\
-M_{LR}^2 A_R^2 & \phi_{XT}^T & \phi_{XT}
\end{bmatrix}
\begin{bmatrix}
\eta_S \\
q_{FT}
\end{bmatrix}
= \begin{bmatrix}
0 \\
Q_{FT}
\end{bmatrix}
\]
The equations of motion are augmented as indicated in equation (2) for aerodynamics, equation (3) for slosh, and equation (6) for relative flow. After incorporating these enhancements to the structural model, the equations of motion have the form: (double dashes indicate the compression of \( \eta_S \) in the first term)

\[
\begin{bmatrix}
I + M_{LSS} & M_{LSF}^T & M_{LSF} \\
\sqrt{M_{SL} \phi ZT} & 1 & 0 \\
M_{LFS} & 0 & M_{LFF}
\end{bmatrix}
\begin{bmatrix}
\eta_S \\
q_{SL} \\
q_{FT}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
Q_{FT}
\end{bmatrix} \tag{6}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\tau_A \omega_A & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\tau_A \omega_S & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\tau_{SL} \omega_{SL} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_A & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega_A^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_S^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega_{SL}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \omega_{SL}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\eta_S,1 \\
\eta_S,2 \\
\eta_S,3 \\
\eta_S',1 \\
\eta_S',2 \\
\eta_S',3 \\
q_{SL} \\
q_{FT}
\end{bmatrix}
= 
\begin{bmatrix}
H_S,1 \\
H_S,2 \\
H_S,3 \\
H_S',1 \\
H_S',2 \\
H_S',3 \\
q_{SL} \\
Q_{FT}
\end{bmatrix} \tag{7}
\]
where the H's are the structural modal forces, I is an identity matrix, and \( \eta_S, \omega_S, b_S \) are the flexible modal amplitudes, frequencies, and dampings. Defining \( q_S \) as the vector \( \eta_S, q_{SL} \), the equation (7) may be summarized as:

\[
\begin{bmatrix}
M_{SS} & M_{ST} \\
M_{TS} & M_{TT}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_S \\
\ddot{q}_{FT}
\end{bmatrix}
+ \begin{bmatrix}
D_{SS} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
q_S \\
q_{FT}
\end{bmatrix}
+ \begin{bmatrix}
K_{SS} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
q_S \\
q_{FT}
\end{bmatrix} = \begin{bmatrix}
Q_S \\
Q_{FT}
\end{bmatrix}
\] (8)

Feedline

Feedline dynamics are of primary importance to the pogo system. The feedline design (fig. 4) was idealized as illustrated in fig. 5. Because of potential interaction of the lateral runs with the structure, primary consideration was given to maintaining the effective lateral run mass and the total system mass. Only one of the two LOX lines was modeled due to symmetry.

A detailed fluid dynamic model was developed of the feedline segments. The feedline model reflects the inertial and compliant characteristics of the fluid and the radial compliance of the line. The model does not include the inertial or compliant properties of the line itself, which are negligible compared with the structure to which it is attached. A compatibility relationship is used to couple the segments. The equations are transformed to relative modal coordinates. An asymmetric equation form is adopted because it facilitates coupling and provides pressures as explicit state variables. The form of the last coordinate is changed to engine weight flow to facilitate coupling to the engine.

The feedline was modeled as 17 segments, each of which was modeled as a discrete mass and spring system. The fluid mass in each segment was divided equally among uniformly spaced discrete masses; the discrete spring rate, which includes the line circumferential elasticity was: \[ (N - 1) E t \beta A_F / L (d \beta + E t) \] where \( N \) is the number of discrete masses, \( L \) is the segment length, \( \beta \) is the liquid bulk modulus, and \( E, t, d, A_F \) are the pipe's Young's modulus, thickness, diameter, and area. Modeling data for the feedline segments identified in fig. 5 are presented in table 2.

Every segment has a mass at each end. To couple the feedline segments, a compatibility relationship is written at each joint stating the flow into the joint equals the flow out. The last mass of the upstream segment is then eliminated using the compatibility relationship. As an example of a joint with more than two segments, the relationship for C-D-P-Q is:

\[
A_{FC} (q_{FC}, C58 - q_{R3}) = A_{FD} (q_{FD}, D1 - q_{R3}) \\
+ A_{FP} (q_{FP}, P1 - q_{R3}) + A_{FQ} (q_{FQ}, Q1 - q_{R3})
\]
Figure 5. Space Shuttle Feedline Idealization
### TABLE 2. SPACE SHUTTLE LOX FEEDLINE MODELING DATA

<table>
<thead>
<tr>
<th>Segment</th>
<th>Discrete Masses</th>
<th>Length m (in)</th>
<th>Pipe Wall Thickness mm (in)</th>
<th>Pipe Diameter m (in)</th>
<th>Pipe Area m² (in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>1.1684 (46.)</td>
<td>6.3500 (0.250)</td>
<td>1.0668 (42.)</td>
<td>0.893837 (1385.45)</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>5.3340 (210.)</td>
<td>1.6510 (0.065)</td>
<td>0.5588 (22.)</td>
<td>0.245245 (380.13)</td>
</tr>
<tr>
<td>C</td>
<td>58</td>
<td>38.1762 (1503.)</td>
<td>1.6510 (0.065)</td>
<td>0.5588 (22.)</td>
<td>0.245245 (380.13)</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>2.5400 (100.)</td>
<td>1.7780 (0.070)</td>
<td>0.4572 (18.)</td>
<td>0.164174 (254.47)</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>1.7780 (70.)</td>
<td>1.7780 (0.070)</td>
<td>0.4572 (18.)</td>
<td>0.164174 (254.47)</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>2.5400 (100.)</td>
<td>1.0922 (0.043)</td>
<td>0.3302 (13.)</td>
<td>0.085632 (132.73)</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>4.3180 (170.)</td>
<td>0.8128 (0.032)</td>
<td>0.2032 (8.)</td>
<td>0.032432 (50.27)</td>
</tr>
<tr>
<td>H, J, L, N</td>
<td>3</td>
<td>1.0160 (40.)</td>
<td>1.0160 (0.040)</td>
<td>0.3048 (12.)</td>
<td>0.072968 (113.10)</td>
</tr>
<tr>
<td>I, K, M, O</td>
<td>6</td>
<td>3.5560 (140.)</td>
<td>1.0160 (0.040)</td>
<td>0.3048 (12.)</td>
<td>0.072968 (113.10)</td>
</tr>
<tr>
<td>P, Q</td>
<td>14</td>
<td>6.7310 (265.)</td>
<td>1.0160 (0.040)</td>
<td>0.3048 (12.)</td>
<td>0.072968 (113.10)</td>
</tr>
</tbody>
</table>

LOX Density 1137.3 kg/m³ (71.00 lb/ft³)

LOX Bulk Modulus 8.62 × 10⁷ N/m² (1.25 × 10⁵ lb/in²)

Pipe Young's Modulus 1.79 × 10¹⁰ N/m² (2.6 × 10⁷ lb/in²)
where \( A_{F,i} \) is the area of segment \( i \), \( q_{F,i,j} \) is the displacement of the \( j \)th discrete mass in segment \( i \), and \( q_{R,i} \) are the displacements of the feedline pipe due to structural motion at reaction coordinate \( i \). There are five reaction coordinates: axial and lateral at the tank bottom, axial at the intersection of segments C, D, P, and Q, and axial and lateral at the lateral manifold, E-F-G. Deflections due to structural rotations were considered small in comparison to lineal deflections and were ignored. The feedline motions at the six engines were assumed to behave identically. This follows from symmetry for pairs I-K, M-O, and P-Q, and is consistent with the small rotation assumption for the remaining groupings, I-K versus M-O, M-O versus P-Q, and I-K versus P-Q. That is, the torque due to thrust differential was neglected.

To verify the assumption of no thrust differential torque, the feedline mode involving fluid motion between I-K and P-Q was isolated and coupled to the closest structural mode. The structural frequency was shifted to maximize the coupling. Both analytical and numerical solutions revealed a highly stable system, verifying the assumption. If the accumulator compliance is eliminated, the feedline mode is raised beyond the frequency range of interest. With these assumptions, the bottom masses of segments I, K, M, O, P, and Q are coupled through six identical springs representing accumulator and cavitation compliance to a single coordinate, \( q_C \), representing the average inertial displacement of fluid into the engines.

The mass and stiffness matrix for the feedline segments were assembled as five reaction coordinates \( q_{R1} \) through \( q_{R5} \), the engine fluid displacement \( q_C \), and 154 discrete coordinates, \( q_{F,Al} \) through \( q_{F,Q14} \). A similarity transformation with a matrix based on the eleven joint compatibility equations was used to couple the segments and reduce the system to \( q_{R1} \) through \( q_{R5} \), \( q_C \), and the 143 independent discrete feedline coordinates. The accumulator-cavitation springs were added to the stiffness matrix at this point. Terms in the first six rows and columns of the mass matrix were neglected. This is
equivalent to modifying the segment model such that the last mass in a segment is zero, and augmenting the first mass in the next segment. After these operations, the feedline equations had the form:

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & M_{FF}
\end{bmatrix}
\begin{bmatrix}
q_R \\
q_C \\
q_F
\end{bmatrix}
+ \begin{bmatrix}
K_{RR} & K_{RC} & K_{RF} \\
K_{CR} & K_{CC} & K_{CF} \\
K_{FR} & K_{FC} & K_{FF}
\end{bmatrix}
\begin{bmatrix}
q_R \\
q_C \\
q_F
\end{bmatrix}
= \begin{bmatrix}
Q_R \\
Q_C \\
Q_F
\end{bmatrix}
\tag{10}
\]

The normal modes of the cantilevered system, \( q_{R1} = \ldots q_{R5} = q_C = 0 \), were calculated. Equation (10) was transformed to relative modal coordinates, and the modal equations were premultiplied by the feedline mode shapes transposed, \( \phi_F^T \) [see equation (11)].

Letting

\[
q_F = \phi_F \eta_F - K_{FF}^{-1} K_{FRC} q_{RC}
\]

where

\[
\begin{bmatrix}
K_{FRC}
\end{bmatrix} = \begin{bmatrix}
K_{FR} & K_{FC}
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
q_{RC}
\end{bmatrix} = \begin{bmatrix}
q_R \\
q_C
\end{bmatrix}
\]

yields

\[
\begin{bmatrix}
-\phi_F^T M_{FF} K_{FF}^{-1} K_{FRC} & 0 & 0 \\
-\phi_F^T & 0 & 0
\end{bmatrix}
\begin{bmatrix}
q_{RC} \\
\eta_F
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 2 \zeta_F \omega_F & 0
\end{bmatrix}
\begin{bmatrix}
q_{RC} \\
\eta_F
\end{bmatrix}
= \begin{bmatrix}
Q_{RC} \\
H_F
\end{bmatrix}
\tag{11}
\]
where \( \omega_F, \eta_F, H_F \) are the feedline natural frequencies, modal amplitudes, and generalized forces and where modal damping, \( \zeta_F = 0.01 \), was introduced in the equations. It may be noted that \( K_{RCRC} - K_{RCF} K_{FF}^{-1} K_{FRC} \), the reduced reaction stiffness, is zero since the fluid is free to move in response to a reaction displacement. This unsymmetric formulation retains the reaction forces explicitly.

The weight flow through the engine, \( W \), is determined from the inertial displacement as \( \rho g A_E (q^*_C - x^*_E) \) where \( \rho \) is the density, \( g \) is the acceleration of gravity, \( A_E \) is the feedline area at the engine, and \( x^*_E \) is the engine axial displacement. With this substitution the coordinate transformation and the feedline equations of motion of equation (11) assume the form:

\[
\begin{bmatrix}
q_F \\
\phi_F
\end{bmatrix} = 
\begin{bmatrix}
-K_{FF}^{-1} K_{FRC} & -\frac{1}{\rho g A_E} & K_{FF}^{-1} K_{FC}
\end{bmatrix}
\begin{bmatrix}
q_{RE} \\
\eta_F \\
W
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
q_{RE} \\
\eta_F \\
W
\end{bmatrix} = 
\begin{bmatrix}
q_R \\
x_E
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\phi_T & 0 & -1 & 0 \\
0 & M_{FF} & K_{FF}^{-1} K_{FRC} & 0 \\
-1 & \rho g A_E & \phi_T & M_{FF} & K_{FF}^{-1} K_{FC} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
q_{RE} \\
\eta_F \\
W
\end{bmatrix} = 
\begin{bmatrix}
K_{RCF} \frac{\phi_F}{\omega^2 F} & 0 \\
0 & -\frac{\phi_F}{\omega^2 F} & 0
\end{bmatrix}
\begin{bmatrix}
q_{RE} \\
\eta_F \\
W
\end{bmatrix} = 
\begin{bmatrix}
Q_{RC} \\
H_F
\end{bmatrix}
\]

(12)
The Space Shuttle pitch plane thrust vector control system was used in this study. A block diagram of this control system is shown in fig. 6. The commanded and actual engine gimbal deflection are assumed identical since actuator resonances are above the frequency range of interest. However inertial effects of the gimbaled engine, sometimes referred to as "tail wags dog", were within the range of interest. The "tail wags dog" equations are developed and incorporated in a matrix format with the control equations from the block diagram.

The mass matrix contributions of the engine were removed from the gimbal plane structural coordinates, \( Z_G \), and \( \theta_G \) and assigned to the center of mass (c.m.) engine coordinates, \( Z_E \) and \( \theta_E \). The additions to the structural equations have the form:

\[
\begin{bmatrix}
-M_E & M_E \lambda & 0 & 0 \\
0 & 0 & M_E & 0 \\
M_E \lambda & - \left( I_E + M_E \lambda^2 \right) & 0 & 0 \\
0 & 0 & 0 & I_E
\end{bmatrix}
\begin{bmatrix}
Z_G^* \\
\theta_G^* \\
Z_E^* \\
\theta_E^*
\end{bmatrix}
= 
\begin{bmatrix}
-F_A + F_C \\
-M_A + F_A \lambda \\
F_A \\
M_A
\end{bmatrix}
\tag{13}
\]

where \( M_E \) and \( I_E \) are the engine mass and inertia, \( F_C \) is the control force, \( F_A \) and \( M_A \) are the actuator force and moment, and \( \lambda \) is the distance from c.m. to gimbal plane. The engine coordinates are related to the commanded gimbal angle, \( \delta \), by: \( \theta_E = \theta_G + \delta \), and \( Z_E = Z_G - \lambda (\theta_G + \delta) \). Equation (13) is transformed to relative coordinates:

\[
\begin{bmatrix}
0 & 0 & -M_E \lambda \\
0 & I_E + M_E \lambda^2 \\
-M_E \lambda & I_E + M_E \lambda^2 & I_E + M_E \lambda^2
\end{bmatrix}
\begin{bmatrix}
Z_G^* \\
\theta_G^* \\
\delta
\end{bmatrix}
= 
\begin{bmatrix}
T \delta \\
0 \\
F \delta
\end{bmatrix}
\tag{14}
\]

or

\[
\begin{bmatrix}
F_{ZG} \\
F_{\theta G}
\end{bmatrix}
= 
\begin{bmatrix}
T \\
0
\end{bmatrix} \delta + 
\begin{bmatrix}
M_E \lambda \\
- \left( I_E + M_E \lambda^2 \right)
\end{bmatrix} \delta
\]
where $F_{ZG}$ and $F_{\theta G}$ are the net lateral force and moment on the gimbal, $T$ is the total thrust, $12 \times 2.45 \times 10^6$ Newtons ($12 \times 550,000$ lb), and where it has been noted that internal actuator forces cancel and that the lateral control force is approximated by the thrust times the angle (in radians) for small gimbal angles. The dynamic equation in $\delta$ is dropped since the actual angle is assumed equal to the commanded angle. The quantities $M_E \alpha$ and $I_E + M_E \lambda^2$ were computed indirectly from the tail wags dog frequencies, which are defined as the frequency at which the modal force is zero.

If the gimbal feedback loop is reduced, the control system may be expressed as a matrix polynomial in the Laplace operator, $s$, and the dummy control variables, $A$, $B$, $C$, identified in fig. 6.

\[ -\delta = \frac{15}{s} \left( C + \frac{3\delta}{1 + 0.016s} \right) \]

gives

\[ (15 + 0.24s)(B - A) + (0.016s^2 + s + 45)\delta = 0 \]

so,

\[
\begin{bmatrix}
1 & 0 & (I_E + M_E \lambda^2)s^2 & 0 & 0 & 0 & 0 \\
0 & 1 & -M_E \lambda s^2 - T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
F_{\theta G} \\
F_{ZG} \\
\delta \\
A \\
B \\
\theta_{RG} \\
\theta_{ARU}
\end{bmatrix} = 0 \quad (15)
\]

where $\theta_{RG}$, $\theta_{ARU}$ are the pitch rotation measured at the rate gyro and the attitude reference unit.
Engine

Engine dynamics are of primary importance to the pogo system. Space Shuttle main engine dynamic characteristics were obtained from North American Rockwell Corporation, Rocketdyne Division. This engine model was of a preliminary nature and neglected cavitation effects, generally important in pump dynamic gains. Measurements on existing engines and other candidate shuttle engine models show substantially greater gains. For these reasons the engine model was a primary candidate for parametric variation. As provided, the transfer function from LOX suction pressure, \( P_S \), to chamber pressure, \( P_C \), and LOX weight flow, \( \dot{W} \), are:

\[
\frac{P_C}{P_S} = \frac{0.25s \left( 1 + \frac{s}{4.6\pi} \right)}{\left( 1 + \frac{s}{\pi} \right)^2},
\]

and

\[
\frac{\dot{W}}{P_S} = \mu \frac{0.075s \left( 1 + \frac{s}{4.6\pi} \right)}{\left( 1 + \frac{s}{\pi} \right)^2}
\]

where

\[
\mu = 6.4516 \times 10^{-4} \frac{m^2}{sec} \left( 1.0 \frac{in.}{sec} \right)
\]

The fuel side transfer functions provided by the engine manufacturer were at least two orders of magnitude smaller and were neglected. Thrust, \( T \), is obtained from chamber pressure by multiplying by the effective area, \( 0.1181m^2 \) (183.0 in. \(^2\)). The transfer function \( P_C/P_S \) is presented as a function of frequency in fig. 7 and fig. 8. It may be noted that a gain of at least 0.618 is required for the thrust oscillation to exceed the pressure times area oscillatory load acting down on the engine. Although the engine never reaches that level, other candidate engines exceed it.

The axial force, \( F_{XG} \), is the sum of the thrust and the product of the suction pressure, \( P_S \), and the line area, \( 12 \times A_E \), so the equation may be written
Figure 7. Space Shuttle Main Engine Gain Versus Frequency.
coupled systems equation

The individual structural, feedline, engine, and control models are combined in one system of equations. To accomplish this, a global state vector was selected, and the feedline and structure are transformed to the state space. The feedline and structure are coupled and the control system is expressed in the global coordinates. Combining the engine, control, and coupled structure-feedline systems gives the final state equations of motion.

The global coordinate system, or state space vector, was formed as follows: structural generalized coordinates, $q_s$, feedline modes, $\eta_F$, LOX tank pressure, $P_T$, engine suction pressure, $P_s$, engine weight flow, $\dot{W}$, total thrust, $T$, gimbal rotation, $\delta$, control variable $A$, control variable $B$. This is a mixed state vector including both displacement and force type elements. The formulation results in some dependent equations which were eliminated automatically by the computer algorithm.

The feedline equations are first transformed to global coordinates. The engine and reaction coordinates are then expressed in terms of the appropriate rows of the composite structural mode shape matrix, $\phi_{SR}$, which include...
rigid, flexible, and slosh modes. The coordinate transformation and the
equations of motion (see equation (12)) assume the form:

if

\[
\{q_{RE}\} = [\phi_{SRC}] \{q_S\}
\]

then

\[
\{q_F\} = \begin{bmatrix} -K_{FF}^{-1} K_{FRC} \phi_{SRC} & \phi_{F}^T & 0 & 0 & \frac{-1}{\rho g_A} K_{FF}^{-1} K_{FC} \\ \phi_{F}^T M_{FF} & K_{FF}^{-1} K_{FRC} \phi_{SRC} & 1 & 0 & \frac{-1}{\rho g_A} \phi_{F}^T M_{FF} K_{FF}^{-1} K_{FC} \end{bmatrix} \begin{bmatrix} q_S \\ \eta_F \\ P_T \\ P_S \\ W \end{bmatrix}
\]

and

\[
\begin{bmatrix} \dot{q}_S \\ \dot{\eta}_F \\ \dot{P}_T \\ \dot{P}_S \\ \dot{W} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_F \\ P_T \\ P_S \\ W \end{bmatrix} + \begin{bmatrix} 0 & K_{RCF} \phi_{F}^T & 0 & 0 & 0 \\ 0 & 0 & \omega_F^2 & 0 & 0 \\ 0 & 0 & 0 & \omega_F & 0 \\ 0 & 0 & 0 & 0 & \omega_F \end{bmatrix} \begin{bmatrix} \eta_F \\ P_T \\ P_S \\ W \end{bmatrix} = \begin{bmatrix} \eta_{RC} \\ \eta_{HF} \end{bmatrix}
\]
The force on the engine fluid displacement, $Q_r$, is the negative of the suction pressure, $-P_S$, times the area, $A_F$. The only source of modal force on the feedline, $H_F$, is the tank pressure, $P_T$, times the area of the feedline at the tank, $A_{FT}$, times the value of the feedline mode shape at the top of the line. Making these substitutions and transforming to modal structural force yields:

Let

$$H_F = \phi_{F}^T A_{FT} P_T$$

then

$$
\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
-\phi_{F}^T M_{FF} K_{FF}^{-1} K_{FC} \phi_{SRC} & 1,0 & 0 & 0 & \cdots \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
0 & 2 \phi_{F} \omega_F & 10 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W
\end{bmatrix}
= \begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W
\end{bmatrix}
$$

(19)

where $\phi_{SR}$ is the first five rows of $\phi_{SRC}$. 

34
Similarly, the structural equations are converted to global coordinates, using the first row of the coordinate transform in equation (18).

\[
\begin{bmatrix}
M_{SS} - M_{ST} K_{FF,1}^{-1} K_{FF,1}^{FRC} & \phi_{SRC} & M_{ST} K_{FF,1}^{-1} & 0 & 0 & -\frac{1}{\rho g A} M_{ST} K_{FF,1}^{-1} K_{FC} \\
\hline
M_{TS} - M_{TT} K_{FF,1}^{-1} K_{FF,1}^{FRC} & \phi_{SRC} & M_{TT} K_{FF,1}^{-1} & 0 & 0 & -\frac{1}{\rho g A} M_{TT} K_{FF,1}^{-1} K_{FC} \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
\eta_F \\
\eta_F \\
PT \\
PS \\
W \\
\end{bmatrix}
\]

\[=\]

\[
\begin{bmatrix}
D_{SS} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\hline
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
\eta_F \\
\eta_F \\
PT \\
PS \\
W \\
\end{bmatrix}
\]

\[+\]

\[
\begin{bmatrix}
K_{SS} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\hline
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
\eta_F \\
\eta_F \\
PT \\
PS \\
W \\
\end{bmatrix}
\]

\[=\]

\[
\begin{bmatrix}
Q_S \\
\eta_F \\
\eta_F \\
\eta_F \\
Q_{FT} \\
\eta_F \\
\eta_F \\
W \\
\end{bmatrix}
\]

The generalized force on the tank fluid, \(Q_{FT}\), is the negative of the pressure, \(-P_T\), times the area of the two feedlines, \(2 \times A_{FT}\). The generalized force on the structure has two sources, the negative of the reaction forces on the two feedlines, \([\text{equation } (19)]\) and the gimbal forces. Combining the feedline equations (19) with the structural equation (20) yields equation (21).

Let

\[
Q_S = -2 \phi_{SR}^T \Omega_R + \phi_{SXG}^T F_{XG} + \phi_{SOG}^T F_{OG} + \phi_{SZG}^T F_{ZG}
\]
Then

\[
\begin{bmatrix}
M_{SS} - M_{ST} K_F^{-1} K_{SRC} 
& M_{ST} F_F 
& 0 \\
M_{TS} - M_{TT} K_F^{-1} K_{SRC} 
& M_{TT} F_F 
& 0 \\
- \rho F_M K_F^{-1} K_{SRC} 
& \rho F_M F_F 
& I 
& 0 \\
0 
& 0 
& 0 
& 0 \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W \\
T \\
F_{XG} \\
F_{\theta G} \\
F_{ZG}
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_{SS} 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& 0 
& 2A_{FT} 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W \\
T \\
F_{XG} \\
F_{\theta G} \\
F_{ZG}
\end{bmatrix}
\]

(21)

\[
\begin{bmatrix}
K_{SS} + 2^T 
& K_{RF} 
& F_F 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& 0 
& 2A_{FT} 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& \omega_F^2 
& F_F A_{FT} 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
0 
& \omega_F^2 
& F_F A_{FT} 
& 0 
& 0 
& 0 
& 0 
& 0 
& 0 \\
\end{bmatrix}
\begin{bmatrix}
q_S \\
\eta_F \\
P_T \\
P_S \\
W \\
T \\
F_{XG} \\
F_{\theta G} \\
F_{ZG}
\end{bmatrix}
\]

where \( \phi_{SXG}, \phi_{\theta G}, \phi_{SZG} \) are the axial, rotational, and lateral structural mode shapes at the gimbal.
Since the coefficients of \( \dot{W} \) and \( W \) are zero, the system can be transformed from a state vector involving \( W \) to one involving \( \dot{W} \) by moving the associated column of the mass matrix to the damping matrix. If this is done and the resulting matrices are denoted \( M_S \), \( D_S \), \( K_S \), the equations may be written:

\[
\begin{bmatrix}
q_S \\
\dot{q}_S \\
\eta_F \\
\dot{\eta}_F \\
P_T \\
\dot{P}_T \\
P_S \\
\dot{P}_S \\
F_{XG} \\
\dot{F}_{XG} \\
F_{\theta G} \\
\dot{F}_{\theta G} \\
F_{ZG} \\
\dot{F}_{ZG}
\end{bmatrix} + \begin{bmatrix}
q_S \\
\dot{q}_S \\
\eta_F \\
\dot{\eta}_F \\
P_T \\
\dot{P}_T \\
P_S \\
\dot{P}_S \\
F_{XG} \\
\dot{F}_{XG} \\
F_{\theta G} \\
\dot{F}_{\theta G} \\
F_{ZG} \\
\dot{F}_{ZG}
\end{bmatrix} + \begin{bmatrix}
q_S \\
\dot{q}_S \\
\eta_F \\
\dot{\eta}_F \\
P_T \\
\dot{P}_T \\
P_S \\
\dot{P}_S \\
F_{XG} \\
\dot{F}_{XG} \\
F_{\theta G} \\
\dot{F}_{\theta G} \\
F_{ZG} \\
\dot{F}_{ZG}
\end{bmatrix} = \{0\} \quad (22)
\]

The control system, equation (15), is expressed in global coordinates by noting that the rotation at the rate gyro and the attitude reference unit can be expressed in terms of the associated rows of the structural mode shape matrix, \( \phi_{SRG} \) and \( \phi_{SARU} \), as

\[
\begin{bmatrix}
\theta_{RG} \\
\dot{\theta}_{ARU}
\end{bmatrix} = \begin{bmatrix}
\phi_{SRG} \\
\phi_{SARU}
\end{bmatrix} \{q_S\}
\]
where $X$ denotes the state vector and $M_C$, $D_C$, $K_C$, the coefficients of the equivalent differential equation, denote the $s^2$, $s$, and constant terms of the matrix polynomial equation.

Combining these control equations (23) with the coupled feedline-structure equations (22) and the engine equations (17) yields:
The equations of motion for the two uncoupled systems, pogo alone and control alone, can be accommodated by dropping the rows and columns associated with $\dot{W}$ and $T$, or with $F_{\theta G}$, $F_{ZG}$, $\delta$, $A$, and $B$, respectively.

\[
\begin{bmatrix}
M_S & 0 & 0 & 0 \\
0 & 0 & 0 & M_{EE} \\
0 & 0 & 0 & 0 \\
M_C & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} + \begin{bmatrix}
D_S & 0 & 0 & 0 \\
0 & 0 & 0 & D_{EE} \\
0 & 0 & 0 & 0 \\
D_C & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} + \begin{bmatrix}
K_S & 0 & 0 & 0 \\
0 & 0 & 0 & K_{EE} \\
0 & 0 & 0 & 0 \\
K_C & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

or

\[
[M] \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} + [D] \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} + [K] \begin{bmatrix}
\dddot{X} \\
\dot{X} \\
X
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\] (24)

The equations of motion for the two uncoupled systems, pogo alone and control alone, can be accommodated by dropping the rows and columns associated with $\dot{W}$ and $T$, or with $F_{\theta G}$, $F_{ZG}$, $\delta$, $A$, and $B$, respectively.

STABILITY INVESTIGATION

The mathematical model developed in the previous section was used to investigate the nature of pogo-control interaction. The model was investigated in four configurations: control system only active, engine system only active, both systems active, and neither system active. This section discusses the method selected for stability investigation and the selection of a
baseline configuration. The sensitivity of the interaction and stability to parametric variations was investigated. The only deviation from a linear relationship between the eigenvalues of the four configurations was observed in the region of very close roots. This interaction was further investigated through multiple parametric variations. The following symbols have been used to identify the curves in the figures: $A$, aero mode (the rigid-body pitch mode), $S_i$, $i$th structural mode (the first three of which are rigid body), and $F_i$, the $i$th feedline mode.

Method

The system stability was evaluated on the basis of closed-loop roots. The type of linear relationship expected is discussed, and an interaction coefficient is defined to evaluate the deviation from linearity, which will be referred to as interaction.

Stability characteristics were evaluated on the basis of the fraction of critical damping associated with the system eigenvalues. The eigenvalues were determined by assuming an exponential solution, $X = X_0 e^{st}$, to the homogeneous system equations. The roots of the characteristic equation, determinant $(s^2 M + sD + K) = 0$, provide the eigenvalues, $s_i$; the non-trivial solutions of $(s_i^2 M + s_i D + K)X_i = 0$ provide the associated eigenvectors, $X_i$. If $a_i$ and $ib_i$ are the real and complex parts of the $i$th eigenvalue, $s_i$, the undamped natural frequency, $\omega_i$, and the fraction of critical damping, $\zeta_i$, are defined by

$$\omega_i = \sqrt{a_i^2 + b_i^2}$$

and

$$\zeta_i = -\frac{a_i}{\omega_i}$$

A negative fraction of critical damping indicates an unstable system; positive values indicate stability.

The eigenvalues of the structural system are dominated by the imaginary part for a lightly damped structure. A small perturbation to the system equations will produce a small variation in the eigenvalues. The change in frequency is small. However, since the real part of the eigenvalue is already small, the change in damping may be relatively large. If two small perturbations are made to the system equations, the variation in the eigenvalues may be approximated by the sum of the two perturbations applied independently. This corresponds to retaining only the linear terms in a power series expansion of the characteristic equation. Since the frequency is nearly constant, the variation in damping is also approximated by the sum of the variations associated with applying the perturbations independently. In particular, if the fractions of critical damping in any particular mode are: $\zeta_O$ for the structural model alone, $\zeta_P$ for the structure with the
engine system, $\zeta_C$ for the structure with control system, and $\zeta_{PC}$ for the combined system, the following relationship should be approximately true:

$$\zeta_{PC} = \zeta_O + (\zeta_P - \zeta_O) + (\zeta_C - \zeta_O)$$

or

$$\zeta_{PC} = \zeta_P + \zeta_C - \zeta_O$$  \hspace{1cm} (26)

To provide a measure of the accuracy of this approximation, an interaction coefficient, $I_C$, is defined for each mode as:

$$I_C \zeta_O = (\zeta_P + \zeta_C - \zeta_O) - \zeta_{PC}$$

$$I_C = \frac{\zeta_P + \zeta_C - \zeta_{PC}}{\zeta_O} - 1$$  \hspace{1cm} (27)

A positive interaction coefficient indicates a less stable combined system than would be expected by the linear combination hypothesis; a negative coefficient, a more stable combined system. For those modes in which the interaction coefficient is small compared to one, the linear combination hypothesis is supported.

Baseline

To improve the computational efficiency, or to make the system more relevant to the interaction problem, a baseline configuration was selected about which subsequent parametric variations would be made. The slosh mode was eliminated, the accumulators were removed, the number of modes retained was reduced, and the 25 percent burn time was selected.

The eigenvalues of the coupled system as developed in the analytical model had a marginally unstable slosh mode with respect to the control system. The source of the instability was traced to locating the slosh degree of freedom too far aft. To evaluate the importance of slosh, the system eigenvalues were recomputed without slosh. A comparison of the eigenvalues with and without slosh showed no significant effect. The 25 percent burn time comparison is presented in fig. 9. The slosh mode was therefore deleted from subsequent runs.
The coupled system now had no instabilities. To evaluate the effect of accumulators, the system eigenvalues were computed with the accumulators removed. The system was still stable as illustrated in fig. 10. It therefore appears that with the preliminary engine model, no accumulators would be required. To enhance the pogo participation for this study, subsequent runs were made with no accumulators.

The natural frequencies and those fractions of critical damping differing significantly from the input values of one percent are presented as a function of burn time in fig. 11 and 12, respectively. The lines trace a system eigenvalue, and the test point symbols identify the predominant input mode in the system eigenvector. The effect of the control system is most pronounced at 25 percent burn, which was therefore selected for the baseline.

The preliminary version of the coupled system included 18 structural (to 15.2 Hz) and 12 feedline (to 44 Hz) modes. To improve computational efficiency, this was reduced to 14 structural (to 9.8 Hz) and two feedline (to 7.4 Hz) modes. The resulting eigenvalues were compared with the more complete set. Both frequency and damping were found to be the same within an error of 2 percent. The reduced set of modes was therefore used in all subsequent calculations.

The baseline system was 25 percent burn time with no slosh and no accumulators. The system eigenvalues are presented in fig. 13 for the coupled system and for the pogo and control systems individually. As expected, the coupled system deviation from the input one percent damping is closely approximated by the sum of the individual system deviations. A numerical calculation confirms this; all interaction coefficients were 0.03 or less. Thus the linear approximation is accurate to within three percent (0.03 percent error in fraction of critical damping) for the baseline system.

Sensitivity

It is recognized that the Space Shuttle configuration and the input parameters are preliminary in nature. Reasonable variations were made to the input parameters to investigate both the stability and the interaction. Parametric variations were made to engine gain and damping, control gain and frequency, feedline and structural damping and gain, and structural and feedline frequencies.

As previously noted, a pump inlet pressure perturbation does not produce a net positive gimbal force for this engine, although it does for other candidate Space Shuttle engines. Typical gains (\(P_C/P_S\)) for other engines are three to five times greater than the preliminary values for this engine. The coupled system damping is plotted as a function of engine gains from one to ten times nominal in fig. 14. The second feedline mode becomes unstable, i.e., pogos, at a little over five times the nominal gain. This indicates accumulators may be required if the engine performs more typically than preliminary models indicate. The coupled system damping for engine dampings
Figure 10. Effect of Feedline Accumulators on System Roots
Figure 11. Coupled System Frequencies Versus Burn Time
Figure 12. Coupled System Stability Versus Burn Time
Figure 14. Coupled System Stability Versus Engine Gain
The control system gain is proportional to $1/\chi$, and the rolloff frequency is proportional to $1/\epsilon$. System damping is presented as a function of control system gain in fig. 16. Reducing the gain substantially reduces the stability of the aero mode, $A$, (rigid-body pitch) while increasing it reduces the stability of the second flexible mode, $S5$. Fig. 17 presents system damping as a function of cutoff frequency. Reducing the cutoff frequency tends to destabilize several flexible modes, while increasing it has little effect. It therefore appears that the control system design is adequate. Modifying the control design affects the coupled system in the same way as the control system alone. Stability with respect to control design can therefore be evaluated based on the control system alone. Again, interaction coefficients were almost as small as those for the baseline, producing less than 6 percent error.

The potential for substantial design modification made substantial variations in structural and feedline parameters desirable. The feedline damping was varied from half to twice nominal (see fig. 18). The associated system roots varied correspondingly. Close structural roots were affected in a similar manner but to a lesser extent. Similarly, the structural damping was varied from half to five times nominal. The coupled roots varied correspondingly as illustrated in fig. 19. Reducing structural damping produces a control instability with the second flexible mode, which is normally only 0.27 percent damped due to control coupling. The feedline roots were largely unaffected and therefore not plotted. Interaction coefficients were all at least as small as for the baseline.

The system stability is affected not only by the input damping but by the modal mass of the input modes. The effective modal mass was varied by scaling the mode shapes, and thus altering the modal gain. The effect of varying feedline gain is illustrated in fig. 20. The predominant effect is to enhance the engine damping in the first feedline mode. Other modes are not strongly affected. The interactions remain as small as for the baseline.

The effect of varying structural gain is illustrated in fig. 21. The effect is similar to varying control gain. The aero mode, $A$, system damping drops with the gain, and the second flexible structural mode, $S5$, destabilizes as the gain is increased. These effects are due to using a control system which is inappropriate for the modified structural system and would be eliminated in the control design cycle. The first significant interaction, a 21 percent deviation from linearity, occurred at four times the nominal structural
Figure 16. Coupled System Stability Versus Control System Gain
Figure 17. Coupled System Stability Versus Control System Cutoff Frequency
Figure 18. Coupled System Stability Versus Feedline Damping
Figure 19. Coupled System Frequencies and Dampings as a Function of Structural Damping
Figure 20. Coupled System Stability Versus Feedline Gain
Figure 21. Coupled System Stability Versus Structural Gain
gain in the third flexible structure mode and the first feedline mode. This interaction is attributed to increased coupling between the two modes. The nature of the interaction is best interpreted after examining the sensitivity to shifts in input frequency.

The root behavior when the structural frequencies are scaled is illustrated in fig. 22. A control instability will develop in the eighth flexible mode, S11, as the structural frequencies are decreased. As the frequency drops, the predominantly first feedline root, Fl, changes character and becomes predominantly the fourth flexible structural mode, S7, and vice versa. In the region of transition, an interaction (not illustrated) of 20 percent occurs in the two modes, S7 and Fl. Interaction again occurs only for the strongly coupled modes. The effects of varying the feedline frequency are illustrated in fig. 23. To maximize the interaction, the feedline frequencies were set at structural frequencies; Fl = S5, S6 and F2 = S11, S12, S13. The character of several of the system roots changes as the input feedline frequencies sweep through the structural frequencies. As expected, substantial interactions occurred at several points where roots were strongly coupled. A 17 percent interaction was observed at Fl = S5, and a 50 percent interaction was observed at Fl = S6.

Two types of instabilities were encountered during the sensitivity investigation. In one the coupled system instability results from a control instability, in the other from a pogo instability. Control instabilities from varying control parameters or structural parameters all occurred when the control system was inappropriate for the associated structure. Pogo instabilities resulting from gains more representative of existing engines would be eliminated by accumulator design. Frequency shifts resulting in strongly coupled, closely spaced roots yielded strong interactions; in all other cases the linear combination of individual dampings was found to be very accurate. To investigate the region of interaction in more detail, the multi-parameter runs described in the next section were made.

Interaction

The nature of the deviation from linearity, which is referred to as interaction, in the region of significant coupling between closely spaced roots was investigated. The root behavior was investigated in detail as the feedline frequency was slowly swept through the structural frequency. The engine model was simultaneously set at four times the gain and one-half of the damping of the nominal engine. This enhances the pogo contribution and makes it more representative of typical engines.

Two regions were investigated in detail, the first and second feedline modes in the vicinity of the second and eighth flexible structural modes respectively. The individual and coupled system root behavior in the vicinity of Fl = S5 and F2 = S11 are illustrated in fig. 24a and fig. 24b, respectively. Fig. 24a illustrates the fact that a stable control system and a stable pogo system can be combined to produce an unstable coupled system. This instability would be predicted on the basis of a linear combination outside the transition region. It may be noted from the figures that the damping crossing occurs at different
Figure 22. Coupled System Roots as a Function of Structural Frequency
Figure 23. Coupled System Stability Versus Feedline Frequency
Figure 24A. Coupled and Uncoupled System Stability Versus Feedline Frequency
Figure 24B. Coupled and Uncoupled System Stability Versus Feedline Frequency
points for each individual system and for the coupled system. The mode shape for each of the root pairs is undergoing substantial changes in response to small variations in the feedline frequency, moreover the transition is occurring at different input values for each of the individual systems and the coupled system. It is not surprising that the linearity assumption breaks down in that case.

The interaction coefficients associated with fig. 24a are presented in fig. 25. The interactions for the root pair are equal in magnitude but opposite in sign and appear to be correlated with the frequency crossing in the control system, however further study would be required to fully understand the behavior of the interaction coefficients. The linear combination of the envelopes of the separate systems gives the envelope of the coupled system roots. The envelope of the root behavior is meant to imply the smooth curve extrapolated from the region on either side of the associated root crossing. Fig. 26 illustrates the extrapolated control and pogo system curves and the linear prediction for the coupled system. The actual coupled root behavior is depicted by the broken line.

CONCLUSIONS

The nature of the pogo-control interaction was investigated using a coupled structural, feedline, control, and engine system model developed for a representative Space Shuttle. To evaluate the sensitivity of the interaction and stability to design modification, the model was subjected to substantial parametric variations. The stability of each system was determined by closed-loop eigenvalue analysis for four conditions: engine system only active, control system only active, both engine and control systems active, and neither engine nor control system active. Based on these investigations, the following conclusions may be made about vehicles with substantial asymmetries or lateral feedline runs:

(1) The coupled pogo-control system may be unstable even though the pogo and control systems are separately stable. Major instabilities may exist even though conventional analysis of both the pogo and control system has shown them to be stable.

(2) The coupled pogo-control system stability can be evaluated on the basis of the separate stabilities of the pogo and control systems as determined by conventional analysis techniques. Specifically, the coupled pogo-control system fraction of critical damping, $\zeta_{PC}$, for any mode is the sum of the damping with the pogo system only, $\zeta_P$, plus the damping with the control system only, $\zeta_C$, less the structural damping with neither pogo nor control systems, $\zeta_O$.

$$\zeta_{PC} = \zeta_P + \zeta_C - \zeta_O$$
Figure 25. Interaction Coefficient Versus Feedline Frequency
Figure 26. Extrapolated System Stability Versus Feedline Frequency
Although this relationship is imprecise for very close roots, it may be used to predict the upper and lower bounds for the root pair. The individual pogo and control system roots are extrapolated through the region of strong coupling from points outside the region. The upper and lower bounds for the coupled root pair are determined by applying the linear relationship to the extrapolated curves.

On the basis of these conclusions it is evident that pogo-control coupling must be evaluated for vehicles with structural asymmetries or long lateral feedline runs. It is suggested that conventional analysis be performed on the separate pogo and control systems, and that the derived relationship be used to verify the stability of the coupled system. Closed-loop eigenvalue analysis was found to be a very effective technique, and is recommended for future stability investigations. Although the understanding of interaction is incomplete, enough insight has been gained to be used effectively in the design process. It is therefore recommended that future effort be concentrated in the less well understood aspects of pogo such as asymmetric tank dynamics and engine dynamics.

McDonnell Douglas Astronautics Company,
Huntington Beach, California,
May 9, 1972
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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