THE CHALLENGE TO UNIFY TREATMENT OF HIGH-TEMPERATURE FATIGUE - A PARTISAN PROPOSAL BASED ON STRAINRANGE PARTITIONING

by S. S. Manson
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER presented at
Symposium on Fatigue at Elevated Temperatures
sponsored by the American Society for Testing Materials
and the American Society of Mechanical Engineers
Staats, Connecticut, June 18-23, 1972
THE CHALLENGE TO UNIFY TREATMENT OF HIGH-TEMPERATURE
FATIGUE - A PARTISAN PROPOSAL BASED ON
STRAINRANGE PARTITIONING
by S. S. Manson

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

SUMMARY

This paper was presented as an overview lecture sum-
marizing the 1972 International Symposium on Fatigue at
Elevated Temperatures held at Storrs, Connecticut. Starting
with the observation of the diversity of subjects covered
and lack of unanimity of approaches used, it became clear
that there exists an urgent need for a unifying framework
around which the many facets can be coherently structured.
It is proposed that the strainrange partitioning concept
has the potential of serving as such a framework. The method
divides the imposed strain into four basic ranges involving
time-dependent and time-independent components. It is shown
that some of the results presented at the Symposium can be
better correlated on the basis of this concept than by alter-
native methods. It is also suggested that methods of data
generation and analysis can be helpfully guided by this
approach. Potential applicability of the concept to the
treatment of frequency and hold-time effects, environmental
influence, crack initiation and growth, thermal fatigue, and
code specifications are then briefly considered. A required
experimental program is outlined.

INTRODUCTION

My original assignment was to prepare an overview of the
Symposium that would serve as its Summary. In reviewing the
papers I was impressed with two major factors: the vastness
of subject content and the diversity of treatment. It became
clear that what was urgently needed was a unified treatment
which would provide direction for the many facets covered by
the Symposium: how best to generate data, how to analyze and
understand the data, and how to use them most effectively for
practical purposes. In preparing this presentation I have
therefore enlarged my scope to include a discussion of the challenge to achieve such unification. Naturally it would be desirable to examine various approaches, and to arrive at an objective conclusion after a careful study of numerous alternatives. However, because of time factors it has been necessary to limit myself to a more modest treatment. Thus I have included the word "partisan" in my title because I shall limit my attempt at "unification" using only the strain-range partitioning approach on which my colleagues and I are presenting a paper at this symposium. The use of other methods as frameworks must be reserved for other occasions.

Because the change in format of my presentation—from one wherein I merely reviewed the contents of the Symposium, to one structured around a unified approach—developed at a rather late date, it was necessary to put together much of what I shall say rather hurriedly. In this connection, I want to acknowledge the very concentrated and devoted efforts of Dr. Gary Halford who not only served as a sounding board for some new concepts that were developed during the preparation, but also contributed many ideas, ran tests, made computations, and assisted in the figure preparation. He and Alfred Nachtigall, who also made calculations and prepared figures, were indispensable to me in the preparation of this paper. I should also emphasize that since the paper was prepared rather hastily, many of the conclusions that will be discussed will have to be regarded as provisional, subject to later change as we obtain more extensive information and make further analyses.

Let me start by listing the major areas covered in this Symposium. Life analysis and prediction is one of the important areas and many different methods have been used at this conference. Many test methods and materials characterization approaches appear in the Symposium; mechanisms and environmental effects have received some attention; fracture mechanics and crack growth were given a considerable amount of attention; and analysis and design codes have already been discussed and will be discussed tomorrow. That treatment of these subjects needs to be unified, and how the strain-range partitioning concept can contribute toward this goal will be the main theme of my presentation.

REVIEW OF LIFE PREDICTION METHODS

Let us first direct our attention to several life prediction methods for high-temperature fatigue analysis. One of
these is the modification of low temperature relations; another is the summation of creep time and fatigue cycle fractions; a third is ductility exhaustion as covered in one paper; and finally, the strainrange partitioning as discussed by Halford.

Looking first at the methods using modified low temperature relations, we note that one of the early concepts was the 10 percent rule. The thought involved here was to account for the accelerated crack initiation and propagation stages brought about by intercrystalline deterioration. The cyclic lives predicted on the basis of the Universal Slopes Equation developed for room temperature behavior were thus divided by a factor of 10 to account for this acceleration. We have applied this method rather extensively over the past seven years, and in many cases, it has given excellent results. However, the method has limitations. In some cases, the creep effect reduces life by even more than a factor of 10; therefore, we proposed a creep-modified 10 percent rule. This rule identified conditions under which the ordinary 10 percent rule is to be modified; an improvement was obtained, but even the modified method had limitations. For example, it could not properly account for the complex stress variation that develops when hold periods are introduced in tension and compression; temperature variations during testing could not properly be accounted for in detail; etc.

The frequency-modified life relation and the characteristic-slope relation were proposed, respectively, by Coffin and by Berling and Conway. These methods also use the equations which characterize behavior in the room temperature range but modify the equation to include a frequency term to account for rate effects at high temperature. Both of these methods can give very good results when applied to situations similar to those for which the input information, from which various constants are determined, closely resemble the real application. However, their use can result in considerable error when extensive hold-periods are present in tension or compression, either separately or combined. It should also be added that the frequency modification approach requires a considerable amount of input data, six or more constants for each temperature, and, of course, each temperature of interest requires separate investigation. How to treat cases wherein temperatures vary in a complex manner during the cycles, and how to establish the number representing the frequency in cases of complex loading, are also open questions.
Another approach that has received attention at this Symposium is the summation of time and cycle fractions, wherein time fractions are used as a measure of creep damage, and cycle fractions are used as a measure of fatigue damage. The basic approach was initially suggested by Robinson and then modified to apply to thermal fatigue problems by Taira, and it has been used for a long time as a basis for the treatment of many problems. Considerable success has been obtained using this method to analyze complex thermal fatigue; for example, the paper by Spera at this Symposium. Successful though the method has been, there are situations in which it has limitations—for example, when the static creep-rupture characteristics of materials are altered as a result of cyclic application of loading, and when ambiguity develops as to how to treat damage due to compressive loading. In order to account for the effect of changes due to creep-rupture during cyclic loading, we introduced the use of a cyclic creep-rupture test, wherein the creep-rupture characteristics of the material are obtained under cyclic loading rather than under static stress. This detail improved our ability to explain experimental observations in many cases; however, it still left open the question of how to treat damage due to compressive stress.

The method of summation of creep-time and fatigue-cycle fractions has also been applied to the study of stress relaxation during strain hold-periods. Campbell presented such an analysis at the ASME Pressure Vessel Piping Conference; Jaske and his coworkers also used this method in a paper presented in this Symposium. In many cases, the method can properly predict experimental behavior, but, as shown by Jaske, considerable error may result in some applications.

The ductility exhaustion concept is another approach for the treatment of creep-fatigue interaction. One of the early studies utilizing this concept was described in reference (14) wherein I considered the case of damage due to alternate applications of cyclic loadings and periods of monotonic plastic straining. The resulting equation was

\[
D_q = \left( - \frac{\left[ \left( D_0 - \delta_1 \right)^2 - n_1 \epsilon_p^2 \right]^{1/2} - \delta_2 \right)^2}{n_2 \epsilon_p^2} \right)^{1/2} - \delta_3
\]
We see here that the remaining ductility, $D_0$, of the material is shown as a rather complex equation involving the number of cycles $n$ for which the completely reversed plastic strain, $\varepsilon_p$, is applied, and the monotonic plastic strain, $\delta$, is imposed between periods of cyclic plastic straining. Space does not permit an extensive discussion of this approach, but this formula suggests that a framework does exist for treating fatigue by accounting for the manner in which increment of loading reduces the remaining ductility. This method may also have applicability in the treatment of ratcheting problems. In this Symposium, we have heard an interesting paper by Polhemus, Spaeth and Vogel in which the interaction of creep and thermal stress was treated by this ductility exhaustion concept. For their treatment, it was necessary to obtain the creep as well as the thermal fatigue characteristics of the material. The combined effect was obtained using a formula resembling equation (1), although the successive solution of a series of equations was involved, rather than the solution by a single closed-form expression. This concept has considerable merit, but to make it useful in a general way will require much further development.

The last of the methods I shall discuss is strain-range partitioning. Since this method will serve as the main theme of my discussion, I shall now provide some background information, show some new relations and procedures that have not been previously published, and apply the method to a number of test results presented at this Symposium. I shall then outline what developments are still required in order to render it suitable for the purpose that I am proposing; namely, as a framework for unifying the treatment of metal fatigue at elevated temperature.
BACKGROUND OF THE STRAINRANGE PARTITIONING METHOD

Inherent in the strainrange partitioning method is the concept that several modes of inelastic deformation must be considered. While it is undoubtedly possible to categorize the deformation modes in fundamental terms -- for example, bulk grain deformation and grain boundary deformation, each mode being appropriately subdivided according to other characterizations -- the approach used in strainrange partitioning is a compromise with engineering practicality. Only two strain modes are chosen: plastic flow and creep. There may, of course, be important differences between the mechanisms involved in these deformation components. Creep probably involves more grain boundary sliding and atomic diffusion than plasticity, while the latter may involve principally slip-plane sliding. Such distinctions are important from the point of view of providing understanding, but for practical purposes these considerations are temporarily bypassed by distinguishing between the components only in relation to their time dependency. "Plastic flow" will be regarded as the sum of all inelastic strain components which occur nearly immediately upon application of stress, while "creep" will be regarded as the sum of all time-dependent components. This distinction alone enables us to separate the components by relatively simple experimental procedures.

Strainrange partitioning is based on the concept that the two modes of deformation may exist separately or concurrently, and that their interaction can influence to a considerable degree the fracture behavior of the material. An important feature of the method relates to the manner in which a tensile component of strain is balanced by a compressive component to close the hysteresis loop during described in reference (15), and will only be described briefly here.

Consider the hysteresis loop shown in figure (1(a)). This hysteresis loop was obtained by loading rapidly to a tensile stress, holding that stress for a period of time during which creep occurred, reversing the stress to a compressive value lower than was used for tension, and providing a longer hold-period in compression to close the hysteresis loop with creep strain. In the tensile portion of the test, there was more plastic flow than in the compressive portion, while the creep component was lower in tension than in compression. In essence therefore, while there was a reversed
creep component and a reversed plastic flow component, there also appeared a strain component wherein tensile plastic flow was reversed by creep.

Considering other possibilities that could develop, it appeared that a minimum of four basic strain components required attention, as shown in the following table:

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>TENSILE STRAIN</th>
<th>COMPRESSION STRAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \varepsilon_{pp}$</td>
<td>Plastic</td>
<td>Plastic</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{pc}$</td>
<td>Plastic</td>
<td>Creep</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{cp}$</td>
<td>Creep</td>
<td>Plastic</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{cc}$</td>
<td>Creep</td>
<td>Creep</td>
</tr>
</tbody>
</table>

Having recognized the four basic components of strain that must be considered, we next set about to devise separate tests in which the major strain component would be one of each of these basic types, and we determined the life relations associated with each one of them. Typical life relations are shown in figure (1(b)). Lines relating strain range to cyclic life on logarithmic scales are straight, analogous to room-temperature fatigue behavior. Each strain component, however, has a different life relation. In figure (1(b)) the $\Delta \varepsilon_{pp}$ component produces the highest life for a given strain range, while the $\Delta \varepsilon_{cp}$ component produces, for this material, the shortest life by a factor of approximately 20 for the same magnitude of strain.

Figure (1(c)) shows that the mechanism of failure is considerably altered by the type of strain range. The top of the figure shows a micrograph of a tested specimen when the $\Delta \varepsilon_{pc}$ strain range was on the order of 1.5 percent. The failure path is transcrystalline; the life was 264 cycles. The bottom of the figure shows a micrograph of another tested specimen, the strain range was of the $\Delta \varepsilon_{cp}$ type, also about 1.5 percent. The life was only 15 cycles; the fracture path is now intercrystalline. Figure (1(d)) shows how the results on two materials, 2-1/4 Cr-1Mo steel, and 316 stainless steel, correlated on the basis of strain range partitioning in tests involving complex cycles. Correlation is within a life factor of approximately two for nearly all cases.

In our paper presented by Halford, the life relations for each of the four components for the two above mentioned materials were studied over a wide temperature range. These could all be represented by straight lines that are reasonably independent of temperature (see fig. 3 of ref. (1)).
What this means is that in these materials the life relations are governed by the nature of the strainranges and not by the temperature at which the strainrange was imposed. If a particular strainrange is imposed at one temperature, the life will be the same as when the same strainrange of the same type is applied at another temperature. It does not, of course, mean that life is independent of temperature. A given imposed load will produce different strains at different temperatures; a given imposed strain will be partitioned in a different manner depending on the temperature. Therefore, a complete life analysis will involve the determination of how much strain is developed and how this strain is partitioned. Once the partitioned components are known, however, the life is essentially established without regard for temperature. From a practical point of view, this also means that the amount of experimental information to be generated is drastically reduced. Furthermore, convenient temperatures can be used to develop the desired strainranges in relatively short periods of time, thereby avoiding extended testing that might be necessary to obtain the input information required for analysis by alternative methods.

It is important to emphasize that for these two materials just discussed the creep and plastic ductilities were not sensitive functions of temperature or of testing time. As I shall later indicate, when we deal with materials in which creep and plastic ductilities are sensitive to temperature or time of application, we would not expect temperature or time-independence of these life relations. I shall provide an illustration of this case in my later discussion.

**SOME NEW RELATIONS AND PROCEDURES APPLICABLE TO STRAIN RANGE PARTITIONING**

So much, then, for background information on strain range partitioning and the status of the development of this method up to this point. As I have already indicated, in order to provide a more substantive presentation, it was necessary to carry the concept a step further. I should now like to discuss several new relations and procedures that were developed for the purpose of analyzing some of the results presented in this Symposium.
A New Interaction Damage Rule

An important step involved in the application of the strainrange partitioning method is the process of combining the effect of several concurrent strain ranges. Figure (2) shows an example wherein a total inelastic strain of 0.01 consists of only two components, $\Delta\varepsilon_{PP} = 0.009$ and $\Delta\varepsilon_{CC} = 0.001$. In the past we used a simple linear damage rule, according to which the life $N_f$ would be determined by equation (3) shown in this figure. A new approach designated in figure (2) as an "interaction damage rule", is simpler to apply and has other advantages. Details of the development will be described in a forthcoming publication (ref. 16). Basically, the derivation is based on the assumption that in the presence of other strainrange components the life relation for each strainrange is altered by suitable displacement. The amount of displacement depends on strainrange component, together with the fraction of the total inelastic strainrange associated with the component involved. As shown in figure (2) by equations (4) to (6) the final result is extremely simple. When several types of strainranges are involved, the first step is to compute a life from the basic life relations for each strainrange as if that strainrange were the entire sum of the strainranges present. This is done for each of the strainranges involved in the analysis. Once these lives have been calculated, the expected life is obtained by weighting each of the lives calculated according to that fraction of the total inelastic strainrange that is truly associated with that strainrange component. Thus, as shown in the last formula in figure (2), the ratio $F_{PP}/N_{PP}$ is obtained by placing in the numerator the fraction of the total inelastic strainrange that is due to $\Delta\varepsilon_{PP}$, and using as the denominator the life $N_{PP}$ which is obtained when the entire inelastic strainrange is considered to be $\Delta\varepsilon_{PP}$. The other terms are similarly established and life is computed on the basis of the weighted life relation shown. We have termed this procedure the "interaction" rule because it is based on the idea that the strainranges interact to cause displacement of the life relations.

Life computed by the "interaction" damage rule and by the ordinary linear damage rule do not differ greatly in most cases, as illustrated in figure (3) for 316 stainless steel. The lines based on computation by both methods are relatively close to each other both at very high frequencies and at very low frequencies. They differ from each other by only a life
factor of two in a relatively small frequency range. The experimental results also agree well with both types of computations and at this time we cannot be certain which damage rule yields more accurate results.

Method of Partitioning Strainranges

Figure (3) also shows an advance that we have made since our last publication regarding the procedure for partitioning a strainrange when only the total value of the inelastic strainrange is known. The concept is applied here to determining the effect of frequency on cyclic life when the total inelastic strainrange is held constant at 0.47 percent. Since we are dealing with continuous cycling, only the balanced components $\Delta \varepsilon_{pp}$ and $\Delta \varepsilon_{cc}$ should be present, but how much of each component is present at each particular frequency must be determined. An experimental approach is used which is illustrated in the figure. The stabilized hysteresis loop is first established after applying a few cycles of loading at the frequency involved. This hysteresis loop provides us with information on the maximum stress that develops both in tension and compression for the applied frequency. Since plastic flow depends primarily on the peak stresses, its magnitude can be determined by conducting another test in which the same maximum and minimum stresses are maintained, but in which the frequency is raised to as high a value as can be achieved. At the high frequency creep will essentially be excluded. The hysteresis loops in the figure were obtained in this way. The inner hysteresis loop in each set shows the amount of inelasticity that develops when the same extreme stresses are applied, but the frequency is sufficiently high to preclude appreciable creep. Note that at condition A the hysteresis loop shows that very little $\Delta \varepsilon_{pp}$ strain is present. On the other hand, at point D, where the frequency is high, practically all of the inelastic strain is $\Delta \varepsilon_{pp}$. At intermediate frequencies, the amount of $\Delta \varepsilon_{pp}$ is determined from rapid cycling, and the $\Delta \varepsilon_{cc}$ component is determined by the difference from the total inelastic strainrange. Although this method involves some experimentation, it should be emphasized that each hysteresis loop is obtained after only a few cycles of loading, and that no test need be carried to fracture. In fact, the same specimen can be used for all the tests involved, involving a total test time of just several days. For more complex type of loadings, involving other types of strainranges, additional procedures may be required; I shall return to this subject later in the discussion.
Tentative Universalized Life Relations

Another new development to be employed in the remaining discussion relates to universalized equations for cyclic life associated with each of the strainrange components, analogous to the Universal Slopes Equation which we have used effectively to treat estimates of fatigue in the subcreep regime. It is recognized that the amount of information available to construct such relations was limited. However, in order to apply the method to some of the data in this Symposium, it was necessary to make estimates of these universalized life relations. Shown in figure (4(a)) is an illustration of how we proceeded to obtain this estimate. First, we normalized the strainrange by dividing by ductility (plastic ductility $D_p$ if the tensile half of the strain range is plasticity, creep ductility $D_c$ if it is creep). The figure shows the results for $\Delta \varepsilon_{cp}$, although similar curves were constructed for each of the strainrange components. The ratio $\Delta \varepsilon_{cp}/D_c$ is plotted against the life $N_{cp}$ for a number of the materials. It is seen that the points fall near a line having a slope of approximately -0.8. Because of the limited amount of information available to construct such a correlation, no attempt was made at further refinement. After making such plots for each of the four components, using data from our own laboratory as well as those contained in the preprints of the reports presented at this Symposium, the lines shown in figure (4(b)) were obtained. It is apparent that any of the strainranges involving a creep component could be represented in such plots by lines of slope -0.8; whereas the line representing the $\Delta \varepsilon_{pp}$ component has a characteristic slope of -0.6, which is equal to that in the Universal Slopes Equation. The coefficients are different for each of the lines.

Figure (5) shows the degree to which these life relations correlated all the data that were used in their construction. Observed life is plotted against the predicted value determined from these simplified relations. The sources for the data are also shown in this figure, and it can be seen that many of the data points were extracted from reports presented in this Symposium. While we do not consider the relations shown here as final ones for estimating life for each strainrange component, the overall correlation is reasonably good. Individual materials, however, may deviate by more than a factor of 2 from their predicted behavior. For example, figure (5(b)) shows that for the 2-1/4Cr-1Mo alloy (square symbols) the $N_{pc}$ life is overestimated by more than a factor
of 2, whereas figure (5(d)) shows that the $N_{cp}$ life is underestimated by more than a factor of 2. The net effect is that, for this alloy, a strain absorbed as $\Delta \varepsilon_{pc}$ is actually more damaging than if absorbed as $\Delta \varepsilon_{cp}$, as was noted in reference (15). However, the life relations of figure (4) imply $\Delta \varepsilon_{cp}$ to be more damaging than $\Delta \varepsilon_{pc}$ for the $D_p$ and $D_c$ ductilities associated with this material. Despite occasional recognized inaccuracies, the tentative relations can be most useful where experimental information is totally lacking for constructing the actual life relations. Use of the tentative relations will be illustrated in some later examples.

APPLICATION OF STRAINRANGE PARTITIONING

TO SUBJECTS COVERED BY SYMPOSIUM

I shall now attempt to demonstrate that the strainrange partitioning approach addresses itself to many of the subjects covered in this Symposium, and that it is useful in overcoming some of the limitations of other methods discussed. As such it has the potential of providing a unifying framework for analyses so urgently needed at this time.

Establishing Bounds on Life

An important use of the concept is the computation of maximum and minimum values that can be expected for a known strainrange. Figure (6) shows two examples. Figure (6(a)) presents data on 316 stainless steel used by Challenger and Moteff, taken from the original publication of Berling and Slot; figure (6(b)) shows results for A-286 referred to by Solomon and Coffin, and also contains some new data generated in our laboratory. The data show fatigue life at various frequencies and temperature. Since continuous cycling was involved in these tests, only the balanced components $\Delta \varepsilon_{pp}$ and $\Delta \varepsilon_{cc}$ should be present. The maximum life would occur if all the inelastic strain were absorbed as $\Delta \varepsilon_{pp}$; the minimum life would occur if the strainrange were all $\Delta \varepsilon_{cc}$.

In figure (6(a)), $N_{pp}$ and $N_{cc}$ bound the 1200 to 1500°F (650 to 815°C) data. At 800°F (425°C), the behavior is similar to that at room temperature. From this figure we observe that there is a difference between the high
temperature line for $\Delta\varepsilon_{pp}$ and the Universal Slopes line at room temperature. The latter line, shown dotted, is higher in life by a factor of about 2 than the $\Delta\varepsilon_{pp}$ line at high temperature. Perhaps this is due to the fact that the oxidizing effect associated with the higher temperatures is absent. Maybe it is just the result of data scatter, or the expected inaccuracies of both the simplified $\Delta\varepsilon_{pp}$ relation and the Universal Slopes Equation. On the other hand, if it is a true behavior pattern, it provides some corroboration to Solomon and Coffin's 19 observations regarding vacuum environment effects which we shall discuss later. What the figure shows mostly, however, is that even if we have no quantitative information on the degree of partitioning involved, we can still bound the life between two lines. In many cases this may be adequate.

Figure (6(b)) shows the concept of data bounding applied to A-286, the material used by Solomon and Coffin 19. Again, the $\Delta\varepsilon_{pp}$ and $\Delta\varepsilon_{cc}$ lines bound all the data within the wide frequency range investigated. Also shown in this figure are some dotted lines that indicate what the lives would be if the strains were partitioned according to the amounts designated along each line. To interpret the results more quantitatively within the framework of the strainrange partitioning approach, we ran several tests on A-286 in which the concept shown in figure (3) was used to partition the strain during continuous cycling. The data points obtained in our program are shown by the triangles. The results will be discussed in the next section. However, we can note here that all the data are indeed bounded by the $N_{pp}$ and $N_{cc}$ lines.

**Frequency Effects**

To obtain more precise information on life, rather than simply determining bounds, we must determine how the total strainrange involved is partitioned into its components. It has already been shown in figure (3) how the frequency effect in 316 stainless steel was analyzed by experimental partitioning of the strainranges. Whether the "linear" or the "interaction" damage rule was applied to the data, the predictions never differed from the experimental values by more than a factor of 2. Notice, incidentally the saturation values of life at both low and high frequency. At the low end of the frequency spectrum the applied strain tends to be absorbed as $\Delta\varepsilon_{cc}$, and further decreases in frequency do not
increase the amount of $\Delta \varepsilon_{cc}$, so life levels off at $N_{cc}$ for the applied strainrange. Similarly, at the highest frequencies, life levels off at $N_{pp}$ for the applied strainrange.

Our analysis of the frequency effect for A-286 is shown in figure (7). This is a material for which the creep ductility varies with rupture time. The information on ductility was obtained by private contact with Dr. Opinsky, who also has presented a paper on A-286 at this Symposium\textsuperscript{20}. Thus, the lower curve shown on this figure represents the $N_{cc}$ value as a function of frequency as determined by the simple formula shown. The curve showing the predicted value of life as a function of frequency was determined by partitioning the strain according to the method previously described; good agreement was obtained with the experimental results shown as triangle points in figure (6(b)).

\textbf{Hold-Time and Relaxation Effects}

The strainrange partitioning method lends itself ideally to the treatment of hold-time effects. During a tensile or compressive strain hold-period, creep occurs and stress relaxes. This creep strain may later be reversed by plastic flow; thus $\Delta \varepsilon_{cp}$ or $\Delta \varepsilon_{pc}$ components may develop. Two applications will be used for illustration.

Figure (8) shows results of Conway and coworkers\textsuperscript{21} for 304 stainless steel involving continuous cycling or tensile hold periods at two strain levels—1/2 and 2 percent. Additional tests, conducted at our laboratory in a short time using a single specimen permitted partitioning in the manner described earlier. Life computations shown by the lines in this figure agree very well with the experimental results. The curves show, as Conway determined experimentally, that there is a saturation life at both very short cycle times as well as at very long cycle times (very high frequency and low frequency, respectively). At the short cycle times, all of the strain is imposed as $\Delta \varepsilon_{pp}$, whereas at the highest of the cycle times the strainranges became $\Delta \varepsilon_{cc}$ or $\Delta \varepsilon_{cp} + \Delta \varepsilon_{pp}$, depending on whether the strain cycling is continuous or whether a hold-time in tension is applied. To reach the point shown on the uppermost curve for the 1/2 percent strainrange would require a total test time of approximately 10,000 hours. Such information can, however, be obtained in a short time if the strainrange partitioning concept is used.
An analysis using the strainrange partitioning approach was also made of the data presented by Jaske, et al. at this Symposium for the alloys Incoloy 800 and 304 stainless steel. These authors investigated the validity of the concept of time and cycle fraction addition. In these tests, involving hold-times in tension, they actually measured the stress relaxation during the hold-period, and made computations based on the static creep-rupture properties. They also computed the cycle ratios based on pure fatigue. Their results, shown in figure (4) of their report show large discrepancies between predicted and experimental lives. To analyze the results on the basis of strainrange partitioning, information on measured stress relaxation was used to compute the $\Delta \varepsilon_{cp}$ strain range component. The results are shown in figure (9). Here, measured fatigue life is plotted against predicted life. Using the time and cycle fraction approach, lives as much as 20 times higher or 5 times lower than predicted were observed, as seen in figure (9(a)). Using the strainrange partitioning approach the experiments agreed with the predictions within factors of 2, as seen in figure (9(b)).

Fatigue Mechanism and Environmental Effects

Both the mechanism of fatigue and environmental effects have received considerable attention at this Symposium. These are both broad subjects, and the strainrange partitioning concept can contribute to both in many ways; we can touch briefly on only two facets of the subject:

Internal ratcheting. - Because of the dual deformation system hypothesized in the strainrange partitioning concept, it becomes possible for an internal ratchet to develop when a $\Delta \varepsilon_{cp}$ or $\Delta \varepsilon_{pc}$ component is present: tensile strain accumulates within one deformation system, compressive strain accumulates within the other system. Although externally there may be little net geometrical change after each cycle, the internal effect may be substantial due to cycle-by-cycle strain accumulation. A repetitive $\Delta \varepsilon_{cp}$ cycle becomes, for example, more like a continuous monotonic tensile creep-rupture test, since every cycle adds tensile creep. Likewise a repetitive $\Delta \varepsilon_{pc}$ cycle simulates a conventional tensile test. It is for this reason that the $\Delta \varepsilon_{pc}$ component is normalized to the tensile ductility $D_p$ in equation (8) of figure (4(b)), and the $\Delta \varepsilon_{cp}$ component is normalized to the creep ductility $D_c$ in equation (10) of figure (4(b)).
To show the detrimental nature of internal ratcheting as compared to balanced strain reversal, consider a simple illustration: a material with a tensile ductility of 0.50 is subjected to a reversed plastic deformation of 0.01 per cycle. Under completely reversed plasticity, using equation (7) of figure (4(b)) the expected life would be 420 cycles. If no plasticity reversal occurred, and a series of tensile strains of 0.01 were imposed, obviously only 50 cycles would exhaust the tensile ductility. We see therefore that in reversed plasticity there is a certain amount of "healing" that takes place during every cycle, as long as the reversed strain is "undoing" some of the damage due to the strain that had occurred in the forward half of the cycle. On the other hand, if the reversal takes place along a different system, the healing is not possible, and we will get a ratcheting effect in each of the directions. This is a possible cause for reduced life in either the $\Delta\varepsilon_{cp}$ or the $\Delta\varepsilon_{pc}$ components. For example, if we consider $\Delta\varepsilon_{cp}$, tensile creep occurs within one system, but the reversal is along another system and therefore does not do any healing. The next cycle of loading simply adds tensile creep during the tensile half of the cycle upon the creep that had already occurred in the previous cycle. Thus, the material ratchets in tensile creep along one system, and in compressive plasticity along the other system, until the material reaches its creep ductility in tension or its plastic ductility in compression. Failure then occurs in a manner analogous to that experienced in a monotonic loading test. If this were the exact case however, the exponent in the $\Delta\varepsilon_{cp}$ and $\Delta\varepsilon_{pc}$ relations would be -1 rather than -0.8. Therefore, some healing due to the reversal does occur if we assume that the -0.8 exponent is an appropriate one. Nevertheless, the concept of strainrange partitioning gives us better insight into the individual effects of the different strain components. When further information is available to derive a better universalized relation, we may be able to quantify this effect more accurately.

Inert environment effects.- Surface interaction with the environment is an additional time-dependent variable in high temperature fatigue. Thus when frequency is lowered, not only is there more time for creep, but more time is available for oxidation or other surface interaction. In figure (13) of Coffin's opening lecture he shows that for A-286 in vacuum at high temperature the effect of frequency becomes very small (except at the very lowest frequencies used); in air the effect is very substantial. In his figure (15) he
also shows that when plots of inelastic strainrange versus life are made of high temperature data obtained on many materials in an inert environment the data fall within the same scatterband as room temperature data on a variety of materials. These and other observations made by Coffin in his opening lecture provide convincing evidence of the importance of oxidation on fatigue life.

The role of creep relative to that of environment needs further clarification. Coffin's plot (fig. 15 of ref. 22) does not normalize the strain data relative to ductility, nor are frequencies cited; thus the wide scatterband (a factor of about 20 in life for a given strainrange) can hide subtleties in behavior that might be more visible on a suitable plot. That a creep effect does occur at high temperature and low frequency is apparent from his figure (13) where the tests at 0.0045 and 0.045 Hz do fall below the "least squares" line for the higher frequency. His joint report with Solomon also demonstrates higher crack growth rates at reduced frequencies even in an inert environment.

Unbalanced stress of the $\Delta \varepsilon_{cp}$ type also tends to emphasize the creep effect. Figure (10) shows an analysis of the data presented by Sheffler and Doble on the tantalum alloys T-111 and ASTAR-811C. These tests were conducted in a hard vacuum of $10^{-8}$ torr, so no environmental interaction is to be expected. The dotted lines in the figure represent Coffin's scatterband. Two types of tests are shown for each alloy. For one, creep and plasticity appear only as balanced strainranges, that is for the case of continuous cycling wherein only $\Delta \varepsilon_{pp}$ and $\Delta \varepsilon_{cc}$ occur. These data fall within Coffin's scatterband although each individual material presents its own line of behavior. On the other hand, when unbalanced creep-plasticity occurs; i.e., when $\Delta \varepsilon_{cp}$ is present (which was introduced by conducting in-phase, thermal-mechanical cycling tests) the lines deviate considerably from the scatter band. Their slope is approximately $-0.8$, as would be expected according to our simplified relationships for these strain components. These curves demonstrate that even in a vacuum environment unbalanced creep-plasticity can cause significant reductions in life.

At this time there is still considerable question regarding the relative roles of creep and environment on high temperature fatigue life. The strainrange partitioning concept can be helpful in defining the effects by suggesting experiments, such as cited above, where the effects are expected to be greatest, so that the studies can be more
definitive. Furthermore, it should be emphasized that even if the environment effect is the dominant one, it would not invalidate the utility of the strainrange partitioning method, since the basic life relations for its use would, in any case, be obtained under the environment in which it is to be applied. Even if all the life relations nearly coincide in an inert environment the approach would still be valid. Finally, it should be emphasized that most practical applications do involve a reactive environment.

Fracture Mechanics and Crack Growth

Next, we shall consider the application of strainrange partitioning to fracture mechanics and crack growth. Recent studies of crack growth have made strong use of concepts of elastic fracture mechanics. In many cases, the conditions for crack growth agree reasonably with those hypothesized in the development of the laws of elastic fracture mechanics; and in such cases, it is appropriate to use the stress intensity in the equations for crack growth. Several applications involving this approach have appeared at this Symposium, and some very interesting results have been presented. At the higher temperatures, where large components of inelastic strain are likely to be present, the concepts of elastic fracture mechanics as derived must be modified. Several examples have appeared at this Symposium wherein plastic strainrange had been used in place of stress when applying fracture mechanics laws to crack growth. These follow the lead of earlier work by Boettner, Laird, and McEvily where the parameter \( \Delta \varepsilon_p - \sqrt{t} \) was used in place of \( \Delta \sigma - \sqrt{t} \) in the crack growth equations. In reference (4) I also applied the \( \Delta \varepsilon_p - \sqrt{t} \) parameter to explain the crack growth laws and the life relations in low cycle fatigue. Strainrange partitioning concepts now help us recognize that in dealing with high temperature, low cycle fatigue we must think in terms of at least four strainrange components, not just the plastic flow that was adequate when dealing with low temperature plasticity.

Before proceeding to crack growth, however, it is desirable to consider the role of strainrange partitioning in explaining the crack initiation equations as well. In reference (4), it was demonstrated that the cycles to both crack initiation and to final fatigue fracture of a laboratory specimen tested in axially reversed loading could be related to plastic strainrange according to a simple power law. When several types of inelastic strainrange components are
possible, a different crack initiation law must be stated for each. Thus we may write

\[
N_0 \begin{array}{c}
pp \\
\end{array} = A_1 (\Delta \epsilon_{pp})^{a_1} \quad N_0 \begin{array}{c}
p_c \\
\end{array} = A_3 (\Delta \epsilon_{p_c})^{a_3} \\
N_0 \begin{array}{c}
c_p \\
\end{array} = A_2 (\Delta \epsilon_{c_p})^{a_2} \quad N_0 \begin{array}{c}
cc \\
\end{array} = A_4 (\Delta \epsilon_{c_c})^{a_4} \\
\end{array}
\]

Each of these relations expresses the number of cycles to crack initiation in terms of the pertinent strain range component, assuming it to be the only component present. When two or more components are present the relations must somehow be combined in a manner as yet to be determined, perhaps using the same concepts as those expressed earlier in equations (2) to (6) in figure (2).

Consider next the equations governing the initial stages of crack growth. First, however, it should be explained that crack initiation is a rather ill-defined term; it is the number of cycles required to develop a crack of arbitrarily specified size. Generally, this size is rather small; in fact, too small for the stress field developed in its vicinity to meet the complete requirements of the laws of fracture mechanics in defining this stress field. Therefore, when we consider the early stages of crack growth, that is, before the crack has grown large enough to develop its own characteristic full stress field, we might reasonably conclude that the same factors that cause the crack to initiate to its current small size will also govern the growth of the crack during the next few cycles of loading. Thus, we may write an equation expressing the fact that for each strain range component \( \frac{dI}{dN} \) is roughly inversely proportional to the number of cycles to initiate the crack. The resulting equations become

\[
\frac{dI}{dN} \propto \frac{1}{N_0^m}
\]

or
\[
\frac{d}{dN}_{pp} = B_1 (\Delta \varepsilon_{pp})^{b_1} \quad \frac{d}{dN}_{pc} = B_3 (\Delta \varepsilon_{pc})^{b_3}
\]
\[
\frac{d}{dN}_{cp} = B_2 (\Delta \varepsilon_{cp})^{b_2} \quad \frac{d}{dN}_{cc} = B_4 (\Delta \varepsilon_{cc})^{b_4}
\]

Here
\[
b_1 \approx -a_1, \quad b_2 \approx -a_2, \quad b_3 \approx -a_3 \quad \text{and} \quad b_4 \approx -a_4
\]

and
\[
\frac{B_1}{A_1} = \frac{B_2}{A_2} = \frac{B_3}{A_3} = \frac{B_4}{A_4}
\]

The fact that \( m \approx 1 \) is verified by thermal fatigue tests reported by Spera, Howes and Bizon.\(^\text{25}\)

Consider, finally, the later stages of crack growth, when a fully developed crack has already appeared, and when the \( \sqrt{\tau} \) term can be introduced, analogous to equations derived from elastic fracture mechanics. These equations become

\[
\frac{d}{dN}_{pp} = C_1 (\Delta \varepsilon_{pp} \sqrt{\tau})^{c_1} \quad \frac{d}{dN}_{pc} = C_3 (\Delta \varepsilon_{pc} \sqrt{\tau})^{c_3}
\]
\[
\frac{d}{dN}_{cp} = C_2 (\Delta \varepsilon_{cp} \sqrt{\tau})^{c_2} \quad \frac{d}{dN}_{cc} = C_4 (\Delta \varepsilon_{cc} \sqrt{\tau})^{c_4}
\]

Both equations (12) and (13) must, of course, be extended to cases where two or more strainrange components are present, and when the hysteresis loop of the bulk material surrounding the crack does not close after each cycle.

Although a number of papers have appeared at this Symposium presenting data on crack growth, it has not been possible, because of time limitations to examine the results in the light of the above equations. Furthermore, no experience is as yet available with their application, and the numerical magnitude of the constants are totally unknown. Finally, it should be recognized that complicated conditions
such as residual stresses can develop at the crack tip when a complex history of temperature and load is imposed. Such complications are characteristic of any treatment by fracture mechanics principles, so the validity of the equations should not be judged on the basis of ad hoc tests not expressly designed for the purpose. The merit of the strain-range partitioning concept is that it can suggest relations such as equations (11) to (13) -- relations that might otherwise elude us -- and test programs that might eventually lead to a unified treatment of crack initiation and growth under the complex conditions commonly encountered in practical high temperature applications. Only after a carefully conceived test program has been conducted will we be able to evaluate the validity of the approach. And even if the results are negative, we will then be in a better position to hypothesize a more proper approach.

Thermal Fatigue

Another subject that has received considerable attention at this Symposium is that of thermal fatigue. We heard this morning a paper by Spera in which he used essentially the time and cycle fraction summation concept to analyze a number of cases of thermal fatigue. In his treatment compressive and tensile stresses were assumed to be equally damaging when determining the creep time ratio. Although excellent results were obtained in all the cases he presented, we have already indicated that the approach using the time and cycle fraction summation may result in error in some cases (see fig. 9(a)). Treatment of creep damage due to compressive stress also remains a moot question. The strain-range partitioning approach is helpful in these respects because the stresses enter into the evaluation only to the extent that they influence the strainrange components that develop. Assuming that we are in a position to establish the strainranges either experimentally or by computation, both these complexities vanish. We shall therefore now indicate how strainrange partitioning can help in unifying the treatment of thermal fatigue.

Rather than using a complex structure it is convenient to use a three-bar assembly (fig. 11(a)) that will enable us to incorporate the basic features of performance without the complexity that attends treatment of a real component. The interior bar is massive and usually constrains thermal expansion or contraction of the two outer bars designated as "edges." All three bars are subject to heating and cooling.
The assembly is, in a sense, analogous to a turbine blade wherein the outside bars represent the edges of the blade and the inside bar represents the massive center of the blade. When the turbine blade is subjected to hot gas, the outside edges are first to heat. Their expansion is, of course, constrained by the massive interior bar which is slow to heat, or may never heat up, depending on external conditions imposed. Thus, inelastic flow occurs in compression on the outer edges upon the first loading. What happens to the strains induced in the edges in subsequent time periods will depend on the manner in which the temperature is varied both in the edges and in the inside bar.

Let us consider several cases. In Case I, shown in figure (11(b)), it is assumed that the edges are heated and cooled rather rapidly, but that the inside massive bar maintains its temperature value throughout the history. This situation is analogous to conventional thermal fatigue type tests wherein the test specimen is constrained between end plates and is repeatedly heated and cooled. If the frequency is reasonably high, the edge bars will simply alternate between tensile and compressive loading, being at all times constrained by the interior bar. The inelastic strain range developed, which is of the $\Delta \varepsilon_{pp}$ form, is shown in Case I of the figure.

In Case II (fig. 11(c)), a hold-period is applied to the edge bars after they are heated. Plastic flow occurs immediately upon heating, developing compressive stress and plastic strain. However, as this strain is held at high temperature, relaxation occurs as shown by the line AB, resulting in compressive creep. When the temperature of the edge is later reduced, tensile plastic flow takes place, but no creep can occur because the reduced temperature is low. The inelastic strain ranges for this case are, therefore, plastic reversed strain $\Delta \varepsilon_{pp}$ and compressive creep balanced by tensile plasticity $\Delta \varepsilon_{pc}$.

In Case III (fig. 11(d)) the edge bars are held at high temperature for an appreciable period just as in Case II. However, the interior bar is now heated to the equilibrium temperature of the edges. This is typical of what might occur in an uncooled turbine blade subjected to extended steady operation at high temperature. A lag occurs, however, between the time when the temperature of the inside bar reaches its maximum value and the time the outside bar reaches its equilibrium value. Compressive plastic flow thus occurs in the edge bars while their temperature has
already achieved maximum value and while the inside bar temperature has not yet begun to rise. No time is allowed, however, for creep. When the inside bar finally reaches the temperature of the edge, it must, of course, force the edge bars to flow plastically in tension, since they had already become shortened by plastic compression in their earlier temperature history. A tensile stress develops at C when the inside bar has reached its maximum temperature. Since, as shown in the figure, both the inside and outside bars are maintained at high temperature for an extended period, creep relaxation of the tensile stress occurs along the line CD producing a $\Delta \varepsilon_{cp}$ type of deformation wherein creep occurs in tension but is balanced later in compressive plastic flow.

Finally, we consider Case IV (fig. 11(e)) wherein, as in Case III the interior bar reaches an equilibrium temperature equal to that of the edge bars; however, the temperature of the interior is constrained from rising rapidly. A "lag" period is introduced during which the edge temperature is maintained at a high value while the inside bar is still cold. While the inside bar is cold and the edge is hot, compressive plastic flow and stress relaxation occurs along EF as in Case II. Later, when both bars reach the same high temperature, tension occurs in the edge, in the manner already described in connection with Case III. Because of the high temperature tensile stress, relaxation occurs along GH, balancing the creep relaxation EF. Thus, the strainrange components present are $\Delta \varepsilon_{pp}$ and $\Delta \varepsilon_{cc}$, as shown in the figure.

The above discussion demonstrates that, depending on the pattern, any combination of these strain components associated with strainrange partitioning may develop. It is clear, therefore, that because of the diversity of strain types that can be developed, depending on test conditions, a wide variety of results can be expected. Indeed, this has been the experience in thermal fatigue testing, namely, that test results are sensitively related to test conditions, and that meaningful results can only be obtained by simulating service conditions very closely.

This very point has been made during the Symposium at an afternoon session of the Low-Cycle Fatigue Subcommittee of ASTM Committee E-9. The question related to the possibility of standardizing fluidized bed thermal fatigue tests, and the conclusion drawn was that the diverse results obtainable from such tests make standardization difficult at this time. Perhaps the strainrange partitioning concept can be helpful here. By designing the tests so as to favor each of
the basic strainrange components (or perhaps only the most mild and most severe as extremes) the relative behavior of materials can be evaluated under easily analyzable conditions. Behavior under more complex conditions could possibly be synthesized from the basic tests.

Another illustration of the usefulness of the strainrange partitioning concept is shown in figure (12). Here the potential beneficial effect of "phasing" the temperature is illustrated. Reference is made to Case IV of figure III(e)) wherein the temperature of the interior of the bar is held back sufficiently long to allow compressive creep to occur. Such compressive creep can then balance the inevitable tensile creep that occurs when the inside bar reaches its maximum temperature coincident with the edge and is held at this value for a long time. Thus, figure (12) illustrates the fact that if creep can be forced to occur in compression by proper temperature phasing between the component parts of an assembly, and if in this way unavoidable tensile creep that occurs later in the cycle can be balanced, it is conceivable that an increase in life could be achieved even though the hysteresis loop is actually widened. A balanced $\Delta \varepsilon_{cc}$ strainrange component may for some materials be less damaging than a smaller value of the unbalanced strainrange $\Delta \varepsilon_{ctv}$. What is plotted on the vertical axis in the lower half of figure (12) is the dimensionless improvement in cyclic life as a function of a parameter which measures the degree of balance of the compressive and tensile creep strains. This parameter is the ratio of relaxed compressive stress to the relaxed tensile stress. Of course, the relaxed compressive stress is achieved by the temperature phasing. It is seen that, if the relaxed compressive stress can be made equal to the relaxed tensile stress, life could be increased by as much as a factor of four for a material complying with the basic behavior shown in figure (4(b)). The computations are not intended to represent any particular realistic physical situation. They do, however, have meaning. For example, the implication is that if tensile creep must occur at the edge of a turbine blade due to a hold period at high temperatures of both the interior and exterior parts of the blade, then perhaps by controlling the rate at which temperature of the interior portion of the blade is increased, a favorably balanced inelastic strain condition can be developed, thus improving the life. Such temperature control might be possible by proper cooling, by surface coatings, or by conductivity control to give the necessary phase relation between the edge temperature and the inside temperature. Again, the method illustrates a case wherein other approaches
such as the time and cycle fraction approach, or the frequency-modified life approach, would not suggest that a life improvement is possible in this temperature phasing procedure. It is certainly food for thought, and another opportunity to check the validity of the strainrange partitioning concept.

The paper of Lawton and Bynum\(^\text{27}\) presented at this Symposium lends some support to the concept discussed above. They conducted cyclic deformation tests at 1050° F (565° C) on welded furnace panels. In one series of tests, a five minute hold period was introduced in one direction of the cycle, and a five minute hold period was subsequently introduced in both directions in a second series of tests. The latter involved favorably balanced creep strain cycles and these tests lasted four times longer on the average than the unbalanced series. Only the strainrange partitioning concept would have suggested the possibility of a life improvement under these conditions; other approaches would have predicted a life reduction.

Structural Analysis and Design Codes

Let us consider now the application to design codes. The strainrange partitioning approach has something to say on this subject as well. It will identify what types of tests should be conducted to properly characterize a material. Not only will it directly provide suggestions as to the types of tests to be run, but the amount of data required might be considerably reduced. For example, if the strainrange-life relations are insensitive to temperature, for materials of interest, higher test temperatures can be used to induce the required creep in less time. The method also suggests that information regarding crack initiation and crack propagation under the different types of strain loadings is required in order to treat all situations that can realistically be encountered.

How to gear the complexity of the analysis with the type of information used in design is also suggested by the strainrange partitioning approach. To illustrate, suppose the most simplified type of analysis is made and that all we can establish from the analysis is the total inelastic strainrange. In that case, if we do not know the nature of the component strainranges, the method suggests that, for conservatism, one would have to use the lowest bounds associated with the most damaging type of strainrange that
could conceivably occur. On the other hand, suppose only a limited analysis is made and it is established that certain strain components are present, but that other types of strainranges are excluded. Then it would be appropriate to use the bounds associated only with the strainrange components present. For example, if there is no $\Delta \varepsilon_{cp}$ present, then one should not use $N_{cp}$ as a lower bound. In connection with continuous strain cycling, the worst situation develops when only the $\Delta \varepsilon_{cc}$ strainrange component is present; then, of course, only the $\Delta \varepsilon_{cc}$ component need be used in first approximation calculations. Finally, if a rigorous analysis is made, the pertinent material characteristics that are required for the analysis are used. Knowing the magnitude of each strainrange component present, it is then possible to determine the life accurately. Of course, the magnitude of each strainrange must first be established. This may require a considerable amount of computation, no different from the computations that would be required for application of other methods of analysis. However, the degree of accuracy required might be considerably reduced when strainrange partitioning is to be applied. Since the total strainrange is often quite accurately known (for example, in thermal stress problems, when the thermal constraint is known), the major purpose of the analysis is to partition the strainrange. But life is not extremely sensitive to small changes in the partitioning; therefore, coarse calculations may suffice. On the other hand, if the summation of the time and cycle fractions are used as a measure of damage, stresses must be determined to a high degree of accuracy because the creep-rupture life is very sensitively related to stress.

**REQUIRED FUTURE PROGRAM**

In the discussions up to now, I have presented a simplified and optimistic approach, hoping in this way to encourage acceptance of the concept proposed and a recognition of its potential usefulness. It is realized, of course, that much needs to be done if the method is to be developed to its full potential. Several directions along which further work is required will now be outlined. Obviously, many materials should be examined to determine the utility, strengths and limitations of this basic method of approach. These should include not only materials of different compositions and of different strengthening mechanisms (e.g., dispersion-strengthened compared to solution strengthened), etc., but materials that have different grain structures.
(directionally solidified, random polycrystalline), eutectics, and other composite structures. Once a large number of materials have been investigated, it may be possible to refine the universalized relations (eqs. (7) to (10)).

Separation of the total cyclic life into its crack initiation and crack propagation phases is another important objective. The laws of crack initiation and propagation must be established for situations involving not only single strainrange components when individually present, but also when several components are collectively present.

We must also learn the most practical methods of partitioning complex strain cycles into their basic components. One method based on an experimental approach has already been mentioned. This method can be extended to consider more complex cases. For example, if other components than \( \Delta \varepsilon_{\text{pp}} \) and \( \Delta \varepsilon_{\text{cc}} \) are present, the same idea can be used, namely by studying the cycle at two frequencies of loading. First, the stresses would be determined experimentally by traversing the temperature and strain history in real time. Then the plastic strain associated with each half of the cycle could be individually established by traversing both the temperature and stress history rapidly to avoid creep. During rapid loading, only plastic strains would develop; subtraction of the plastic strain from the real-time inelastic strain would lead to the creep component. Once the strain components are known for each half of the cycle, the net strainranges for the complete cycle can be determined by properly combining the strains according to the basic definition of each strainrange component. An alternate approach is to seek a rheological model, a model consisting of springs, sliders, and dashpots by which the basic behavior of the material can be simulated. The strains associated with each type of cycle can thus be determined directly from the model. Finally, the more conventional approach is to formulate the basic and complete stress-strain analysis using the constitutive relations and equilibrium and compatibility equations. Analysis is then the same as if other methods of life computation were to be used. The strain components would then be determined—how much is creep? how much is plastic flow? how much is in tension? how much is in compression?—and then they would be combined in a way so as to determine the net values \( \Delta \varepsilon_{\text{pp}}, \Delta \varepsilon_{\text{pp}}, \Delta \varepsilon_{\text{cc}}, \Delta \varepsilon_{\text{cc}}, \) and \( \Delta \varepsilon_{\text{cc}} \) in order to substitute into the life relations.

A problem not yet discussed is what happens when the elastic component is large relative to the inelastic strain.
The cases thus far treated have been those in which most of the strain involved is inelastic. In a sense this problem is analogous to the treatment in the Universal Slopes Equation of the elastic and plastic components. However, in the present case we must consider four inelastic components. When these components are very small, it obviously will not be appropriate to use an experimental approach to measure them, but instead they may be computed by analytical procedures. In a sense, when the analysis is made on the basis of time and cycle fractions, the individual creep strains that occur during the various stress loadings are being computed. The results of such computations might then be applied in connection with strainrange partitioning. The damage rules, the method of combining not only the components that occur within a given cycle of loading but components of strain that vary from cycle to cycle must also be investigated.

Material behavior which changes as a function of time and temperature, oxidation environments, coatings, and radiation must also be extensively investigated. Whether the strainrange partitioning approach is valid when complex histories of numerous variables are involved is, of course, very important. How to use basic data to make effective computations is equally important.

The mechanistic studies, that is, the studies of the deformation and fracture mechanisms involved, still require considerable investigation. We have outlined how the dual mode of deformation can explain reduced life in terms of ratcheting. Obviously, this is a highly simplified picture. We must study more accurately what happens during the deformation process and how these deformation components interact with each other. The environmental effect and how it can introduce behavior patterns that are not anticipated by ordinary analysis is also of interest, and should be considered. A host of other variables, a few of which are multiaxial effects, applications to ratcheting, the effects of mean stress or mean strain, and numerous other parameters must, of course, also be studied. Before this method, or any other, can be accepted as a unified approach much effort will be required; however, a useful start has been made.

CONCLUDING REMARKS

An attempt has been made to present a case for the potential applicability of the strainrange partitioning concept to a host of aspects of the treatment of high-temperature
fatigue. It has been the contention in this Summary talk that this concept provides a framework around which treatment of high-temperature metal fatigue could perhaps be unified. The view presented is admittedly partisan. No attempt has been made to be objective, nor have analogous treatments for alternative points of view been presented. It can, however, be said that the viewpoint expressed here was not hurriedly formulated. Over the years, the author has been associated with various attempts to develop suitable methods of analysis. Already cited is the progression of our efforts, starting with the 10-percent rule and its modifications, the introduction of cyclic-creep curves to be used in conjunction with the summation of time and cycle fractions, and various other methods. The view on the potential of strainrange partitioning comes, therefore, after a considerable amount of concentration on the subject. At this point, I am reasonably convinced of its utility; but, of course, it is recognized that further work is required to bring the method to full fruition. We at the NASA-Lewis Research Center intend to pursue the approach vigorously; hopefully others will join with us in completing the program.
REFERENCES


(a) HYSTERESIS LOOP WITH CREEP AND PLASTICITY.

TENSILE PLASTIC FLOW, COMPRESSIVE CREEP, \( \Delta e_{pc} = 0.0162, N = 264 \).

TENSILE CREEP, COMPRESSIVE PLASTIC FLOW, \( \Delta e_{cp} = 0.0147, N = 15 \).

(c) PHOTOMICROGRAPHS OF FRACTURED SPECIMENS OF 316 STAINLESS STEEL TESTED IN CREEP-FATIGUE. CREEP AT 1300°F (705°C), PLASTIC FLOW AT 600°F (315°C). (Ref. 15.)

(b) SUMMARY OF PARTITIONED STRAINRANGE-LIFE RELATIONS.

(d) LIFE PREDICTABILITY BY STRAINRANGE PARTITIONING.

Figure 1. - Background of strainrange partitioning concept.
EXAMPLE:

\[ \Delta \varepsilon_{\text{in}} = 0.010 \]
\[ \Delta \varepsilon_{\text{pp}} = 0.009, \Delta \varepsilon_{\text{CC}} = 0.001 \]
\[ \alpha_{\text{pp}} = 0.009, \alpha_{\text{CC}} = 0.001 \cdot 0.1 \]

BY CONVENTIONAL DAMAGE RULE:

\[ \frac{1}{N_{\text{pp}}} + \frac{1}{N_{\text{CC}}} + \frac{1}{N_{f}} \]

IN THIS CASE:

\[ \frac{1}{N_{A}} + \frac{1}{N_{B}} + \frac{1}{N_{f}} \]

BY NEW "INTERACTION" DAMAGE RULE:

\[ \frac{F_{\text{pp}} + F_{\text{CC}}}{N_{\text{pp}}} + \frac{F_{\text{pp}}}{N_{\text{CC}}} + \frac{F_{\text{CC}}}{N_{f}} \]

IN THIS CASE:

\[ \frac{0.90 + 0.10}{N_{D}} + \frac{1}{N_{C}} + \frac{1}{N_{f}} \]

IN GENERAL CASE:

\[ \frac{F_{\text{pp}} + F_{\text{CC}}}{N_{\text{pp}}} + \frac{F_{\text{CC}}}{N_{\text{CC}}} + \frac{1}{N_{f}} \]

Figure 2. - Relations for determining life when two or more strainrange components are present.

Figure 3. - Method of partitioning using real-time hysteresis loop and rapid-cycling hysteresis loop between same stress limits. Data for 316 stainless steel tested at 1500°F (815°C), \( \Delta \varepsilon_{\text{in}} = 0.0047 \), triangular strain waveform. Data from ref. 16.
Table: Material, Test Temperature, Data Source

<table>
<thead>
<tr>
<th>Material</th>
<th>Test Temperature</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAR M-200</td>
<td>1700°F (925°C)</td>
<td>NASA (NEW DATA)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1200°F (650°C)</td>
<td>(REF. 17)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1200°F (650°C)</td>
<td>(REF. 17)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1300°F (705°C)</td>
<td>(REF. 1)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1300°F (705°C)</td>
<td>(REF. 1)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1500°F (815°C)</td>
<td>(REF. 17)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1500°F (815°C)</td>
<td>(REF. 17)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1500°F (815°C)</td>
<td>(REF. 17)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1200°F (650°C)</td>
<td>(REF. 21)</td>
</tr>
<tr>
<td>316 SS</td>
<td>1200°F (650°C)</td>
<td>(REF. 21)</td>
</tr>
<tr>
<td>304 SS</td>
<td>1200°F (650°C)</td>
<td>NASA (NEW DATA)</td>
</tr>
<tr>
<td>304 SS</td>
<td>1500°F (815°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>304 SS</td>
<td>1500°F (815°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>304 SS</td>
<td>1200°F (650°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>INCO 800</td>
<td>1000°F (540°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>INCO 800</td>
<td>1200°F (650°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>INCO 800</td>
<td>1400°F (760°C)</td>
<td>(REF. 13)</td>
</tr>
<tr>
<td>ZIRCOLOY-2</td>
<td>570°F (300°C)</td>
<td>(REF. 26)</td>
</tr>
<tr>
<td>ASTAR 811C</td>
<td>2100°F (1150°C)</td>
<td>(REF. 23)</td>
</tr>
</tbody>
</table>

Figure 4. - Tentative universalized relations for the four strainrange components.

Figure 5. - Data correlation using tentative strainrange partitioning life relations.
Figure 6. - Use of strainrange partitioning to establish bounds on life.

Figure 7. - Application of strainrange partitioning to frequency effect for a material with time-dependent creep ductility, A-286 tested at 1100°F (595°C), $\Delta \varepsilon_{\text{in}} = 0.009$. 

(a) 316 STAINLESS STEEL

(b) AGED A-286 TESTED IN AIR AT 1100°F (595°C).

$\Delta \varepsilon_{\text{pp}} = 0.75 D_{\text{pp}} N_{\text{pp}}^{-0.6}$

$\Delta \varepsilon_{\text{CC}} = 0.75 D C N_{\text{CC}}^{-0.8}$

$N_{\text{pp}} = \left( \frac{0.75 D_{\text{pp}}}{\Delta \varepsilon_{\text{pp}}} \right)^{1/0.6}$

$N_{\text{cc}} = \left( \frac{0.75 D C}{\Delta \varepsilon_{\text{cc}}} \right)^{1/0.8}$

<table>
<thead>
<tr>
<th>FREQUENCY, Hz</th>
<th>CYCLES TO FAILURE</th>
</tr>
</thead>
</table>
Figure 8. - Application of strainrange partitioning to data for 304 stainless steel tested at 1200°F (650°C) taken from reference 21.

Figure 9. - Predictability of life relaxation results for tensile strain hold-time cycles. Data on Incoloy 800 and 304 SS from ref. 13 for tensile strain hold-time cycles.

Figure 10. - Effect of unbalanced creep on life relations in vacuum.
Figure 11. Strainrange components produced in edge by several thermal histories.

Figure 12. Potential benefit of temperature phasing.