A COMPUTER PROGRAM FOR
MODELING NON-SPHERICAL ECLIPSING
BINARY STAR SYSTEMS

D. B. WOOD

DECEMBER 1972

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
A Computer Program
for
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Binary Star Systems

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Advanced Plans Staff

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ABSTRACT

The accurate analysis of eclipsing binary light curves is fundamental to obtaining information on the physical properties of stars. Until recently, the analysis of these stars has been performed using the relatively simple "spherical model." This model, however, does not account for a number of complications which we know to exist.

The model described in this document accounts for the important geometric and photometric distortions such as rotational and tidal distortion, gravity brightening, and reflection effect. This permits a more accurate analysis of interacting eclipsing star systems.

The model is designed to be useful to anyone with "moderate" computing resources. The programs, written in FORTRAN IV for the IBM 360, consume about 80k bytes of core. On the 360/75, about 93 milliseconds (ms) is required to predict a point on the light curve to an accuracy of ± 0.0002 magnitude. Solution for 10 unknowns, using 100 normal points, requires about 65 seconds per iteration.

The FORTRAN program listings are provided, and the computational aspects are described in some detail. Implementation of the programs as they are presently written should present no problems. In addition, it should be fairly straightforward to modify the programs to suit the users' individual requirements.
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A COMPUTER PROGRAM FOR MODELING NON-SPHERICAL ECLIPSING BINARY STAR SYSTEMS

I. INTRODUCTION

Most of our knowledge about the physical properties of stars comes from the analysis of eclipsing binary stars. We observe the nature of the change of light intensity with time, and "solve" this light curve to determine such physical quantities as mass, radius, surface temperature, luminosity, density gradient, variation of intensity across the surface, etc. Until recently, the analysis of light curves has been done via the "spherical model," wherein the stars are represented as non-interacting spheres. Any deviations from this simple picture were "rectified" out of the light curve.

In recent literature, considerable emphasis has been placed on the advantage of analyzing eclipsing binary systems with models which are more realistic than the traditional "spherical model." Some of these new models are quite general (Hill and Hutchings 1970; Wilson and Devinney 1971), whereas others are more specific (e.g. Lucy 1968; Mochnacki and Doughty 1972; Horak 1966; Rucinski 1969). Unfortunately, however, these new models have not been made generally available to observers for the analysis of their data.

The intent of this publication is to provide a relatively simple, inexpensive (in terms of computer use) modern model which anyone can use to analyze an observed light curve and arrive at physical parameters of the stars. Various aspects of this model have already been described (Wood 1969, 1971a, 1971b, 1971c, 1971d). In order to provide a complete description, much of the material in these earlier publications will be repeated here.

The model is programmed in the FORTRAN IV language for the IBM 360. Most calculations are in the single precision. The programs occupy about 80k bytes of core and require an average of about 93 ms on the 360/75 to calculate one point on a light curve to a precision of ± 0.0002 magnitudes (73 ms out of eclipse; 110 ms for partial or annular; 37 ms for total). The deletion of reflection effect from the calculation decreases the compute time by about 40%. Solution for 10 unknowns using 100 normal points requires about 65 seconds per iteration. If convergence does not occur by about 4 iterations, it is not likely to occur at all with that parameter set. Almost one third of the compute time is devoted to the extraction of square roots.

This model takes into account the best understood geometric and photometric distortions, including rotational and tidal distortion, limb darkening,
gravity brightening, and reflection. The major simplifications are 1) each
cstar is treated as a triaxial ellipsoid, 2) reflection effect is approximated, and
3) the stars are considered to rotate in their orbital plane with a period equal
to the orbital period. The first simplification is the main detail which sepa-
rates this model from other non-spherical models, and permits the relatively
fast compute time. This approximation is probably quite valid except for ex-
tremely close systems, since the limiting potential (Roche) surface would de-
fine the extent of the stars' outer atmosphere, not the photosphere. Geomet-
ically, the reflection is very good. Astrophysically, it is consistent with
gray atmosphere approximation. As we will see later, the constraint on rota-
tion can be relaxed.

In this publication we will first describe the model and its parameters.
Then the computational technique will be outlined. A description of the impor-
tant features of each computer subprogram follows. Finally, the details of
running the program are described

II. THE MODEL

A. Physical Description

Basically, the model is defined by the following parameters:

Orbital parameters

- Period of revolution $P$
- Time of conjunction $T_c$
- Semimajor axis or orbit $R_o$
- Orbital eccentricity $e$
- Longitude of periastron $\omega$
- Inclination $i$

Geometric parameters

- Semiaxes of stars A and B $a_A, b_A, c_A$
- $a_B, b_B, c_B$

Photometric parameters

- Surface intensity (see text) $\bar{I}_A, \bar{I}_B$
- Limb darkening coefficient $u_A, u_B$
Gravity brightening coefficient $v_A, v_B$
Reflection coefficient $w_A, w_B$

The two stars are not identified in the usual way as "larger" and "smaller" but rather as star "A" and star "B". Star A is eclipsed at the deeper eclipse and is considered to be the central star about which B revolves.

It is convenient to replace the six geometric parameters (the stellar semi-axes) with six dimensionless parameters:

$$a, k_a, \epsilon_A, \epsilon_B, \tau_A, \tau_B$$

which are related to the actual axes by the following:

$$
\begin{align*}
 a_A &= aR_0 \\
 b_A &= \epsilon_A aR_0 \\
 c_A &= (1 + \tau_A) \epsilon^2_A aR_0 \\
 a_B &= k_a aR_0 \\
 b_B &= \epsilon_B k_a aR_0 \\
 c_B &= (1 + \tau_B) \epsilon^2_B k_a aR_0 \\
\end{align*}
$$

Note that primary eclipse is an occultation if $k_a < 1$ and a transit if $k_a > 1$. The semi-major axis of the orbit, $R_0$, is the unit of length, and is usually set to unity.

From the work of Chandrasekhar (1933) the six axes of the stars can be expressed as functions of the mass ratio and polytropic indices. If we retain terms only to the third order in stellar radius divided by separation, then the stars are triaxial ellipsoids with axes given by:

$$
\begin{align*}
 a_A &= \nu_A \left[ 1 + \frac{1}{6} (1 + 7q) \Delta_{\nu_A}^{2} \right] \\
 b_A &= \nu_A \left[ 1 + \frac{1}{6} (1 - 2q) \Delta_{\nu_A}^{2} \right] \\
 c_A &= \nu_A \left[ 1 - \frac{1}{6} (2 + 5q) \Delta_{\nu_A}^{2} \right]
\end{align*}
$$

for star A. Here $q$ is the mass ratio (star B to star A) and $\nu_A$ is given by:

$$\nu_A = a_{oA}/R_0$$

where $a_{oA}$ is the "unperturbed radius" of star A; i.e. the radius of a sphere of equivalent volume. The parameter $\Delta_{\nu_A}$ is a slowly varying function of the polytropic index, $n$, and asymptotically approaches 1 as $n$ approaches 5.
For star B we have, similarly,

\[
\begin{align*}
\alpha_B &= \nu_B \left[ 1 + \frac{1}{6} (1 + \frac{7}{q}) \Delta_{2B} \nu_B^3 \right] \\
\beta_B &= \nu_B \left[ 1 + \frac{1}{6} (1 - \frac{2}{q}) \Delta_{2B} \nu_B^3 \right] \\
\gamma_B &= \nu_B \left[ 1 - \frac{1}{6} (2 + \frac{5}{q}) \Delta_{2B} \nu_B^3 \right]
\end{align*}
\] (4)

where:

\[ \nu_B = \frac{a_o B}{R_o} = \frac{a_o A k_v}{R_o} \] (5)

The six stellar axes or the six dimensionless geometric parameters may thus be expressed as functions of:

\[ a_o A, k_v, q, n \]

where we here assume, for convenience, that one n applies to both stars.

Limb darkening is expressed by the usual linear law

\[ I = I_o (1 - u + u \cos \gamma) \] (6)

where \( \gamma \) is the foreshortening angle. \( I_o \) is the intensity of the "sub-earth" point, where the line of sight from the observer is normal to the stellar surface and \( \gamma = 0 \). Arbitrarily, the intensity ratio is defined at time \( T_Q = T_c + P/4 \) as

\[ j = \frac{I_B}{I_A} \] (7)

where \( I \) is the value of \( I_o \) at \( T_Q \).

If the observations are taken over a narrow wavelength region, then we can write:

\[ j = \frac{\exp \left( \frac{c}{T_A} \right) - 1}{\exp \left( \frac{c}{T_B} \right) - 1} \] (8)

where \( T_A, T_B \) are the effective surface temperatures at the sub-earth point at \( T_Q \). Here \( C = \frac{hc}{k \lambda} = 1.43879 / \lambda \) where \( \lambda \) is the wavelength of observation. For a bolometric light curve:

\[ j = \left( \frac{T_B}{T_A} \right)^4 \] (9)
The gravity brightening coefficient \( v \), is defined to be used analogously to limb darkening so that

\[
I_0 = \bar{I} \left[ 1 - v + v \left( \frac{r}{\bar{r}} \right) \right]
\]

(10)

where \( r \) is the local radius and \( \bar{r} \) is the radius to the sub-earth point at \( T_Q \), where \( \bar{I} \) is defined.

If the local effective temperature, \( T \), varies as local gravity to some power \( \beta \), then we may write \( v \) in terms of \( \beta \):

\[
v = \left( \frac{\Delta_2 - 5}{\Delta_2} \right) \frac{\beta C}{T} \left( \frac{e^{\sigma/T}}{e^{\sigma/T} - 1} \right)
\]

(11)

where \( C = 1.43879/\lambda \).

Reflection is treated by determining the local incident intensity, \( L^* \), at any point and reflecting a fraction, \( w \), of this uniformly over the out-going hemisphere. Thus the local emergent intensity, \( I_o \), has added to it the intensity \( I^* \), given by:

\[
I^* = wL^*/2\pi(1 - u/2).
\]

(12)

The incident radiation \( L^* \), can be calculated in detail by integrating over the source star as described by Chen and Rhein (1969) and by Wood (1971b, 1971d). The temperature distributions produced in this manner agree quite closely between Chen and Rhein, Wood, and Napier (1968). Unfortunately, such an exact treatment is prohibitively time consuming. Over the range of interest (\( a_A \) and \( a_B < 0.5 \)) a fairly good approximation is possible which avoids the necessity of integration (Wood 1972). See Appendix II.

B. Perturbations on Basic Model

1. Extended Atmospheres

Normally, the stars are considered to have very small scale heights, so that they can be considered as having sharp edges. If the scale height becomes appreciable (greater than about \( 10^{-3} \) of the stellar radius) then the star must be considered to have an extended atmosphere. A light ray passing through this atmosphere will be attenuated according to:

\[
j = j_o e^{-r}
\]

(13)
where, for a simple exponential atmosphere, the optical depth, \( \tau \), is given by:

\[
\tau = \tau_o e^{-r/h}
\]  

where, for a simple exponential atmosphere, the optical depth, \( \tau \), is given by:

\( \tau = \tau_o e^{-r/h} \)  

\( \tau_o \) is the optical depth at the "surface," where a ray grazes the limb. The apparent distance of a ray above the limb is given by \( r \), and \( h \) is the scale height. The effect of such an extended atmosphere has been described by Wood (1971b, 1971e).

2. Orbital Skew

The basic model assumes that the stars rotate in the orbital plane with the major axes aligned on the apse. Deviations to this geometry can be introduced; however, they must be constant over the entire orbital period.

The stars' major axis may be made to lag or lead periastron passage by an amount \( \sigma \). The star's pole may be tilted from the orbital pole by an amount \( \iota \).

C. System Luminosity

The total observed luminosity of a star is given by an integral over the apparent ellipse:

\[
L = \int \int (I_o + I^*)(I - u + u \cos \gamma) dA
\]  

The total system luminosity at any time \( t \) is

\[
L_{TOT}(t) = L_A(t) + L_B(t) - L_{ECL}(t)
\]  

The system luminosity is normalized at \( t = T_Q \):

\[
L_{TOT}(T_Q) = L_A(T_Q) + L_B(T_Q) = 1
\]

III. OUTLINE OF COMPUTATIONAL TECHNIQUE

A. General

Calculation of a light curve is just the evaluation of equation (16) for various values of \( t \). This involves, generally, three double integrations of equation (15); one over star A, one over star B, and one over the overlapping area.
To prepare for these integrations, the computer programs must perform two fundamental tasks: 1) determine the outlines of the stars and their overlapped area as projected on the plane of the sky; and 2) determine the light intensity at any point on the apparent stars.

The solution of a light curve, that is going from an observed light curve to orbital elements, uses least squares differential corrections. The light curve intensity is a function of time and some number, n, of parameters:

\[
\text{Intensity} = I(t, X_1, X_2, X_3, \ldots, X_n).
\]

Taking a linearized Taylor expansion around approximate values of \(X_i\), we may write:

\[
I_{\text{OBS}} - I_{\text{COMP}} = \Delta I = \frac{\partial I}{\partial x_1} \delta x_1 + \cdots + \frac{\partial I}{\partial x_n} \delta x_n = \sum_{i=1}^{n} \frac{\partial I}{\partial x_i} \delta x_i \tag{17}
\]

\(I_{\text{OBS}}\) is the observed intensity at \(t\) and \(I_{\text{COMP}}\) is the intensity computed by the model at time \(t_0\) for the approximate values of \(x_i\). \(\delta x_i\) is then the difference \(x_{\text{TRUE}} - x_{\text{APPROX}}\), and is a differential correction to the approximate \(x_i\). We have a different equation (17) for each observed time, \(t\), and can solve them by least squares to get the best \(\delta x_i\). Hopefully, the process can be repeated to converge upon the best set of \(X_i\) which most nearly represent the light curve.

Convergence, especially in the presence of observational error, is very hard to accomplish. Exactly which parameters to allow as variables, and what initial values to assign, is still a black art. Completely automated solution is not generally possible.

Each partial derivative in equation (17) is calculated numerically by varying \(x_i\) to determine its effect on \(I\). Fortunately some shortcuts are available, for potentially, to solve a light curve of \(m\) observations, we could have to evaluate the system intensity \(m(2n + 1)\) times per iteration.

B. Coordinate Systems

The fundamental coordinate system used in most calculations is a rectangular system centered on star A. The x-axis points along the line of sight; the plane of the sky is the y-z plane, with z directed northward. This is the \((x, y, z)_A\) system. Some calculations, for star B, are performed in a similar \((x, y, z)_B\) system, centered on star B.

Each star contains a coordinate system imbedded within it with the \(x', y', z'\) axes coincident with the principle axes of the stars; the \((x', y', z')_A\) and
(x', y', z')_B systems. Except for reflection, there is octant symmetry, so the relative sense of (x', y', z')_A and (x', y', z')_B does not matter.

The apparent stars have principle axes which are rotated from the (y, z) axes, so each star has its own coordinate system which is coincident with the principle axes of the apparent ellipse: (y, z)_A and (y, z)_B.

C. Model Parameters

The model can be considered as having parameters defined either as "model space" or "astrophysics space." These are shown in Table I. All of the parameters have been previously described.

A light curve may be generated by inputting either model or astrophysical parameters. The solution of a given light curve can be done only in astrophysics space.

Table I. Model Parameters

<table>
<thead>
<tr>
<th>Astrophysics Space</th>
<th>Model Space</th>
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</thead>
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<tr>
<td><strong>Orbital Elements</strong></td>
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</tr>
<tr>
<td>e sin ω, e cos ω, P, T_C, R_o, i</td>
<td>e sin ω, e cos ω, P, T_C, R_o, i</td>
</tr>
<tr>
<td>a_oA, k_y, q, n</td>
<td>a, k_a, e_A, e_B, f_A, f_B</td>
</tr>
<tr>
<td><strong>Geometric Elements</strong></td>
<td></td>
</tr>
<tr>
<td>T_A, T_B, β_A, β_B</td>
<td>j, v_A, v_B</td>
</tr>
<tr>
<td>u_A, u_B, w_A, w_B</td>
<td>u_A, u_B, w_A, w_B</td>
</tr>
<tr>
<td><strong>Photometric Elements</strong></td>
<td></td>
</tr>
<tr>
<td>h_A, h_B, τ_oA, τ_oB</td>
<td></td>
</tr>
<tr>
<td><strong>Atmospheric Parameters</strong></td>
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<tr>
<td>h_A, h_B, τ_oA, τ_oB</td>
<td></td>
</tr>
<tr>
<td><strong>Orbital Perturbation Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>σ_A, σ_B, t_A, t_B</td>
<td>σ_A, σ_B, t_A, t_B</td>
</tr>
</tbody>
</table>
IV. THE COMPUTER PROGRAM

The computer program is subdivided into a number of subroutines and functions. Some parameters are passed through CALL statements, but most are contained in COMMON blocks. Figure 1 shows the subprogram interrelationships. Common variables are defined in Tables II, III and IV. Appendix III contains program listings, written in FORTRAN-IV for the IBM S-360. In the following paragraphs the general functioning of each subprogram will be described.

A. Main Program (WINK)

The main program, WINK, performs all initializations, input, and ordinary output. Coarse logic flow is shown in Table V. As part of the initialization process, WINK uses the following three subroutines:

- WINK
- ASTROQ
- GRID
- GEOMET
  - ORBIT A
  - ORBIT B
  - PARAM
  - ZONES (BYPASSED WHEN IN SOLVE 1)
  - TOTINT (BYPASSED WHEN IN SOLVE 1)
- ENERGY
  - OUTPUT
- BRIGHT
- REFL (BYPASSED WHEN IN SOLVE 1)

Figure 1a. Sub-program Interrelationships (Initialization/Normalization).
Figure 1b. Sub-program Interrelationships (Light-Prediction Function LUMC).

GRID  -  to set up the Gauss integration coefficients and weights
GEOMET  -  to set all time-independent system variables
ASTROQ  -  to convert astrophysical parameters to model parameters

The system luminosity at any specified time is calculated by the function LUMC. Thus, in LUMC are contained all the computations of equations (16) and (17). The total stellar energy radiated over 4π steradians is calculated by ENERGY. The calculation is approximate, and is for display purposes only. Light curve solution is done through the differential-corrector program SOLVE1.

B. Gauss Quadrature Constants (GRID)

GRID loads COMMON/GAUSS/ with weights, coefficients and (1-coefficients^2)^1/2 for the accuracy specified (4-point, 6-point or 16-point integration).
In addition, COMMON/GAUXX/ is always loaded for 16-point integration to be used by ENERGY and ATMECL. The approximate errors in luminosity are 0.6% for 4-point; 0.19% for 6-point; 0.012% for 16-point. The computing time required goes roughly as 0.6:1:5.1.

C. Generation of Model Parameters from Astrophysical Parameters (ASTROQ and ASTROX)

The subroutine ASTROQ is used to generate model parameters from astrophysical parameters, using equations (2), (4), (8) and (11). It is programmed
Table II. Input Parameter Codes

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<th>Code</th>
<th>FORTRAN Variable</th>
<th>Use</th>
<th>Default Code</th>
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<tr>
<td>1</td>
<td>AINCL</td>
<td>inclination in degrees</td>
<td>90.0</td>
</tr>
<tr>
<td>2</td>
<td>ESINW</td>
<td>e sin ω</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>ECOSW</td>
<td>e cos ω</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>UA</td>
<td>limb darkening star A</td>
<td>0.6</td>
</tr>
<tr>
<td>5</td>
<td>UB</td>
<td>limb darkening star B</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>semi-major axis star A (model); radius unperturbed sphere of star A (ap.)</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>RATIOK</td>
<td>ratio of star B to star A; semi-axes (model) or radii (ap.)</td>
<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>ELLIP</td>
<td>ellipticity star A (model); gravity exponent star A (ap.)</td>
<td>0.25</td>
</tr>
<tr>
<td>9</td>
<td>ELLIPB</td>
<td>ellipticity star B (model); gravity exponent star B (ap.)</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>EPSI</td>
<td>differential ellipticity star A (model); surface temperature star A (ap.)</td>
<td>10000.0</td>
</tr>
<tr>
<td>11</td>
<td>EPSIB</td>
<td>differential ellipticity star B (model); surface temperature star B (ap.)</td>
<td>10000.0</td>
</tr>
<tr>
<td>12</td>
<td>TCONJ</td>
<td>time of conjunction</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>VA</td>
<td>gravity bright. coefficient star A (model); mass ratio (ap.)</td>
<td>1.0</td>
</tr>
<tr>
<td>14</td>
<td>VB</td>
<td>gravity bright. coefficient star B (model); not used for input (ap.)</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>RATIOJ</td>
<td>relative surface brightness (model) not used for input</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>WA</td>
<td>reflection albedo star A</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>WB</td>
<td>reflection albedo star B</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>PERIOD</td>
<td>period</td>
<td>1.0</td>
</tr>
<tr>
<td>23</td>
<td>STAR3</td>
<td>third star light (intensity)</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>QUAD</td>
<td>quadrature magnitude</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>RNOT</td>
<td>orbital semi-major axis length</td>
<td>1.0</td>
</tr>
<tr>
<td>26</td>
<td>WAVE</td>
<td>wavelength of observations</td>
<td>5500.0</td>
</tr>
<tr>
<td>27</td>
<td>POLYX</td>
<td>polytropic index (if = 0 interpret input as 'model', not 'ap')</td>
<td>5.0</td>
</tr>
<tr>
<td>31</td>
<td>SCALE(1)</td>
<td>scale height of atmos. star A</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>SCALE(2)</td>
<td>scale height of atmos. star B</td>
<td>0</td>
</tr>
<tr>
<td>33</td>
<td>SURF(1)</td>
<td>optical depth at edge star A</td>
<td>0</td>
</tr>
<tr>
<td>34</td>
<td>SURF(2)</td>
<td>optical depth at edge star B</td>
<td>0</td>
</tr>
</tbody>
</table>
Table II. Input Parameter Codes (Continued)

<table>
<thead>
<tr>
<th>Code</th>
<th>FORTRAN Variable</th>
<th>Use</th>
<th>Default Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>SLIP(1)</td>
<td>phase shift star A (radians)</td>
<td>0</td>
</tr>
<tr>
<td>36</td>
<td>SLIP(2)</td>
<td>phase shift star B (radians)</td>
<td>0</td>
</tr>
<tr>
<td>37</td>
<td>TILT(1)</td>
<td>tilt of pole star A (radians)</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>TILT(2)</td>
<td>tilt of pole star B (radians)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table III. Other Input Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>FORTRAN Variable</th>
<th>Use</th>
<th>Default Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>NTEG</td>
<td>Integration precision code</td>
<td>1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1. --4 × 4 integration</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. --6 × 6 integration</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. --16 × 16 integration</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>S</td>
<td>start time for light curve prediction</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>TINT</td>
<td>time interval for light curve prediction (0 to read time cards)</td>
<td>0.1</td>
</tr>
<tr>
<td>22</td>
<td>END</td>
<td>end time for light curve prediction</td>
<td>0.5</td>
</tr>
<tr>
<td>84</td>
<td>IFFY</td>
<td>maximum number of iterations</td>
<td>6.</td>
</tr>
<tr>
<td>86</td>
<td>ICRD</td>
<td>for punched output of predicted curve</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0--no punched output</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.--punched output in mag.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.--punched output in lum.</td>
<td></td>
</tr>
<tr>
<td>87</td>
<td>ILUM</td>
<td>interpret input light curve as lum</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0--magnitude</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.--luminosity</td>
<td></td>
</tr>
<tr>
<td>88</td>
<td>JUMP</td>
<td>Flip/Flop for printout of orbital elements (no data field)</td>
<td>print</td>
</tr>
<tr>
<td>99</td>
<td>---</td>
<td>Stop</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>---</td>
<td>begin light curve prediction</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>---</td>
<td>begin light curve solution read obs.</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>---</td>
<td>begin light curve solution, use old observations</td>
<td></td>
</tr>
</tbody>
</table>

(NOTE. For 0, -1, -2, the data field, if non-zero, specifies the quadrature magnitude.)
### Table IV. Other Important Variables

<table>
<thead>
<tr>
<th>FORTRAN Name</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCL</td>
<td>$i$; orbital inclination in radians</td>
</tr>
<tr>
<td>E</td>
<td>$e$; orbital eccentricity</td>
</tr>
<tr>
<td>T0</td>
<td>$T_0$; time of periastron passage</td>
</tr>
<tr>
<td>Q</td>
<td>$q; (1 - e^2)^{1/2}$</td>
</tr>
<tr>
<td>MU</td>
<td>$\mu$; mean daily motion (type REAL)</td>
</tr>
<tr>
<td>ALPHA</td>
<td>$\alpha$; angle, in plane of sky between semi-major axis of one apparent ellipse and the center of the other</td>
</tr>
<tr>
<td>THETA</td>
<td>$\theta$; orbital longitude</td>
</tr>
<tr>
<td>CHI</td>
<td>$\chi$; angle, in plane of sky, between $z$-axis of $(x, y, z)_A$ system and center of star B</td>
</tr>
<tr>
<td>R</td>
<td>orbital radius</td>
</tr>
<tr>
<td>DELTA</td>
<td>$\delta$; apparent separation of centers of stars</td>
</tr>
<tr>
<td>PARA</td>
<td>constants which define the ellipsoids</td>
</tr>
<tr>
<td>AAXIS</td>
<td>semi-major axis of apparent ellipses</td>
</tr>
<tr>
<td>BAXIS</td>
<td>semi-minor axis of apparent ellipses</td>
</tr>
<tr>
<td>PHASE</td>
<td>orbital phase</td>
</tr>
<tr>
<td>PHI</td>
<td>$\phi$; angle, in plane of sky, between axes of $\theta$; apparent ellipse and $(y, z)$ axes</td>
</tr>
<tr>
<td>THETAP</td>
<td>$\theta'$; orbital longitude of each star</td>
</tr>
<tr>
<td>QINT</td>
<td>surface intensity at $T_Q$</td>
</tr>
<tr>
<td>AA</td>
<td>a-axes of ellipsoids</td>
</tr>
<tr>
<td>BB</td>
<td>b-axes of ellipsoids</td>
</tr>
<tr>
<td>CC</td>
<td>c-axes of ellipsoids</td>
</tr>
<tr>
<td>RBAR</td>
<td>$\bar{r}$; radius of ellipsoid in line of sight direction at $T_Q$</td>
</tr>
<tr>
<td>UMA</td>
<td>$1 - u$</td>
</tr>
<tr>
<td>VMA</td>
<td>$1 - v$</td>
</tr>
<tr>
<td>WMA</td>
<td>$w/2 \pi(1 - u/2)$</td>
</tr>
<tr>
<td>BOLOJ</td>
<td>bolometric intensity ratio</td>
</tr>
<tr>
<td>TOTAL</td>
<td>system luminosity at $T_Q$</td>
</tr>
<tr>
<td>STAR 1</td>
<td>luminosity of star A at $T_Q$</td>
</tr>
<tr>
<td>STAR2</td>
<td>luminosity of star B at $T_Q$</td>
</tr>
<tr>
<td>IFSPH</td>
<td>set to 1 if star A spherical, 2 if star B spherical, 3 if both spherical, 0 if none spherical</td>
</tr>
<tr>
<td>JTYPE</td>
<td>set to 1 if annular eclipse, 2 if total, 3 if partial, 4 if atmospheric</td>
</tr>
<tr>
<td>KSTAR</td>
<td>set to 1 if star A eclipsed, 2 if star B eclipsed, 3 if no eclipse</td>
</tr>
</tbody>
</table>
### Table IV. Other Important Variables (Continued)

<table>
<thead>
<tr>
<th>FORTRAN Name</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST</td>
<td>logical variable identifying which parameters are &quot;variable&quot;</td>
</tr>
<tr>
<td>NOBS</td>
<td>number of observational data cards read</td>
</tr>
<tr>
<td>NZONE</td>
<td>for detailed reflection, contains number of zones per octant - set to 0 for shortcut reflection</td>
</tr>
<tr>
<td>MREF</td>
<td>set to 1 when integrating total luminosity of star A, 2 for star B, and 3 when integrating partial eclipse</td>
</tr>
<tr>
<td>NREF</td>
<td>counter which keeps track of point in integration grid - used for reflection</td>
</tr>
<tr>
<td>IREF</td>
<td>set to 2 when calculating partial derivatives, otherwise set to 1</td>
</tr>
<tr>
<td>NINC</td>
<td>set to 3 when calculating normalization, otherwise zero</td>
</tr>
<tr>
<td>LST</td>
<td>indicates variable for which partial derivative is currently being calculated</td>
</tr>
<tr>
<td>SINI</td>
<td>( \sin i )</td>
</tr>
<tr>
<td>COSI</td>
<td>( \cos i )</td>
</tr>
<tr>
<td>SISQ</td>
<td>( \sin^2 i )</td>
</tr>
<tr>
<td>CISQ</td>
<td>( \cos^2 i )</td>
</tr>
<tr>
<td>SINJ</td>
<td>( \sin (i + \epsilon) )</td>
</tr>
<tr>
<td>COSJ</td>
<td>( \cos (i + \epsilon) )</td>
</tr>
<tr>
<td>SINT</td>
<td>( \sin ) orbital longitude of each star (( \theta' ))</td>
</tr>
<tr>
<td>COST</td>
<td>( \cos ) orbital longitude of each star (( \theta' ))</td>
</tr>
<tr>
<td>COTH</td>
<td>( \cos ) (true anomaly - mean anomaly) for each star (( \Theta ))</td>
</tr>
<tr>
<td>SINP</td>
<td>( \sin \phi )</td>
</tr>
<tr>
<td>COSP</td>
<td>( \cos \phi )</td>
</tr>
<tr>
<td>DCHI</td>
<td>( \delta \sin \chi )</td>
</tr>
<tr>
<td>ECHI</td>
<td>( \delta \cos \chi )</td>
</tr>
<tr>
<td>WT</td>
<td>Gauss weights, ( W ), in COMMON/GAUSS/ also observational weights, in COMMON/OBS/</td>
</tr>
<tr>
<td>X</td>
<td>Gauss coordinates, ( X )</td>
</tr>
<tr>
<td>XC</td>
<td>( (1 - X^2)^{1/2} )</td>
</tr>
<tr>
<td>L</td>
<td>the number of Gauss points</td>
</tr>
<tr>
<td>TIME</td>
<td>times - observed or internally-generated</td>
</tr>
<tr>
<td>LUM</td>
<td>observed luminosity (type REAL)</td>
</tr>
<tr>
<td>CLUM</td>
<td>computed luminosities (also used internally in LSQS)</td>
</tr>
</tbody>
</table>
Table V. Main Program Logic Flow

<table>
<thead>
<tr>
<th>Statement Number</th>
<th>Sub Program</th>
<th>Computational Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>ASTROQ</td>
<td>Initializations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Input parameters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Convert astrophysical input to model parameters. Decide if prediction or solution. Prediction: generate times (calculate or read) and go to 52</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>Solution: read observations</td>
</tr>
<tr>
<td>203</td>
<td></td>
<td>Read card to see which parameters are variables</td>
</tr>
<tr>
<td>204</td>
<td></td>
<td>Generate normalized intensities and go to 52</td>
</tr>
<tr>
<td>52</td>
<td>GRID</td>
<td>Set up Gaussian integration constants.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set IREF flag (1 means not calculating partial derivatives)</td>
</tr>
<tr>
<td></td>
<td>GEOMET</td>
<td>Compute time-independent quantities; including normalization</td>
</tr>
<tr>
<td></td>
<td>ENERGY</td>
<td>Approximate total stellar luminosity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set up output quantities.</td>
</tr>
<tr>
<td>105</td>
<td>LUMC</td>
<td>Output model parameters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If solution mode, go to 150.</td>
</tr>
<tr>
<td>150</td>
<td>SOLVE1</td>
<td>Solution: perform one iteration.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Go to 52 for next iteration, or to 66 if through iterating.</td>
</tr>
</tbody>
</table>

D. Time-Independent Ellipsoid Calculations (GEOMET)

GEOMET computes the star axes from the geometric parameters. See Table VI. The axes are screened for fatal errors (semi-axes ≤ 0). A call to subroutine ORBITA establishes the orbital eccentricity, time of periastron and mean daily motion.
Table VI. GEOMET Logic Flow

<table>
<thead>
<tr>
<th>Statement Number</th>
<th>Sub Program</th>
<th>Computational Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td></td>
<td>If calculating partial derivatives (IREF = 2) for photometric variables (LST &lt; 11), bypass much of calculation—go to 5. Calculate ellipsoid axes and abort if ( \leq 0 ). Also abort if inclination or period ( \leq 0 ). Calculate sine and cosine functions of inclination (orbital and individual stars). Set flag for possible short cuts if one or both stars spherical. ORBITA Set up basic orbital parameters Define luminosity of star A as 1 Define luminosity of star B as j Set up functions of u, v, and w ORBITB Orbital mechanical solution for quadrature time Calculate ( \bar{r} ) Calculate ellipsoid parameters at quadrature. Set flag NINC for &quot;normalization&quot; (this is used in BRIGHT in conjunction with reflection) Initialize reflection only if not calculating partial derivatives. ZONES Get total light of stars at quadrature. TOTINT Set system normalization value, TOTAL. Set flag NINC for normal operation.</td>
</tr>
</tbody>
</table>

To prepare for intensity normalization, the system luminosity at time = \( T_Q \) must be calculated. The two subroutines used are:

ORBITB calculates time-dependent orbital quantities
PARAM calculates time-dependent ellipsoid quantities

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Reflection is initialized through a call to ZONES, but only when partial derivations are not being calculated. The system luminosity for normalization is then calculated by obtaining the total light of each star, through two calls to TOTINT.

Note that in the case where partial derivatives are being calculated for variables which do not alter systems geometry, a large portion of GOEMET is bypassed.

E. Calculation of Luminosity (LUMC)

LUMC serves primarily as a dispatcher, to arrange all necessary time-dependent calculations, to determine the nature of the eclipse, if any, and to call the appropriate integration routines. See Table VII. Thus, LUMC calls for the following subroutines:

- ORBITB calculates time-dependent orbital quantities
- PARAM calculates time-dependent ellipsoid quantities
- SCREEN determines nature of eclipse (partial, total, annular, atmospheric; primary, secondary)

The integrations are performed by one of the following functions:

- TOTINT for total uneclipsed light of one star
- ANNECL for annular eclipse light loss
- ECLINT for partial eclipse light loss
- ATMECL for atmospheric eclipses (replaces TOTINT for total light).

F. Time-Independent Orbital Parameters (ORBITA)

ORBITA loads COMMON/ORBIT/ with the basic fixed quantities eccentricity $e$; time of periastron $T_o$; mean daily motion $\mu$; and $q = (1 - e^2)^{1/2}$.

G. Time-Dependent Orbital Parameters (ORBITB)

ORBITB performs the orbital mechanics and sets up a number of necessary angles. The following important calculations are performed using the equations shown in Appendix I:

1. Solution of Kepler's equation
2. Set radius vector, scaled by $R_o (R)^*$

*Normally $R_o$, the system unit of length, is unity. However, if a stellar axis is extremely small, $R_o$ may have to be greater than one to preserve computational accuracy.
Table VII.  LUMC Logic Flow

<table>
<thead>
<tr>
<th>Statement Number</th>
<th>Sub Program</th>
<th>Computational Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>ORBIT B</td>
<td>Calculate orbital mechanical quantities at time t</td>
</tr>
<tr>
<td>4</td>
<td>PARAM</td>
<td>Calculate ellipsoid quantities at time t</td>
</tr>
<tr>
<td>5</td>
<td>SCREEN</td>
<td>Determine nature of eclipse, if any</td>
</tr>
<tr>
<td>10</td>
<td>TOTINT</td>
<td>If no eclipse, get total light of each star; go to 50</td>
</tr>
<tr>
<td>25</td>
<td>TOTINT</td>
<td>If total eclipse, total light is just star in front; go to 50</td>
</tr>
<tr>
<td>35</td>
<td>ANNECL</td>
<td>If annular eclipse, light loss given by ANNECL; go to 36</td>
</tr>
<tr>
<td>45</td>
<td>ECLINT</td>
<td>If partial eclipse, light loss given by ECLINT; go to 36</td>
</tr>
<tr>
<td>36</td>
<td></td>
<td>Test scale height of atmosphere</td>
</tr>
<tr>
<td>39</td>
<td>TOTINT</td>
<td>If no atmosphere, star behind calculated by TOTINT; go to 38</td>
</tr>
<tr>
<td>37</td>
<td>ATMECL</td>
<td>If atmosphere, star behind calculated by ATMECL; go to 38</td>
</tr>
<tr>
<td>38</td>
<td>TOTINT</td>
<td>Total light is sum of each star less eclipse; go to 50</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>For final light, add 3rd star and normalize</td>
</tr>
</tbody>
</table>

3. Set orbital longitude (θ)
4. Set apparent separation of centers (δ)
5. Set the (y, z) projections of apparent separation (δ sin χ; δ cos χ)
6. Set sin and cos of rotation angle of stars (θ') (orbital longitude plus any phase shift).

H. Time-Dependent Ellipsoid Calculations (PARAM)

PARAM determines all the necessary ellipsoid parameters and puts them into the matrix PARA in COMMON/TVARS/. In addition, the following important operations are performed:

1. Establish semi-axes of apparent ellipses
2. Set sin and cos of rotation of (ŷ, ẑ) axes from (y, z) axes (φ)
3. Set angle from y axis of star A to center of star B(α).
The equations used are shown in Appendix I.

I. Identification of Eclipse (SCREEN)

SCREEN sets two important flags in COMMON/FLAGS/. KTYPE tells if there is an eclipse and of which star; JTYPE tells the type of eclipse (annular, total, partial, atmospheric).

The subroutine LIMITY is used for fine screening once it is determined that the stars are potentially close to eclipse (when the apparent separation of centers is less than the sum of radius vectors). If the eclipse is partial, LIMITY returns the y-limits of the overlapping area, which SCREEN in turn returns to LUMC.

J. Determination of Geometry of Overlapping Stars (LIMITY)

LIMITY is logically the most complex subroutine. It is called upon when there is reason to believe there is an eclipse.

In general, two ellipses may intersect in as many as four points. This subroutine uses a vertical "scan wire" (a line y = constant) to sample the nature of the intersections of the two apparent stars. Refer to Figures 2 and 3. Only if the "scan wire" finds two roots for each ellipse (four total roots) is it possible to have an eclipse. For some y = constant, let the four roots be \( Z_{A1}, Z_{A2} \) (for star A) and \( Z_{B1}, Z_{B2} \) (for star B). The way in which these roots interleave determines if there is an eclipse, and if it is partial. If \( Z_{A1} \) and \( Z_{A2} \) are both always greater (or less) than \( Z_{B1} \) and \( Z_{B2} \), then there is no eclipse (Figure 2, case III). For a partial eclipse, at least part of the time the four roots must alternate; e.g. \( Z_{A1} > Z_{B1} > Z_{A2} > Z_{B2} \). The scan wire is stepped from right to left to determine a change in the interleaving of the four roots or a change in the number of roots. When a change occurs, the scan is reversed at 1/10 of the step size. This is repeated to determine the y-coordinate of the limits of integration to about \( 10^{-8} \) of the semi-major axis.

Certain special situations are sought for:

1. If the eclipse is potentially shallow, a smaller step size and fewer scan reversals are used.

2. If the potentially shallow eclipse is to the left hand end of the star (-y axis), the negative limit is sought first.

3. If partial derivatives of photometric parameters are being calculated, the previously-determined limits are used.
CASE I
ONLY 2 ROOTS;
NO ECLIPSE

CASE II
4 ROOTS, BUT
NO ECLIPSE

CASE III
4 ROOTS; ECLIPSE
TOTAL OR ANNULAR

CASE IV
4 ROOTS; PARTIAL
ECLIPSE WITH STAR
LIMBS AS INTEGRATION
LIMITS

CASE V
4 ROOTS; PARTIAL
ECLIPSE WITH
INTERSECTIONS AS
INTEGRATION LIMITS

CASE VI
4 ROOTS; PARTIAL
ECLIPSE WITH
THE GENERAL SITUATION
OF 4 INTERSECTIONS

Figure 2. Cases Encountered by Subroutine Limity in Determining Partial Eclipse Integration Limits.

4. If partial derivatives of geometric parameters are being calculated, the previously-determined limits are used to set initial values.

K. Determination of Z Limits of Partial Eclipse (LIMITZ)

LIMITZ is another entry in LIMITY, used by ECLINT to determine the limits of integration in the z direction. Referring to Figure 2, cases IV, V and VI, notice that of the four roots for any given y within the region of overlap, the two "inner" roots are the z limits.

L. Integrations (TOTINT, ANNECL, ECLINT, ATMECL)

Integration by Gaussian quadrature over an ellipse of semi-axes a, b is given by:
Figure 3. Scan Line Search for Integration Limits, $y_H$ and $y_L$. Line a Finds Only Two Roots; Line b Finds Four Roots but They do not Intermesh; Line c Finds Four Roots Which Intermesh. Thus $y_H$ is Found by Reversing the Direction of Scan (and Reducing the Step Size) Whenever the Scan Line Changes From Situation b to c or Vice Versa. Line d Finds Only Two Roots; Line e Finds Four Roots, and One Pair is Contained Within Another. Thus $y_L$ is found by going Back and Forth Between Situation d and e.
\[ I = ab \sum_{j=1}^{n} W_j \sqrt{1 - X_j^2} \sum_{i=1}^{n} W_i I_p(aX_j, X_i b \sqrt{1 - X_j^2}) \tag{18} \]

where \( I_p(y, z) \) is the intensity, along the line of sight, at point \( y, z \). This is the argument \((I_o + I^*)(1 - u + u \cos \gamma)\) of equation (15). This argument is calculated by the function \textsc{bright}. The \( W \) and \( X \) are the Gaussian weights and coordinates, set by \textsc{grid}.

Integration over an eclipsed area, which, in this case is bounded by elliptical arcs, is given by:

\[ I = Y_D \sum_{j=1}^{n} W_j Z_D \sum_{i=1}^{n} W_i I_p(Y_j X_j + Y_s, Z_D X_i + Z_s) \tag{19} \]

where:

\[
\begin{align*}
Y_D &= \frac{1}{2} (Y_H - Y_L) \\
Y_S &= \frac{1}{2} (Y_H + Y_L) \\
Z_D &= \frac{1}{2} (Z_H - Z_L) \\
Z_S &= \frac{1}{2} (Z_H + Z_L)
\end{align*}
\tag{20}
\]

\( Y_H \) and \( Y_L \) are the \( y \) limits of integration. \( (Y_H > Y_L) \) from \textsc{limity}; \( Z_H \) and \( Z_L \) are the \( Z \) limits, from \textsc{limitz}, and are functions of:

\[ Y_j = Y_D X_j + Y_S. \]

The function \textsc{bright} operates in the plane of the sky \((y, z)\) coordinate system. Except for \textsc{eclint}, integration is performed in the \((\tilde{y}, \tilde{z})\) coordinate system, aligned with the ellipse axes. Thus, in most cases, a coordinate rotation is necessary. Additionally, coordinate translations must be performed in the case of \textsc{annecl}, because the integration is performed over the area of one star, but with the intensity of the points on the other star. Note that in \textsc{eclint}, if star B is eclipsed, this translation must also be performed, since the integration limits are in the \((x, y, z)_A\) system, centered on star A.

Whenever an atmospheric eclipse can occur, \textsc{atmecl} is used in lieu of \textsc{totint} for the eclipsed star. Every point which is behind the atmosphere of the eclipsing star is calculated by \textsc{bright} but then attenuated by \textsc{absorb}. Note that the portion of the eclipsed star which will be physically eclipsed is not attenuated, since that light will be removed by \textsc{eclint} or \textsc{annecl}. Integration precision in \textsc{atmecl} is relatively poor.

23
Schematic grid points (for $4 \times 4$ integration) are shown in Figure 4.

M. **Intensity at a Point (BRIGHT)**

BRIGHT is used by the integration routines TOTINT, ANNECL, ECLINT and ATMECL to evaluate the argument of the integration; that is the apparent intensity at a point on a star. Normally, BRIGHT calls the reflection function REFL. However, when partial derivatives are being calculated, certain shortcuts are desirable. In the matrix RINTS are stored five sets of the results of the function REFL; for star A at $t$, star B at $t$, eclipsed area at $t$, star A at $T_Q$, and star B at $T_Q$. When a numerical derivative is calculated, these "old" stored values of incident energy are used to calculate reflection. That is, it is assumed that the effect of a change in incident energy is second order when a parameter is perturbed. See Table VIII.

<table>
<thead>
<tr>
<th>Statement Number</th>
<th>Sub Program</th>
<th>Computational Milestones</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>REFL</td>
<td>Step MREF, which counts the calls to BRIGHT for this particular integration.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set NN. NREF is 1 if total integration of star 1; 2 if total integration of star 2. 3 if eclipse integration.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NINC is 3 for normalization integrations; 0 for all others. NN will thus always be 1, 2, 3, 4 or 5.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculate $\cos \gamma$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set up RAD for call to REFL. The parameters $x, y, z, \sqrt{x^2 + y^2 + z^2}$ are passed via COMMON/RINT.</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>If this is not a partial derivative calculation, load RINTS with new reflection values. Otherwise, the pre-stored values are used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Calculate intensity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If the $y, z$ coordinates in the call to BRIGHT fall off the star, then $x$ is imaginary. In this case $x$ is set to zero and a message printed. This may occur occasionally due to rounding error, in which case $x = 0$ is a good approximation.</td>
</tr>
</tbody>
</table>
Figure 4. Schematic Representation of Integration Grid as Used in Integration Functions
N. Incident Energy for Reflection (REFL)

The energy incident upon one star is assumed to be a function of the following properties of the other (source) star:

1. Apparent angular extent as seen from the first star,
2. Intensity at the end of the a-axis,
3. Limb darkening,
4. Apparent zenith distance as seen from the first star.

This approximation has been empirically determined from more exact integrations over the source star. The more rigorous treatment increases computing time by about an order of magnitude.

O. Initialization of Reflection (ZONES)

ZONES remains because historically it was necessary when detailed reflection was used. It is preserved in case the user wants to incorporate a more exact reflection model. The present ZONES calculates the emergent intensity at the end of the a-axis.

P. Calculation of Total Stellar Radiation (OUTPUT and ENERGY)

These routines are used only for display purposes. The total 4π steradian stellar energy output is calculated to more nearly represent the relative stellar brightnesses than the surface intensity at time $T_Q$. The calculation has an accuracy of 2 to 3%.

Q. Atmospheric Absorption (ABSORB)

ABSORB attenuates any intensity (as calculated by BRIGHT) according to equations (13) and (14).

R. Differential Corrector (SOLVE1)

The differential corrector logic flow is shown in Table IX. The program has the following traits:

1. The outer loop is over the individual observations; so that for a particular time of observation, $t_i$, all necessary partial derivatives are calculated.
2. The logical vector TEST determines which system parameters are to be considered as variables.
<table>
<thead>
<tr>
<th>Statement Number</th>
<th>Sub Program</th>
<th>Computational Milestones</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>GEOMET</td>
<td>Start outer loop over all observations. When all observations are done, go to 250</td>
</tr>
<tr>
<td></td>
<td>LUMC</td>
<td>Set IREF flag to 1 for calculation of $\Delta I$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compute time-independent quantities; including normalization</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obtain calculated intensity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Form $\Delta I$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set IREF flag to 2 for calculation of partials</td>
</tr>
<tr>
<td>401–415</td>
<td>GEOMET</td>
<td>Check each parameter to see if designated as a variable; if so, continue. When no more variables, go to next observation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Special coding for each variable to set up high and low values for partials. From here control goes to 440, 450, 480, 4400, or 4800</td>
</tr>
<tr>
<td>4400, 4800</td>
<td>ASTROX</td>
<td>If this is astrophysical variable, convert to model parameters, and calculate new normalization</td>
</tr>
<tr>
<td></td>
<td>GEOMET</td>
<td>Calculate partial derivative; go to 200</td>
</tr>
<tr>
<td>4530</td>
<td>LUMC</td>
<td>Calculate partial derivative; go to 200</td>
</tr>
<tr>
<td>440, 450</td>
<td>GEOMET</td>
<td>Calculate new normalization for non-astrophysical variable</td>
</tr>
<tr>
<td>480</td>
<td></td>
<td>Calculate partial derivative; go to 200</td>
</tr>
<tr>
<td>520</td>
<td>LSQS</td>
<td>Output time vs. $\Delta I$</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>Solve normal equations</td>
</tr>
<tr>
<td>533</td>
<td></td>
<td>Screen size of corrections; reduce if necessary. If not converged, step INDIC counter. If extremely small correction, delete from further consideration by changing logical vector</td>
</tr>
<tr>
<td>710</td>
<td>ASTROX</td>
<td>Screen values of new parameters</td>
</tr>
<tr>
<td>1720, 1730</td>
<td></td>
<td>Insure that all new parameters are installed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set QUAD with new value</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If, upon entry, there were no &quot;TRUE&quot; variables; or if convergence has failed, output time and delta magnitude</td>
</tr>
</tbody>
</table>
3. These variables are arranged (by integer vector VV) so that "photometric" variables precede "geometric" variables in order of treatment.

4. Since internally all subprograms operate in "model space," whereas SOLVE1 operates in "astrophysical space," all astrophysical variable perturbations must be "translated" by a call to ASTROX. The two types of variables are differentiated by the integer vector V.

5. After being calculated by the least squares subroutine LSQS, the differential corrections are subjected to a variety of filters, any of which the user may feel he wants to alter. These are:
   a. Restrict all differential corrections to 25% of the current value of the variable (or to 0.25 if the variable is zero).
   b. Consider a variable as "over-converged," and delete it as a variable, if the differential correction is less than 0.01% of the current value (or less than 0.001% if the current value is less than 0.001).
   c. Flag a variable as not converged if the differential correction is greater than 1% (or greater than 0.001 if the variable is zero). Convergence is accepted only if no variable is so flagged.
   d. Examine the new values of the variables (old value plus differential correction) to see if they lie in an acceptable range. The upper limits are in vector SA and the lower limits in vector SB.

6. Note that system quadrature magnitude (i.e., the normalization) is available as an adjustable parameter, but it is handled in a different manner than the other variables.

S. Least Squares (LSQS and MAMUL)

LSQS and the matrix multiplication routine MAMUL, have been taken from an old IBM 704 SHARE routine.

T. Common Blocks

The common blocks are organized in the following manner:

1. ORBE main orbital elements.
2. AUXE auxiliary or secondarily derived orbital elements.
3. TVARS quantities which are time-dependent.
4. FLAGS a variety of flags and counters.
5. const $\pi$, $2\pi$, and $\pi/2$.
6. OBS vectors set aside primarily to hold observational data (up to 101 observations).
7. VARIB variables for "astrophysics space."
8. ORBIT fixed celestial mechanical parameters.
9. ROTAT coordinate rotation and translation constants.
10. GAUSS Gauss quadrature constants.
11. GAUXX Guass $16 \times 16$ quadrature constants.
12. TRIG sin and cos functions of important angles.
13. RINT communication between BRIGHT and REFL.

V. Operation of the Computer Program

The computer program can be used for two distinct calculations; predicting a light curve for given parameters or solving an observed light curve for the parameters which produce the best fit.

A. Input of Parameters

Orbital parameters, whether for prediction or as initial approximations for solution, are input one per input record. The nature of the parameter is specified by a two-digit code, and the value in a 10-digit field, which must have the decimal point specified. Table II indicates the code, the parameter meant, and the default values. Note that the interpretation of several of the input parameters (astrophysical vs. model) is determined by the value specified for the polytropic index: an index of zero specifies model parameters. Input is terminated by a code of 1, -1, or -2. Table III lists a number of control parameters and their interpretations.

B. Light Curve Prediction

A light curve may be predicted either in astrophysics or model space. The use of model space may be useful for "unusual" distortions, for example, simulating a rapidly-rotating star by an oblate spheroid.

The prediction mode is specified by terminating input with a code of zero. Prediction may be either for equally spaced times over any interval, or at individually specified times.

1. Prediction at equally spaced times—specify start time, interval, and end time (see Table III). The end time may equal the start time, but it may not be earlier than the start time. The interval must not be zero.

2. Prediction at specified times—specify a time interval of zero. Then, following the card which ends parameter input, specify times, one per card, in a 10 wide field. The decimal must be specified. End of input is specified by a negative time.
C. **Light Curve Solution**

Light curve solution is possible only in astrophysics space. Solution mode is specified by terminating parameter input with an input code of -1. This card is then followed by the observations, one per card. An observation card consists of three fields of ten (specify the decimal in all fields):

- **col 1-10:** time
- **col 11-20:** observed magnitude or intensity (See input code 87 in Table III)
- **col 21-30:** weight of observation

Note that the least squares subroutine will give all observations unit weight if the first observation is specified to have a weight of zero. In spite of possibly intrinsically lower accuracy, it is not advisable to give in-eclipse observations lower weight, since their low accuracy is compensated by their high information content.

Observational data input is ended by a data card with a negative time. This card is ten followed by a "T/F" card which specifies which parameters are "variable." This card consists of seventeen 2-character fields. A "T" indicates the parameter is to be variable. An "F" (or blank) means it is to be held at its specified value. The seventeen fields, which mostly correspond to input data codes 1 through 17, are described in Table X. Note that another solution of the same observational data can be made by terminating parameter input with input code -2. No data cards are read, so the next card must be the T/F card.

If the T/F card specifies all parameters false (a blank card) then observed minus computed magnitudes are printed for each observed time.

D. **Data Card Arrangement**

Data input decks are shown schematically in Figure 5.

1. Card 1--title card. Must have first four letters "WINK". Any other four characters (including blanks) terminates the program.
2. Any number of parameter cards, the last card containing a code of 1, -1, or -2. The requirement for any other cards is determined by this number.
3. Observation cards
   a. If the last parameter card had code zero, and the interval is not zero, no more cards are read. The next card should be a new title card ('WINK' to go on, anything else, e.g., 'STOP', to stop).
Table X. T/F Card Codes

<table>
<thead>
<tr>
<th>column</th>
<th>A &quot;T&quot; in column makes this a variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Inclination</td>
</tr>
<tr>
<td>4</td>
<td>$e \sin \omega$</td>
</tr>
<tr>
<td>6</td>
<td>$e \cos \omega$</td>
</tr>
<tr>
<td>8</td>
<td>$u_A$</td>
</tr>
<tr>
<td>10</td>
<td>$u_B$</td>
</tr>
<tr>
<td>12</td>
<td>$A_{oA}$</td>
</tr>
<tr>
<td>14</td>
<td>$k_{\nu}$</td>
</tr>
<tr>
<td>16</td>
<td>$\beta_A$</td>
</tr>
<tr>
<td>18</td>
<td>$\beta_B$</td>
</tr>
<tr>
<td>20</td>
<td>$T_A^*$</td>
</tr>
<tr>
<td>22</td>
<td>$T_B^*$</td>
</tr>
<tr>
<td>24</td>
<td>$T_{CONJ}$</td>
</tr>
<tr>
<td>26</td>
<td>mass ratio</td>
</tr>
<tr>
<td>28</td>
<td>quadrature magnitude</td>
</tr>
<tr>
<td>30</td>
<td>(not used)</td>
</tr>
<tr>
<td>32</td>
<td>$w_A$</td>
</tr>
<tr>
<td>34</td>
<td>$w_B$</td>
</tr>
</tbody>
</table>

b. If the last parameter card had code zero, and the interval is zero, observation cards specifying times for prediction are needed. The last card must have a negative time. The next card would be a new title card.

c. If the last parameter card had code -1, observation cards are necessary; the last card must have a negative time. This card must be followed by a T/F card, specifying which parameters are variable. The next card would be a new title card.

d. If the last parameter card was -2, the next card must be the T/F card; followed by a new title card.

E. Sample Data Run

To assist in checking out the program on another computer, Appendix IV contains sample input/output for different runs.

*It is never advisable to allow both temperatures as variables: There is not that much information in one light curve. Fix the temperature best determined by spectroscopy or multicolor photometry.
PREDICT LIGHT CURVE FOR EQUALLY-SPACED POINTS (PARAMETER 21 ≠ 0)

PREDICT LIGHT CURVE FOR INDIVIDUALLY SPECIFIED TIMES (PARAMETER 21=0)

SOLVE LIGHT CURVE

SOLVE LIGHT CURVE WHICH IS ALREADY READ IN

Figure 5. Input Card Decks.
REFERENCES

Wood, D. B., 1971e, invited paper, IAU Colloquium No. 16, to be published.
APPENDIX I

EQUATIONS

A. Orbital Mechanical Equations Used in ORBITB

1. Time of periastron, \( T_o \), is found from:

\[
\cos^{-1} \left[ \frac{e \sin \omega + e^2}{e \left(1 + e \sin \omega \right)} \right] - q \frac{e \cos \omega}{1 + e \sin \omega} = \mu (T_c - T_o)
\]

where:

\[
q = \sqrt{1 - e^2} \\
\mu = 2 \pi / P
\]

2. The mean anomaly, \( M \), is given by:

\[
M = \mu (t - T_o)
\]

3. Kepler's equation, \( E - e \sin E = M \), is conveniently solved by iteration when expressed in the form:

\[
\Delta E = \frac{e \sin E - E + M}{1 - e \cos E}
\]

where the first approximation, for \( e \leq 0.75 \), is \( E = M \).

4. The true anomaly, \( v \) is found from:

\[
\cos v = \frac{\cos E - e}{1 - e \cos E} \\
\sin v = \frac{q \sin E}{1 - e \cos E}
\]

5. The orbital longitude \( \theta \) follows from:

\[
e \sin \theta = e \sin \omega \sin v - e \cos \omega \cos v \\
e \cos \theta = e \cos \omega \cos v + e \cos \omega \sin v
\]

6. The radius vector, \( R \), is given by

\[R = R_o (1 - e \cos E)\]
7. For an eccentric orbit, the star, rotating in uniform motion, will deviate from $\theta$ by a small angle $\Theta$ where:

$$\sin \Theta = \sin v \cos M - \cos v \sin M$$

The star's orbital longitude, $\theta'$, is given by:

$$\theta' = \theta - \Theta$$

8. The apparent separation of centers is given by:

$$\delta = R\delta'$$

where:

$$\delta' = (\sin^2 \theta + \cos^2 \theta \cos^2 i)^{1/2}$$

9. The angle, $\chi$, from the Z-axis to the center of the other star is given by:

$$\cot \chi = \cos i \cot \theta'$$

B. Ellipsoidal Star Equations Used in PARAM

1. The triaxial ellipsoid in 3-dimensions is generally written as:

$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Eyz + 2Fxz = a^2b^2c^2$$

where:

$$A = b^2c^2 \cos^2 \theta' \sin^2 i + a^2c^2 \sin^2 \theta' \sin^2 i + a^2b^2 \cos^2 i$$

$$B = b^2c^2 \sin^2 \theta' + a^2c^2 \cos^2 \theta'$$

$$C = b^2c^2 \cos^2 \theta' \cos^2 i + a^2c^2 \sin^2 \theta' \cos^2 i + a^2b^2 \sin^2 i$$

$$D = (b^2c^2 - a^2c^2) \sin \theta' \cos \theta' \sin i$$

$$E = (b^2c^2 - a^2c^2) \sin \theta' \cos \theta \cos i$$

$$F = (b^2c^2 \cos^2 \theta' + a^2c^2 \sin^2 \theta' - a^2b^2) \sin i \cos i$$

2. The cosine of the projection angle for limb darkening is:

$$\cos \gamma = (Ax + Dy + Fz)/T$$
where $T = [x^2(A^2 + D^2 + F^2) + y^2(B^2 + D^2 + E^2) + z^2(C^2 + E^2 + F^2) + 2xy(AD + BD + EF) + 2xz(AF + CF + DE) + 2yz(BE + CE + DF)]^{1/2}$

3. To project onto the plane of the sky, $x$ must be eliminated by the expression:

$$x = -\frac{Dy + Fz}{A} + \frac{1}{A} \left[ Ny^2 + Pz^2 + 2Ryz + S \right]^{1/2}$$

where:

- $N = D^2 - AB$
- $P = F^2 - AC$
- $R = DF - AE$
- $S = Aa^2 b^2 c^2$

4. The axes of the apparent ellipse—the outline of the ellipsoid on the plane of the sky—are:

$$a' = \left[ \frac{2S}{-(P - N)^2 + (2R)^2} \right]^{1/2}$$

$$b' = \left[ \frac{2S}{[(P - N)^2 + (2R)^2]} \right]^{1/2}$$

5. The major axis of this ellipse is rotated from the $y$-axis by angle $\phi$ given by:

$$\cot 2 \phi = \frac{N - P}{2R}$$

6. The angle, $\alpha$ between the major axis of one apparent ellipse and the center of the other is given by:

$$\cos \alpha = (\sin \theta' \cos \phi - \cos \theta' \cos i \sin \phi)/\delta'$$

$$\sin \alpha = - (\sin \theta' \sin \phi + \cos \theta' \cos i \cos \phi)/\delta'$$

C. Other Important Equations

1. Mean stellar radius (to sub-earth point at quadrature)

$$\bar{r} = \left[ \sin^2 i \left( \frac{\sin^2 \Theta}{a^2} + \frac{\cos^2 \Theta}{b^2} \right) + \frac{\cos^2 i}{c^2} \right]^{-1/2}$$
2. Partial derivative $\frac{\partial I}{\partial Q}$ where $Q$ is the normalization factor:

$$\frac{\partial I}{\partial Q} = 0.92061 \ I_{\text{COMP}}$$

3. Approximation to total bolometric energy radiated over $4\pi$ steradians:

$$E_{\text{bolo}} = 8 \left( \frac{a}{2} \right) \left( \frac{b}{2} \right) \sum_{j=1}^{n} w_j \frac{1}{2} \sqrt{4 - (1 + X_j)^2} \sum_{i=1}^{n} w_i I_N(x_N, y_N)$$

where:

$$x_N = \left( \frac{a}{2} \right) (1 + X_j)$$

$$y_N = \frac{1}{2} \left( \frac{b}{2} \right) (1 + X_j) \sqrt{4 - (1 + X_j)^2}$$

and the normal emergent intensity, $I$, at $(x_N, y_N)$ is given by:

$$I_N = I \left( 1 - \nu + \nu \frac{r_N}{r} \right) G$$

The radius, $r_N$ to point $(x_N, y_N)$ is given by:

$$r_N = \left[ c^2 - X_N \left( \frac{c^2}{a^2} - 1 \right) \right]^{1/2}$$

The quantity $G$ is the area element:

$$G = \left[ \frac{x_N^2}{a^2} \left( \frac{c^2}{a^2} - 1 \right) + \frac{y_N^2}{b^2} \left( \frac{c^2}{b^2} - 1 \right) \right]^{1/2}$$
APPENDIX II

REFLECTION APPROXIMATION

The amount of radiation, $L^*$, which is incident upon the reflecting star is assumed to be a function of:

1) the intensity of the substellar point on the source star, $I_s$.
2) the apparent area of the source star, $A_s$
3) the limb darkening of the source star, $u_s$
4) the cosine of the zenith distance of the source star, $\cos \lambda'$

Thus, we seek a function of the form:

$$L^* = I_s A_s f(u_s) g(\cos \lambda')$$

If the source star were at an infinite distance, we would expect the limb darkening dependence to be $(1 - u/3)$. However, since the star is quite close when reflection is important, its radiation is more strongly dependent upon limb darkening. Empirically, the dependence is found to be very well approximated by:

$$f(u_s) = 1 - u_s/2,$$

which is reasonable since:

$$\int_0^{\pi/2} (1 - u + u \cos \theta) \sin \theta d\theta = 1 - u/2$$

The function $g$ is more complex. For a given geometry, $g$ can be approximated by two straight lines, with the change of slope occurring when the source star starts to set. The slopes and intercepts of these lines are functions of the sizes of the source star, $a_s$, and the reflecting star, $a_r$. The approximation used is:

$$g(\cos \lambda') = -0.065354 + a_r g_1 + (\cos \lambda') (2.044 + a_r g_2) + C$$

where:

$$g_1 = 0.224935 - 0.761696 a_s + 3.81425 a_s^2$$
$$g_2 = -0.170831 + 1.231707 a_s - 9.955083 a_s^2$$

$C$ is the larger of 0 or:
The zenith angle is given by:

$$\cos \lambda' = (\lambda \Delta x + \mu \Delta y + \nu \Delta z)/d$$

where:

$$\lambda, \mu, \nu$$ are the direction cosines of the local normal and $$(\Delta x, \Delta y, \Delta z)$$ are the direction numbers of the center of the source star. The distance between the point on the reflecting star and the center of the source star, $$d$$, is given by:

$$d = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}.$$

The apparent area of the source star is given by calculating:

$$\lambda = r_s/d$$ and $$\lambda_2 = \lambda - \pi/2 + \lambda'.$$

Then:

1) If $$0 < \lambda_2 < \lambda$$

$$A = \lambda^2 \cos^{-1}\left(\frac{\lambda - \lambda_2}{\lambda}\right) - (\lambda - \lambda_2)(2 \lambda \lambda_2 - \lambda_2^2)^{1/2}$$

2) If $$\lambda \leq \lambda_2 < 2\lambda$$

$$A = \pi \lambda^2 - \lambda^2 \cos^{-1}\left(\frac{\lambda - \lambda_1}{\lambda}\right) + (\lambda - \lambda_1)(2 \lambda \lambda_1 - \lambda_1^2)^{1/2}$$

3) If $$\lambda_2 \geq 2\lambda$$

$$A = \pi \lambda^2$$

4) If $$\lambda_2 \leq 0$$, $$A = 0$$

Note that $$\lambda_1 = \lambda + \pi/2 - \lambda'.$$
APPENDIX III

COMPUTER LISTINGS
C MAIN PROGRAM FOR ECLIPSING BINARY CURVE PREDICTION OR SOLUTION.
C THIS VERSION CONSTRUCTED FOR LEAST SQUARE SOLUTION WITH
C ASTROPHYSICAL VARIABLES
C WRITTEN BY D. B. WOOD
C REVISED FOR MINIMUM IBM S360 CONFIG ON 30 SEPT 1972
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIQK, ELLIP, ELLIPB, 1 EPSI, EPSIB, TCONJ, VA, VB, RATIJO, WA, WB, PERIOD
COMMON/CONST/ PI, TW0PI, HALPI
COMMON/ORBIT/ E, TO, Q, MU
COMMON/TVARS/ALPHA(2), THETA, CHIR, DELTA, PARA(19, 2), AAXIS(2)
1 BAXIS(2), PHASE, PHI(2), THETA(2)
COMMON/OBS/ TIME(101), LUM(101), WT(101), CLUM(101)
COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), TILT(2), QINT(2), AA(2), BB(2)
1 CC(2), RBAR(2), UMA(2), VMA(2), MMA(2), RNOT, BOLOJ, QUAD, TOTAL
2 STAR1, STAR2, STAR3
COMMON/FLAGS/IFSPH, JTYPE, KSTAR, TEST(17), NOBS, NZONE, MREF, NREF
1 IREF, NINC, LST
COMMON/VARIB/BLQ(18)
DIMENSION BLOAD(18), BL(18), SKAL(2), TP(2)
DIMENSION AUX(8), WORD(15), TITLE(15), LARL(3)
EQUIVALENCE (AUX(1), SCALE(1)), (BL(1), INCL)
DATA LABL / 'A', 'B', 'I'/
DATA SKAL / 0, 0/
DATA STOP / 'WINK'/
REAL IMX(3) / '4X4', '6X6', '16X16' /
DATA BLOAD / 90.2*0,2*.6,25,1.2*25,2*1E4,0.1,4*0,1,/ DATA WORD / 'PARK', 'OSPH', 'URAL', 'OTAL', 'TIAL', 'ERIC', 'I'/
REAL INCL, MU, LUM, LUMC
LOGICAL TEST
C ONE-TIME INITIALIZATIONS
DEGRAD = 1.745329E-2
BLOAD(1) = BLOAD(1)*DEGRAD
DO 10 I=1,18
BL(I) = BLOAD(I)
10 BLQ(I) = BL(I)
DO 11 I=1,18
11 AUX(I) = 0.
RNOT = 1.
NTEG = 1
S = 0.
TINT = 0.1
END = 0.5
QUAD = 0.
ONE = 1.
WAVE = 5500.
POLYX = 5.
JUMP = 1
ILUM = 0
ICRD = 0
PI = 3.141592
TWOPI = PI+PI
HALPI = PI/2.
50 NFLAG = 1
C RE-ENTRY POINT
66 READ (5,965) (TITLE(I),I=1,15)
WRITE (6,966) (TITLE(I),I=1,15)
C STOP IF FIRST WORD OF TITLE IS NOT 'WINK'
IF (TITLE(1) .NE. 'WINK') STOP
47 DO 48 I=1,17
BL(I) = BLQ(I)
48 TEST(I) = .FALSE.
IFFY = 6
51 READ (5,805) I,DATA
WRITE (6,951) I,DATA
COl 1-2, INT. CODE -- COl 3-12, PARAMETER
***REMARK***
C N<0 QUAD * INITIATE SOLVE MODE
C (USE N=-1 TO REQUEST OBS. DATA INPUT. USE N=-2 IF OBS. IN CORE)
C =0 QUAD * INITIATE PREDICT MODE
C =1 I (DEGREES) CODE 1-18 FOR ORB. PARAM.
C =2 E SIN OMEGA
C =3 E COS OMEGA
C =4 UA
C =5 UB
C =6 A (UNPERTURBED SPHERE)
C =7 K (UNPERTURBED SPHERE)
C =8 ELLIP A (BETA: GRAV. EXP.)
C =9 ELLIP B (BETA: GRAV. EXP.)
C =10 EPSI A (EQUATORIAL TEMP)
C =11 EPSI B (EQUATORIAL TEMP)
C =12 T CONJ
C =13 VA (MASS RATIO)
C =14 VB (USED INTERNALLY FOR QUAD MAG)
C =15 J (NOT USED FOR AP SOLN)
C =16 WA
C =17            WB
C =18            PERIOD
C =19            INTEGRATION PRECISION CODE
C =20            FOR LIGHT CURVE PREDICTION
C =21            DELTA TIME
C =22            FOR LIGHT CURVE PREDICTION
C =23            END TIME
C =24            'THIRD' STAR LIGHT
C =25            QUAD *
C =26            SET QUADRATURE MAGNITUDE
C =27            WAVELENGTH
C =28            WAVELENGTH OF OBS (ANGSTROMS)
C =29            POLYTROP I
C =30            POLYTROPIC INDEX OF STARS
C =31            SCALE HT A
C =32            STAR ATMOSPHERE
C =33            SCALE HT B
C =34            STAR ATMOSPHERE
C =35            TAU A
C =36            OPTICAL DEPTH OF ATMO AT SURFACE
C =37            TAU B
C =38            OPTICAL DEPTH OF ATMO AT SURFACE
C =39            SLIP A
C =40            ORBITAL PHASE SHIFT (RADIANS)
C =41            SLIP B
C =42            ORBITAL PHASE SHIFT (RADIANS)
C =43            TILT A
C =44            STAR EQUATOR TILT (RADIANS)
C =45            TILT B
C =46            STAR EQUATOR TILT (RADIANS)
C =47            IFFY
C =48            MAX. ITERATIONS
C =49            0*1,2
C =50            I FOR PUNCHED OUTPUT IN MAG (2:LUM)
C =51            0*1
C =52            1 TO READ INPUT AS LUMINOSITY
C =53            --
C =54            FLIP/FLOP FOR ORB. ELEMENT PRINTOUT
C =55            --
C =56            LEAVE PROGRAM - RETURN TO EXEC SYSTEM
C (ANY UNIDENTIFIABLE CODE IS IGNORED)
C * NOTE THAT 'QUAD' IS LIGHT AT QUADRATURE - MAY BE INPUT WITH '24'
C OR WITH COMPUTATION INITIATION CODE (I * LE. 0). IN THIS LATTER
C CASE, QUAD=0 IS IGNORED AND PREVIOUS VALUE USED.
C NOTE THAT CODES, * LE. 0 TERMINATE INPUT AND START COMPUTATION
IF (I * LE. 0) GO TO 55
IF (I * EQ. 20) S = DATA
IF (I * EQ. 21) TINT = DATA
IF (I * EQ. 22) END = DATA
IF (I * EQ. 23) ONE = 1. - DATA
IF (I * EQ. 24) QUAD = DATA
IF (I * EQ. 25) RNOT = DATA
IF (I * EQ. 26) WAVE=DATA
IF (I * EQ. 27) POLYX=DATA
IF (I * GT. 30 . AND. I * LT. 39) GO TO 70
IF (I * EQ. 39) NTEG = DATA
IF (I * EQ. 44) IFFY = DATA
IF (I * EQ. 46) RNOT = DATA
IF (I * EQ. 47) ILUM = DATA
IF (I * EQ. 48) JUMP = 3 - JUMP
IF (I .EQ. 99) STOP
IF (I .GT. 18) GO TO 51
BL(I) = DATA
IF (I .EQ. 1) INCL = INCL*DEGRAD
GO TO 51
55 IF (DATA) 5,6,5
5 QUAD = DATA
6 CALL ASTROQ(WAVE,POLYX)
STAR3 = 1. - ONE
SCALE(1) = SKAL(1)*RNOT*A
SCALE(2) = SKAL(2)*RNOT*A*RATIOK
IF (I .LT. 0) GO TO 200
II = 0
IF (END .LT. S) GO TO 60
C IF TINT=0, GO READ TIMES AS INPUT DATA, USING LOGIC WHICH READS
C LIGHT CURVE FOR SOLUTION
IF (TINT) 60,200,53
53 NOBS = 1
TIME(1) = S
7 NOBS = NOBS + 1
IF (NOBS .EQ. 102) GO TO 160
TIME(NOBS) = TIME(NOBS-1) + TINT
IF (TIME(NOBS) .LE. END) GO TO 7
100 CONTINUE
NOBS = NOBS - 1
C LOOP REENTRY POINT FOR SOLUTION OR NEW PARAMETERS
52 CONTINUE
CALL GRID(NTEG)
AINCL = INCL/DEGRAD
IREF = 1
CALL GEOMET
RATIO = STAR2/STAR1
SLB = ONE*RATIO/(1. + RATIO)
SLA = ONE - SLB
GO TO (103,102), JUMP
103 CONTINUE
EA = ENERGY(1)
EB = ENERGY(2)
ER = EB/EA
ELB = ER/(1. + ER)
ELA = 1. - ELB
OMEGA = 0
IF (E .NE. 0.) OMEGA = ARSIN(ESINW/E)/DEGRAD
GNU2 = BLQ(6)*BLQ(7)
AZ1 = RNOT*BLQ(6)
AZ2 = RNOT*GNU2
IF (POLYX .EQ. 0) GO TO 105

CALCULATE POLAR TEMPERATURE (FOR DISPLAY ONLY)
C = 1.43879E8/WAVE
DO 104 I=1,2
TP(I) = EXP(C/BLQ(9+I)) - 1.
104 TP(I)=C/(ALOG(1.+1./(VMA(I)+BLQ(12+I)*CC(I)/BB(I))*TP(I)))
105 CONTINUE
WRITE (6,901) PERIOD,TCONJ,AINCL,ESINK,ECOSW,OMEGA,E,RNOT
WRITE (6,902) LABL(1),BLO(6),AZ1,BLOQ(8),BLOQ(10),TP(1)
WRITE (6,903) WAVE,A,RATIOK,BLOQ(7),RATIOJ,BLOQJ,BLOQ(13),QUAD
WRITE (6,904) LABL(2),ELLIP,EPsi,UA,VA,WA,AA(1),BB(1),CC(1)
WRITE (6,905) LABL(2),ELLIPB,EPsib,UB,VB,WA,AA(2),BB(2),CC(2)
WRITE (6,906) RATIO,ER
WRITE (6,907) (LABL(I),SCALE(I),SURF(I),SLIP(I),TILT(I),I=1,2)
WRITE (6,908) IMX(NTEG)
102 IF (11 .LT. 0) GO TO 150

COMPUTE THEORETICAL LIGHT CURVE
WRITE (6,909)
DO 110 I=1,NOBS
CLUM(I) = LUMC(TIME(I))
CINT = -2.5*ALOG1O(CLUM(I)) + QUAD
IF (KSTAR .EQ. 3) JTYPE = 5
1 WRITE (6,910) WORD(JTYPE),WORD(JTYPE+5),WORD(JTYPE+10),
WRITE (6,911) LABL(KSTAR),TIME(I),PHASE,CLUM(I),CINT
IF (ICRD .EQ. 0) GO TO 110
IF (ICRD .EQ. 2) GO TO 107
C PUNCH EITHER IN LUMINOSITY OR MAGNITUDE
WRITE (7,980) TIME(I),CINT
GO TO 110
107 WRITE (7,980) TIME(I),CLUM(I)
110 CONTINUE
GO TO 66
C LIGHT CURVE SOLUTION
150 CONTINUE
CALL SOLVE1(IFFY)
155 IF (IFFY) 59,58,52
58 WRITE (6,925)
GO TO 66
59 IF (IFFY+2) 66,66,46
46 WRITE (6,926)
DO 49 I=1,17
IF (.NOT. TEST(I)) GO TO 49
WRITE (6,949) I, BLQ(I)
49 CONTINUE
GO TO 66
70 J = I - 30
IF (J .LE. 2) GO TO 75
AUX(J) = DATA
GO TO 51
75 SKAL(J) = DATA
GO TO 51
C SOLUTION DESIRED, READ OBSERVATIONAL DATA
200 IF (I .LT. -1) GO TO 203
NOBS = 1
WRITE (6,921)
201 READ (5,800) TIME(NOBS), LUM(NOBS), WT(NOBS)
WRITE (6,900) TIME(NOBS), LUM(NOBS), WT(NOBS)
IF (TIME(NOBS) .LT. 0) GO TO 202
NOBS = NOBS + 1
IF (NOBS .EQ. 102) GO TO 260
GO TO 201
202 NOBS = NOBS - 1
IF (I .EQ. 0) GO TO 52
C 'T' MAKES THAT A VARIABLE TO ITERATE (PARAM. ARE IN ORDER 1-18)
C IF BLANK LINE RETURNED, PROGRAM OUTPUTS TIME VS DEL MAG
203 READ (5,812) (TEST(J), J=1,17)
WRITE (6,922) NOBS, (TEST(J), J=1,17)
IF (I .LT. -1) GO TO 206
F = EXP(-.921034*QUAD)
WRITE (6,915)
DO 207 J=1,NOBS
IF (ILUM) 204,204,210.
210 CINT = LUM(J)
GO TO 205
204 CINT = EXP(-.921034*LUM(J))
205 LUM(J) = CINT/F
207 WRITE (6,916) TIME(J), LUM(J)
206 II = I
GO TO 52
60 WRITE (6,960)
STOP
160 WRITE (6,961)
GO TO 100
260 WRITE (6,961)
GO TO 202
805 FORMAT (12,F10.3)
FUNCTION ABSORB (Y,Z,K)
CALCULATES ABSORPTION FOR GIVEN Y,Z IN ATMOSPHERE OF STAR N(=1,2)
C ABSORPTION EXPRESSED AS FRACTION TRANSMITTED (UNITY FOR NO ABSORB)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1  *CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNDT,BOLDJ,QUAD,TOTAL
2  *STAR1,STAR2,STAR3
COMMON/TVARS/ALPHA(2),THETA,CHI,R,DELTA,PARA(19,2),AAXIS(2)
1  *BAXIS(2),PHASE,PHI(2),THETAP(2)
N = K
ABSORB = 1.
C IF HT NEGATIVE, THIS PART WILL BE PHYSICALLY ECLIPSED. NO ABSORB.
3  IF (SCALE(N)) 6,6,3
  CONTINUE
  DSQ = Y**2 + Z**2
  COSQ = Y**2/DSQ
  RD = 1./SQRT(COSQ/AAXIS(N)**2 + (1.-COSQ)/BAXIS(N)**2)
  HT = (SQRT(DSQ) - RD)/SCALE(N)
  IF (HT) 6,6,5
C IF HT POSITIVE, (X,Y) IS ABOVE THE SURFACE
5  TAU = EXP(-HT)*SURF(N)
  IF (TAU .LT. 1.E-4) GO TO 8
  ABSORB = EXP(-TAU)
6  RETURN
C FOR SMALL TAU, APPROXIMATE EXPONENTIAL
8  ABSORB = 1. - TAU
GO TO 6
END

FUNCTION ANNECL (N)
C GAUSSIAN INTEGRATION FOR ANNULAR ECLIPSE OF STAR N (1 OR 2)
C INTEGRATION IS OF STAR N OVER BOUNDARY OF OTHER STAR (K)
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
COMMON /GAUSS/ WT(16),X(16),L,XC(16)
COMMON/TVARS/ALPHA(2),THETA,CHI,R,DELTA,PARA(19,2),AAXIS(2)
1  *BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1  *IREF,NINC,LST
MREF = 0
NREF = 3
DCHI = SIGN(1.,ECHI)*DCHI
M = N
K = 3 - M
SUM2 = 0.
DO 100 J=1,L
50
SUML = 0.
6 Y = AAXIS(K)*X(J) + DCHIP
8 DO 50 I=1,L
  Z = BAXIS(K)*X(I)*XC(J) + ABSECHI
  YP = Y*COSP(M) - Z*SINP(M)
  ZP = Y*SINP(M) + Z*COSP(M)
50 SUM1 = SUM1 + WT(I)*BRIGHT(YP,ZP,M)
100 SUM2 = SUM2 + WT(J)*XC(J)*SUM1
110 ANNECL = AAXIS(K)*BAXIS(K)*SUM2
RETURN
END

**************************************************************************

SUBROUTINE ASTROQ(WAVE,POLY)
C SUBROUTINE TO CONVERT PHYSICAL VARIABLES TO MODEL PARAMETERS
COMMON/VARIB/BLQ(17)
COMMON/ORBE/BLK(18)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1, CC(2), RBAR(2), UMA(2), VMA(2), WMA(2), RNOT, BOLQJ, QUAD, TOTAL
2, STAR1, STAR2, STAR3
DIMENSION D(5), T(2), NU(2), EP(2), Z(2), V(2), AO(2), QU(2), E(2)
1, BETA(2), AP(2)
EQUIVALENCE (BLQ(6), GNU), (BLQ(7), RATIOK), (BLQ(13), Q)
EQUIVALENCE (BLK(8), EP(1)), (BLK(10), Z(1)), (BLK(13), V(1))
1 (BLK(15), RATIOJ)
DATA D/1.51985, 1.1482, 1.0289, 1.00267, 1.0/
C D IS SET FOR INTEGRAL POLYTROPES ONLY
REAL NU
FCNA(A,B) = 1.(1.+7.*A)*D(IX)*B**3/6.*
FCNB(A,B) = 1.(1.-2.*A)*D(IX)*B**3/6.*
FCNC(A,B) = 1..(-1.+2.5*A)*D(IX)*B**3/3.*
C INITIALIZATIONS
IX = INT(POLY)
IF (IX) 2, 2, 1
1 PK = (D(IX)-5.)/D(IX)
C = 1.43879E8/WAVE
C SET UP COMMON/VARIB/ FROM INPUT IN COMMON/ORBE/
2 DO 3 I=1,17
  3 BLQ(I) = BLK(I)
  BLQ(14) = QUAD
C CONVERT TO MODEL PARAMETERS
5 IF (IX) 12, 12, 6
NU(1) = GNU
NU(2) = GNU*RATIOK
QU(1) = Q
QU(2) = 1./Q
DO 10 I=1,2
BETA(I) = BLQ(7+I)
T(I) = BLQ(9+I)
AP(I) = FCNA(QU(I),NU(I))
EP(I) = FCNB(QU(I),NU(I))/AP(I)
Z(I) = FCNC(QU(I),NU(I))/(AP(I)*EP(I)**2) - 1.0
CEX = C/T(I)
E(I) = EXP(CEX)
V(I) = PK*BETA(I)*CEX*E(I)/(E(I) - 1.)
10 CONTINUE
RATIOJ = (E(1)-1.)/(E(2)-1.)
BOLoji = (T(2)/T(1))**4
BLK(6) = AP(1)*GNU
BLK(7) = RATIOK*AP(2)/AP(1)
12 CONTINUE
RETURN
ENTRY ASTROX
GO TO 5
END

FUNCTION ATMECL (N)
C GAUSSIAN INTEGRATION OVER ENTIRE AREA OF STAR N (1 OR 2)
C REPLACES TOTINT TO CALCULATE 'TOTAL' INTENSITY IF ATMOS. ECL.
C PORTION OF STAR N WHICH IS BEHIND ATMOSPHERE OF OTHER STAR
C (STAR K) IS GIVEN ATTENUATION BY 'ABSORB' FUNCTION.
C 'ABSORB' ATTENUATES OUT-OF-GEOMETRIC-ECLIPSE LIGHT ONLY.
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
COMMON /GAUXX/ WT(16),X(16),L,XC(16)
COMMON/TVARS/ALPHA(2),THETA,CHI,R,DELTA,PARA(19,2),AAXIS(2)
1 ,BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/FLAGS/IFSPH,JTYPEKSTARTEST(17),NOBS,NZONE,MREF,NREF
1 ,IREF,NINC,LST
MREF = 0
NREF = N
SUM2 = 0.
K = 3 - NREF
6 DO 100 J=1,L
SUM1 = 0.
Y = AAXIS(NREF)*X(J)
YP = Y - DCHI
DO 10 I=1,L
Z = BAXIS(NREF)*X(I)*XC(J)
YP = Y*COSP(NREF) - Z*SINP(NREF)
ZP = Y*SINP(NREF) + Z*COSP(NREF)
ZPP = Z - ECHI
10 SUM1 = SUM1 + WT(I)*BRIGHT(YP,ZP,NREF)*ABSORB(YPZPNREF)*ABSORB(YPPZPPK)
100 SUM2 = SUM2 + WT(J)*XC(J)*SUM1
ATMECL = AAXIS(NREF)*BAXIS(NREF)*SUM2
RETURN
END

FUNCTION BRIGHT(YQZQN)
CALCULATES BRIGHTNESS IN L.O.S. FOR GIVEN Y,Z ON STAR N (=1,2)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1 ,CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNOTELOJ,QUAD,TOTAL
2 ,STAR1,STAR2,STAR3
COMMON/TVARS/ALPHA(2),THETA,CHI,R,DELTA,PARA(19,2),AAXIS(2)
1 ,BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/FLAGS/IFSPH,KTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1 ,IREF,NINC,LST
COMMON/ORBE/INCL,ESINW,ECOSW,UA,UB,A,RATIOK,ELLIP,ELLIPB,
1 EPSI,EPSIB,TCONJ,VA,VB,RATIOJ,WA,WB,PERIOD
COMMON/RINT/X,Y,Z,RAD
C RINTS STORE REFLECTION VALUES TO ABBREVIATE CALC FOR SOLUTION
DIMENSION RINTS (5,256)
DIMENSION U(2),V(2),W(2)
EQUIVALENCE (U(1),UA),(V(1),VA),(W(1),WA)
REAL INCL
REAL LOCINT
MREF = MREF + 1
NN = NREF+NINC
Y = YQ
Z = ZQ
II = N
3 Q1 = PARA(4,II)*Y + PARA(6,II)*Z
YSQ = Y*Y
ZSQ = Z*Z
YZ = Y*Z

53
Q2 = PARA(13,II)*YSQ + PARA(14,II)*ZSQ + PARA(16,II)
Q2 = Q2 + 2.*PARA(15,II)*YZ
IF (Q2 .LE. 0.) GO TO 22
X = (SQRT(Q2) - Q1)/PARA(1,II)
10 T = SQRT(PARA(7,II)*X*X + PARA(8,II)*YSQ + PARA(9,II)*ZSQ + 2.*
1 (PARA(10,II)*X*Y + PARA(11,II)*X*Z + PARA(12,II)*YZ))
COSGAM = (PARA(1,II)*X + Q1)/T
COSGAM IS LIMB DARKENING PROJECTION ANGLE
RAD = SQRT(X**2 + YSQ + ZSQ)
CALL FOR REFLECTED LIGHT IF IREF=1, OTHERWISE USE STORED VALUES
IF (IREF .EQ. 1) RINTS(NN,MREF) = REFL(II)
16 RRATIO = RAD/RBAR(II)
LOCINT = QINT(II)*(VMA(II) + V(II)*RRATIO) + WMA(II)*RINTS
1 (NN,MREF)
BRIGHT = LOCINT*(UMA(II) + U(II) * COSGAM)
20 RETURN
C IF DESCIMINATE NEGATIVE (X IMAGINARY) WRITE MSG AND RETURN ZERO
22 BRIGHT = 0.
WRITE (6,900) Q2,YZ,IIPHASE
GO TO 20
900 FORMAT(' DESC.=',E17.8,' AT',2E17.8,' ON STAR',13,' AT PHASE',
1 ,F8.5)
END

FUNCTION ECLINT (N,YH,YL)
C GAUSSIAN INTEGRATION OVER ECLIPSED AREA OF STAR N (1 OR 2)
C ECLIPSED AREA IS THAT COMMON TO TWO INTERSECTING ELLIPSES
C YH AND YL ARE THE Y LIMITS OF THE INTERSECTION
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
COMMON /GAUSS/ WT(16),X(16),XC(16)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1 ,IREF,NINC,LST
MREF = 0
NREF = 3
M = N
DIF = 0.5*(YH - YL)
SUM = 0.5*(YH + YL)
SUM2 = 0.
DO 100 J=1,L
SUM1 = 0.
Y = DIF*X(J) + SUM
100 CONTINUE
C 'LIMITZ' OBTAINS THE Z LIMITS (ZH, ZL) FOR GIVEN Y COORDINATE.
CALL LIMITZ(M, Y, ZH, ZL)
YY = Y
IF (M .EQ. 2) YY = Y-DCHI
6 CONTINUE
DIFZ = 0.5*(ZH - ZL)
SUM2 = 0.5*(ZH + ZL)
DO 10 I=1,L
Z = DIFZ*X(I) + SUMZ
C 'BRIGHT' CALCULATES BRIGHTNESS OF STAR N AT POINT (YY, Z)
10 SUM1 = SUM1 + WT(I)*BRIGHT(YY, Z, M)
100 SUM2 = SUM2 + WT(J)*DIFZ*SUM1
ECLINT = DIF*SUM2
RETURN
END

**************************************************************************************************

FUNCTION ENERGY(N)
C CALCULATES TOTAL STELLAR RADIATION OF STAR N (1 OR 2)
COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), QINT(2), AA(2), BB(2)
1 ,CC(2), RBAR(2), UMA(2), VMA(2), WMA(2), RNbT, BOLOJQUAD, TOTAL
2 ,STAR1, STAR2, STAR3
COMMON /GAUXX/ WT(16), X(16), L, XC(16)
M = N
AX = 0.5*AA(M)
BX = 0.5*BB(M)
SUM2 = 0.
DO 100 J=1,L
SUM1 = 0.
XJ = X(J) + 1.
XJS = 0.5*SQRT(4. - XJ**2)
XX = AX*XJ
DO 10 I=1,L
YY = BX*XJS*(X(I) + 1.)
10 SUM1 = SUM1 + WT(I)*OUTPUT(XX, YY, M)
100 SUM2 = SUM2 + WT(J)*XJS*SUM1
ENERGY = 8.*AX*BX*SUM2
RETURN
END

**************************************************************************************************
SUBROUTINE GEOMET
CALCULATE GEOMETRICAL FACTORS BASED ON ORBITAL ELEMENTS
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD
COMMON/FLAGS/IFSPH, JTYPE, KSTAR, TEST(17), NOBS, NZONE, MREF, NREF
1, IREF, NINC, LST
COMMON/TRIG/ SINI, COSI, SISQ, CISQ, SINJ(2), COSJ(2), SIN(2), COST(2)
, COTH(2)
COMMON/CONST/ PI, TWOPI, HALPI
COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), TILT(2), QINT(2), AA(2), BB(2)
 COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), TILT(2), QINT(2), AA(2), BB(2)
1, CC(2), RBAR(2), UMA(2), VMA(2), WMA(2), RNOT, BOLOJ, QUAD, TOTAL
2, STAR1, STAR2, STAR3
EQUIVALENCE (ISUM, IFSPH)
REAL INCL
IF ((IREF *EQ. 2) .AND. (LST *LT. 11)) GO TO 5
AA(1) = A*RNOT
AA(2) = AA(1)*RATIOK
BB(1) = AA(1)*ELLIP
BB(2) = AA(2)*ELLIPB
CC(1) = BB(1)*ELLIP*(1.+EPSI)
CC(2) = BB(2)*ELLIPB*(1.+EPSIB)
C CHECK FOR BAD INPUT DATA
IF ((AA(1) .LE. 0) .OR. (AA(2) .LE. 0)) GO TO 777
IF ((BB(1) .LE. 0) .OR. (BB(2) .LE. 0)) GO TO 777
IF ((CC(1) .LE. 0) .OR. (CC(2) .LE. 0)) GO TO 777
IF ((INCL .LE. 0) .OR. (PERIOD .LE. 0)) GO TO 777
SINI = SIN(INCL)
SISO = SINI**2
CISO = 1. - SISO
COSI = SQRT(CISO)
ANG = TILT(1) + INCL
SINJ(1) = SIN(ANG)
COSJ(1) = COS(ANG)
ANG = TILT(2) + INCL
SINJ(2) = SIN(ANG)
COSJ(2) = COS(ANG)
ISUM = 0
CHECK FOR SPHERICAL STARS AND SET ISUM ACCORDINGLY
IF (((1. - ELLIP) .LE. 1.E-6) .AND. (ABS(EPSI) .LE. 1.E-6))
1 ISUM = 1
IF (((1. - ELLIPB) .LE. 1.E-6) .AND. (ABS(EPSIB) .LE. 1.E-6))
1 ISUM = ISUM + 2
CALL ORBITA
QINT(1) = 1.
5 CONTINUE
QINT(2) = RATIOJ
UMA(1) = 1. - UA
UMA(2) = 1. - UB
VMA(1) = 1. - VA
VMA(2) = 1. - VB
WMA(1) = WA/(TWOPI*(1. - .5*UA))
WMA(2) = WB/(TWOPI*(1. - .5*UB))
T = TCONJ. + PERIOD/4.
CALL ORBITB(T)
DO 6 I=1,2
CSQ2 = COTH(I)**2
SSQ2 = 1. - CSQ2
SJQ = SINJ(I)**2
CJSQ = COSJ(I)**2
6 RBAR(I) = 1./SQRT(SJQ*(SSQ2/AA(I)**2 + CSQ2/BB(I)**2) +
1 CJSQ/CC(I)**2)
CALL PARAM
NINC = 3
IF (IREF .EQ. 1) CALL ZONES
STAR1 = TOTINT(1)
STAR2 = TOTINT(2)
TOTAL = STAR1 + STAR2 + STAR3
NINC = 0
RETURN
777 WRITE (6,977) INCL,PERIOD,ELLIP,(AA(I),BB(I),CC(I),I=1,2)
STOP
977 FORMAT ('*** ABORT *** *** ***/(3F15.6)
END

***********************************************************************

SUBROUTINE GRID(N)
C SETS INTEGRATION GRID SIZE
C N=1 FOR VERY COARSE GRID (INITIAL CONVERGENCE)
C N=2 FOR INTERMEDIATE GRID (PREDICTION AND FINAL CONVERGENCE)
C N=3 FOR FINE GRID (DETAILED PREDICTION ONLY)
C THE FINE GRID IS ALWAYS SET UP FOR USE FOR ATMOSPHERIC ECLIPSES
COMMON /GAUSS/ WT(16),X(16),L,XC(16)
COMMON /GAUXX/ WTX(16),XX(16),LX,XCX(16)
C A,C,E ARE THE GAUSS WEIGHTS
C B,D,F ARE THE GAUSS COORD.
REAL*8 C(6) / .17132449,.36076157,.46791393,.46791393,1,.36076157,.17132449/, D(6) /-.93246951,-.66120939,-.23861919,.23861919,.66120939,.93246951/ 
REAL*8 A(16) / .0271524594,.0622535239,.0951585117,.124628971,1,.149595989,.169156519,.182603415,2,.18945061,.182603415,2,.169156519,3,.149595989,.124628971,.0951585117,.0622535239,.0271524594/ 
REAL*8 E(4)/.347854845,.652145155,.652145155,.347854845/ 
REAL*8 F(4)/-.86113631,-.33998104,.33998104,.86113631/ 
DATA M4/4/, M6/6/, M16/16/
GO TO (4,6,16), N
C COARSE GRID
4 DO 5 I=1,M4
   WT(I) = E(I)
   X(I) = F(I)
5 XC(I) = DSORT(1. - F(I)**2)
   L = M4
GO TO 40
C INTERMEDIATE GRID
6 DO 10 I=1,M6
   WT(I) = C(I)
   X(I) = D(I)
10 XC(I) = DSORT(1. - D(I)**2)
   L = M6
GO TO 40
C FINE GRID
16 DO 20 I=1,M16
   WT(I) = A(I)
   X(I) = B(I)
20 XC(I) = DSQRT(1. - B(I)**2)
   L = M16
40 DO 50 I=1,M16
   WTX(I) = A(I)
   XX(I) = B(I)
50 X CX(I) = DSQRT(1. - B(I)**2)
   LX = M16
RETURN
END

***************************************************************************************
SUBROUTINE LIMITY(JTAG,YH,YL)
C DETERMINES INTEGRATION LIMITS FOR INTERSECTING ELLIPSES
C MAIN (INITIAL) ENTRY FOR Y LIMITS
C LIMITZ ENTRY FOR Z LIMITS
COMMON/Tvars/ALPHA(2),THETA,CHI,R,DELTA,PARA(19,2),AAXIS(2)
 1  ,BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
 1  ,IREF,NINC,LST
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
DIMENSION Z(2,2),TOP(2),BOT(2),Q1(2),Q2(2),Q3(2),Q4(2),Q5(2),Q6(2)
  Q7(2),ZZ(4),YSQ(2),LTAG(2)
EQUIVALENCE (Z(l,l),ZZ(1)),(YSQ(1),HOLD)
LOGICAL QB
  MM = 1
  JS = 0
  AXE = AAXIS(1)
C IF IREF=2, THEN ARE CALCULATING PARTIAL DERIVITIVES, AND THE SEARCH
C CAN BE SHORTENED (ELIMINATED IF IT IS A NON-GEOMETRIC PARAMETER)
  IF (IREF .EQ. 1) GO TO 200
  IF (LST .LT. 11) GO TO 55
  AXE = YHIGH + 0.03*AAXIS(1)
200 JS = JTAG
C UPON ENTRY NORMALLY JTAG=0. BUT SET TO 1 (OR -1) IF POTENTIALLY
C SHALLOW ECLIPSE (SIGN TELLS WHICH END TO EXPECT IT ON)
  IF (JS) 201,202,203
201 F = -.001
  MM = 2
  N = 5
  GO TO 205
203 F = .001
  N = 5
  GO TO 205
202 F = .01
  N = 7
205 DEL = -F*AAXIS(1)
  L = 1
  M = 1
  JSET = 0
C JSET SET TO 1 IF 4 ROOTS FOUND AT ANY POINT
C JSET SET TO -1 IF TOTAL/ANNULAR ECLIPSE IS POSSIBLE
C IF YL IS ANTICIPATED TO BE NEAR EDGE, START SEEKING LOW ROOT FIRST
  Y = SIGN(AXE,F)
  DO 5 I=1,2
```
Q1(1) = 0.5/Para(14,I)
Q2(I) = -Para(15,I)*Q1(I)
Q3(I) = Para(15,I)**2 - 4.*Para(14,I)*Para(13,I)
Q4(I) = -4.*Para(14,I)*Para(16,I)

C N MUST BE AN ODD INTEGER TO END SCAN INSIDE STAR

DO 30 K=1,N
  KK = 0
  Q8 = Y - DCHI
  Q7(2) = Q8*Q2(2) + ECHI
  Q5(2) = Q3(2)*Q8*Q8 + Q4(2)
  IF (Q5(2)) 8,7,7
  Q7(1) = Y*Q2(1)
  Q5(1) = Q3(1)*Y*Y + Q4(1)
  IF (Q5(1)) 8,9,9

C IF L=2, THEN HAVE JUST PASSED OFF STAR EDGE

C ALL ROOTS HAVE BEEN CALCULATED, NOW SIZE THEM
  IF (Z(1,1) - Z(1,2)) 11,12,12
  TOP(1) = Z(1,2)
  BOT(1) = Z(1,1)
  GO TO 13
  TOP(1) = Z(1,1)
  BOT(1) = Z(1,2)
  IF (Z(2,1) - Z(2,2)) 14,15,15
  TOP(2) = Z(2,2)
  BOT(2) = Z(2,1)
  GO TO 16
  TOP(2) = Z(2,1)
  BOT(2) = Z(2,2)

C UPPER/LOWER ESTABLISHED FOR EACH FUNCTION, SET UP AND TEST TRUTH TABL
  IF (TOP(1) - TOP(2)) 17,18,18
  I=2
  J=1
  GO TO 20
  I=1
  J=2

C INDEX I FOR UPPER FUNCTION (HAS LARGEST ROOT), J FOR OTHER

QB = .TRUE.

60
```
LTAG(M) = J
IF (BOT(I) .GT. TOP(J)) QB = .FALSE.
   IF (L .EQ. 2) GO TO 23
C L IS FORWARD/BACKWARD SWITCH (1 TO SEE INTERMESH, 2 TO SEEK NON MESH)
22 IF (QB) GO TO 25
   GO TO 24
23 IF (.NOT. QB) GO TO 25
C STEP SCAN LINE BY ONE INCREMENT
24 IF (ABS(Y) .GT. 1.1*AAXIS(1)) GO TO 65
   Y = Y + DEL
   IF (K .GT. 1) KK = KK+1
   IF (KK .LE. 10) GO TO 6
C KK EXCEEDS 10 ONLY IF NO LONGER SENSITIVE TO DEL
C INTERSECTION OR EDGE PASSED, REVERSE SCAN AT 0.1 INTERVAL
   DEL = -0.1*DEL
   Y = Y + DEL
   L = 3 - L
C IF L=1 SET IT TO 2, IF L=2 SET IT TO 1
30 CONTINUE
   IF (BOT(I) .GE. BOT(J)) LTAG(M) = 3
   IF (MM .EQ. 2) GO TO 50
C WHEN MM=1, HAVE HIGH Y ROOT. WHEN MM=2, HAVE LOW Y ROOT.
32 YHIGH = Y
   IF (M .EQ. 2) GO TO 53
33 L = 1
   M = 2
   MM = 3-MM
   DEL = F*AAXIS(1)
   IF (JS) 40,34,40
   Y = -AXE
   GO TO 4
C WHEN ECLIPSE IS SHALLOW, START SEARCH FOR 2ND LIMIT NEAR 1ST
   Y = Y - 0.2*SIGN(AAXIS(1),F)
   GO TO 4
34 YLOW = Y
   IF (M .EQ. 1) GO TO 33
53 CONTINUE
   IF ((LTAG(1) .EQ. 3) .OR. (LTAG(2) .EQ. 3)) GO TO 55
   IF (LTAG(1) .EQ. LTAG(2)) GO TO 60
C WHEN ECLIPSE IS SHALLOW, START SEARCH FOR 2ND LIMIT NEAR 1ST
   Y = YLOW
   YH = YHIGH
   YL = YLOW
   JTAG = JSET
   RETURN
C APPARENTLY NO INTERSECTION SO ECLIPSE MAY BE TOTAL OR ANNULAR

61
60   JSET = -1
     GO TO 55
C    NO INTERSECTION - FALSE ALARM - HAVE RUN OFF STAR EDGE
65   JSET = 0
     GO TO 55
C
ENTRY LIMITZ(NX,YY,ZHIGH,ZLOW)
Y = YY
YSQ(1) = Y*Y
Q8 = Y - DCHI
YSQ(2) = Q8*Q8
Q7(1) = Y*Q2(1)
Q7(2) = Q8*Q2(2) + ECHI
Q9 = 0.
   IF (NX .EQ. 2)   Q9 = -ECHI
   DO 100  I = 1,2
Q6(I) = Q1(I)*SQRT(YSQ(I)*Q3(I) + Q4(I))
Z(I,1) = Q7(1) + Q6(I)
100 Z(I,2) = Q7(1) - Q6(I)
C Z LIMITS ARE INNER TWO Z VALUES. DISCARD HIGHEST AND LOWEST BY SORT
   DO 150  J = 1,4
JJ = 4-J
JEXIT = 0
   DO 110 I = 1, JJ
     IF (ZZ(I) - ZZ(I+1)) 110, 110, 105
105  HOLD = ZZ(I)
     ZZ(I) = ZZ(I+1)
     ZZ(I+1) = HOLD
     JEXIT = 1
110 CONTINUE
   IF (JEXIT) 160, 160, 150
150 CONTINUE
160 ZHIGH = ZZ(3) + Q9
     ZLOW = ZZ(2) + Q9
RETURN
END

**************************************************************************

SUBROUTINE LSQS(A,C,D,W,M,N,ERR,DE,IX)
C USES MATRIX MULTIPLICATION SUBROUTINE 'MAMUL'
C A = OBSERVATION EQUATION MATRIX
C D = OBSERVATION EQUATION VECTOR (RESIDUALS UPON EXIT)
C = ANSWERS VECTOR (EXIT)
W = WEIGHT VECTOR. 1ST IS ZERO FOR UNIT WEIGHTING
M = NUMBER OF UNKNOWNS
N = NUMBER OF OBSERVATION EQUATIONS
ERR = R.M.S. ERROR + INDIVIDUAL ERRORS (EXIT)
DE = DETERMINANT OF NORMAL EQUATIONS (EXIT)
IX = 0 USUALLY. NOT 0 IF SAME OBSERVATION EQUATION MATRIX
DIMENSIONS SHOULD BE 
A(N*M), AT(M*N), B(M*M)
C(M), D(N), DC(N), W(N), WT(N), ERR(M+1)
INDEX(M,2), PIVOT(M)
DIMENSION A(100,17), AT(17,100), B(17,17)
DIMENSION DC(100), ERR(18), PIVOT(17), INDEX(17,2), PIVOT(17)
COMMON/OBS/ DUMMY(303), WT(101)
EQUIVALENCE (IROW, JROW), (ICOLUM, JCOLUMN), (AMAX, T, SWAP)
1000 IF (IX) 1200, 1010, 1200
1010 IF (W(1)) 1040, 1040, 1020
DO 1030 J=1,N
WT(J) = SQRT(W(J))
D(J) = D(J)*WT(J)
DO 1030 I=1,M
A(J*I) = A(J*I)*WT(J)
DO 1050 I=1,M
DO 1050 J=1,N
AT(I,J) = A(J,I)
CALL MAMUL(AT, A, D, N, M, M)
CALL MAMUL(AT, D, C, N, M, I)
SOLVE NORMAL EQUATIONS
MATRIX: INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
INITIALIZATION
DE = 1.0
DO 20 J=1,M
20 IPIVOT(J) = 0
DO 550 I=1,M
SEARCH FOR PIVOT ELEMENT
AMAX = 0.0
45 DO 105 J=1,M
50 IF (IPIVOT(J)-1) 60, 105, 60
60 DO 100 K=1,M
70 IF (IPIVOT(K)-1) 80, 100, 740
80 IF (ABS(AMAX) - ABS(B(J,K))) 85,100,100
85 IROW=J
90 ICOLUM=K
95 AMAX = B(J,K)
100 CONTINUE
105 CONTINUE
110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1

C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
130 IF(IROW-ICOLUM) 140, 260, -140
140 DE = -DE
150 DO 200 L=1,M
160 SWAP = B(IROW,L)
170 B(IROW,L) = B(ICOLUM,L)
200 B(ICOLUM,L) = SWAP
220 SWAP = C(IROW)
230 C(IROW) = C(ICOLUM)
250 C(ICOLUM) = SWAP
260 INDEX(I,1)=IROW
270 INDEX(I,2)=ICOLUM
310 PIVOT(I) = B(ICOLUM,ICOLUM)
320 DE = DE*PIVOT(I)
C
C DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
330 B(ICOLUM,ICOLUM) = 1.0
340 DO 350 L=1,M
350 B(ICOLUM,L) = B(ICOLUM,L)/PIVOT(I)
370 C(ICOLUM) = C(ICOLUM)/PIVOT(I)
C
C REDUCE NON-PIVOT ROWS
C
380 DO 550 LI=1,M
390 IF(L1-ICOLUM) 400, 550, 400
400 T = B(L1,ICOLUM)
420 B(L1,ICOLUM) = 0.0
430 DO 450 L=1,M
450 B(L1,L) = B(L1,L) - B(ICOLUM,L)*T
500 C(L1) = C(L1) -C(ICOLUM)*T
550 CONTINUE
C INTERCHANGE COLUMNS

600 DO 710 I=1,M
610 L = M+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUMN=INDEX(L,2)
650 DO 705 K=1,M
660 SWAP = B(K,JROW)
670 B(K,JROW) = B(K,JCOLUMN)
700 B(K,JCOLUMN) = SWAP
705 CONTINUE
710 CONTINUE

CCCC FORM RESIDUALS
740 DO 1070 J=1,N
750 SWAP = 0.0
760 DO 1060 K=1,M
1060 SWAP = SWAP + A(J,K)*C(K)
770 DC(J) = SWAP
780 SWAP = 0.0
790 DO 1080 J=1,N
800 D(J) = D(J) - DC(J)
1080 SWAP = SWAP + D(J)**2
810 FNM = N-M
820 IF (FNM) 1090, 1090, 1100
1090 ERR(1) = 0.0
GO TO 1110
1100 ERR(1) = SQRT(SWAP/FNM)

CCCC CALCULATE INDIVIDUAL ERRORS
1110 DO 1120 I=1,M
1120 ERR(I+1) = ERR(1)*SQRT(ABS(B(I,I)))

CCCC HERE IF IX-NOT 0, FOR ITERATIVE LSQ, NORM EQN MTRX IS SAME
1200 IF (W(I)) 1230, 1230, 1210
1210 DO 1220 J=1,N
1220 D(J) = D(J)*WT(J)
1230 CALL MAMUL(AT,D,DC,N,M,1)
1250 DO 1240 J=1,M
1240 SWAP = SWAP + B(J,K)*DC(K)
1250 C(J) = SWAP

END
FUNCTION LUMC(T)
CALCULATE SYSTEM LUMINOSITY AT TIME T
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
   ,IREF,NINC,LST
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
   ,CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNOT,BOLOJ,QUAD,TOTAL
2   ,STAR1,STAR2,STAR3
EQUIVALENCE (JJ,JTYPE),(KK,KSTAR)
REAL LUMC
CALL ORBITB(T)
CALC ORBITAL PARAMETERS
   CALL PARAM
CALC ELLIPSE PARAMETERS
C
CALC LUMINOSITY
   CALL SCREEN(YH,YL)
   LL = 3 - KK
C DETERMINE TYPE OF ECLIPSE (JJ,KK SET BY SCREEN)
C JJ: 1=ANNULAR, 2=TOTAL, 3=PARTIAL, 4=ATMOSPHERIC
C KK: 1=STAR 1, 2=STAR 2, 3=NO ECLIPSE
   GO TO (20,20,10), KK
C NO ECLIPSE
10   ALUM = TOTINT(1) + TOTINT(2)
   GO TO 50
   20   GO TO (35,25,45,55), JJ
C TOTAL ECLIPSE
25   ALUM = TOTINT(LL)
   GO TO 50
C ANNULAR ECLIPSE
35   ECLIPS = ANNECL(KK)
C IF THERE IS NO ATMOSPHERE, USE TOTINT - OTHERWISE ATMECL
36   IF (SCALE(LL)) 39,39,37
39   TOTKK = TOTINT(KK)
   GO TO 38
37   TOTKK = ATMECL(KK)
38   ALUM = TOTINT(LL) + TOTKK - ECLIPS
   GO TO 50
C PARTIAL ECLIPSE
45   ECLIPS = ECLINT(KK,YH,YL)
   GO TO 36
**C ATMOSPHERIC ECLIPSE ONLY**

55 ALUM = TOTINT(LL) + ATMECL(KK)

50 LUMC = (ALUM + STAR3)/TOTAL

RETURN

END

************************************************

SUBROUTINE MAMUL(A,B,C,NCA,NRA,NCB)
DIMENSION A(17,100),B(100,17),C(17,17)
DO 300 I=1,NCB
   DO 300 J=1,NRA
      SUMC = 0.0
      DO 200 K=1,NCA
         200 SUMC = SUMC + A(J,K)*B(K,I)
   300 C(J,I) = SUMC
RETURN
END

************************************************

SUBROUTINE ORBITA
CALCULATES BASIC QUANTITIES FOR ORBITAL MECHANICS CALCULATIONS
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
   EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD
COMMON/CONST/ PI,TWOPI,HALPI
COMMON/ORBIT/ E,TO,Q,MU
REAL INCL
REAL MU
MU = TWOPI/PERIOD
ESQ = ESINW**2 + ECOSW**2
IF (ESQ) 20,20,5
   5 E = SQRT(ESQ)
   Q = SQRT(1. - ESQ)
   IF (ECOSW) 6,25,6
   CONTINUE
   Q1 = 1. + ESINW
   Q2 = ARCCOS((ESINW + ESQ)/(E*Q1))
   Q3 = ECOSW*Q/Q1
   IF (Q3) 10,10,12
   10 Q4 = Q3 + Q2 + TWOPI
GO TO 15
12 Q4 = Q3 - Q2
15 TO = TCONJ + Q4/MU
18 RETURN
CIRCULAR ORBIT
20 Q = 1.
   E = 0
   TO = TCONJ
   GO TO 18
25 Q4 = TWOPI
   GO TO 15
END

********************************************************************************************************

SUBROUTINE ORBITB(T)
CALCULATES ORBITAL MECHANICS PORTION - R AND LONGITUDE FOR GIVEN TIME
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB, 1
EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD
COMMON/CONST/ PI, TWOPI, HALPI
COMMON/ORBIT/ E, TO, Q, MU
COMMON/ROTAT/ SINP(2), COSP(2), DCHIECHI
COMMON/TRIG/ SINI, COSI, SISQ, CISQ, SINJ(2), COSJ(2), SINT(2), COST(2)
   , COTH(2)
COMMON/TVARS/ALPHA(2), THETA, CHI, R, DELTA, PARA(19,2), AAXIS(2)
   , BAXIS(2), PHASE, PHI(2), THETAP(2)
COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), TILT(2), QINT(2), AA(2), BB(2)
   , CC(2), RBAR(2), UMA(2), VMA(2), WMA(2), RROT, BOLO, QUAD, TOTAL
2 , STAR1, STAR2, STAR3
DIMENSION CHIFCN(2)
REAL MU, INCL, MP
   M = MU*(T - TO)
C FORCE MEAN ANOMALY (M) TO LIE BETWEEN + OR - 2*PI
5 IF (ABS(M) .LT. TWOPI) GOTO 10
    M = SIGN((ABS(M) - TWOPI), M)
GO TO 5
C GET FIRST GUESS FOR SOLUTION OF KEPLER EQUATION
10 IF (E .GT. 0.75) GOTO 15
    ECC = M
C IF CIRCULAR (E=0) SOLUTION IS TRIVIAL
IF (E) 40,40,16
15 ECC = 0.5*(M + SIGN(PI,M))
C PROCEDURE FOR SOLUTION OF KEPLER EQUATION
16 COSE = COS(ECC)
   Q1 = 1. - E*COSE
   SINE = SIN(ECC)
   DELE = (E*SINE - ECC + M)/Q1

C EQUATION SOLVED WHEN CORRECTION TO ECCENTRIC ANOMALY LESS THAN
   10**-6
   IF (ABS(DELE) .LT. 1.0E-6) GO TO 20
   ECC = ECC + DELE
   GO TO 16

CONVERGED

C R=RADIUS VECTOR, V=TRUE ANOMALLY, W=OMEGA=LONGITUDE OF PERIASTRON
C THETA = ORBITAL LONGITUDE
20 R = Q1
   COSV = (COSE - E)/Q1
   SINV = (SINE*Q)/Q1
   SINTH = (ESINW*SINV - ECOSW*COSV)/E
   COSTH = (ESINW*COSV + ECOSW*SINV)/E
   THETA = ARCOS(COSTH)
   IF (SINTH) 25,28,28
25 THETA = -(THETA - TWOPI)
28 V = ARCOS(COSV)
   IF (SINV) 30,35,35
30 V = -(V - TWOPI)
35 CONTINUE
   W = ARCOS(ECOSW/E)
   IF (ESINW) 36,37,37
36 W = -(W - TWOPI)
37 R = R*RN0T
   Q6 = SQRT(SISO*SINTH**2 + CISQ)
   IF (ABS(SINH) .GE. 1.0E-6) GO TO 102
   CHIFCN(1) = SINTH
   CHIFCN(2) = COSTH
   CHI = HALPI - THETA
   GO TO 104
102 CHIFCN(1) = SINH/Q6
   CHIFCN(2) = COSTH*COSI/Q6
   CHI = ARSIN(CHIFCN(2))
   IF (CHIFCN(1)) 103,104,104
103 CHI = -(CHI - TWOPI)
C DELTA = APPARENT SEPARATION OF CENTERS
104 DELTA = Q6*R
   DCHI = R*SINTH
   ECHI = R*COSTH*COSI
   DO 110 J=1,2
      MP = M + SLIP(J)
   END
COTH(J) = COS(V - MP)
THETAP(J) = W - HALPI + MP
SINT(J) = SIN(THETAP(J))
COST(J) = COS(THETAP(J))
110 CONTINUE
PHASE = (T - TCONJ)/PERIOD
RETURN
40 THETA = M
CIRCULAR ORBIT
R = 1.
W = HALPI
SINTH = SIN(THETA)
COSTH = COS(THETA)
V = M
GOTO 37
END

FUNCTION OUTPUT(XY,N)
C CALCU LATES ENERGY OUTPUT OF STAR N AT COORD (X,Y) ON SURFACE
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1 ,CC(2),RBAR(2),UMA(2),WMA(2),QINT(2),AA(2),BB(2),RIBOJ,QUAD,TOTAL
2 +STAR1,STAR2,STAR3
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
1 EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD
DIMENSION V(2)
EQUIVALENCE (V(1),VA)
M = N
Q1 = (CC(M)/AA(M))**2 - 1.
Q2 = (CC(M)/BB(M))**2 - 1.
XSQ = X**2
YSQ = Y**2
5 R = SQRT(CC(M)**2 - XSQ*Q1 - YSQ*Q2)
XX = XSQ/AA(M)**2
YY = YSQ/BB(M)**2
G = SQRT((1.+XX*Q1+YY*Q2)/(1.-XX-YY))
10 OUTPUT = QINT(M)*(VMA(M)+V(M)*R/RBAR(M))*G
RETURN
END

***********************************************************************
SUBROUTINE PARAM
CALCULATES PARAMETERS FOR ELLIPSODIAL STARS
C II=1(2) IF STAR 1(2) SPHERICAL
C II=0 IF NO STAR SPHERICAL =3 IF BOTH STARS SPHERICAL
COMMON/CONST/ PI,TWOPI,HALPI
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
COMMON/TRIG/ SIN1,COS1,SISO,CISO,SINJ(2),COSJ(2),SINT(2),COST(2)
   ,COTH(2)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
   ,CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),ROT0,BOLOJ,QUAD,TOTAL
COMMON/TVARS/ALPHA(2),THETACHIR,DELTAPARA(19),AAXIS(2)
   ,BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/FLAGS/IFSPH,JTYPE,KSTART(17),NOBS,NZONE,MREF,NREF
DIMENSION X(6,6),ASQ(2),BSQ(2),CSQ(2),ABSQ(2),ACSQ(2),BCSQ(2)
EQUIVALENCE
   (ASQ(I),X(2,1)),(BSQ(I),X(4,1)),(CSQ(I),X(3,2)),
   (ABSQ(I),X(5,2),ACSQ(I),X(4,3)),(BCSQ(I),X(5,4))
4 IX = IFSPH + 1
GO TO (41,42,43,100), IX
5 DO 10 I=1L,1U
   ASQ(I) = A(I)**2
   BSQ(I) = B(I)**2
   CSQ(I) = C(I)**2
   ABSQ(I) = ASQ(I)*BSQ(I)
   ACSQ(I) = ASQ(I)*CSQ(I)
10 BCSQ(I) = BSQ(I)*CSQ(I)
15 DO 30 I=1L,1U
   SIS = SINJ(I)**2
   CIS = COSJ(I)**2
   SCI = SINJ(I)*COSJ(I)
   STSQ = SINT(I)**2
   CSTQ = COST(I)**2
   SCT = SINT(I)*COST(I)
   Q1 = ACSQ(I)*STSQ
   Q2 = BCSQ(I)*CSTQ + Q1
   PARA(1,I) = Q2*SIS + CIS*ABSQ(I)
   PARA(2,I) = BCSQ(I)*STSQ + ACSQ(I)*CSTQ
   PARA(3,I) = Q2*CIS + SIS*ABSQ(I)
   Q3 = (BCSQ(I) - ACSQ(I))*SCT
   PARA(4,I) = SINJ(I)*Q3
   PARA(5,I) = COSJ(I)*Q3
   PARA(6,I) = (Q2 - ABSQ(I))*SCI

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CROSS PRODUCTS

DO 20 J=1,6
DO 18 K=J,6
  18 X(J,K) = PARA(J,I)*PARA(K,I)
  20 CONTINUE

PARA(7,I) = X(1,1) + X(4,4) + X(6,6)
PARA(8,I) = X(2,2) + X(4,4) + X(5,5)
PARA(9,I) = X(3,3) + X(5,5) + X(6,6)
PARA(10,I) = X(1,4) + X(2,4) + X(5,6)
PARA(11,I) = X(1,6) + X(3,6) + X(4,5)
PARA(12,I) = X(2,5) + X(3,5) + X(4,6)
PARA(13,I) = X(4,4) - X(1,2)
PARA(14,I) = X(6,6) - X(1,3)
PARA(15,I) = X(4,6) - X(1,5)
PARA(16,I) = PARA(1,1)*ASQ(I)*BCSQ(I)
PARA(17,I) = -PARA(13,I)/PARA(16,I)
PARA(18,I) = -PARA(14,I)/PARA(16,I)
PARA(19,I) = -PARA(15,I)/PARA(16,I)
Q4 = SORT((PARA(17,I) - PARA(18,I))**2 + (2.*PARA(19,I))**2)
Q5 = PARA(17,I) + PARA(18,I)
AAXIS(I) = SORT(2./(Q5 - Q4))
BAXIS(I) = SORT(2./(Q5 + Q4))
Q7 = PARA(13,I) - PARA(14,I)
TAN2P = 2.*PARA(15,I)/Q7
IF (ABS(TAN2P) .GT. 1.0E-6) GO TO 26
C SPECIAL CASE FOR COT2P NEAR OR EQUAL INFINITY
C PHI IS NEARLY ZERO UNLESS STAR ON END (Q7 NEGATIVE)
IF (Q7 .LT. 0.) GO TO 25
PHI(I) = 0.5*TAN2P
SINP(I) = PHI(I)
COSP(I) = 1.0 - 0.5*PHI(I)**2
GO TO 27
25 COSP(I) = 0.5*TAN2P
SINP(I) = 1.0 - 0.5*COSP(I)**2
PHI(I) = 0.5*PI - COSP(I)
GO TO 27
26 COT2P = 1./TAN2P
Q8 = SORT(1.0 + COT2P**2)
TANP = -COT2P + SIGN(Q8,PARA(15,I))
PHI(I) = ATAN(TANP)
COSP(I) = COS(PHI(I))
SINP(I) = COSP(I)*TANP
27 CONTINUE
ALPHA(I) = CHI - PHI(I)
30 CONTINUE
IF (IL-IU) 35,102,35
35 RETURN
C SET UP FOR SPHERICAL/NON-SPHERICAL STARS
41 IL = 1
   IU = 2
   GO TO 5
42 IL = 2
   IU = 2
   GO TO 5
43 IL = 1
   IU = 1
   GO TO 5
100 IL = 1
   IU = 2
   GO TO 110
102 IL = IFSPH
   IU = IFSPH
C C SPECIAL CALCULATION FOR SPHERICAL STARS
110 DO 120 I=IL,IU
   Q2 = AA(I)**2
   Q1 = AA(I)**4
   PARA(1,I) = Q1
   PARA(2,I) = Q1
   PARA(3,I) = Q1
   PARA(7,I) = Q1**2
   PARA(8,I) = Q1**2
   PARA(9,I) = Q1**2
   PARA(13,I) = -Q1**2
   PARA(14,I) = -Q1**2
   PARA(16,I) = Q2*Q1**2
   PARA(17,I) = -1./Q2
   PARA(18,I) = -1./Q2
   PARA(4,I) = 0.
   PARA(5,I) = 0.
   PARA(6,I) = 0.
   PARA(10,I) = 0.
   PARA(11,I) = 0.
   PARA(12,I) = 0.
   PARA(15,I) = 0.
   PARA(19,I) = 0.
   PHI(I) = 0.
   SINP(I) = 0.
   COSP(I) = 1.
   ALPH(A(I)) = CHI
AAXIS(I) = AA(I)
BAXIS(I) = AA(I)
120 CONTINUE
GO TO 35
END

FUNCTION REFL(N)
CALCULATES REFLECTED LIGHT ONTO STAR N AT COORD X,Y,Z
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
1 EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD
COMMON/TRIG/ SINI, COSI, SISQ, CISQ, SINJ(2), COSJ(2), SINT(2), COST(2)
1 , COTH(2)
COMMON/RINT/X,Y,Z,RAD
COMMON/CONST/PI, TWOPHI, HALPI
COMMON/FLAGS/IFSPH, JTYPE, KSTAR, TEST(17), NOBS, NZONE, MREF, NREF
1 , IREF, NINC, LST
COMMON/AUXE/SCALE(2), SURF(2), SLIP(2), TILT(2), QINT(2), AA(2), BB(2)
1 , CC(2), RBAR(2), UMA(2), VMA(2), WMA(2), RNOTBOLOJ, QUAD, TOTAL
2 , STAR1, STAR2, STAR3
COMMON/TVARS/ALPHA(2), THETA, CHI, R, DELTA, PARA(19,2), AAXIS(2)
1 , BAXIS(2), PHASE, PHI(2), THETAP(2)
COMMON/TABLE/ENDINT(2)
DIMENSION U(2), V(2), W(2)
REAL INCL, LAMB, LAMP
3 II = N
IF (W(II)) 5, 36, 5
5 STP = SINT(II)
CTP = COST(II)
IF (II .NE. 2) GO TO 4
STP = -STP
CTP = -CTP
4 CONTINUE
XX = X*SINI*CTP + Y*STP + Z*COSI*CTP
YY = -X*SINI*STP + Y*CTP - Z*COSI*STP
ZZ = -X*COSI + Z*SINI
RINV = 1./RAD
COSL = XX*RINV
COSM = YY*RINV
COSN = ZZ*RINV
COSL, COSM, COSN = DIRCOS NORMAL TO STAR AT X,Y,Z

74
L = 3 - II

100 CONTINUE
C SHORTCUT VERSION - APPROXIMATES LIGHT REFLECTED ONTO N AT X, Y, Z
XDEL = R*COTH(L) - XX
YDEL = -YY
ZDEL = -ZZ.
DINV = 1./SQRT(XDEL**2 + YDEL**2 + ZDEL**2)
LAMB = ARSIN(RBAR(L)*DINV)
CLP = DINV*(XDEL*COSL + YDEL*COSM + ZDEL*COSN)
LAMP = HALPI + LAMB - ARCOS(CLP)
IF (LAMP .LE. 0) GO TO 36
IF (LAMP .LT. LAMB) GO TO 110
IF (LAMP .LT. 2.*LAMB) GO TO 120
AREA = PI*LAMB**2
GO TO 150
110 AREA = (LAMB**2)*ARCS((LAMB-LAMP)/LAMB)-(LAMB-LAMP)*
1 SQRT(2.*LAMB*LAMP-LAMP**2)
GO TO 150
120 LAMP = 2.*LAMB - LAMP
AREA = (LAMB**2)*((PI-ARCS((LAMB-LAMP)/LAMB))+(LAMB-LAMP)*
1 SQRT(2.*LAMB*LAMP-LAMP**2)
150 C = .38736 + AA(II)*(1.43431*AA(L)-.82442) + CLP*(AA(II)*
1 (.9378*AA(L)-.43316)-1.22172)
IF (C .LT. 0) C=0
FM =2.044+AA(II)*(-.170831+AA(L)*(1.231707-AA(L)*9.955083))
FB=-.065354+AA(II)*(.224935+AA(L)*(-.761696+AA(L)*3.81425))
REFL = ENDINT(L)*AREA*(1.-.5*U(L))*(C+FB+CLP*FM)
RETURN
36 REFL = 0.
RETURN
END

***********************************************************************
SUBROUTINE SCREEN(YH,YL)
C FOR EACH ORBITAL POSITION, SCREEN STARS TO FIND IF ECLIPSE
C SET KK SET JJ
C =1 STAR 1 ECLIPSED =1 ANNULAR ECLIPSE
C =2 STAR 2 ECLIPSED =2 TOTAL ECLIPSE
C =3 NO ECLIPSE =3 PARTIAL ECLIPSE
C =4 ATMOSPHERIC ECLIPSE
C YH,YL = RIGHT AND LEFT HAND LIMITS FOR PARTIAL ECLIPSE
COMMON/ROTAT/ SINP(2),COSP(2),DCHI,ECHI
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1 ,IREF,NINC,LST
EQUIVALENCE (JJ,JTYPE),(KK,KSTAR)
COMMON/TVARS/ALPHA(2),THETA,CHI,R,DELTA,PARA(192),AAXIS(2)
1 ,BAXIS(2),PHASE,PHI(2),THETAP(2)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1 ,CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNOT,BOLIQ,QUAD,TOTAL
2 ,STAR1,STAR2,STAR3
DIMENSION RV(2)
II = 0
JJ = 3
KK = 1
20 IF (ABS(THETA) .GT. 1.570796 .AND. ABS(THETA) .LT. 4.712289) 1
1 KK = 2
C KK=1 WHEN STAR 2 ECLIPSES STAR 1. KK=2 WHEN STAR 1 ECLIPSES STAR 2
DO 5 I=1,2
  COSA = COS(ALPHA(I))**2
  RV(I) = 1./SQRT(COSA/AAXIS(I)**2 + (1.-COSA)/BAXIS(I)**2)
SEPARATION = RV(1) + RV(2) - DELTA
C NO ECLIPSE CAN OCCUR IF SEPARATION EXCEEDS SUM OF RADIUS VECTORS
LL = 3 - KK
IF (SEP) 6,6,12
C TEST TO SEE IF INSIDE 10 SCALE HEIGHTS OF ATMOSPHERE
6 IF (SEP + 10.*RV(LL)*SCALE(LL)) 10,10,7
C SET JJ=4 FOR ATMOSPHERIC ECLIPSE AND NO PHYSICAL ECLIPSE
7 JJ = 4
   GO TO 50
10 KK = 3
RETURN
12 IF (SEP .LT. 0.1*AAXIS(1)) II = IFFIX(SIGN(1.,DCHI))
C FOR ENTERING LIMITY, SET II=1 IF YH IS POTENTIALLY NEAR LIMB
C -1 IF YL IS POTENTIALLY NEAR LIMB
CALL LIMITY(II,YH,YL)
C II = 0 WHEN NO SET OF 4 ROOTS FOUND
C II = 1 FOR INTERSECTION (PARTIAL ECLIPSE)
C II = -1 FOR POSSIBLE TOTAL/ANNULAR ECLIPSE
IF (II) 15,10,50
C HERE TO EXAMINE TOTAL/ANNULAR ECLIPSE CASE
15 IF (DELTA - ABS(RV(1) - RV(2))) 17,50,50
17 JJ = 1
   IF (AAXIS(1) - AAXIS(2)) 22,23,24
22 IF (KK-1) 23,23,50
23 JJ=2
C SET JJ TO 2 FOR TOTAL ECLIPSE
   GO TO 50
24 IF (KK-1) 23,50,23
50 CONTINUE
120 RETURN
END

**************************************************************************************************

SUBROUTINE SOLVE1(INDEX)
C DIFFERENTIAL CORRECTOR TO OPERATE WITH ASTROPHYSICAL VARIABLES
COMMON/ORBE/BLKX(18)
COMMON/CONST/PI,TWOPI,HALPI
COMMON/OBS/ TIME(101),LUM(101),WT(101),CLUM(101)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1 +IREF,NINC,LST
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
1 +CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNOT,BOLOJ,QUAD,TOTAL
2 +STAR1,STAR2,STAR3
COMMON/VARIB/BLKV(17)
DIMENSION V(8),VV(17),SA(17),SB(17),SC(17)
DATA SA /1.570796,2*.9,2*1.,.9,100.,2*1.,2*1.E5,0,50.,10.
1 +,0,2*2./
DATA SB /2.,-9.,-9.6*0.2*1.E3,2*0.5.,3*0/
DATA SC /2.,2*9.,2*0.2*001.2*0.2*1.E3,0.01,-5.,3*0/
DIMENSION BLKS(17),BLKA(17),ERR(18),D(100,17),DELT1(100)
REAL LUM,LOW,LUMC
LOGICAL TEST
INTEGER V,VV
DATA V /1,2,3,4,5,12,16,17/
C V CONTAINS THE INDICES OF THE NON-ASTROPHYSICAL VARIABLES
DATA VV /4,5,8,9,10,11,14,15,16,17,1,2,3,6,7,12,13/
C VV CONTAINS ORDER IN WHICH PARTIALS ARE TO BE CALCULATED.
C THEY ARE ARRANGED SO THAT THE FIRST 10 ARE ALL NON-GEOMETRIC.
INDIC = INDEX-1
IF (INDEX .LE. 0) GO TO 1720
INDIC = 0
DO 250 I=1,NOBS
   IREF = 1
   CALL GEOMET
   CINT = LUMC(TIME(I))
   DELT1(I) = LUM(I) - CINT
   IREF = 2
   NVAR = 0
250 DO 200 LST=1,17
L = VV(LST)
BLKS(L) = BLKV(L)
IF (TEST(L)) GO TO 210
200 CONTINUE
IF (NVAR) 1730,1730,205
205 CONTINUE
250 CONTINUE
WRITE (6,732)
WRITE (6,730) (TIME(J),DELTI(J),J=1,NOBS)
SUM=0
DO 260 J=1,NOBS
260 SUM = SUM + DELTI(J)**2
GO TO 500
210 NVAR = NVAR + 1
CLUMH = CINT
CLUML = CINT
NN = 1
GO TO (401,402,404,404,406,407,408,408,410,410,412,413,
1 414,415,404,404), L
C SUB-BLOCKS FOR EACH VARIABLE
401 HIGH = BLKS(L)
LOW = BLKS(L) - .01
GO TO 440
402 HIGH = BLKS(L) + .00125
LOW = BLKS(L) - .00125
GO TO 450
404 IF (BLKS(L)-.1) 4042,4042,4041
4041 HIGH = BLKS(L)
LOW = BLKS(L) - .1
GO TO 440
4042 HIGH = .1
LOW = 0.
GO TO 450
406 HIGH = 1.025*BLKS(L)
LOW = BLKS(L)
GO TO 4800
407 HIGH = 1.025*BLKS(L)
LOW = BLKS(L)
GO TO 4800
408 HIGH = BLKS(L)
LOW = BLKS(L) - .025
GO TO 4400
410 HIGH = BLKS(L) + 100.
LOW = BLKS(L)
GO TO 4800
412   TD = .001*BLKX(18)
       HIGH = BLKS(L) + TD
       LOW  = BLKS(L) - TD
       GO TO 450
413   HIGH = BLKS(L) + .1
       LOW  = BLKS(L)
       GO TO 4800
414   D(I,NVAR) = 0.92061*CINT

C SPECIAL CALC OF PARTIAL DERIV FOR QUADRATURE MAG
       GO TO 200
C THERE IS NO VARIABLE FOR L=15
415   NVAR = NVAR-1
       GO TO 200
C FORM PARTIAL DERIVITIVES
440   BLX(L) = LOW
       CALL GEOMET
       CLUML = LUMC(TIME(I))
       GO TO (453,480), NN
450   NN = 2
       GO TO 440
480   BLX(L) = HIGH
       CALL GEOMET
       CLUMH = LUMC(TIME(I))
453   D(I,NVAR) = (CLUMH - CLUML)/(HIGH - LOW)
       BLX(L) = BLKS(L)
       GO TO 200
C SPECIAL TREATMENT FOR ASTROPHYSICAL VARIABLES
4400  BLK(L) = LOW
       CALL ASTROX
       CALL GEOMET
       CLUML = LUMC(TIME(I))
       GO TO 4530
4800  BLK(L) = HIGH
       CALL ASTROX
       CALL GEOMET
       CLUMH = LUMC(TIME(I))
4530  D(I,NVAR) = (CLUMH - CLUML)/(HIGH - LOW)
       BLK(L) = BLKS(L)
       CALL ASTROX
       GO TO 200
C SOLVE OBS EQUATIONS
500   CALL LSQS (D, BLKA, DELT, WT, NVAR, NOBS, ERR, DET, O)
       WRITE (6,911) ERR(1), SUM, DET
       J = 0
       DO 511 LST=1,17

79
L = VV(LST)
IF (TEST(L)) GO TO 520
510 CONTINUE
511 CONTINUE
GO TO 700
520 J = J+1
IF (BLKV(L)) 524,523,524
523 IF (ABS(BLKA(J)) *GT. .25) BLKA(J) = SIGN(.25,BLKA(J))
GO TO 522
524 IF (ABS(BLKA(J)/BLKV(L)) *GT. .25)
1     BLKA(J) = SIGN(0.25*BLKV(L),BLKA(J))
522 BLKV(L) = BLKV(L) + BLKA(J)
WRITE (6,910) L,BLKA(J),ERR(J+1),BLKV(L)
IF (ABS(BLKV(L)) *LT. .001) GO TO 530
IF (ABS(BLKA(J)/BLKV(L)) *LT. .01) INDIC=INDIC+1
C DROP VARIABLES THAT HAVE VERY SMALL CORRECTIONS
IF (ABS(BLKA(J)/BLKV(L)) *LT. .0001) TEST(L) = .FALSE.
GO TO 533
530 IF (ABS(BLKA(J)) *LT. .0001) INDIC=INDIC+1
IF (ABS(BLKA(J)) *LT. .00001) TEST(L) = .FALSE.
533 CONTINUE
IF (L .EQO. 12) GO TO 612
IF (BLKV(L) *GT. SA(L)) BLKV(L) = SA(L)
IF (BLKV(L) *LE. SB(L)) BLKV(L) = SC(L)
GO TO 510
612 IF (ABS(BLKA(J)/BLKX(18)) *LT. .1) GO TO 510
BLKV(L)=BLKV(L)-BLKA(J)*SIGN(.1*BLKX(18),BLKA(J))
GO TO 510
700 IF (INDIC .EQO. 0) INDEX=0
DO 710 I=1,8
710 BLKX(V(I)) = BLKV(V(I))
CALL ASTROX
QUAD = BLKV(14)
1710 RETURN
1720 INDEX = -1
GO TO 1721
1730 INDEX = -2
1721 IREF = 1
DO 1731 J=1,NOBS
CINT = LUMC(TIME(J))
1731 DELTI(J) = -2.5*ALOG10(LUM(J)/CINT)
WRITE (6,731)
WRITE (6,730) (TIME(J),DELTI(J),J=1,NOBS)
GO TO 1710
910 FORMAT (3X,13,F11.5,F10.5,F14.5)
FUNCTION TOTINT (N)
C GAUSSIAN INTEGRATION OVER ENTIRE AREA OF STAR N (1 OR 2)
C STAR IS ELLIPSE WITH AXES (AAXIS,BAXIS) ROTATED FROM
C (YP,ZP) AXES BY ANGLE SPECIFIED BY SINP,COSP.
COMMOM /ROAT/ SINP(2),COSP(2),DCH1,ECH1
COMMON /GAUSS/ WT(16),X(16),L,XC(16)
COMMON /TVARS/ALPHA(2),THETA1,CH1,R,DELTA,PARA(19,2),AAXIS(2)
1,BAXIS(2),PHASE,PHI(2),THETAPI(2)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1,IREF,NINC,LST
MREF = 0
NREF = N
SUM2 = 0.
DO 100 J=1,L
SUM1 = 0.
Y = AAXIS(NREF)*X(J)
DO 10 I=1,L
Z = BAXIS(NREF)*X(I)*XC(J)
YP = Y*COSP(NREF) - Z*SINP(NREF)
ZP = Y*SINP(NREF) + Z*COSP(NREF)
10 SUM1 = SUM1 + WT(I)*BRIGHT(YP,ZP,NREF)
100 SUM2 = SUM2 + WT(J)*XC(J)*SUMI
TOTINT = AAXIS(NREF)*BAXIS(NREF)*SUM2
RETURN
END

SUBROUTINE ZONES
C ESTABLISHES TABLE OF COORD AND LOCAL INTENSITY FOR REFLECTION CALC
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
1 EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD

911 FORMAT ('0------- RMS ERR',E13.5,5X,'SUM RESID SQ',E13.5,5X,
1 'DET',E13.5,'0 VAR DELTA',6X,'ERR',9X,'NEW VAL')
730 FORMAT (4(F12.5,F9.5))
731 FORMAT (1X,4(4X,'TIME',5X,'DEL-MAG.'))
732 FORMAT (1HO,4(4X,'TIME',6X,'DELTA I'))
END

******************************************************************************************************************
FUNCTION TOTINT (N)
C GAUSSIAN INTEGRATION OVER ENTIRE AREA OF STAR N (1 OR 2)
C STAR IS ELLIPSE WITH AXES (AAXIS,BAXIS) ROTATED FROM
C (YP,ZP) AXES BY ANGLE SPECIFIED BY SINP,COSP.
COMMOM /ROAT/ SINP(2),COSP(2),DCH1,ECH1
COMMON /GAUSS/ WT(16),X(16),L,XC(16)
COMMON /TVARS/ALPHA(2),THETA1,CH1,R,DELTA,PARA(19,2),AAXIS(2)
1,BAXIS(2),PHASE,PHI(2),THETAPI(2)
COMMON/FLAGS/IFSPH,JTYPE,KSTAR,TEST(17),NOBS,NZONE,MREF,NREF
1,IREF,NINC,LST
MREF = 0
NREF = N
SUM2 = 0.
DO 100 J=1,L
SUM1 = 0.
Y = AAXIS(NREF)*X(J)
DO 10 I=1,L
Z = BAXIS(NREF)*X(I)*XC(J)
YP = Y*COSP(NREF) - Z*SINP(NREF)
ZP = Y*SINP(NREF) + Z*COSP(NREF)
10 SUM1 = SUM1 + WT(I)*BRIGHT(YP,ZP,NREF)
100 SUM2 = SUM2 + WT(J)*XC(J)*SUMI
TOTINT = AAXIS(NREF)*BAXIS(NREF)*SUM2
RETURN
END

SUBROUTINE ZONES
C ESTABLISHES TABLE OF COORD AND LOCAL INTENSITY FOR REFLECTION CALC
COMMON/ORBE/INCL, ESINW, ECOSW, UA, UB, A, RATIOK, ELLIP, ELLIPB,
1 EPSI, EPSIB, TCONJ, VA, VB, RATIOJ, WA, WB, PERIOD

81
COMMON/TABLE/ENDINT(2)
COMMON/AUXE/SCALE(2),SURF(2),SLIP(2),TILT(2),QINT(2),AA(2),BB(2)
  ,CC(2),RBAR(2),UMA(2),VMA(2),WMA(2),RNOT,BOLDJ,QUAD,TOTAL
DIMENSION V(2),AREA(2)
EQUIVALENCE (V(1),VA)
QLITE(VV) = QINT(J)*(VMA(J) + V(J)*VV/RBAR(J))
100 DO 110 J=1,2
110 ENDINT(J) = QLITE(AA(J))
RETURN
END

******************************************************************************
APPENDIX IV

SAMPLE COMPUTER OUTPUT
WINK- SAMPLE RUN

INPUT DATA
1  88.0000
6  0.383700
7  0.739500
10 10000.0
11  8660.00
13  0.500000
27  5.00000
19  2.00000
21  0.250000E-01
0  0.0
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