THEORETICAL ANALYSIS OF SWELLING CHARACTERISTICS OF CYLINDRICAL URANIUM DIOXIDE FUEL PINS WITH A NIOBNIUM - 1-PERCENT-ZIRCONIUM CLAD

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The relations between clad creep strain (hoop and axial) and fuel volume swelling are shown for cylindrical UO₂ fuel pins with a Nb-1Zr clad. These relations were obtained by using the computer code CYGRO-2. These clad-strain - fuel-volume-swelling relations may be used with any fuel-volume-swelling model, provided the fuel volume swelling is isotropic and independent of the clad restraints. The effects of clad temperature (over a range from 1118 to 1642 K (2010° to 2960° R)), pin diameter, clad thickness, and central hole size in the fuel have been investigated. In all calculations the irradiation time was 50 000 hours. The burnup rate was varied.
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SUMMARY

A preliminary theoretical analysis has been made to determine the strains imposed on the cladding of cylindrical fuel pins as a result of fission-induced swelling of the fuel. The CYGRO-2 computer code was used, and fuel swelling values were taken from BUBL-1 calculations. The results apply only to uranium dioxide (UO₂) fuel pins with a niobium - 1-percent-zirconium (Nb-1Zr) clad. The effects of clad temperature, pin diameter, relative clad thickness (ratio of clad wall thickness to clad outside diameter), and relative hole size (ratio of fuel inside diameter to fuel outside diameter) have been investigated.

This analysis was performed to determine the suitability of the UO₂-Nb1Zr fuel-clad combination for use in a proposed compact reactor for space power with an operating life of 50,000 hours. Therefore, the irradiation time in all calculations was 50,000 hours.

The particular relations between clad strain (hoop and axial) and fuel swelling given in this report greatly simplify the analysis of the swelling behavior of UO₂ fuel pins with a Nb-1Zr clad. These relations occur because the fuel is much stronger than the clad; they may not occur when the clad is strong relative to the fuel. Each fuel-clad combination must be considered separately. Any source of swelling values may be used in these clad-strain - fuel-swelling relations provided the fuel swelling rates are isotropic, uniform throughout the fuel, and independent of clad restraint.

The clad hoop strain must be limited to some safe value to prevent interference with coolant flow in the channels between closely spaced fuel pins or to prevent clad rupture. Calculations based on the clad-strain - fuel-swelling relations given in this report show that reactor core size, as determined by the limitation on clad strain, is minimized by using solid fuel pins of the smallest practical diameter. This occurs because the fuel is much stronger than the clad, and the swelling characteristics are not significantly affected by a central hole in the fuel. However, other design considerations, such as fission gas removal or redistribution of the fuel through a vaporization-redeposition process, may require a central hole in the fuel.
INTRODUCTION

The relations between fuel swelling and clad strain are very important in a compact reactor designed for high temperature and long life. The fuel pins are closely spaced in a compact reactor, so excessive radial swelling of the fuel would produce flow interference. Long exposure of clad materials to irradiation generally reduces their ductility. Thus, clad strain must be limited to some safe value to prevent rupture. If the clad ruptures, fuel and fission products will be released and contaminate the reactor coolant loop. Core reactivity may also be adversely affected, and dose rates at points around the reactor would be increased.

The fuel-clad combination considered here is uranium dioxide (UO$_2$) and niobium - 1-percent zirconium (Nb-1Zr). This combination is being studied for application in a proposed compact reactor for space power (ref. 1). This analysis was carried out to determine the clad-strain - fuel-swelling relations for cylindrical fuel elements. The fuel material may be either solid or have a central hole. The computer program CYGRO-2 (refs. 2 and 3) was used to obtain the clad-strain - fuel-swelling relations and the results were compared with experimental data (ref. 4).

The mechanism of fuel swelling is very complicated and is not well understood. A model has been proposed by Warner and Nichols (ref. 5) to predict the free swelling of UO$_2$. These predicted values are independent of fuel geometry. The swelling values used in this analysis were obtained from reference 5 and were used as input data to the CYGRO-2 program. The clad-strain - fuel-swelling relations thus obtained are independent of the source of swelling values. Any swelling model could have been used to obtain these relations, provided the swelling rates are isotropic, uniform throughout this fuel, and independent of the clad restraint.

According to Warner and Nichols, fuel swelling results from the production of gaseous and solid fission products. The gaseous fission products are primarily xenon and krypton. Fission gas bubbles form in the fuel and migrate along the thermal gradient until they reach the center of the fuel, where they release their gas. Saturation in gaseous swelling occurs when the gas generation rate is equal to the gas release rate.

There are several solid fission products formed during irradiation. The swelling from these elements is dependent on their concentration and chemical state. The solid-fission-product swelling rate for UO$_2$ is assumed to be constant over the range of temperatures and burnups considered in this analysis.

This report discusses the effects of various parameters on the clad-strain - fuel-swelling and clad-strain - burnup relations. The parameters considered are (1) clad temperature, (2) clad outside diameter, (3) the ratio of clad wall thickness to clad outside diameter, and (4) the ratio of fuel inside diameter to fuel outside diameter.

Three fuel-pin sizes were considered - 6.350, 9.525, and 12.700 millimeters
outside diameter. The ratio of clad wall thickness to clad outside diameter varied from 0 (no clad) to 0.20. The ratio of fuel inside diameter to fuel outside diameter varied from 0 (solid fuel) to 0.75. And the clad surface temperature varied from 1118 to 1642 K (2010° to 2960° R).

The relations between fuel swelling and clad strain obtained from the analysis of the UO₂-Nb-1Zr pins were used to study an alternate to the reactor described in reference 1.

**CALCULATION MODEL**

A sketch of the cross section of a fuel pin is shown in figure 1. The fuel pin consists of a hollow cylinder of UO₂ fuel surrounded by Nb-1Zr clad. A radial gap exists between the fuel and the clad at room temperature, but this gap is closed during operation at power because of differential thermal expansion between the fuel and the clad. This closed gap results in an interference fit between the clad and the fuel during operation of the reactor. During irradiation, the fuel swells and interacts with the clad. This interaction is affected by fission gas pressure between the fuel and the clad, coolant pressure outside the clad, fission-induced swelling of the fuel, irradiation-induced swelling of the clad, and differential thermal expansion between the fuel and the clad.

In this study, the effects of direct pressure loads on the clad due to fission gas accumulation and coolant pressure were minimized by arbitrarily equating the two. Irradiation-induced clad swelling was assumed to be negligible. Fission-induced UO₂ swelling values were obtained from the work of Warner and Nichols (ref. 5). The fuel-clad interaction caused by fuel swelling (obtained from ref. 5) and differential thermal expansion were calculated by using the CYGRO-2 computer code (ref. 3).

The major assumptions in CYGRO-2 are

1. Axial and azimuthal symmetry of temperature and stresses
2. Steady-state temperature calculations
3. No fuel pellet cracking
4. Fuel-clad contact always operating without slip
5. No mass transfer in fuel
6. Stationary bubble model (bubbles not allowed to migrate)
7. Fuel cylinder may be in radial contact only with the clad. The fuel may never contact the ends of the clad.
8. Stress and strain equations based on the assumption of plane strain

Although fuel swelling can be calculated by using the CYGRO-2 computer code, the swelling model in the code is not expected to be valid under the operating conditions in this reactor because of the amount of fission product gas mobility expected within the fuel.
Fission-induced UO₂ fuel swelling has been calculated (ref. 5) by Warner and Nichols as a function of average fuel temperature and burnup by using the BUBL-1 computer code. This computer code analyzes fission gas bubble nucleation, mobility, and interactions at dislocations and grain boundaries by a Monte Carlo technique.

The average temperature of the UO₂ fuel required to obtain fuel swelling values is given by equation (1). All terms used in this report are defined in appendix A.

\[
\overline{T} = T_w + 109BD_p^2(1 - 2Y)^2\psi \tag{1}
\]

where

\[
\psi = \frac{1}{2} - \frac{3}{2}(\varphi)^2 + \frac{\varphi^4 \ln\left(\frac{1}{\varphi}\right)^2}{1 - \varphi^2} \quad \text{(see fig. 2(b))} \tag{2}
\]

\[
\varphi = \frac{D_i}{D_0}
\]

This equation is derived in appendix B.

This equation for average fuel temperature does not consider the radial temperature drop in the clad. The temperature drop in the clad for the range of values of relative clad thickness Y and burnup B considered here was calculated and was found to be small. Equation (1) can be written in the following form and is plotted for three pin diameters as shown in figure 2:

\[
\frac{\overline{T} - T_w}{B\psi} = 109D_p^2(1 - 2Y)^2 \tag{3}
\]

Fuel swelling results from the production of gaseous and solid fission products. Reference 5 predicts that for a given burnup the amount of gaseous swelling in UO₂ increases with increasing temperature to about 1773 K (3190° R). Above this temperature, saturation of gaseous swelling occurs. Saturation of gaseous swelling occurs when the fission gas production rate is equal to the gas release rate. After saturation occurs, further swelling is caused by the solid fission products only. The solid volume expansion rate in UO₂ has been calculated to be 0.13 percent \(\Delta V/V\) per \(10^{20}\) fissions per cubic centimeter (ref. 6). This value is used in reference 5.

The relations between fuel swelling and fuel temperature calculated in reference 5 for burnups from 1 to 8 percent is shown in figure 3. These curves are based on a uniform fuel temperature. In this analysis the average fuel temperature \(\overline{T}\) calculated by
using equation (1) is assumed to be equivalent to the uniform temperature used in reference 5. The curves in figure 3 must not be extrapolated past the temperature at which saturation of swelling occurs (about 1773 K).

Some attempts have been made to verify this model of fuel swelling and clad deformation by comparison with experiments. However, there is only a meager amount of experimental clad-strain - fuel-swelling data (ref. 7) for this particular combination of fuel and clad.

Irradiation data on 0.635-centimeter- (1/4-in.-) diameter UO$_2$ fuel pins clad with W-26Re are reported in reference 4. One specimen (number 342 in capsule HT-BRR-9) irradiated to about 0.7 percent burnup at about 1600 K exhibited about 6 percent fuel swelling and 2.2 percent hoop strain in the clad. The volume swelling obtained from figure 3(a) is 4 percent. The clad hoop strains calculated with this model are 1.14 and 1.52 percent for 4.0 and 6.0 percent volume swelling.

This model neglects the affect of clad restraint on fuel swelling and would not be expected to be a valid model with a combination of strong clad and weak fuel. However, the creep strength of the Nb-1Zr clad ($1.52 \times 10^7$ N/M$^2$) is much less than that of the UO$_2$ ($7.52 \times 10^7$ N/M$^2$) for 1.0 percent strain in 50 000 hours at 1228 K (2210° R). Thus, for the range of parameters investigated in this report, the clad has little effect on fuel swelling.

The thermophysical properties of the UO$_2$ fuel and Nb-1Zr clad are given in appendix C. Changes in properties due to irradiation are neglected. Both first- and second-stage creep strains are included.

**PARAMETRIC ANALYSIS OF FUEL PINS**

Several curves have been plotted directly from the CYGRO-2 calculations and are presented in this section. These curves are also compared with experimental data and with the results of an analysis (see appendix D) where the fuel is treated as a rigid body with respect to external loads and the swelling is uniform throughout the fuel and isotropic.

An examination of the results of the CYGRO-2 calculations shows that the relations between clad strain (hoop, $\Delta D/D$; and axial, $e_z$) and fuel swelling $\Delta V/V$ are very nearly linear for burnups to 8 percent. The irradiation time in these calculations was 50 000 hours. (The fuel-pin parameters analyzed in this report by using CYGRO-2 are given in table I.) These relations between clad strain (hoop and axial) and fuel swelling do not differ significantly over the range of parameters given in table I for a clad temperature range of 1118 to 1642 K (2010° to 2960° R). The parameters given in table I give a fuel volume fraction that varies from a maximum of 63 percent to a minimum of
15 percent for a relative pin spacing $S/D_p$ of 1.10.

The relation between clad hoop strain $\Delta D/D$ and fuel volume swelling $\Delta V/V$ with relative clad thickness $Y$ as the parameter is given in figure 4(a). For an unclad fuel pin ($Y = 0$), the slope of the curve is 1/3 for small values of fuel swelling (about 5 percent or less). The slope of the curve decreases slightly as the fuel swelling increases. In this case ($Y = 0$), $\Delta D/D$ is the diametrical expansion of the fuel.

The slope of the curves relating clad hoop strain (at the clad outside diameter) to fuel swelling decreases as the relative clad thickness increases. Thus, for a given amount of fuel swelling, the hoop strain at the clad outside diameter decreases as the relative clad thickness increases.

An analysis using equations developed in reference 8 confirms that the relation between clad hoop strain and fuel swelling is approximately linear and that the slope decreases with increasing values of relative clad thickness. The analysis also shows that, for a given relative clad thickness, the clad hoop strain is maximum at the inside diameter and minimum at the outside diameter. Also, the clad hoop strain at the outside diameter decreases with increasing relative clad thickness for a given amount of fuel swelling.

Calculations were made to determine the effects of changes in clad temperature on the preceding relation. The clad temperature was varied from 1118 to 1642 K (2010° to 2960° R). The same values of $\Delta V/V$ for each case were used at all temperatures. Keeping the fuel swelling constant while varying the temperature is accomplished by varying the burnup (see fig. 3). The calculations showed that, for a given amount of fuel swelling $\Delta V/V$, clad strain (hoop and axial) is independent of temperature over the preceding temperature range.

The experimental data cited in the previous section are for a large relative clad thickness ($Y = 0.355$), which prevented any radial growth of the clad ($\Delta D/D = 0$). This model (fig. 4(a)) predicts very small values of clad hoop strain for large values of relative clad thickness and moderate values of fuel swelling. Thus, the data of reference 7 tend to confirm the clad-hoop-strain - fuel-swelling curves of figure 4(a).

As noted earlier in this section, an analysis was performed where the fuel was treated as a rigid body with respect to external loads. This analysis (appendix D) shows that the relation between clad hoop strain and fuel swelling was independent of the pin diameter and the size of the central hole in the fuel. This relation is approximately linear for the range of values of relative clad thickness (0.04 to 0.20) and fuel swelling (to 32 percent) considered.

In figure 4(b) the results of the rigid-body analysis are compared with the curves obtained from the CYGRO-2 calculations (fig. 4(a)). For an unclad fuel pin ($Y = 0$), the rigid-body analysis and CYGRO-2 give identical results for both diametrical ($\Delta D/D$) and axial ($e_z$) expansion of the fuel. For a relative clad thickness from 0.04 to 0.16, fig-
ure 4(b) shows that the clad hoop strains obtained from the rigid-body analysis are lower than the CYGRO-2 values. The maximum difference is about 6.7 percent. For a relative clad thickness of 0.20, the clad hoop strains obtained from the rigid-body analysis are less than the CYGRO-2 values for fuel swelling values of 16 percent or less. For higher fuel swelling values, the rigid-body values exceed the CYGRO-2 values. For 30 percent fuel swelling, there is a difference of about 21 percent.

The relation between axial clad strain $e_z$ and fuel swelling $\Delta V/V$ is shown in figure 4(c). The theoretical axial strain curve obtained from the rigid-body analysis is also shown. The axial clad strain values calculated by CYGRO-2 can be represented by two straight lines. The two lines intersect at 12.5 percent $\Delta V/V$. For values of $\Delta V/V$ less than 12.5 percent, the curve obtained from the CYGRO-2 calculations agrees very well with the rigid-body model. And for values of $\Delta V/V$ greater than 12.5 percent, the CYGRO-2 values are slightly less than the rigid-body model. The reduction in axial strain at the higher $\Delta V/V$ values appears to be associated with a significant reduction of the clad wall thickness caused by plastic flow in the clad. Note that the axial clad strain is a function of fuel volume swelling only. It is independent of the clad diameter, the clad thickness, and the size of the central hole in the fuel.

An examination of the CYGRO-2 calculations shows that the principal strains in the fuel are approximately equal. Since isotropic forced swelling values were used as input to the CYGRO-2 program, it is apparent that the clad did not significantly affect the swelling behavior of the fuel. Therefore, the swelling characteristics of the fuel can be considered independent of clad restraint for this fuel-clad combination. Thus, the rigid-body analysis gives clad strain values that agree well with the CYGRO-2 values over most of the range of parameters considered herein. The simple relations between clad strain and fuel swelling given herein will not occur for all fuel-clad combinations. Each combination must be considered separately.

We will now discuss fuel-pin behavior, with burnup as the independent variable rather than fuel volume swelling. This change of variables is being made because burnup is a more useful parameter in reactor analysis. Burnup can be related to core size and control requirements. It can also be used to determine the average fuel temperature in a pin. And when the burnup and the average fuel temperature are known, the fuel volume swelling can be determined.

The relation between clad hoop strain and burnup for 6.350-, 9.525-, and 12.700-millimeter- (0.250-, 0.375-, 0.500-in.-) diameter fuel pins is shown in figures 5 to 10. The clad temperature for these curves is 1228 K ($2210^\circ$ R). This is the clad temperature for the proposed reactor mentioned previously.

The curves presented in figures 5 to 7 are plotted for given values of relative clad thickness and with relative hole size as the parameter. These curves show that for a given burnup the clad hoop strain decreases as the relative hole size $\varphi$ increases. A
solid pin gives the highest hoop strain. The clad hoop strain decreases as the relative hole size increases because increasing the relative hole size reduces the average fuel temperature (burnup constant). This in turn reduces the fuel swelling and the clad hoop strain.

The relation between clad hoop strain and burnup is also shown in figures 8 to 10 but in a somewhat different manner. In these figures, the relative clad thickness \( Y \) is the parameter. Again, the clad temperature is 1228 K \( (2210^0 R) \). These curves show that for a given burnup, clad hoop strain decreases with increasing relative clad thickness.

The reduction in clad hoop strain (at the clad outside diameter) occurs because of the effects described earlier in discussing the results illustrated in figure 4(a). Increasing the relative clad thickness at constant burnup also decreases the average fuel temperature and reduces the fuel swelling.

**EFFECTS OF FUEL-PIN DESIGN PARAMETERS ON ALLOWABLE BURNUP**

One of the important parameters in fuel-pin design is clad strain. The strain must be limited to some safe value to prevent clad rupture. This safe value is determined by the clad's physical properties, temperature, and operating time. Excessive radial swelling in a compact reactor with closely spaced fuel pins would produce flow interference. Excessive fuel-pin swelling would also require large radial clearances between the fuel pins and their lateral restraints (ref. 1). Thus, the thermal bowing of the fuel pins could be large enough to affect the cooling of the pins and the control of the reactor.

One criterion for maximum allowable clad strain is hoop strain \( \Delta D/D_0 \). This criterion may not be adequate when there is considerable axial strain. However, sufficient data are not available to evaluate biaxial clad creep rupture in an irradiation environment. In the discussion that follows, clad hoop strain is considered to be the controlling parameter. Two values of allowable hoop strain \( (0.5 \text{ and } 1.0 \text{ percent}) \) were arbitrarily selected for this discussion.

Clad strain is not a useful parameter in reactor design calculations. Therefore, clad strain is discussed here in terms of the burnup required to produce a given clad strain rather than in terms of clad strain itself. So the effects of the parameters relative hole size \( \varphi \) and relative clad thickness \( Y \) on burnup for the two values of clad hoop strain given are now discussed. The clad temperature is 1228 K \( (2210^0 R) \). This is the clad temperature of the proposed fast reactor under study. A value of 1.10 for the relative pin spacing \( S/D_p \) was selected. This appears to be a reasonable value to avoid any undesirable flow or temperature distributions in the core.

The fuel volume fraction \( V_F \) of a unit cell in a reactor core with equilateral pin
spacing is given by the following equation (see appendix E):

\[ V_F = \frac{0.907}{2} \left( \frac{S}{D_p} \right)^2 (1 - 2Y)^2 (1 - \varphi^2) \]  \hspace{1cm} (4)

This equation is for 100-percent-dense fuel. The leading constant is reduced proportionately for lower fuel densities. A plot of equation (4) is given in figure 11 for a relative pin spacing of 1.10. The fuel volume fraction for other relative pin spacings can be obtained easily from figure 11.

Fuel volume fraction is a function of two pin parameters: relative hole size and relative clad thickness. In figure 11(a) the fuel volume fraction is plotted as a function of relative hole size with relative clad thickness as the parameter. Figure 11(a) shows that for any value of relative hole size, the fuel volume fraction decreases as the relative clad thickness increases. In figure 11(b) the fuel volume fraction is plotted as a function of relative clad thickness with relative hole size as the parameter. Figure 11(b) shows that for any value of relative clad thickness, the fuel volume fraction decreases as the relative hole size increases. It is apparent that a solid pin with the thinnest possible clad will give the maximum fuel volume fraction.

A useful way to determine the effects of the fuel-pin design parameters presented in this analysis is to plot allowable burnup (the burnup required to produce a given clad strain) as a function of fuel volume fraction. The fuel volume fraction is a parameter used in reactor design calculations and is indicative of the fuel loading in a unit cell.

The relation between allowable burnup and fuel volume fraction was plotted for pin diameters of 6.350, 9.525, and 12.700 millimeters (0.250, 0.375, and 0.500 in.) and clad hoop strains of 0.5 and 1.0 percent. Figures 12 and 13 show this relation with relative hole size and relative clad thickness, respectively, as the parameter. These curves were obtained by using figures 5 to 11.

The effects of changing relative clad thickness \( Y \) and holding the relative hole size \( \varphi \) constant can be determined by examining figure 12. Note that when the relative hole size and the relative pin spacing \( S/D_p \) are fixed (as for the curves in fig. 12), the fuel volume fraction is a function of relative clad thickness only (see fig. 11(a)). Figure 11(a) shows that the fuel volume fraction decreases as the relative clad thickness increases.

Figure 12 shows that the allowable burnup increases as the fuel volume fraction decreases (relative clad thickness increases). An examination of figure 4(a)) shows that, for a given clad hoop strain, the allowable fuel swelling increases with increasing values of relative clad thickness. As noted in the previous section, increasing the relative clad thickness decreases the average fuel temperature which decreases the fuel
swelling. And for a given value of fuel swelling, increasing the relative clad thickness decreases the hoop strain at the clad outside diameter. Therefore, the allowable burnup must increase to produce the same clad strain.

The effects of changing the relative hole size in the fuel and holding the relative clad thickness constant can be determined by examining figure 13. With the relative clad thickness and relative pin spacing fixed (as in fig. 13), the fuel volume fraction is a function of relative hole size only. Thus, the fuel volume fraction is maximum when there is no hole in the center of the fuel \((\varphi = 0)\). This is shown by the right-hand end points of the parametric curves in figure 13.

The slopes of the curves in figure 13 are relatively flat. The swelling of these pins is weakly restrained, and the only effect is the change in fuel swelling due to the change in average fuel temperature. The change in average fuel temperature increases with pin diameter.

A comparison of figures 12 and 13 shows that allowable burnup is affected less by changes in relative hole size than by changes in relative clad thickness. It was shown earlier that the relation between clad hoop strain and fuel swelling is essentially independent of relative hole size and strongly dependent on relative clad thickness. The effects of changes in relative hole size and relative clad thickness on burnup are discussed in the preceding section, PARAMETRIC ANALYSIS OF FUEL PINS.

There is also a size effect that should be noted. The burnup required for given values of clad hoop strain, relative clad thickness, and relative hole size decreases as the pin diameter increases, as shown in figures 12 and 13. The radial temperature gradient in the fuel increases as the pin diameter increases. Since the clad wall temperature is fixed, the internal fuel temperatures must increase. This raises the average temperature of the fuel. Fuel swelling is a function of burnup and average fuel temperature (fig. 3). Thus, the burnup required to produce a given clad strain (or fuel swelling) decreases as pin diameter increases.

Space power reactors must be as compact as possible. Factors that affect size are power, operating life, and allowable burnup. Core size decreases as allowable burnup increases for constant total power and life. Since allowable burnup is influenced by relative clad thickness \(Y\), relative hole size \(\varphi\), and pin diameter, it is necessary to know what the best combination is - a thick clad and no hole (or a small hole), or a thin clad and a large hole. The influence of these variables \((Y, \varphi)\) on allowable burnup for two pin diameters is shown in table II. These values were obtained by using figures 11(b) and 12. A fuel volume fraction of 50 percent was arbitrarily selected.

It is apparent from table II that allowable burnup is increased by thickening the clad and reducing (or eliminating) the central hole. The size effect mentioned previously is also shown.
CONCLUDING REMARKS

The simple relations between clad strain and fuel swelling given in this report greatly simplify the analysis of the swelling behavior of UO₂ fuel pins with a Nb-1Zr clad. These relations occur because the fuel is much stronger than the clad; they may not occur when the clad is strong relative to the fuel. Each fuel-clad combination must be considered separately. The results given herein apply only to this particular fuel-clad combination.

The relations between clad strain (hoop and axial) and fuel swelling are independent of clad temperature over a range from 1118 to 1642 K (2010° to 2910° R). The clad hoop strain can be represented as a linear function of fuel swelling, with the ratio of clad thickness to clad outside diameter as a parameter. As this ratio increases (the clad becomes thicker), the slope of the curves of hoop strain as a function of fuel swelling decreases. This relation is independent of the pin diameter and the size of the central hole for the range of values considered.

The clad axial strain can also be represented as a linear function of fuel swelling. This relation is independent of the pin diameter, the clad thickness, and the size of the central hole in the fuel over the range of values considered. This linear relation is not continuous. The slope decreases at 12.5 percent fuel volume swelling. This change in slope appears to be caused by necking of the clad wall.

A particular fuel swelling model was used in the calculations to obtain these relations. But the relations between clad strain and fuel swelling are independent of the fuel swelling model used. Any source of swelling values may be used to determine the corresponding clad strains, provided the swelling characteristics are independent of the clad restraints.

An analysis was also made treating the fuel as a rigid body with respect to external loads. It was found that the relation between clad hoop strain and fuel swelling was independent of the pin diameter and the size of the central hole in the fuel. This relation is approximately linear for the range of values of relative clad thickness and fuel swelling considered.

Clad hoop strain values obtained from the rigid-body analysis are lower than the CYGRO-2 curves for a relative clad thickness from 0.04 to 0.16. The maximum difference is about 6.7 percent. For a relative clad thickness of 0.20, the clad hoop strains obtained from the rigid-body analysis are less than the CYGRO-2 values for fuel swelling values of about 16 percent or less. For higher fuel swelling values, the rigid-body values exceed the CYGRO-2 values. For a fuel swelling of 30 percent, there is a difference of about 21 percent.

Reactor size is a prime consideration in the design of a space nuclear power system. And the fuel pins must be designed to minimize core size. This is done by making
both the fuel volume fraction of a unit cell in the core and the allowable burnup as large as possible.

Calculations show that for 1.0 percent clad hoop strain and an arbitrarily selected value of fuel volume fraction of 50 percent, allowable burnup is 3.20 percent for a 6.350-millimeter- (0.250-in.-) diameter pin and 2.45 percent for a 12.700-millimeter- (0.500-in.-) diameter pin. The allowable burnup for a hollow pin with a relative hole size of 0.50 is 2.20 percent for a 6.350-millimeter- (0.250-in.-) diameter pin and 1.90 percent for a 12.700-millimeter- (0.500-in.-) diameter pin. Clad temperature is held constant at 1228 K (2210° R).

It is apparent from the preceding discussion that a solid pin of the smallest practical diameter is the most desirable. But in some cases a central hole may be required. The fission gas bubbles form in the fuel and migrate along the radial thermal gradient until they reach the center of the fuel, where they are released. Also, at high fuel temperatures a central void may form in a solid pin through a vaporization-redeposition process. It may be desirable to design a central hole in the fuel in order to lower the temperature and reduce the vaporization-redeposition process. There are many other factors involved in the design of a minimum-size reactor core than have been discussed herein. A few of these factors are (1) the number of fuel pins required; the number of pins increases as the pin size decreases; (2) fabrication feasibility of fuel pins and support structure; (3) maintainence of critical tolerances in individual fuel pins and fuel-pin assemblies; (4) design and fabrication of core support structure; (5) pressure-drop, flow-distribution, and heat-transfer characteristics of the core.

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APPENDIX A

SYMBOLS

B \quad \text{fuel burnup in 50 000 hr, percent}

D_i \quad \text{inside diameter of fuel pellet, cm (in.)}

D_o \quad \text{outside diameter of fuel pellet, cm (in.)}

D_p \quad \text{outside diameter of clad, cm (in.)}

\Delta D_i \quad \text{change in inside diameter of fuel pellet, cm (in.)}

\Delta D_o \quad \text{change in outside diameter of fuel pellet, cm (in.)}

\Delta D_p \quad \text{change in outside diameter of clad, cm (in.)}

\Delta D/D \quad \text{hoop strain in clad at outside diameter}

e_z \quad \text{axial strain in clad}

k \quad \text{thermal conductivity of fuel, W/cm-K(Btu/hr-ft-°F)}

L \quad \text{length of reactor core, cm (in.)}

l \quad \text{axial length of fuel or clad, cm (in.)}

\Delta l \quad \text{change in axial length of fuel or clad, cm (in.)}

q \quad \text{internal heat generation rate of fuel, W/cm^3(Btu/hr-ft^3)}

R_i \quad \text{inside radius of fuel pellet}

R_o \quad \text{outside radius of fuel pellet}

R_p \quad \text{outside radius of clad, cm (in.)}

r \quad \text{radius of any point in fuel pellet}

S \quad \text{spacing of fuel pins on a triangular pitch, cm (in.)}

S/D_p \quad \text{relative pin spacing}

T \quad \text{temperature in rod at some distance } r \text{ from center}

\overline{T} \quad \text{mean radial temperature of fuel pin, K (°R)}

T_w \quad \text{temperature of clad at outside diameter, K (°R)}

t \quad \text{clad thickness, cm (in.)}

V_F \quad \text{fuel volume fraction of unit cell in core, percent}

V'_F \quad \text{volume of fuel in a unit cell}
\( V_T \) \hspace{1em} \text{volume of a unit cell}

\( \Delta V/V \) \hspace{1em} \text{fuel volume change caused by solid and gaseous fission products}

\( Y \) \hspace{1em} \text{ratio of clad wall thickness to clad outside diameter, } t/D_p

\( \varphi \) \hspace{1em} D_i/D_o

\( \psi \) \hspace{1em} \text{defined by eq. (B11)}
APPENDIX B

DERIVATION OF AVERAGE TEMPERATURE IN FUEL PIN

The differential equation for steady-state axisymmetric heat transfer in a cylindrical rod with an internal heat source and no axial heat transfer is

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = - \frac{q}{k} \tag{B1}$$

The general solution of this equation is

$$T = \frac{qr^2}{4k} + C_1 \ln r + C_2 \tag{B2}$$

where $T$ is the temperature in the rod at some distance $r$ from the center. The constants $C_1$ and $C_2$ are evaluated for the following boundary conditions:

$$\frac{dT}{dr} = 0 \quad \text{at} \quad r = R_i$$

$$T = T_w \quad \text{at} \quad r = R_o$$

The temperature drop in the clad is neglected. Thus, the particular solution for the radial temperature distribution in a cylindrical rod with an internal heat source is

$$T = T_w + \frac{q}{4k} \left( \frac{R_o^2 - r^2}{R_i^2} - 2R_i^2 \ln \frac{R_o}{R_i} \right) \tag{B3}$$

The average radial temperature of the rod is determined by the following relation:

$$\overline{AT} = \int_A T \, dA \tag{B4}$$

where $dA = 2\pi r \, dr$. So
After solving the equation (B5), the following equation for $\overline{T}$ is obtained:

$$\overline{T} = T_w + \frac{qR_o^2}{4k} \left[ \frac{1}{2} - \frac{3}{2} \left( \frac{R_i}{R_o} \right)^2 + \frac{\left( \frac{R_i}{R_o} \right)^4 \ln \left( \frac{R_o}{R_i} \right)^2}{1 - \left( \frac{R_i}{R_o} \right)^2} \right]$$

But

$$R_p = R_o + t$$

or $D_p = D_o + 2t$. Thus,

$$R_o^2 = \frac{D_o^2}{4} = \frac{(D_o - 2t)^2}{4}$$

By squaring the preceding term and letting $Y = t/D_p$, the following expression is obtained:

$$R_o^2 = \frac{D_p^2}{4} (1 - 2Y)^2$$

The equation for $\overline{T}$ may now be written in the following form:

$$\overline{T} = T_w + \frac{qD_p^2}{16k} (1 - 2Y)^2 \psi$$

where
An average value of the thermal conductivity \( k \) of the fuel is assumed to be 0.0251 W/cm-K (1.45 Btu/hr-ft-°F).

The volumetric heat generation rate in the fuel for 50,000 hours of reactor operation is

\[
q = 43.9 \text{ B W/cm}^3 \tag{B12}
\]
or

\[
q = 4.23 \times 10^6 \text{ B Btu/hr-ft}^3 \tag{B13}
\]

Therefore, the final form of equation (B10) is

\[
\overline{T} = T_w + 109BD_p^2\psi(1 - 2Y)^2 \tag{B14}
\]

where \( D_p \) is in centimeters and the temperature in kelvin, or

\[
\overline{T} = T_w + 1266BD_p^2\psi \tag{B15}
\]

where \( D_p \) is in inches and the temperature is in degrees R.
APPENDIX C

THERMOPHYSICAL PROPERTIES OF FUEL (UO₂) AND CLAD (Nb-1Zr)

Coefficient of Thermal Expansion

Fuel. - The coefficient of thermal expansion of the fuel is

\[ \alpha = [6.797 + 0.00579(T - 273)] \times 10^{-6} \]

where \( \alpha \) is in cm/cm/K and \( T \) is in kelvin. (See ref. 9.)

Clad. - The thermal expansion of the clad is as follows:

<table>
<thead>
<tr>
<th>Temperature, K (°R)</th>
<th>Coefficient of thermal expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>294 (530)</td>
<td>2.28×10^{-6} (4.1×10^{-6})</td>
</tr>
<tr>
<td>1368 (2460)</td>
<td>2.50×10^{-6} (4.5×10^{-6})</td>
</tr>
</tbody>
</table>

(See ref. 10.)

Modulus of Elasticity

Fuel. - The modulus of elasticity of the fuel is as follows:

<table>
<thead>
<tr>
<th>Temperature, K (°R)</th>
<th>Modulus of elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N/m²</td>
</tr>
<tr>
<td>294 530</td>
<td>19.30×10^{10}</td>
</tr>
<tr>
<td>1073 1472</td>
<td>16.55</td>
</tr>
</tbody>
</table>

(See ref. 11.)

Clad. - The modulus of elasticity of the clad is as follows:
Temperature

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Modulus of elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>°R</td>
</tr>
<tr>
<td>297</td>
<td>530</td>
</tr>
<tr>
<td>1645</td>
<td>2960</td>
</tr>
<tr>
<td>1747</td>
<td>3160</td>
</tr>
<tr>
<td>1922</td>
<td>3460</td>
</tr>
</tbody>
</table>

(See ref. 12.)

**Thermal Conductivity**

**Fuel.** - The thermal conductivity of the fuel is

$$k = \frac{1}{11.75 + 0.0235(T - 273)} \text{ W/cm-K}$$

where $T$ is in kelvin. (See ref. 13.)

**Clad.** - The thermal conductivity of the clad is as follows:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Thermal conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>°R</td>
</tr>
<tr>
<td>297</td>
<td>530</td>
</tr>
<tr>
<td>422</td>
<td>760</td>
</tr>
<tr>
<td>700</td>
<td>1260</td>
</tr>
<tr>
<td>1369</td>
<td>2460</td>
</tr>
</tbody>
</table>

(See ref. 10.)

**Poisson's Ratio**

Poisson's ratio is assumed to be 0.3 for both fuel and clad and to be independent of temperature.
Creep Rate

**Fuel.** - The creep rate of the fuel is

\[ \dot{\varepsilon} = B \sigma^{4.5} e^{-Q/RT} \]

where

- \( \dot{\varepsilon} \) creep rate, hr\(^{-1}\)
- \( B \) \( 4.13 \times 10^{-10} \) in./hr-lbf
- \( \sigma \) applied stress, psi
- \( Q \) \( 364 \, 008 \) J/mole (87 000 cal/mole)
- \( R \) \( 3.3144 \) J/mole/K (1.9872 cal/mole/K)
- \( T \) temperature, K

(See ref. 14.)

**Clad.** - The creep properties of the clad are as follows:

<table>
<thead>
<tr>
<th>Larson-Miller parameter</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N/m(^2)</td>
</tr>
<tr>
<td>38</td>
<td>( 4.35 \times 10^7 )</td>
</tr>
<tr>
<td>42</td>
<td>2.07</td>
</tr>
<tr>
<td>44</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Creep properties are for 1.0 percent strain. (See ref. 15.) The Larson-Miller parameter is given by \( T(15 + \log t) \times 10^{-3} \). The temperature is in degrees R.

**Stress**

**Fuel.** - The stress of the fuel is as follows:
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Ultimate tensile stress</th>
<th>Yield stress</th>
<th>Plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>°R</td>
<td>kg/cm$^2$</td>
<td>psi</td>
</tr>
<tr>
<td>300</td>
<td>540</td>
<td>1100</td>
<td>15650</td>
</tr>
<tr>
<td>1365</td>
<td>2460</td>
<td>1360</td>
<td>19350</td>
</tr>
<tr>
<td>1591</td>
<td>2860</td>
<td>1550</td>
<td>22000</td>
</tr>
<tr>
<td>1923</td>
<td>3460</td>
<td>1000</td>
<td>14220</td>
</tr>
</tbody>
</table>

(See ref. 16.)

**Clad.** - The stress of the clad is as follows:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Yield stress$^a$</th>
<th>Tensile stress</th>
<th>Elongation in 2.54 cm (1 in.), percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>°R</td>
<td>N/m$^2$</td>
<td>psi</td>
</tr>
<tr>
<td>297</td>
<td>535</td>
<td>2.44×10$^8$</td>
<td>35400</td>
</tr>
<tr>
<td>1221</td>
<td>2200</td>
<td>1.62</td>
<td>23500</td>
</tr>
<tr>
<td>1368</td>
<td>2460</td>
<td>1.43</td>
<td>20700</td>
</tr>
</tbody>
</table>

$^a$0.2 Percent offset.

(See ref. 12.)
APPENDIX D

RIGID-BODY ANALYSIS OF FUEL PINS

If the fuel is treated as a rigid body with respect to external loads, the strength of the clad has no effect on the isotropic swelling of the fuel. Thus, the swelling of the fuel can be considered independently of the clad.

The relation between fuel volume change and the dimensional changes of the fuel is given by

\[
\frac{\Delta V}{V} = \frac{[(D_0 + \Delta D_0)^2 - (D_1 + \Delta D_1)^2](l + \Delta l) - (D_0^2 - D_1^2)l}{(D_0^2 - D_1^2)l}
\]  

(D1)

Expanding this equation and canceling like terms gives

\[
\frac{\Delta V}{V} = \left[1 + 2\left(\frac{\Delta D_0}{D_0} - \frac{\Delta D_1}{D_1}\right) + \frac{\Delta D_0^2}{D_0^2} - \frac{\Delta D_1^2}{D_1^2}\right] \left(1 + \frac{\Delta l}{l}\right) - 1
\]  

(D2)

The swelling of the fuel is assumed to be uniform throughout the fuel and isotropic. Thus,

\[
\frac{\Delta D_0}{D_0} = \frac{\Delta D_1}{D_1} = \frac{\Delta l}{l}
\]

Substituting these relations into equation (D2) gives

\[
\frac{\Delta V}{V} = \left[1 + 2\frac{\Delta D_0}{D_0} + \left(\frac{\Delta D_0}{D_0}\right)^2\right] \left(1 + \frac{\Delta D_0}{D_0}\right) - 1
\]

(D3)

But
Thus,\[
\frac{\Delta V}{V} = \left( 1 + \frac{\Delta D_0}{D_0} \right)^3 - 1 \quad \text{(D4)}
\]
or\[
\frac{\Delta D_0}{D_0} = \left( 1 + \frac{\Delta V}{V} \right)^{1/3} - 1 \quad \text{(D5)}
\]

Also\[
e_z = \left( 1 + \frac{\Delta V}{V} \right)^{1/3} - 1 \quad \text{(D6)}
\]

The distortion of the clad is now considered. The clad is assumed not to swell, and the fuel and clad are assumed to be in radial contact. Thus, the inside diameter of the clad is equal to the outside diameter of the fuel:

\[
0 = \left[ (D_p + \Delta D_p)^2 - (D_o + \Delta D_o)^2 \right] (1 + \Delta l) - \left( D_p^2 - D_o^2 \right) l 
\quad \text{(D7)}
\]

Expanding this equation and canceling like terms gives

\[
0 = \left[ 1 + 2 \left( \frac{\Delta D_p}{D_p} \frac{\Delta D_o}{D_o} + D_p^2 \left( \frac{\Delta D_p}{D_p} \right)^2 - D_o^2 \left( \frac{\Delta D_o}{D_o} \right)^2 \right) \frac{D_p^2 - D_o^2}{D_p^2 - D_o^2} \right] (1 + \frac{\Delta l}{l}) - 1 \quad \text{(D8)}
\]
or

23
where $\Delta l/l$ is now the clad axial strain.

Assume there is no slipping between the fuel and the clad. (This assumption is also made in the CYGRO-2 program.) Thus, the axial strains in the fuel and clad are equal. Also let $\Delta D/D = \Delta D_p/D_p$. Thus,

$$0 = \left\{ \frac{D_p^2}{D_p^2 - D_0^2} \left[ 2 \frac{\Delta D_p}{D_p} + \left( \frac{\Delta D_p}{D_p} \right)^2 \right] - \frac{D_0^2}{D_0^2} \left[ 2 \frac{\Delta D_0}{D_0} + \left( \frac{\Delta D_0}{D_0} \right)^2 \right] \right\} \left( 1 + \frac{\Delta l}{l} \right) - 1 \quad (D9)$$

The equation for clad hoop strain can now be expressed in the following form by using equations (D5) and (D12):

$$0 = \left[ 1 - \left( \frac{D_0}{D_p} \right)^2 \right] \left( \frac{\Delta D_0}{D_0} \right) \left[ \left( 1 + \frac{\Delta D}{D} \right)^2 - 1 \right] - \left( \frac{D_0}{D_p} \right)^2 \left[ \left( 1 + \frac{\Delta D_0}{D_0} \right)^2 - 1 \right] \quad (D10)$$

or

$$\left( 1 + \frac{\Delta D}{D} \right)^2 = 1 + \left( \frac{D_0}{D_p} \right)^2 \left[ \left( 1 + \frac{\Delta D_0}{D_0} \right)^3 - 1 \right] - \frac{\Delta D_0}{D_0} \left( 1 + \frac{\Delta D_0}{D_0} \right) \quad (D11)$$

$$D_0 = D_p - 2t$$

$$\frac{D_0}{D_p} = 1 - 2Y \quad (D12)$$

The equation for clad hoop strain can now be expressed in the following form by using equations (D5) and (D12):
\[
\frac{\Delta D}{D} = \left\{ \frac{(1 - 2Y)^2}{1 + (1 - 2Y)^2} \left[ \frac{\Delta V}{V} \right] - \left( \frac{1 + \Delta V}{V} \right)^{1/3} - 1 \right\}^{1/2} - 1
\] (D13)
APPENDIX E

DERIVATION OF FUEL VOLUME FRACTION OF A UNIT CELL

This derivation is for a reactor core with equilateral pin spacing. A sketch of a unit cell is shown in figure 14. The axial length of the cell is unity.

The volume of the unit cell $V_T$ is given by the following equation:

$$V_T = \frac{\sqrt{3} \, s^2}{4} \quad \text{(E1)}$$

The volume of fuel $V'_F$ is the unit cell is

$$V'_F = \frac{\pi (D_o^2 - D_i^2)}{8} \quad \text{(E2)}$$

The fuel volume fraction $V_F$ is defined as the volume of fuel in a unit cell. Thus,

$$V_F = \frac{V'_F}{V_T} = \frac{0.907(D_o^2 - D_i^2)}{S^2} \quad \text{(E3)}$$

or

$$V_F = \frac{0.907 \, D_o^2}{S^2} \left[ 1 - \left(\frac{D_i}{D_o}\right)^2 \right] \quad \text{(E4)}$$

The final form of this equation is obtained by using the following expressions:

$$\phi = \frac{D_i}{D_o}$$

$$D_p = D_o + 2t$$

$$Y = \frac{t}{D_p}$$
Therefore,

\[ V_F = \frac{0.907(1 - 2Y)^2(1 - \varphi^2)}{(\frac{S}{D_p})^2} \]  

(E5)

Note that this equation is for 100-percent-dense fuel.
REFERENCES


28


### TABLE I. - FUEL-PIN PARAMETERS

<table>
<thead>
<tr>
<th>Clad outside diameter, ( D_p )</th>
<th>Clad thickness, ( t )</th>
<th>Relative clad thickness, ( Y )</th>
<th>Burnup, percent</th>
<th>Relative hole size, ( \varphi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>in.</td>
<td>mm</td>
<td>in.</td>
<td></td>
</tr>
<tr>
<td>6.350</td>
<td>0.250</td>
<td>0.508</td>
<td>0.020</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.762</td>
<td>.030</td>
<td>.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.016</td>
<td>.040</td>
<td>.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.270</td>
<td>.050</td>
<td>.02</td>
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<tr>
<td></td>
<td></td>
<td>.508</td>
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<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.762</td>
<td>.030</td>
<td>.12</td>
</tr>
<tr>
<td>9.525</td>
<td>0.375</td>
<td>0.762</td>
<td>.030</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.016</td>
<td>.040</td>
<td>.107</td>
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<td></td>
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<td>1.270</td>
<td>.050</td>
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<td></td>
<td></td>
<td>.762</td>
<td>.030</td>
<td>.08</td>
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<td></td>
<td>1.016</td>
<td>.040</td>
<td>.107</td>
</tr>
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<td>.050</td>
<td>.133</td>
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<td>.08</td>
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<td>1.016</td>
<td>.040</td>
<td>.107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.270</td>
<td>.050</td>
<td>.133</td>
</tr>
<tr>
<td>12.700</td>
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<td>0.508</td>
<td>0.020</td>
<td>0.04</td>
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<td>.030</td>
<td>.06</td>
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<td>.040</td>
<td>.08</td>
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<tr>
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<td></td>
<td>1.524</td>
<td>.060</td>
<td>.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.032</td>
<td>.080</td>
<td>.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.016</td>
<td>.040</td>
<td>.08</td>
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<td></td>
<td></td>
<td>1.524</td>
<td>.060</td>
<td>.12</td>
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<td>.080</td>
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<td>.08</td>
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<tr>
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<td></td>
<td>1.524</td>
<td>.060</td>
<td>.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.032</td>
<td>.080</td>
<td>.16</td>
</tr>
</tbody>
</table>
### TABLE II. - ALLOWABLE BURNUP

(a) Pin diameter, 6.350 millimeters (0.250 in.)

<table>
<thead>
<tr>
<th>Relative hole size, $\varphi$</th>
<th>Relative clad thickness, $Y$</th>
<th>0.5 Percent clad hoop strain</th>
<th>1.0 Percent clad hoop strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.091</td>
<td>1.85</td>
<td>3.20</td>
</tr>
<tr>
<td>.25</td>
<td>0.078</td>
<td>1.65</td>
<td>3.00</td>
</tr>
<tr>
<td>.50</td>
<td>0.03</td>
<td>1.20</td>
<td>2.20</td>
</tr>
</tbody>
</table>

(b) Pin diameter, 12.700 millimeters (0.500 in.)

<table>
<thead>
<tr>
<th>Relative hole size, $\varphi$</th>
<th>Relative clad thickness, $Y$</th>
<th>0.5 Percent clad hoop strain</th>
<th>1.0 Percent clad hoop strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.091</td>
<td>1.50</td>
<td>2.45</td>
</tr>
<tr>
<td>.25</td>
<td>0.078</td>
<td>1.40</td>
<td>2.35</td>
</tr>
<tr>
<td>.50</td>
<td>0.03</td>
<td>1.10</td>
<td>1.90</td>
</tr>
</tbody>
</table>
(a) Temperatures in kelvin, pin diameter in centimeters.
\[ \frac{T - T_w}{B\psi} = 109 \, d_p^2 \left(1 - 2Y^2\right). \]

(b) Temperatures in degrees Rankine, pin diameter in inches.
\[ \frac{T - T_w}{B\psi} = 1266 \, d_p^2 \left(1 - 2Y^2\right). \]

(c) Fuel-pin parameter \( \psi \) as function of relative hole size.

Figure 2. - Average fuel temperature as function of relative clad thickness with burnup and fuel-pin parameter \( \psi \) as variables.
Figure 3. - Fuel volumetric swelling as function of average fuel temperature with burnup as parameter.

(a) Fuel temperature in kelvin.

(b) Fuel temperature in degrees Rankine.
Figure 4. - Clad strain as function of fuel volumetric swelling.
Figure 5. - Clad hoop strain as function of burnup with relative hole size as parameter. Clad outside diameter, 6.325 mm (0.250 in.); clad wall temperature, 1228 K (2210° R).
Figure 6. - Clad hoop strain as function of burnup with relative hole size as parameter. Clad outside diameter, 9.525 mm (0.375 in.); clad wall temperature, 1228 K (2210°F).
Figure 7. - Clad hoop strain as function of burnup with relative hole size as parameter. Clad outside diameter, 12.700 mm (0.500 in.); clad wall temperature, 1228 K (2210°F).
Figure 8. - Clad hoop strain as function of burnup with relative clad thickness as parameter. Clad outside diameter, 6.350 mm (0.250 in.); clad wall temperature, 1228 K (2210° F).
Figure 9. Clad hoop strain as function of burnup with relative clad thickness as parameter. Clad outside diameter, 9.525 mm (0.375 in.); clad wall temperature, 1228 K (2210° R).
Figure 10. - Clad hoop strain as function of burnup with relative clad thickness as parameter. Clad outside diameter, 12.700 mm (0.500 in.); clad wall temperature, 1228 K (2210° R).
Figure 11 - Fuel volume fraction for unit cell. Cylindrical fuel pins with equilateral spacing; 100-percent-dense fuel. $V_f\left(\frac{S/D_p}{l/D_p}\right)^2 = \frac{0.907}{(1 - 2\gamma)^2(1 - \varphi^2)}$. (10)
Figure 12. - Allowable burnup as function of fuel volume fraction with relative hole size as parameter. Clad hoop strain, 0.5 and 1.0 percent; clad wall temperature, 1228 K (2210° R); relative pin spacing, 1.10.
Figure 13. - Allowable burnup as function of fuel volume fraction with relative clad thickness as parameter. Clad strain, 0.5 and 1.0 percent; clad wall temperature, 1228 K (2210° R); relative pin spacing, 1.10.

Figure 14. - Unit cell for reactor core with equilateral fuel-pin spacing.
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